Chapter 5 Suggested Problems Solutions

3. When we use discounted payback, we need to find the value of all cash flows today. The value today of the project cash flows for the first four years is:

```
Value today of Year 1 cash flow = \$5,000/1.14 = \$4,385.96 Value today of Year 2 cash flow = \$5,500/1.14^2 = \$4,232.07 Value today of Year 3 cash flow = \$6,000/1.14^3 = \$4,049.83 Value today of Year 4 cash flow = \$7,000/1.14^4 = \$4,144.56
```

To find the discounted payback, we use these values to find the payback period. The discounted first year cash flow is \$4,385.96, so the discounted payback for an initial cost of \$8,000 is:

```
Discounted payback = 1 + (\$8,000 - 4,385.96)/\$4,232.07 = 1.85 years
```

```
For an initial cost of $12,000, the discounted payback is:
Discounted payback = 2 + ($12,000 - 4,385.96 - 4,232.07)/$4,049.83 = 2.84 years
```

Notice the calculation of discounted payback. We know the payback period is between two and three years, so we subtract the discounted values of the Year 1 and Year 2 cash flows from the initial cost. This is the numerator, which is the discounted amount we still need to make to recover our initial investment. We divide this amount by the discounted amount we will earn in Year 3 to get the fractional portion of the discounted payback.

```
If the initial cost is $16,000, the discounted payback is: Discounted payback = 3 + (\$16,000 - 4,385.96 - 4,232.07 - 4,049.83) / \$4,144.56 = 3.80 years
```

12. *a.* The profitability index is the PV of the future cash flows divided by the initial investment. The cash flows for both projects are an annuity, so:

```
PI_{I} = \$18,000(PVIFA_{10\%,3}) / \$30,000 = 1.492

PI_{II} = \$7,500(PVIFA_{10\%,3}) / \$12,000 = 1.554
```

The profitability index decision rule implies that we accept project II, since PI_{II} is greater than the PI_{I} .

b. The NPV of each project is:

```
\begin{split} NPV_I = & -\$30,\!000 + \$18,\!000(PVIFA_{10\%,3}) = \$14,\!763.34 \\ NPV_{II} = & -\$12,\!000 + \$7,\!500(PVIFA_{10\%,3}) = \$6,\!651.39 \end{split}
```

The NPV decision rule implies accepting Project I, since the NPV_I is greater than the NPV_{II}.

- c. Using the profitability index to compare mutually exclusive projects can be ambiguous when the magnitudes of the cash flows for the two projects are of different scales. In this problem, project I is 2.5 times as large as project II and produces a larger NPV, yet the profitability index criterion implies that project II is more acceptable.
- **16.** *a.* The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

```
AZM Mini-SUV:
Cumulative cash flows Year 1 = $320,000 = $320,000
```

```
Cumulative cash flows Year 2 = \$320,000 + 180,000 = \$500,000
```

Payback period = 1 + \$130,000 / \$180,000 = 1.72 years

AZF Full-SUV:

```
Cumulative cash flows Year 1 = \$350,000 = \$350,000 Cumulative cash flows Year 2 = \$350,000 + 420,000 = \$770,000 Cumulative cash flows Year 2 = \$350,000 + 420,000 + 290,000 = \$1,060,000
```

Payback period = 2+ \$30,000 / \$290,000 = 2.10 years

Since the AZM has a shorter payback period than the AZF, the company should choose the AZM. Remember the payback period does not necessarily rank projects correctly.

b. The NPV of each project is:

$$NPV_{AZM} = -\$450,000 + \$320,000 / 1.10 + \$180,000 / 1.10^2 + \$150,000 / 1.10^3$$

$$NPV_{AZM} = \$102,366.64$$

$$NPV_{AZF} = -\$800,000 + \$350,000 / 1.10 + \$420,000 / 1.10^2 + \$290,000 / 1.10^3 \\ NPV_{AZF} = \$83,170.55$$

The NPV criteria implies we accept the AZM because it has the highest NPV.

c. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the AZM is:

$$0 = -\$450,000 + \$320,000 / (1 + IRR) + \$180,000 / (1 + IRR)^2 + \$150,000 / (1 + IRR)^3$$

$$\rightarrow IRR_{AZM} = 24.65\%$$

And the IRR of the AZF is:

```
0 = -\$800,000 + \$350,000 / (1 + IRR) + \$420,000 / (1 + IRR)^2 + \$290,000 / (1 + IRR)^3 \\ \Rightarrow IRR_{AZF} = 15.97\%
```

The IRR criteria implies we accept the AZM because it has the highest IRR. Remember the IRR does not necessarily rank projects correctly.

