CHAPTER

11

Return and Risk

The Capital Asset Pricing Model (CAPM)

McGraw-Hill/Irwin

Key Concepts and Skills

- Know how to calculate expected returns
- Know how to calculate covariances, correlations, and betas
- Understand the impact of diversification
- Understand the systematic risk principle
- Understand the security market line
- Understand the risk-return tradeoff
- Be able to use the Capital Asset Pricing Model (CAPM)

Chapter Outline

11.1 Individual Securities

11.2 Expected Return, Variance, and Covariance

11.3 The Return and Risk for Portfolios

11.4 The Efficient Set for Two Assets

11.5 The Efficient Set for Many Assets (SKIM)

11.6 Diversification

11.7 Riskless Borrowing and Lending (SKIM)

11.8 Market Equilibrium

11.9 Relationship between Risk and Required Return (CAPM)

11.1 Individual Securities

- The characteristics of individual securities that are of interest are the:
  - Expected Return
  - Variance and Standard Deviation
  - Covariance and Correlation (to another security or index)

11.1 Individual Securities

- The characteristics of individual securities that are of interest include:
  - Expected Return, E(R)
    - The return that investors expect to earn over the next period.
    - The probability-weighted average of possible returns.

$$E(R) = \sum_{s=1}^{K} p_s R_s$$

Where K is the number of states of the economy (or possible returns)

p_s is the probability of the realization of state s

R_s is the possible return if state s occurs

-Variance and Standard Deviation, σ^2 and σ

-They are measures of total risk of an asset (or security).

-Total risk is relevant for a stand-alone investment environment.

-The dispersion of possible returns around E(R)

-The probability-weighted average of squared deviations of possible returns from E(R)

$$\sigma^2 = \sum_{s=1}^{K} p_s (R_s - E(R))^2$$

and Standard deviation is the positive square root of variance

11.2 Expected Return, Variance, and Covariance

Consider the following two risky asset world. There is a 1/3 (33.3%) chance of each state of the economy, and the only assets are a stock fund and a bond fund.

		Rate of Return	
Scenario	***Probability***	***Stock Fund***	***Bond Fund***
Recession	33.3%	-7%	17%
Normal	33.3%	12%	7%
Boom	33.3%	28%	-3%

Expected Return

	Stock Fund		Bond Fund	
	Rate of	***Squared***	***Rate of***	***Squared***
Scenario	***Return***	***Deviation***	***Return***	***Deviation***
Recession	-7%	0.0324	17%	0.0100
Normal	12%	0.0001	7%	0.0000
Boom	28%	0.0289	-3%	0.0100
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

Expected Return

Scenario	Stock Fund Rate of Return	Stock Fund Squared Deviation	Bond Fund Rate of Return	Bond Fund Squared Deviation
Recession	-7%	0.0324	17%	0.0100
Normal	12%	0.0001	7%	0.0000
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Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$E(r_S) = \tfrac{1}{3} \times (-7\%) + \tfrac{1}{3} \times (12\%) + \tfrac{1}{3} \times (28\%)$$

$$E(r_S) = 11\%$$

Variance

Scenario	Stock Fund Rate of Return	Stock Fund Squared Deviation	Bond Fund Rate of Return	Bond Fund Squared Deviation
Recession	-7%	0.0324	17%	0.0100
Normal	12%	0.0001	7%	0.0000
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Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$(-0.07 - 0.11)^2 = 0.0324$$

Variance

Scenario	Stock fund Rate of Return	Stock fund Squared Deviation	Bond Fund Rate of Return	Bond Fund Squared Deviation
Recession	-7%	3.24%	17%	1.00%
Normal	12%	0.01%	7%	0.00%
Boom	28%	2.89%	-3%	1.00%
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$(0.12 - 0.11)^2 = 0.0001$$

Variance

Scenario	Stock fund Rate of Return	Stock fund Squared Deviation	Bond Fund Rate of Return	Bond Fund Squared Deviation
Recession	-7%	3.24%	17%	1.00%
Normal	12%	0.01%	7%	0.00%
Boom	28%	2.89%	-3%	1.00%
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$(0.28 - 0.11)^2 = 0.0289$$

Variance

Scenario	Stock fund Rate of Return	Stock fund Squared Deviation	Bond Fund Rate of Return	Bond Fund Squared Deviation
Recession	-7%	3.24%	17%	1.00%
Normal	12%	0.01%	7%	0.00%
Boom	28%	2.89%	-3%	1.00%
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

0.0205 = 0.333*0.0324 + 0.333*0.0001 + 0.333*0.0289

Standard Deviation

Scenario	Stock Fund Rate of Return	Stock Fund Squared Deviation	Bond Fund Rate of Return	Bond Fund Squared Deviation
Recession	-7%	0.0324	17%	0.0100
Normal	12%	0.0001	7%	0.0000
Boom	28%	0.0289	-3%	0.0100
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$14.3\% = \sqrt{0.0205}$$

Covariance

Scenario	Stock Deviation	Bond Deviation	Product	Weighted
Recession	-18%	10%	-0.0180	-0.0060
Normal	1%	0%	0.0000	0.0000
Boom	17%	-10%	-0.0170	-0.0057
Sum				-0.0117
Covariance				-0.0117

"Deviation" compares return in each state to the expected return.

"Weighted" takes the product of the deviations multiplied by the probability of that state.

Covariance (σ_{AB}) and Correlation (ρ_{AB})

Covariance (σ_{AB}) and Correlation (ρ_{AB}) between the returns on two securities

-- These two statistics measure how any pair of securities interrelate with each other.

$$\sigma_{AB} = \sum_{s=1}^{K} p_s (R_{A_s} - E(R_A))(R_{B_s} - E(R_B))$$

and $\rho_{AB} = \sigma_{AB} / (\sigma_A * \sigma_B)$

-- Example with the stock and bond funds:

σ_{SB} = 0.333*(-0.07-0.11)(0.17-0.07) + 0.333*(0.12-0.11)(0.07-0.07) + 0.333*(0.28-0.11)(-0.03-0.07) = -0.0117

and ρ_{SB} = -0.0117/(0.1432*0.0819) = -1.0

11.3 The Return and Risk for Portfolios

	Stock Fund		Bond Fund	
Scenario	**Rate of Return**	**Squared Deviation**	**Rate of Return**	**Squared Deviation**
Recession	-7%	0.0324	17%	0.0100
Normal	12%	0.0001	7%	0.0000
Boom	28%	0.0289	-3%	0.0100
Expected return	11.00%		7.00%	
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Note that stocks have a higher expected return than bonds and higher risk. Let us turn now to the risk-return tradeoff of a portfolio that is 50% invested in bonds and 50% invested in stocks, $w_B = w_S = 50\%$.

Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.160%
Normal	12%	7%	9.5%	0.003%
Boom	28%	-3%	12.5%	0.123%
Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

The rate of return on the portfolio is a weighted average of the returns on the stocks and bonds in the portfolio:

$$R_P = w_B R_B + w_S R_S$$

$$5\% = 50\% \times (-7\%) + 50\% \times (17\%)$$

Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.160%
Normal	12%	7%	9.5%	0.003%
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The rate of return on the portfolio is a weighted average of the returns on the stocks and bonds in the portfolio:

$$R_P = w_B R_B + w_S R_S$$

$$9.5\% = 50\% \times (12\%) + 50\% \times (7\%)$$

Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.160%
Normal	12%	7%	9.5%	0.003%
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Expected return	11.00%	7.00%	9.0%	
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The rate of return on the portfolio is a weighted average of the returns on the stocks and bonds in the portfolio:

$$R_P = w_B R_B + w_S R_S$$

$$12.5\% = 50\% \times (28\%) + 50\% \times (-3\%)$$

Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.160%
Normal	12%	7%	9.5%	0.003%
Boom	28%	-3%	12.5%	0.123%
Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

The *expected* rate of return on the portfolio is a weighted average of the *expected* returns on the securities in the portfolio.

$$E(R_P) = w_B E(R_B) + w_S E(R_S)$$

$$9\% = 50\% \times (11\%) + 50\% \times (7\%)$$

Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.0016
Normal	12%	7%	9.5%	0.0000
Boom	28%	-3%	12.5%	0.0012
Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

The variance of the rate of return on the two risky assets portfolio is

$$\sigma_P^2 = (w_B \sigma_B)^2 + (w_S \sigma_S)^2 + 2(w_B \sigma_B)(w_S \sigma_S)\rho_{BS}$$

where ρ_{BS} is the correlation coefficient between the returns on the stock and bond funds. Recall that $\rho_{SB} = -1.0$ in this example!

Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.0016
Normal	12%	7%	9.5%	0.0000
Boom	28%	-3%	12.5%	0.0012
Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

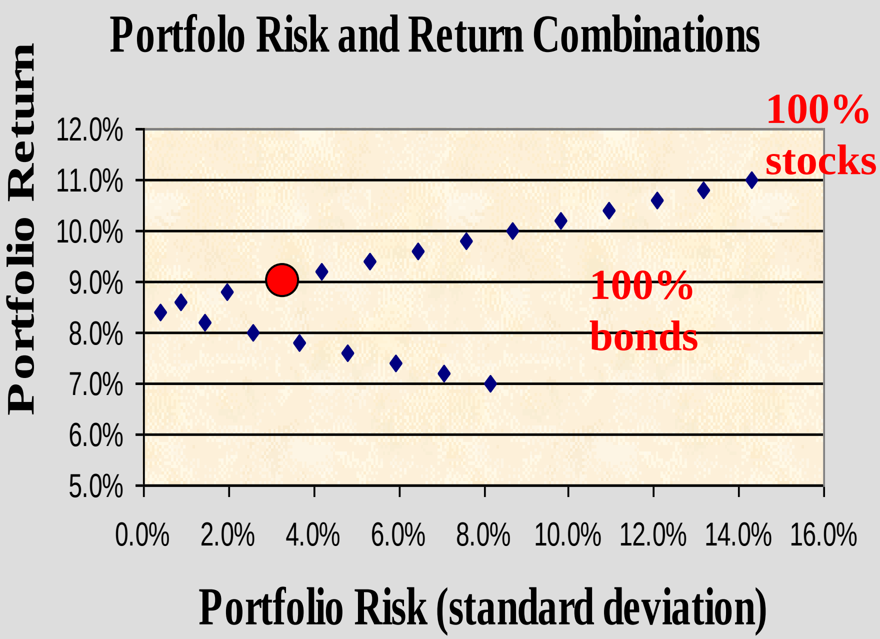
Observe the decrease in risk that diversification offers.

An equally weighted portfolio (50% in stocks and 50% in bonds) has less risk than either stocks or bonds held in isolation.

Recall that these two funds are perfectly negatively correlated, ρ=-1

11.4 The Efficient Set for Two Assets

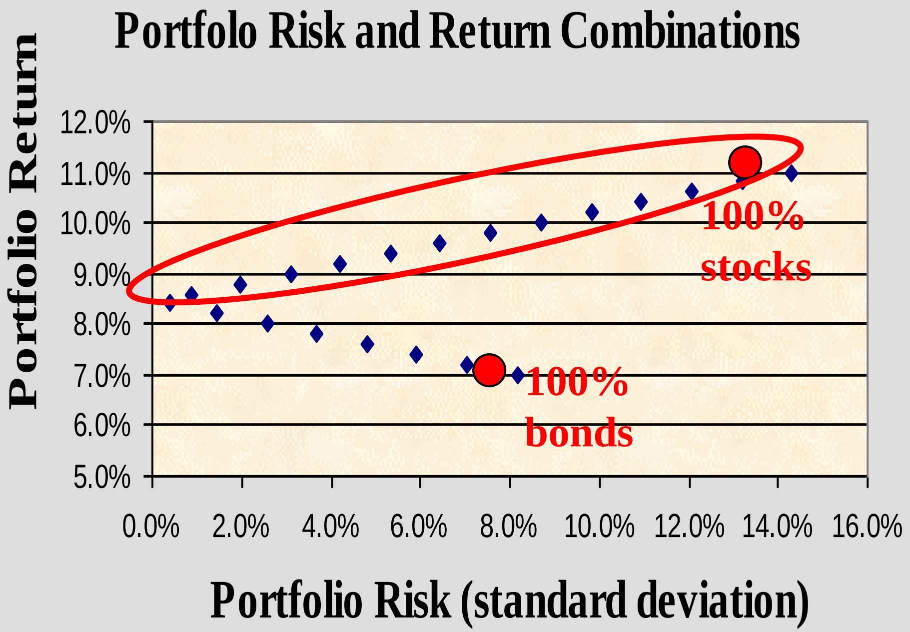
% in stocks	Risk	Return
0%	8.2%	7.0%
5%	7.0%	7.2%
10%	5.9%	7.4%
15%	4.8%	7.6%
20%	3.7%	7.8%
25%	2.6%	8.0%
30%	1.4%	8.2%
35%	0.4%	8.4%
40%	0.9%	8.6%
45%	2.0%	8.8%
50.00%	**3.08%**	**9.00%**
55%	4.2%	9.2%
60%	5.3%	9.4%
65%	6.4%	9.6%
70%	7.6%	9.8%
75%	8.7%	10.0%
80%	9.8%	10.2%
85%	10.9%	10.4%
90%	12.1%	10.6%
95%	13.2%	10.8%
100%	14.3%	11.0%



