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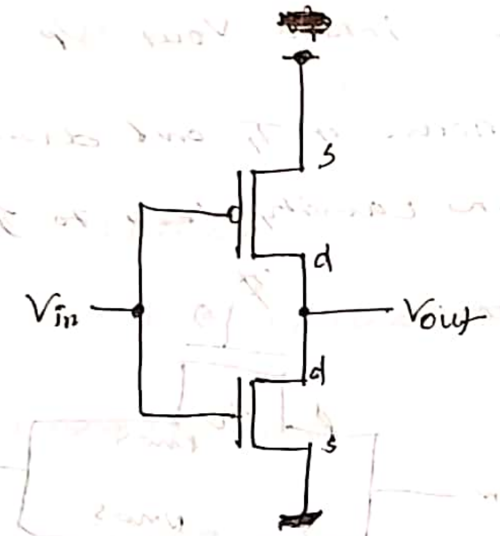
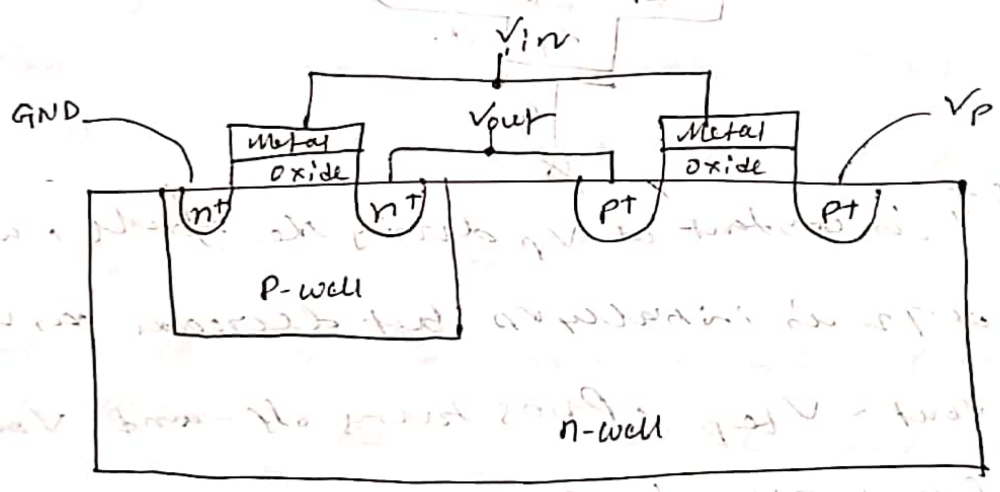
## CMOS power dissipation

1. Static
2. Dynamic
   Short Circuit

## Static Power Dissipation



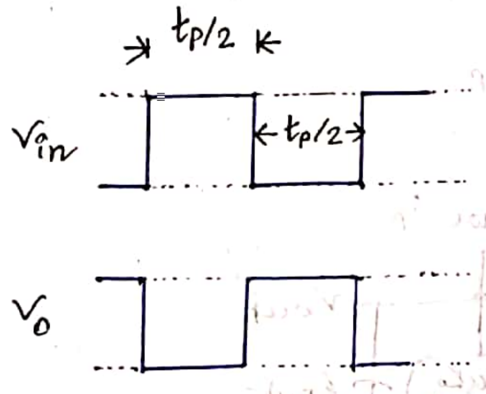



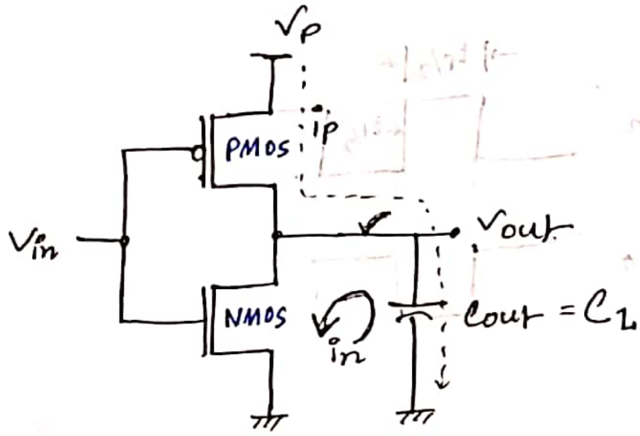

$$P_s = \sum_{i}^{n} (\text{leakage current} \times \text{supply voltage})$$

n = number of devices

leakage current = small amount of current in reverse bias

# Dynamic Power Dissipation




## Average Dynamic Power Dissipation, $P_d$

$$P_d = \frac{1}{t_p}\left[\int_0^{t_p/2} i_n(t) V_o \, dt + \int_{t_p/2}^{t_p} i_p(t)(V_p - V_o)\, dt\right]$$

