One-sample or paired tests

1   Examples

Example 1.7 (Spatial Ability Scores of Students)

- Data on a student's spatial ability using four tests of visualization.

- For each student, a single score representing their overall measure of spatial ability.

- The spatial ability scores for 68 female and 82 male high school students enrolled in advanced placement calculus classes in Florida.

- What is the distribution of spatial ability scores for the population represented by this sample of data?

- Does the distribution for the male students appear to possess different characteristics than that of the female students?

- These questions are problems in density estimation.

Example 1.8 (Sunspots)

- Data on mean monthly sunspot observations collected at the Swiss Federal Observatory in Zurich and the Tokyo Astronomical Observatory from the years 1749 to 1983.

- Excessive variability over time, obscuring any underlying trend in the cycle of sunspot appearances.

- No apparent analytical form or simple parametric model.

- Powerful method for obtaining the trend from noise in this case is wavelet estimation and thresholding.

2   Preliminaries

Fundamentals of Statistical Hypothesis Testing

A statistical hypothesis test is a formal procedure for using observed data to assess evidence against a null hypothesis about a population, model, or parameter.

Hypotheses.   A test involves two competing hypotheses:

$$H_0 \quad \text{(null hypothesis)}, \qquad H_1 \quad \text{(alternative hypothesis)}.$$

The null hypothesis represents the default or baseline assumption, while the alternative hypothesis represents a departure from H_0.

Test statistic. A test statistic is a function of the observed data,

$$T = T(X_1, \ldots, X_n),$$

constructed so that large or extreme values of T indicate evidence against H_0. The distribution of T under H_0 is known exactly or asymptotically.

Sampling distribution and rejection rule. Under the null hypothesis, the test statistic has a known sampling distribution. A rejection region is chosen so that

$$P_{H_0}(\text{reject } H_0) = \alpha,$$

where $\alpha \in (0, 1)$ is the significance level of the test.

p-value. The p-value is defined as the probability, computed under H_0, of observing a test statistic at least as extreme as the one obtained from the data. The null hypothesis is rejected if the p-value is less than or equal to α.

Errors and power. A Type I error occurs when H_0 is rejected while it is true, with probability α. A Type II error occurs when H_0 is not rejected while H_1 is true. The power of a test is the probability of correctly rejecting H_0 when H_1 is true.

Exact and asymptotic tests. Exact tests have null distributions that are valid for finite sample sizes. Asymptotic tests rely on limiting distributions that are valid as the sample size tends to infinity.

Parametric and nonparametric tests. Parametric tests assume a specific parametric form for the population distribution, while nonparametric tests avoid such assumptions. Some nonparametric tests are distribution-free, meaning their null distributions do not depend on the underlying population distribution.

2.1   Distribution-Free vs. Nonparametric Methods

Nonparametric methods. A statistical method is called *nonparametric* if it does not assume that the underlying population distribution belongs to a finite-dimensional parametric family (such as the normal family indexed by (μ, σ^2)). Instead, the distribution is treated as an unknown, possibly infinite-dimensional object. Nonparametric methods may still rely on structural assumptions such as independence, continuity, symmetry, or smoothness.

Examples: rank-based tests, kernel density estimation, nonparametric regression, spline smoothing, and the empirical distribution function.

Distribution-free methods. In classical usage, distribution-free methods are almost always statistical tests. A statistical test is called *distribution-free* if the null distribution of its test statistic does not depend on the underlying population distribution (within a specified class, such as continuous distributions). In this case, critical values and p-values are valid without knowing the exact form of the population distribution.

Examples: the sign test, Wilcoxon signed-rank test, Mann–Whitney U test, and the Kolmogorov–Smirnov test (under continuity).

Key distinction. Nonparametric refers to the *model* (no parametric family is assumed), whereas distribution-free refers to the *sampling distribution of the test statistic.* All classical distribution-free tests are nonparametric, but many nonparametric methods are not distribution-free.

2.2 Asymptotic Distribution-Free Tests

Many commonly used statistical tests are not distribution-free in finite samples, but become distribution-free *asymptotically.* Specifically, a test statistic T_n is called *asymptotically distribution-free* if

$$T_n \xrightarrow{d} T,$$

where the limiting distribution T does not depend on the underlying population distribution.

