Chapter 9 Suggested Problems Solutions

11. This stock has a constant growth rate of dividends, but the required return changes twice. To find the value of the stock today, we will begin by finding the price of the stock at Year 6, when both the dividend growth rate and the required return are stable forever. The price of the stock in Year 6 will be the dividend in Year 7, divided by the required return minus the growth rate in dividends. So:
$P_6 = D_6 (1 + g) / (R – g) = D_0 (1 + g)^7 / (R – g) = \$3.10(1.06)^7 / (.11 – .06) = \$93.23$

Now we can find the price of the stock in Year 3. We need to find the price here since the required return changes at that time. The price of the stock in Year 3 is the PV of the dividends in Years 4, 5, and 6, plus the PV of the stock price in Year 6. The price of the stock in Year 3 is:
$P_3 = \$3.10(1.06)^4 / 1.13 + \$3.10(1.06)^5 / 1.13^2 + \$3.10(1.06)^6 / 1.13^3 + \$93.23 / 1.13^3 = \$74.37$

Finally, we can find the price of the stock today. The price today will be the PV of the dividends in Years 1, 2, and 3, plus the PV of the stock in Year 3. The price of the stock today is:
$P_0 = \$3.10(1.06) / 1.15 + \$3.10(1.06)^2 / (1.15)^2 + \$3.10(1.06)^3 / (1.15)^3 + \$74.37 / (1.15)^3$
$P_0 = \$56.82$

12. Here we have a stock that pays no dividends for 9 years. Once the stock begins paying dividends, it will have a constant growth rate of dividends. We can use the constant growth model at that point. It is important to remember that the general form of the constant dividend growth formula is:
$P_t = [D_t \times (1 + g)] / (R – g)$

This means that since we will use the dividend in Year 10, we will be finding the stock price in Year 9. The dividend growth model is similar to the PVA and the PV of a perpetuity: The equation gives you the PV one period before the first payment. So, the price of the stock in Year 9 will be:
$P_9 = D_{10} / (R – g) = \$15.00 / (.13 – .055) = \$200$

The price of the stock today is simply the PV of the stock price in the future. We simply discount the future stock price at the required return. The price of the stock today will be:
$P_0 = \$200 / 1.13^9 = \$66.58$

13. The price of a stock is the PV of the future dividends. This stock is paying five dividends, so the price of the stock is the PV of these dividends using the required return. The price of the stock is:
$P_0 = \$15 / 1.12 + \$18 / 1.12^2 + \$21 / 1.12^3 + \$24 / 1.12^4 + \$27 / 1.12^5 = \$73.26$

14. With differential dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the PV of the future stock price, plus the PV of all dividends during the differential growth period. The stock begins constant growth in Year 5, so we can find the price of the stock in Year 4, one year before the constant dividend growth begins, as:
$P_4 = D_4 (1 + g) / (R – g) = \$2.75(1.05) / (.13 – .05) = \$36.09$

The price of the stock today is the PV of the first four dividends, plus the PV of the Year 4 stock price. So, the price of the stock today will be:
$P_0 = \$10 / 1.13 + \$7 / 1.13^2 + \$6 / 1.13^3 + (\$2.75 + 36.09) / 1.13^4 = \$42.31$

15. With differential dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the PV of the future stock price, plus the PV of all dividends during the differential growth period. The stock begins constant growth in Year 4, so we can find the price of the stock in Year 3, one year before the constant dividend growth begins as:

$P_3 = D_3 (1 + g) / (R - g) = D_0 (1 + g_1)^3 (1 + g_2) / (R - g_2) = \$2.80(1.20)^3(1.05) / (.12 - .05) = \$72.58$

The price of the stock today is the PV of the first three dividends, plus the PV of the Year 3 stock price. The price of the stock today will be:

$P_0 = \$2.80(1.20) / 1.12 + \$2.80(1.20)^2 / 1.12^2 + \$2.80(1.20)^3 / 1.12^3 + \$72.58 / 1.12^3 = \$61.32$

16. Here we need to find the dividend next year for a stock experiencing differential growth. We know the stock price, the dividend growth rates, and the required return, but not the dividend. First, we need to realize that the dividend in Year 3 is the current dividend times the FVIF. The dividend in Year 3 will be:

$D_3 = D_0 (1.30)^3$

And the dividend in Year 4 will be the dividend in Year 3 times one plus the growth rate, or:

$D_4 = D_0 (1.30)^3 (1.18)$

The stock begins constant growth after the 4th dividend is paid, so we can find the price of the stock in Year 4 as the dividend in Year 5, divided by the required return minus the growth rate. The equation for the price of the stock in Year 4 is:

$P_4 = D_4 (1 + g) / (R - g)$

Now we can substitute the previous dividend in Year 4 into this equation as follows:

$P_4 = D_0 (1 + g_1)^3 (1 + g_2) (1 + g_3) / (R - g_3) = D_0 (1.30)^3 (1.18) (1.08) / (.11 - .08) = 93.33D_0$

When we solve this equation, we find that the stock price in Year 4 is 93.33 times as large as the dividend today. Now we need to find the equation for the stock price today. The stock price today is the PV of the dividends in Years 1, 2, 3, and 4, plus the PV of the Year 4 price. So:

$P_0 = D_0(1.30)/1.11 + D_0(1.30)^2/1.11^2 + D_0(1.30)^3/1.11^3 + D_0(1.30)^3(1.18)/1.11^4 + 93.33D_0/1.11^4$

We can factor out D_0 in the equation, and combine the last two terms. Doing so, we get:

$P_0 = \$65.00 = D_0\{1.30/1.11 + 1.30^2/1.11^2 + 1.30^3/1.11^3 + [(1.30)^3(1.18) + 93.33] / 1.11^4\}$

Reducing the equation even further by solving all of the terms in the braces, we get:

$\$65 = \$67.34 * D_0 \quad \rightarrow \quad D_0 = \$65.00 / \$67.34 = \$.97$

This is the dividend today, so the projected dividend for the next year will be:

$D_1 = \$.97(1.30) = \$1.25$

17. The constant growth model can be applied even if the dividends are declining by a constant percentage, just make sure to recognize the negative growth. So, the price of the stock today will be:

$P_0 = D_0 (1 + g) / (R - g) = \$9(1 - .04) / [(.11 - (-.04)] = \$57.60$

18. We are given the stock price, the dividend growth rate, and the required return, and are asked to find the dividend. Using the constant dividend growth model, we get:

$P_0 = \$58.32 = D_0 (1 + g) / (R - g) \quad \rightarrow \quad D_0 = \$58.32(.115 - .05) / (1.05) = \$3.61$

19. The price of a share of preferred stock is the dividend payment divided by the required return. We know the dividend payment in Year 5, so we can find the price of the stock in Year 4, one year before the first dividend payment. Doing so, we get:

$P_4 = \$8.00 / .056 = \$142.86$

The price of the stock today is the PV of the stock price in the future, so the price today will be:

$P_0 = \$142.86 / (1.056)^4 = \$114.88$

22. Here we have a stock paying a constant dividend for a fixed period, and an increasing dividend thereafter. We need to find the present value of the two different cash flows using the appropriate quarterly interest rate. The constant dividend is an annuity, so the present value of these dividends is:

$PVA = C(PVIFA_{R,t}) = \$0.80(PVIFA_{2.5\%,12}) = \$8.21$

Now we can find the present value of the dividends beyond the constant dividend phase. Using the present value of a growing annuity equation, we find:

$P_{12} = D_{13} / (R - g) = \$0.80(1 + .01) / (.025 - .01) = \$53.87$

This is the price of the stock immediately after it has paid the last constant dividend. So, the present value of the future price is:

$PV = \$53.87 / (1 + .025)^{12} = \$40.05$

The price today is the sum of the present value of the two cash flows, so:

$P_0 = \$8.21 + 40.05 = \$48.26$

25. First, we need to find the annual dividend growth rate over the past four years. To do this, we can use the future value of a lump sum equation, and solve for the interest rate. Doing so, we find the dividend growth rate over the past four years was:

$FV = PV(1 + R)^t \rightarrow \quad \$1.77 = \$1.35(1 + R)^4$

⇨ $R = (\$1.77 / \$1.35)^{1/4} - 1 = .0701$, or 7.01%

We know the dividend will grow at this rate for five years before slowing to a constant rate indefinitely. So, the dividend amount in seven years will be:

$D_7 = D_0(1 + g_1)^5(1 + g_2)^2 = \$1.77(1 + .0701)^5(1 + .05)^2 = \$2.74$

26. *a.* We can find the price of all the outstanding company stock by using the dividends the same way we would value an individual share. Since earnings are equal to dividends, and there is no growth, the value of the company's stock today is the present value of a perpetuity, so:

$P = D / R = \$950{,}000 / .12 = \$7{,}916{,}666.67$

The price-earnings ratio is the stock price divided by the current earnings, so the price-earnings ratio of each company with no growth is:

P/E = Price / Earnings = $7,916,666.67 / $950,000 = 8.33 times

b. Since the earnings have increased, the price of the stock will increase. The new price of the outstanding company stock is:

$P = D / R = (\$950{,}000 + 100{,}000) / .12 = \$8{,}750{,}000$

The price-earnings ratio is the stock price divided by the current earnings, so the price-earnings with the increased earnings is:

P/E = Price / Earnings = $8,750,000 / $950,000 = 9.21 times

c. Since the earnings have increased, the price of the stock will increase. The new price of the outstanding company stock is:
P = D / *R* = ($950,000 + 200,000) / .12 = $9,583,333.33

The price-earnings ratio is the stock price divided by the current earnings, so the price-earnings with the increased earnings is:
P/E = Price / Earnings = $9,583,333.33 / $950,000 = 10.09 times

27. *a.* If the company does not make any new investments, the stock price will be the present value of the constant perpetual dividends. In this case, all earnings are paid as dividends, so, applying the perpetuity equation, we get:
P = Dividend / *R* = $9.40 / .12 = $78.33

b. The investment is a one-time investment that creates an increase in EPS for two years. To calculate the new stock price, we need the cash cow price plus the NPVGO. In this case, the NPVGO is simply the present value of the investment plus the present value of the increases in EPS. So, the NPVGO will be:
$\text{NPVGO} = C_1 / (1 + R) + C_2 / (1 + R)^2 + C_3 / (1 + R)^3$
$\text{NPVGO} = -\$1.95 / 1.12 + \$2.75 / 1.12^2 + \$3.05 / 1.12^3 = \$2.62$

So, the price of the stock if the company undertakes the investment opportunity will be:
P = $78.33 + 2.62 = $80.96

c. After the project is over, and the earnings increase no longer exists, the price of the stock will revert back to $78.33, the value of the company as a cash cow.

28. *a.* The price of the stock is the present value of the dividends. Since earnings are equal to dividends, we can find the present value of the earnings to calculate the stock price. Also, since we are excluding taxes, the earnings will be the revenues minus the costs. We simply need to find the present value of all future earnings to find the price of the stock. The present value of the revenues is:
$PV_{\text{Revenue}} = C_1 / (R - g) = \$7,500,000(1 + .05) / (.13 - .05) = \$98,437,500$

And the present value of the costs will be:
$PV_{\text{Costs}} = C_1 / (R - g) = \$3,400,000(1 + .05) / (.13 - .05) = \$44,625,000$

Since there are no taxes, the present value of the company’s earnings and dividends will be:
$PV_{\text{Dividends}} = \$98,437,500 - 44,625,000 = \$53,812,500$

Note that since revenues and costs increase at the same rate, we could have found the present value of future dividends as the present value of current dividends. Doing so, we find:
$D_0 = \text{Revenue}_0 - \text{Costs}_0 = \$7,500,000 - 3,400,000 = \$4,100,000$

Now, applying the growing perpetuity equation, we find:
$PV_{\text{Dividends}} = C_1 / (R - g) = \$4,100,000(1 + .05) / (.13 - .05) = \$53,812,500$

This is the same answer we found previously. The price per share of stock is the total value of the company’s stock divided by the shares outstanding, or:

P = Value of all stock / Shares outstanding = $53,812,500 / 1,000,000 = $53.81

b. The value of a share of stock in a company is the present value of its current operations, plus the present value of growth opportunities. To find the present value of the growth opportunities, we need to discount the cash outlay in Year 1 back to the present, and find the value today of the increase in earnings. The increase in earnings is a perpetuity, which we must discount back to today. So, the value of the growth opportunity is:

NPVGO = $C_0 + C_1 / (1 + R) + (C_2 / R) / (1 + R)$

NPVGO = –$17,000,000 – $6,000,000 / (1 + .13) + ($4,200,000 / .13) / (1 + .13)

NPVGO = $6,281,143.64

To find the value of the growth opportunity on a per share basis, we must divide this amount by the number of shares outstanding, which gives us:

$NPVGO_{\text{Per share}}$ = $6,821,143.64 / 1,000,000 = $6.28

The stock price will increase by $6.28 per share. The new stock price will be:

New stock price = $53.81 + 6.28 = $60.09

29. *a.* If the company continues its current operations, it will not gro, so we can value the company as a cash cow. The total value of the company as a cash cow is the present value of the future earnings, which are a perpetuity, so:

Cash cow value of company = C / R = $71,000,000 / .12 = $591,666,666.67

The value per share is the total value of the company divided by the shares outstanding, so:

Share price = $591,666,666.67 / 15,000,000 = $39.44

b. To find the value of the investment, we need to find the NPV of the growth opportunities. The initial cash flow occurs today, so it does not need to be discounted. The earnings growth is a perpetuity. Using the present value of a perpetuity equation will give us the value of the earnings growth one period from today, so we need to discount this back to today. The NPVGO of the investment opportunity is:

NPVGO = $C_0 + C_1 / (1 + R) + (C_2 / R) / (1 + R)$

NPVGO = –$16,000,000 – 5,000,000 / (1 + .12) + ($11,000,000 / .12) / (1 + .12)

NPVGO = $61,380,952.38

c. The price of a share of stock is the cash cow value plus the NPVGO. We have already calculated the NPVGO for the entire project, so we need to find the NPVGO on a per share basis. The NPVGO on a per share basis is the NPVGO of the project divided by the shares outstanding, which is:

NPVGO per share = $61,380,952.38 / 15,000,000 = $4.09

This means the per share stock price if the company undertakes the project is:

Share price = Cash cow price + NPVGO per share = $39.44 + 4.09 = $43.54

30. *a.* Using the equation to calculate the price of a share of stock with the PE ratio:

P = Benchmark PE ratio × EPS

So, with a PE ratio of 21, we find: P = 21($2.35) = $49.35

b. First, we need to find the earnings per share next year, which will be:

$EPS_1 = EPS_0(1 + g) = \$2.35(1 + .07) = \$2.51$

Using the equation to calculate the price of a share of stock with the PE ratio:
P_1 = Benchmark PE ratio × EPS_1= 21($2.51)= $52.80

c. To find the implied return over the next year, we calculate the return as:
$R = (P_1 – P_0) / P_0$ = ($52.80 – 49.35) / $49.35 = .07, or 7%

Notice that the return is the same as the growth rate in earnings. Assuming a stock pays no dividends and the PE ratio is constant, this will always be true when using price ratios to evaluate the price of a share of stock.

31. We need to find the enterprise value of the company. We can calculate EBITDA as sales minus costs, so:
EBITDA = Sales – Costs = $28,000,000 – 12,000,000 = $16,000,000

Solving the EV/EBITDA multiple for enterprise value, we find:
Enterprise value = $16,000,000(7.5) = $120,000,000

The total value of equity is the enterprise value minus any outstanding debt and cash, so:
Equity value = Enterprise value – Debt – Cash = $120,000,000 – 54,000,000 – 18,000,000
Equity value = $48,000,000

So, the price per share is:
Stock price = $48,000,000 / 950,000 = $50.53

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31. We need to find the enterprise value of the company. We can calculate EBITDA as sales minus costs, so:

EBITDA = Sales – Costs = $28,000,000 – 12,000,000 = $16,000,000

Solving the EV/EBITDA multiple for enterprise value, we find:

Enterprise value = $16,000,000(7.5) = $120,000,000

Equity value = Enterprise value – Debt + Cash = $120,000,000 – 54,000,000 + 18,000,000

Equity value = $84,000,000

So, the price per share is:

Stock price = $84,000,000 / 950,000 = $88.42

36. Here we have a stock with differential growth, but the dividend growth changes every year for the first four years. We can find the price of the stock in Year 3 since the dividend growth rate is constant after the third dividend. The price of the stock in Year 3 will be the dividend in Year 4, divided by the required return minus the constant dividend growth rate. So, the price in Year 3 will be: $P_3 = \$3.85(1.20)(1.15)(1.10)(1.05) / (.13 – .05) = \$76.71$

The price of the stock today will be the PV of the first three dividends, plus the PV of the stock price in Year 3, so:

$$P_0 = \$3.85(1.20) / (1.13) + \$3.85(1.20)(1.15) / 1.13^2 + \$3.85(1.20)(1.15)(1.10) / 1.13^3 + \$76.71 / 1.13^3 = \$65.46$$