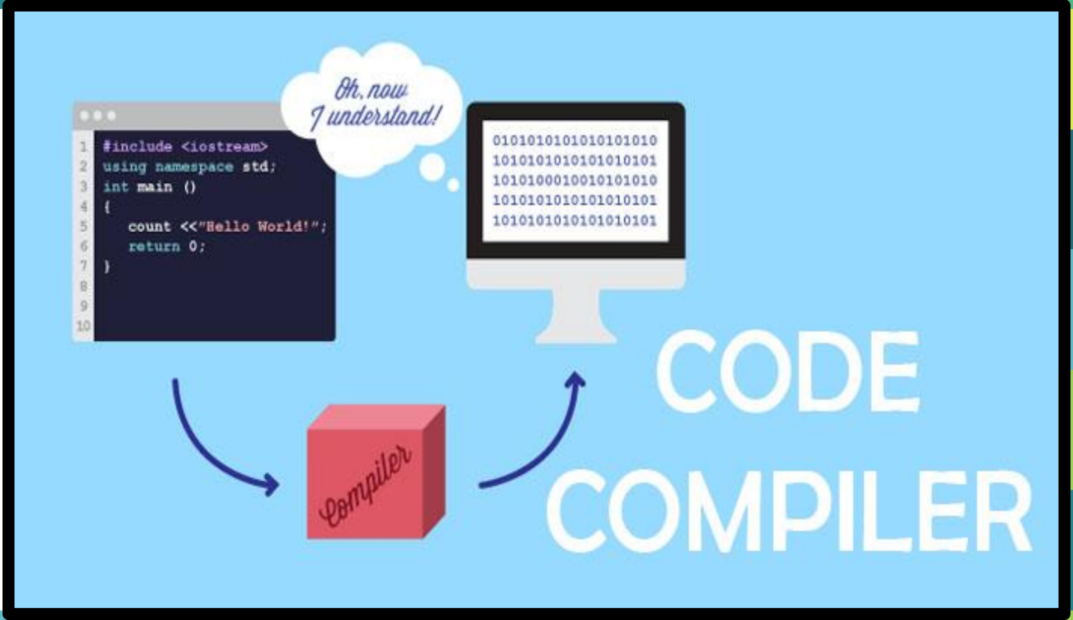
CSE- 303

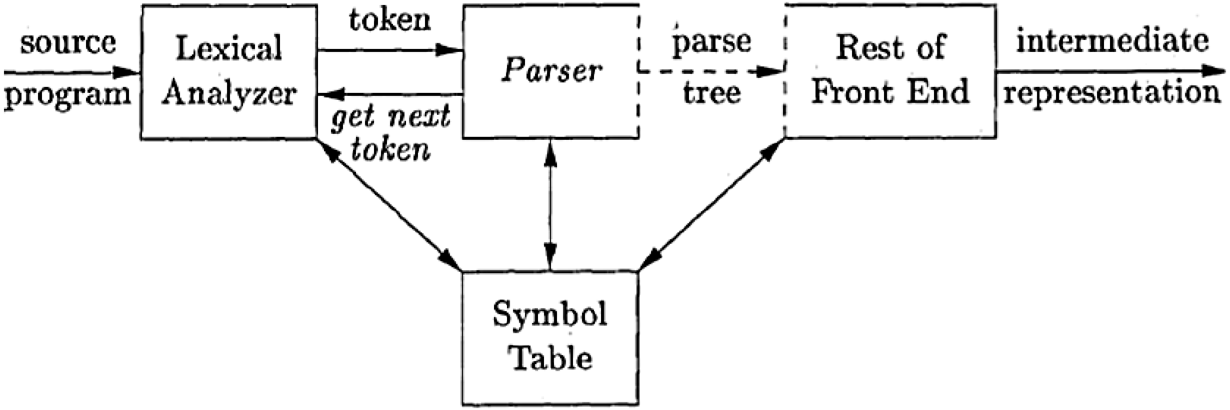
Compiler

Chapter - 4



The Role of the Parser

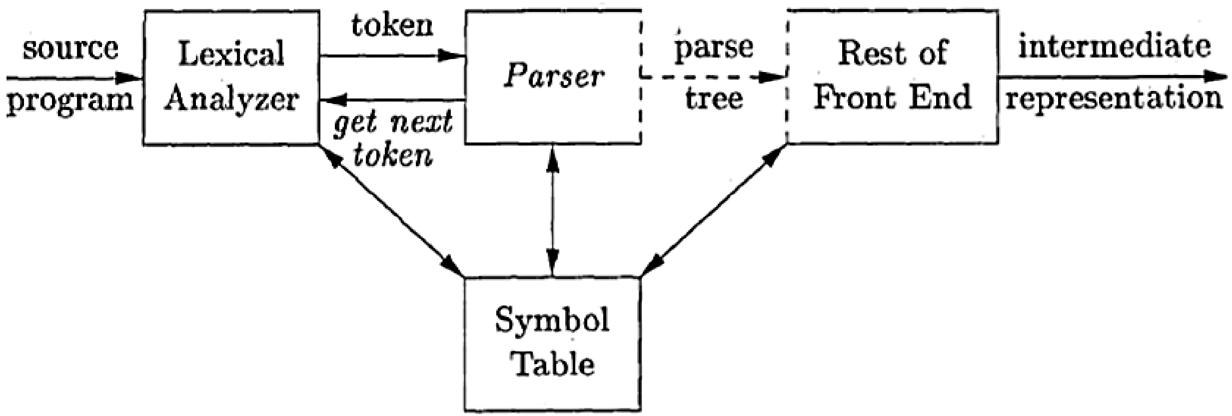
- In our compiler model, the parser obtains a string of tokens from the lexical analyzer.
- It then verifies that the string of token names can be generated by the grammar for the source language.



Position of parser in compiler model

The Role of the Parser

- ❑ We expect the parser
- ➢ to report any syntax errors in an intelligible fashion and
- ➢ to recover from commonly occurring errors to continue processing the remainder of the program.



Position of parser in compiler model

The Role of the Parser

- Conceptually, for well-formed programs, the parser constructs a parse tree and passes it to the rest of the compiler for further processing.
- In fact, the parse tree need not be constructed explicitly.
- Since checking and translation actions can be interspersed with parsing.
- Thus, the parser and the rest of the front end could well be implemented by a single module.

The Role of the Parser

- There are three general types of parsers for grammars:

1) **Universal:** can parse any type of grammar; not very efficient. Universal parsing methods are Cocke-Younger-Kasami algorithm and Earley's algorithm.
2) **Top-down:** build parse trees from the top (root) to the bottom (leaves); Ex: Recursive descent parsing, Backtracking
3) **Bottom-up:** start from the leaves and work their way up to the root

- The commonly used methods in compilers are top-down or bottom-up.
- In either case, the input to the parser is scanned from left to right, one symbol at a time.

The Role of the Parser