This phenomenon is typically a consequence of limit theorems such as the Central Limit Theorem, functional central limit theorems, or likelihood-based asymptotic results.

Examples:

- Wald or Z tests, where the standardized estimator converges to $N(0, 1)$.

- Likelihood ratio tests, which converge to a χ^2 distribution under regularity conditions (Wilks' theorem).

- Many rank- and permutation-based tests, whose limiting null distributions are universal.

Remark. Asymptotic distribution-freeness holds only under appropriate regularity conditions (e.g., independence, finite variance, smoothness). In finite samples, these tests generally depend on the underlying data-generating distribution.

Summary:

- Nonparametric methods avoid specifying a parametric family for the population distribution.

- Distribution-free methods have test statistics whose null distributions do not depend on the population distribution.

- Many parametric and nonparametric tests are distribution-free only in the asymptotic sense.

Distribution-free asymptotic test statistic

- Test statistic: $T(\cdot) = T(X_1, \ldots, X_n)$, a function of the data.

Example

$$T = \frac{\bar{X} - \mu}{s/\sqrt{n}}, \qquad \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \qquad s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n - 1},$$

where μ is known under H_0.

Distribution-free test statistic

- $T_1 = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

- $T_2 = \dfrac{\bar{X} - \mu}{s/\sqrt{n}} \sim t_{n-1}$

The distributions of T_1 and T_2 hold for any distributional properties of the data.

Nonparametric distribution-free test statistic

- The class U, $T(\cdot)$ is distribution free over contains more than one distributional forms.

- Distribution-free confidence interval, distribution-free multiple comparison procedure, distribution-free confidence band, asymptotically distribution-free test statistic, asymptotically distribution-free multiple comparison procedure, and asymptotically distribution-free confidence band.

3 Rank statistic

- **Absolute rank:** For any random variable $Z_1, \ldots, Z_n$, the absolute rank of Z_i, denoted by R_i, is the rank of $|Z_i|$ among $|Z_1|, \ldots, |Z_n|$.

- **Rank statistic:** A statistic $T(R)$ based only on the ranks of a sample is a rank statistic.

    - $T(R)$ is distribution-free over i.i.d. joint continuous distribution.

- **Signed rank:** The signed rank of Z_i is $R_i \psi_i$, where

$$\psi_i = \begin{cases} 1, & Z_i > 0, \\ 0, & Z_i < 0. \end{cases}$$

- **Signed rank statistic:** A statistic $T(\psi, R) = T(R_1\psi_1, \ldots, R_n\psi_n)$ that is a function of $Z_1, \ldots, Z_n$ only through the signed ranks is a signed rank statistic.

    - $T(\psi, R)$ is distribution-free over i.i.d. joint continuous distribution symmetric about 0.

4 Sign test (Fisher) – paired replicates data / one-sample data

Sign test

- $Z_1, \ldots, Z_n$ random sample from a continuous population that has a common median θ.

- If $Z_i \sim F_i$, then $F_i(\theta) = P(Z_i \leq \theta) = P(Z_i > \theta) = 1 - F_i(\theta)$.

- Hypothesis testing:
$$H_0 : \theta = 0 \qquad \text{versus} \qquad H_A : \theta \neq 0.$$

Sign test (Cont.)

- Sign test statistic:
$$B = \sum_{i=1}^{n} \psi_i.$$

- Motivation:
  - When θ is larger than 0, there will be larger number of positive Z_i's $\to$ big B value $\to$ reject H_0 in favor of $\theta > 0$.

- Under H_0, $B \sim (n, 1/2)$.

- Significance level α: probability of rejecting H_0 when it is true.

- Note:
  - choices of α are limited to possible values of the $B \sim (n, 1/2)$ cdf
  - compare the distribution of B under H_0 and the observed test statistic value.

Rejection regions

- $H_A : \theta > 0$, Reject H_0 if $B \geq b_{\alpha;n,1/2}$.

- $H_A : \theta < 0$, Reject H_0 if $B \leq n - b_{\alpha;n,1/2}$.

- $H_A : \theta \neq 0$, Reject H_0 if $B \geq b_{\alpha/2;n,1/2}$ or $B \leq n - b_{\alpha/2;n,1/2}$.

