CHAPTER

4

Discounted Cash Flow Valuation

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Key Concepts and Skills

- Be able to compute the future value and/or present value of a single cash flow or series of cash flows
- Be able to compute the return on an investment
- Be able to use a financial calculator and/or spreadsheet to solve time value problems
- Understand perpetuities and annuities

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Chapter Outline

4.0 Time Value of Money (TVM)

4.1 Valuation: The One-Period Case

4.2 The Multiperiod Case

4.3 Compounding Periods and EAR

4.4 Simplifications for Multiple Cash Flows

4.5 Loan Amortization

4.6 What Is a Firm Worth?

4.7 Summary and Conclusion

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4.0 Time Value of Money (TVM)

- Basic Concept of TVM
  - A dollar in the future does not have the same value as a dollar today. Be specific, a future dollar is less valuable than a today dollar. WHY?

Implications

  - Cash flows that are located at different time points on a time line have different TVM. As such, they should ***not*** be added together or subtracted from one another ***horizontally***. On the other hand, we ***can*** add cash flows together or subtract them from one another ***vertically*** at any time point on a time line because they have the same TVM.

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4.1 The One-Period Case

- If you were to invest $10,000 at 5-percent interest for one year, your investment would grow to $10,500.

$500 would be interest ($10,000 × .05)

<u>$10,000</u> is the principal repayment ($10,000 × 1)

$10,500 is the total due. It can be calculated as:

$$\$10,500 = \$10,000 \times (1.05)$$

- The total amount due at the end of the investment is call the *Future Value* (*FV*).

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Future Value

- In the one-period case, the formula for *FV* can be written as:

$$FV = C_0 \times (1 + r)$$

Where C_0 is cash flow today (time zero), and r is the appropriate interest rate.

In general, FV is the value of a sum after investing (or growing) over one or more periods of time, when compounding at a given interest (or growth) rate.

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Present Value

- If you were to be promised $10,000 due in one year when interest rates are 5-percent, your investment would be worth $9,523.81 in today's dollars.

$$\$9{,}523.81 = \frac{\$10{,}000}{1.05}$$

- The amount that a borrower would need to set aside today to be able to meet the promised payment of $10,000 in one year is called the *Present Value* (*PV*).

Note that $10,000 = $9,523.81×(1.05).

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Present Value

- In the one-period case, the formula for *PV* can be written as:

$$PV = \frac{C_1}{1+r}$$

Where C_1 is cash flow at date 1, and

r is the appropriate discount (or interest) rate, which is determined according to the ***Opportunity Cost Principle***, i.e., **the rate of return on the best available alternative investment of equal risk**.

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Net Present Value

- The Net Present Value (*NPV*) of an investment is the present value of the expected cash flows, less the cost of the investment.
- Suppose an investment that promises to pay $10,000 in one year is offered for sale for $9,500. Your interest rate is 5%. Should you buy?

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Net Present Value

$$NPV = -\$9{,}500 + \frac{\$10{,}000}{1.05}$$

$$NPV = -\$9{,}500 + \$9{,}523.81$$

$$NPV = \$23.81$$

The present value of the cash inflow is greater than the cost. In other words, the Net Present Value is positive, so the investment should be purchased.

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Net Present Value

In the one-period case, the formula for *NPV* can be written as:

$$NPV = -Cost + PV$$

If we had *not* undertaken the positive *NPV* project considered on the last slide, and instead invested our $9,500 elsewhere at 5 percent, our *FV* would be less than the $10,000 the investment promised, and we would be worse off in *FV* terms :

$$\$9{,}500 \times (1.05) = \$9{,}975 < \$10{,}000$$

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4.2 The Multiperiod Case

- The general formula for the future value of an investment over many periods can be written as:

$$FV = C_0 \times (1 + r)^T$$

Where

C_0 is cash flow at date 0,

r is the appropriate interest rate, and

T is the number of periods over which the cash is invested.

Note: FV calculations involve moving cash flow(s) from left to right on the time line.

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Future Value

- Suppose a stock currently pays a dividend of $1.10, which is expected to grow at 40% per year for the next five years.
- What will the dividend be in five years?

$$FV = C_0 \times (1 + r)^T$$

$$\$5.92 = \$1.10 \times (1.40)^5$$

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Future Value and Compounding

- Notice that the dividend in year five, \$5.92, is considerably higher than the sum of the original dividend plus five increases of 40-percent on the original \$1.10 dividend:

$$\$5.92 > \$1.10 + 5\times[\$1.10\times.40] = \$3.30$$

This is due to *compounding,* i.e., the process of investing (or growing) beyond one compounding period and hence earning interest on interest.

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Future Value and Compounding

$\$1.10 \times (1.40)^5$

$\$1.10 \times (1.40)^4$

$\$1.10 \times (1.40)^3$

$\$1.10 \times (1.40)^2$

$\$1.10 \times (1.40)$

0	1	2	3	4	5
\$1.10	\$1.54	\$2.16	\$3.02	\$4.23	\$5.92

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4.2 The Multiperiod Case: Present Value

- The general formula for the present value of a cash flow discounted back many periods can be written as:

$$PV = \frac{C_T}{(1+r)^T}$$

Where

C_T is cash flow at date T,

r is the appropriate discount (or interest) rate, and

T is the number of periods over which the cash is discounted back.

