COMPSCI 590N
Lecture 10: Probability 2

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Outline

Histograms and Counting

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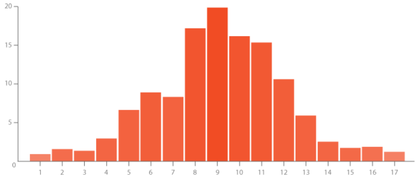

Histograms and Counting

Numpy provides functions for counting and computing histograms.

```
>>> import numpy as np
>>> A = np.random.randn(1000)
>>> np.histogram(A,nbins=5)
>>> counts,bin_edges = np.histogram(A,bins=7)
>>> counts
array([ 38, 180, 381, 289,  98,  11,   3])
>>> bin_edges
array([-2.75925765, -1.77917762, -0.7990976 ,
    0.18098242,  1.16106244,
        2.14114246,  3.12122248,  4.1013025 ])
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  - `Seaborn` is an extension to `matplotlib` design explicitly for plotting distributions.

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- **Many** other randomized algorithms.

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- Example: To generate samples from a Bernoulli distribution (binary), flip a bunch of coins. The result of each coin flip is a sample from the Bernoulli.

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- Slice sampling.
- The list goes on.

Cumulative Distribution Functions

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Cumulative Distribution Function

The CDF $F(x)$ for a distribution $p(x)$ is given by,

$$F(x) = \int_{-\infty}^{x} p(x')dx'$$

for continuous variables and by

$$F(x) = \sum_{x'=-\infty}^{x} P(x')$$

for discrete variables.

Cumulative Distribution Functions: Multinomial

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Inverse CDF Sampling

Let $F(x)$ be an invertible CDF. Then we can sample from the associated distribution by:

1. Sample $u^{(s)}$ from a Uniform(0,1).
2. $x^{(s)} = F^{-1}(U)$

Monte Carlo Integration

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- The core idea is based on approximating **expected values**.

Expected Values

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Expected Value

Given a distribution $p(x)$ over a random variable X and a function of the random variable $f : \Omega \to \mathbb{R}$, the expected valued of the function is defined as

$$\mathbb{E}[f(X)] = \int_{\Omega} p(x)f(x)dx$$

where the integral is a sum when X is discrete.

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$$\begin{aligned}\mathbb{E}[X] &= \sum_x P(X = x) \cdot X \\ &= \sum_{x=1}^{6} \frac{1}{6} \cdot x \\ &= \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6) = 3.5\end{aligned}$$

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$$\begin{aligned}\mathbb{E}[X^2] &= \int_x \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) x^2 dx \\ &= \frac{1}{\sigma\sqrt{2\pi}} \int_x \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) x^2 dx \\ &= \frac{1}{\sigma\sqrt{2\pi}} \int_x \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) x^2 dx\end{aligned}$$

The Importance of Expected Values

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  3. Sampling based approximations (Monte Carlo Integration).

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Let X be a random variable drawn from $p(x)$, let $f(x)$ be a function of X, and let $x^{(1)}, \ldots, x^{(S)}$ be samples from $p(x)$, then

$$\mathbb{E}(f(x)) \approx \frac{1}{S} \sum_{s} x^{(s)}$$

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- This works for multidimensional integrals as well.

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A circle is defined by $x^2 + y^2 = r^2$ so we can define a piecewise function that tells us whether we are inside or outside of the circle.

$$f(x,y) = \begin{cases} 1 & : x^2 + y^2 \leq r^2 \\ 0 & : x^2 + y^2 > r^2 \end{cases}$$

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The area of a circle is given by

$$A_r = \int_{-r}^{r} \int_{-r}^{r} f(x,y) dx dy$$

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- How do we sample uniformly from a square?
- Sample x and y independently from $Uniform(-r, r)$

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Now we can estimate the area of a circle as:

$$A_r = \int_{-r}^{r}\int_{-r}^{r} f(x,y)dxdy$$
$$\approx 2r^2 \frac{1}{S}\sum_{s} f(x^{(s)}, y^{(s)})$$

where

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  2. Pseudo-random number generation

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- The rate at which these can be harvested may be limited.

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- Many such sequences exist.
- Given the seed, computing pseudo-random number is extremely fast.

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```
>>> np.random.rand(5)
array([ 0.05321291,  0.7837193 ,  0.21213337,
    0.26499685,  0.4338131 ])
>>> np.random.randn(5)
array([-1.23805748, -0.18105506,  0.96736693,
    1.25355742, -0.08038946])
>>> np.random.randint(1,100,5)
array([28, 35, 32, 19, 94])
```

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Demo