Overview and Learning Objectives

Overview

This chapter covers five methods for evaluating capital budgeting problems:

1. The Net Present Value (NPV) method.
2. The Payback Period method (two versions).
3. The Average Accounting Return (AAR) method.
4. The Internal Rate of Return (IRR) method.
5. The Profitability Index (PI) method.

For each method, we learn what it measures, its computation, and how we use it to make decisions on independent and mutually exclusive projects. We also develop four criteria, which are based on the factors that determine value, to evaluate each capital budgeting method in the context of the objective of a firm-value maximization.

Learning Objectives

After reading course materials on this chapter, students should be able to:

- Explain the reasoning underlying the criteria for evaluating capital budgeting methods (or investment rules).
- Use various capital budgeting methods to make the right investment decision on independent projects as well as mutually exclusive projects.
- Evaluate each of the five capital budgeting methods in the context of the objective of the firm.
- Explain why the NPV method is the conceptually most correct method in making investment decisions.
- Explain why and what are the problems associated with using the IRR method in investment decision-making, especially selecting among mutually exclusive projects.
- Apply the incremental project analysis to solve the problems of the IRR method.
- Apply the McKinsey's Modified internal rate of return (MIRR) method to solve the problems of using IRR to choose among mutually exclusive projects with difference in in cash flow timing.

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Numerical Example

Compute the IRR, NPV, PI, and payback period for the following two projects. Assume the required return is 10%.

Year	Project A	Project B
0	-$200	-$150
1	$200	$50
2	$800	$100
3	-$800	$150

Solution

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Evaluation Criteria and Project Types

In chapter one, we learned that the primary goal of a firm is to maximize firm value, and there are three aspects of cash flows, size, timing, and risk, that determine value. In addition, among the three major types of financial decisions, investment decision is the key in increasing firm value. As such, a good capital budgeting method should enable us to make the right decision in choosing among investment projects that lead to value maximization.

Four Evaluation Criteria

To address the above issue, we introduce the following four evaluation criteria to evaluate each capital budgeting method to determine whether the method leads us to the right investment decision.

A good capital budgeting method, i.e., one that can assist us in making investment decisions that lead to firm value maximization, should:

A. Include all relevant cash flows, not accounting earnings, in the analysis. (Consideration of the size of cash flows).

B. Recognize the time value of money, i.e., involve present value calculations. (Consideration of the timing of cash flows).

C. Discount cash flows at the appropriate discount rate, or cost of capital, or required return, which is determined according to the opportunity cost of capital. (Consideration of the risk of cash flows).

D. Lead us to select the most value-creating project among mutually exclusive alternatives. (The primary goal of a firm and its investment decision).

Note that the first three criteria, **A**, **B** and **C**, are necessary conditions for the right investment decision, and the last criterion, **D**, is the sufficient condition.

When we apply these four criteria to evaluate each capital budgeting method later in this chapter, we will just refer to them by their letters, i.e., **A**, **B**, **C**, and **D**.

In making capital budgeting decisions, we need to recognize that there are two types of projects:

- Independent projects
  - The acceptance or rejection of one project does not affect the decision on the other projects.
  - Accept a project if it exceeds a minimum acceptance criterion!

- Mutually exclusive projects
  - There is at most one project, among all projects under consideration, can be accepted. In other words, the acceptance of a project excludes the acceptance of all other alternatives under consideration.
  - Rank all alternatives and select the best acceptable one!

Throughout this chapter, we use the following information about two projects for numerical illustrations as we discuss the various capital budgeting methods:

	Expected after-tax net cash flows (in $M)	
Year (t)	Projects S	Project L
0	(9)	(9)
1	7	2
2	4	4
3	2	8

Assume that the discount rate for both projects is 10%.

We first consider these two projects as independent projects and then as mutually exclusive projects in the numerical illustration of each capital budgeting method.

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The Net Present Value (NPV) Method (Ref: Section 5.1)

- For a project, its net present value is defined as: Net Present Value (NPV) = Total PV of future CF's − Initial Investment (CF_0)

  $\mathbf{NPV = \sum^{n}_{t=1} CF_t/(1+r)^t - CF_0}$

Decision Rules

- **Minimum Acceptance Criteria:** Accept if NPV > 0 (for independent projects)
- **Ranking Criteria:** Choose the highest NPV > 0 (for mutually exclusive projects)

Reinvestment assumption: the NPV method assumes that all expected cash flows can be reinvested at the appropriate discount rate, i.e., the opportunity cost of capital.