Note: **PV calculations involve moving cash flow(s)** **<u>from right to left</u> on the time line.**

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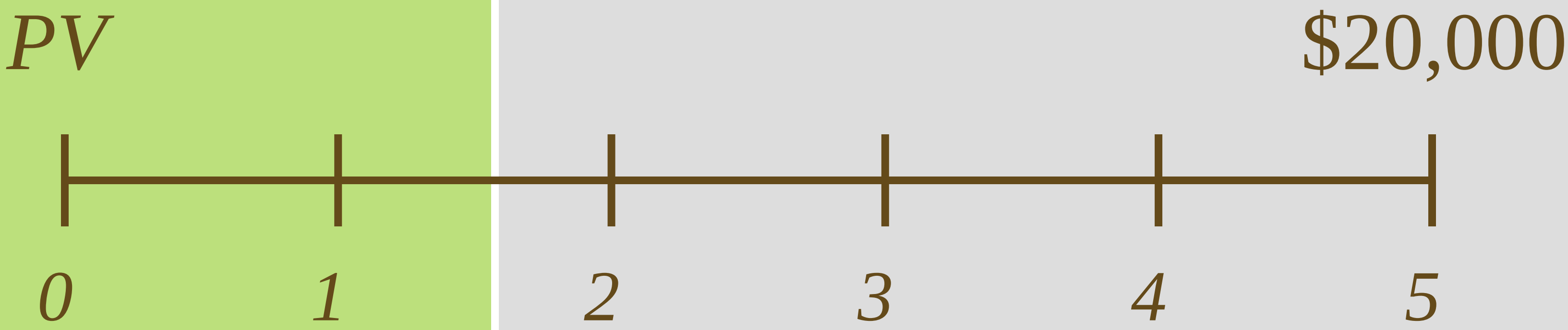
Present Value and Discounting

- How much would an investor have to set aside today in order to have $20,000 five years from now if the current rate is 15%?



$$\$9{,}943.53 = \frac{\$20{,}000}{(1.15)^5}$$

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Properties of PV and FV

Other factors being equal,

- The longer the holding horizon, the larger (or smaller) the Future (or Present) Value!
- The higher the interest rate, the larger (or smaller) the Future (or Present) Value!

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How Long is the Wait?

If we deposit $5,000 today in an account paying 10%, how long does it take to grow to $10,000?

$$FV = C_0 \times (1+r)^T \qquad \$10{,}000 = \$5{,}000 \times (1.10)^T$$

$$(1.10)^T = \frac{\$10{,}000}{\$5{,}000} = 2$$

$$\ln(1.10)^T = \ln(2)$$

$$T = \frac{\ln(2)}{\ln(1.10)} = \frac{0.6931}{0.0953} = 7.27 \text{ years}$$

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What Rate Is Enough?

Assume the total cost of a college education will be $50,000 when your child enters college in 12 years. You have $5,000 to invest today. What rate of interest must you earn on your investment to cover the cost of your child's education?

About 21.15%.

$$FV = C_0 \times (1+r)^T$$

$$\$50{,}000 = \$5{,}000 \times (1+r)^{12}$$

$$(1+r)^{12} = \frac{\$50{,}000}{\$5{,}000} = 10$$

$$(1+r) = 10^{1/12}$$

$$r = 10^{1/12} - 1 = 1.2115 - 1 = .2115$$

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Calculator Keys

- Texas Instruments BA-II Plus
  - FV = future value
  - PV = present value
  - I/Y = interest (or discount) rate
    - P/Y must equal 1 for the I/Y to be the periodic rate
    - Interest is entered as a percent, not a decimal
  - N = number of periods
  - Remember to clear the registers (CLR TVM) after each problem
  - Other calculators are similar in format

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Uneven Cash Flows

- Consider an investment that pays $200 one year from now, with cash flows increasing by $200 per year through year 4. If the interest rate is 12%, what is the present value of this stream of cash flows?
- If the issuer offers this investment for $1,500, should you purchase it?