We can consider other portfolio weights besides 50% in stocks and 50% in bonds …

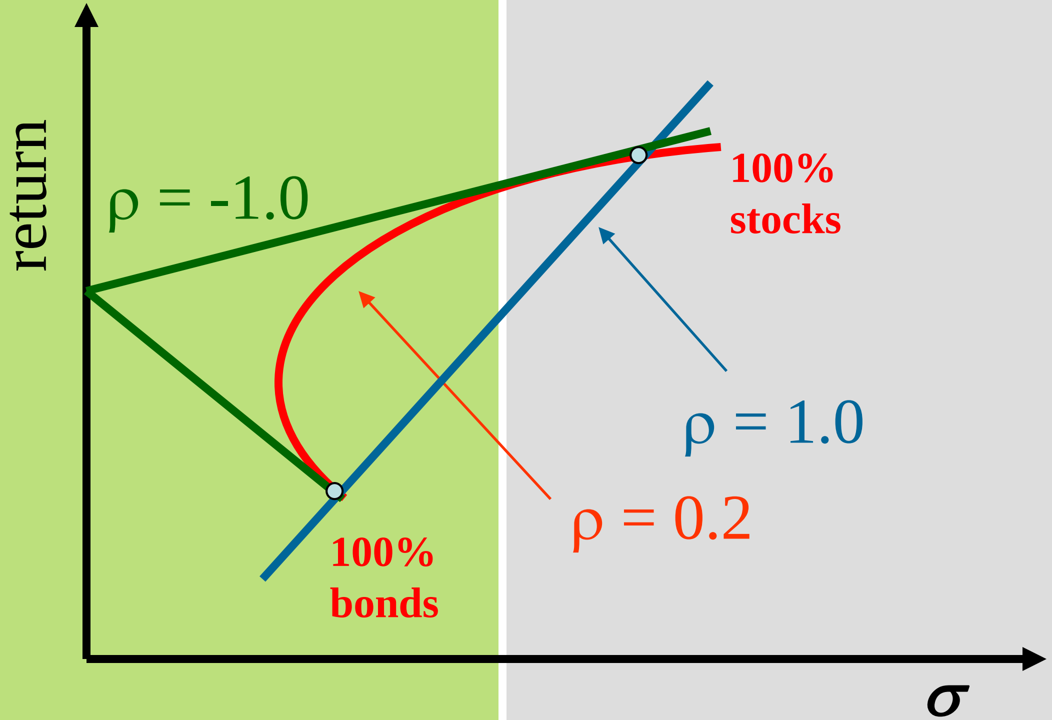
The Efficient Set for Two Assets

% in stocks	*Risk*	*Return*
0%	8.2%	7.0%
5%	7.0%	7.2%
10%	5.9%	7.4%
15%	4.8%	7.6%
20%	3.7%	7.8%
25%	2.6%	8.0%
30%	1.4%	8.2%
35%	0.4%	8.4%
40%	0.9%	8.6%
45%	2.0%	8.8%
50%	3.1%	9.0%
55%	4.2%	9.2%
60%	5.3%	9.4%
65%	6.4%	9.6%
70%	7.6%	9.8%
75%	8.7%	10.0%
80%	9.8%	10.2%
85%	10.9%	10.4%
90%	12.1%	10.6%
95%	13.2%	10.8%
100%	14.3%	11.0%



Note that some portfolios are “better” than others. They have higher returns for the same level of risk or less. These portfolios form the *efficient frontier.*

Portfolios with Various Correlations



- Relationship depends on correlation coefficient

  $-1.0 \leq \rho \leq +1.0$

- If ρ = +1.0, no risk reduction is possible
- If ρ = –1.0, complete risk reduction is possible

The Diversification Effect

- The diversification effect depends on how the returns on securities are correlated, i.e., correlation between security returns counts!
- Most securities are positively correlated, $0 < \rho < +1.0$
- The lower the correlation, i.e., the less positive its value, the greater the risk reduction potential (or diversification benefit).
- If $\rho = +1.0$, no risk reduction is possible.
- Adding securities with lower correlations with existing securities in the portfolio can lower the total risk of the portfolio.
- NOTE: Correlation has NO impact on the expected return of the portfolio!

The Diversification Effect

Consider the total risk of the equally weighted portfolio of the stock and bond funds with different correlation coefficients –

For $\rho_{SB} = +1.0$, $\sigma_P = 0.1125$

➔ Diversification benefit = 0

For $\rho_{SB} = +0.2$, $\sigma_P = 0.0893$

➔ Diversification benefit = 0.1125 – 0.0893 = 0.0232

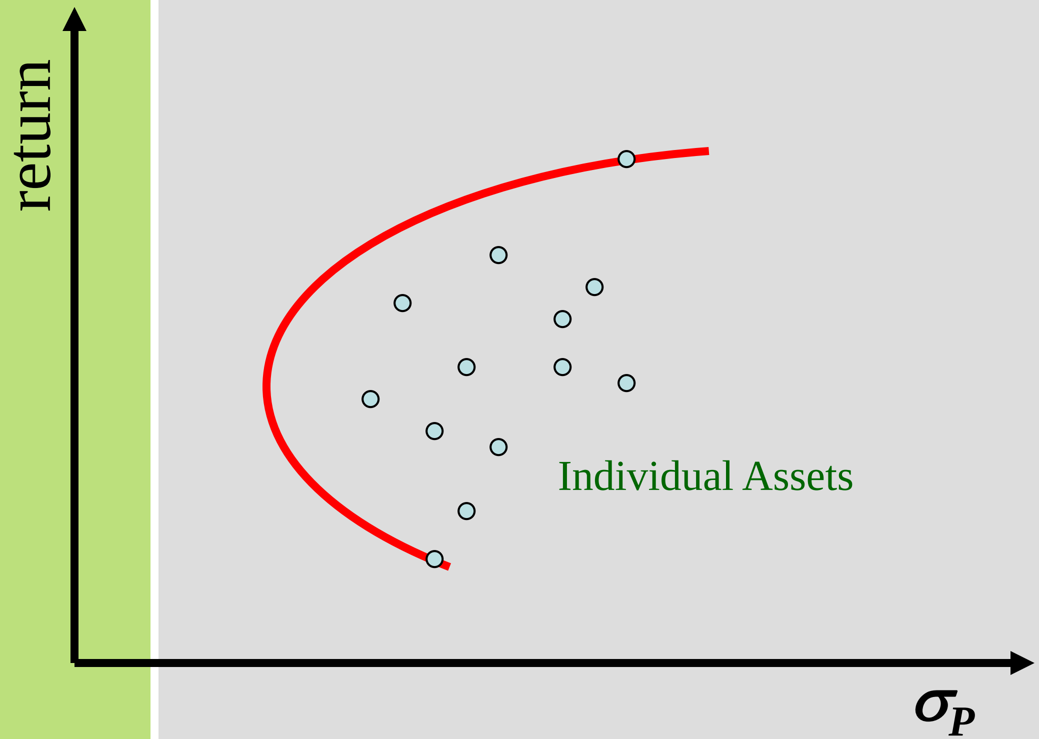
For $\rho_{SB} = -0.2$, $\sigma_P = 0.0750$

