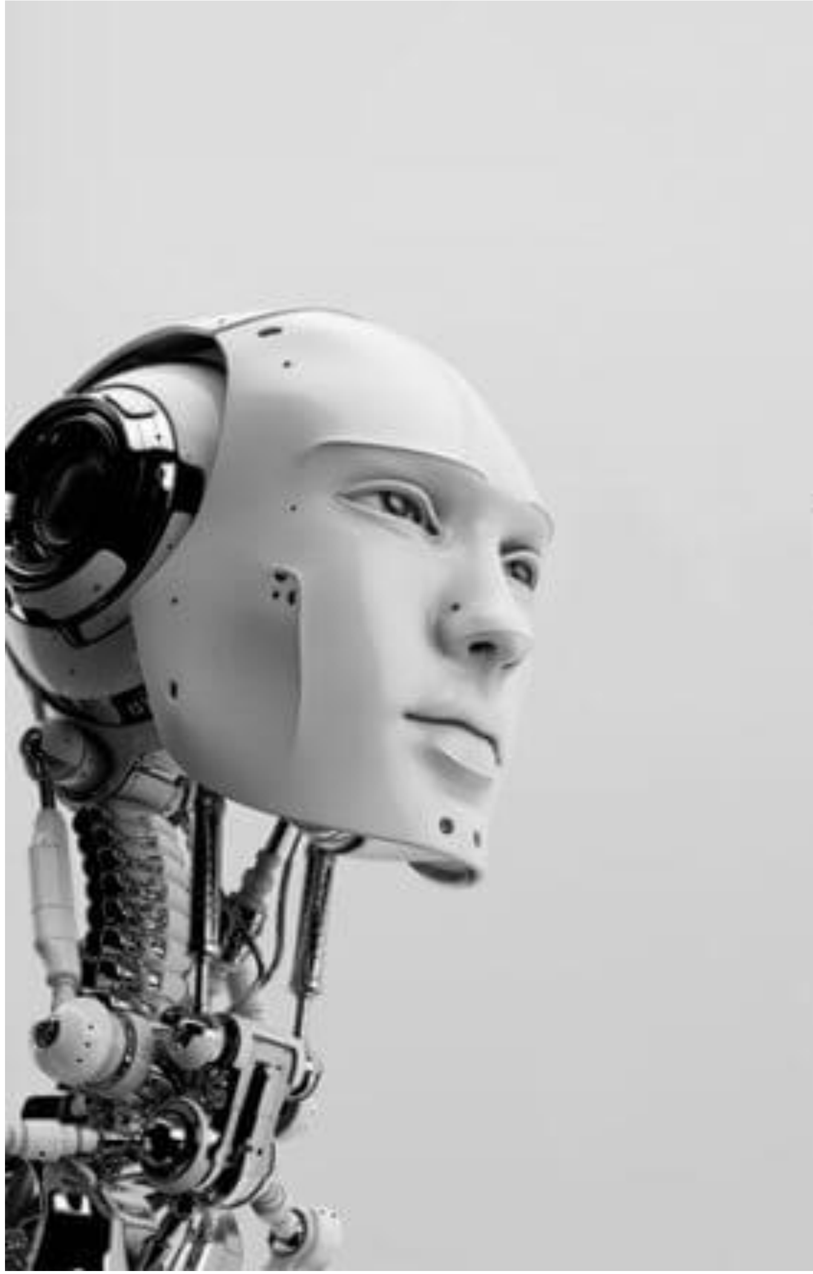

Artificial Intelligence

4 Beyond Classical Search

Russell & Norvig, AI: A Modern Approach, 3rd Ed

(Local Search)

