Overview and Learning Objectives

Overview

This chapter explains the concept and algebra of the time value of money (TVM) and net present value (NPV). NPV depends upon the **size, timing, and riskiness of expected cash flows**. In order to focus on the time value of money, we postpone the discussion on the relationship between risk and discount rate till a later time.

There are three ways to solve TVM problems: with a financial calculator, with formulas, and with time value factor tables. We use both the formulas and financial calculator approaches in our discussion. We encourage students to use financial calculators during examinations. However, a good understanding of the formulas is necessary to value more complex cash flow streams. The financial calculator approach is based on the Texas Instrument Business Analyst II Plus (TI BA II Plus) model.

Learning Objectives

After reading course materials on this chapter, students should be able to:

- Explain clearly the basic concept of TVM and its implications on solving TVM problems.
- Describe the basics of the TVM relationship.
- Apply the Opportunity Cost Principle in determining the appropriate discount rate in present value (PV) calculations.
- Identify different cash flows patterns.
- Identify and solve for the output variable in TVM problems with various cash flows patterns, using the formula and/or financial calculator approaches.
- Distinguish between the properties of present value and future value.
- Explain the difference between simple and compound interests.
- Calculate different types of interest rates and how they are different.
- Choose the right type of interest rates for solving TVM problems based on different periods and interest rates.
- Explain how the NPV rule relates to the firm value in the context of the objective of a firm, and the related calculations.

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Time Value of Money (TVM): Conceptual Basics

Basic Concept of TVM

A dollar in the future does not have the same value as a dollar today. In specific, a future dollar is less valuable than a today dollar. **Why?**

- Inflation erodes the purchasing power of future dollars such that a future dollar has a smaller claim on resources than a today dollar.
- By putting a today dollar into an investment account that generates interest, the today dollar can grow into a larger sum in the future and hence more valuable than a future dollar.
- A future dollar may be uncertain while a today dollar is certain.

Implications on TVM Calculations

- **Cash flows that are located at different time points on a time line have different time values of money, and hence they are different commodities. As such, they should NOT be added together or subtracted from one another *horizontally*. On the other hand, we can add or subtract cash flows *vertically* at any time point on a time line because they have the same time value of money.**
- This is a critical rule that we have to follow in any TVM calculations.

Intuitions of TVM Calculations

- **Future value (FV)** calculations, the **compounding** process, move the cash flow(s) **from the present into the future**, i.e., moving them **from left to right** on the time line.
- **Present value (PV)** calculations, the **discounting** process, bring the cash flow(s) **back from the future**, i.e., moving them from **right to left** on the time line.
- FV and PV calculations are reciprocal relationships.
- Using PV and/or FV calculations, we can bring any cash flow(s) to any specific time point on the time line such that they all have the same TVM and can be added together or subtracted from one another **when they are at the SAME time point**.

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Time Value of Money (TVM) Calculations: Basics with Single Cash Flow & Simple Nominal Interest Rate (Ref: Sections 4.1, 4.2 & 4.6)

The TVM Relationship

$FV = C_0 * (1 + r)^T$

Where:

> **r** is the interest rate per period.
> **T** is the duration of the holding horizon.
> C_0 (also called PV) is the value of the cash flow at period 0 (beginning of horizon)
> FV (also called C_T) is the value of the cash flow at period T (end of horizon)

The above TVM relationship can be generalized as:

> $C_{t+T} = C_t * (1 + r)^T$

Where:

> C_t and C_{t+T} are, respectively, the values of the cash flow at period t (beginning of horizon) and at period T+t (end of horizon).

There are four variables, FV, PV, r and T, in a single cash flow problem. We will discuss solving for each of them using both the formula and financial calculator approaches.

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Solving for Future Value (FV or C_T or C_{t+T}) and Compounding

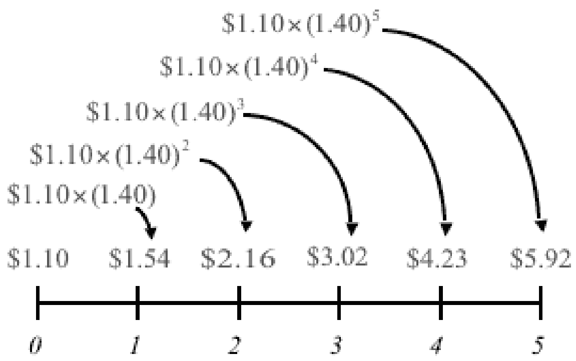
In general, FV is the value of cash flow, C_0 or C_t, after investing (or growing) over the holding horizon, T, ***compounding** at **a given** interest (or growth) rate, r.*

Let us consider this example - Suppose Jay Ritter invested in an investment fund managed by the Modigliani Company. Ritter invests $1.10 in the Modigliani Fund, which is expected to grow at 40% annually for the next five years. What will the value of Ritter's investment be in five years?

Formula Approach

FV (or C_T) = C_0 * (1 + r)^T

$5.92 = $1.10 * (1.40)^5



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Solving for Future Value (FV or C_T or C_{t+T}) and Compounding (cont'd.)

