Chapter 8 Suggested Problems Solutions

9. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

```
(1+R) = (1+r)(1+h) \rightarrow h = [(1+.14)/(1+.10)] - 1 = .0364, or 3.64%
```

15. Any bond that sells at par has a YTM equal to the coupon rate. Both bonds sell at par, so the initial YTM on both bonds is the coupon rate, 7 percent. If the YTM suddenly rises to 9 percent:

```
P_{Laurel} = $35(PVIFA<sub>4.5%,4</sub>) + $1,000(PVIF<sub>4.5%,4</sub>) = $964.12

P_{Hardy} = $35(PVIFA<sub>4.5%,30</sub>) + $1,000(PVIF<sub>4.5%,30</sub>) = $837.11
```

The percentage change in price is calculated as:

Percentage change in price = (New price – Original price) / Original price

$$\Delta P_{Laurel}\% = (\$964.12 - 1,000) / \$1,000 = -.0359, \text{ or } -3.59\%$$

 $\Delta P_{Hardy}\% = (\$837.11 - 1,000) / \$1,000 = -.1629, \text{ or } -16.29\%$

If the YTM suddenly falls to 5 percent:

```
\begin{array}{ll} P_{Laurel} &= \$35(PVIFA_{2.5\%,4}) + \$1,000(PVIF_{2.5\%,4}) = \$1,037.62 \\ P_{Hardy} &= \$35(PVIFA_{2.5\%,30}) + \$1,000(PVIF_{2.5\%,30}) = \$1,209.30 \\ \Delta P_{Laurel}\% &= (\$1,037.62 - 1,000) / \$1,000 = +.0376, \text{ or } 3.76\% \\ \Delta P_{Hardy}\% &= (\$1,209.30 - 1,000) / \$1,000 = +.2093, \text{ or } 20.93\% \end{array}
```

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates. Notice also that for the same interest rate change, the gain from a decline in interest rates is larger than the loss from the same magnitude change. For a plain vanilla bond, this is always true.

16. Initially, at a YTM of 10 percent, the prices of the two bonds are:

```
P_{Faulk} = \$30(PVIFA_{5\%,24}) + \$1,000(PVIF_{5\%,24}) = \$724.03

P_{Gonas} = \$70(PVIFA_{5\%,24}) + \$1,000(PVIF_{5\%,24}) = \$1,275.97
```

If the YTM rises from 10 percent to 12 percent:

```
\begin{array}{ll} P_{Faulk} &= \$30(PVIFA_{6\%,24}) + \$1,000(PVIF_{6\%,24}) &= \$623.49 \\ P_{Gonas} &= \$70(PVIFA_{6\%,24}) + \$1,000(PVIF_{6\%,24}) &= \$1,125.50 \\ \end{array}
```

The percentage change in price is calculated as:

```
Percentage change in price = (New price – Original price) / Original price \Delta P_{\text{Faulk}}\% = (\$623.49 - 724.03) / \$724.03 = -.1389, or -13.89\%
```

$$\Delta P_{\text{Gonas}}\% = (\$1,125.50 - 1,275.97) / \$1,275.97 = -.1179, \text{ or } -11.79\%$$

If the YTM declines from 10 percent to 8 percent:

```
\begin{array}{lll} P_{Faulk} &= \$30(PVIFA_{4\%,24}) + \$1,000(PVIF_{4\%,24}) &= \$847.53 \\ P_{Gonas} &= \$70(PVIFA_{4\%,24}) + \$1,000(PVIF_{4\%,24}) &= \$1,457.41 \\ \Delta P_{Faulk}\% &= (\$847.53 - 724.03) / \$724.03 &= +.1706, \text{ or } 17.06\% \\ \Delta P_{Gonas}\% &= (\$1,457.41 - 1,275.97) / \$1,275.97 &= +.1422, \text{ or } 14.22\% \end{array}
```

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

20. Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are four months until the next coupon payment, so two months have passed since the last coupon payment. The accrued interest for the bond is:

Accrued interest = $$59/2 \times 2/6 = 9.83

And we calculate the dirty price as:

Dirty price = Clean price + Accrued interest = \$1,053 + 9.83 = \$1,062.83

21. To find the number of years to maturity for the bond, we need to find the price of the bond. Since we already have the coupon rate, we can use the bond price equation, and solve for the number of years to maturity. We are given the current yield of the bond, so we can calculate the price as:

Current yield = $.0842 = \$90/P_0$ \Rightarrow $P_0 = \$90/.0842 = \$1,068.88$

Now that we have the price of the bond, the bond price equation is:

 $P = \$1,068.88 = \$90\{[(1 - (1/1.0781)^t] / .0781\} + \$1,000/1.0781^t\}$

We can solve this equation for *t* as follows:

 $1,068.88 (1.0781)^{t} = 1,152.37 (1.0781)^{t} - 1,152.37 + 1,000$

 $152.37 = 83.49(1.0781)^t$

 $1.8251 = 1.0781^t$

 $t = \log 1.8251 / \log 1.0781 = 8.0004 \approx 8 \text{ years}$

The bond has 8 years to maturity.

22. The bond has 9 years to maturity, so the bond price equation is:

 $P = \$1,053.12 = \$36.20(PVIFA_{R\%,18}) + \$1,000(PVIF_{R\%,18})$

 $\Rightarrow R = 3.226\%$

This is the semiannual interest rate, so the YTM is:

 $YTM = 2 \times 3.226\% = 6.45\%$

The current yield is the annual coupon payment divided by the bond price, so:

Current yield = \$72.40 / \$1,053.12 = .0687, or 6.87%

24. The price of a zero coupon bond is the PV of the par, so:

a.
$$P_0 = \$1,000/1.035^{50} = \$179.05$$

b. In one year, the bond will have 24 years to maturity, so the price will be:

$$P_1 = \$1,000/1.035^{48} = \$191.81$$

The interest deduction is the price of the bond at the end of the year, minus the price at the beginning of the year, so:

Year 1 interest deduction = \$191.81 - 179.05 = \$12.75

The price of the bond when it has one year left to maturity will be:

 $P_{24} = \$1,000/1.035^2 = \933.51

Year 25 interest deduction = \$1,000 - 933.51 = \$66.49

c. Previous IRS regulations required a straight-line calculation of interest. The total interest received by the bondholder is:

Total interest = \$1,000 - 179.05 = \$820.95

The annual interest deduction is simply the total interest divided by the maturity of the bond, so the straight-line deduction is:

Annual interest deduction = \$820.95 / 25 = \$32.84

- d. The company will prefer straight-line methods when allowed because the valuable interest deductions occur earlier in the life of the bond.
- **25.** *a.* The coupon bonds have a 6 percent coupon which matches the 6 perent required return, so they will sell at par. The number of bonds that must be sold is the amount needed divided by the bond price, so:

Number of coupon bonds to sell = 45,000,000 / 1,000 = 45,000

The number of zero coupon bonds to sell would be:

Price of zero coupon bonds = $\$1,000/1.03^{60} = \169.73

Number of zero coupon bonds to sell = 45,000,000 / 169.73 = 265,122

b. The repayment of the coupon bond will be the par value plus the last coupon payment times the number of bonds issued. So:

Coupon bonds repayment = 45,000(\$1,030) = \$46,350,000

The repayment of the zero coupon bond will be the par value times the number of bonds issued, so:

Zeroes: repayment = 265,122(\$1,000) = \$265,122,140

c. The total coupon payment for the coupon bonds will be the number bonds times the coupon payment. For the cash flow of the coupon bonds, we need to account for the tax deductibility of the interest payments. To do this, we will multiply the total coupon payment times one minus the tax rate. So:

Coupon bonds: (45,000)(\$60)(1-.35) = \$1,755,000 cash outflow

Note that this is cash outflow since the company is making the interest payment.

