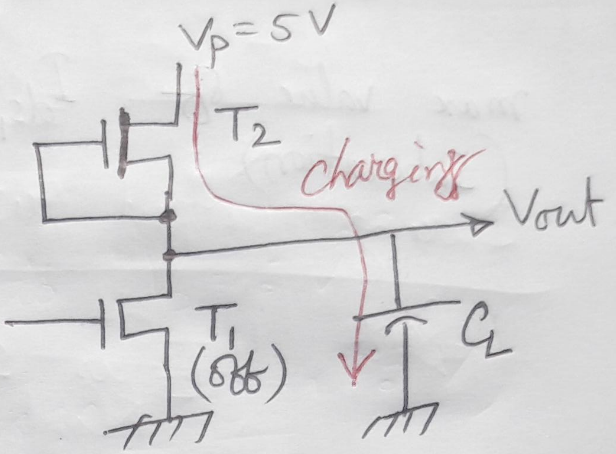
2.10 Edge times for NMOS inverter with a Depletion Load

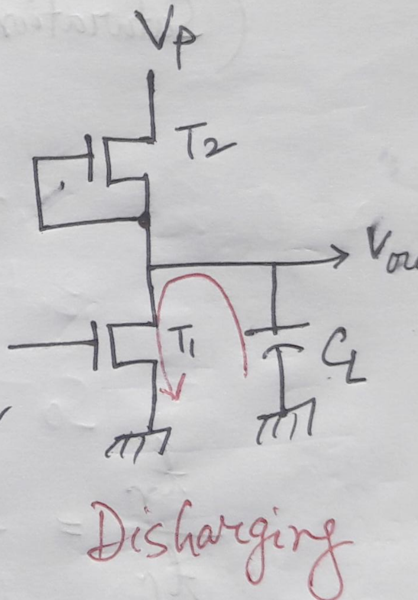


$\boxed{\text{Charging}}$ of Load capacitor (C_L) is done from V_P to C_L through transistor T_2.

$V_{in} = 0V$

Thus, $V_{out} = 5V$ after charging finishes.

So I_{ds_2} plays the role while charging.



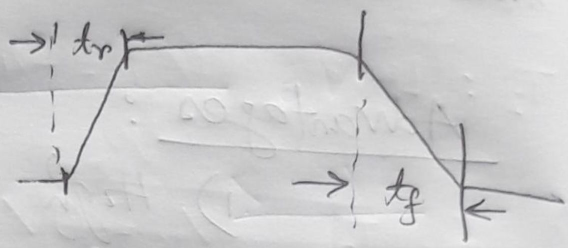
$\boxed{\text{Discharging}}$ When $V_{in} = 5V$, T_1 is on

$V_{in} = 5V$

Now C_L is discharged through T_1 to ground (0V)

So I_{ds_1} plays the role while discharging.

t_r = rise time
= output voltage rises from 0 to 5V



The more value of I_{ds_2},
the quicker V_{out} rises

So $t_r \propto \frac{1}{I_{ds_2}}$

Similarly $t_f \propto \frac{1}{I_{ds_1}}$
fall time

max value of I_{ds_1} (Saturation) $= \frac{\epsilon \mu_n}{D} \times \frac{W_1}{L_1} \times \frac{1}{2}(V_{gs_1} - V_t)^2$

$= 30 \times \frac{W_1}{L_1} \times \frac{1}{2}(5-1)^2$

$= 240 \left(\frac{W_1}{L_1}\right) \mu A$

max value of I_{ds_2} (Saturation) $= \frac{\epsilon \mu_n}{D} \times \frac{W_2}{L_2} \times \frac{1}{2}(V_{gs_2} - V_{td})^2$