Financial Calculator Approach

'N' key: the length of the holding horizon for single cash flow problems, T.
- In this example, input N = 5 for the 5-year holding horizon.

'I/Y' key: the interest (or growth) rate, r, in the percentage format.
- In this example, input I/Y = 40 for the annual return of 40%.

'PV' key: the present value of the cash flow, PV or C_0 or C_t.
- In this example, input PV = -1.10 for the original investment of $1.10.

'PMT' key: the fixed amount payment in an annuity, PMT.
- For single cash flow problems, always input PMT = 0.

'FV' key: the future value of the cash flow, FV or C_T or C_{t+T}.
- In this example, we COMPUTE (CPT) FV → 5.92, the output variable.

Note that the sign of the inputs for the PV, PMT and FV keys only denotes whether the corresponding variable is a cash outflow, i.e., with the minus (-) sign, or a cash inflow. It is the magnitude that represents the amount!

When examining this example closely, we notice that the value of Ritter's investment in year five, $5.92, is considerably higher than the sum of the original investment plus five increases of 40% on the original investment of $1.10:

$5.92 > $(1.10 + 5 * (1.10 * 0.40)) = $3.30

This is due to compounding, i.e., the process of investing or growing beyond one compounding period and hence earning interest on (previously earned) interest.

In this example, the total simple interest earned on the original investment of $1.10 over the 5-year holding horizon is 5*(1.10*0.40)) = $2.20. The difference between the ending value and the sum of the original investment and the total simple interest earned is:

$5.92 − $(1.10 + 5 * (1.10 * 0.40)) = $2.62

which represents the compound interest or interest on interest. That is, the $2.62 represents the amount of interest being earned on the previously earned interest. Note that in this example, the interest on interest component, $2.62, is larger than the total simple interest component, $2.20, even though the horizon is only 5 years. This illustrates "the power of compound interest", and how we can use the money to work for us.

Properties of Future Value (FV)

Other factors being equal:

- **FV increases as the interest rate (r) increases.**
  - By interpreting the interest rate as the speed we drive, the faster we drive, the farther we can go.

- **FV increases as the holding horizon (T) lengthens.**
  - By interpreting the holding horizon as the time we spend on driving, the longer time we drive, the farther we can go.

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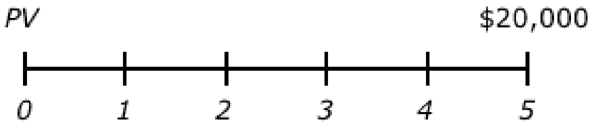
Solving for Present Value (PV or C_0 or C_t) and Discounting

In general, PV is the beginning value of a future cash flow, C_{t+T} or C_T, to be received at the end of the horizon, T, ***discounting*** *at* ***the appropriate discount*** *(or interest) rate, r*.

It is important to note that in PV calculations, the choice of the discount rate should be determined according to the **Opportunity Cost Principle**, which states that:

The appropriate discount rate should be the rate of return on the best available alternative investment, i.e., the highest return, of equal risk.

Let us consider this example:
How much would an investor have to set aside today in order to have $20,000 five years from now if the current rate is 15%?



Formula Approach

PV (or C_0) = C_T / $(1 + r)^T$ = C_T*$(1 + r)^{-T}$

$9,943.53 = $20,000 / $(1.15)^5$= $20,000 * $(1.15)^{-5}$

The investor has to set aside $9,943.53 today to have $20,000 in five years.

Financial Calculator Approach

- input N = 5 for the 5-year holding horizon.

- input I/Y = 15 for the discount rate of 15%.

- input FV = 20,000 for the future amount of $20,000.

- For single cash flow problems, always input PMT = 0.

- COMPUTE (CPT) PV → 9,943.53, the output variable.

Properties of Present Value (PV)

Other factors being equal:

- **PV decreases as the interest rate (r) increases.**
  - The higher the interest rate the investment account offers, the less money we need to put in this account today to get the same amount in the future.

- **PV decreases as the holding horizon (T) lengthens.**
  - The longer time we have to grow our money, the less money we need to put down today to get the same amount in the future.

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Solving for the interest rate (r)

Consider this example:
Assume the total cost of a college education will be $50,000 when your child enters college in 12 years. You have $5,000 to invest today. What rate of interest must you earn on your investment to cover the cost of your child's education?

Formula Approach

$$\mathbf{r = (C_{t+T} / C_t)^{1/T} - 1}$$

$$r = (\$50{,}000 / \$5{,}000)^{1/12} - 1$$

r = **0.2115** = 21.15% per year

Financial Calculator Approach

- input N = 12 for the 12-year holding horizon

- input PV = -5,000 for the amount you put into the investment today

- input FV = 50,000 for the future amount of $50,000

- For single cash flow problems, always input PMT = 0

- COMPUTE (CPT) I/Y → 21.15, the output variable

Financial Management I (FIN 531)

Solving for the holding horizon (T)

Consider this example:
If we deposit $5,000 today in an account paying 10%, how long does it take to grow to $10,000?

