An Example of APV and the Tax Subsidy to Debt

Since students are familiar with the Modigliani-Miller assumptions, our example takes advantage of the simplicity in the MM world. Suppose PMM Inc. has an investment that costs $10,000,000 with expected EBIT (cash flows from operations) of $2,531,646 per year forever. The investment can be financed either (i) with $10,000,000 in equity or (ii) financed with $4,969,630 of debt at the 10% interest rate and the remaining amount in internally generated equity capital. The financing mix reflects the target debt ratio of 0.45. The discount rate on an all-equity-financed project, r_0 in this business risk class is 20%. The firm's marginal tax rate is 21%.

1. **All-equity value, NPV_U**

   Annual after tax cash flows to unlevered equity are $UCF = EBIT*(1-T_C) = \$2,531,646*(1-.21) = \$2,000,000$. The net present value of the project if financed with internal equity only is therefore:

   $$NPV_U = (\$2,000,000/.20) - \$10,000,000 = \$0$$

   Here we have assumed the MM world, with perpetual cash flows and no maturity date for the debt. Thus, we value the cash flows to unlevered equity using the perpetuity formula, pmt/i.

   Since NPV_U = $0, the all-equity (unlevered) firm should be indifferent to accepting or rejecting the project.

2. **Financing side effect: Tax Subsidy, NPVF**

   In our example, the annual interest payment is $r_B*B = .10*\$4,969,630 = \$496,963$. The annual tax subsidy is $T_C*r_B*B = .21*.10*\$4,969,630 = \$104,362$, and the present value of this financing side-effect discounted at 10% (the market cost of debt) is:

   $$NPVF = \$104,362/.10 = \$1,043,622$$

   Again, we use the perpetuity formula to value the cash flows.

   The **Adjusted Present Value (APV)** of the project is then:

   $$\mathbf{APV = NPV_U + NPVF = \$0 + \$1,043,622 = \$1,043,622}$$

   After including the value of the tax subsidy, stockholders can expect to gain $1,043,622. The firm should accept the project if it is financed with

$4,969,630 in debt at the interest rate of 10%.

Of course, with perpetual cash flows and no bankruptcy costs, this is simply the result of MM Proposition I with corporate taxes where the value of tax benefits equals $T_C*B = .21*\$4,969,630 = \$1,043,622$.

3. **Calculation of the debt amount, B, given the target debt ratio, X**

   Recall from MM Propostion I with corporate taxes that $V_L = V_U + T_C*B$, and $V_U = [EBIT*(1-T_C)]/r_0$. Given the target debt ratio, $X = B/V_L$, we can express the debt amount as $B = X * V_L$. By substitution, we can express MM Proposition I with corporate taxes as $V_L = V_U + T_C*(X*V_L)$. By rearranging the terms, we can solve for **$V_L = V_U / (1 - T_C*X)$**!

   In this example, V_U is calculated above as \$10,000,000, and T_C=21% and X=0.45 (both given).
   Hence, $V_L = \$10,000,000/(1-.21*.45) = \$11,043,622$

   ==> **B = .45*\$11,043,622 = \$4,969,630**.

Reference Lecture Slides 5 - 7 and 38 - 39 for additional numerical examples on the APV approach.