- The NPV method directly measures the contribution of the project to the value of the firm.

Evaluation

A: YES B: YES C: YES D: YES

The NPV method is conceptually the most correct capital budgeting method!

Example

Project S

$NPV_S = -\$9M + \$7M/1.10 + \$4M/1.10^2 + \$2M/1.10^3 = \$2.17M$

Project L

$NPV_L = \$2.13M$ (check it out yourself!)

Accept both S and L if they are independent projects because both have positive NPV! Accept S if they are mutually exclusive projects because S can add more value to the firm than L.

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The 'Pure' Payback Period (PP) Method (Ref: Section 5.2)

The 'Pure' Payback Period:

- The (pure) payback period of a project is defined as the length of time its future cash flows take to recover its initial investment. It is computed by adding the cash flows up until the accumulated cash flows sum up to zero, i.e., summing up expected future cash flows (CF_t) until they equal to the initial investment (CF_0).

  Find T where:

  $\sum_{t=0}^{T} CF_t = 0$
- The payback period is a liquidity measure of the project – how fast can we recover our investment?

Decision Rules

- **Minimum Acceptance Criteria:** Accept if its payback period is shorter than the arbitrarily specified cutoff horizon (for independent projects)
- **Ranking Criteria:** Choose the shortest payback period if it is shorter than the arbitrarily specified cutoff horizon (for mutually exclusive projects)

Evaluation

A: NO B: NO C: NO D: NO

Disadvantages

- Ignores the time value of money
- Ignores cash flows after the payback period
- Biased against long-term projects
- Requires an arbitrary acceptance criteria
- A project accepted based on the payback criteria may not have a positive NPV

Advantages

- Easy to understand
- Biased toward liquidity

Example

Project S

t=	0	1	2	3
CF_t=	-9	7	4	2
Cum. CF_t=	-9	-2	2	

Since the cumulative cash flow is positive by the end of Year 2, implying that the payback period is shorter than two years. We determine the approximate timing of payback by dividing the deficit amount at the beginning of the period when payback occurs (i.e., \$2M) by the amount of cash flow for that period (i.e., \$4M). Hence, the payback period of Project S is:

1 + \$2M/\$4M = 1.5 years.

Project L

The payback period is:
1 + 1 + \$3M/\$8M = 2.375 years. (check it out!)

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The Discounted Payback Period (DPP) Method (Ref: Section 5.3)

The Discounted Payback Period (DPP) Method is:

- The discounted payback period of a project is computed by adding the project's discounted cash flows up until the accumulated discounted cash flows sum up to zero, i.e., summing up the present values of expected future cash flows ($CF_t/(1+r)_t$) until they equal to the initial investment (CF_0).

  Find T where:

  $\sum^{T}_{t=0} CF_t/(1+r)^t = 0$

- The discounted payback period method is a refined version of the (pure) payback period method because it takes into consideration the timing and risk of cash flows.

Decision Rules

- **Minimum Acceptance Criteria:** Accept if its discounted payback period is shorter than the arbitrarily specified cutoff horizon (for independent projects)
- **Ranking Criteria:** Choose the shortest discounted payback period if it is shorter than the arbitrarily specified cutoff horizon (for mutually exclusive projects)

Evaluation

A: NO B: YES C: YES D: NO

Example

Project S

t=	0	1	2	3
$PV(CF_t)$=	-9	6.36	3.31	
$Cum.PVCF_t$=	-9	-2.64	0.67	

Since the cumulative discounted cash flow is positive by the end of Year 2, implying that the discounted payback period is shorter than two years. We determine the approximate timing of discounted payback by dividing the deficit amount at the beginning of the period when payback occurs ($2.64M), by the amount of discounted cash flow for that period (i.e., $3.31M). Hence, the discounted payback period of Project S is:

1 + $2.64M/$3.31M = **1.80 years**

Project L

The discounted payback period is:

1 + 1 + $3.88M/$6.01M = **2.65 years** (check it out!)

Note: In Chapter 4, we learned that the present value of a future cash flow is usually lower than its face value. Thus, it is not surprising that the discounted payback period of a project is typically longer than its payback period!

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The Average Accounting Return (AAR) Method

The Average Accounting Return (AAR) Method is:

- The average accounting return of an investment is defined as its average net income, NI, divided by its average book value, BV.

  i.e., AAR = $[\sum^{n}_{t=1} NI_t/n] / [(BV_0 + BV_n)/2]$

- This method measures the return on asset (ROA) of the investment.

