

বাংলাদেশ ইউনিভার্সিটি অব প্রফেশনালস

সেকশন/গ্রুপ... A (Section-A)



ইনভিজিলেটরের স্বাক্ষর

মোট পৃষ্ঠা সংখ্যা... 9 ...টি

BSc. in CSE-17, Final Exam (Fall), Dec-20 পরীক্ষা (Examination), 20 20

বিষয় (Subj): Applied Statistics and Queuing Theory পত্র/কোর্স নং (Paper/Course No): CSE-407

পত্র/কোর্সের নাম (Paper/Course Name): CSE-17 কেন্দ্র (Center): MIST

রেজিঃ নম্বর (Regn No): 131401170018 শিক্ষাবর্ষ (Session): 2019-2020

রোল নম্বর (Roll No): 201714018 তারিখ (Date): 23-12-2020

INSTRUCTIONS FOR EXAMINEE

পরীক্ষক কর্তৃক প্রণীত

- Examinees are forbidden to write their names either on outer cover page or anywhere of the answer scripts. In case of violation, the answer script will not be evaluated.
- Examinees must mention their roll and registration number along with session on the outer cover page of the answer scripts clearly. Otherwise, answer scripts may not be evaluated.
- Students will write his examination roll number on the top left corner and section-A/B on the top right corner of each page. All pages must be numbered chronologically at the bottom center in x of y format. (for example: 1 of 21)
- All rough works should be done in the same paper used as answer scripts. Answer scripts should be submitted intact. Papers used for rough work should be pen through by the examinees.
- In no case, an examinee will be allowed to start the examination half an hour after the commencement of examination.
- Examinees must abide by the instructions of chief invigilator if there are no definite instructions on any subject/matter.
- No examinee will be allowed to leave the examination session until an hour has elapsed from the commencement of examination.
- Legal action will be taken against the examinees those are caught for copying and found guilty for any breach of discipline as per rule.

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INSTRUCTIONS FOR EXAMINEE

9. Smoking is strictly prohibited during examination.
10. The Camera of the examinee MUST always be ON during the examination and answer script submission. If Camera is OFF then that online examination will be treated as CANCELLED.
11. The answer scripts submitted beyond specified time will be treated as CANCELLED.
12. The examinee has to share his/her computer screen to the invigilator throughout the examination time.
13. The focus of the camera should be such that the invigilator(s) can see the script and examinee with his/her surroundings.
14. The examinee will send his/her scanned examination script in PDF format to the following e-mail addresses:
 - (a) e-mail address of subject invigilator/examiner.
 - (b) Central Database Scheme (coursecode@mist.ac.bd)
Example: EECE433@mist.ac.bd
15. The examinee has to preserve the original answer script of every examination and be ready to submit whenever asked for.
16. Answer script should be the A4 size papers with a cover page provided by Department. Examinee has to fill up his/her necessary details on the cover page. Section A and section B must be clearly marked on the cover page like. **Section A** or **Section B**
17. Examination duration for each subject will be two hours (section-A for one hour + section B for One hour). In between students will get 20 minutes time to submit the answer script of section A and 10 minutes time to issue the question for section B . After completion of 01 hour examination time for section B, students will get 20 minutes to submit the answer script of section B.
18. After completion of written examination (online/physical), viva will be conducted by the respective faculty of that subject.

Section-AAns. to the ques. no.-01(a)

The given statement is known as the Inclusion-Exclusion Identity theorem.

We can prove the theorem simply first then the general theorem can be derived.

We know,

If, E and F are two events, then,

$$P(E) + P(F) = P(E \cup F) + P(EF)$$

$$\Rightarrow P(E \cup F) = P(E) + P(F) - P(EF) \quad \text{--- (1)}$$

Eqⁿ(1) is for two events unions together.

Now we calculate for three events E, F, G from eqⁿ(1) i.e.

$$\begin{aligned} P(E \cup F \cup G) &= P((E \cup F) \cup G) \\ &= P(E \cup F) + P(G) - P((E \cup F)G) \quad [\text{Eq}^n(1)] \\ &= P(E \cup F) + P(G) - P(EG \cup FG) \\ &= P(E) + P(F) - P(EF) + P(G) - P(EG \cup FG) \\ &= P(E) + P(F) + P(G) - P(EF) - P(EG) - P(FG) \\ &\quad + P(EGF) \\ &= P(E) + P(F) + P(G) - P(EF) - P(EG) - P(FG) \\ &\quad + P(EFG) \quad \text{--- (2)} \end{aligned}$$

From eqⁿ (1) and eqⁿ (2) Now we can generalize the theorem as following:

$$P\left(\bigcup_{i=1}^n E_i\right) = \sum_{i=1}^n P(E_i) - \sum_{i < j} P(E_i E_j) + \sum_{i < j < k} P(E_i E_j E_k) - \dots + (-1)^{n+1} P(E_1 E_2 \dots E_n)$$

- - - (3)

Eqⁿ (3) is called the Inclusion - Exclusion Theorem and which states that:

The probability of the union of n events equals to the sum of probabilities of these events taken one at a time, minus the sum of probabilities two at a time, plus sum of probabilities taken three at a time, and soon ...

Ans. to the ques. no.-01(b)

Let,

E be the event when test result is "Positive".
and,

D be the event when person has COVID-19.