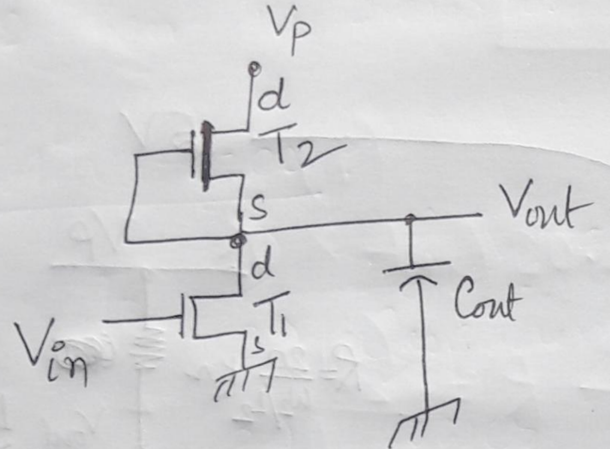
$= 25 \times \frac{W_2}{L_2} \times \frac{1}{2}(0+4)^2$

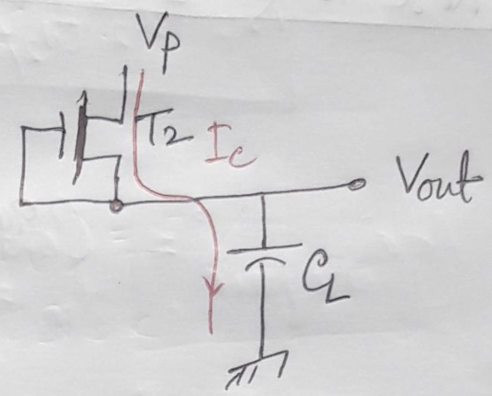
$= 200 \left(\frac{W_2}{L_2}\right) \mu A$

$$\frac{t_r}{t_f} = \frac{I_{ds_1}}{I_{ds_2}} = \frac{\overset{6}{\cancel{240}}}{\underset{5}{\cancel{200}}} \times \frac{(W_1/L_1)}{(W_2/L_2)}$$

$$\therefore \boxed{\frac{t_r}{t_f} = \frac{6K}{5}}$$

Let total output capacitance = C_{out}





Let $V_{in} = 5V$, $V_{out} = 0.3\,V$

Now if $V_i = 0.3V$, then

T₁ turns off (see red)