➔ Diversification benefit = 0.1125 – 0.0750 = 0.0375

For $\rho_{SB} = -1.0$, $\sigma_P = 0.0308$

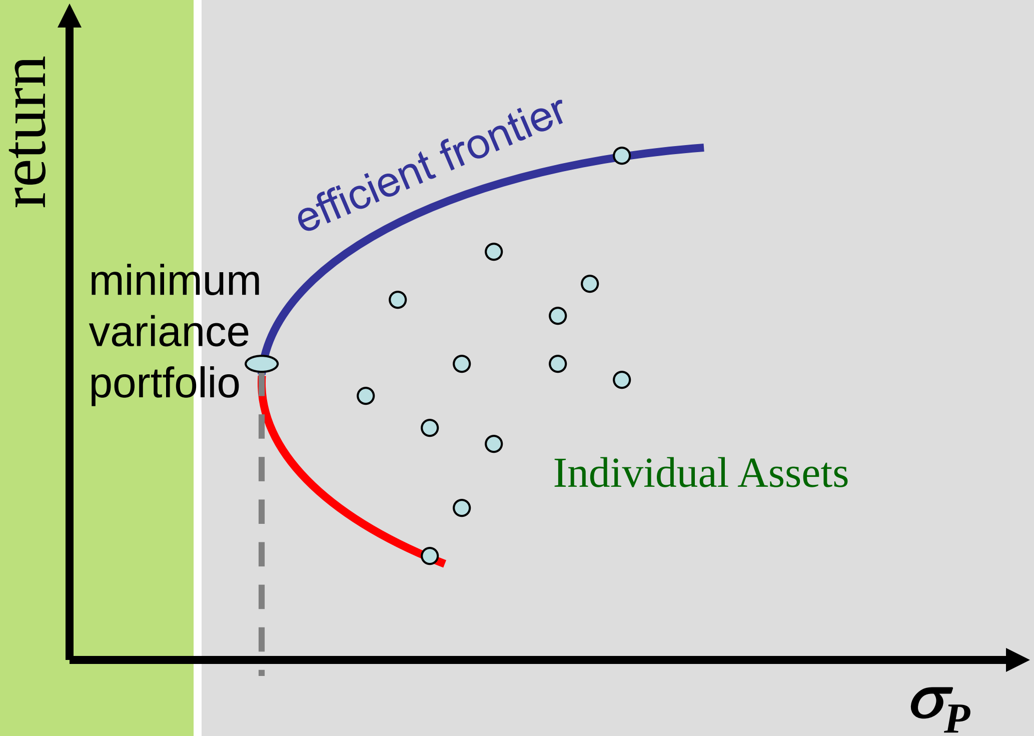
➔ Diversification benefit = 0.1125 – 0.0308 = 0.0817

11.5 The Efficient Set for Many Securities



Consider a world with many risky assets; we can still identify the *opportunity set* of risk-return combinations of various portfolios.

The Efficient Set for Many Securities



The section of the opportunity set above the minimum variance portfolio is the efficient frontier.

Announcements, Surprises, and Expected Returns

- The return on any security consists of two parts.
  - First, the expected returns
  - Second, the unexpected or risky returns
- A way to write the return on a stock in the coming month is:

$$R = \overline{R} + U$$

where

$\overline{R}$ is the expected part of the return

U is the unexpected part of the return

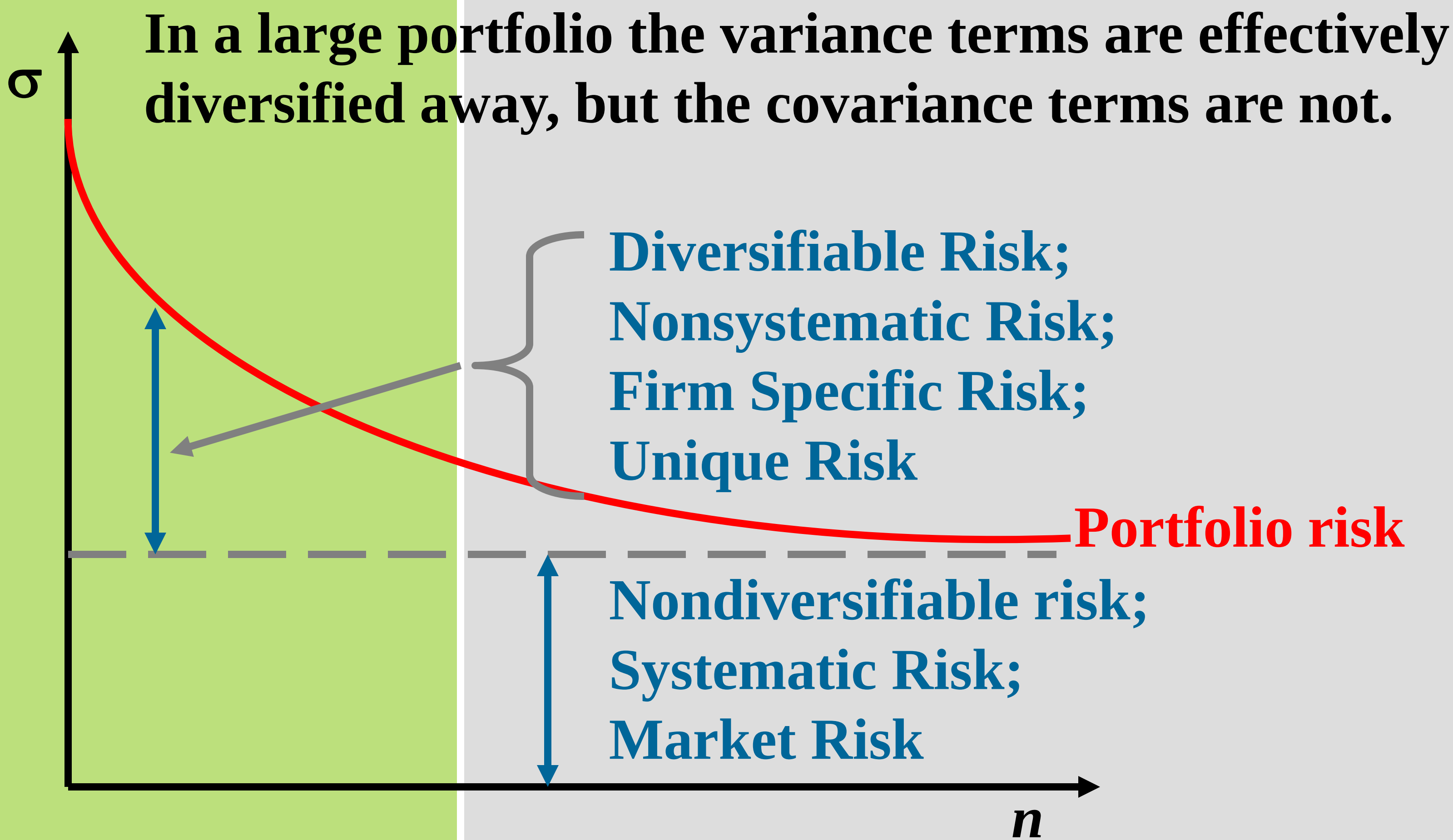
Announcements, Surprises, and Expected Returns

- Any announcement can be broken down into two parts, the anticipated (or expected) part and the surprise (or innovation):
  - Announcement = Expected part + Surprise.
- The expected part of any announcement is the part of the information the market uses to form the expectation, $\bar{R}$, of the return on the stock.
- The surprise is the news that influences the unanticipated return on the stock, U.

