Chapter 11 Suggested Problems Solutions

9. *a.* This portfolio does not have an equal weight in each asset. We first need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

Boom: $R_p = .30(.24) + .40(.45) + .30(.33) = .3510$, or 35.10%
Good: $R_p = .30(.09) + .40(.10) + .30(.15) = .1120$, or 11.20%
Poor: $R_p = .30(.03) + .40(–.10) + .30(–.05) = –.0460$, or –4.60%
Bust: $R_p = .30(–.05) + .40(–.25) + .30(–.09) = –.1420$, or –14.20%

And the expected return of the portfolio is:
$E(R_p) = .20(.3510) + .35(.1120) + .30(–.0460) + .15(–.1420) = .0743$, or 7.43%

b. To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation the portfolio is:
$\sigma_p^2 = .20(.3510 – .0743)^2 + .35(.1120 – .0743)^2 + .30(–.0460 – .0743)^2 + .15(–.1420 – .0743)^2$
$\sigma_p^2 = .02717 \quad \Rightarrow \quad \sigma_p = (.02717)^{1/2} = .1648$, or 16.48%

10. The beta of a portfolio is the sum of the weight of each asset times the beta of each asset. So, the beta of the portfolio is:
$\beta_p = .10(.75) + .35(1.90) + .20(1.38) + .35(1.16) = 1.42$

11. The beta of a portfolio is the sum of the weight of each asset times the beta of each asset. If the portfolio is as risky as the market it must have the same beta as the market. Since the beta of the market is one, we know the beta of our portfolio is one. We also need to remember that the beta of the risk-free asset is zero. It has to be zero since the asset has no risk. Setting up the equation for the beta of our portfolio, we get:
$\beta_p = 1.0 = {}^1/_3(0) + {}^1/_3(1.65) + {}^1/_3(\beta_X)$
Solving for the beta of Stock X, we get: $\beta_X = 1.35$

12. CAPM states the relationship between the risk of an asset and its expected return. CAPM is:
Required $R_i = R_f + [E(R_M) – R_f] \times \beta_I = .05 + (.11 – .05)(1.15) = .1190$, or 11.90%

16. *a.* Again, we have a special case where the portfolio is equally weighted, so we can sum the returns of each asset and divide by the number of assets. The expected return of the portfolio is:
$E(R_p) = (.121 + .05)/2 = .0855$, or 8.55%

b. We need to find the portfolio weights that result in a portfolio with a β of 0.50. We know the β of the risk-free asset is zero. We also know the weight of the risk-free asset is one minus the weight of the stock since the portfolio weights must sum to one, or 100 percent. So:
$\beta_p = 0.50 = X_S(1.13) + (1 – X_S)(0)$

⇨ $0.50 = 1.13X_S + 0 - 0X_S$ ➔ $X_S = 0.50/1.13 = .4425$

And, the weight of the risk-free asset is:
$X_{Rf} = 1 - .4425 = .5575$

c. We need to find the portfolio weights that result in a portfolio with an expected return of 10 percent. We also know the weight of the risk-free asset is one minus the weight of the stock since the portfolio weights must sum to one, or 100 percent. So:
$E(R_p) = .10 = .121X_S + .05(1 - X_S)$
⇨ $.10 = .121X_S + .05 - .05X_S = .7042$
So, the β of the portfolio will be:
$\beta_p = .7042(1.13) + (1 - .7042)(0) = 0.796$

d. Solving for the β of the portfolio as we did in part *b*, we find:
$\beta_p = 2.26 = X_S(1.13) + (1 - X_S)(0)$
⇨ $X_S = 2.26/1.13 = 2 = 1 - 2 = -1$
The portfolio is invested 200% in the stock and –100% in the risk-free asset. This represents borrowing at the risk-free rate to buy more of the stock.