For the zero coupon bonds, the first year interest payment is the difference in the price of the zero at the end of the year and the beginning of the year. The price of the zeroes in one year will be:

 $P_1 = \$1,000/1.03^{58} = \180.07

The year 1 interest deduction per bond will be this price minus the price at the beginning of the year, which we found in part b, so:

Year 1 interest deduction per bond = \$180.07 - 169.73 = \$10.34

The total cash flow for the zeroes will be the interest deduction for the year times the number of zeroes sold, times the tax rate. The cash flow for the zeroes in year 1 will be:

Cash flows for zeroes in Year 1 = (265,122)(\$10.34)(.35) = \$959,175.00

Notice the cash flow for the zeroes is a cash inflow. This is because of the tax deductibility of the imputed interest expense. That is, the company gets to write off the interest expense for the year even though the company did not have a cash flow for the interest expense. This reduces the company's tax liability, which is a cash inflow.

During the life of the bond, the zero generates cash inflows to the firm in the form of the interest tax shield of debt. We should note an important point here: If you find the PV of the cash flows from the coupon bond and the zero coupon bond, they will be the same. This is because of the much larger repayment amount for the zeroes.

26. To find the capital gains yield and the current yield, we need to find the price of the bond. The current price of Bond P and the price of Bond P in one year is:

```
P: P_0 = \$90(PVIFA_{7\%,10}) + \$1,000(PVIF_{7\%,10}) = \$1,140.47

P_1 = \$90(PVIFA_{7\%,9}) + \$1,000(PVIF_{7\%,9}) = \$1,130.30

Current yield = \$90 / \$1,140.47 = .0789, or 7.89%

Capital gains yield = (New price – Original price) / Original price

Capital gains yield = (\$1,130.30 - 1,140.47) / \$1,140.47 = -.0089, or -0.89\%
```

The current price of Bond D and the price of Bond D in one year is:

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D: P_0 = $50(PVIFA_{7\%,10}) + $1,000(PVIF_{7\%,10}) = $859.53

P_1 = $50(PVIFA_{7\%,9}) + $1,000(PVIF_{7\%,9}) = $869.70

Current yield = $50 / $859.53 = 0.0582 or 5.82%

Capital gains yield = ($869.70 - 859.53) / $859.53 = .0118, or 1.18%
```

All else held constant, premium bonds pay a high current income while having price depreciation as maturity nears; discount bonds pay a lower current income but have price appreciation as maturity nears. For either bond, the total return is still 7%, but this return is distributed differently between current income and capital gains.

27. *a*. The rate of return you expect to earn if you purchase a bond and hold it until maturity is the YTM. The bond price equation for this bond is:

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P_0 = \$930 = \$56(PVIFA_{R\%,10}) + \$1,000(PVIF_{R\%,10})

\Rightarrow R = YTM = 6.58\%
```

b. To find our HPY, we need to find the price of the bond in two years. The price of the bond in two years, at the new interest rate, will be:

```
P_2 = \$56(PVIFA_{5.58\%,8}) + \$1,000(PVIF_{5.58\%,8}) = \$1,001.44
```

To calculate the HPY, we need to find the interest rate that equates the price we paid for the bond with the cash flows we received. The cash flows we received were \$90 each year for two years, and the price of the bond when we sold it. The equation to find our HPY is:

```
P_0 = $930 = $56(PVIFA_{R\%,2}) + $1,001.44(PVIF_{R\%,2})
```

 \Rightarrow R = HPY = 9.68%

The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

28. The price of any bond (or financial instrument) is the PV of the future cash flows. Even though Bond M makes different coupons payments, to find the price of the bond, we just find the PV of the cash flows. The PV of the cash flows for Bond M is:

$$\begin{split} P_{M} &= \$800 (PVIFA_{4\%,16}) (PVIF_{4\%,12}) + \$1,000 (PVIFA_{4\%,12}) (PVIF_{4\%,28}) + \$30,000 (PVIF_{4\%,40}) \\ P_{M} &= \$15,200.77 \end{split}$$

Notice that for the coupon payments of \$800, we found the PVA for the coupon payments, and then discounted the lump sum back to today.