Large-Sample Approximation (Sign test)

$$B^* = \frac{B - E_0(B)}{V_0(B)^{1/2}} \sim N(0,1) \quad \text{as } n \to \infty,$$

where
$$E_0(B) = \frac{n}{2}, \qquad V_0(B) = \frac{n}{4}.$$

Rejection regions:

- $H_A : \theta > 0$, Reject H_0 if $B^* \geq z_\alpha$.

- $H_A : \theta < 0$, Reject H_0 if $B^* \leq -z_\alpha$.

- $H_A : \theta \neq 0$, Reject H_0 if $B^* \geq z_{\alpha/2}$ or $B^* \leq -z_{\alpha/2}$.

Ties (Sign test)

- Discard zero Z values and redefine n.

- If too many zeros, choose alternative statistical procedure (Chapter 10).

5 Example (Sign test)

Example (HWC: Chapter 3, Example 3.5, pg. 65) – paired sample sign test

- Beak-Clapping Counts.

- Subjects: chick embryos.

- X = average number of claps per minute during the dark period.

- Y = average number of claps per minute during the period of illumination.

- Test responsivity of a chick embryo to a light stimulus.

- $H_A : \theta > 0$.

```
df = data.frame(X = c (5.8,13.5,26.1,7.4,7.6,23,10.7,9.1,
19.3,26.3,17.5,17.9,18.3,14.2,55.2,15.4,30,21.3,
26.8 , 8.1 , 24.3 , 21.3,18.2,22.5,31.1),
Y = c(5,21,73,25,3,77,59,13,36,46,9,25,
59,38,70,36,55,46,25,30,29,46,71,31,33))
head(df)
```

```
library(dplyr)
df = mutate(df, Z= Y-X, Psi = ifelse(Z > 0 , 1 , 0 ) )
head(df)
```

- lower.tail=F provides $P(B > b_{\alpha=.05}) = .05$

```
qbinom(p = .05, size = length(df$Psi),
       prob = 1/2, lower.tail = FALSE)
```

- We need $P(B \geq b) = .05$. Therefore, Reject H_0 if $B \geq 18$. However, the significance level is not .05.

```
1 - pbinom((18-1), size = length(df$Psi),
           prob = 1/2, lower.tail = TRUE)
```

- Observed value of test statistic is

```
sum(df$Psi)
```

- We reject in favor of $\theta > 0$ at the $\alpha = .05$ level.

- Didn't use actual Z_i.

- Actual magnitude of the Z_i's will be necessary for distribution-free point and interval estimates of θ associated with sign test.

Built-in function `SIGN.test` in package `BSDA`

```
library(BSDA)
SIGN.test(df$Y, df$X, alt = "greater")
```

P-value using `pbinom` and large-sample approximation

```
1 - pbinom((21-1), size = length(df$Psi),
          prob = 1/2, lower.tail = TRUE)
B.star <- (21-25/2)/sqrt(25/4)
B.star
1-pnorm(B.star)
```

- Both the exact test and the large-sample approximation indicate that there is strong evidence that chick embryos are indeed responsive to a light stimulus, as measured by an increase in the frequency of beak-claps.

- To test $H_0 = \theta_0$, compute $Z_1 - \theta_0, \ldots, Z_n - \theta_0$ and do sign test on the Z's.

6 Parametric t-test

Q: Is rank test always less useful then parametric t-test?

- Let $Z_i \sim N(\theta, \sigma^2)$.

- $H_0 : \theta = 0$ versus $H_A : \theta > 0$.

- Test statistic:
$$T = \frac{\bar{Z} - \theta}{s/\sqrt{n}}.$$

- T is Studentized t-distribution with degrees of freedom $n - 1$.

- t_0: the observed value of test statistic.

- P-value: $P(T \geq t_0)$.

7 Wilcoxon signed rank test

Assumptions

$$Z_i = Y_i - X_i \sim F, \quad \text{where } F \text{ is symmetric about common median } \theta.$$

Test statistic

Let $S_i = \text{Sign}(Z_i)$

$$T^+ = \sum_{i=1}^{n} R_i S_i,$$

sum of positive signed ranks.

- no-closed form distribution.

- use iterative algorithms.

We have $T^+ + T^- = \frac{n(n+1)}{2}$. Under null, we must have $T^+ = T^-$ and thus if null is true, we should have $T^+ = T^- \approx \frac{n(n+1)}{4}$. This forms the basis for the following rejection regions.