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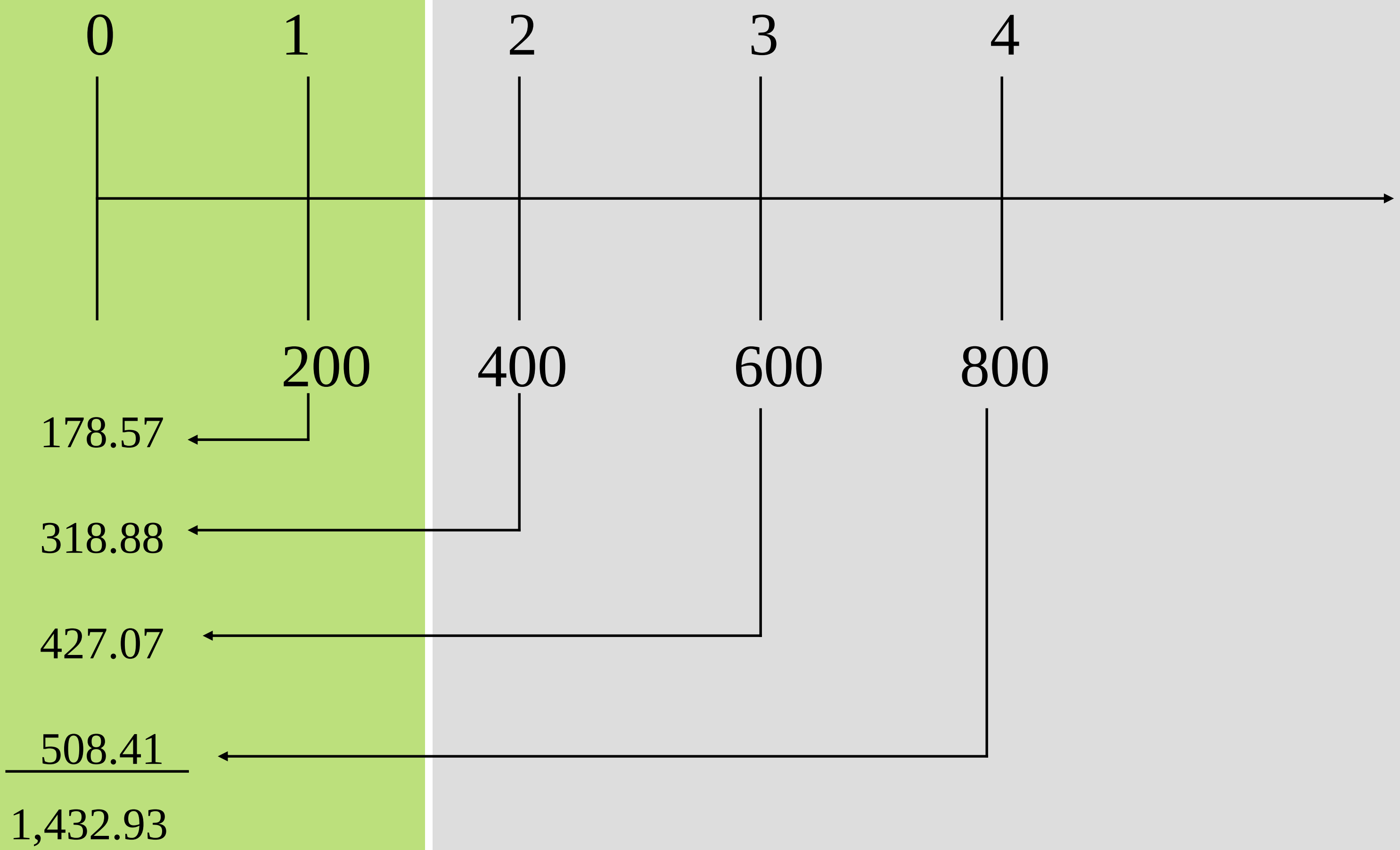
Uneven Cash Flows



Present Value < Cost → Do Not Purchase

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Valuing "Lumpy" Cash Flows

First, set your calculator to 1 payment per year.

Then, use the cash flow menu:

CF0	0	CF3	600	I	12
CF1	200	F3	1	NPV	1,432.93
F1	1	CF4	800		
CF2	400	F4	1		
F2	1				

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4.3 Compounding Periods and Effective Annual Interest Rate

Compounding an investment *m* times a year for *T* years provides for future value of wealth:

$$FV = C_0 \times \left(1 + \frac{r}{m}\right)^{m \times T}$$

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Compounding Periods

- For example, if you invest $50 for 3 years at 12% compounded semi-annually, your investment will grow to

$$FV = \$50 \times \left(1 + \frac{.12}{2}\right)^{2\times 3} = \$50 \times (1.06)^6 = \$70.93$$

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Effective Annual Rates of Interest (EAR)

A reasonable question to ask in the above example is "what is the effective *annual* rate of interest on that investment?"

$$FV = \$50 \times (1 + \frac{.12}{2})^{2\times3} = \$50 \times (1.06)^6 = \$70.93$$

The Effective Annual Rate (EAR) of interest is the annual rate that would give us the same end-of-investment wealth after 3 years:

$$\$50 \times (1 + EAR)^3 = \$70.93$$

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Effective Annual Rates of Interest

$$FV = \$50 \times (1 + EAR)^3 = \$70.93$$

$$(1 + EAR)^3 = \frac{\$70.93}{\$50}$$

$$EAR = \left(\frac{\$70.93}{\$50}\right)^{1/3} - 1 = .1236$$

So, investing at 12.36% compounded annually is the same as investing at 12% compounded semi-annually.

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Effective Annual Rates of Interest

- Find the Effective Annual Rate (EAR) of an 18% APR loan that is compounded monthly.
- What we have is a loan with a monthly interest rate of 1½%.
- This is equivalent to a loan with an annual interest rate of 19.56%.

$$\left(1+\frac{r}{m}\right)^{n\times m} = \left(1+\frac{.18}{12}\right)^{12} = (1.015)^{12} = 1.1956$$

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Continuous Compounding

- The general formula for the future value of an investment compounded continuously over many periods can be written as:

$$FV = C_0 \times e^{rT}$$

Where

C_0 is cash flow at date 0,

r is the stated annual interest rate,

T is the number of years, and

e is a transcendental number approximately equal to 2.718. e^x is a key on your calculator.

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More on Periods and Interest Rates

- Compounding (or Discounting) Period
  - The length of time to earn (or to be charged with) interest once.
  - Examples: day; week; month; quarter; year; etc.
- Compounding (or Discounting) Frequency, m
  - The number of compounding (or discounting) periods within a year.
- Payment Period
  - The length of time to pay or receive a payment (or a cash flow).
  - Examples: daily; weekly; monthly; quarterly; etc.