Bond N is a zero coupon bond with a \$30,000 par value; therefore, the price of the bond is the PV of the par, or:

 $P_N = \$30,000(PVIF_{4\%,40}) = \$6,248.67$

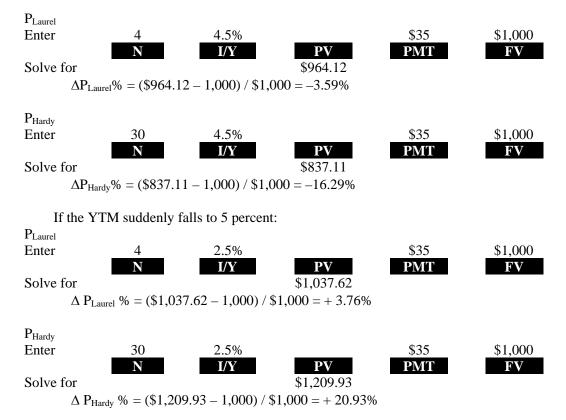
Calculator Solutions

1. a. Enter Solve for	30 N	2.5% I/Y	PV \$476.74	PMT	\$1,000 FV
b.EnterSolve for	30 N	5% I/Y	PV \$231.38	РМТ	\$1,000 FV
c. Enter Solve for	30 N	7.5% I/Y	PV \$114.22	PMT	\$1,000 FV
2. a. Enter	30 N	3.5% I/Y	PV	\$35 PMT	\$1,000 FV
Solve for b. Enter Solve for	30 N	4.5% I/Y	\$1,000.00 PV \$837.11	\$35 PMT	\$1,000 FV
c. Enter	30 N	2.5% I/Y	PV \$1,209.30	\$35 PMT	\$1,000 FV
3. Enter Solve for	$\frac{26}{N}$ 923% × 2 = 5.85%	I/Y 2.923%	±\$1,050 PV	\$32 PMT	\$1,000 FV
4. Enter Solve for \$4	$ \begin{array}{c} 23 \\ \hline \mathbf{N} \end{array} $ $ \begin{array}{c} 1.96 \times 2 = \$83.92 \\ 3.92 / \$1,000 = 8.3 \end{array} $	3.8% I/Y	±\$1,060 PV	PMT \$41.96	\$1,000 FV
5. Enter Solve for	19 N	3.90% I/Y	PV €1,079.48	€45 PMT	€1,000 FV

6. Enter	21		±¥92,000	¥2,800	¥100,000	
Solve for	N	I/Y 3.34%	\mathbf{PV}	PMT	\mathbf{FV}	
13. P ₀ Enter	50 N	3.5% I/X	PV	PMT	\$1,000 FV	
Solve for P ₁			\$179.05			
Enter	48 N	3.5% I/Y	PV	PMT	\$1,000 FV	
Solve for \$191.81	- 179.05 = \$12	.75	\$191.81			
14. Miller C	orporation					
Enter	26 N	3% I/Y	PV	\$40 PMT	\$1,000 FV	
Solve for			\$1,178.77			
P ₁ Enter	24 N	3% I/Y	\mathbf{PV}	\$40 PMT	\$1,000	
Solve for	N	1/ Y	\$1,169.36	PMI	\mathbf{FV}	
P ₃ Enter	20	3%		\$40	\$1,000	
Solve for	N	I/Y	PV \$1,148.77	PMT	\mathbf{FV}	
P ₈ Enter	10	3%		\$40	\$1,000	
Solve for	N	I/Y	PV \$1,085.30	PMT	FV	
P ₁₂ Enter	2	3%		\$40	\$1,000	
Solve for	2 N	I/Y	PV \$1,019.13	PMT	\$1,000 FV	
Modigliani Company						
P ₀ Enter	26	4%	DV.	\$30	\$1,000	
Solve for	N	I/Y	PV \$840.17	PMT	${f FV}$	
\mathbf{P}_1						

