Exam-1

Show clear works for partial credits. Typeset your answer. Feel free to ask questions if there is anything unclear.

Qn1: State the main characteristics that one should take into account while running a longitudinal data analysis.

Qn2: Derive the contrast for equality of the difference between the average response at occasions 2 through n and the baseline value in the two groups when the regression coefficient is $\boldsymbol{\kappa} = (\alpha, \theta, \beta_2, \ldots, \beta_n, \gamma_2, \ldots, \gamma_n)$ for the model $y_{i,j,g} = \alpha + \theta g + \beta_j + \gamma_j * g + \epsilon_{i,j}$ for $j = 1, \ldots, n$ and $g = 0, 1$ with $\beta_1 = \gamma_1 = 0$. (For reference 'the average response at occasions 2 through n and the baseline value' is discussed in 199 of the slides pdf. However, the contrast stated in the slide will change here and should be expressed based on $\boldsymbol{\kappa}$ as in the practice problem for AUCMB.)

Qn3: What are the appropriate covariance models for Orthodontic Measurements on Children data based on the observation times? First, examine the covariance by itself. Later fit the linear mixed model from Qn 2 with all possible covariance structures that are reasonable and compare either using LRT or AIC and get the most suited model. While fitting the linear mixed with random effects, consider adding both a random intercept and slope with respect to time.

Qn4: Compare linear mixed model and two-stage analysis for the Orthodontic Measurements on Children data to study the effect of 'sex' on 'dental growth'. Considering g in Qn2 as 'sex', compute the contrast-based effect of 'sex' and test for its significance.

Qn5: Why random effect distribution is assumed to have 0 expectation? Why only time-varying predictors should be considered as covariates in individual-specific random effects?

<u>Answer</u> 1 Random effect expectation

The zero expectation of the random effects is by choice and it is only appropriate when a fixed effect term of the same predictor is also included in the model to estimate the population average effect. Like the following model, the random effect term $b_{2,i}$ *should not be assumed to have zero expectation, but the* *expectation of* $b_{1,i}$ *can be assumed to be zero as* α *will capture population average intercept.*

$y_{i,j} = \alpha + b_{1,i} + b_{2,i}t_{i,j} + \epsilon_{i,j}$

Time-varying predictors

Take the two models

1: $y_{i,j,g} = \alpha + \theta g + \beta t_{i,j} + b_{1,i} + b_{2,i}t_{i,j} + b_{3,i}g + \epsilon_{i,j,g}$ *with two groups* $g = 0$ *and* $g = 1$.

2: $y_{i,j,g} = \alpha + \theta g + \beta t_{i,j} + b_{1,i} + b_{2,i}t_{i,j} + \epsilon_{i,j,g}$

A given subject can only belong to one of the two groups. $b_{1,i}, b_{2,i}, b_{3,i}$ *are all subject-specific terms. If subject 1 is group '0' its individual-specific intercept is* $b_{1,1}^{(1)}$ *and if subject 2 is group '1', its individual-specific intercept is* $b_{1,2}^{(1)} + b_{3,2}^{(1)}$ *under model 1. Under model 2, these terms are* $b_{1,1}^{(2)}$ *and* $b_{1,2}^{(2)}$ *respectively and thus* $b_{1,1}^{(2)} = b_{1,1}^{(1)}$ *and* $b_{1,2}^{(2)} = b_{1,2}^{(1)} + b_{3,2}^{(1)}$ *without any information loss. Since individual-specific specific treatment effects are usually not of any direct interest, model 2 and model 1 are equivalent.*