11.6 Diversification and Portfolio Risk

- Diversification can substantially reduce the variability of returns without an equivalent reduction in expected returns.
- This reduction in risk arises because worse than expected returns from one asset are offset by better than expected returns from another.
- However, there is a minimum level of risk that cannot be diversified away, and that is the systematic portion.

Portfolio Risk and Number of Stocks



Systematic Risk

- Risk factors that affect a large number of assets
  - Also known as non-diversifiable risk or market risk
- Examples include uncertainty about general economic conditions such as changes in GDP, inflation, interest rates, etc.
- Only the systematic risk, the risk that is inherent in the market, of the risky security will be compensated because such risk cannot be diversified away, i.e., cannot be avoided, even when the risky security is part of a well-diversified portfolio.

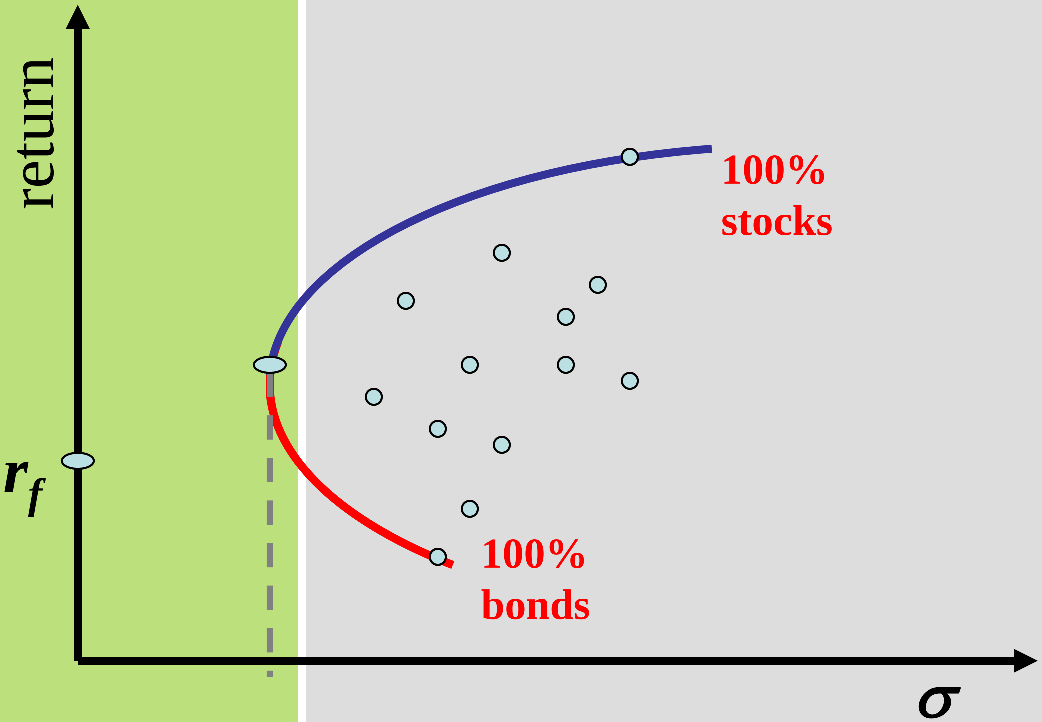
Unsystematic (Diversifiable) Risk

- Risk factors that affect a single asset or a limited number of assets
  - Also known as unique risk and asset-specific risk
- Examples company-specific announcements or events such as labor strikes, part shortages, etc.
- The risk that can be eliminated by combining assets into a portfolio
- If we hold only one asset, or assets in the same industry, then we are exposing ourselves to risk that we could diversify away.

Total Risk

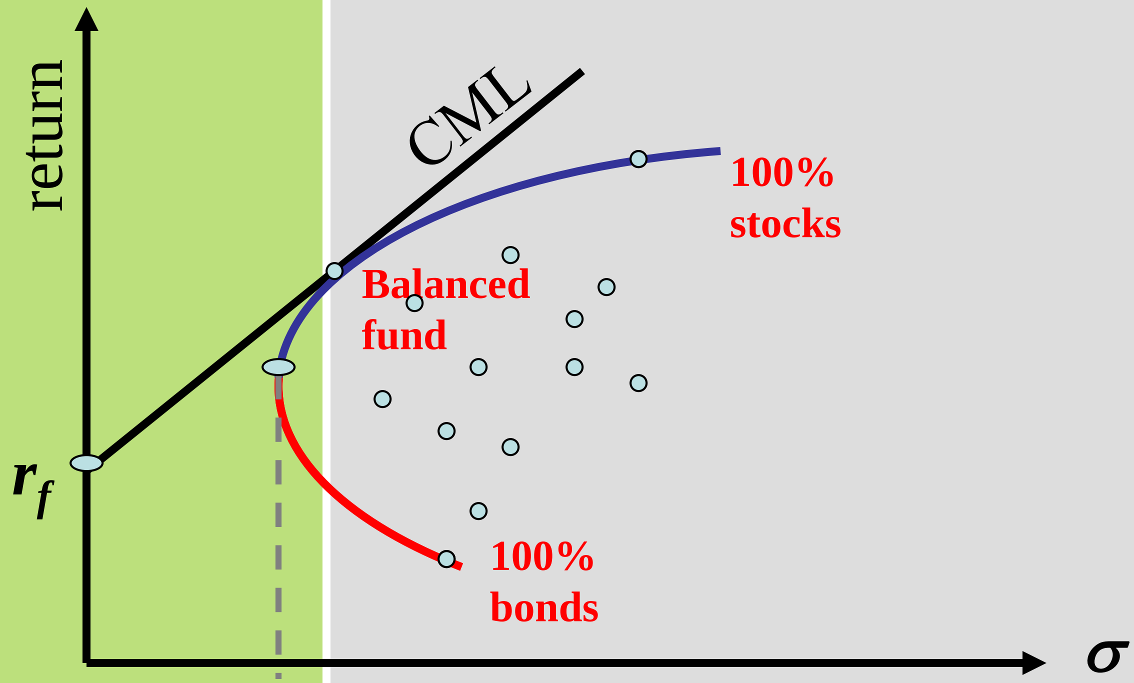
- Total risk = systematic risk + unsystematic risk
- The standard deviation of returns is a measure of total risk.
- For well-diversified portfolios, unsystematic risk is very small.
- Consequently, the total risk for a diversified portfolio is essentially equivalent to the systematic risk.

