HW-1

Qn 2. Let there be two groups with 2 longitudinal data points for each subject with n subjects in each group. In notation, $X_{i,1}, X_{i,2}$ be the two visits for i-th subject in group 1 and $Y_{j,1}, Y_{j,2}$ be the two visits for j-th subject in group 2. Now, to compare the change over time, we usually look at the differences $\bar{X}_1 - \bar{X}_2$, where $\bar{X}_k = \frac{1}{n}\sum_i X_{i,k}$ and for cross-group difference, it is $\bar{X} - \bar{Y}$, where $\bar{X} = \frac{1}{2n}\sum_{i,k} X_{i,k}$ and $\bar{Y} = \frac{1}{2n}\sum_{i,k} Y_{i,k}$. Now compare the variances of $\bar{X}_1 - \bar{X}_2$ and $\bar{X} - \bar{Y}$ with and without correlation among the repeated measures from the same subject. Based on that, what would be its impact on the inferences if the repeated measures are positively correlated?

<u>Answer</u> 1 $V(\bar{X}_1 - \bar{X}_2) = V(\bar{X}_1) + V(\bar{X}_2) - 2cov(\bar{X}_1, \bar{X}_2)$

$V(\bar{X} - \bar{Y}) = V(\frac{\bar{X}_1+\bar{X}_2}{2} - \frac{\bar{Y}_1+\bar{Y}_2}{2}) = \frac{1}{4}\{V(\bar{X}_1) + V(\bar{X}_2) + 2cov(\bar{X}_1, \bar{X}_2) + V(\bar{Y}_1) + V(\bar{Y}_2) + 2cov(\bar{Y}_1, \bar{Y}_2)$ *and other terms* $cov(\bar{X}_\ell, \bar{Y}_k) = 0$ *due across subject independence.*

A positive correlation among the repeated measures will lead to a positive covariance, $cov(\bar{X}_1, \bar{X}_2) > 0$ *which will decrease the first variance, but increase the second one. Hence, having correlated measurements affects different test statistics differently.*