Formula Approach

$$\mathbf{T = \ln(C_{t+T} / C_t) / \ln(1 + r)}$$

$$T = \ln(\$10{,}000/\$5{,}000) / \ln(1.10)$$

$T = 7.27 \rightarrow$ It takes 7.27 years for $5,000 to grow to $10,000.

Financial Calculator Approach

- input I/Y = 10 for the interest rate of 10%.

- input PV = -5,000 for the amount you put into the investment today.

- input FV = 10,000 for the future amount of $10,000.

- For single cash flow problems, always input PMT = 0.

- COMPUTE (CPT) N → 7.27; the output variable.

The Rule of 72

For doubling the investment value, r (in percentage format) * T ≈ 72

This is a handy formula that helps us to find the approximate length of holding horizon (T), given the interest rate (r), in order to double the value of the investment. Or the approximate interest rate needed over a specified horizon in order to double the value of the investment.

The previous example is about finding the length of horizon needed to double the investment, i.e., from $5,000 to $10,000, given an interest rate of 10%. By applying the rule of 72, the approximate value for T is 72/10 = 7.2 years, a close approximation to the 7.27 years according to the formula for solving for T.

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Net Present Value (NPV) Rule

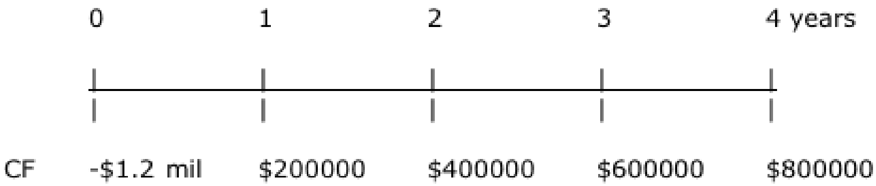
Consider this example:
The net present value (NPV) of an investment is the present value of the expected cash flows, less the costs of the investment. **According to the NPV rule, a positive NPV project will add value to the firm and hence should be accepted.**

The general formula for calculating NPV:

$$\mathbf{NPV = -C_0 + C_1 \div (1+r) + C_2 \div (1+r)^2 + .. + C_T \div (1+r)^T}$$

The initial cash flow (C_0) is typically an investment and is subtracted to compute the NPV. Note that the formula from the text assumes that the discount rate (r) is the same in each period. Even though this is often the case, the above formula can be modified for different discount rates across the horizon by allowing the discount rate variable, r, takes on different values for different periods.

Another Example:
You have an opportunity to invest in a business that will pay $200,000 in one year, $400,000 in two years, $600,000 in three years and $800,000 in four years. You can earn 12% per year compounded annually on a mutual fund that has similar risk. If it costs $1.2 million to start this business, should you invest?



The appropriate discount rate = 12%

$$NPV = -C_0 + C_1 \div (1+r) + C_2 \div (1+r)^2 + .. + C_T \div (1+r)^T$$

$$= -\$1{,}200{,}000 + \$200{,}000 \div (1.12)^1 + \$400{,}000 \div (1.12)^2 + \$600{,}000 \div (1.12)^3 + \$800{,}000 \div (1.12)^4$$
= **$232,931.59**

According to the NPV rule, we should accept this project and lead to an increase of firm value by $232,931.59.

Implication on the Value of a Firm (Ref: Section 4.6)

The value of a firm can be considered as the sum of the NPVs of all its investment projects. As such, the firm value is determined by the size, timing and riskiness of the cash flows associated with its investment projects.

Financial Management I (FIN 531)

Multiple Cash Flows Problems (Ref: Section 4.4)

As illustrated in the NPV example in the previous section, TVM applications may involve more than one cash flow. In fact, most TVM applications are multiple cash flows, rather than single cash flow, problems. By considering a multiple cash flows problem as a combination of many single cash flow problems, we can solve the multiple cash flows application by applying the appropriate formula discussed in section 4.3 to each cash flow component individually as illustrated in the NPV example.

A different set of keys is used on the financial calculator when solving multiple cash flows problems – the CF and NPV keys. We use the CF key to input all cash flow data and the NPV key to compute the present value of the multiple cash flows.

When inputting cash flows data, we treat each cash flow individually such that the frequency of each cash flow is always equal to one, i.e., F01 = F02 = …..= Fn = 1, the default value. We also need to include the minus (-) sign for cash outflows

After we finish inputting all cash flow data, we move to the NPV key and input the discount rate information to the 'I' variable. Then, we scroll down to the 'NPV' variable and press the CPT key to compute the present value of the multiple cash flows.

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Multiple Cash Flows Problems (cont'd.)

Unlike the five TVM keys (N; I/Y; PV; PMT and FV) that automatically register the value that we type in, the CF and NPV keys require us to press the ENTER key to register the input value. We know that the financial calculator has registered our input for the variables when we see an equal sign that links the input value to the variable.

Let us use the financial calculator to redo the NPV example in section 4.1.2.5 as an application of multiple cash flows problem. First, we press the 'CF' key to begin inputting the cash flows data:

CF_0 = - 1,200,000; C01 = 200,000 & F01 = 1; C02 = 400,000 & F02 = 1; C03 = 600,000 & F03 = 1; C04 = 800,000 & F04 = 1.