- d. Incremental IRR analysis is not necessary. The AZM has the smallest initial investment, and the largest NPV, so it should be accepted.
- **17.** *a*. The profitability index is the PV of the future cash flows divided by the initial investment. The profitability index for each project is:

```
\begin{split} PI_A &= \left[\$110,000 \, / \, 1.12 \, + \, \$110,000 \, / \, 1.12^2 \right] \, / \, \$150,000 = 1.24 \\ PI_B &= \left[\$200,000 \, / \, 1.12 \, + \, \$200,000 \, / \, 1.12^2 \right] \, / \, \$300,000 = 1.13 \\ PI_C &= \left[\$120,000 \, / \, 1.12 \, + \, \$90,000 \, / \, 1.12^2 \right] \, / \, \$150,000 = 1.19 \end{split}
```

b. The NPV of each project is:

```
NPV_A = -\$150,000 + \$110,000 / 1.12 + \$110,000 / 1.12^2 = \$35,905.61

NPV_B = -\$300,000 + \$200,000 / 1.12 + \$200,000 / 1.12^2 = \$38,010.20

NPV_C = -\$150,000 + \$120,000 / 1.12 + \$90,000 / 1.12^2 = \$28,890.31
```

- c. Accept projects A, B, and C. Since the projects are independent, accept all three projects because the respective profitability index of each is greater than one.
- d. Accept Project B. Since the Projects are mutually exclusive, choose the Project with the highest PI, while taking into account the scale of the Project. Because Projects A and C have the same initial investment, the problem of scale does not arise when comparing the profitability indices. Based on the profitability index rule, Project C can be eliminated because its PI is less than the PI of Project A. Because of the problem of scale, we cannot compare the PIs of Projects A and B. However, we can calculate the PI of the incremental cash flows of the two projects, which are:

Project
$$C_0$$
 C_1 C_2 $B-A$ $-\$150,000$ $\$90,000$ $\$90,000$

When calculating incremental cash flows, remember to subtract the cash flows of the project with the smaller initial cash outflow from those of the project with the larger initial cash outflow. This procedure insures that the incremental initial cash outflow will be negative. The incremental PI calculation is:

$$PI(B - A) = [\$90,000 / 1.12 + \$90,000 / 1.12^{2}] / \$150,000 = 1.014$$

The company should accept Project B since the PI of the incremental cash flows is greater than one.

- e. Remember that the NPV is additive across projects. Since we can spend \$450,000, we could take two of the projects. In this case, we should take the two projects with the highest NPVs, which are Project B and Project A.
- **19.** *a.* The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

NP-30:

Cumulative cash flows Year 1 = \$185,000 = \$185,000 Cumulative cash flows Year 2 = \$185,000 + 185,000 = \$370,000 Cumulative cash flows Year 3 = \$185,000 + 185,000 + 185,000 = \$555,000

Payback period = 2 + (\$180,000/\$185,000) = 2.97 years

NX-20:

Payback period = 3 + (\$19,000/\$133,100) = 3.14 years

Since the NP-30 has a shorter payback period than the NX-20, the company should choose the NP-30. Remember the payback period does not necessarily rank projects correctly.

b. The IRR is the interest rate that makes the NPV of the project equal to zero, so the IRR of each project is:

NP-30:

$$0 = -\$550,000 + \$185,000(\{1 - [1/(1 + IRR)^5]\} / IRR)$$
 \Rightarrow $IRR_{NP-30} = 20.27\%$
And the IRR of the NX-20 is:
 $0 = -\$350,000 + \$100,000 / (1 + IRR) + \$110,000 / (1 + IRR)^2 + \$121,000 / (1 + IRR)^3 + \$133,100 / (1 + IRR)^4 + \$146,410 / (1 + IRR)^5$ \Rightarrow $IRR_{NX-20} = 20.34\%$

The IRR criteria implies accepting the NX-20.