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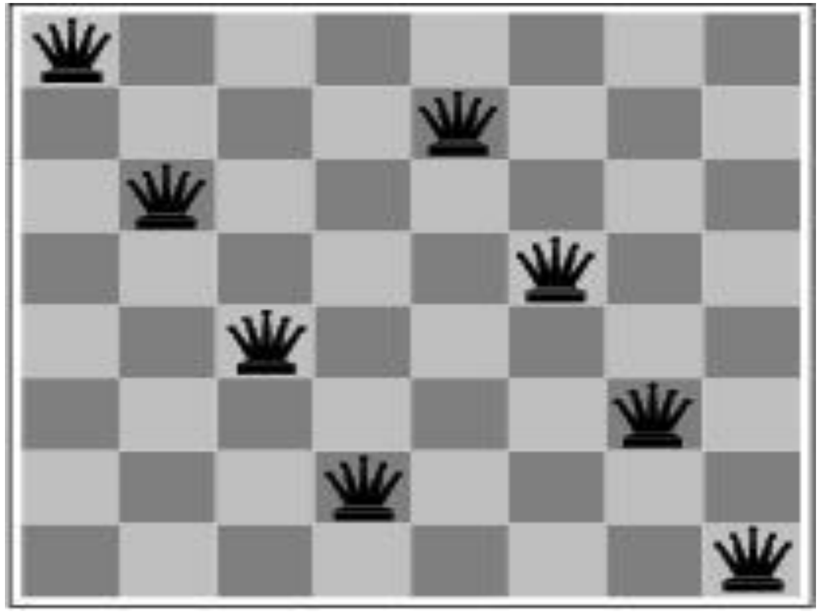
Search Heuristics

Local search techniques and optimization

- ✓ Hill-climbing
- ✓ Simulated annealing
- ✓ Local Beam Search
- ✓ Genetic algorithms
- ✓ Issues with local search

Local search and optimization

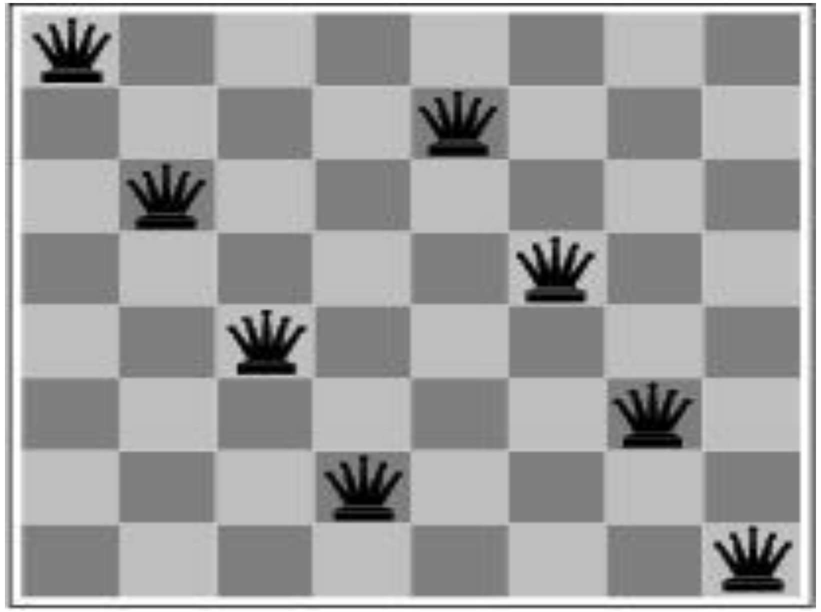
- Previously: systematic exploration of search space.
  - Path to goal is solution to problem
- YET, for some problems path is irrelevant.
  - E.g 8-queens
- Different algorithms can be used
  - Local search



Local search and optimization

Applications:

- Integrated-circuit design
- Factory-floor layout
- Job-shop scheduling
- Automatic programming
- Telecommunications
- Network Optimization
- Vehicle Routing



Local search and optimization

- Local search
  - Keep track of single current state
  - Move only to neighboring states
  - Ignore paths