- The most efficient top-down and bottom-up methods work only for subclasses of grammars.
- But several of these classes, particularly, LL and LR grammars, are expressive enough to describe most of the syntactic constructs in modern programming languages.
- Parsers implemented by hand often use LL grammars. The predictive-parsing approach works for LL grammars.
- Parsers for the larger class of LR grammars are usually constructed using automated tools.

Syntax Error Handling

- If a compiler had to process only correct programs, its design and implementation would be simplified greatly.
- However, a compiler is expected to assist the programmer in locating and tracking down errors that inevitably creep into programs, despite the programmer’s best efforts.
- Strikingly, few languages have been designed with error handling in mind, even though errors are so commonplace.
- Most programming language specifications do not describe how a compiler should respond to errors.
- Error handling is left to the compiler designer.
- Planning the error handling right from the start can both simplify the structure of a compiler and improve its handling of errors.

Syntax Error Handling

- Common programming errors can occur at many different levels.
- **Lexical errors** include misspellings of identifiers, keywords, or operators — e.g., the use of an identifier elipsesize instead of ellipsesize — and missing quotes around text intended as a string.

Syntax Error Handling

- Common programming errors can occur at many different levels.
- **Syntactic errors** include misplaced semicolons or extra or missing braces, that is, "{" or "}". As another example, in C or Java, the appearance of a case statement without an enclosing switch is a syntactic error.