c. The profitability index is the present value of all subsequent cash flows, divided by the initial investment, so the profitability index of each project is:

$$\begin{split} PI_{NP\text{-}30} &= (\$185,000\{[1-(1/1.15)^5] \ / \ .15 \ \}) \ / \ \$550,000 = 1.128 \\ PI_{NX\text{-}20} &= [\$100,000 \ / \ 1.15 \ + \ \$110,000 \ / \ 1.15^2 \ + \ \$121,000 \ / \ 1.15^3 \ + \ \$133,100 \ / \ 1.15^4 \\ &\quad + \ \$146,410 \ / \ 1.15^5] \ / \ \$350,000 \ = 1.139 \end{split}$$

The PI criteria implies accepting the NX-20.

d. The NPV of each project is:

$$\begin{split} NPV_{NP\text{-}30} &= -\$550,\!000 + \$185,\!000\{[1 - (1/1.15)^5] / .15]\} = \$70,\!148.69 \\ NPV_{NX\text{-}20} &= -\$350,\!000 + \$100,\!000 / 1.15 + \$110,\!000 / 1.15^2 + \$121,\!000 / 1.15^3 \\ &+ \$133,\!100 / 1.15^4 + \$146,\!410 / 1.15^5 = \$48,\!583.79 \end{split}$$

The NPV criteria implies accepting the NP-30.

23. *a.* Here the cash inflows of the project go on forever, which is a perpetuity. Unlike ordinary perpetuity cash flows, the cash flows here grow at a constant rate forever, which is a growing perpetuity. The PV of the future cash flows from the project is:

```
PV of cash inflows = C_1/(R - g) = \$290,000/(.11 - .05) = \$4,833,333.33
```

NPV is the PV of the outflows minus by the PV of the inflows, so the NPV is:

NPV of the project =
$$-\$3,900,000 + 4,833,333.33 = \$933,333.33$$

The NPV is positive, so we would accept the project.

b. Here we want to know the minimum growth rate in cash flows necessary to accept the project. The minimum growth rate is the growth rate at which we would have a zero NPV. The equation for a zero NPV, using the equation for the PV of a growing perpetuity is:

$$0 = -\$3,900,000 + \$290,000/(.11 - g)$$

$$\Rightarrow g = 3.56\%$$

25. First, we need to find the future value of the cash flows for the one year in which they are blocked by the government. So, reinvesting each cash inflow for one year, we find:

```
Year 2 cash flow = $285,000(1.04) = $296,400
Year 3 cash flow = $345,000(1.04) = $358,800
Year 4 cash flow = $415,000(1.04) = $431,600
```

Year 5 cash flow = \$255,000(1.04) = \$265,200

So, the NPV of the project is:

$$NPV = -\$950,000 + \$296,400/1.11^2 + \$358,800/1.11^3 + \$431,600/1.11^4 + \$265,200/1.11^5$$

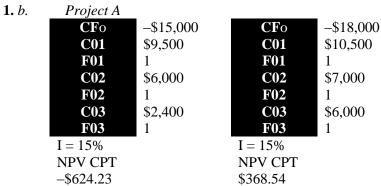
 $NPV = -\$5,392.06$

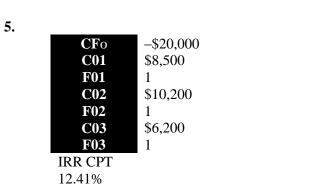
And the IRR of the project is:

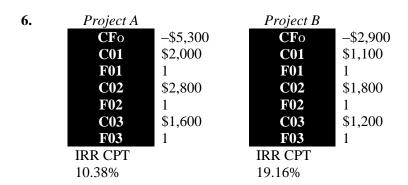
$$0 = -\$950,000 + \$296,400/(1 + IRR)^2 + \$358,800/(1 + IRR)^3 + \$431,600/(1 + IRR)^4 + \$265,200/(1 + IRR)^5$$

While this may look like a MIRR calculation, it is not a MIRR, rather it is a standard IRR calculation. Since the cash inflows are blocked by the government, they are not available to the company for a period of one year. Thus, all we are doing is calculating the IRR based on when the cash flows actually occur for the company.

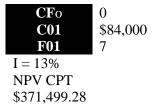
Calculator Solutions





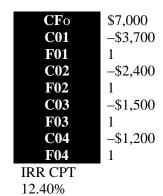


7.



PI = \$371,499.28 / \$385,000 = 0.965

10.