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More on Periods and Interest Rates

- Nominal Rate, i
  - The simple, quoted, or contracted annual interest rate.
- Periodic Rate, PER
  - The simple interest rate for the compounding (or discounting) period.
  - Defined as PER = i/m;   m: compounding frequency
- Effective Annual Rate, EAR
  - The annual rate of interest actually being earned or charged.
  - Defined as $EAR = (1+PER)^m - 1$
- Effective Periodic Rate, EPR
  - The rate of interest actually being earned or charged for the payment (or cash flow) period.
  - Defined as $EPR = (1+PER)^n – 1$; n: number of compounding (or discounting) per payment period

Rules for Choice of Interest Rates

1. For single cash flow problems, always use the EAR as the interest rate, r, in TVM calculations.
2. For multiple cash flows problems,

-- If the payment period matches with the compounding (or discounting) period, use the PER as the interest rate, r, in TVM calculations.

-- If the payment period does not match with the compounding (or discounting) period, use the EAR (for annual payment period) ***or*** EPR (for non-annual payment period) as the interest rate, r, in TVM calculations.

3. Do NOT use the nominal rate, i, directly as the interest rate, r, in TVM calculations.

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4.4 Simplifications

- Perpetuity
  - A constant stream of cash flows that lasts forever
- Growing perpetuity
  - A stream of cash flows that grows at a constant rate forever
- Annuity
  - A stream of constant cash flows that lasts for a fixed number of periods
- Growing annuity
  - A stream of cash flows that grows at a constant rate for a fixed number of periods

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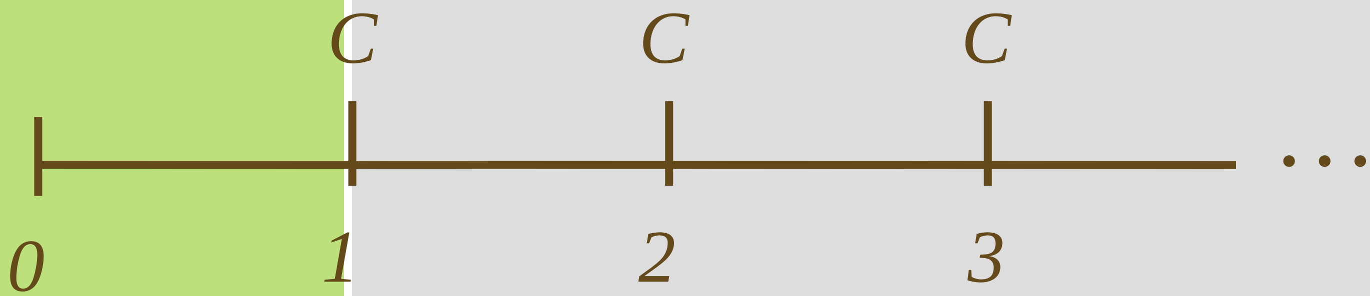
Perpetuity

A constant stream of cash flows that lasts forever



$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r}$$

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Perpetuity: Example

What is the value of a British consol that promises to pay £15 every year for ever?

The interest rate is 10-percent.

$$PV = \frac{£15}{.10} = £150$$

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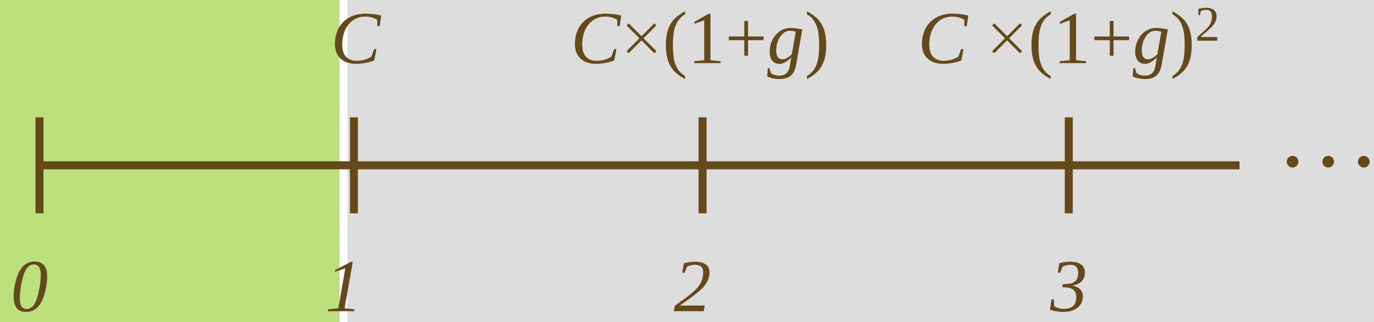
Growing Perpetuity

A growing stream of cash flows that lasts forever



$$PV = \frac{C}{(1+r)} + \frac{C\times(1+g)}{(1+r)^2} + \frac{C\times(1+g)^2}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r-g}$$