Optimal Portfolio with a Risk-Free Asset



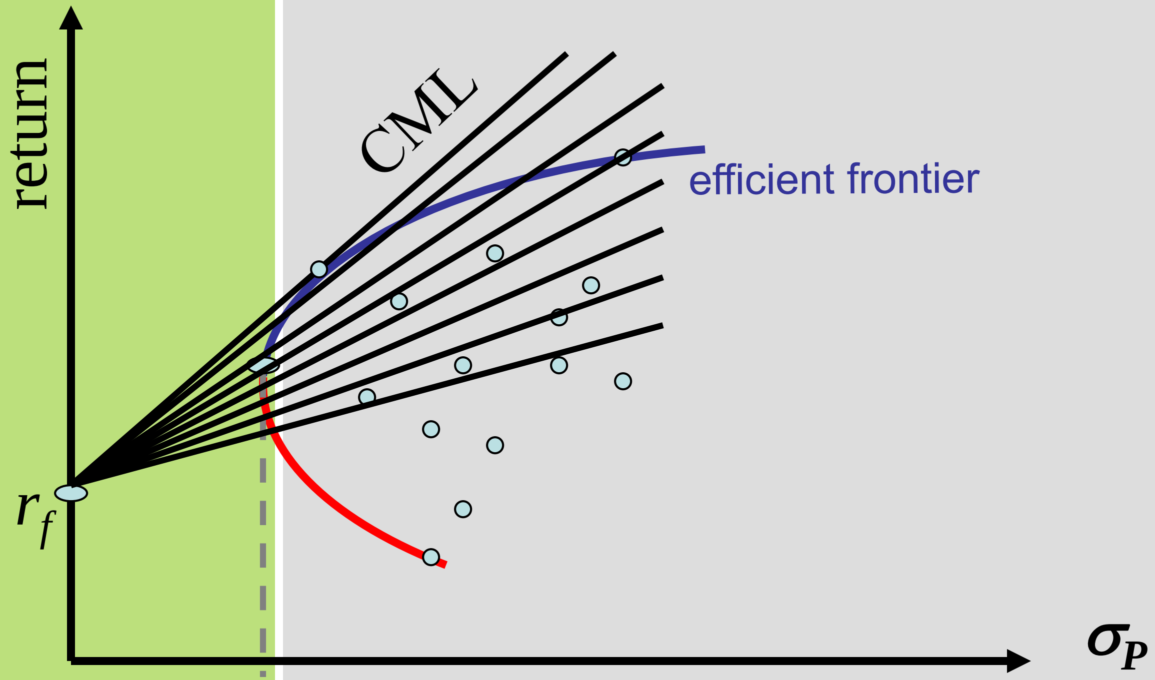
In addition to stocks and bonds, consider a world that also has risk-free securities like T-bills.

11.7 Riskless Borrowing and Lending



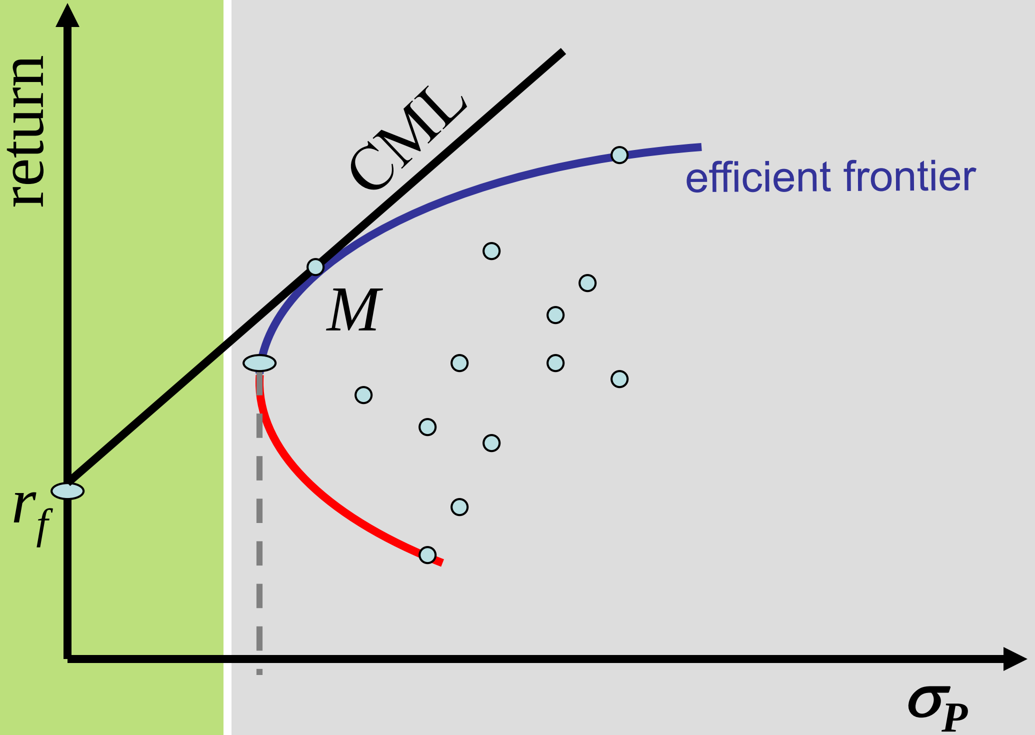
Now investors can allocate their money across the T-bills and a balanced mutual fund.

Riskless Borrowing and Lending



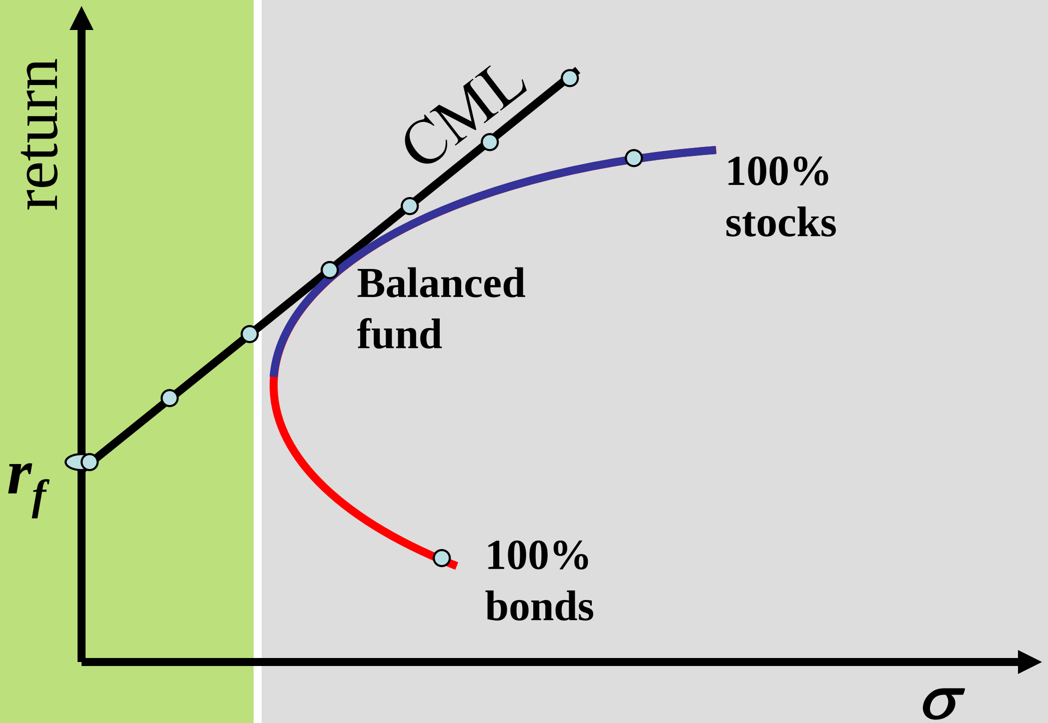
With a risk-free asset available and the efficient frontier identified, we choose the capital allocation line with the steepest slope – the Capital Market Line (CML).