22. *a.* We need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:
Boom: $E(R_p) = .4(.20) + .4(.25) + .2(.60) = .3000$, or 30.00%
Normal: $E(R_p) = .4(.15) + .4(.11) + .2(.05) = .1140$, or 11.40%
Bust: $E(R_p) = .4(.01) + .4(-.15) + .2(-.50) = -.1560$, or –15.60%

And the expected return of the portfolio is:
$E(R_p) = .30(.30) + .45(.114) + .25(-.156) = .1023$, or 10.23%

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, than add all of these up. The result is the variance. So, the variance and standard deviation of the portfolio is:
$\sigma^2_p = .30(.30 - .1023)^2 + .45(.114 - .1023)^2 + .25(-.156 - .1023)^2$
$\sigma^2_p = .02847$ ➔ $\sigma_p = (.02847)^{1/2} = .1687$, or 16.87%

b. The risk premium is the return of a risky asset, minus the risk-free rate. T-bills are often used as the risk-free rate, so:
$RP_i = E(R_p) - R_f = .1023 - .038 = .0643$, or 6.43%

c. The approximate expected real return is the expected nominal return minus the inflation rate, so:
Approximate expected real return = .1023 – .035 = .0673, or 6.73%

To find the exact real return, we will use the Fisher equation. Doing so, we get:
$1 + E(R_i) = (1 + h)[1 + e(r_i)]$ ➔ $1.1023 = (1.0350)[1 + e(r_i)]$
⇨ $e(r_i) = (1.1023/1.035) - 1 = .0650$, or 6.50%

The approximate real risk-free rate is:
Approximate expected real return = .038 – .035 = .003, or 0.30%
And using the Fisher effect for the exact real risk-free rate, we find:

$1 + E(R_i) = (1 + h)[1 + e(r_i)]$ ➔ $1.038 = (1.0350)[1 + e(r_i)]$
⇨ $e(r_i) = (1.038/1.035) – 1 = .0029$, or 0.29%

The approximate real risk premium is the approximate expected real return minus the risk-free rate, so:
Approximate expected real risk premium = .0673 – .003 = .0643, or 6.43%

The exact real risk premium is the exact real return minus the risk-free rate, so:
Exact expected real risk premium = .0650 – .0029 = .0621, or 6.21%

23. We know the total portfolio value and the investment of two stocks in the portfolio, so we can find the weight of these two stocks. The weights of Stock A and Stock B are:
X_A = \$180,000 / \$1,000,000 = .18
X_B = \$290,000/\$1,000,000 = .29

Since the portfolio is as risky as the market, the β of the portfolio must be equal to one. We also know the β of the risk-free asset is zero. We can use the equation for the β of a portfolio to find the weight of the third stock. Doing so, we find:
$\beta_p = 1.0 = X_A(.85) + X_B(1.40) + X_C(1.45) + X_{Rf}(0)$ ➔ $X_C = .30413793$
So, the dollar investment in Stock C must be:
Invest in Stock C = .30413793(\$1,000,000) = \$304,137.93

We also know the total portfolio weight must be one, so the weight of the risk-free asset must be one minus the asset weight we know, or:
$1 = X_A + X_B + X_C + X_{Rf}$ ➔ $1 = .18 + .29 + .30413793 + X_{Rf}$ ➔ $X_{Rf} = .22586207$
So, the dollar investment in the risk-free asset must be:
Invest in risk-free asset = .22586207(\$1,000,000) = \$225,862.07

24. We are given the expected return and β of a portfolio and the expected return and β of assets in the portfolio. We know the β of the risk-free asset is zero. We also know the sum of the weights of each asset must be equal to one. So, the weight of the risk-free asset is one minus the weight of Stock X and the weight of Stock Y. Using this relationship, we can express the expected return of the portfolio as:
$E(R_p) = .1122 = X_X(.1535) + X_Y(.0940) + (1 – X_X – X_Y)(.045)$
And the β of the portfolio is:
$\beta_p = .96 = X_X(1.55) + X_Y(0.70) + (1 – X_X – X_Y)(0)$

We have two equations and two unknowns. Solving these equations, we find that:
$X_X = –0.2838710$
$X_Y = 2.0000000$
$X_{Rf} = –0.7161290$

The amount to invest in Stock X is:
Investment in stock X = –0.28387(\$100,000) = –\$28,387.10

A negative portfolio weight means that you short sell the stock. If you are not familiar with short selling, it means you borrow a stock today and sell it. You must then purchase the stock at a later date to repay the borrowed stock. If you short sell a stock, you make a profit if the stock decreases in value. The negative weight on the risk-free asset means that we borrow money to invest.