Rejection regions

- $H_A : \theta > 0$, Reject H_0 if $T^+ \geq t_\alpha$.

- $H_A : \theta < 0$, Reject H_0 if $T^+ \leq \dfrac{n(n+1)}{2} - t_\alpha$.

- $H_A : \theta \neq 0$, Reject H_0 if $T^+ \geq t_{\alpha/2}$ or $T^+ \leq \dfrac{n(n+1)}{2} - t_{\alpha/2}$.

Ties

- Discard zero values among the Z_i's.

- If there are ties, assign each of the observations in a tied group the average of the integer ranks that are associated with the tied group.

- not exact test

8 Theoretical distribution of T^+

Large-Sample Approximation of the Wilcoxon Signed-Rank Test

Setup

Let
$$Z_1, \ldots, Z_n \overset{\text{i.i.d.}}{\sim} F$$
be paired differences (or one-sample centered observations). We consider the hypothesis
$$H_0 : \ \text{median}(Z) = 0 \quad \text{versus} \quad H_1 : \ \text{median}(Z) \neq 0.$$
Assume throughout that under H_0, the distribution of Z_i is continuous and symmetric about zero, so that
$$\mathbb{P}(Z_i = 0) = 0 \quad \text{and ties occur with probability zero.}$$
Define
$$A_i = |Z_i|, \qquad S_i = \text{sign}(Z_i) \in \{-1, +1\},$$
and let R_i denote the rank of A_i among $A_1, \ldots, A_n$, so that
$$R_i \in \{1, \ldots, n\}, \qquad \sum_{i=1}^{n} R_i = \frac{n(n+1)}{2}.$$

The Wilcoxon signed-rank statistic is defined as
$$T = \sum_{i:Z_i>0} R_i = \sum_{i=1}^{n} R_i \mathbf{1}\{S_i = +1\}.$$

Key Distributional Property Under H_0

Under H_0, conditional on the absolute values $(A_1, \ldots, A_n)$ (and hence on the ranks $(R_1, \ldots, R_n)$), the signs $S_1, \ldots, S_n$ are independent Rademacher random variables:

$$\mathbb{P}(S_i = +1 \mid A_1, \ldots, A_n) = \mathbb{P}(S_i = -1 \mid A_1, \ldots, A_n) = \frac{1}{2}.$$

Equivalently,

$$X_i := \mathbf{1}\{S_i = +1\} \quad \text{are i.i.d. Bernoulli} \left(\frac{1}{2}\right),$$

independent of the ranks.

Mean of the Signed-Rank Statistic

Conditioning on the ranks,

$$\mathbb{E}[T \mid R_1, \ldots, R_n] = \sum_{i=1}^{n} R_i \mathbb{E}[X_i] = \frac{1}{2} \sum_{i=1}^{n} R_i = \frac{n(n+1)}{4}.$$

Since this quantity is nonrandom,

$$\boxed{\mathbb{E}_0[T] = \frac{n(n+1)}{4}.}$$

Variance of the Signed-Rank Statistic

Again conditioning on the ranks and using independence,

$$\text{Var}(T \mid R_1, \ldots, R_n) = \sum_{i=1}^{n} R_i^2 \, \text{Var}(X_i) = \frac{1}{4} \sum_{i=1}^{n} R_i^2.$$

Using the identity

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6},$$

we obtain

$$\boxed{\text{Var}_0(T) = \frac{n(n+1)(2n+1)}{24}.}$$

Asymptotic Normal Approximation

Write the centered statistic as

$$T - \mathbb{E}[T \mid R_1, \ldots, R_n] = \sum_{i=1}^{n} R_i \left(X_i - \frac{1}{2}\right).$$

This is a sum of independent, mean-zero random variables with total variance

$$s_n^2 = \frac{n(n+1)(2n+1)}{24}.$$

Since $\max_i R_i = n$ and $s_n^2 = O(n^3)$, the Lindeberg condition holds, and by the Lindeberg–Feller central limit theorem,

$$\frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} \xrightarrow{d} \mathcal{N}(0,1).$$

Thus, for large n,

$$\boxed{T \approx \mathcal{N}\left(\frac{n(n+1)}{4}, \frac{n(n+1)(2n+1)}{24}\right) \quad \text{under } H_0.}$$

Continuity Correction

Because T is discrete, a continuity-corrected test statistic is often used:

$$Z_{\text{cc}} = \frac{T - \frac{n(n+1)}{4} \mp 0.5}{\sqrt{\frac{n(n+1)(2n+1)}{24}}},$$

where the sign is chosen to move the numerator toward zero.