11.8 Market Equilibrium



With the capital allocation line identified, all investors choose a point along the line—some combination of the risk-free asset and the market portfolio *M*. In a world with homogeneous expectations, *M* is the same for all investors.

Market Equilibrium



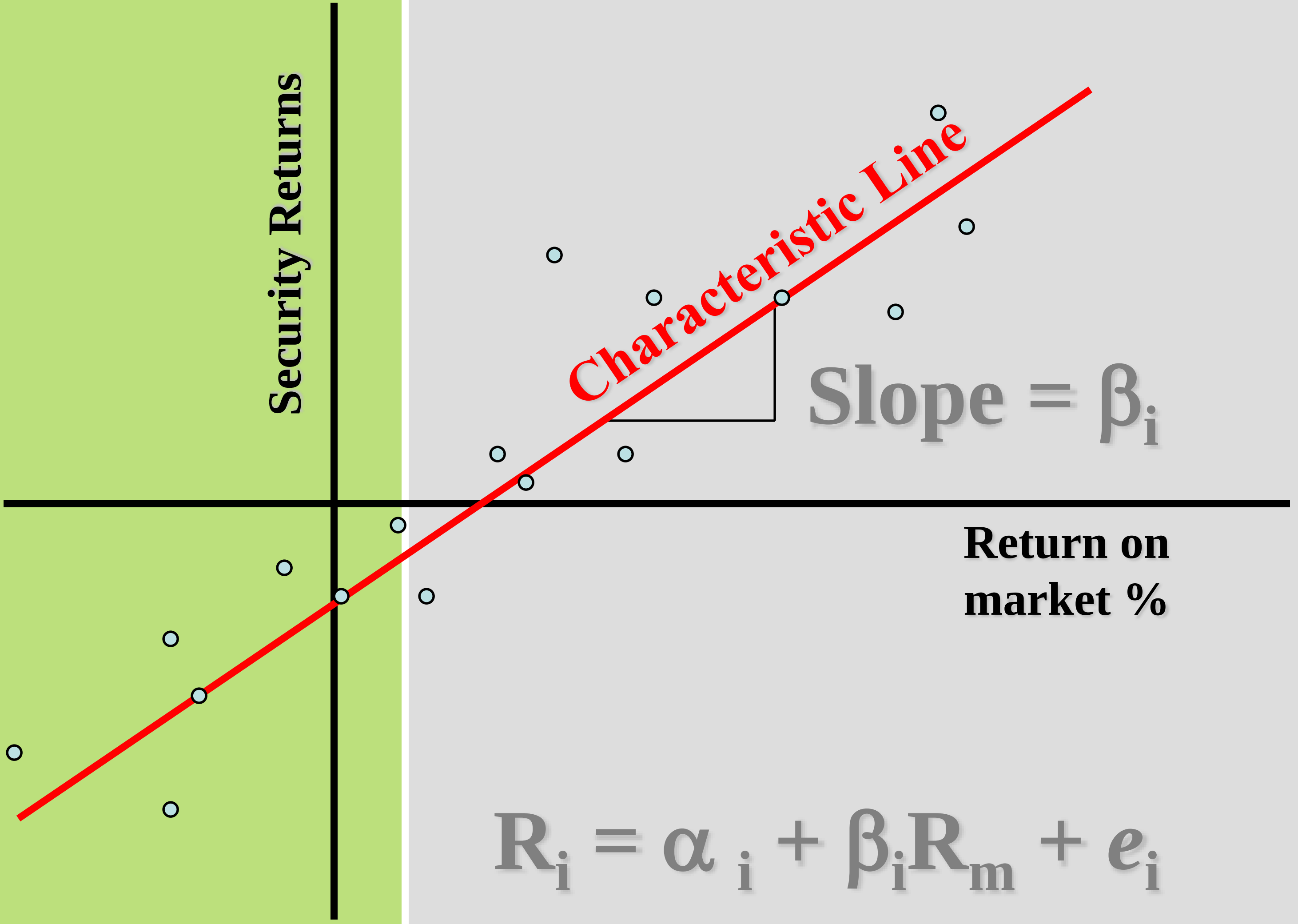
Where the investor chooses along the Capital Market Line depends on his risk tolerance. The big point is that all investors have the same CML.

Risk When Holding the Market Portfolio

- Researchers have shown that the best measure of the risk of a security in a large portfolio is the *beta* (β)of the security.
- Beta measures the responsiveness of a security to movements in the market portfolio (i.e., systematic risk).

$$\beta_i = \frac{Cov(R_i, R_M)}{\sigma^2(R_M)}$$

Estimating β with Regression



$$R_i = \alpha_i + \beta_i R_m + e_i$$

The Formula for Beta

$$\beta_i = \frac{Cov(R_{i,} R_M)}{\sigma^2(R_M)} = \rho \frac{\sigma(R_i)}{\sigma(R_M)}$$

Clearly, your estimate of beta will depend upon your choice of a proxy for the market portfolio.

11.9 Relationship between Risk and Required Return (CAPM)

- Required Return on the Market:

$$\bar{R}_M = R_F + \text{Market Risk Premium}$$

- Required return on an individual security:

$$\bar{R}_i = R_F + \beta_i \times \underbrace{(\bar{R}_M - R_F)}_{\text{Market Risk Premium}}$$

This applies to individual securities held within well-diversified portfolios.

Required Return on a Security

- This formula is called the Capital Asset Pricing Model (CAPM):
  - CAPM depicts the systematic risk, β, and required return, *R*, relationship of individual securities and portfolios in a market portfolio framework.