25. The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each stock is:

$E(R_A) = .33(.102) + .33(.115) + .33(.073) = .0967$, or 9.67%

$E(R_B) = .33(–.045) + .33(.148) + .33(.233) = .1120$, or 11.20%

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation of Stock A are:

$\sigma^2 =.33(.102 – .0967)^2 + .33(.115 – .0967)^2 + .33(.073 – .0967)^2 = .00031$

$\Rightarrow \quad \sigma = (.00031)^{1/2} = .0176$, or 1.76%

And the standard deviation of Stock B is:

$\sigma^2 =.33(–.045 – .1120)^2 + .33(.148 – .1120)^2 + .33(.233 – .1120)^2 = .01353$

$\Rightarrow \quad \sigma = (.01353)^{1/2} = .1163$, or 11.63%

To find the covariance, we multiply each possible state times the product of each assets' deviation from the mean in that state. The sum of these products is the covariance. So, the covariance is:

$$\text{Cov}(A,B) = .33(.102 – .0967)(–.045 – .1120) + .33(.115 – .0967)(.148 – .1120)$$
$$+ .33(.073 – .0967)(.233 – .1120) = –.001014$$

And the correlation is:

$\rho_{A,B} = \text{Cov}(A,B) / \sigma_A \sigma_B = –.001014 / (.0176)(.1163) = –.4964$

26. The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each stock is:

$E(R_A) = .25(–.020) + .60(.138) + .15(.218) = .1105$, or 11.05%

$E(R_B) = .25(.034) + .60(.062) + .15(.092) = .0595$, or 5.95%

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation of Stock A are:

$\sigma_A^2 =.25(–.020 – .1105)^2 + .60(.138 – .1105)^2 + .15(.218 – .1105)^2 = .00644$

$\Rightarrow \quad \sigma_A = (.00644)^{1/2} = .0803$, or 8.03%

And the standard deviation of Stock B is:

$\sigma_B^2 =.25(.034 – .0595)^2 + .60(.062 – .0595)^2 + .15(.092 – .0595)^2 = .00032$

$\Rightarrow \quad \sigma_B = (.00032)^{1/2} = .0180$, or 1.80%

To find the covariance, we multiply each possible state times the product of each assets' deviation from the mean in that state. The sum of these products is the covariance. So, the covariance is:

$$\text{Cov}(A,B) = .25(–.020 – .1105)(.034 – .0595) + .60(.138 – .1105)(.062 – .0595)$$
$$+ .15(.218 – .1105)(.092 – .0595) = .001397$$

And the correlation is:
$\rho_{A,B}$ = Cov(A,B) / σ_A σ_B = .001397 / (.0803)(.0180) = .9658

27. *a.* The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:
$E(R_P) = X_F E(R_F) + X_G E(R_G) = .30(.10) + .70(.15) = .1350$, or 13.50%

b. The variance of a portfolio of two assets can be expressed as:
$\sigma_P^2 = X_F^2 \sigma_F^2 + X_G^2 \sigma_G^2 + 2X_F X_G \sigma_F \sigma_G \rho_{F,G}$
$\sigma_P^2 = .30^2(.43^2) + .70^2(.62^2) + 2(.30)(.70)(.43)(.62)(.25) = .23299$

So, the standard deviation is:
$\sigma_P = (.23299)^{1/2} = .4827$, or 48.27%

28. *a.* The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:
$E(R_P) = X_A E(R_A) + X_B E(R_B) = .35(.09) + .65(.15) = .1290$, or 12.90%

The variance of a portfolio of two assets can be expressed as:
$\sigma_P^2 = X_A^2 \sigma_A^2 + X_B^2 \sigma_B^2 + 2X_A X_B \sigma_A \sigma_B \rho_{A,B}$
$\sigma_P^2 = .35^2(.36^2) + .65^2(.62^2) + 2(.35)(.65)(.36)(.62)(.50) = .22906$

