Assignment 1

Group - 04

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1. (a) A gambler has in his pocket a fair coin and two-headed coin. He selects one of the coins at random, and he flips it, it shows heads. What is the probability that it is the fair coin?

Ans : For the events

F = the fair coin is chosen

F^c = the two headed coin is chosen

H_i = ith coin toss flips heads

T_i = ith coin toss flips tails

Probabilities:

Since a coin is randomly chosen:

$$P(F) = P(F^c) = \frac{1}{2}$$

If we note the number of heads in each coin :

$$P(H_i|F) = \frac{1}{2} \qquad P(T_i|F) = \frac{1}{2}$$

$$P(H_i|F^c) = 1 \qquad P(T_i|F^c) = 0$$

Note: for fair coin the probability

$P(H_i|F) = P(T_i|F) = \frac{1}{2}$.

For two headed coin it's alway head

So, $P(H_i|F^c) = 1$

and it is never tail. So,

$P(T_i|F^c) = 0$

We have to calculate $P(F|H_1)$.

Applying Bayes formula

$$P(F|H_1) = \frac{P(H_1|F)P(F)}{P(H_1|F)P(F) + P(H_1|F^c)P(F^c)}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{2}}{\frac{1}{2}\cdot\frac{1}{2} + 1\cdot\frac{1}{2}}$$

$$= \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{2}}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$P(F|H_1) = \frac{1}{3}$$

Ans

(b) Suppose that she flips the same coin a third time and it shows tails. Now what is the probability that it is the fair coin?

<u>Ans:</u> From (a) we have

$$P(H_i|F) = \frac{1}{2} \qquad P(T_i|F) = \frac{1}{2}$$

$$P(H_i|F^c) = 1 \qquad P(T_i|F^c) = 0$$

We have to find, $P(F|H_1H_2T_3)$.

From Bayes formula.

$$P(F|H_1H_2T_3) = \frac{P(H_1H_2T_3|F)\,P(F)}{P(H_1H_2T_3|F)P(F) + P(H_1H_2T_3|F^c)\,P(F^c)}$$

Since, H_1, H_2 and T_3 are independent conditionally on F or F^c

$$P(H_1|F) = \frac{1}{2}$$

$$P(H_2|F) = \frac{1}{2}$$

$$P(T_3|F) = \frac{1}{2}$$

So, $P(H_1H_2T_3|F) = P(H_1|F)P(H_2|F).P(T_3|F)$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$

$$= \frac{1}{8}$$

$$P(H_1 H_2 T_3 | F^c) = P(H_1 | F^c) . P(H_2 | F^c) . P(T_3 | F)$$

$$= 1 . 1 . 0$$

$$= 0$$

$$\cancel{P(F | H_1 H_2 T_3)} = \frac{\cancel{P(H_1 H_2 T_3) \, P(F)}}{\cancel{P(H_1 H_2 T_3 | F) . P(F)}}$$

$$P(F | H_1 H_2 T_3) = \frac{P(H_1 H_2 T_3 | F) . P(F)}{P(H_1 H_2 T_3 | F) . P(F) + P(H_1 H_2 T_3 | F^c) . P(F^c)}$$

$$= \frac{\frac{1}{8} . \frac{1}{2}}{\frac{1}{8} . \frac{1}{2} + 0 . \frac{1}{2}}$$

$$= \frac{\frac{1}{16}}{\frac{1}{16}}$$

$$P(F | H_1 H_2 T_3) = 1$$

Ans

Ans. No. 2

If a family has two children and the born child is equally likely to be a boy or a girl then the possible cases are four:

(B, B), (B, G), (G, B), (G, G)

B = Boy
G = Girl

case of both being girl is one (G,G)

(a) Cases when the eldest is a girl are two: (G, B), (G, G)

Let, assume the incident both are girls be, E → {(G, G)}
and "S" is all possible cases

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$$

Let, the eldest is a girl is the incident, $E_a \rightarrow \{(G,B),(G,G)\}$

$$\therefore P(E \cap E_a) = \{(G,G)\}$$

$$\therefore P(E \cap E_a) = \frac{n(E \cap E_a)}{n(S)}$$

$$= \frac{1}{4}$$

and $$P(E_a) = \frac{n(E_a)}{n(S)}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

$\therefore$ The probability that both are girls given that the eldest is a girl is, $$P\left(\frac{E}{E_a}\right) = \frac{P(E \cap E_a)}{P(E_a)}$$

$$= \frac{1/4}{1/2}$$

$$= \frac{1}{2} \text{ (Ans)}$$

(b) From 'a',

$n(E) = 1$, $n(S) = 4$

Let, the incident where at least one is a girl is, $E_b = \{(B,G), (G,B), (G,G)\}$

$\therefore$ Probability of E_b, $P(E_b) = \frac{n(E_b)}{n(S)}$

$= \frac{3}{4}$

Here, $(E \cap E_b) = \{(G,G)\}$

It's the incident where at a time at least one is a girl and both are girls.

$\therefore P(E \cap E_b) = \frac{n(E \cap E_b)}{n(S)}$

$= \frac{1}{4}$

$\therefore$ The probability of both are girls given that at least one is a girl is,

$P\left(\frac{E}{E_b}\right) = \frac{P(E \cap E_b)}{P(E_b)} = \frac{1/4}{3/4}$

$= \frac{1}{3}$ (Ans)