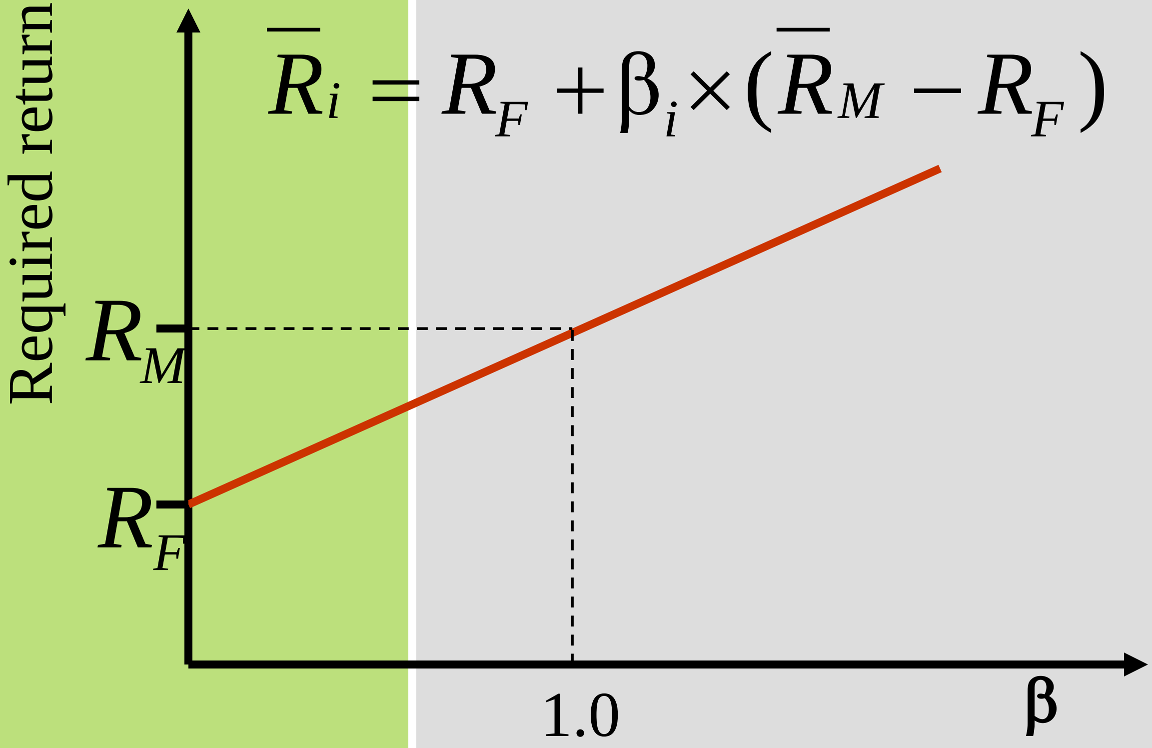
$$\bar{R}_i = R_F + \beta_i \times (\bar{R}_M - R_F)$$

Required return on a security = Risk-free rate + Beta of the security × Market risk premium

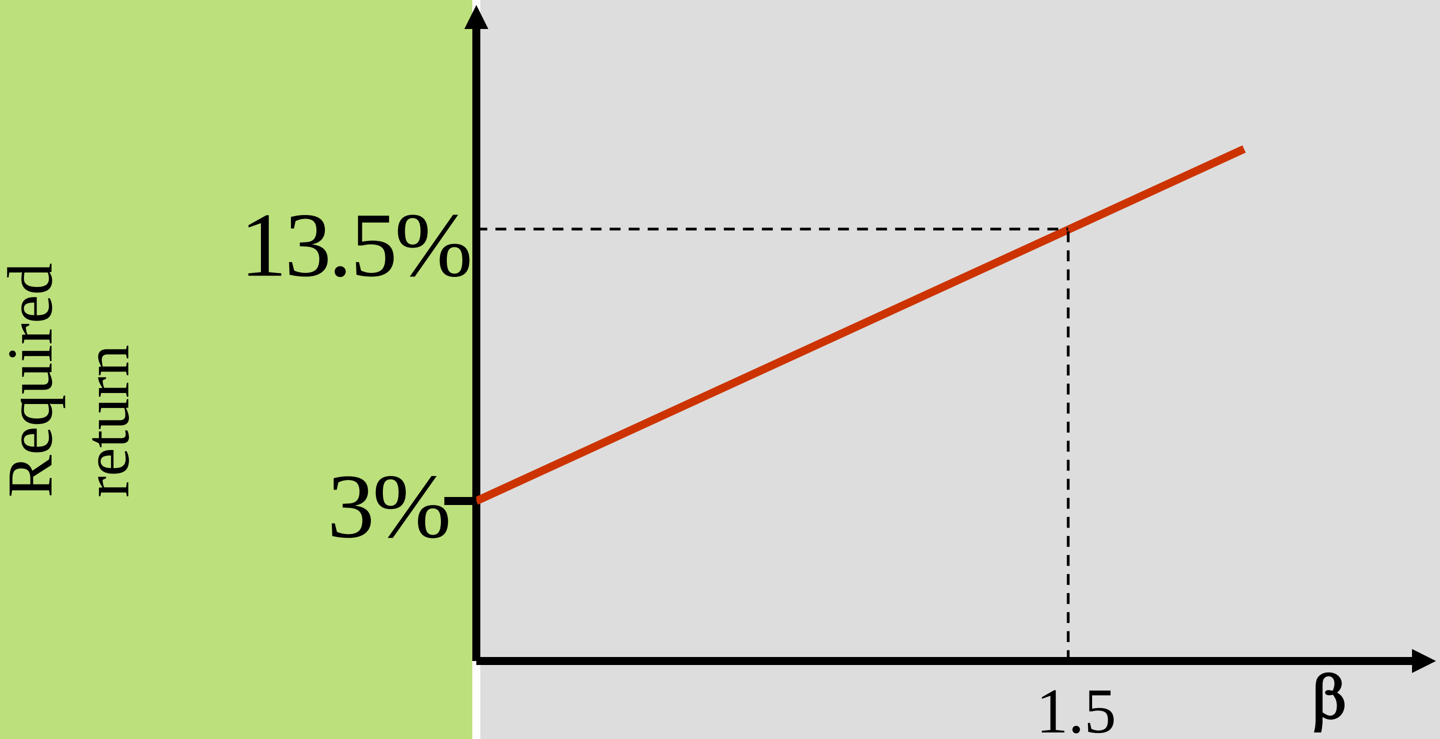
- Assume $\beta_i = 0$, then the required return is R_F.
- Assume $\beta_i = 1$, then $\bar{R}_i = \bar{R}_M$

Relationship Between Risk & Return

$$\bar{R}_i = R_F + \beta_i \times (\bar{R}_M - R_F)$$



Relationship Between Risk & Return



$$\beta_i = 1.5 \qquad R_F = 3\% \qquad \bar{R}_M = 10\%$$

$$\bar{R}_i = 3\% + 1.5 \times (10\% - 3\%) = 13.5\%$$

CAPM Analysis

Another approach to identify mispriced securities –

If Required (CAPM) Return > Expected Return, then the security is OVER-priced!

If Required (CAPM) Return < Expected Return, then the security is UNDER-priced!

Estimation of CAPM

The CAPM:

$$R_i = R_F + \beta_i \times (R_M - R_F)$$

Input Parameters:

-- R_F: Yield on short-term Treasury securities.

-- $(R_M – R_F)$: Historical market risk premium (Chpt. 10)

-- β_i: Historical beta, the slope coefficient of the market model (or the Characteristic Line)

Quick Quiz

- How do you compute the expected return and standard deviation for an individual asset? For a portfolio?
- What is the difference between systematic and unsystematic risk?
- What type of risk is relevant for determining the expected return?
- Consider an asset with a beta of 1.2, a risk-free rate of 5%, and a market return of 13%.
  - What is the required return on the asset?