So, the standard deviation is: $\sigma_P = (.22906)^{1/2} = .4786$, or 47.86%

b. $\sigma_P^2 = X_A^2 \sigma_A^2 + X_B^2 \sigma_B^2 + 2X_A X_B \sigma_A \sigma_B \rho_{A,B}$
$\sigma_P^2 = .35^2(.36^2) + .65^2(.62^2) + 2(.35)(.65)(.36)(.62)(–.50) = .12751$

So, the standard deviation is: $\sigma = (.12751)^{1/2} = .3571$, or 35.71%

c. As Stock A and Stock B become less correlated, or more negatively correlated, the standard

35. *a.* The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the expected return and standard deviation of each stock are:

Asset 1:
$E(R_1) = .15(.20) + .35(.15) + .35(.10) + .15(.05) = .1250$, or 12.50%
$\sigma_1^2 = .15(.20 – .1250)^2 + .35(.15 – .1250)^2 + .35(.10 – .1250)^2 + .15(.05 – .1250)^2 = .00213$
$\sigma_1 = (.00213)^{1/2} = .0461$ or 4.61%

Asset 2:

$E(R_2) = .15(.20) + .35(.10) + .35(.15) + .15(.05) = .1250$, or 12.50%

$\sigma_2^2 = .15(.20 – .1250)^2 + .35(.10 – .1250)^2 + .35(.15 – .1250)^2 + .15(.05 – .1250)^2 = .00213$

$\sigma_2 = (.00213)^{1/2} = .0461$ or 4.61%

Asset 3:

$E(R_3) = .15(.05) + .35(.10) + .35(.15) + .15(.20) = .1250$, or 12.50%

$\sigma_3^2 = .15(.05 – .1250)^2 + .35(.10 – .1250)^2 + .35(.15 – .1250)^2 + .15(.20 – .1250)^2 = .00213$

$\sigma_3 = (.00213)^{1/2} = .0461$ or 4.61%

b. To find the covariance, we multiply each possible state times the product of each assets' deviation from the mean in that state. The sum of these products is the covariance. The correlation is the covariance divided by the product of the two standard deviations. So, the covariance and correlation between each possible set of assets are:

Asset 1 and Asset 2:

$\text{Cov}(1,2) = .15(.20 – .1250)(.20 – .1250) + .35(.15 – .1250)(.10 – .1250)$
$+ .35(.10 – .1250)(.15 – .1250) + .15(.05 – .1250)(.05 – .1250)$

$\text{Cov}(1,2) = .00125$

$\rho_{1,2} = \text{Cov}(1,2) / \sigma_1 \sigma_2 = .00125 / (.0461)(.0461) = .5882$

Asset 1 and Asset 3:

$\text{Cov}(1,3) = .15(.20 – .1250)(.05 – .1250) + .35(.15 – .1250)(.10 – .1250)$
$+ .35(.10 – .1250)(.15 – .1250) + .15(.05 – .1250)(.20 – .1250)$

$\text{Cov}(1,3) = –.002125$

$\rho_{1,3} = \text{Cov}(1,3) / \sigma_1 \sigma_3 = –.002125 / (.0461)(.0461) = –1$

Asset 2 and Asset 3:

$\text{Cov}(2,3) = .15(.20 – .1250)(.05 – .1250) + .35(.10 – .1250)(.10 – .1250)$
$+ .35(.15 – .1250)(.15 – .1250) + .15(.05 – .1250)(.20 – .1250)$

$\text{Cov}(2,3) = –.00125$

$\rho_{2,3} = –.00125 / (.0461)(.0461) = –.5882$

c. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 1 and Asset 2:

$E(R_P) = X_1E(R_1) + X_2E(R_2) = .50(.1250) + .50(.1250) = .1250$, or 12.50%

The variance of a portfolio of two assets can be expressed as:

$\sigma_P^2 = X_1^2 \sigma_1^2 + X_2^2 \sigma_2^2 + 2X_1X_2\sigma_1\sigma_2\rho_{1,2}$