So,
 D_c be the event when person is healthy.

We have to find,

person has the disease given that his
test result is positive.

So, $P(D|E)$ we have to find.

Here, Given,

$$P(E|D) = 96\% = 0.96$$

$$P(E|D_c) = 1.5\% = 0.015$$

$$P(D) = 0.9\% = 0.009$$

$$P(D_c) = 99.1\% = 0.991$$

Now,

$$\begin{aligned} P(D|E) &= \frac{P(E|D)P(D)}{P(E|D)P(D) + P(E|D_c)P(D_c)} \\ &= \frac{0.96 \times 0.009}{(0.96 \times 0.009) + (0.015 \times 0.991)} \\ &= 0.368 \end{aligned}$$

So, the probability of person has disease given test result is
positive = 0.368
(Ans)

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Ans. to the ques. no. - 01(c)

Given that,

Shahriar hit with probability, $P(S_h) = 0.7$

Samia hit with probability, $P(S_a) = 0.2$

Sujan hit with probability, $P(S_u) = 0.2$

They are independent so,

(i) Ans:

Given that exactly one shot hit the target, probability that it was

$$\text{Sujan's shot} = \underbrace{(1-0.7)}_{\text{Shahriar}} \times \underbrace{(1-0.2)}_{\text{Samia}} \times \underbrace{0.2}_{\text{Sujan.}}$$

$$= 0.048.$$

(ii) Ans:

Given that target is hit, probability that Samia hit it is = $(1-0.7) \times 0.2 \times (1-0.2)$

$$= 0.048.$$

Ans. to the ques. no.-03(a)

X is a random variable with PDF

given by:

$$f_X(x) = \begin{cases} cx^2 & |x| \leq 1 \\ \text{otherwise} & \end{cases}$$

We know,

$$F(x) = \begin{cases} \frac{1}{3-x} \\ 0 \end{cases}$$

Probability distribution function .

(1) Ans.)

So,

$$F(x) = \frac{d}{dx} (cx^2)$$

$$1 = c$$

Ans. to the ques. no. - 03(b)

Derivation of the Bayes' Formula for two mutually exclusive events E and F are given below:

We know that,

$$F \cup F_c = S$$

[S is Sample space and $P(S) = 1$]

So, we can write,

$$E = E(F \cup F_c) = EF \cup EF_c.$$

Here, EF and EF_c are mutually exclusive, so we can write from inclusion-exclusion theorem:

$$P(E) = P(EF \cup EF_c)$$

$$= P(EF) + P(EF_c) \quad \dots \quad (1)$$

Now from conditional probability we know:

$$P(E|F) = \frac{P(EF)}{P(F)}$$

$$\Rightarrow P(EF) = P(E|F) P(F)$$

Similarly, we can write:

$$P(EF_c) = P(E|F_c) P(F_c)$$

putting these two on Eqn (1) we get!

P.T.O.

$$\begin{aligned} P(E) &= P(E|F)P(F) + P(E|F_c)P(F_c) \\ &= P(E|F)P(F) + P(E|F_c)(1 - P(F)) \end{aligned}$$

And that's the Bayes' Formula for
E, F mutually exclusive two events.
with this we can write,

$$\begin{aligned} P(F|E) &= \frac{P(EF)}{P(E)} \\ &= \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F_c)(1 - P(F))} \end{aligned}$$

Ans. to the ques. no. - 03(c)

Let the event that 4 of us picked our own hats be respectively E_1, E_2, E_3, E_4 .

So,

$$P(E_1) = \frac{1}{4}$$

$$P(E_1, E_2) = P(E_2 | E_1) P(E_1)$$

$$= \frac{1}{3} \times \frac{1}{4}$$

$$= \frac{1}{12}$$

Let $E_1, E_2 = E$ So,

$$P(E_1, E_2, E_3) = P(E, E_3)$$

$$= P(E_3 | E) P(E)$$

$$= \frac{1}{2} \times \frac{1}{12}$$

$$= \frac{1}{24}$$

Let, $E_1, E_2, E_3 = F$ So,

$$P(E_1, E_2, E_3, E_4) = P(F, E_4)$$

$$= P(E_4 | F) P(F)$$

$$= 1 \times \frac{1}{24}$$

$$= \frac{1}{24}$$

$$P(E_1 \cup E_2 \cup E_3 \cup E_4) = P(E_1) + P(E_2) + P(E_3) + P(E_4) \\ - [P(E_1 E_2) + P(E_2 E_3) + P(E_3 E_4)] + P(E_1 E_4)$$

+

$$P(E_1 \cup E_2 \cup E_3 \cup E_4) = \sum_{i=1}^4 P(E_i) \\ - \sum_{i < j} P(E_i E_j) + \sum_{i < j < k} P(E_i E_j E_k) \\ - P(E_1 E_2 E_3 E_4)$$

$$= 4 \times \frac{1}{4} - 4 \times \frac{1}{12} + 2 \times \frac{1}{24} \\ - \frac{1}{24}$$

$$= \frac{3}{4}$$

$$\text{So, } \text{Ans}(\bar{E}_1) = 1 - \frac{3}{4} = \frac{1}{4}$$

So, none of them got ^(Ans) own hat at $\frac{1}{4}$ probability