Afterwards, we scroll through the CF sheet to make sure that the input is complete and correct. Then, we press the 'NPV' key and enter I=12 for the appropriate discount rate of 12%. Now, we scroll down to NPV and press the 'CPT' key and the output should be **NPV = 232,931.59**.

Note that the CF and NPV keys help us to compute the present value of a multiple cash flows problem. If you want to compute the future value of this multiple cash flows problem, you can simply treat the output from this step as the present value of a single cash flow and then apply the formula or the financial calculator procedures for computing its future value as discussed in earlier section.

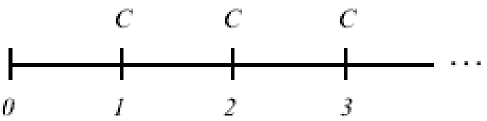
The above approach is practical for multiple cash flows problems with manageable horizons. For applications that have long horizons, it will be challenging to solve each cash flow individually. As such, consider the following categories of special simplified multiple cash flows patterns in the next few sections.

Financial Management I (FIN 531)

Perpetuity

Definition

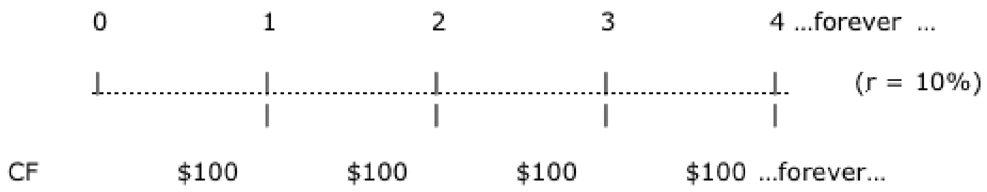
A stream of constant cash flows (C) that lasts forever.



$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots$$

Formulas

PV = C ÷ r
FV = ∞ (undefined)
C = PV * r



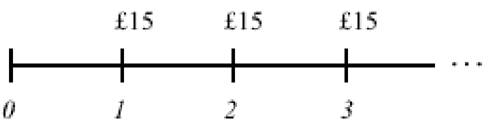
PV = $100 ÷ 0.1 = **$1,000**

This formula is rather intuitive. Each period a coupon (interest) payment equal to C = PV x r (e.g. $1,000 x 0.1 = $100) can be paid without reducing the PV (principal).

The British consol bond and the fixed rate preferred stock are examples of perpetuities.

What is the value of a British consol that promises to pay £15 each year, every year until the sun turns into a red giant and burns the planet to a crisp?

The interest rate is 10%.

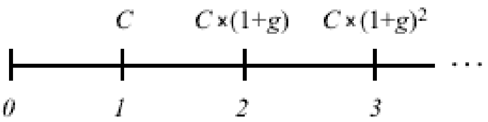


$$PV = \frac{£15}{.10} = £150$$

Growing Perpetuity

Definition

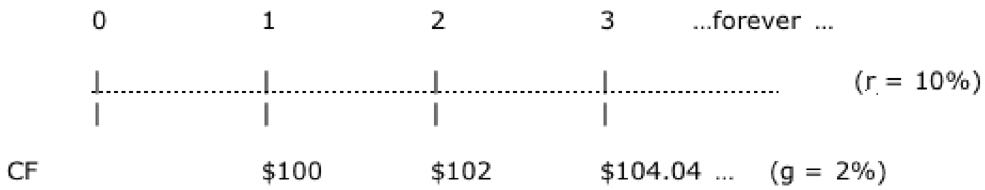
A stream of cash flows that grows at a constant rate (g) forever.



$$PV = \frac{C}{(1+r)} + \frac{C\times(1+g)}{(1+r)^2} + \frac{C\times(1+g)^2}{(1+r)^3} + \cdots$$

Formulas

$PV_t = C_{t+1} \div (r - g)$
FV = ∞ (undefined)



PV_0 = \$100 ÷ (0.10 - 0.02) = **\$1,250**

Note that:

$C_2 = C_1$ x (1+g) = \$100 x (1+0.02) = \$102
$C_3 = C_2$ x (1+g) = C_1 x $(1+g)^2$ = \$102 x (1+0.02) = \$100 x $(1+0.02)^2$ = \$104.04

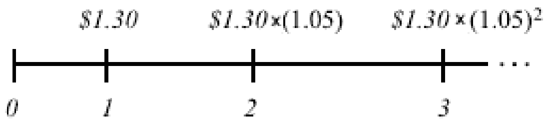
In general:

$C_{t+T} = C_t \times (1 + g)^T$

Common stocks of mature companies are examples of growing perpetuities.

The expected dividend next year is \$1.30 and dividends are expected to grow at 5% forever.