$i_p = i$
$i_n = -i$

$$= \frac{1}{t_p}\left\{\int_0^{t_p/2} -i(t) V_o \, dt + \int_{t_p/2}^{t_p} i(t)(V_p - V_o)\, dt\right\}$$

$i(t) = C_L \frac{dV_o}{dt}$
$\Rightarrow i(t)dt = C_L dV_o$

$$= \frac{1}{t_p}\left\{\int_0^{t_p/2} -(i(t)dt)\, V_o + \int_{t_p/2}^{t_p} (i(t)dt)(V_p - V_o)\right\}$$

$t = 0 \rightarrow V_o = V_p$
$t = t_p/2 \rightarrow V_o = 0$
$t = t_p \rightarrow V_o = V_p$

$$= \frac{1}{t_p}\left\{\int_{V_p}^{0} -(C_L\, dV_o)\, V_o + \int_0^{V_p} (C_L\, dV_o)(V_p - V_o)\right\}$$

$$= \frac{C_L}{t_p}\left\{\int_0^{V_p} V_o \, dV_o + \int_0^{V_p} (V_p - V_o)\, dV_o\right\}$$

$$= \frac{C_L}{t_p}\left\{\int_0^{V_p} (V_o + V_p - V_o)\, dV_o\right\}$$

$$= \frac{C_L}{t_p}\left\{\int_0^{V_p} V_p \, dV_o\right\}$$

$$= \frac{C_L \cdot V_p}{t_p}\int_0^{V_p} dV_o$$

$$= \frac{C_L \cdot V_p}{t_p}\left[V_o\right]_0^{V_p}$$

$$= \frac{C_L V_P}{t_P}\,[V_P - 0]$$

$$= \frac{C_L \cdot V_P \cdot V_P}{t_P}$$

$$= \frac{C_L \cdot V_P^2}{t_P}$$

$$\boxed{\begin{aligned} P_d &= \frac{C_L \cdot V_P^2}{t_P} \\ P_d &= C_L \cdot f_P \cdot V_P^2 . \end{aligned}}$$

$$\frac{1}{t_P} = f_P$$

Proof & maths are required.