CFo	\$7,000
C01	-\$3,700
F01	1
C02	-\$2,400
F02	1
C03	-\$1,500
F03	1
C04	-\$1,200
F04	1
I = 10%	-

I = 10% NPV CPT -\$293.70

	CFo	\$7,000
	C01	-\$3,700
	F01	1
	C02	-\$2,400
	F02	1
	C03	-\$1,500
	F03	1
	C04	-\$1,200
	F04	1
-	2001	-

I = 20% NPV CPT \$803.24

11. a. Deepwater fishing

Deepwater justing		
CFo	-\$950,000	
C01	\$370,000	
F01	1	
C02	\$510,000	
F02	1	
C03	\$420,000	
F03	1	
TD D CDT		

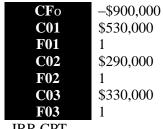
IRR CPT 17.07%

Submarine ride

Submarine riae		
CFo	-\$1,850,000	
C01	\$900,000	
F01	1	
C02	\$800,000	
F02	1	
C03	\$750,000	
F03	1	
IDD CDE		

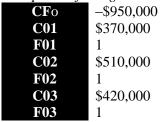
IRR CPT 16.03%





IRR CPT 14.79%

c. Deepwater fishing



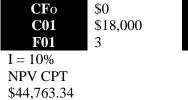
I = 14% NPV CPT \$50,477.88

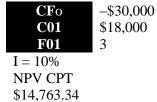
Submarine ride

Juditut itte ittee		
CFo	-\$1,850,000	
C01	\$900,000	
F01	1	
C02	\$800,000	
F02	1	
C03	\$750,000	
F03	1	

I = 14% NPV CPT \$61,276.34

12. Project I





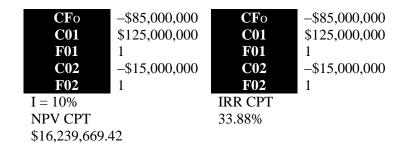
PI = \$44,763.34 / \$30,000 = 1.492

Project II

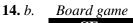
1 10 100 11			
CFo	\$0	CFo	-\$12,000
C01	\$7,500	C01	\$7,500
F01	3	F01	3
I = 10%		I = 10%	
NPV CPT		NPV CPT	
\$18,651.39		\$6,651.39	

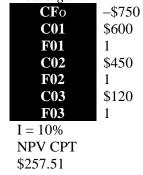
PI = \$18,651.39 / \$12,000 = 1.554

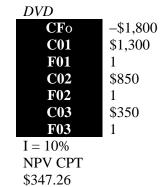
13.



Financial calculators will only give you one IRR, even if there are multiple IRRs. Using trial and error, or a root solving calculator, the other IRR is -86.82%.