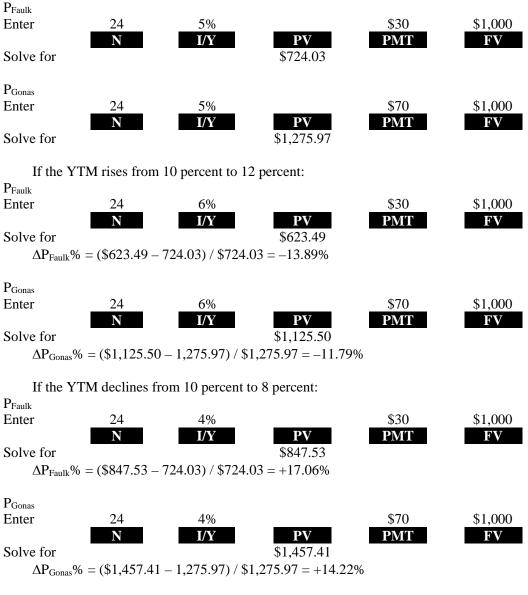
Enter	24	4%		\$30	\$1,000
Solve for	N	I/Y	PV \$847.53	PMT	\mathbf{FV}
P ₃ Enter	20 N	4% I/Y	PV	\$30 PMT	\$1,000 FV
Solve for	- \	1 / 1	\$864.10		- v
P ₈ Enter	10	4%		\$30	\$1,000
Solve for	N	I/Y	PV \$918.89	PMT	\mathbf{FV}
P_{12}					
Enter	2	4%		\$30	\$1,000
Solve for	N	I/Y	PV \$981.14	PMT	\mathbf{FV}

15. If both bonds sell at par, the initial YTM on both bonds is the coupon rate, 7 percent. If the YTM suddenly rises to 9 percent:



All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.

16. Initially, at a YTM of 10 percent, the prices of the two bonds are:



All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

17. Enter 18
$$\pm \$1,050$$
 $\$31$ $\$1,000$ Solve for 2.744% \times 2 = 5.49%

18. The company should set the coupon rate on its new bonds equal to the required return; the required return can be observed in the market by finding the YTM on outstanding bonds of the company.

Solve for 3.	±\$1,063 I/Y 218%		,000 F V			
$3.218\% \times 2 = 6.44\%$	D 04.040.00					
21. Current yield = $.0842 = $90/$	P_0 ; $P_0 = \$1,068.88$					
Enter 7	.81% ±\$1,068.88 I/Y PV		,000			
Solve for 8.0004 8 years	I/Y PV	PMT	FV			
22.						
Enter 18	±\$1,053.12 PV		,000 F V			
	226%	FIVII	. V			
24.						
a. Po	707 /0	Φ.1	000			
Enter 50 N	7%/2 I/Y PV		,000 TV			
Solve for	\$179.05					
<i>b</i> . P ₁						
Enter 48	7%/2 I/Y PV		,000 TV			
Solve for	\$191.81	LIVII	' V			
year 1 interest deduction = $$191.81 - 179.05 = 12.75						
P_{24}						
Enter 2 7	%/2%		,000			
Solve for	I/Y PV \$933.51	PMT	\mathbf{FV}			
year 25 interest deduction = $\$1,000 - 933.51 = \66.49						

- c. Total interest = \$1,000 179.05 = \$820.95Annual interest deduction = \$820.95 / 25 = \$32.84
- d. The company will prefer straight-line method when allowed because the valuable interest deductions occur earlier in the life of the bond.
- **25.** *a*. The coupon bonds have a 6% coupon rate, which matches the 6% required return, so they will sell at par; # of bonds = \$45,000,000/\$1,000 = 45,000.

For the zeroes:

Solve for

Enter 60 6%/2 \$1,000

N I/Y PV PMT FV

Solve for \$169.73

45,000,000/169.73 = 265,122 will be issued.

- b. Coupon bonds: repayment = 45,000(\$1,030) = \$46,350,000 Zeroes: repayment = 265,122(\$1,000) = \$265,122,140
- c. Coupon bonds: (45,000)(\$60)(1 .35) = \$1,755,000 cash outflow Zeroes:

Enter 58 6%/2 \$1,000

N I/Y PV PMT FV

Solve for \$180.07

year 1 interest deduction = \$180.07 - 169.73 = \$10.34(265,122)(\$10.34)(.35) = \$959,175.00 cash inflow

During the life of the bond, the zero generates cash inflows to the firm in the form of the interest tax shield of debt.