$\sigma_P^2 = .50^2(.0461^2) + .50^2(.0461^2) + 2(.50)(.50)(.0461)(.0461)(.5882) = .001688$

And the standard deviation of the portfolio is: $\sigma_P = (.001688)^{1/2} = .0411$ or 4.11%

d. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 1 and Asset 3:

$E(R_P) = .50(.1250) + .50(.1250) = .1250$, or 12.50%

$\sigma_P^2 = .50^2(.0461^2) + .50^2(.0461^2) + 2(.50)(.50)(.0461)(.0461)(–1) = .000000$

Since the variance is zero, the standard deviation is also zero.

e. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 2 and Asset 3:

$E(R_P) = .50(.1250) + .50(.1250) = .1250$, or 12.50%

$\sigma_P^2 = .50^2(.0461^2) + .50^2(.0461^2) + 2(.50)(.50)(.0461)(.0461)(–.5882) = .000438$

And the standard deviation of the portfolio is: $\sigma_P = (.000438)^{1/2} = .0209$ or 2.09%

f. As long as the correlation between the returns on two securities is below 1, there is a benefit to diversification. A portfolio with negatively correlated securities can achieve greater risk reduction than a portfolio with positively correlated securities, holding the expected return on each stock constant. Applying proper weights on perfectly negatively correlated securities can reduce portfolio variance to 0.

37. *a.* A typical, risk-averse investor seeks high returns and low risks. For a risk-averse investor holding a well-diversified portfolio, beta is the appropriate measure of the risk of an individual security. To assess the two stocks, we need to find the expected return and beta of each of the two securities.

Stock A:

Since Stock A pays no dividends, the return on Stock A is simply: $(P_1 – P_0) / P_0$. So, the return for each state of the economy is:

$R_{Recession}$ = (\$64 – 75) / \$75 = –.147, or –14.70%

R_{Normal} = (\$87 – 75) / \$75 = .160, or 16.00%

$R_{Expanding}$ = (\$97 – 75) / \$75 = .293, or 29.30%

The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of the stock is:

$E(R_A) = .20(–.147) + .60(.160) + .20(.293) = .1253$, or 12.53%

And the variance of the stock is:

$\sigma_A^2 = .20(–0.147 – 0.1253)^2 + .60(.160 – .1253)^2 + .20(.293 – .1253)^2 = 0.0212$

$\Rightarrow \sigma_A = (0.0212)^{1/2} = .1455$, or 14.55%

Now we can calculate the stock's beta, which is:

$\beta_A = (\rho_{A,M})(\sigma_A) / \sigma_M = (.70)(.1455) / .18 = .566$

For Stock B, we can directly calculate the beta from the information provided. So, the beta for Stock B is:

$\beta_B = (.24)(.34) / .18 = .453$

The expected return on Stock B is higher than the expected return on Stock A. The risk of Stock B, as measured by its beta, is lower than the risk of Stock A. Thus, a typical risk-averse investor holding a well-diversified portfolio will prefer Stock B. Note, this situation implies that at least one of the stocks is mispriced since the higher risk (beta) stock has a lower return than the lower risk (beta) stock.

b. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

$E(R_P) = X_A E(R_A) + X_B E(R_B) = .70(.1253) + .30(.14) = .1297$, or 12.97%

To find the standard deviation of the portfolio, we first need to calculate the variance. The variance of the portfolio is:

$\sigma_P^2 = X_A^2\sigma_A^2 + X_B^2\sigma_B^2 + 2X_AX_B\sigma_A\sigma_B\rho_{A,B}$

$\sigma_P^2 = (.70)^2(.1455)^2 + (.30)^2(.34)^2 + 2(.70)(.30)(.1455)(.34)(.36) = .02825$

And the standard deviation of the portfolio is: $\sigma_P = (0.02825)^{1/2} = .1681$ or 16.81%

c. The beta of a portfolio is the weighted average of the betas of its individual securities. So the beta of the portfolio is:

$\beta_P = .70(.566) + .30(0.453) = .532$