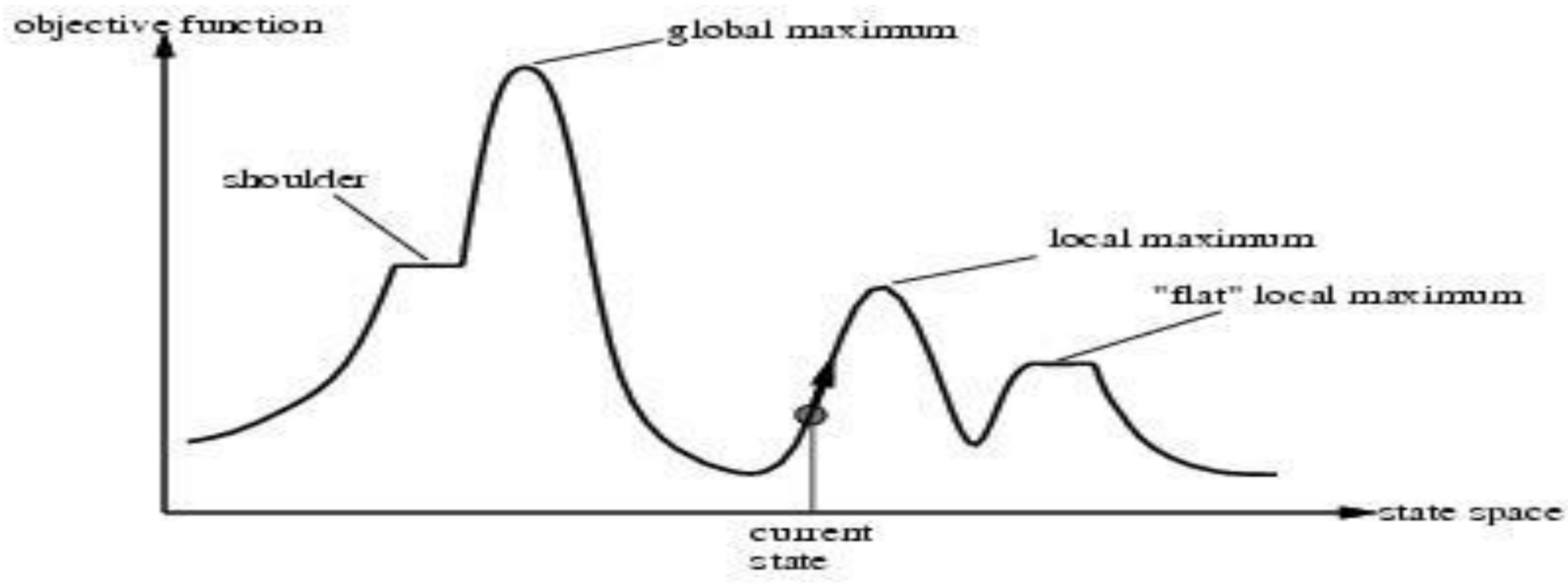
- Advantages:
  - Use very little memory (Usually Constant Amount)
  - Can often find reasonable solutions in large or infinite (continuous) state spaces.

Local search and optimization

- Local Search algorithms are very useful for solving "Pure Optimization" problems.

- "Pure optimization" problems
    - All states have an objective function (OF)
    - Goal is to find state with max (or min) objective value
    - Does not quite fit into path-cost/goal-state formulation
    - Local search can do quite well on these problems.

“Landscape” of search



A complete local search algorithm always finds a goal if one exists!!

Hill-climbing search

function HILL-CLIMBING(*problem*) **return** a state that is a local maximum

input: *problem*, a problem

local variables: *current*, **a node.**

neighbor, **a node.**

current ← MAKE-NODE(INITIAL-STATE[*problem*])

loop do

neighbor ← a highest valued successor of *current*

if VALUE [*neighbor*] ≤ VALUE[*current*] **then return** STATE[*current*]

current ← *neighbor*

Hill-climbing (Steepest-Ascent Version) search

- “A loop that continuously moves in the direction of increasing value” – to Uphill
  - terminates when a peak is reached
  - Aka greedy local search

- Value can be either
  - Objective function value
  - Heuristic function value

- Hill climbing does not look ahead of the immediate neighbors of the current state.

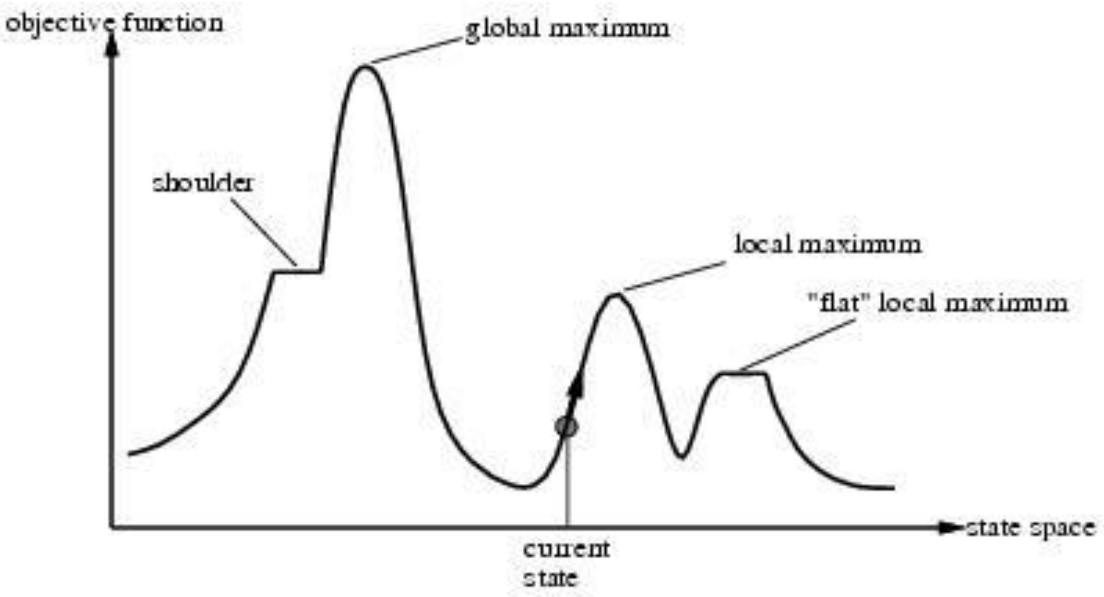
- Can randomly choose among the set of best successors, if multiple have the best value