  However, this situation is usually allowed by the parser and caught later in the processing, as the compiler attempts to generate code.

Syntax Error Handling

- Common programming errors can occur at many different levels.
- **Semantic errors** include type mismatches between operators and operands. An example is a return statement in a Java method with result type void.

Syntax Error Handling

- Common programming errors can occur at many different levels.

- **Logical errors** can be anything from incorrect reasoning on the part of the programmer to the use in a C program of the assignment operator = instead of the comparison operator ==. The program containing = may be well formed; however, it may not reflect the programmer's intent.

Syntax Error Handling

- The precision of parsing methods allows syntactic errors to be detected very efficiently.
- Several parsing methods, such as the LL and LR methods, detect an error as soon as possible.
- That is, when the stream of tokens from the lexical analyzer cannot be parsed further according to the grammar for the language.
- More precisely, they have the viable-prefix property, meaning that they detect that an error has occurred as soon as they see a prefix of the input that cannot be completed to form a string in the language.

Syntax Error Handling

- The error handler in a parser has goals that are simple to state but challenging to realize:
- Report the presence of errors clearly and accurately.
- Recover from each error quickly enough to detect subsequent errors.
- Add minimal overhead to the processing of correct programs.

Syntax Error Handling

- Fortunately, common errors are simple ones.
- A relatively straightforward error-handling mechanism often suffices.
- How should an error handler report the presence of an error?
- At the very least, it must report the place in the source program where an error is detected.
- There is a good chance that the actual error occurred within the previous few tokens.
- A common strategy is to print the offending line with a pointer to the position at which an error is detected.

Derivation

- A derivation is basically a sequence of production rules, in order to get the input string.
- During parsing, we take two decisions for some sentential form of input:

1) Deciding the non-terminal which is to be replaced.
2) Deciding the production rule, by which, the non-terminal will be replaced.

- Two ways of derivation:

1) Left-most Derivation: input is scanned and replaced from left to right
2) Right-most Derivation: scan and replace the input with production rules, from right to left

Derivation

- Input: id + id * id
- Production rules:

$E \rightarrow E + E$
$E \rightarrow E * E$
$E \rightarrow id$

Left-most Derivation

$E \rightarrow E * E$
$E \rightarrow E + E * E$
$E \rightarrow id + E * E$
$E \rightarrow id + id * E$
$E \rightarrow id + id * id$

Right-most Derivation

$E \rightarrow E + E$
$E \rightarrow E + E * E$
$E \rightarrow E + E * id$
$E \rightarrow E + id * id$
$E \rightarrow id + id * id$

LL(k) Grammar

LL (1) Grammar

Read one symbol at a time
Leftmost Derivation
Left to Right Scanning

What about LL (k) Grammar?
What about LR (1) Grammar?

*****Determine if a grammar is LL (1)*****

LL(1) grammar is not left-recursive!!! Then check for the other three conditions…

Representative Grammars

Ambiguity: A grammar G is said to be ambiguous if it has more than one parse tree for at least one string.

- The following grammar treats + and $*$ alike.

$$E \rightarrow E + E \quad | \quad E * E \quad | \quad (E) \quad | \quad \textbf{id}$$

- So it is useful for illustrating techniques for handling ambiguities during parsing.
- Here, E represents expressions of all types.
- This grammar permits more than one parse tree for expressions like $a + b * c$.

Representative Grammars

- Constructs that begin with keywords like while or int, are relatively easy to parse.
- We therefore concentrate on expressions, which present more of challenge, because of the associativity and precedence of operators.