Decision Rules

- **Minimum Acceptance Criteria:** Accept if AAR > the arbitrarily specified level (for independent projects).
- **Ranking Criteria:** Choose the highest AAR > the arbitrarily specified level (for mutually exclusive projects).

Evaluation

A: NO   B: NO   C: NO   D: NO

Disadvantages

- Ignores the time value of money.
- Uses an arbitrary benchmark cutoff rate.
- Based on accounting earnings and book values, not cash flows and market values.
- A project accepted based on the AAR criteria may not have a positive NPV.

Advantages

- The accounting information is usually available.
- Easy to calculate.

Example

- In ACC 505, you learned that cash flow = net income + depreciation expense. Given the cash flow and depreciation information, we can calculate the net income by using NI = CF − Dep.
- For simplification, we depreciate the assets of both projects with the straight-line method with no salvage value at the end of the project.
  - Annual depreciation expense, Dep = \$9M / 3 = \$3M
  - The average investment or book value is \$(9M + 0)/2 = \$4.5M

Project S

t =	1	2	3
CF =	7	4	2
NI =	4	1	-1

--> Average net income = \$(4M + 1M − 1M) / 3 = \$1.33M
--> AAR_S = \$1.33M / \$4.5M = **29.6%**

Project L

AAR_L = \$1.67M / \$4.5M = 37.1% (check it out!)

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The Internal Rate of Return (IRR) Method (Ref: Section 5.4)

The Internal Rate of Return Method:

- The internal rate of return of a project is defined as the discount rate that equates the discounted value of its expected cash flows with its initial investment.
  - In other words, the IRR is the discount rate that makes the 'NPV' equal to zero.

Find IRR such that $\sum^{n}_{t=1} CF_t/(1+IRR)^t = CF_0$ (→ "NPV"=0!!!)

- Reinvestment assumption: the IRR method assumes that all expected cash flows can be reinvested at the same rate as the internal rate return of the project.
- The IRR measures the (expected) rate of return, or discounted cash flow return, on the project.

Decision Rules

- **Minimum Acceptance Criteria:** Accept if IRR > r (for independent projects)
- **Ranking Criteria:** Choose the highest IRR > r (for mutually exclusive projects)

Evaluation

A: YES B: YES C: NO D: NO

Reinvestment Assumption:

- All future cash flows assumed reinvested at the IRR.

Disadvantages:

- Does not distinguish between investing and borrowing.
- IRR may not exist or there may be multiple IRR.
- Problems with mutually exclusive investments.
  - Its ranking of projects does NOT guarantee the selection of the most value creating alternative.

Advantages:

- Easy to understand and communicate.

Example

Project S

Using the CF key on the financial calculator, we input CF = -9; C01 = 7; C02 = 4 and C03 = 2. Then, we press the IRR key and compute (by pressing the CPT key), → IRR_S = **26.7(%)**.

Project L

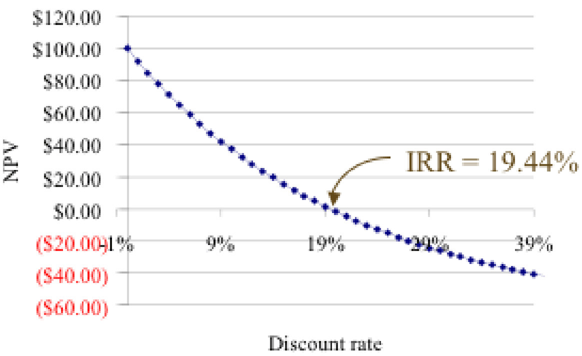
IRR_L = **20.4%** (check it out)

Accept both S and L if they are independent projects because both have IRR > r! Accept S if they are mutually exclusive projects because S has a higher IRR than L.

Note that, without the assistance of the financial calculator, we estimate the IRR by the trial and error method and the NPV payoff profile approach.

The NPV payoff profile is the graph that plots the NPV of a project against the discount rate. The IRR of the project is the x-intercept term of the NPV payoff profile.

Discount Rate	NPV
0%	$100.00
4%	$71.04
8%	$47.32
12%	$27.79
16%	$11.65
20%	($1.74)
24%	($12.88)
28%	($22.17)
32%	($29.93)
36%	($36.43)
40%	($41.86)



Despite that the IRR method is widely used in practice, and its framework is similar to that of the NPV method, there are several issues concerning this method that we need to understand in order to use this popular method correctly in investment decision making.