If the discount rate is 10%, what is the value of this promised dividend stream?



$$PV = \frac{\$1.30}{.10 - .05} = \$26.00$$

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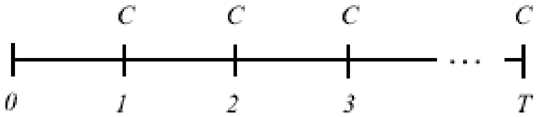
(Delayed or Ordinary) Annuity

Definition

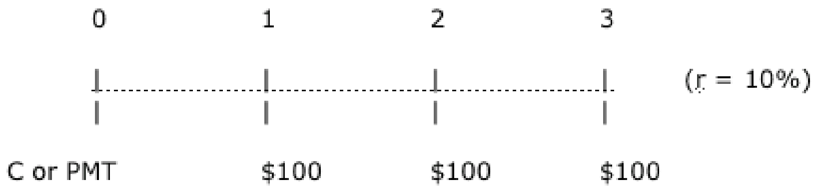
A stream of constant payments (PMT or C) that lasts for a finite horizon, i.e., a specified finite number of periods.



$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots \frac{C}{(1+r)^T}$$

Formulas

PV = C * [1 − (1/(1+r)^T)] / r
FV = C * [(1+r)^T − 1] / r
C = FV *r / [(1+r)^T − 1] or **C = PV *r / [1 − (1/(1+r)^T)]**



1. What is the present value (PV) of this 3-year annuity?

Formula Approach

PV = \$100 x [1 – (1 ÷ 1.1^3)] / 0.1 = **\$248.69**

Note that this is equivalent to summing up the present values of the three single cash flows in this 3-year annuity, i.e., PV = \$100/1.1 + \$100/1.1^2 + \$100/1.1^3 = \$248.69.

Financial Calculator Approach

'N' key: the number of payments for annuity problems, T.

- input N = 3 for the 3-year annuity.

input I/Y = 10 for the discount rate of 10%.

'PMT' key: the fixed amount payment in an annuity, PMT.

- input PMT = 100 for the annual constant payment of \$100.

'FV' key: the future value of the annuity, FV.

- For PV of an annuity problem, the payments take the place of the future value such that the input value for FV is always zero; input FV = 0.

'PV' key: the present value of the annuity, PV.

- COMPUTE (CPT) PV = -248.69; the output variable.

2. What is the future value (FV) of this 3-year annuity?

The Formula approach

FV = \$100 x [$1.1^3 - 1$] / 0.1 = **\$331**

Note that this is equivalent to summing up the future values of the three single cash flows in this 3-year annuity, i.e., FV = \$100*1.1 + \$100*1.1^2 + \$100*1.1^3 = **\$331**.

The Financial Calculator approach

input N = 3 for the 3-year annuity.

input I/Y = 10 for the interest rate of 10%.

'PV' key: the present value of the annuity, PV.

- For FV of an annuity problem, the payments take the place of the present value such that the input value for PV is always zero; input PV = 0.

input PMT = -100 for the annual constant deposit of \$100.

'FV' key: the future value of the annuity, FV.

- COMPUTE (CPT) FV = **331**; the output variable.

3. What is the annuity payment (PMT or C) for this 3-year annuity given its present value or its future value is known?

The Formula approach

C = \$248.69 × 0.1 / [$1 - (1 \div 1.1^3)$] = \$100, given PV!
OR C = \$331 × 0.1 / [$1.1^3 - 1$] = \$100, given FV!

The Financial Calculator approach

input N = 3 for the 3-year annuity.

input I/Y = 10 for the interest rate of 10%.

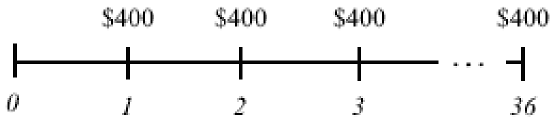
EITHER input PV = 0 and input FV = 331; if FV is given.

OR input FV = 0 and input PV = 248.69; if PV is given.

COMPUTE (CPT) PMT → -100; the output variable.

Installment loan payments and bonds are examples of annuities.

If you can afford a $400 monthly car payment, how much car can you afford if interest rates are 7% on 36-month loans?



$$PV = \frac{\$400}{.07/12}\left[1-\frac{1}{(1+.07/12)^{36}}\right] = \$12{,}954.59$$

Annuity Due or Annuity in Advance

The only difference between an annuity due and an (ordinary) annuity is the timing of the payment:

- For an (ordinary) annuity, payments are at the end of each period.
- **For an annuity due, payments are at the beginning of each period.**

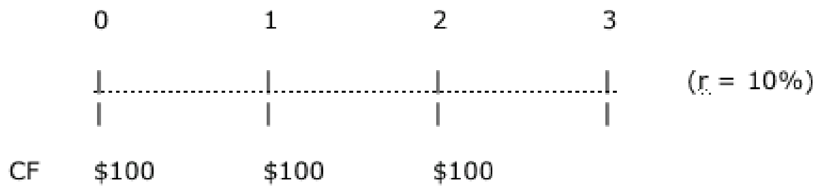
Note that for FV calculations, each payment in an annuity due has one additional period to earn interest, relative to the (ordinary) annuity. For PV calculations, each payment in an annuity due is discounted one less period than each (ordinary) annuity payment. These lead to the same adjustment factor, (1+r), to be applied to both FV and PV calculations when converting from an (ordinary) annuity into an annuity due.

