COMPSCI 590N
Lecture 3: Classes and Representing Numbers

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Outline

Importing Functions

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import <module_name> # imports a module
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DEMO

Object Oriented Programming

DEMO

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```
class <class_name>:
    def __init__(self,<args>):
        self.<member_variable> = <expression>
        <body>

    def <member_function>(self,<args>):
        <body>
```

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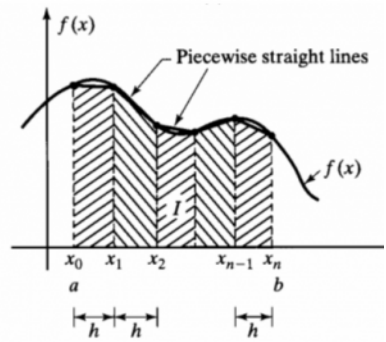
- **Instance variables**:
    - `name` and `tricks`
- **Methods:**
    - `bark`, `teach_trick`, and `do_trick`

Outline

Numerical Computing

Numerical computing is the approximation of continuous values and functions on a computer with finite precision.



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- The set of real numbers is continuous and uncountably infinite.
- But the set of numbers representable by a computer is finite...

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- Shift the mapping to store negative numbers: $0000000_2 = -125$ and $11111111_2 = 126$.

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Binary Integers

Given n bits, $b_1, ..., b_n \in \{0, 1\}$, we map binary values to integer values as follows:

$$(b_n b_{n-1} ... b_2 b_1 b_0)_2 = \sum_{i=0}^{n} b_i 2^i = x$$

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For example:

$$0101_2 = 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 5$$

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- This is equivalent to shifting the decimal.
- d is usually a multiple of 2.
- Restricted by the range of representable integers.
- **Observation:** Between 0 and 1, we would usually like a finer discretization, but between 1000 and 1001, we may be ok with a rough discretization.

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Floating Point

Rewrite a number, x, in scientific notation $x = a \cdot 10^b$. Then, x can be stored by storing a as a fixed point and b as an integer. Floating point numbers use a fixed number of digits for a, known as the *mantissa*, and b, known as the *exponent*:

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- Floating point numbers naturally have finer granularity nearer zero.

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  - The distance between 1.0 and the next largest number is $\approx 10^{-16}$.
- Because we can only represent finite numbers, we must rely on rounding and approximation which can lead to errors if you are not careful.

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- Incorrect timestamps were then used to incorrectly calculate distance and speed of an incoming missile.