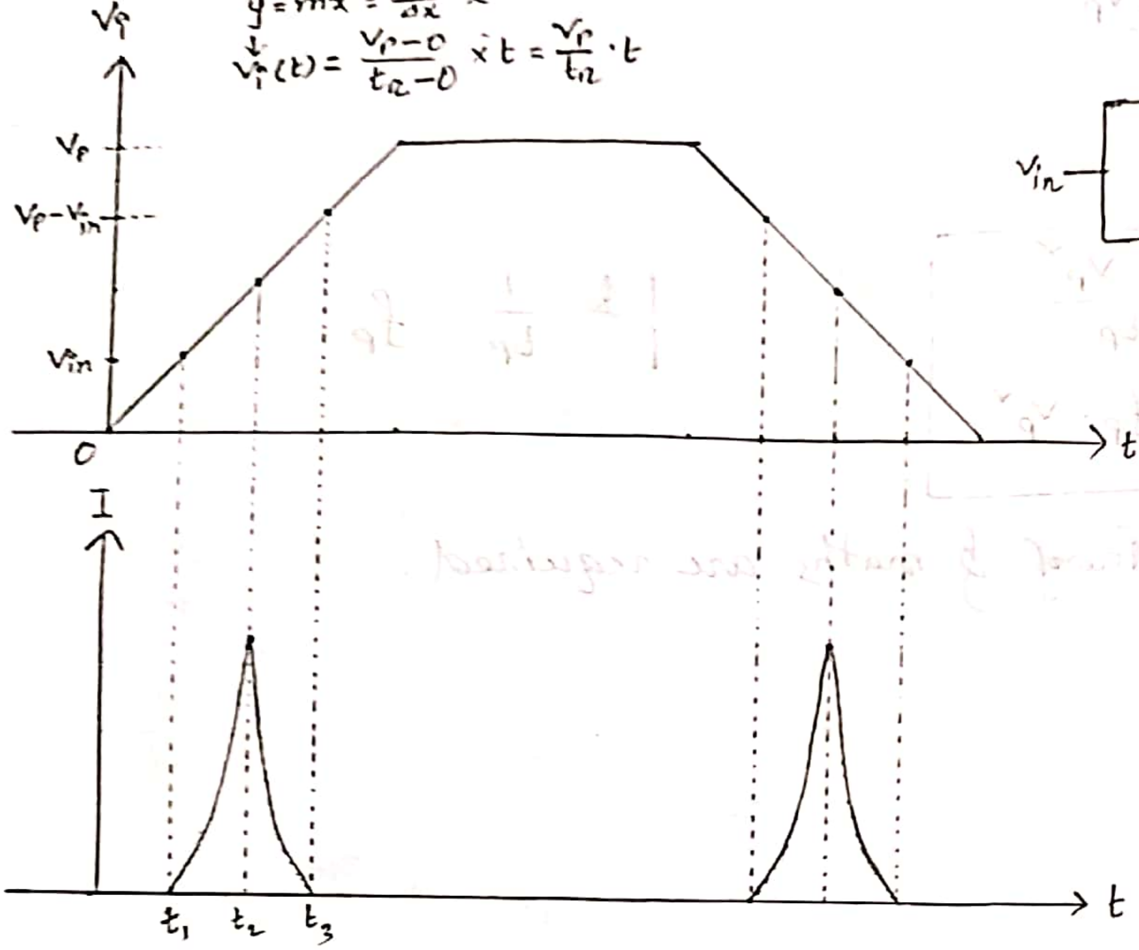
# Short Circuit Power Dissipation

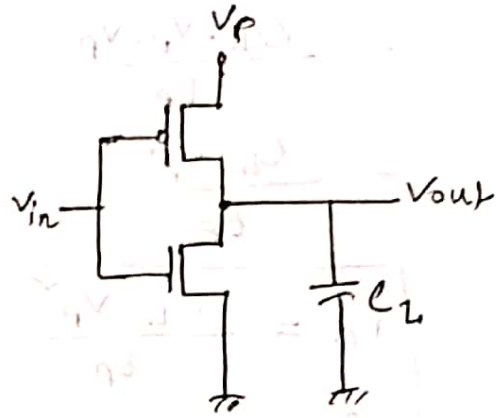
Q. When this happens? Ans:- When both nMOS and pMOS transistor are on.

Short circuit power dissipation $= I * V$

$$y = mx = \frac{\Delta y}{\Delta x} x$$

$$V_i(t) = \frac{V_p - 0}{t_{2} - 0} \times t = \frac{V_p}{t_2} \cdot t$$




$$I_{mean} = \frac{1}{t_p} \int_0^{t_p} I(t)\,dt$$

$$= \frac{2}{t_p} \int_0^{t_3} I(t)\,dt$$

$$= \frac{4}{t_p} \int_{t_1}^{t_2} I(t)\,dt$$

$$= \frac{4}{t_p} \int_{t_1}^{t_2} \left\{ \frac{\epsilon \mu_n}{D} \frac{W}{L} \frac{(V_{gs}(t) - V_{tn})^2}{2} \right\} dt$$

$$= \frac{4}{t_p} \left( \frac{\epsilon \mu_n}{D} \frac{W}{L} \right) \cdot \frac{1}{2} \int_{t_1}^{t_2} (V_{gs}(t) - V_{tn})^2\, dt$$

$$I_{mean} = \frac{2}{t_p} \beta \int_{t_1}^{t_2} (V_i(t) - V_{tn})^2 \, dt$$

let, $\frac{\epsilon \mu_n}{D} \cdot \frac{W}{L} = \beta$

Now, $V_i(t) = \frac{V_P}{t_r} \cdot t$

$t_2 = \frac{t_r}{2}$

When, $t = t_1$, $V_i = V_{tn}$

So, $V_{tn} = \frac{V_P}{t_r} \cdot t_1$

$\Rightarrow t_1 = \frac{V_{tn} \cdot t_r}{V_P}$

So,

$$I_{mean} = \frac{2\beta}{t_p} \int_{\frac{V_{tn} \cdot t_r}{V_P}}^{t_r/2} \left( \frac{V_P}{t_r} \cdot t - V_{tn} \right)^2 dt$$

$$= \frac{2\beta}{t_p} \left[ \frac{\left( \frac{V_P}{t_r} t - V_{tn} \right)^3}{3 \cdot \frac{V_P}{t_r}} \right]_{\frac{V_{tn} \cdot t_r}{V_P}}^{t_r/2}$$

$$= \frac{2\beta}{t_p} \frac{t_r}{3 \cdot V_P} \left[ \left( \frac{V_P}{t_r} t - V_{tn} \right)^3 \right]_{\frac{V_{tn} \cdot t_r}{V_P}}^{t_r/2}$$

$$= \frac{2\beta t_r}{3 t_p V_P} \left[ \left( \frac{V_P}{t_r} \cdot \frac{t_r}{2} - V_{tn} \right)^3 - \left( \frac{V_P}{t_r} \cdot \frac{V_{tn} \cdot t_r}{V_P} - V_{tn} \right)^3 \right]$$

$$= \frac{2\beta t_r}{3 t_p V_P} \left[ \left( \frac{V_P}{2} - V_{tn} \right)^3 - (V_{tn} - V_{tn})^3 \right]$$

