

Overview and Learning Objectives

Overview

This chapter covers the historic return statistics of the U.S. capital market. In addition to defining return and risk, this chapter also provides quantifiable measures of return and risk for a single asset. Lastly, the risk-free rate and risk premiums are introduced.

In earlier chapters, we discuss how managers can achieve the goal of the firm, i.e., shareholders' wealth maximization, by accepting positive NPV projects. To compute NPV, we need future cash flows and a discount rate. So far, the discount rate is treated as a given. To estimate the appropriate discount rate, we have to first define the relationship between return and risk. In this chapter, we will look at the history of returns on different asset classes. Intuitively, and confirmed by history, investors demand higher returns for riskier investments.

The materials in this chapter involve applications of basic statistical concepts that you have learned in an introductory statistics course, an admission prerequisite for the webMBA program.


Learning Objectives

After reading course materials on this chapter, students should be able to:


- Calculate total dollar returns and percentage returns on investments.
- Define and compute the holding period return.
- Define and compute the arithmetic and geometric average returns, and explain their strengths and weaknesses.
- Identify the sample or historic return and risk measures and their calculations.
- Estimate future returns based on the return history of the U.S. capital market.
- Explain the concepts of risk premium and excess return, and the return-risk relationship among various asset classes and evidence from the U.S. capital market.

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Single Period Returns (Ref: Section 10.1)

Total dollar return has two components: cash distributions and capital gain/loss, i.e., change in value of the investment ( Slide).

- For bonds, cash distribution takes the form of coupon payments.
- For stocks, cash distribution takes the form of cash dividends.
- For mutual funds, cash distributions take the forms of dividend distribution and (realized) capital gain distribution.

Percentage return, R_{t+1} , is defined as the total dollar return divided by the beginning value of the investment, P_t . For stocks, it is composed of the dividend yield, i.e., D_{t+1}/P_t , and the capital gain yield, i.e., $(P_{t+1}-P_t)/P_t$ ( Slide).

- Percentage return is preferred to total dollar return when comparing among investment alternatives because this measure removes the scale effect in the total dollar return due to the amount of initial investment.

Let us consider this numerical example for total dollar return and percentage return on a stock:

Suppose you bought 100 shares of Wal-Mart (WMT) one year ago today at \$25. Over the last year, you received \$20 in dividends (dividend = 20 cents per share \times 100 shares). At the end of the year, the stock sells for \$30. How did you do?

You did pretty well! Your initial investment was $\$25 \times 100 = \$2,500$

Dividend = \$20 and capital gain = $\$(30-25) \times 100 = \500

- Total dollar return = $\$(20+500) = \520
- Percentage return = $\$520/\$2,500 = 20.8\%$.

- Dividend yield = $\$20/\$2,500 = 0.8\%$.
- Capital gain yield = $\$(30-25)/\$25 = 20\%$.

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Holding-Period Return (Ref: Sections 10.2 and 10.6)

Holding-period return is the cumulative return over the holding horizon of T years. It takes into account compounding interest in relating the beginning market value to the ending market value of the investment ([Slide](#)).

$$\text{Holding-Period Return} = (1 + R_1) \times (1 + R_2) \times \dots \times (1 + R_T) - 1$$

where R_i is the percentage return for year i .

$$\text{Ending Market value} = \text{Beginning Market Value} \times (1 + \text{Holding-Period Return})$$

There are two approaches to compute the average periodic return of the investment: geometric average return (R_G) and arithmetic average return (R_A).

The geometric average return is consistent with the buy and hold strategy. This strategy assumes that all interim investment returns are reinvested in the same investment. This measure provides the answer for the question: "What was your average compound return earned per year over a particular holding period of multiple years?"

$$\text{Geometric Average Return: } R_G = [(1 + R_1) \times (1 + R_2) \times (1 + R_3) \dots \times (1 + R_T)]^{1/T} - 1$$

The arithmetic average return assumes a rebalancing strategy. This strategy assumes that at the beginning of EACH period over the investment horizon, the investment will be reset to its initial amount. This measure answers the question: "What was your return earned in an average year over a particular holding period of multiple years?"

$$\text{Arithmetic Average Return: } R_A = \frac{(R_1 + R_2 + R_3 \dots + R_T)}{T}$$

The geometric average return ([Slide](#)) takes into account the effects of compounding whereas the arithmetic average return is a simple average. The geometric average return is usually less than the arithmetic average return ([Slide](#)) EXCEPT if there is **no** variation in percentage returns during the holding horizon, then both arithmetic and geometric average returns will be the same. The discrepancy between these two average return measures increases with the variation of percentage returns during the holding horizon.