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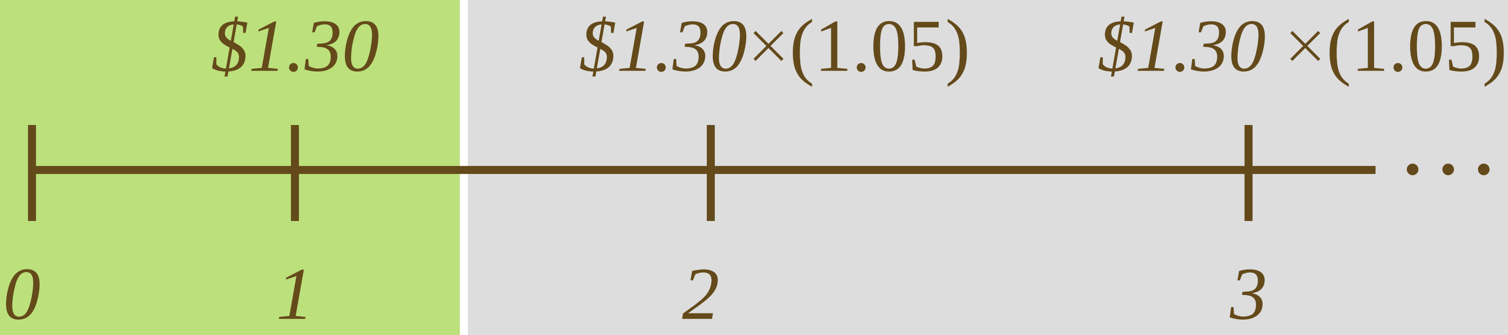
Growing Perpetuity: Example

The expected dividend next year is $1.30, and dividends are expected to grow at 5% forever.

If the discount rate is 10%, what is the value of this promised dividend stream?



$$PV = \frac{\$1.30}{.10 - .05} = \$26.00$$

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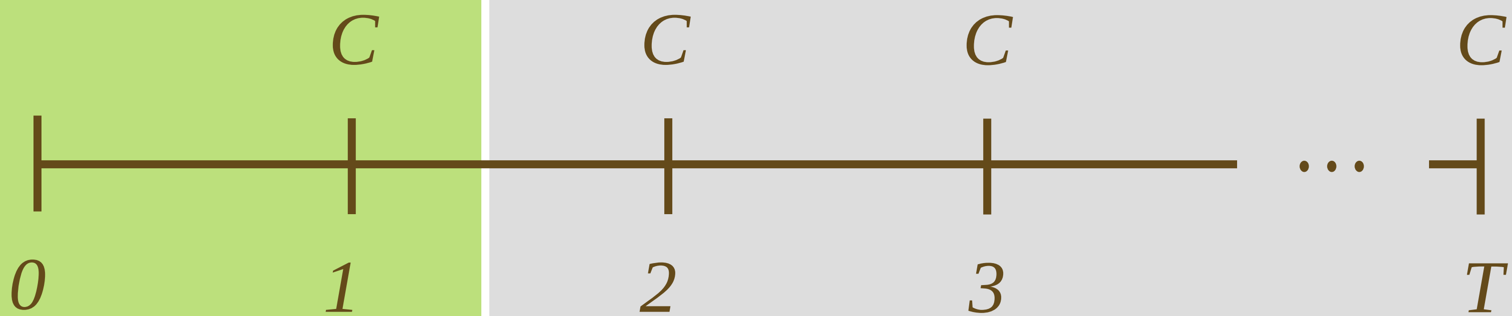
Annuity

A constant stream of cash flows with a fixed maturity



$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots \frac{C}{(1+r)^T}$$

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

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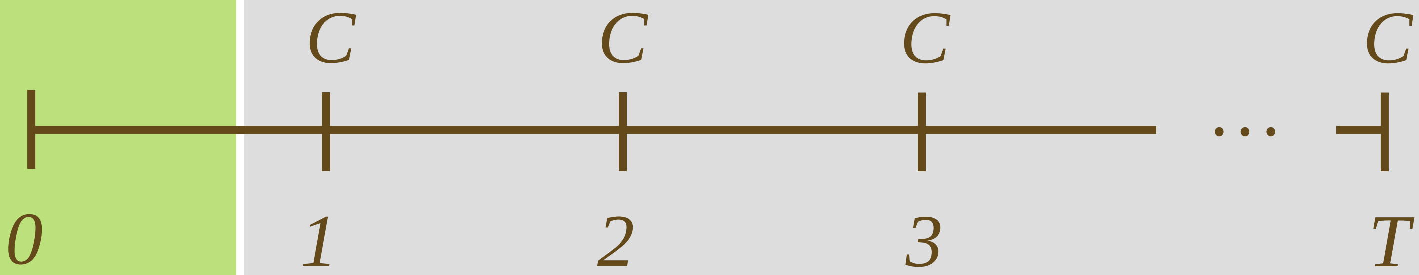
Annuity Intuition

- 

An annuity is valued as the difference between two perpetuities:

one perpetuity that starts at time *1*

less a perpetuity that starts at time *T* + *1*

$$PV = \frac{C}{r} - \frac{\left(\frac{C}{r}\right)}{(1+r)^T}$$

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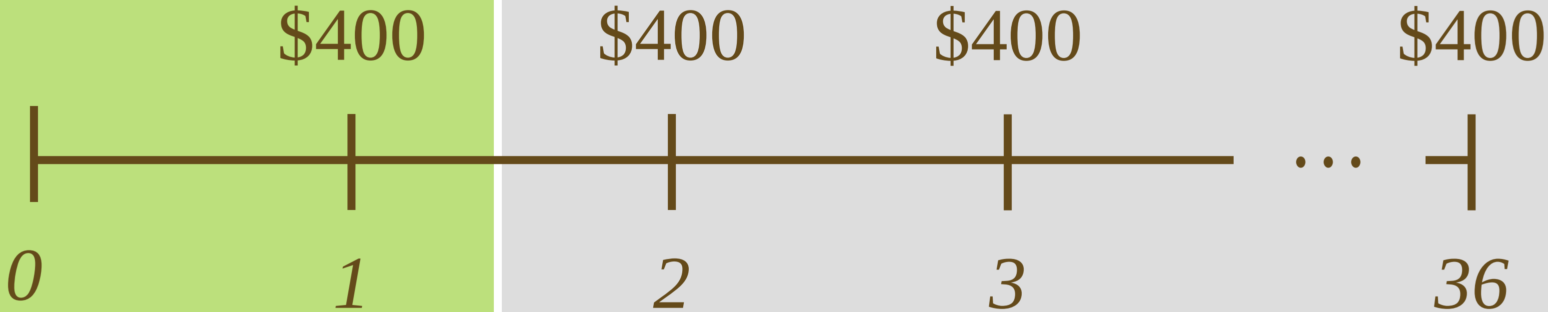
Annuity: Example