$$= \frac{2\beta t_r}{3 t_p V_P} \cdot \left( \frac{V_P}{2} - V_{tn} \right)^3$$

$$= \frac{2\beta t_r}{3 t_p V_P} \cdot \frac{1}{2^3} (V_P - 2V_{tn})^3$$

$$\boxed{I_{mean} = \frac{\beta t_r}{12 t_p V_P} (V_P - 2V_{tn})^3}$$

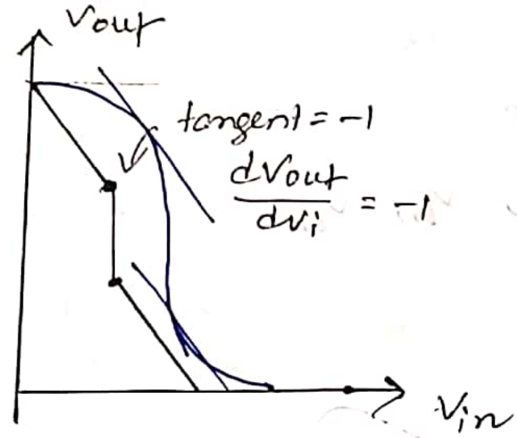
→ Short Circuit Power Dissipation Average Current.

$$P_{sc} = I_{mean} \times V_P$$

$$\boxed{\Rightarrow P_{sc} = \frac{\beta t_r}{12 t_p} (V_P - 2V_{tn})^3}$$

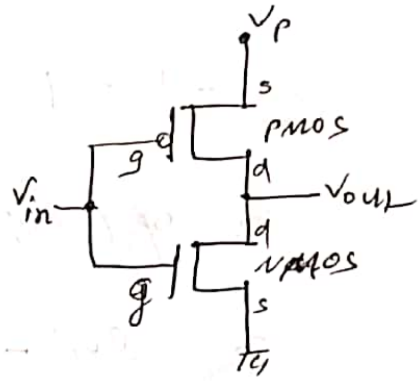
# NOISE MARGIN (CMOS)

$V_{IL}$ = Highest voltage reliably recognized as LOW.




Blue graph → correct

NMOS → Saturation
PMOS → Resistive




$$I_P\,(res) = I_N\,(sat^n)$$

$$\Rightarrow \frac{\epsilon \mu_P}{D}\frac{W_P}{L_P}\left[(V_{sg}-V_{tP})V_{sd}-\frac{V_{sd}^2}{2}\right] = \frac{\epsilon \mu_N}{D}\frac{W_N}{L_N}\frac{(V_{gs}-V_{tn})^2}{2}$$

now,

$$\mu_P = \frac{1}{2}\mu_N$$

$$\Rightarrow \mu_P/\mu_N = \frac{1}{2}$$

let, $\frac{W_P}{L_P} = 2 \cdot \frac{W_N}{L_N}$

$$\frac{W_P/L_P}{W_N/L_N} = 2$$

$$\Rightarrow \frac{1}{2}\cdot 2\left[(V_{sg}-V_{tP})V_{sd}-\frac{V_{sd}^2}{2}\right] = \frac{(V_{gs}-V_{tn})^2}{2}$$

$$\Rightarrow (V_P - V_i - V_{tP})(V_P - V_{out}) - \frac{(V_P - V_{out})^2}{2} = \frac{(V_i - V_{tn})^2}{2}$$

$$\Rightarrow 2(V_P - V_i - V_{tP})(V_P - V_{out}) - (V_P - V_{out})^2 = (V_i - V_{tn})^2$$

$$\Rightarrow \underbrace{(V_P - V_{out})^2}_{x} - 2(V_P - V_i - V_{tP})\underbrace{(V_P - V_{out})}_{x} + (V_i - V_{tn})^2 = 0$$

$$V_P - V_{out} = \frac{2(V_P - V_i - V_{tP}) \pm \sqrt{4(V_P - V_i - V_{tP})^2 - 4(V_i - t_n)^2}}{2}$$

$$\Rightarrow V_P - V_{out} = \frac{2(V_P - V_i - V_{tP}) \pm 2\sqrt{(V_P - V_i - V_{tP})^2 - (V_i - V_{tn})^2}}{2}$$

$$\Rightarrow V_p - V_{out} = V_p - V_i - V_{tp} \pm \sqrt{(V_p - V_i - V_{tp} + V_p - V_{tn})(V_p - V_p - V_{tp} - V_i + V_{tn})}$$

$$\Rightarrow -V_{out} = -V_i - V_{tp} \pm \sqrt{(V_p - V_{tp} - V_{tn})(V_p - 2V_i - V_{tp} + V_{tn})}$$

$$\Rightarrow V_{out} = V_i + V_{tp} \pm \sqrt{(V_p - V_{tp} - V_{tn})}\ \sqrt{(V_p - 2V_i - V_{tp} + V_{tn})}$$