For individuals who follow the buy and hold strategy in their investments, the geometric average is more appealing because using the geometric average return to compute future value will give the actual ending value. Using the arithmetic average return to compute future value will overstate the actual ending value because the effects of compounding are not included in the calculation of the arithmetic average return. Hence, we should not compare a geometric average return against an arithmetic average return.

However, most financial publications, including Stocks, Bills and Inflation: 20xx Yearbook, by Roger G. Ibbotson and Rex A. Sinquefeld, provide only arithmetic average returns ([Slide](#)). This practice is supported by the statistical property that the arithmetic average of a sample is an unbiased estimator of the population mean.

Further, the geometric average tells you the return you earned per year over the time period based on annual compounding. The arithmetic average tells you what you earned in an average year.

Hence, the appropriate average depends on the question you are asking. If you are using estimates of annual returns to determine future values, then the arithmetic average is probably too high if you have a

long horizon, and the geometric average is probably too low if you have a short horizon. The arithmetic average is probably best for short planning horizons, and the geometric average is probably best for very long planning horizons.

However, we can combine the information provided in both measures in forecasting the average annual rate of return for a T-period holding horizon, $R(T)$, by using the Blume's formula ([Slide](#)) :

$$R(T) = (T-1)/(N-1) * \text{Geometric Average} + (N-T)/(N-1) * \text{Arithmetic Average}$$

where N is the number of years of historical annual return data that the forecast is based on.

For example, over the last 86 years, the arithmetic and geometric average annual returns on U.S. large company stocks were, respectively, 11.8% and 9.8%. What is the average annual return forecast for U.S. large company stocks for the next ten years?

In this example, $N=86$ and $T=10$, hence, $R(10) = (10-1)/(86-1)*9.8\% + (86-10)/(86-1)*11.8\% = 11.59\%$! In other words, for the next 10 years, we forecast that U.S. large company stocks offer an annual rate of return of 11.59%.

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Return and Risk Statistics (Ref: Sections 10.3 and 10.5)

When we look to the past to learn about return and risk, we must keep in mind that history may not repeat itself. Historic, Ex Post, return and risk statistics are useful places to start but may not be sufficient for estimating discount rates to be used for evaluating future projects.

The historic (ex post) average return, \bar{R} , is the arithmetic average return:

$$\bar{R} = \frac{(R_1 + R_2 + R_3 \dots + R_T)}{T} \quad (= R_A)$$

Through out Chapter 10 and in most places in the text, the term average return refers to the arithmetic average return.

The historic (ex post) return variance:

$$\sigma^2 = \frac{\sum_{t=1}^T (R_t - \bar{R})^2}{T - 1}$$

where \bar{R} is the arithmetic average return

The historic (ex post) return standard deviation: $\sigma = \sqrt{\sigma^2}$

Both variance and standard deviation are measures of TOTAL risk, which is the appropriate risk consideration if we only invest in one security.

Let us consider the following sample of NYSE Composite Index returns as a numerical illustration regarding the historic return and risk statistics:

Year	Return on the NYSE Composite Index
1990	-8.84%
1991	28.45%
1992	4.82%
1993	8.00%
1994	-2.84%
1995	31.42%
1996	18.27%
1997	31.23%

Historic average (arithmetic) return:

$$\bar{R} = (-8.84\% + 28.45\% + 4.82\% + 8.00\% - 2.84\% + 31.42\% + 18.27\% + 31.23\%) / 8 = 13.81\%$$

Historic return variance:

$$\sigma^2 = [(-.0884 - .1381)^2 + (.2845 - .1381)^2 + (.0482 - .1381)^2 + (.0800 - .1381)^2 + (-.0284 - .1381)^2 + (.3142 - .1381)^2 + (.1827 - .1381)^2 + (.3123 - .1381)^2] / (8 - 1)$$

$$\sigma^2 = .0250$$

Historic standard deviation:

$$\sigma = 0.1582 = 15.82\%$$

Let us also examine the difference between geometric and arithmetic average returns.

Geometric mean return:

$$R_G = (.9116 \times 1.2845 \times 1.0482 \times 1.08 \times .9716 \times 1.3142 \times 1.1827 \times 1.3123)^{1/8} - 1$$

$$R_G = 0.1283 = 12.83\%$$

Note that for the NYSE Composite Index, a standard deviation of 15.82%, suggests there was quite a bit of variation in annual returns during the sample period. The difference between the arithmetic and geometric average returns was quite large, i.e., 13.81% versus 12.83%.