- Characterized as “trying to find the top of Mount Everest in a thick fog while suffering from amnesia”

Hill climbing and local maxima

- When local maxima exist, hill climbing is suboptimal

- Simple (often effective) solution
    - Multiple random restarts



Hill-climbing example

- 8-queens problem, complete-state formulation
  - All 8 queens on the board in some configuration

- **Successor function:**
  - move a single queen to another square in the same column.

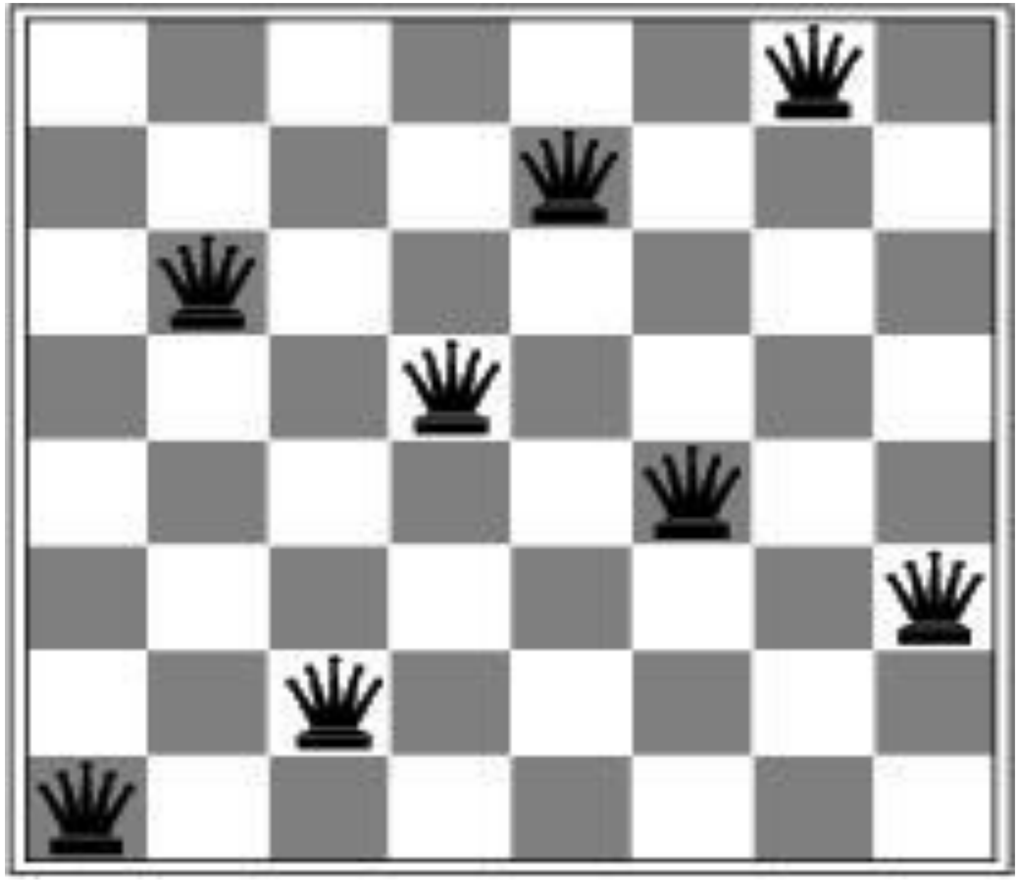
- Example of a heuristic function *h(n)*:
  - the number of pairs of queens that are attacking each other (directly or indirectly)
  - (so we want to minimize this)