- **Associativity:** If an operand has operators on both sides, the side on which the operator takes this operand is decided by the associativity of those ope-rators.
  - Left-associative: +, -, *, / ; Ex: (id + id) + id
  - Right-associative: Exponentiation; Ex: id ^ (id ^ id)

- **Precedence:** If two different operators share a common operand, the precedence of operators decides which will take the operand.
  - Precedence Sequence: * / have higher precedence than + - ; Ex: 2+(3*4)

Representative Grammars

- Associativity and precedence are captured in the following grammar.
- E represents expressions consisting of terms separated by $+$ signs.
- T represents terms consisting of factors separated by $*$ signs.
- F represents factors that can be either parenthesized expressions or identifiers:

$$
\begin{aligned}
E &\rightarrow E + T \mid T \\
T &\rightarrow T * F \mid F \\
F &\rightarrow (E) \mid \mathbf{id}
\end{aligned}
$$

Representative Grammars

- The above grammar belongs to the class of LR grammars that are suitable for bottom-up parsing.
- This grammar can be adapted to handle additional operators and additional levels of precedence.
- However, it cannot be used for top-down parsing because it is left recursive.

$$
\begin{aligned}
E &\rightarrow E + T \mid T \\
T &\rightarrow T * F \mid F \\
F &\rightarrow (E) \mid \textbf{id}
\end{aligned}
$$

Elimination of Left Recursion

- A grammar is left recursive if it has a nonterminal A such that there is a derivation $A \stackrel{+}{\Rightarrow} Aa$ for some string a.
- Top-down parsing methods cannot handle left-recursive grammars, so a transformation that eliminates left recursion is needed.
- In simple left recursion there was one production of the form $A \rightarrow A\alpha$.
- In general case, left recursion:

$$A \longrightarrow Bac$$
$$B \longrightarrow Ccd$$
$$C \longrightarrow Abd$$

Elimination of Left Recursion

- Left-recursive pair of productions $A \to A\alpha \mid \beta$ can be replaced by the non-left-recursive productions

$$\begin{aligned} A &\to \beta A' \\ A' &\to \alpha A' \mid \epsilon \end{aligned}$$

without changing the set of strings derivable from A.

- This rule by itself suffices in many grammars.

Elimination of Left Recursion

$$A \to A\alpha \mid \beta$$

to be replaced by

$$\begin{aligned} A &\to \beta A' \\ A' &\to \alpha A' \mid \epsilon \end{aligned}$$

- Grammar for arithmetic expressions,

$$\begin{aligned} E &\to E + T \mid T \\ T &\to T * F \mid F \\ F &\to (\ E\ ) \mid \mathbf{id} \end{aligned}$$

- Eliminating the immediate left recursions we obtain,

$$\begin{aligned} E &\to TE' \\ E' &\to +TE' \mid \epsilon \\ T &\to FT' \\ T' &\to *FT' \mid \epsilon \\ F &\to (E) \mid \mathbf{id} \end{aligned}$$

Elimination of Left Recursion

- No matter how many *A*-productions there are, we can eliminate immediate left recursion from them.
- First, we group the *A*-productions as,

$$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid A\alpha_3 \mid \ldots \mid A\alpha_m \mid \beta_1 \mid \beta_2 \mid \beta_3 \mid \ldots \mid \beta_n$$

where no β_i, begins with an *A*.

- Then, we replace the *A*-productions by,

$$A \rightarrow \beta_1 A_0 \mid \beta_2 A_0 \mid \beta_3 A_0 \mid \ldots \mid \beta_n A_0$$
$$A_0 \rightarrow \alpha_1 A_0 \mid \alpha_2 A_0 \mid \alpha_3 A_0 \mid \ldots \mid \alpha_m A_0 \mid$$

- It does not eliminate left recursion involving derivations of two or more steps.

Elimination of Left Recursion

- Consider the grammar,

$$
\begin{aligned}
S &\rightarrow Aa \mid b \\
A &\rightarrow Ac \mid Sd \mid \epsilon
\end{aligned}
$$

- The nonterminal S is left-recursive because **$S \Rightarrow Aa \Rightarrow Sda$**, but it is not immediately left recursive.

- In such cases, we need to use the algorithm shown next.

Algorithm

Input: Grammar G with no cycles or ϵ-productions.

Output: An equivalent grammar with no left recursion.

METHOD: Apply the algorithm to G. Note that the resulting non-left-recursive grammar may have ϵ-productions.