In the earlier section, we recognize that variance and standard deviation, which measure the dispersion of outcomes around the mean of a distribution, are measures of total risk. For the case of a normal distribution, these measures identify the percentage of the distribution that lies within $\pm 1 \sigma$ (68%), $\pm 2 \sigma$ (95%), and $\pm 3 \sigma$ (99.7%) from the mean [Slide 2](#). For example, if returns on the NYSE Composite Index are normally distributed with mean = 13.81% and $\sigma = 15.82\%$, there is a 68% chance that the actual return in a given year will be between -2.01%, i.e., 13.81% - 15.82%, and 29.63%, i.e., 13.81% + 15.82%. Another way to put this is that there is only a 32% chance that actual return in a given year will be lower than -2.01% or higher than 29.63%. In fact, there is only a 0.15% chance that the actual return will be below -33.65%, i.e., 13.81% - 3*15.82%, or higher than 61.27%, i.e., 13.81% + 3*15.82%.

Table 10.2 (slide 12) summarizes the annual return history of five major asset classes in the U.S. capital market as well as the annual inflation rate statistics over the period of 1926 – 2011. The statistics show a positive relationship between return and risk. For example, small company stocks generated an average annual return (16.5%) that was about three times that of long-term government bonds (6.1%). However, their level of total risk (32.5%) was over three times of those associated with long-term government bonds (9.8%).

From Table 10.2, we can compute the historic annual return statistics in real terms by applying the Fisher equation presented in Chapters 6 and 8.

Asset Classes	Nominal Average Return	Real Average Return	Standard Deviation
Large Company Stocks	11.8%	8.7%	20.3%
Small Company Stocks	16.5%	13.4%	32.5%
Long-Term Corporate Bonds	6.4%	3.3%	8.4%
Long-Term Government Bonds	6.1%	3.0%	9.8%
U.S. Treasury Bills	3.6%	0.5%	3.1%
Inflation	3.1%		4.2%

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Average Stock Returns and Risk-Free Returns

(Ref: Sections 10.4 and 10.7)

We assume that individuals are risk-averse investors, that means individual investors dislike risk and will not take risk unless they are adequately compensated for bearing risk. The compensation, i.e., the risk premium or excess return, takes the form of additional return that is over and above the risk-free rate of return.

Risk Premium (or Excess Return) = Returns on Risky Securities – Risk-free Return

Market Risk Premium = Return on the Stock Market – Risk-free Return

In the U.S., a treasury bill is considered a risk-free security and hence the yield on treasury bills is usually used as the risk-free return. treasury bills are U.S. federal government short-term debt that will mature in one year or less.

Based on the return and risk statistics presented in Table 10.2, the average return of the large-company stocks (stock market) was 11.8% and that of Treasury Bills was 3.6%, and hence the market risk premium was $(11.8\% - 3.6\%) = 8.2\%$. More examples of the calculation of risk premium on various asset classes:

- The *Risk Premium* is the added return (over and above the risk-free rate) resulting from bearing risk.
- One of the most significant observations of stock market data is the long-run excess of stock return over the risk-free return.
 - The average excess return from large company common stocks for the period 1926 through 2011 was: $8.2\% = 11.8\% - 3.6\%$
 - The average excess return from small company common stocks for the period 1926 through 2011 was: $12.9\% = 16.5\% - 3.6\%$
 - The average excess return from long-term corporate bonds for the period 1926 through 2011 was: $2.8\% = 6.4\% - 3.6\%$

The graph in this [slide](#) also shows a positive relationship between return and risk among various asset classes. This shows that the return history of the U.S. capital market is consistent with the intuition that rational risk-averse investors will demand a higher return for a riskier investment. In the next chapter, we will discuss the capital asset pricing model that provides a formal definition of the relationship between return and risk.

Let us now look at a numerical illustration on how we can use the historical information to help us estimate future return on a risky asset class: Suppose The Wall Street Journal announced that the current rate for one-year treasury bills is 2%. What is the expected return on the market of small-company stocks?

- Recall from the above examples that the average excess return from small-company common stocks for the period 1926 through 2011 was 12.9%
- Given a risk-free rate of 2%, we have an expected return on the small-company stocks of $(2\% + 12.9\%) = 14.9\%$

The U.S. Equity Risk Premium: Historical and International Perspectives (Section 10.7)

Over the period (1926-2011) examined, the U.S. equity premium has been quite large (8.2% for large cap stocks and higher for small cap stocks) compared to earlier years in the U.S., as well as to, a lesser extent, the premiums earned in foreign countries.

For example, using U.S. data from 1802, the historical equity risk premium was 5.4%. The overall world equity risk premium for 1900 to 2010 is 6.9%, versus 7.2% for the U.S.

A good estimate for the future risk premium in the U.S. may be 7%, although somewhat higher or lower estimates could also be considered reasonable.

2008: A Year of Financial Crisis (Section 10.8)

The S&P500 index plunged -37 percent in 2008, which is behind only 1931 at -43 percent. Moreover, there were 18 days during 2008 on which the value of the S&P changed by more than 5 percent. From 1956 to 2007 there were only 17 such days.

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