Problems with the IRR Approach

- Multiple IRRs
  - A project may have multiple IRRs when its cash flows change sign more than once in its entire life.
    - One IRR for each change in sign!
- Are We Borrowing or Lending?
  - For Borrowing, accept a project if IRR < discount rate
- The Scale Problem
  - In favor of smaller projects
- The Timing Problem
  - In favor of projects that have most of their cash inflows coming in early, i.e. 'earlier' projects

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Problems with the Internal Rate of Return (IRR) Method (Ref: Section 5.5)

The economic reason underlying the problems of the IRR method is its **faulty reinvestment rate assumption (i.e., its own IRR)**, which is based on an unrealistic condition that there is an abundant supply of projects with the same profitability as the one being considered. This leads to systematic biases in selecting among mutually exclusive projects when the following situations apply:

The Scale Problem

The IRR method biases in favor of smaller projects, i.e., projects with smaller initial investments, when choosing among mutually exclusive projects.

- When using the IRR decision rule to choose among mutually exclusive projects –
- Would you rather make 100% or 50% on your investments?
- What if the 100% return is on a $1 investment while the 50% return is on a $1,000 investment?

Example

t=	0	1	2
Small (S): CF =	-$100	$140	$140
Large (L): CF =	-$1,000	$1,250	$1,250

--> IRR_S = 107.48% versus IRR_L = 90.59%

The IRR method leads us to select S over L due to its higher IRR. However, with r=15%, we find that project L (NPV = $1,032.14) adds more value to the firm than project S (NPV = $127.60). As such, we should select the large project according to the NPV method in order to maximize firm value.

Incremental Project Analysis

We can resolve the problem of the IRR method in choosing among mutually exclusive projects by applying the **Incremental Project Analysis -**

We first construct an incremental project by subtracting the cash flows stream of project S from that of project L. Thus, cash flows for the incremental project (L − S) are:

t=	0	1	2
CF =	-$900	-$1,110	-$1,110

Then, we compute the IRR of this incremental project, IRR=88.70%, and apply the IRR method to decide whether we should accept this incremental project. In this example, the IRR of the incremental project is greater than the discount rate, implying that this incremental project should be accepted. As such, the IRR method on the incremental project indicates that we should switch from accepting Project SMALL to accepting Project LARGE. This is consistent with the decision based on the NPV method, and hence resolving the conflicting recommendation of the IRR method.

The Timing Problem

The IRR method biases in favor of 'shorter' projects, i.e., projects that have major cash inflows during the earlier stage of their lives, when choosing among mutually exclusive projects.

Example (Slide)

t=	0	1	2	3
'Short' (S): CF =	-$10,000	$10,000	$1,000	$1,000
'Long' (L): CF =	-$10,000	$1,000	$1,000	$12,000

--> IRR_S = 16.04% versus IRR_L = 12.94%

The IRR method leads us to select project S over project L due to its higher IRR. However, with r=10%, we find that project L (NPV = $751.31) adds more value to the firm than project S (NPV = $668.68). As such, we should select project L according to the NPV method in order to maximize firm value.

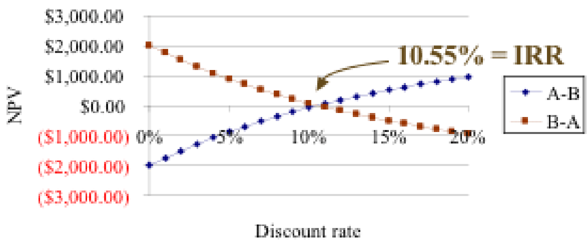
Similar to the scale problem, we can also resolve the above conflicting recommendation problem by applying the Incremental Project Analysis.(Slide)*

We first construct an incremental project by subtracting the cash flows stream of project S from that of project L. Thus, cash flows for the incremental project (L – S) are:

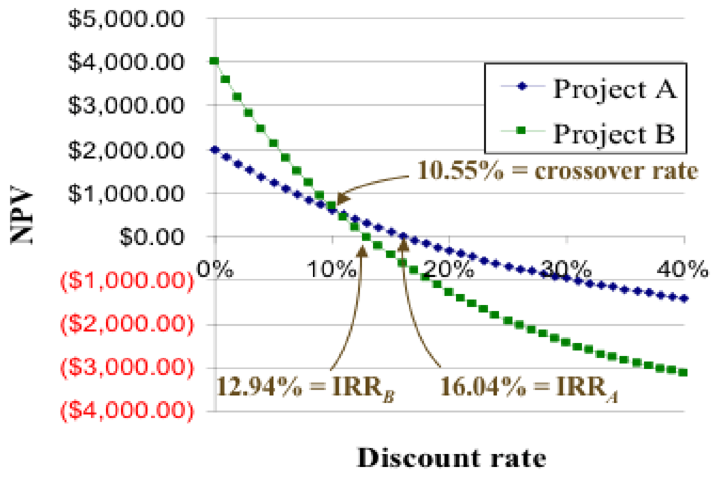
t=	0	1	2	3
CF =	0	-$9,000	$0	$11,000

Then, we compute the IRR of this incremental project, IRR=10.50%, which is greater than the discount rate, implying that this incremental project should be accepted. As such, the IRR method on the incremental project indicates that we should switch from accepting project S to accepting project L. This is again consistent with the decision based on the NPV method.

Year	Project A	Project B	Project A-B	Project B-A
0	($10,000)	($10,000)	$0	$0
1	$10,000	$1,000	$9,000	($9,000)
2	$1,000	$1,000	$0	$0
3	$1,000	$12,000	($11,000)	$11,000



By constructing the NPV payoff profiles of these two projects, we notice that the IRR of the incremental project is also the discount rate at which the two payoff profiles crossover. As such, the IRR of the incremental project is also called the 'crossover' rate.



If we examine the NPV payoff profiles closely, we notice that the IRR method will agree with the NPV method in selecting mutually exclusive projects if the discount rate is greater than the crossover rate. This can be illustrated by this example:

Example

Here are the net cash flows (in millions) of two projects:

Expected after-tax net cash flows

Year (t)	Project S	Project L
0	(9)	(9)
1	7	2
2	4	4
3	2	8

Assume that the discount rate for both projects is 10%.

Determine whether one or both of these two projects are acceptable, and which one should be accepted, according to the various decision rules.

The crossover rate for those two projects is 9.54%, which is less than the discount rate of 10%. As such, we should reject the incremental project and stay with project S, as the NPV method recommends. On the other hand, if the discount rate is less than the crossover rate, the IRR method will mislead our investment-decision making like the one in the example of 'short' versus 'long' projects - crossover rate (10.5%) > discount rate (10%); or the example of 'small' versus 'large' projects – crossover rate (88.7%) > discount rate (10%).

NOTE that when applying the Incremental Project Analysis to resolve the scale or timing problem of the IRR method, you are advised to construct a 'lending/investing' type of project, i.e., the first nonzero cash flow of the incremental project has to be negative! This can avoid one from swapping one problem of the IRR method, i.e., the scale or timing problem, for another problem, i.e., the 'lending/investing' versus 'borrowing/financing' project. Recall that a project is of the 'borrowing/financing' type if its first nonzero cash flow is positive, implying that it generates cash inflow upfront such as borrowing a loan. For the 'borrowing/financing' type of investments, one should accept such project if its IRR is less than the discount rate, i.e., IRR < r, implying that the cost of borrowing implied in the concerned project is lower than the alternatives!

The Modified IRR (MIRR) Method

The Modified IRR method offers an alternate approach that resolves the problem of the IRR method in choosing among mutually exclusive projects with differences in timing of cash flows. Recall that the IRR method applies the faulty and unrealistic reinvestment rate on future cash flows in calculating a project's internal rate of return (IRR).

According to the McKinsey's article on the fallacy of IRR, the Modified IRR (MIRR) method addresses IRR's faulty reinvestment rate assumption problem with a 2-step procedure. **First, we compute the future value of each future cash flow (i.e., excluding the initial cash outlay) with the appropriate discount rate, i.e., the cost of capital, as the reinvestment rate. Second, we solve for the "discount rate" that equates the present value of the total value, which is defined as the sum of future values of all non-initial cash flows at the end of the project's life, with the initial cash outlay of the project. And this "discount rate" is the Modified IRR of the project!**

Numerical Illustration - Solving the Timing Problem with the Modified IRR (MIRR) Method

- First, we calculate the total value of Project S's non-initial cash flows as $10000*1.1^2 + $1000*1.1 + $1000 = $14200 at t=3.
- Second, we find the "discount rate" that equates the present value of Project S's total value, i.e., $14,200 (at t=3), with its initial cash flow of $10,000 (at t=0). In other words, Project S's MIRR = ($14,200/$10,000)^(1/3) - 1 = 12.40%.