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) for (each i from 1 to n) {
3) for (each j from 1 to $i-1$) {
4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

Algorithm

- We eliminate left-recursion in three steps.
  - eliminate ϵ-productions
  - eliminate cycles ($A \overset{+}{\Rightarrow} A$)
  - eliminate left-recursion

arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
for (each i from 1 to n) {
    for (each j from 1 to $i - 1$) {
        replace each production of the form $A_i \to A_j\gamma$ by
            the productions $A_i \to \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$
            where $A_j \to \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the
            current A_j-productions
    }
    eliminate the immediate left recursion among
        the A_i-productions;
}

Algorithm

- Non-terminal *A* is nullable if $A \overset{*}{\Rightarrow} \epsilon$

$$
\begin{aligned}
S &\rightarrow XZ \\
X &\rightarrow aXb \mid \epsilon \\
Z &\rightarrow aZ \mid ZX \mid \epsilon
\end{aligned}
$$

Find out the nullable non-terminals in the above grammar!

- Grammar is with cycles if $A \overset{+}{\Rightarrow} A$

$$
\begin{aligned}
S &\rightarrow X \mid Xb \mid SS \\
X &\rightarrow S \mid a
\end{aligned}
$$

Find out the non-terminals which form cycle!

Algorithm

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) **for** (each i from 1 to n) {
3)   **for** (each j from 1 to $i - 1$) {
4)   replace each production of the form $A_i \rightarrow A_j\gamma$ by
            the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$
            where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the
            current A_j-productions
5)   }
6)   eliminate the immediate left recursion among
        the A_i-productions;
7) }

- In the first iteration for $i = 1$, the outer for-loop of lines (2) through (7) eliminates any immediate left recursion among A_1-productions.
- Any remaining A_1-productions of the form $A_1 \rightarrow A_l\alpha$ must therefore have $l > 1$

Algorithm

arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
for (each i from 1 to n) {
  for (each j from 1 to $i-1$) {
    replace each production of the form $A_i \rightarrow A_j\gamma$ by
      the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$
      where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the
      current A_j-productions
  }
  eliminate the immediate left recursion among
    the A_i-productions;
}

- After the 1st iteration of the outer for-loop, all nonterminals A_k , where k < i, are “cleaned”.
- That is, any production $A_k \rightarrow A_l\alpha$, must have l > k.

Algorithm

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) for (each i from 1 to n) {
3) for (each j from 1 to $i - 1$) {
4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

- As a result, on the i^{th} iteration, the inner loop of lines (3) through (5) progressively raises the lower limit in any production $A_i \rightarrow A_m\alpha$, until we have $m \geq I$
- Then, eliminating immediate left recursion for the A_i productions at line (6) forces m to be greater than i.

Algorithm- Example

- We apply the procedure to grammar,

$$\begin{aligned} S &\rightarrow Aa \mid b \\ A &\rightarrow Ac \mid Sd \mid \epsilon \end{aligned}$$

- Technically, the algorithm is not guaranteed to work, because of the ϵ-production.
- But in this case the production $A \rightarrow \epsilon$ turns out to be harmless.
- To remove A_j from the right-hand side of the A_i production $A_i \rightarrow A_j\gamma$ replace $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \delta_3\gamma \mid \ldots\ldots\ldots\ldots \mid \delta_k\gamma$
  where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \delta_3 \mid \ldots\ldots\ldots\ldots \mid \delta_k$ and $\delta_i \neq \epsilon$

Algorithm- Example

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.

Left-Recursive Grammar

$$
\begin{aligned}
S &\rightarrow Aa \mid b \\
A &\rightarrow Ac \mid Sd \mid \epsilon
\end{aligned}
$$

- We order the non-terminals *S, A*
- *A1 = S*, *A2 = A*

Algorithm- Example

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) **for** (each i from 1 to n) {
3) **for** (each j from 1 to $i-1$) {
4) replace each production of the form $A_i \to A_j\gamma$ by the productions $A_i \to \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \to \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

- *$i = 1$, $A_1 = S$*
- *$j = 1$ to $j = i - 1 = 1-1 = 0$, the loop is not entered*
- *There is no immediate left recursion among the S-productions, so nothing happens for the case $i = 1$. ($A_1 = S$)*

Algorithm- Example

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) **for** (each i from 1 to n) {
3) **for** (each j from 1 to $i - 1$) {
4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