Hill-climbing example

Current state: h=17

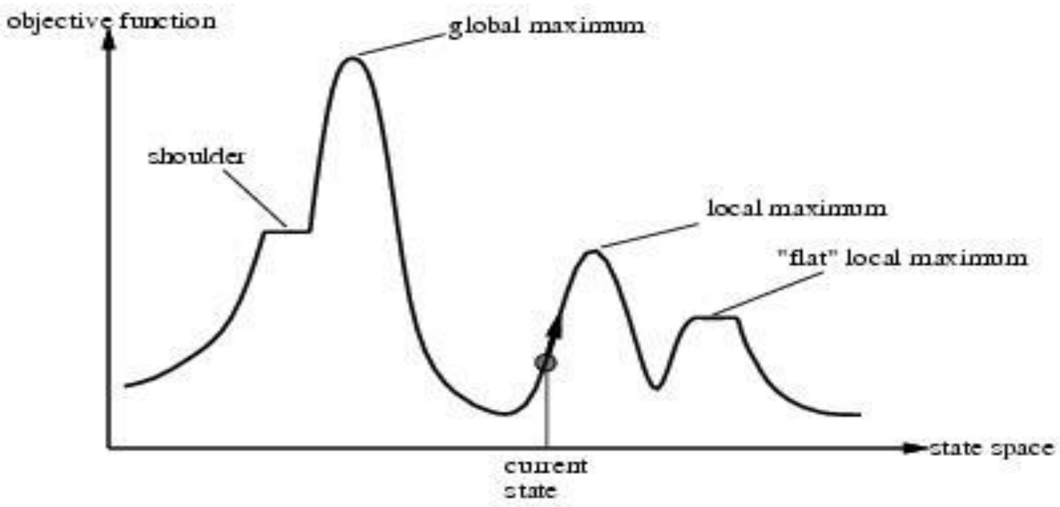
Shown is the h-value for each possible successor in each column

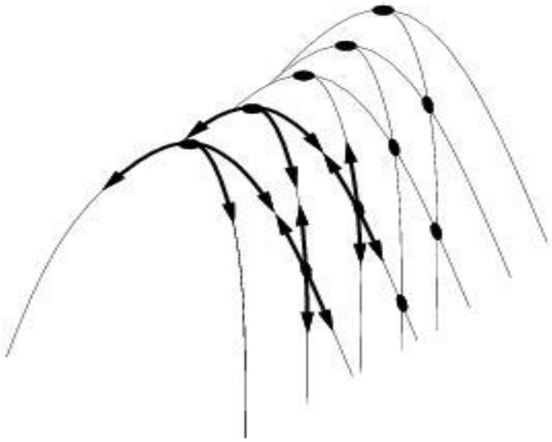
A local minimum for 8-queens



A local minimum in the 8-queens state space (h=1)

Other drawbacks





- Ridge = sequence of local maxima difficult for greedy algorithms to navigate

- Plateau = an area of the state space where the evaluation function is flat.

Performance of hill-climbing on 8-queens

- Randomly generated 8-queens starting states…
- 14% the time it solves the problem
- 86% of the time it get stuck at a local minimum
- However…
  - Takes only 4 steps on average when it succeeds
  - And 3 on average when it gets stuck
  - (for a state space with ~17 million states)

Possible solution…sideways moves

- If no downhill (uphill) moves, allow sideways moves in hope that algorithm can escape
    - Need to place a limit on the possible number of sideways moves to avoid infinite loops

- For 8-queens
    - Now allow sideways moves with a limit of 100
    - Raises percentage of problem instances solved from 14 to 94%

    - However….
        - 21 steps for every successful solution
        - 64 for each failure

Hill-climbing variations

- Stochastic hill-climbing
  - Random selection among the uphill moves.
  - The selection probability can vary with the steepness of the uphill move.
  - Usually converges more slowly than steepest-ascent but in some state space, it finds better solutions

- First-choice hill-climbing
  - stochastic hill climbing by generating successors randomly until a better one is found
  - Useful when there are a very large number of successors

- Random-restart hill-climbing
  - Tries to avoid getting stuck in local maxima.