By applying the same 2-step procedure to Project L, its MIRR will be 12.69%. (Check it out!) Since Project L's MIRR is higher than that of Project S, we will choose Project L over Project S according to the Modified IRR method, given that both MIRRs are higher than the discount rate of 10%. And this recommendation is consistent with that of the NPV method. Hence, the Modified IRR method is useful in choosing among mutually exclusive projects with the timing problem!

While it is not discussed here, the Modified IRR method, similar to the IRR method, also provides biased recommendation in favor of smaller projects when choosing among mutually exclusive projects with different scales.

The Multiple IRRs Problem

- Another type of problem associated with the IRR method is due to its mathematical characteristic – there are as many solutions to an IRR problem as there are changes in sign in the cash flow stream. For example, a multi-phase project may have two cash outlays such that it will have two IRRs.
- NOTE that a change in sign of cash flows is defined as either from a positive cash flow this year to a negative cash flow next year, or from a negative cash flow this year to a positive cash flow next year. In other words, a positive or negative cash flow that is followed by zero cash flow is NOT considered as a change in sign.

Example

Consider the cash flow stream [-$100,+$230,-$132] for years 0, 1, and 2.

$$0 = -\$100 + \$230\,[1/(1 + IRR)] - \$132\,[1/(1 + IRR)]^2$$

This is a simple quadratic equation. By replacing [1/(1 + IRR)] with x, we can restate the equation as:

$$0 = a + bx + cx^2$$
$$0 = -100 + 230x - 132x^2$$

The roots of a quadratic equation are:

$x = [-b +/- (b^2 - 4ac)]^{(1/2)} / 2a$
Hence x = 1.1 and 1.2 → IRR = 10% and 20%.

In fact, the IRR solution can contain imaginary ($i = -1^{(1/2)}$) roots or no roots at all. For example, the cash flow stream [+$1,+$2,+$3] has no internal rate of return.

In practice, this type of IRR problem may not be detrimental because many project cash flows change sign only once (negative cash flow for initial investment and positive cash flows thereafter).

Numerical Illustration - Solving Multiple IRRs Problem with the Modified IRR (MIRR) Method

- Consider a project that has the following cash flow stream [-$100,+$150,-$200,+$300] for years 0, 1, 2 and 3. Let us assume that the appropriate discount rate for this project is 10%. Hence,
- First, we calculate the total value of the project's non-initial cash flows as $150*1.1^2 + (-$200)*1.1 + $300 = $261.50 at t=3.
- Second, we find the "discount rate" that equates the present value of the project's total value, i.e., $261.50 (at t=3), with its initial cash flow of $100 (at t=0). In other words, MIRR = ($261.50/$100)^(1/3) - 1 = 37.78%. And we accept this project because its MIRR is higher than its discount rate of 10%!

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The Profitability Index (PI) Method (Ref: Section 5.6)

The Profitability Index (PI) Method:

- The profitability index of a project is defined as the ratio of the PV of the expected cash flows, i.e., intrinsic value, to the initial investment, of the project.

  $PI = \sum^{n}_{t=1} CF_t(1+r)^{-t} / CF_0$ OR $PI=(NPV/CF_0)+1$

Note: We only use the amount of CF_0, i.e., ignoring its sign, in computing PI!

- The profitability index is the benefit-cost ratio of the project – what is the bang for the buck of the project?

Decision Rules

- **Minimum Acceptance Criteria:** Accept if PI > 1 (for independent projects)
- **Ranking Criteria:** Choose the highest PI > 1 (for mutually exclusive projects)

Evaluation

A: YES B: YES C: YES D: NO

The PI method also biases in favor of smaller projects when choosing among mutually exclusive projects.

Disadvantages

- Problems with mutually exclusive investments.

Advantages

- May be useful with limited capital for investment.
- Easy to understand and communicate.
- Correct decision when evaluating independent projects.

Example

Project S

$PI_S = (NPV/CF_0) + 1 = \$(2.17M/9.0M) + 1 = \mathbf{1.2411}$

Project L

$PI_L = \$(2.14M/9.0M) + 1 = \mathbf{1.2367}$

Accept both S and L if they are independent projects because both have PI > 1! Accept S if they are mutually exclusive projects because S has a higher PI than L.

Reference the text for the discussion on the basic idea of how one can use the profitability index method in addressing the capital rationing situation commonly faced in practice. Note that mathematically programming may be needed when the PI method is applied to address the capital rationing situation that involves numerous investment alternatives.

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