- *$i = 2$, $A_2 = A$*
- *$j = 1$ to $j = i - 1 = 2-1 = 1$, the loop is entered*

Algorithm- Example

4) replace each production of the form $A_i \to A_j\gamma$ by the productions $A_i \to \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \to \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions

Left-Recursive Grammar

$$S \to Aa \mid b$$
$$A \to Ac \mid Sd \mid \epsilon$$

- *$i = 2$, $A2 = A$, $j = 1$, $A1 = S$*
- *We need to*
  - put productions of the form $S \to \delta 1 \mid \delta 2 \mid \delta 3 \ldots\ldots\ldots\ldots \mid \delta k$
  - in productions of the form $A \to S\gamma$
- Production(s) with S at the left-hand-side, $S \to Aa \mid b$
- Productions(s) with A at the left side and right side beginning with S is (are), $A \to Sd$

Algorithm- Example

4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions

Left-Recursive Grammar

$$S \rightarrow Aa \mid b$$
$$A \rightarrow Ac \mid Sd \mid \epsilon$$

- $S \rightarrow Aa \mid b$ to be put in A, $A \rightarrow Sd$
- We substitute $S \rightarrow Aa \mid b$ in $A \rightarrow Sd$ to get the following A-productions,

$$\boldsymbol{A \rightarrow Aad \mid bd}$$

Algorithm- Example

6) eliminate the immediate left recursion among the A_i-productions;

- *All* $A_i = A_2 = A$*-productions together,*

$$A \rightarrow Ac \mid Aad \mid bd \mid \epsilon$$

- Eliminating the immediate left recursion among the A-productions yields the following,

$$A \rightarrow bdA' \mid A'$$
$$A' \rightarrow cA' \mid adA' \mid \epsilon$$

Algorithm- Example

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) **for** (each i from 1 to n) {
3) **for** (each j from 1 to $i - 1$) {
4) replace each production of the form $A_i \to A_j\gamma$ by the productions $A_i \to \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \to \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

- *i has attained the value of $n = 2$ and the loops are no more entered.*

Algorithm- Example

Left-Recursive Grammar

$$
\begin{aligned}
S &\rightarrow Aa \mid b \\
A &\rightarrow Ac \mid Sd \mid \epsilon
\end{aligned}
$$

- Put together we get the following non-left-recursive grammar,

$$
\begin{aligned}
S &\rightarrow Aa \mid b \\
A &\rightarrow bdA' \mid A' \\
A' &\rightarrow cA' \mid adA' \mid \epsilon
\end{aligned}
$$

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) **for** (each i from 1 to n) {
3) **for** (each j from 1 to $i - 1$) {
4) replace each production of the form $A_i \to A_j\gamma$ by the productions $A_i \to \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \to \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

Conceptual Technique Summary (Again)

- Put some order in the nonterminals.
- Start by making first nonterminal productions left-recursion-free.
- Put the first nonterminal left-recursion-free productions into those of the second one.
- Now make the productions of second nonterminal left-recursion-free.
- Thus keep on growing the set of left-recursion-free productions.