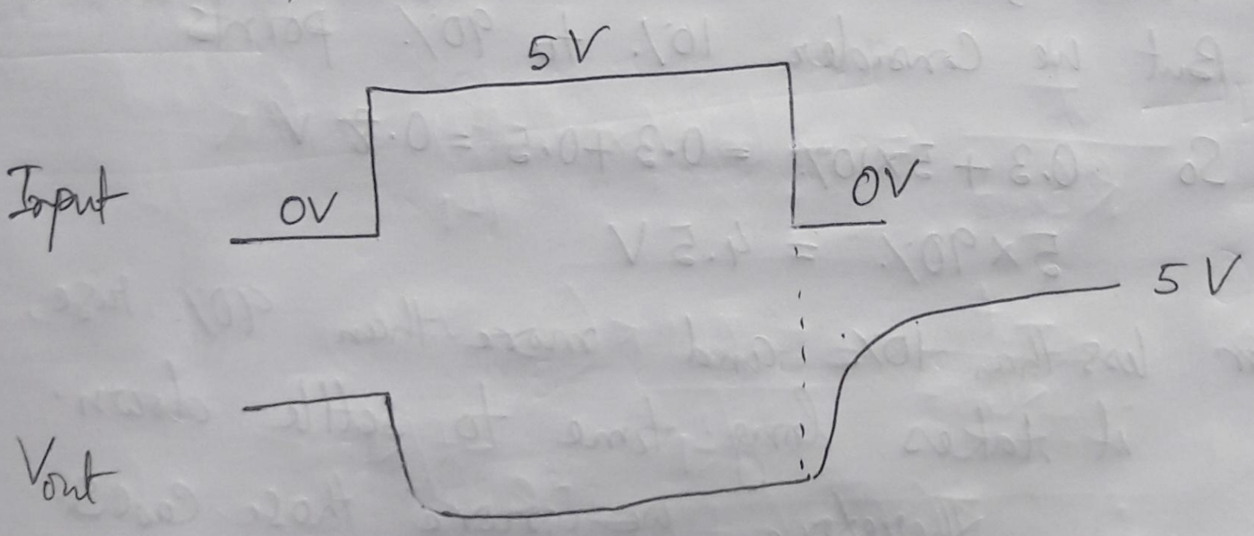
T₂ remains ON

C_L gradually charges to 5 V

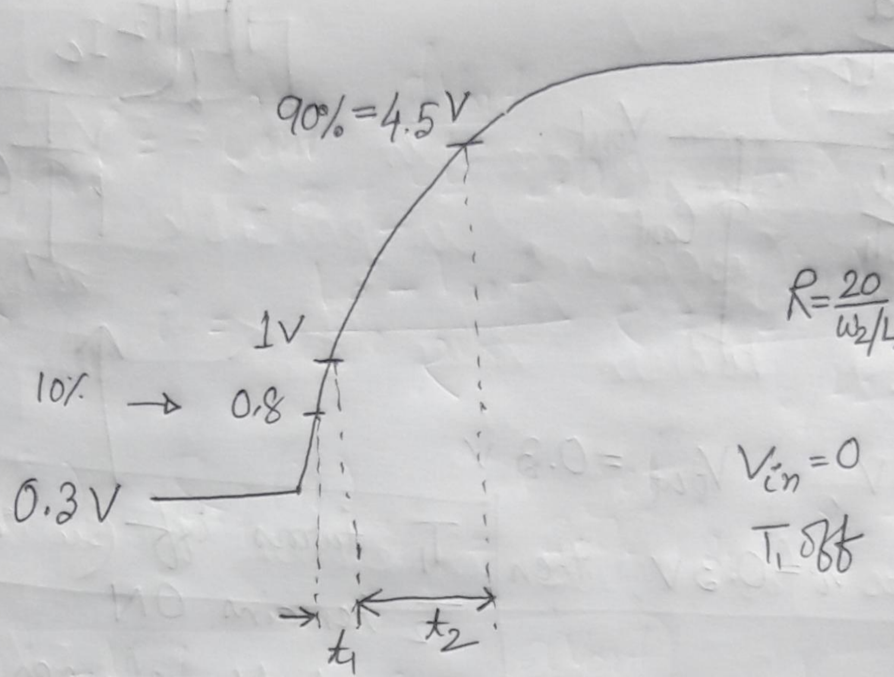
The current I_c is proportional to rate of change of output voltage

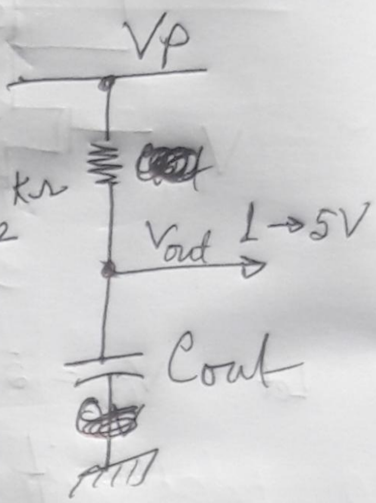
$$I_c = C_{out} \frac{dV_{out}}{dt}$$

The rise time depends on current I_c. Thus, in turn, the rise time depends on T_2



Rise time behavior





$$t_r = t_1 + t_2$$

During output rise

Case (i) T_2 is saturated, when $V_{out} = 0.8$ to $1\,V$

Case (ii) T_2 is in resistive region when $V_{out} = 1\,V$ to $5\,V$

* Output rises from 0.3 to 5V.
  But we consider 10% to 90% points

  So $0.3 + 5 \times 10\% = 0.3 + 0.5 = 0.8\,V$

  $5 \times 90\% = 4.5\,V$

  for less than 10% and more than 90% rise, it takes long time to settle down. Therefore, we ignore those cases.

(i) When $V_{out} = 0.8$ to 1.0 Volt

$V_{ds} = V_d - V_s = 5 - 0.8 = 4.2$

$V_{gs} - V_{td} = 0 + 4 = 4$

$\therefore V_{ds} > V_{gs} - V_{td}$

So T_2 is in sat^n region

(ii) When $V_{out} = \underset{\downarrow}{1.0}$ to 4.5

$V_{ds} = V_d - V_s = 5 - 1 = 4$ or less

$V_{gs} - V_{td} = 0 + 4 = 4$

$\therefore V_{ds} \leq V_{gs} - V_{td}$, So T_2 is in resistive

$t_r = t_1 + t_2$

$\approx t_2$

[t_1 very small, and sat^n current Cause t_1 to be very low

So resistive region

$$R = \frac{\text{pinch-off voltage}}{\text{pinch-off current}}$$

$$= \frac{(V_{gs} - V_{td})}{\frac{\epsilon \mu_n}{D} \times \frac{W_2}{L_2} \times \frac{1}{2}(V_{gs} - V_{td})^2}$$