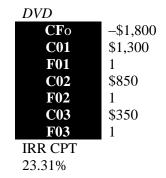




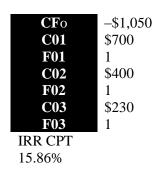


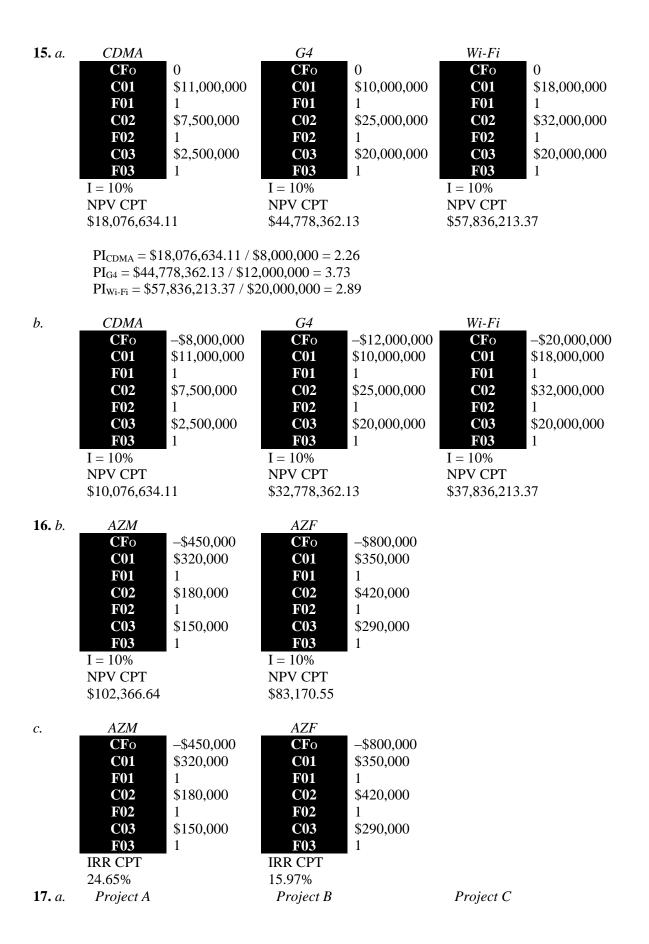
c. Board game

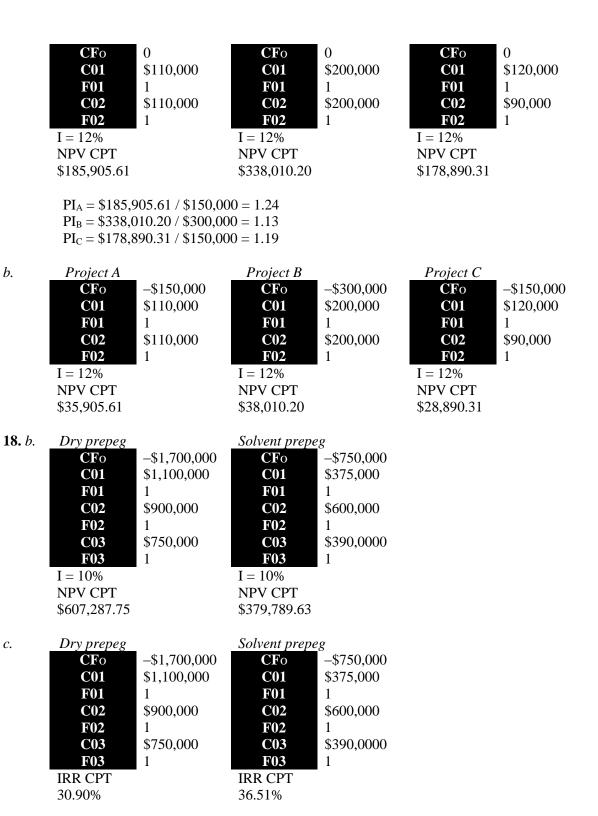
\mathbf{CFo}	-\$750
C01	\$600
F01	1
C02	\$450
F02	1
C03	\$120
F03	1
IRR CPT	
33.79%	



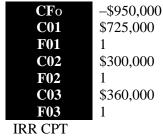
d.







d.



IRR CPT 25.52%

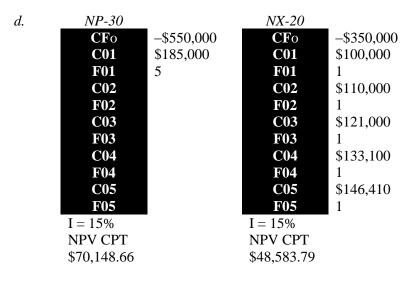
19. <i>b</i> .	NP-30		NX-20	
	CFo	-\$550,000	CFo	-\$350,000
	C01	\$185,000	C01	\$100,000
	F01	5	F01	1
	C02		C02	\$110,000
	F02		F02	1
	C03		C03	\$121,000
	F03		F03	1
	C04		C04	\$133,100
	F04		F04	1
	C05		C05	\$146,410
	F05		F05	1
	IRR CPT		IRR CPT	
	20.27%		20.34%	

c. NP-30 NX-20 **CF**o -\$550,000 -\$350,000 **CF**o C01 \$185,000 **C01** \$100,000 F01 5 F01 1 **C02 C02** \$110,000 F02 F02 C03 C03 \$121,000 F03 F03 1 **C04 C04** \$133,100 F04 F04 1 C05 C05 \$146,410 F05 F05 I = 15%I = 15%NPV CPT NPV CPT

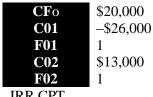
 $\begin{aligned} PI_{NP\text{-}30} &= \$620,\!148.69 \ / \ \$550,\!000 = 1.128 \\ PI_{NX\text{-}20} &= \$398,\!583.79 \ / \ \$350,\!000 = 1.139 \end{aligned}$

\$398,583.79

\$620,148.69



28.



IRR CPT ERROR 7