If you can afford a $400 monthly car payment, how much car can you afford if interest rates are 7% on 36-month loans?



$$PV = \frac{\$400}{.07/12}\left[1 - \frac{1}{(1+.07/12)^{36}}\right] = \$12{,}954.59$$

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How to Value Annuities with a Calculator

N	36
I/Y	7/12 = .5833
PV	12,954.59
PMT	–400
FV	0

For installment loans, use periodic rate for I/Y!

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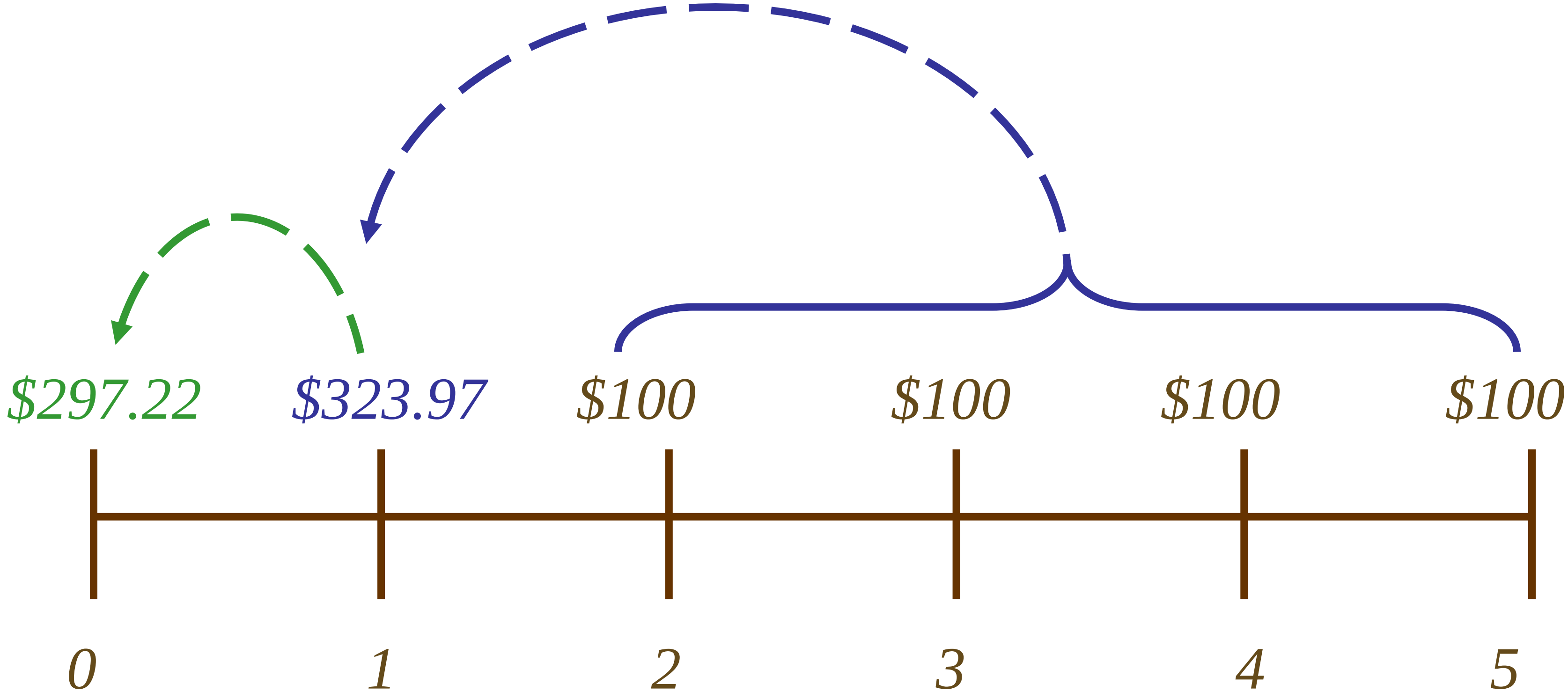
What is the present value of a four-year annuity of $100 per year that makes its first payment two years from today if the discount rate is 9%?

$$PV_1 = \sum_{t=1}^{4} \frac{\$100}{(1.09)^t} = \frac{\$100}{(1.09)^1} + \frac{\$100}{(1.09)^2} + \frac{\$100}{(1.09)^3} + \frac{\$100}{(1.09)^4} = \$323.97$$



$$PV_0 = \frac{\$327.97}{1.09} = \$297.22$$

How to Value “Lumpy” Cash Flows

First, set your calculator to 1 payment per year.
Then, use the cash flow menu:

CF0	0	I	9
CF1	0	NPV	297.22
F1	1		
CF2	100		
F2	4		

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Growing Annuity

A growing stream of cash flows with a fixed maturity

$$\begin{array}{ccccc} & C & C\times(1+g) & C\times(1+g)^2 & C\times(1+g)^{T-1} \\ \vdash & \vert & \vert & \vert \cdots & \dashv \\ 0 & 1 & 2 & 3 & T \end{array}$$

$$PV = \frac{C}{(1+r)} + \frac{C\times(1+g)}{(1+r)^2} + \cdots + \frac{C\times(1+g)^{T-1}}{(1+r)^T}$$

$$PV = \frac{C}{r-g}\left[1-\left(\frac{1+g}{(1+r)}\right)^T\right]$$

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Growing Annuity: Example

A defined-benefit retirement plan offers to pay $20,000 per year for 40 years and increase the annual payment by 3% each year. What is the present value at retirement if the discount rate is 10%?

