Proposition I (no tax world)

The market value of any firm is independent of its capital structure and is given by capitalizing its expected cash flows at the rate of return appropriate to its business risk class.

Equation of Proposition I (no tax world)

Value of the levered firm (V_L) = Value of the unlevered firm (V_U) = earnings before interest and taxes divided by weighted average cost of capital. In notation form we have:

$$V_L = V_U = EBIT / WACC$$

Where:

$$WACC = (B/(B+S))r_b + (S/(B+S))r_s$$

Note that this is the WACC equation that you learned in FIN 531 or its equivalent.

In simpler terms, the value of a firm is based on the present value of future expected operating income. How the firm is financed does not affect its value. Remember, this result is based on the simplifying assumptions made by Modigliani and Miller in their 1958 paper. This paper is cited in footnote 2 in Chapter 16 of your text.

An example is used below to illustrate MM's Proposition I.

Example: Equilibrium Values

	Unlevered Firm	**Levered Firm**
Net Operating Income	$500,000	$500,000
interest paid	$0	$150,000
Net Income	$500,000	$350,000
required return	10.00%	10.00%
total firm value	$5,000,000	$5,000,000
required ROE	10.00%	14.00%
MV equity	$5,000,000	$2,500,000

interest rate	na	6.00%
MV debt	$0	$2,500,000

The unlevered firm is financed with common equity only. Thus its interest expense is zero. The value of the unlevered firm is $5,000,000. The notation for the unlevered firm is V_u.

$$V_u = \$500,000/.10 = \$5,000,000$$

The levered firm is financed with 50% debt. Its interest expense can be calculated follows:

$$.06 * \$2,500,000 = \$150,000$$

The value of the levered firm's equity is $2,500,000. The notation for the levered firm is V_L.

$$S_L = \$350,000/.14 = \$2,500,000$$

Notice that the required return on equity (ROE) for the levered firm is 14%. When a firm adds debt financing to its capital structure, its equity becomes more risky. This is because the interest on the debt must be paid before the stockholders receive any cash flows. The stockholders have a lower priority claim to the firm's cash flows than the debtholders.

The overall weighted average cost of capital for the levered firm is the same as for the unlevered firm, 10%. The levered firm has more of the cheapest capital component, debt. This pushes the overall cost of capital down. However, the cost of equity rises as more debt is used to finance the firm. This pushes the overall cost of capital up. These two factors moving in opposite directions exactly cancel each other out. The overall cost of capital stays the same, in the 1958 MM model, as the debt level rises.

The WACC formula is applied to our example below:

$$.50\ (.06) + .50\ (.14) = .10$$

Modigliani and Miller argued that this situation would have to hold in equilibrium. The value of the unlevered firm would have to equal the value of the levered firm. If one was worth more than the other, investors would not buy shares of the one that was more expensive. Market forces would push its price down and the price of the cheaper firm up, until they were equal.

The example below shows a situation where values are not equal for the unlevered and levered firms.

Disequilibrium Values

	Unlevered Firm	Levered Firm
Net Operating Income	$500,000	$500,000
interest paid	$0	$150,000
Net Income	$500,000	$350,000
required return	10.00%	9.23%
total firm value	$5,000,000	$5,416,667
required ROE	10.00%	12.00%
MV equity	$5,000,000	$2,916,667
interest rate	na	6.00%
MV debt	$0	$2,500,000

V_U = $500,000/.10 = $5,000,000

V_L = $350,000/.12 + $150,000/.06 = $2,916,667 + $2,500,000 = $5,416,667

WACC = ($2,500,000/$5,416,667 * .06) + ($2,916,667/$5,416,667 * .12) = .0923

Here the levered firm (L) has a higher value than the unlevered firm (U). If investors can lever up on their own account (i.e., **Homemade Leverage**) and buy shares of the cheaper unlevered firm, they can make a riskless, or arbitrage, profit (i.e., **Capital Structure Arbitrage**). As investors seek risk-free profits, they will sell the "relatively overpriced" levered firm and buy the "relatively underpriced" unlevered firm in this example.

Example for Capital Structure Arbitrage:

Fred short sells 10% of the "overpriced" Levered Firm's equity for 0.1*$2,916,667 = $291,667, and partially finances the Capital Structure Arbitrage by borrowing 0.1*$2,500,000 = $250,000 (i.e., an amount equivalent to 10% of the levered firm's debt) on his personal account, i.e., Homemade Leverage. These transactions leave him

with $541,667. He uses this amount of cash to buy 10% of the "underpriced" Unlevered Firm for 0.1*$5,000,000 = $500,000. Notice that Fred is ahead by $41,667 with zero investment upfront! In other words, Fred constructs the Capital Structure Arbitrage that demands zero out-of-pocket investment but generates an arbitrage profit, i.e., the *first condition* of a successful Capital Structure Arbitrage!

Since Fred owns 10% of the "underpriced" unlevered firm, he claims 10% of its total net income of $500,000. Thus, $50,000 is his share of the net income. He must pay $15,000 (i.e., .06 * $250,000) of interest on his personal account, i.e., "homemade" leverage. This leaves him with $35,000. This amount perfectly offsets the amount of net income that he owes due to his short position in the "overpriced" levered firm's equity, i.e., 10%*$350,000=$35,000. As such, there is zero net cash flow afterwards, which is the *second condition* for a successful Capital Structure Arbitrage!

In summary, Fred constructed with ZERO investment the arbitrage portfolio by short selling the equity of the relatively overpriced Levered Firm, investing in the equity of the relative underpriced Unlevered Firm, and financing the arbitrage portfolio with personal borrowing, i.e., Homemade Leverage. By doing so, Fred collected the arbitrage profit of $41,667! This fulfils the *first condition* for a successful Capital Structure Arbitrage! Fred also satisfied the *second condition* for a risk-free Capital Structure Arbitrage in that the net cash flow for his arbitrage portfolio is ZERO!

Other investors will see this opportunity to earn an arbitrage profit and also sell Firm L. The selling activity will push the price of Firm L down. The new demand for Firm U will push its price up. The economy will be in equilibrium when both firms have the same price despite different capital structures. This proves the **Capital Structure Irrelevance conclusion of MM Proposition I in a no-tax world**!

Here is a summary of the procedures and conditions for constructing a successful **Capital Structure Arbitrage**:

1. Calculate V_U ; S_L ; B ; and V_L
2. Determine the direction of relative misvaluation of Firm U versus Firm L
3. Construct the Capital Structure Arbitrage strategy to reap the (positive) arbitrage profit associated with the relative misvaluation of the two firms by taking the appropriate opposite positions in the two

firms along with the homemade leverage/unleverage with ZERO up-front investment, i.e., the ***first condition***

4. Show that there is ZERO net cash flow generated from your arbitrage portfolio afterwards, i.e., the ***second condition***