Differentiating w.r.t $V_i$,

$$\frac{dV_{out}}{dV_i} = 1 + 0 \pm \sqrt{V_p - V_{tp} - V_{tn}}\ \frac{-2}{2\sqrt{V_p - 2V_i - V_{tp} + V_{tn}}}$$

$$\Rightarrow -1 = 1 \pm \frac{\sqrt{V_p - V_{tp} - V_{tn}}}{\sqrt{V_p - 2V_i - V_{tp} + V_{tn}}}$$

$$\Rightarrow 2 = \pm \frac{\sqrt{V_p - V_{tp} - V_{tn}}}{\sqrt{V_p - 2V_i - V_{tp} + V_{tn}}}$$

$$\Rightarrow 4(V_p - 2V_i - V_{tp} + V_{tn}) = V_p - V_{tp} - V_{tn}$$

$$\Rightarrow 3V_p - 8V_i - 3V_{tp} + 5V_{tn} = 0$$

$$\Rightarrow 8V_i = 3V_p - 3V_{tp} + 5V_{tn}$$

$$\Rightarrow V_i = \frac{3V_p - 3V_{tp} + 5V_{tn}}{8} = V_{IL}$$

Beyond this value, $V_i$ can not be considered as low.

$NM(0) = V_{IL} - 0$

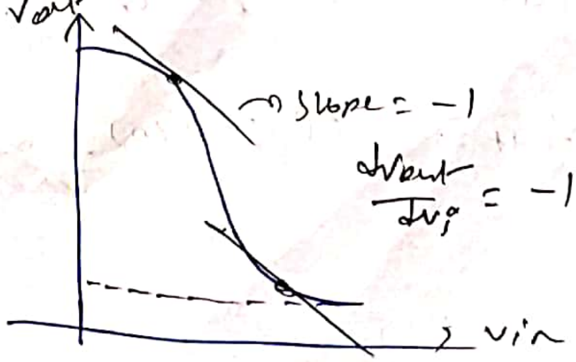
$= 2.125 - 0$

$= 2.125\,V$

$$V_{IL} = \frac{3\cdot 5 - 3\cdot 1 + 5\cdot 1}{8} = \frac{17}{8} = 2.125\,V$$

$V_p = 5$, $V_{tp} = 1$, $V_{tn} = 1$

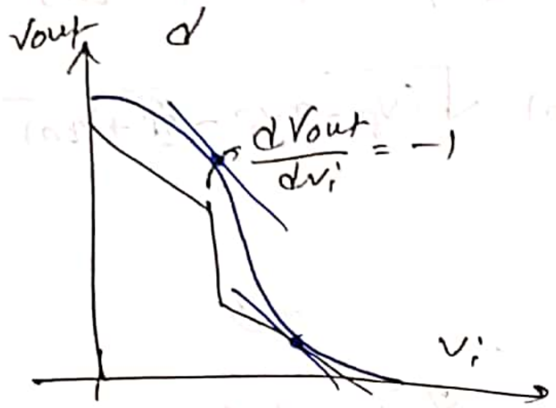
$NM(0) = 2.125\,V$



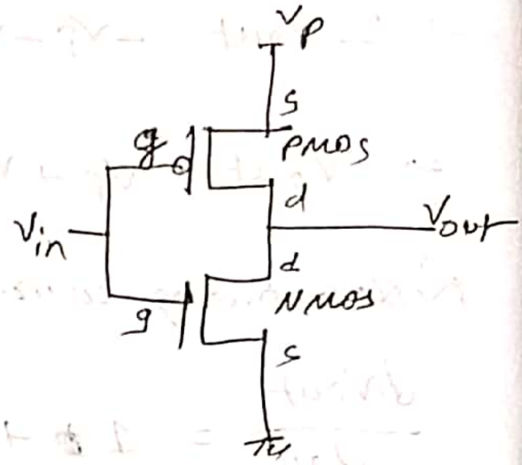
Home work $\quad NM(1) : -V_{IH} = \dfrac{5V_P - 5V_{tP} + 3V_{tn}}{8}$

Proof:

Here,




* blue graph correct




PMOS → saturation

NMOS → Resistive

$$I_N(\text{res}) = I_P(\text{sat}^n)$$

$$\Rightarrow \frac{\epsilon\mu_N}{D}\frac{W_N}{L_N}\left[(V_{gs}-V_{tn})V_{ds} - \frac{V_{ds}^2}{2}\right] = \frac{\epsilon\mu_P}{D}\frac{W_P}{L_P}\frac{(V_{sg}-V_{tP})^2}{2}$$

we know, $\mu_N = 2\mu_P$

$\Rightarrow \mu_N/\mu_P = 2$ $\quad$ Let, $\cancel{\dfrac{W_N}{L_N}}\ \dfrac{W_P}{L_P} = 2 \cdot \dfrac{W_N}{L_N}$