$\$20{,}000 \qquad \$20{,}000\times(1.03) \qquad \$20{,}000\times(1.03)^{39}$

0   *1*   *2*   …   *40*

$$PV = \frac{\$20{,}000}{.10-.03}\left[1-\left(\frac{1.03}{1.10}\right)^{40}\right] = \$265{,}121.57$$

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Growing Annuity: Example

You are evaluating an income generating property. Net rent is received at the end of each year. The first year's rent is expected to be $8,500, and rent is expected to increase 7% each year. What is the present value of the estimated income stream over the first 5 years if the discount rate is 12%?

$\$8{,}500 \times (1.07) =$ $\$9{,}095$

$\$8{,}500 \times (1.07)^2 =$ $\$9{,}731.65$

$\$8{,}500 \times (1.07)^3 =$ $\$10{,}412.87$

$\$8{,}500 \times (1.07)^4 =$ $\$11{,}141.77$

0	1	2	3	4	5
	$8,500	$9,095	$9,731.65	$10,412.87	$11,141.77
$34,706.26					

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PV of Growing Annuity: Cash Flow Keys

First, set your calculator to 1 payment per year.

Then, use the cash flow menu:

CF0	0	I	12
CF1	8,500.00	NPV	$34,706.26
CF2	9,095.00		
CF3	9,731.65		
CF4	10,412.87		
CF5	11,141.77		

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PV of a delayed growing perpetuity

Your firm is about to make its initial public offering of stock and your job is to estimate the correct offering price. Forecast dividends are as follows.

Year:	**1**	**2**	**3**	**4**
Dividends per share	\$1.50	\$1.65	\$1.82	5% growth thereafter

If investors demand a 10% return on investments of this risk level, what price will they be willing to pay?

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PV of a delayed growing perpetuity

Year	0	1	2	3	4	…
Cash flow		\$1.50	\$1.65	\$1.82	\$1.82×1.05	

The first step is to draw a timeline.

The second step is to decide on what we know and what it is we are trying to find.

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PV of a delayed growing perpetuity

Year	0	1	2	3
Cash flow		\$1.50	\$1.65	\$1.82 dividend + P_3 = \$1.82 + \$38.22
PV of cash flow	\$32.81			

$$P_3 = \frac{1.82 \times 1.05}{.10 - .05} = \$38.22$$

$$P_0 = \frac{\$1.50}{(1.10)} + \frac{\$1.65}{(1.10)^2} + \frac{\$1.82 + \$38.22}{(1.10)^3} = \$32.81$$

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Payment Due versus Delayed Payment

- If the cash flows occur at the beginning, rather than at the end, of each period, we adjust both the FV and PV calculations by multiplying the corresponding equations (discussed in previous slides) with the same interest factor, (1+r).

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4.5 Loan Amortization

- Pure Discount Loans are the simplest form of loan. The borrower receives money today and repays a single lump sum (principal and interest) at a future time.
- Interest-Only Loans require an interest payment each period, with full principal due at maturity.
- Amortized Loans require repayment of principal over time, in addition to required interest.

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Pure Discount Loans

- Treasury bills are excellent examples of pure discount loans. The principal amount is repaid at some future date, without any periodic interest payments.
- If a T-bill promises to repay $10,000 in 12 months and the market interest rate is 7 percent, how much will the bill sell for in the market?

  ➔ PV = 10,000 / 1.07 = 9,345.79

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Interest-Only Loan

- Consider a 5-year, interest-only loan with a 7% interest rate. The principal amount is $10,000. Interest is paid annually.
  - What would the stream of cash flows be?
    - Years 1 – 4: Interest payments of .07(10,000) = 700
    - Year 5: Interest + principal = 10,700
- This cash flow stream is similar to the cash flows on corporate bonds, and we will talk about them in greater detail later (Chapter 8).

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Amortized Loan with Fixed Principal Payment

- Consider a $50,000, 10 year loan at 8% interest. The loan agreement requires the firm to pay $5,000 in principal each year plus interest for that year.

