

DS632 System Simulation

Assignment 1

Team

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Question 1 : Calculate the 5 fundamental performance metrics, as done in example exercise 1

Answer :

For Exponential distribution

Intermediate arrival time = 1.25 minutes

Mean service time = 1 minute

$\lambda = 1 \text{ entity} / 1.25 \text{ min}$ (arrival rate)

$\lambda = 0.8 / \text{min}$

$\mu = 1 \text{ entity} / 1 \text{ min}$ (service rate)

$\mu = 1 / \text{min}$

$$\begin{aligned}\rho &= \lambda / c\mu \\ &= 0.8 / \text{min} / 1 / \text{min} \\ &= 0.8\end{aligned}$$

$$\begin{aligned}L &= \rho / 1 - \rho \\ &= 0.8 / (1 - 0.8) \\ &= 4 \text{ entities}\end{aligned}$$

$$\begin{aligned}W &= L / \lambda \\ &= 4 / 0.8 / \text{min} \\ &= 5 \text{ min}\end{aligned}$$

$$\begin{aligned}W_q &= W - E(s) \\ &= 5 \text{ min} - 1 \text{ min} \\ &= 4 \text{ min}\end{aligned}$$

$$L_q = \lambda W_q$$

$$= 0.8 / \text{min} * 4 \text{ min}$$

$$= 3.2 \text{ entities}$$

For Uniform Distribution:

Intermediate arrival time = 1.25 min

Mean service time = 1 min

Arrival Rate ; $\lambda = 1 \text{ entity} / 1.25 \text{ min (arrival rate)} = 0.8 \text{ entities} / \text{min}$

Service Rate ; $\mu = 1/E(s) = 1 \text{ entity} / \text{min}$

a= 0.1

b= 1.9

$$\text{Standard Deviation} = \sqrt{(b-a)^2/12}$$

$$= \sqrt{(1.9-0.1)^2/12}$$

$$= 0.5196$$

$$\rho = \lambda/\mu$$

$$= 0.8/\text{min} / 1/\text{min} = 0.8$$

$$= 0.8$$

$$W_q = \lambda(SD^2 + 1/\mu^2) / 2(1 - \lambda/\mu)$$

$$= 0.8(0.5196^2 + 1^2) / (2 * (1 - 0.8/1))$$

$$= 2.54 \text{ min}$$

$$W = W_q + E(s)$$

$$= 2.54 + 1$$

$$= 3.54 \text{ mins}$$

$$L_q = \lambda W_q$$

$$= 0.8 * 2.54$$

$$= 2.032 \text{ entities}$$

$$L = \lambda W$$

$$= 0.8 \times 3.54$$

$$= 2.832 \text{ entities}$$

For Triangular distribution:

$$a = 0.1$$

$$b = 1.9$$

$$m = 1.0$$

$$\text{Arrival Rate; } \lambda = 1 \text{ entity} / 1.25 \text{ min (arrival rate)} = 0.8 \text{ entities} / \text{min}$$

$$\text{Expected Service Time; } E(s) = (a+m+b)/3 = (0.1+1.0+1.9)/3 = 1 \text{ min}$$

$$\text{Service rate; } \mu = 1/E(s) = 1 \text{ entity} / \text{min}$$

$$\begin{aligned} \text{Standard Deviation} = \theta &= \sqrt{((a^2+m^2+b^2) - am - ab - bm)/18} \\ &= \sqrt{((0.1)^2 + (1)^2 + (1.9)^2 - 0.1 \cdot 1.0 - 0.1 \cdot 1.9 - 1 \cdot 1.9)/18} \\ &= \sqrt{(2.43)/18} \\ &= \sqrt{0.135} \\ &= 0.367 \text{ min} \end{aligned}$$

$$\begin{aligned} \rho &= \lambda/\mu \\ &= 0.8/\text{min} / 1/\text{min} \\ &= 0.8 \end{aligned}$$

$$\begin{aligned} W_q &= \lambda(SD^2 + 1/\mu^2)/2(1-\lambda/\mu) \\ &= 0.8(0.367^2 + 1/1^2)/(2 \cdot (1-0.8/1)) \\ &= 2.27 \text{ min} \end{aligned}$$

$$\begin{aligned} W &= W_q + E(s) \\ &= 2.27 + 1 \\ &= 3.27 \text{ minutes} \end{aligned}$$

$$\begin{aligned} L_q &= \lambda W_q \\ &= 0.8 (2.27) \\ &= 1.816 \text{ entities} \end{aligned}$$

$$\begin{aligned}
 L &= \lambda W \\
 &= 0.8 / \text{min} * 3.27 \text{ min} \\
 &= 2.616 \text{ entities}
 \end{aligned}$$

Sr. No.	Service time distribution	L (entity)	L _Q (entity)	W (min)	W _Q (min)	ρ
1	Exponential	4.00	3.20	5.00	4.00	0.8
2	Uniform	2.832	2.032	3.54	2.54	0.8
3	Triangular	2.616	1.816	3.27	2.27	0.8

Question 2 : Comment on whether this queuing system, relative to the one in exercise 1, performs better or worse

Answer : In Comparison with the exponential distribution and triangular distribution, **Triangular distribution performs better**. As in Triangular distribution service time has the smallest steady-state with average number of entities in queue, L_Q and average number of entities in the system L. For the same utilization of the system and for the same expected service times [E(s)], less waiting times and shorter queue implies the better service for customers and better overall performance.

Question 3 : Write *one* sentence specifying your understanding of why -- in a conceptual sense -- this system performs better or worse.

Answer : Reason for better performance in Triangular Distribution is less variance in service time resulting in reduced average number of entities in queue and reduced average wait time in queue.