$$= \frac{(0+4)}{25 \times 10^{-6} \times \frac{W_2}{L_2} \times \frac{1}{2}(0+4)^2} = \frac{20 \times 10^3}{W_2/L_2}\ \Omega$$

$$R = \frac{20}{W_2/L_2} \; k\Omega$$

Again $\quad i = C_{out} \frac{dV_{out}}{dt} \quad \ldots (i)$

$\Rightarrow \quad i = \frac{V_P - V_{out}}{R} \quad \ldots (ii)$

From (i) & (ii), we get

$$\frac{V_P - V_{out}}{R} = C_{out} \frac{dV_{out}}{dt}$$

$$\Rightarrow \quad \frac{dt}{RC_{out}} = \frac{dV_{out}}{V_P - V_{out}}$$

$$\int \frac{dt}{RC_{out}} = \int \frac{dV_{out}}{V_P - V_{out}}$$

Say $\ln K =$

$$\Rightarrow \quad \frac{t}{RC_{out}} = -\ln(V_P - V_{out}) - \ln K$$

$$\Rightarrow \quad \frac{-t}{RC_{out}} = \ln K(V_P - V_{out})$$

$$\therefore \; K(V_P - V_{out}) = e^{-t/RC_{out}} \quad \ldots (iii)$$

Now when $t=0$, $V_{out} = V_i$ — V_i = initial voltage across Capacitor C_{out}

$$\therefore \quad K(V_p - V_i) = e^{-0/RC_{out}} = 1$$

$$\therefore \quad K = \frac{1}{V_p - V_i}$$

putting value of K in equn (iii), we get

$$\frac{1}{V_p - V_i}\left(V_p - V_{out}\right) = e^{-t/RC_{out}}$$

$$\Rightarrow \quad V_p - V_{out} = \left(V_p - V_i\right) e^{-t/RC_{out}}$$

$$\Rightarrow \quad \boxed{V_{out} = V_p - \left(V_p - V_i\right) e^{-t/RC_{out}}} \quad \cdots (IV)$$

Now at time $t = t_2$ $V_{out} = 4.5\,V$, and initial voltage of $1V$ in Eqn (iv)

$$4.5 = 5 - (5-1)\, e^{-t_2/RC_{out}}$$

$$\Rightarrow \quad 4e^{-t_2/RC_{out}} = 5 - 4.5 = 0.5$$

$$\Rightarrow \quad e^{-t_2/RC_{out}} = \frac{0.5}{4}$$

$$\Rightarrow \quad \log_e e^{-t_2/RC_{out}} = \log_e\left(\frac{0.5}{4}\right)$$

$$\Rightarrow \quad -t_2/RC_{out} = -2.08$$

$$\therefore \boxed{t_2 = 2.08\, R\, C_{out}}$$

Therefore, $t_r \approx t_2 = 2.08\, C_{out} \times R$

putting value of $R = \frac{20}{W_2/L_2}\ k\Omega$

we get $t_r = 2.08\, C_{out} \times \frac{20}{W_2/L_2}$

$$\boxed{t_r = \frac{42\, C_{out}}{W_2/L_2}\ ns}$$

$R = k\Omega$
$C_{out} = $ pico Farad
$R C_{out} = k \times pico$
$= 10^{-9} = n$

$$\frac{t_r}{t_f} = \frac{6K}{5}$$

$$\Rightarrow t_f = \frac{5 t_r}{6K} = \frac{5}{6K} \times \frac{42\, C_{out}}{W_2/L_2}$$

$$= \frac{5}{\cancel{6}} \times \frac{\cancel{(W_2/L_2)}}{W_1/L_1} \times \frac{\overset{7}{\cancel{42}}\, C_{out}}{\cancel{(W_2/L_2)}}$$

$$\boxed{t_f = \frac{35\, C_{out}}{W_1/L_1}}$$