Alternative Signed Form

Define the signed-rank sum

$$W = \sum_{i=1}^{n} R_i S_i.$$

Since $S_i = 2X_i - 1$,

$$W = 2T - \frac{n(n+1)}{2}.$$

Under H_0,

$$\mathbb{E}[W] = 0, \qquad \text{Var}(W) = \frac{n(n+1)(2n+1)}{6},$$

and

$$\frac{W}{\sqrt{\frac{n(n+1)(2n+1)}{6}}} \xrightarrow{d} \mathcal{N}(0,1).$$

Remarks on Zeros and Ties

If some $Z_i = 0$, they are removed and the above results apply with n replaced by $n' = \#\{i : Z_i \neq 0\}$. If ties occur among $|Z_i|$, average ranks are used and the variance is reduced accordingly.

Enumerate all 2^n possible outcomes for sample size three $n = 3$:

```
library(gtools)
x <- c(0,1)
df <- permutations(n=2, r=3, v=x,
                 repeats.allowed=TRUE) %>% data.frame
df
```

```
T.plus = apply(df, 1,
function(x){sum(x %*% seq(1,3))} )
df = mutate(df, T.plus = T.plus)
df
```

```
table(df$T.plus)/sum(table(df$T.plus))
```

9   Monte Carlo Simulation

Compare Monte Carlo simulation results with the theoretical results:

```
n = 3; nsim = 10000
Z = matrix(rnorm(n*nsim), ncol=n)
T.plus.mc = apply(Z, 1,
function(x) {sum(rank(abs(x)) * (x>0))} )
table(T.plus.mc)/nsim
```

10   Example (Wilcoxon signed rank test)

- Data are from nine patients who received tranquilizer.

- X (pre) factor IV value was obtained at the first patient visit after initiation of therapy.

- Y (post) factor IV value was obtained at the second visit after initiation of therapy.

- Test improvement due to tranquilizer that corresponds to a reduction in factor IV values.

```
pre = c (1.83 , .50 , 1.62, 2.48, 1.68, 1.88,
        1.55, 3.06, 1.30)
post = c (.878 , .647, .598, 2.05, 1.06, 1.29,
         1.06, 3.14, 1.29)
wilcox.test(post, pre, paired=TRUE,
          alterative = "less")
```

```
df <- data.frame(X=pre, Y=post)
df <- mutate(df, Z = Y-X, R=rank(abs(Z)),
          psi = ifelse(Z>0,1,0), Rpsi = R*psi)
df
```

P-value is $P(T^+ \leq 5)$:

```
psignrank(q=sum(df$Rpsi), n=9, lower.tail = TRUE)
```

- There is strong evidence that tranquilizer does lead to patient improvement at $\alpha = .05$, as measured by a reduction in the Hamilton scale factor IV values.

11   Point and interval estimates

- All three tests (sign test, Wilcoxon signed rank, and t-test) have an associated estimate and confidence interval for the location parameter θ.

- Order statistic: $Z_{(1)} < Z_{(2)} < \cdots < Z_{(n)}$.

- $Z_{(1)}$ is the minimum.

- $Z_{(n)}$ is the maximum.

- Quantile: equally spaced splitting points of continuous intervals with equal probabilities.

Point and interval estimate of θ associated with the sign test statistic

- median: $\tilde{\theta} = \text{median}\{Z_i, i = 1, \ldots, n\}$.

- Let $Z_{(1)}, \ldots, Z_{(n)}$ denote the ordered Z_i and if

    - n is odd, $\tilde{\theta} = Z_{(k+1)}$, where $k = (n-1)/2$.
    - n is even, $\tilde{\theta} = \dfrac{Z_{(k)} + Z_{(k+1)}}{2}$, where $k = n/2$.

- $100(1 - \alpha)\%$ confidence interval associated with two-sided test:

$$\left(Z_{(n+1-b_{\alpha/2;n,1/2})}, \ Z_{(b_{\alpha/2;n,1/2})} \right),$$

where $b_{\alpha/2;n,1/2}$ is the upper $\alpha/2$ percentile of the null distribution of B (sign test statistic).