Hill-climbing with random restarts

- Different variations
  - For each restart: run until termination v. run for a fixed time
  - Run a fixed number of restarts or run indefinitely

- Analysis
  - Say each search has probability p of success
    - E.g., for 8-queens, $p = 0.14$ with no sideways moves

  - Expected number of restarts?
  - Expected number of steps taken?

Expected number of restarts

- Probability of Success = p
- Number of restarts = 1 / p
- This means 1 successful iteration after (1/p – 1) failed iterations
- Let avg. number of steps in a failure iteration = f

  and avg. number of steps in a successful iteration = s
- Therefore, expected number of steps in random-restart hill climbing = 1 * s + (1/p – 1) f

So for 8-queens, p = 14%, s = 4, f = 3,

Expected no of moves = 1 * 4 + (1/0.14 -1) * 3 = 22

With sideways moves, p = 94%, s = 21, f = 64

Expected no of moves = 1 * 21 + (1/0.94 -1) * 64 = 25

Local beam search

- Keep track of k states instead of one
  - Initially: k randomly selected states
  - Next: determine all successors of k states
  - If any of successors is goal $\rightarrow$ finished
  - Else select k best from successors and repeat.

- Major difference with random-restart search
  - Information is shared among k search threads.

- Can suffer from lack of diversity.
  - Stochastic beam search
    - choose k successors proportional to state quality (randomly).

Search using Simulated Annealing

- Simulated Annealing = hill-climbing with non-deterministic search (random walk)

- Basic ideas:
  - like hill-climbing identify the quality of the local improvements
  - instead of picking the best move, pick one randomly
  - say the change in objective function is d
  - if d is positive, then move to that state
  - otherwise:
    - move to this state with probability proportional to d
    - thus: worse moves (very large negative d) are executed less often
  - however, there is always a chance of escaping from local maxima

Physical Interpretation of Simulated Annealing

- A Physical Analogy:
  - Imagine letting a ball roll downhill on the function surface
    - ➢ this is like hill-climbing (for minimization)
  - Now imagine shaking the surface, while the ball rolls, gradually reducing the amount of shaking
    - ➢ this is like simulated annealing
    - ➢ If we shake the surface, we can bounce the ball out of local minimum but hard enough to dislodge from global minimum

- Annealing = physical process used to temper or harden metals glass by heating them to a high temperature and then gradually cooling them, thus allowing the material to reach a low-energy crystalline state.

Simulated annealing

function SIMULATED-ANNEALING(*problem, schedule*) **return** a solution state

input: *problem*, a problem

schedule, a mapping from time to temperature

local variables: *current***,** a node.

*next***,** a node.

*T***,** a "temperature" controlling the probability of downward steps

current $\leftarrow$ MAKE-NODE(INITIAL-STATE[*problem*])

for t $\leftarrow$ 1 to ∞ do

$T \leftarrow schedule[t]$

if $T = 0$ **then return** *current*

next $\leftarrow$ a randomly selected successor of *current*

$\Delta E \leftarrow$ VALUE[*next*] - VALUE[*current*]

if $\Delta E > 0$ **then** *current* $\leftarrow$ *next*

else *current* $\leftarrow$ *next* only with probability $e^{\Delta E / T}$

More Details on Simulated Annealing

- Lets say there are 3 moves available, with changes in the objective function of d1 = -0.1, d2 = 0.5, d3 = -5. (Let T = 1).
- pick a move randomly:
  - if d2 is picked, move there.
  - if d1 or d3 are picked, probability of move = exp(d/T)
  - move 1: prob1 = exp(-0.1) = 0.9,
    - i.e., 90% of the time we will accept this move
  - move 3: prob3 = exp(-5) = 0.05
    - i.e., 5% of the time we will accept this move

- T = "temperature" parameter
  - high T => probability of "locally bad" move is higher
  - low T => probability of "locally bad" move is lower
  - typically, T is decreased as the algorithm runs longer
    - i.e., there is a "temperature schedule"

Simulated Annealing in Practice

Method proposed in 1983 by IBM researchers for solving VLSI layout problems (Kirkpatrick et al, *Science*, 220:671-680, 1983).