Formulas

PV of an annuity due = PV of an (ordinary) annuity * (1+r)
= C * (1+r) * [1 −; (1/(1+r)^T)] / r

FV of an annuity d FV of an (ordinary) annuity * (1+r)
= C * (1+r) * [(1+r)^T − 1] / r

Payment of an annuity due = Payment of an (ordinary) annuity / (1+r)
i.e., C = (FV/(1+r)) *r / [(1+r)^T − 1]
OR C = (PV/(1+r)) *r / [1 − (1/(1+r)^T)]



Formula Approach

PV = $100 * (1.1) × [1 – (1 ÷ 1.1^3)] / 0.1 = **$273.55**
FV = $100 * (1.1) × [$1.1^3$ − 1] / 0.1 = **$364.10**
C = ($248.68/1.1) × 0.1 / [1 – (1 ÷ 1.1^3)] = **$90.91**, given PV!
OR C = ($331/1.1) × 0.1 / [$1.1^3$ − 1] = **$90.91**, given FV!

Financial Calculator Approach

We first set the financial calculator from its default END mode to the BGN mode, then we follow the respective procedures for an (ordinary) annuity to calculate PV, FV or PMT.

For switching from the END mode to the BGN mode, we first press the 2nd key and then the BGN key. Then, we press the 2nd key and then the SET key. We should see a big BGN as well as a small BGN, indicating that the financial calculator is now in the BGN mode. Now, we press the 2nd key and then the QUIT key. We should still see the small BGN symbol.

As a good practice, you should always switch back to the END mode once you finish an annuity due problem.

With the financial calculator in the BEG mode, we compute:

PV: Input N = 3; I/Y = 10; PMT = -100; FV = 0; → CPT PV = 273.55
FV: Input N = 3; I/Y = 10; PMT = -100; PV = 0; → CPT FV = 364.10
PMT: Input N = 3; I/Y = 10; PV = 248.69; FV = 0;
OR: Input N = 3; I/Y = 10; PV = 0; FV = 331.00;
→ CPT PMT = -90.91

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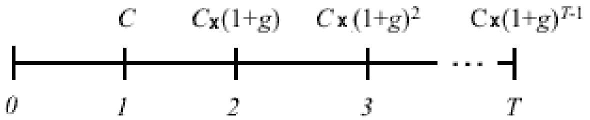
Growing Annuity

Definition

A stream of cash flows that grows at a constant rate (g) for a finite horizon, i.e., a specified finite number of periods.



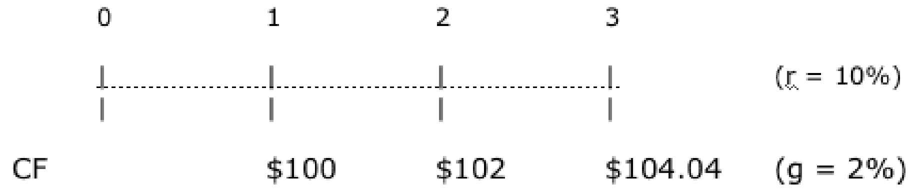
$$PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \cdots + \frac{C \times (1+g)^{T-1}}{(1+r)^T}$$

Formulas

$PV_t = C_{t+1} * [1 - ((1+g)/(1+r))^T] / (r-g)$; OR

$PV_t = C_{t+1} * [1 - ((1+g)^T/(1+r)^T)] / (r-g)$

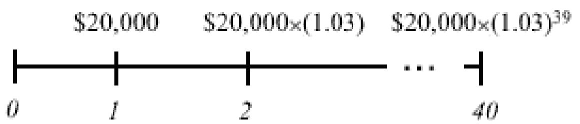
$FV_{t+T} = C_{t+1} * [(1+r)^T - (1+g)^T] / (r-g)$



$PV_0 = \$100 \times [1 - ((1+0.02) \div (1+0.10))^3] / (0.10-0.02) = \$253.37$

$FV_3 = \$100 \times [(1+0.10)^3 - (1+0.02)^3] / (0.10-0.02) = \$337.24$

Another example:
A defined-benefit retirement plan offers to pay $20,000 per year for 40 years and increase the annual payment by 3% each year. What is the present value at retirement if the discount rate is 10%?



$$PV = \frac{\$20{,}000}{.10 - .03}\left[1 - \left(\frac{1.03}{1.10}\right)^{40}\right] = \$265{,}121.57$$

Compounding Periods and Interest Rates (Ref: Section 4.3)

Remember that compounding refers to the process of interest being earned on the original principal amount and the previously earned interest. **The more frequently the compounding occurs, the faster your money grows. Here are two examples that compare compounding that occurs once per year (annually) with compounding that occurs twice per year (semi-annually).**

Suppose you invest $1,000 at 3% annual interest for five years. What amount will your money grow to with annual versus semi-annual compounding?

- **For annual compounding: You will have $1,000 * 1.03^5 = $1,159 in your account after five years at 3% interest with annual compounding.**
- **For semi-annual compounding: You will have $1,000 * 1.015^{10} = $1,161 in your account after five years at 3% interest with semi-annual compounding. With semi-annual compounding, the interest rate is 1.5% for a half-year period. The number of periods is ten because there are 10 half-year periods in five years.**

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Definitions for Periods and Interest Rates

Compounding (or Discounting) Period

- **The length of time to earn (or be charged with) interest once.**
  - Examples, a day; a week; a month; a quarter; a year, etc.