Point and interval estimate of θ associated with the Wilcoxon signed rank statistic

- Hodges–Lehmann estimator:

$$\hat{\theta} = \text{median} \left\{ \frac{Z_i + Z_j}{2} : i \leq j, \ i, j = 1, \ldots, n \right\}.$$

- Walsh averages: $(Z_i + Z_j)/2$, $i \leq j$.

- $M = n(n+1)/2$ Walsh averages.

- $W_{(1)} \leq \cdots \leq W_{(M)}$ denote the ordered values of $(Z_i + Z_j)/2$.

- If

    - M is odd, $\hat{\theta} = W_{(k+1)}$, where $k = (M-1)/2$.
    - M is even, $\hat{\theta} = \dfrac{W_{(k)} + W_{(k+1)}}{2}$, where $k = M/2$.

- $100(1 - \alpha)\%$ confidence interval associated with two-sided test:

$$\left(W_{\left(\frac{n(n+1)}{2} + 1 - t_{\alpha/2} \right)}, \ W_{(t_{\alpha/2})} \right),$$

where $t_{\alpha/2}$ is the upper $\alpha/2$ percentile of the null distribution of T^+.

- The percentile points can be found using the R function `psignrank`.

12 Relationship between Wilcoxon signed rank test statistic and Walsh averages (Tukey (1949))

- HWC page 57, comment 17.

- Wilcoxon test statistic: $T^+ = \sum_{i=1}^{n} R_i \psi_i$.

- Number of Walsh averages greater than $\hat{\theta}$:

$$W^+ = \# \left\{ \frac{Z_i + Z_j}{2} > \hat{\theta} \right\}.$$

- Prove $T^+ = W^+$ by induction.

- Base of the Induction:

  - Assume that $\hat{\theta}$ is greater than all $Z_1, \ldots, Z_n$, then $\hat{\theta}$ is greater than all Walsh averages. Thus, $W^+ = 0$.
  - Then, $Z_i - \hat{\theta}$ are all negative. Thus, $T^+ = 0$.

Induction Steps

- Move $\hat{\theta}$ to the left passing through $Z_1, \ldots, Z_n$ one and two at the time and show that

  - W^+ changes value when moving past a Walsh average by the same amount.
  - T^+ changes value when
    * ranks of some $|Z_i - \hat{\theta}|$ change or
    * sign of some rank change by the same amount.

13 Comparison

- Power of a statistical test: the probability of rejecting the null hypothesis when it is false.

- The power of the sign test can be low relative to t-test.

- The power of signed-rank Wilcoxon test is nearly that of the t-test for normal distributions and generally greater than that of the t-test for distributions with heavier tails than the normal distribution.

Note: Read HWC page 71, comment 35 (power results for sign test).

14 Empirical power calculation

```
power.compute <- function(n = 30,
df = 2,
nsims = 1000,
theta = 0){
wil.sign.rank = rep(0, nsims)
ttest = rep(0,nsims)
```

```
Z = matrix((rt(n*nsims,df) + theta),
ncol = n,nrow = nsims)
wil.sign.rank = apply(Z, 1 , function(x){
wilcox.test(x)$p.value})
ttest = apply(Z, 1 , function(x){t.test(x)$p.value})
pow.wil.sign.rank = mean(wil.sign.rank <=.05)
pow.ttest = mean(ttest <=.05)
rt = c(pow.wil.sign.rank, pow.ttest)
names(rt) = c("Wilcoxon.signed.rank.power",
"t.test.power")
return(rt)
}
```

$\theta = 0$

```
power.compute.val = power.compute(n=30, df =2,
nsims =1000, theta = 0)
power.compute.val
```

$\theta = 0.5$

```
power.compute.val = power.compute(n=30, df =2,
nsims =1000, theta = 0.5)
power.compute.val
```

$\theta = 1$

```
power.compute.val = power.compute(n=30, df =2,
nsims =1000, theta = 1)
power.compute.val
```

15   Summary

- Assumptions on F_i:

  - Sign Test: any continuous distribution.
  - Signed-Rank Test: any symmetric continuous distribution.
  - t-test: any normal distribution.

- The continuity assumption assures that ties are impossible: With probability one we have $Z_i \neq Z_j$ when $i \neq j$.

- The continuity assumption is only necessary for exact hypothesis tests not for estimates and confidence intervals.