$$\Rightarrow \frac{W_N/L_N}{W_P/L_P} = \frac{1}{2}$$

$$\Rightarrow 2 \cdot \frac{1}{2}\left[(V_{gs}-V_{tn})V_{ds} - \frac{V_{ds}^2}{2}\right] = \frac{(V_{sg}-V_{tP})^2}{2}$$

$$\Rightarrow \left[(V_i - V_{tn})V_{out} - \frac{V_{out}^2}{2}\right] = \frac{(V_P - V_i - V_{tP})^2}{2}$$

$$\Rightarrow 2(V_i - V_{tn})V_{out} - V_{out}^2 = (V_P - V_i - V_{tP})^2$$

$$\Rightarrow V_{out}^2 - 2(V_i - V_{tn})V_{out} + (V_P - V_i - V_{tP})^2 = 0$$

$$V_{out} = \frac{2(V_i - V_{tn}) \pm \sqrt{4(V_i - V_{tn})^2 - 4(V_P - V_i - V_{tP})^2}}{2}$$

$$\Rightarrow V_{out} = \frac{2(V_i - V_{tn}) \pm 2\sqrt{(V_i - V_{tn})^2 - (V_P - V_i - V_{tP})^2}}{2}$$

$$\Rightarrow V_{out} = (V_i - V_{tn}) \pm \sqrt{(V_i - V_{tn} + V_p - V_i - V_{tp})(V_i - V_{tn} - V_p + V_i + V_{tp})}$$

$$\Rightarrow V_{out} = V_i - V_{tn} \pm \sqrt{V_p - V_{tn} - V_{tp}}\ \sqrt{2V_i - V_p - V_{tn} + V_{tp}}$$

Differentiating w.r.t. $V_i$

$$\frac{dV_{out}}{dV_i} = 1 - 0 \pm \sqrt{V_p - V_{tn} - V_{tp}}\ \frac{2}{2\sqrt{2V_i - V_p - V_{tn} + V_{tp}}}$$

$$\Rightarrow -1 = 1 \pm \frac{\sqrt{V_p - V_{tn} - V_{tp}}}{\sqrt{2V_i - V_p - V_{tn} + V_{tp}}}$$

$$\Rightarrow 2 = \pm \frac{\sqrt{V_p - V_{tn} - V_{tp}}}{\sqrt{2V_i - V_p - V_{tn} + V_{tp}}}$$

$$\Rightarrow 4(2V_i - V_p - V_{tn} + V_{tp}) = V_p - V_{tn} - V_{tp}$$

$$\Rightarrow 8V_i - 5V_p - 3V_{tn} + 5V_{tp} = 0$$

$$\Rightarrow 8V_i = 5V_p + 3V_{tn} - 5V_{tp}$$

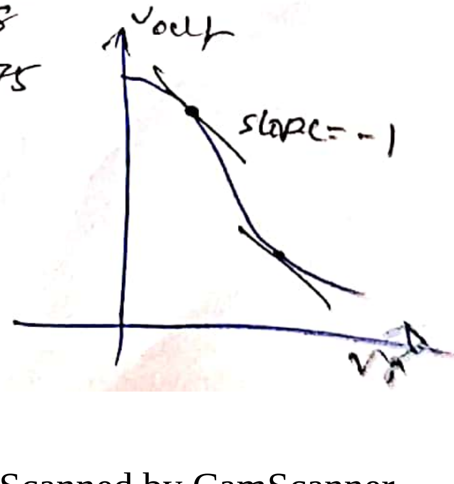
$$\Rightarrow V_i = \frac{5V_p - 5V_{tp} + 3V_{tn}}{8} = V_{IH}$$

$NM(1) = V_{IH} - 5$
$= 2.875 - 5$
$= -2.125$

$5 - V_{IH}$
$= 5 - 2.875$
$= 2.125$

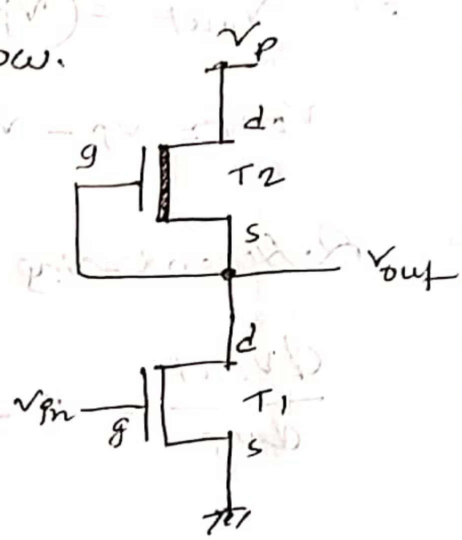
I'm confused !!

$$V_{IH} = \frac{5\cdot5 - 5\cdot1 + 3\cdot1}{8} = \frac{23}{8} = 2.875$$



## NOISE MARGIN (NMOS inverter)

$V_{IL}$ = Highest voltage reliably recognized as LOW.