26. Bond P P_0 Enter 10 7% \$90 \$1,000 I/Y **PMT** Solve for \$1,140.47 \mathbf{P}_{1} Enter 7% \$90 \$1,000 N **PMT** Solve for \$1,130.30 Current yield = \$90 / \$1,140.47 = 7.89%Capital gains yield = (\$1,130.30 - 1,140.47) / \$1,140.47 = -0.89%

Bond D
P₀
Enter 10 7% \$50 \$1,000

\$859.53

P₁
Enter 9 7% \$50 \$1,000

N I/Y PV PMT FV

Solve for \$869.70

Current yield = \$50 / \$859.53 = 5.82%

Capital gains yield = (\$869.70 - 859.53) / \$859.53 = +1.18%

All else held constant, premium bonds pay a higher current income while having price depreciation as maturity nears; discount bonds pay a lower current income but have price appreciation as maturity nears. For either bond, the total return is still 7%, but this return is distributed differently between current income and capital gains.

27. a. Enter 10 $\pm \$930$ \$56 \$1,000 N I/Y PV PMT FV Solve for 6.58%

This is the rate of return you expect to earn on your investment when you purchase the bond.

b. Enter 5.58% \$56 \$1,000 Solve for \$1,001.44 The HPY is: Enter ±\$930 \$56 \$1,001.44 PV \mathbf{PMT} \mathbf{FV} Solve for 9.68%

The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

28. P_M

CFo C01 \$0 F01 12 **C02** \$800 F02 16 **C03** \$1,000 F03 11 **C04** \$31,000 F04 I = 4%NPV CPT

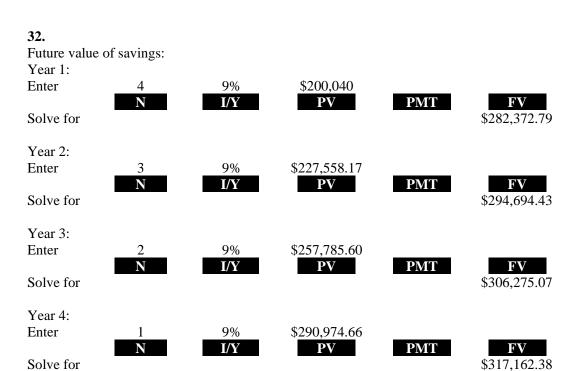
\$15,200.77

 P_N Enter \$30,000 40 4% **PMT** Solve for \$6,248.67 31. Real return for stock account: 1 + .12 = (1 + r)(1 + .04); r = 7.6923%7.6923% 12 C/Y NOM BIBIB Solve for 7.4337% Real return for bond account: 1 + .07 = (1 + r)(1 + .04); r = 2.8846%Enter 2.8846% 12 NOM IDIDID C/Y Solve for 2.8472% Real return post-retirement: 1 + .08 = (1 + r)(1 + .04); r = 3.8462%Enter 3.8462% BBBB C/Y NOM Solve for 3.7800% Stock portfolio value: Enter \$800 12×30 7.4337% / 12 **PMT** Solve for \$1,196,731.96 Bond portfolio value: \$400 Enter 12×30 2.8472% / 12 I/Y \mathbf{PV} **PMT** \mathbf{FV} Solve for \$170,316.78 Retirement value = \$1,196,731.96 + 170,316.78 = \$1,367,048.74Retirement withdrawal: 3.7800% / 12 Enter \$1,367,048.74 25×12 \mathbf{FV} Solve for \$7,050.75 The last withdrawal in real terms is: Enter 30 + 254% \$7,050.75

Solve for

PMT

\$60,963.34



Future value = \$282,372.79 + 294,694.43 + 306,275.07 + 317,162.38 + 327,400.96Future value = \$1,527,905.64

He will spend \$500,000 on a luxury boat, so the value of his account will be:

Value of account = \$1,527,905.64 - 500,000

Value of account = \$1,027,905.64

Enter 25 9% \$1,027,905.64 N I/Y PV PMT FV
Solve for \$104,647.22