- theoretically will always find the global optimum (the best solution)

Useful for some problems, but can be very slow

- Slowness comes about because T must be decreased very gradually to retain optimality
- In practice how do we decide the rate at which to decrease T? (this is a practical problem with this method)

Genetic Algorithms (GA)

- Different approach to other search algorithms
  - A successor state is generated by combining two parent states rather than modifying the current state (sexual reproduction rather than asexual reproduction)
- A state is represented as a string over a finite alphabet (e.g. most commonly 0s and 1s, binary)
  - 8-queens
    - State = position of 8 queens each in a column
      => 8 x log(8) bits = 24 bits (for binary representation)
    - Alternatively, the state could be represented as digits, ranges 1 to 8

- Like Beam Search, GA starts with *k* randomly generated states (called the population)
- Objective function (fitness function).
  - Should return higher values for better states.
  - Opposite to heuristic function, e.g., # non-attacking pairs in 8-queens
- Produce the next generation of states by "simulated evolution"
  - Random selection
  - Crossover
  - Random mutation

Genetic algorithms

- 23/(24+23+20+11) = 29% etc

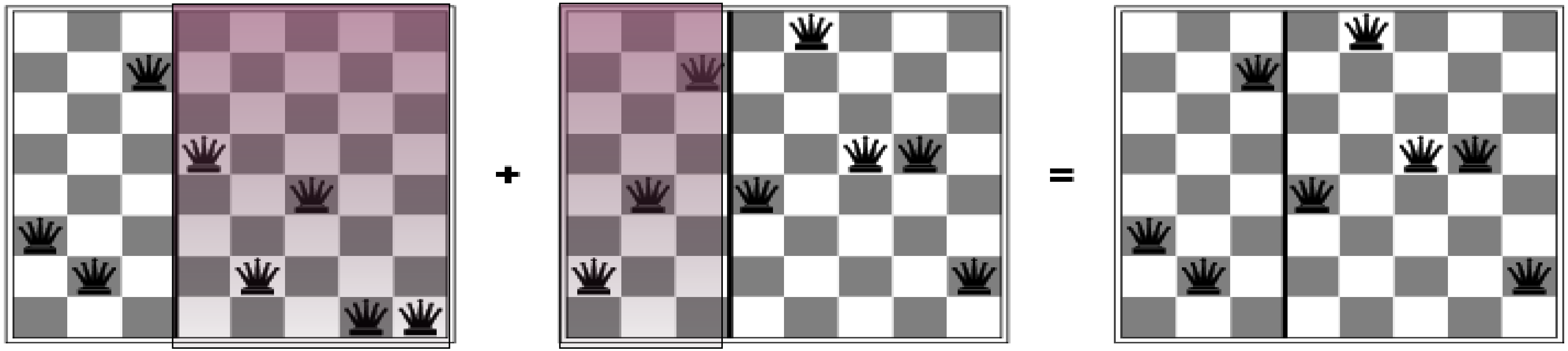
4 states for 8-queens problem

2 pairs of 2 states randomly selected based on fitness. Random crossover points Selected using Culling

New states after crossover

Random mutation applied

Genetic algorithms



Has the effect of “jumping” to a completely different new part of the search space (quite non-local)

Genetic algorithm pseudocode

```
function GENETIC-ALGORITHM(population, FITNESS-FN) returns an individual
   inputs: population, a set of individuals
           FITNESS-FN, a function that measures the fitness of an individual

   repeat
      new_population ← empty set
      for i = 1 to SIZE(population) do
         x ← RANDOM-SELECTION(population, FITNESS-FN)
         y ← RANDOM-SELECTION(population, FITNESS-FN)
         child ← REPRODUCE(x, y)
         if (small random probability) then child ← MUTATE(child)
         add child to new_population
      population ← new_population
   until some individual is fit enough, or enough time has elapsed
   return the best individual in population, according to FITNESS-FN
```

```
function REPRODUCE(x, y) returns an individual
   inputs: x, y, parent individuals

   n ← LENGTH(x); c ← random number from 1 to n
   return APPEND(SUBSTRING(x, 1, c), SUBSTRING(y, c + 1, n))
```

Comments on genetic algorithms

- Positive points
  - Random exploration can find solutions that local search can't
    - (via crossover primarily)
  - Appealing connection to human evolution
    - E.g., see related area of genetic programming, Bioinformatics

- Negative points
  - Large number of "tunable" parameters
    - Difficult to replicate performance from one problem to another

  - Lack of good empirical studies comparing to simpler methods

  - Useful on some (small?) set of problems but no convincing evidence that GAs are better than hill-climbing w/random restarts in general