Enhancement NMOS ($T_1$) → $Sat^n$

Depletion NMOS ($T_2$) → Resistive

$$I_D(res) = I_E(sat^n)$$

$$\Rightarrow \frac{\epsilon\mu_D}{D}\frac{W_D}{L_D}\left[(V_{gs_2}-V_{tD})V_{ds_2}-\frac{V_{ds_2}^2}{2}\right] = \frac{\epsilon\mu_E}{D}\frac{W_E}{L_E}\frac{(V_{gs_1}-V_t)^2}{2}$$

For calculation purpose;

let, $\mu_E \simeq \mu_D$

$$\frac{W_E/L_E}{W_D/L_D} = K$$

$$\Rightarrow (V_{gs_2}-V_{tD})V_{ds_2}-\frac{V_{ds_2}^2}{2} = K\frac{(V_{gs_1}-V_t)^2}{2}$$

$$\Rightarrow \cancel{(V_i-V_{tD})V_{ds}}$$

$$\Rightarrow (0-V_{tD})(V_p-V_{out}) - \frac{(V_p-V_{out})^2}{2} = K\frac{(V_i-V_t)^2}{2}$$

$$\Rightarrow -2V_{tD}(V_p-V_{out}) - (V_p-V_{out})^2 = K(V_i-V_t)^2$$

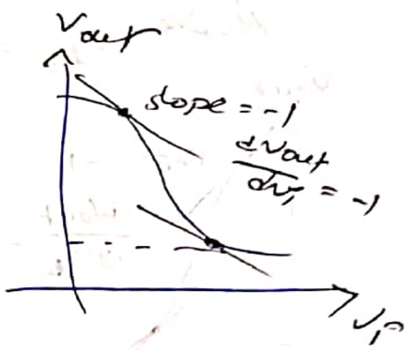
$$\Rightarrow (V_p-V_{out})^2 + 2V_{tD}(V_p-V_{out}) + K(V_i-V_t)^2 = 0$$

$$V_p-V_{out} = \frac{-2V_{tD}\pm\sqrt{4V_{tD}^2-4K(V_i-V_t)^2}}{2}$$

$$\Rightarrow V_p-V_{out} = -V_{tD}\pm\sqrt{V_{tD}^2-K(V_i-V_t)^2}$$

$$\Rightarrow -V_{out} = -V_p-V_{tD}\pm\sqrt{V_{tD}^2-K(V_i-V_t)^2}$$

$$\Rightarrow V_{out} = V_i + V_{tD} \pm \sqrt{V_{tD}^2 - k(V_i - V_t)^2}$$




Differentiating w.r.t $V_i$

$$\frac{dV_{out}}{dV_i} = 0 + 0 \pm \frac{-2(V_i - V_t)k}{2\sqrt{V_{tD}^2 - k(V_i - V_t)^2}}$$

$$\Rightarrow -1 = \pm \frac{(V_i - V_t)\,k}{\sqrt{V_{tD}^2 - k(V_i - V_t)^2}}$$

$$\Rightarrow V_{tD}^2 - k(V_i - V_t)^2 = (V_i - V_t)^2 k^2$$

$$\Rightarrow V_{tD}^2 = (V_i - V_t)^2 k^2 + k(V_i - V_t)^2$$

$$\Rightarrow V_{tD}^2 = (V_i - V_t)^2 (k^2 + k)$$

$$\Rightarrow (V_i - V_t)^2 = \frac{V_{tD}^2}{k^2 + k}$$

$$\Rightarrow V_i - V_t = \sqrt{\frac{V_{tD}^2}{k^2 + k}}$$

$$\Rightarrow V_i = \sqrt{\frac{V_{tD}^2}{k^2 + k}} + V_t$$

$$\boxed{V_{IL} = V_t + \sqrt{\frac{V_{tD}}{k^2 + k}} = V_t + \frac{V_{tD}}{\sqrt{k^2 + k}}}$$

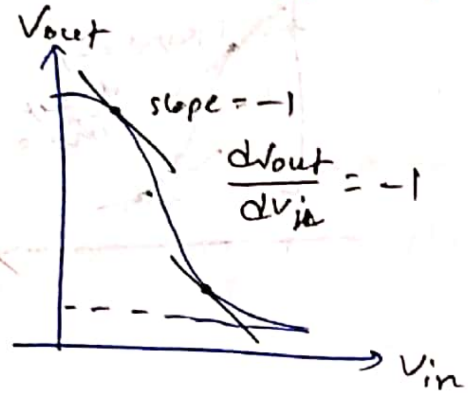
$$\left.\begin{array}{l} V_t = 1 \\ k = 6 \\ V_{tD} = 4 \end{array}\right\} \quad V_{IL} = 1 + \frac{4}{\sqrt{6^2 + 6}} = 1 + \frac{4}{\sqrt{42}} = 1.62$$