Compounding (or Discounting) Frequency, m

- **The number of compounding (or discounting) periods within a year.**
  - Examples, m=4 for quarterly compounding; m=52 for weekly compounding; m=365 for daily compounding; etc.

Payment (or Cash Flow) Period

- **The length of time to pay or receive a payment (or a cash flow).**
  - Examples, a day; a week; a month; a quarter; a year, etc.

Nominal Rate, i

- **The simple, quoted, stated or contracted annual interest rate.**
  - Example, the 3% in the previous examples!

Periodic Rate, PER

- **The simple interest rate for the compounding (or discounting) period.**
- Defined as nominal rate/compounding frequency, i.e., PER = i/m.
  - Example, the 1.5% (=3%/2) in the previous examples!

Effective Annual Rate, EAR

- **The annual rate of interest actually being earned or charged.**
- It takes into account the effect of earning or being charged with interest more than once per year.
- $EAR = (1 + i/m)^m - 1 = (1 + PER)^m - 1$

Effective Periodic Rate, EPR

- **The rate of interest actually being earned or charged during the payment (or cash flow) period.**
- It takes into account the effect of earning or being charged with interest more than once per payment (or cash flow) period.
- $EPR = (1 + i/m)^n - 1 = (1 + PER)^n - 1$
  - Where n is the number of compounding per payment period.

The table below shows the nominal rate (i), the periodic rate (PER), and the effective annual rate (EAR) for various compounding periods.

Nominal Rate, i	Compounding Period	Compounding Frequency, m	Periodic Rate, PER	Effective Annual Rate, EAR
10%	Annual	1	10%	10.00%
10%	Semi-annual	2	5%	10.25%
10%	Quarterly	4	2.5%	10.38%
10%	Monthly	12	0.83%	10.47%
10%	Daily	365	0.0274%	10.52%

e.g., for quarterly compounding period, EAR = $(1 + 0.10/4)^4 - 1 = 0.1038 = 10.38\%$

Observations

- For annual compounding, nominal rate = periodic rate = effective annual rate.
- The shorter the compounding period, the more frequent is compounding, i.e., more opportunities to earn interest on interest, and hence the higher the EAR.

Since the EAR measures the actual rate of return being earned or charged within the year, it should be used in comparing investment and financing alternatives. In the next example, we will look at how the EAR calculation can be used to compare loans with different compounding periods.

Marcus Smith, M.D. is comparing interest rates at several banks as he seeks financing. He wishes to borrow funds for one year. East Federal Bank offers to lend money at a 10% nominal rate of interest with semi-annual compounding. Bank Two typically calculates interest using quarterly compounding. Calculate the nominal Bank Two rate at which Dr. Smith would be indifferent between the loans at East Federal and Bank Two.

The East Federal's effective annual rate (EAR):

- $(1 + 0.10/2)^2 - 1 = 0.1025$ or 10.25%
- Calculate the Bank Two nominal rate of interest at which Dr. Smith would be indifferent between the two loans:
  - We use the EAR equation, with Bank Two's nominal rate as the unknown. The EAR we are aiming for is the EAR provided by East Federal Bank.
    - EAR = $(1 + i / 4)^4 - 1 = 0.1025$
    $\rightarrow (1 + i / 4) = 1.1025.25$
    $\rightarrow (i / 4) = .0247$
    $\rightarrow i = 4 * 0.0247 = 0.0988 = 9.88\%$

The nominal Bank Two rate at which Dr. Smith would be indifferent between the two loans is 9.88%.

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Rules for the Choice of Interest Rates

1. **For single cash flow problems, always use the EAR as the interest rate, r, in TVM calculations.**
2. For multiple cash flows problems:
   - **If the payment period matches the compounding (or discounting) period, use the PER as the interest rate.**
   - **If the payment period does not match the compounding (or discounting) period, use the EAR (for annual payment period) or the EPR (for non-annual payment period) as the interest rate.**
3. **For installment loan problems, use the PER as the interest rate.**
4. **Do NOT use the nominal rate directly as the interest rate, r, in TVM calculations.**

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More TVM Applications

When approaching TVM applications/problems, I would suggest the following series of questions to help you identify the nature of the problem and the procedures you should use in solving the problem.

- How many implicit and/or explicit parts are there in the problem?
- For each identified part
    - What is the cash flow pattern?
        - Single cash flow? General multiple cash flows? Annuity (delayed or due)? Growing annuity? Perpetuity? Growing perpetuity?
    - Is it in the PV or FV context?
    - What is the output variable?
    - Does the payment period match with the compounding (or discounting) period? Is its payment period annual? Is it an installment loan?

Then, work through each identified part one at a time and put all parts together and you are done.

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Distributions of Inheritance Trust Fund

Your grandparents left an inheritance of $5 millions in a trust fund that has a nominal annual rate of return of 9%, compounded daily. What is the maximum monthly amount that this trust fund can distribute such that it will last forever? Assume a 360-day year and a 30-day month in your calculations!