Year	Beginning Balance	Interest Payment	Principal Payment	Total Payment	Ending Balance
1	50,000	4,000	5,000	9,000	45,000
2	45,000	3,600	5,000	8,600	40,000
3	40,000	3,200	5,000	8,200	35,000
4	35,000	2,800	5,000	7,800	30,000
5	30,000	2,400	5,000	7,400	25,000
6	25,000	2,000	5,000	7,000	20,000
7	20,000	1,600	5,000	6,600	15,000
8	15,000	1,200	5,000	6,200	10,000
9	10,000	800	5,000	5,800	5,000
10	5,000	400	5,000	5,400	0

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Amortized Loan w/ Fixed Payment

- Each payment covers the interest expense plus reduces principal
- Consider a 4 year loan with annual payments. The interest rate is 8% ,and the principal amount is $5,000.
  - What is the annual payment?
    - 4 N
    - 8 I/Y
    - 5,000 PV
    - CPT PMT

    = -1,509.60

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	5,000.00	1,509.60	400.00	1,109.60	3,890.40
2	3,890.40	1,509.60	311.23	1,198.37	2,692.03
3	2,692.03	1,509.60	215.36	1,294.24	1,397.79
4	1,397.79	1,509.60	111.82	1,397.78	0.01
Totals		6,038.40	1,038.41	4,999.99	

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4.6 What Is a Firm Worth?

- Conceptually, a firm should be worth the present value of the firm's cash flows.
- The tricky part is determining the size, timing, and *risk* of those cash flows.

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4.7 Summary and Conclusions

- Two basic concepts, future value and present value are introduced in this chapter.
- Interest rates are commonly expressed on an annual basis, but semi-annual, quarterly, monthly and even continuously compounded interest rate arrangements exist.
- The formula for the net present value of an investment that pays $*C* for *N* periods is:

$$NPV = -C_0 + \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+r)^N} = -C_0 + \sum_{t=1}^{N} \frac{C}{(1+r)^t}$$

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4.7 Summary and Conclusions (continued)

- We presented four simplifying formulae:

$$\text{Perpetuity}: PV = \frac{C}{r}$$

$$\text{Growing Perpetuity}: PV = \frac{C}{r-g}$$

$$\text{Annuity}: PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

$$\text{Growing Annuity}: PV = \frac{C}{r-g}\left[1 - \left(\frac{1+g}{(1+r)}\right)^T\right]$$

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How do you get to Carnegie Hall?

- Practice, practice, practice.
- It's easy to watch Olympic gymnasts and convince yourself that you are a leotard purchase away from a triple back flip.
- It's also easy to watch your finance professor do time value of money problems and convince yourself that you can do them too.
- There is no substitute for getting out the calculator and flogging the keys until you can do these correctly and quickly.

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This is my calculator. This is my friend!

- Your financial calculator has two major menus that you *must* become familiar with:
  - The time value of money keys:
    - N; I/YR; PV; PMT; FV
    - Use this menu to value things with level cash flows, like annuities *e.g*. student loans.
    - It can even be used to value growing annuities.
  - The cash flow menu
    - CF_j *et cetera*
    - Use the cash flow menu to value “lumpy” cash flow streams.

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Problems

- You have $30,000 in student loans that call for monthly payments over 10 years.
  - $15,000 is financed at seven percent APR
  - $8,000 is financed at eight percent APR and
  - $7,000 at 15 percent APR
- What is the interest rate on your portfolio of debt?

Hint: don't even think about doing this:

$$= \frac{15{,}000}{30{,}000} \times 7\% + \frac{8{,}000}{30{,}000} \times 8\% + \frac{7{,}000}{30{,}000} \times 15\%$$

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Problems

Find the payment on each loan, add the payments to get your total monthly payment: $384.16.

Set PV = $30,000 and solve for I/Y = 9.25%

N	120		120		120		120
I/Y	.5833		.6667		1.25		9.25 = .771*12
PV	15,000	+	8,000	+	7,000	=	30,000
PMT	–174.16	+	–97.06	+	–112.93	=	–384.16
FV	0		0		0		0

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Problems

- You are considering the purchase of a prepaid tuition plan for your 8-year old daughter. She will start college in exactly 10 years, with the first tuition payment of $12,500 due at the start of the year. Sophomore year tuition will be $15,000; junior year tuition $18,000, and senior year tuition $22,000. How much money will you have to pay today to fully fund her tuition expenses? The discount rate is 14%

CF0	0
CF1	0
F1	9
CF2	$12,500
F2	1
CF3	15,000
F3	1
CF4	$18,000
F4	1
CF4	$22,000
F4	1
I	14
NPV	$14,662.65

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Problems

You are thinking of buying a new car. You bought you current car exactly 3 years ago for $25,000 and financed it at 7% APR for 60 months. You need to estimate how much you owe on the loan to make sure that you can pay it off when you sell the old car.

N	60
I/Y	.5833
PV	25,000
PMT	–495.03
FV	0

N	24
I/Y	.5833
PV	11,056
PMT	–495.03
FV	0

N	36
I/Y	.5833
PV	25,000
PMT	–495.03
FV	11,056

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Problems

You have just landed a job and are going to start saving for a down-payment on a house. You want to save 20 percent of the purchase price and then borrow the rest from a bank.

You have an investment that pays 10 percent APR. Houses that you like and can afford currently cost $100,000. Real estate has been appreciating in price at 5 percent per year and you expect this trend to continue.

How much should you save every month in order to have a down payment saved five years from today?

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Problems

- First we estimate that in 5 years, a house that costs $100,000 today will cost $127,628.16
- Next we estimate the monthly payment required to save up that much in 60 months.

N	5	N	60
I/Y	5	I/Y	.8333
PV	100,000	PV	0
PMT	0	PMT	–329.63
FV	127,628.16	FV	$25,525.63 = 0.20×$127,628.16

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Quick Quiz

- How is the future value of a single cash flow computed?
- How is the present value of a series of cash flows computed.
- What is the Net Present Value of an investment?
- What is an EAR, and how is it computed?
- What is a perpetuity? An annuity?

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