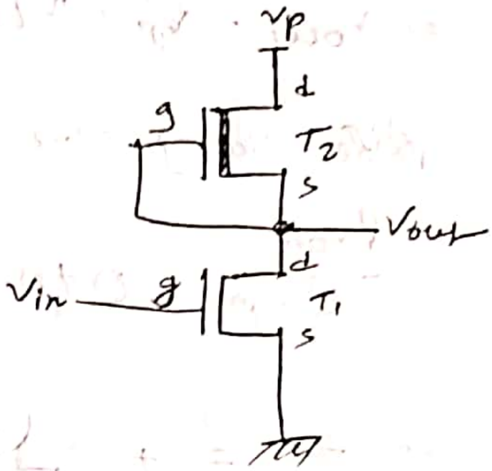
$$NM(0) = 1.62 - 0.2$$

$$= 1.42$$

Homework

Derive, $V_{IH} = V_t + \dfrac{2V_{tD}}{\sqrt{3K}}$




Enhancement NMOS ($T_1$) → Resistive

Depletion NMOS ($T_2$) → Saturation

$$I_E\,(\text{res}) = I_D\,(\text{sat}^n)$$

$$\Rightarrow \frac{\epsilon\mu_E}{D}\frac{W_E}{L_E}\left[(V_{gs_1} - V_t)V_{ds_1} - \frac{V_{ds_1}^2}{2}\right] = \frac{\epsilon\mu_D}{D}\frac{W_D}{L_D}\frac{(V_{gs_2} - V_{tD})^2}{2}$$

For calculation purpose, ~~we know,~~

let, $\mu_E \simeq \mu_D$

$$\frac{W_E/L_E}{W_D/L_D} = K$$

$$\Rightarrow K\left[(V_{gs_1} - V_t)V_{ds_1} - \frac{V_{ds_1}^2}{2}\right] = \frac{(V_{gs_2} - V_{tD})^2}{2}$$

$$\Rightarrow \left[(V_i - V_t)V_{out} - \frac{V_{out}^2}{2}\right] = \frac{(0 - V_{tD})^2}{2K}$$

$$\Rightarrow \left[(V_i - V_t)V_{out} - \frac{V_{out}^2}{2}\right] = \frac{V_{tD}^2}{2K}$$

$$\Rightarrow \left[2(V_i - V_t)V_{out} - V_{out}^2\right] = \frac{V_{tD}^2}{K}$$

$$\Rightarrow V_{out}^2 - 2(V_i - V_t)V_{out} + \frac{V_{tD}^2}{K} = 0$$

$$V_{out} = \frac{2(V_i - V_t) \pm \sqrt{4(V_i - V_t)^2 - 4V_{tD}^2/K}}{2}$$

$$V_{out} = (V_i - V_t) \pm \sqrt{(V_i - V_t)^2 - V_{tD}^2/k}$$

Differentiating w.r.t $V_i$,

$$\frac{dV_{out}}{dV_i} = (1-0) \pm \frac{2(V_i - V_t)}{2\sqrt{(V_i - V_t)^2 - V_{tD}^2/k}}$$

$$\Rightarrow -1 = 1 \pm \frac{2(V_i - V_t)}{2\sqrt{(V_i - V_t)^2 - V_{tD}^2/k}}$$

$$\Rightarrow 2 = \pm \frac{(V_i - V_t)}{\sqrt{(V_i - V_t)^2 - V_{tD}^2/k}}$$

$$\Rightarrow 4\left[(V_i - V_t)^2 - V_{tD}^2/k\right] = (V_i - V_t)^2$$

$$\Rightarrow 4(V_i - V_t)^2 - 4\frac{V_{tD}^2}{k} = (V_i - V_t)^2$$

$$\Rightarrow 3(V_i - V_t)^2 = 4\frac{V_{tD}^2}{k}$$

$$\Rightarrow \cancel{V_{tD}} (V_i - V_t)^2 = \frac{4V_{tD}^2}{3k}$$

$$\Rightarrow V_i - V_t = \frac{\cancel{2V_{tD}}}{} \frac{2V_{tD}}{\sqrt{3k}}$$

$$\Rightarrow V_i = V_t + \frac{2V_{tD}}{\sqrt{3k}} = V_{IH}$$

$$V_{IH} = V_t + \frac{2V_{tD}}{\sqrt{3k}}$$

$$V_{IH} = 1 + \frac{2 \cdot 4}{\sqrt{3 \times 6}}$$

$$= 2.89\,V$$

$$NM(1) = 5 - V_{IH}$$
$$= 5 - 2.89$$
$$= 2.11\,V$$

ঠিক নয়

অথবা

$$NM(1) = V_{IH} - 5$$
$$= 2.89 - 5$$
$$= -2.11$$

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