There is only one part in this problem – a perpetuity in the PV context. The output variable is the perpetual monthly payment. Since the monthly payment period does not match the daily discounting period, we should use the EPR in our calculations.

EPR = $(1 + 0.09/360)^{30} - 1 = 0.007527 = 0.7527\%$
→ C = $5,000,000 * 0.007527 = **$37,636.25 per month**.

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Value of Employment Contract

Jack Ferguson has signed a 3-year contract to work for a computer software company. He expects to receive a base salary of $5,000 a month and a bonus of $10,000 at year-end. All payments are made at the end of periods. What is the present value of the contract if the stated annual interest rate, compounded monthly, is 12 percent?

There are two parts in this problem:

- an (ordinary) annuity corresponding to the monthly salary
- an (ordinary) annuity corresponding to the annual bonus.

Both parts are in the PV context with the present value being the output variable.

For the monthly salary annuity, the monthly payment period matches with the monthly discounting period. As such, the PER will be used in our calculations.

> PER = 0.12/12 = 0.01 and N = 3 * 12 = 36 payments
> → PV = $5,000 * (1 – $(1+0.01)^{-36}$) / 0.01 = **$150,537.53**
> OR N = 3*12 = 36; I/Y = PER = 12/12 = 1; PMT = 5,000; FV = 0;
> → CPT PV = **-150,537.53**

For the annual bonus annuity, the annual payment period does not match with the monthly discounting period. As such, the EAR will be used in our calculations.

> EAR = (1 + 0.12/12)12 − 1 = 0.1268 and N = 3 * 1 = 3 payments
> → PV = $10,000 * (1 − $(1+0.1268)^{-3}$) / 0.1268 = **$23,740.42**
> OR N = 3*1 = 3; I/Y = EAR = $(1+12/12)^{12}$ − 1 = 12.68; PMT = 10,000; FV = 0; →
> CPT PV = **-23,740.42**

Hence, the present value of this contract is $(150,537.53 + 23,740.42) = $174,274.95

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Retirement Planning

You just turn 30 and start to plan for your retirement in 30 years. You recently open a 401k account that will generate an annual rate of return of 9%, compounded annually. You will make your first annual contribution of $8,000 at the end of the year. Your annual contribution is expected to grow with your salary at a long-term steady rate of 5%. How large will your nest egg be at your retirement?

There is only one component in this problem – a growing annuity in the FV context. The output variable is the FV. Since the annual payment period matches with the annual compounding period, the PER will be used in our calculations.

PER = 0.09/1 = 0.09; g = 0.05; T = 30; and C_{t+1} = $8,000
→ FV = $8,000 * $((1+0.09)^{30} - (1+0.05)^{30}) / (0.09 - 0.05)$ = **$1,789,147.22**

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Mortgage Loan Payment and Amortization (Section 4.5)

Mae Smith is planning to purchase a home for $250,000. She has a down payment of $50,000 and plans to borrow the rest. She can lock in a 30 year fixed mortgage rate of 7.125% (interest rate per year) today. Calculate the amount of her monthly payments.

There is only one part in this problem – an (ordinary) annuity in the PV context. This is an installment loan application with the monthly payment as the output variable.
Since this is an installment loan problem, we use the PER in our calculations,

PER = 7.125% / 12 = 0.5938 % and N = 30 * 12 = 360 payments
→ PMT = $200,000 * 0.005938 / [1 − (1 / (1.005938)360)] = **$1,347.52**
OR N = 30*12 = 360; I/Y = PER = 7.125%/12 = 0.5938; PV = 200,000; FV = 0;
→ CPT PMT = **-1,347.52**

Amortization Schedule

An amortization schedule is a chart of loan payments, with a breakdown of how much principal and interest is paid each period. The interest can be calculated by multiplying the interest rate by the remaining balance on the loan at the end of the most recent period. As the loan being gradually paid off, the interest amount decreases over the life of the loan. The principal paid off, i.e., principal repayment, in a specific period can be calculated by subtracting the interest for the period from the payment amount. As months and years go by, the interest component in each period gets smaller and the principal repayment component gets larger. The last loan payment is almost entirely principal repayment. The amortization schedule for Mae Smith's mortgage loan is shown below. By the end of the loan, the principal balance remaining is zero since the loan will be completely paid off.

Month	Payment	Interest	Principal	Principal Balance Remaining
1	1347.52	1187.60	159.92	199840.08
2	1347.52	1186.65	160.87	199679.21
3	1347.52	1185.70	161.82	199517.39
4	1347.52	1184.73	162.78	199354.61
.	1347.52	.	.	.
.	1347.52	.	.	.
360	1347.52	7.95	1339.57	0

Suppose Mae makes extra payments on her mortgage loan. Her neighbor tells her she is giving the bank interest free use of her money. Is he correct?

No, he is not correct. If Mae makes extra payments, this money will reduce her loan balance outstanding. She will be charged less interest, since she has paid back more of the money she borrowed. Her loan payment will stay the same, at $1,347.44 per month. However, her loan will be completely paid back early.

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