Left Factoring

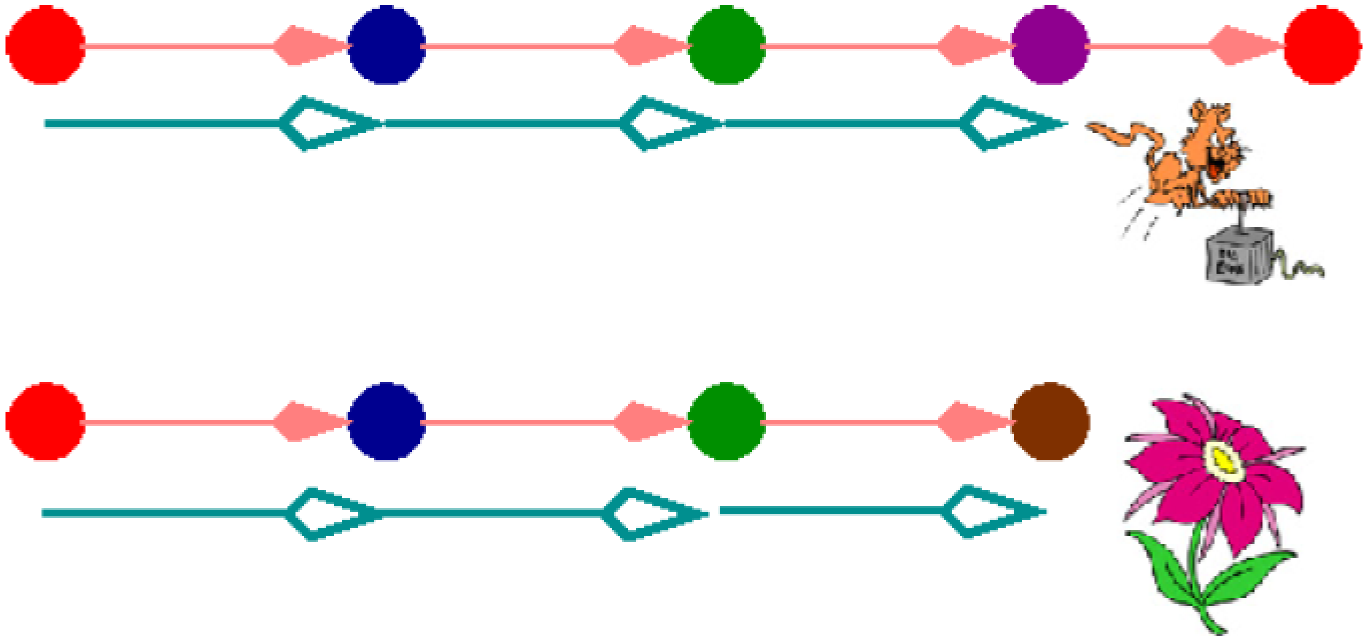
- Left factoring is a grammar transformation that is useful for producing a grammar suitable for predictive parsing.

- The basic idea is that sometimes it is not clear which of two alternative productions to use to expand a nonterminal A.

- We may be able to rewrite the *A*-productions to defer the decision until we have seen enough of the input to make the right choice.

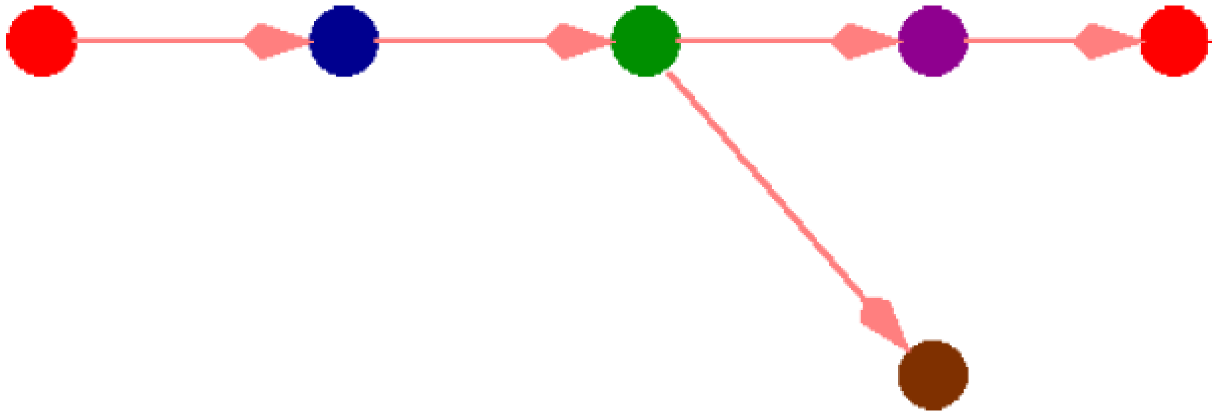
Left Factoring

Road Direction: *Red* $\rightarrow$ *Blue* $\rightarrow$ *Green* $\rightarrow$ *Brown*



Left Factoring

Defer the decision until we have seen enough of the input to make the right choice.



Left Factoring

- We have the two productions,

$$\begin{array}{lcl} stmt & \rightarrow & \textbf{if } expr \textbf{ then } stmt \textbf{ else } stmt \\ & | & \textbf{if } expr \textbf{ then } stmt \end{array}$$

- On seeing the input token **if**, we cannot immediately tell which production to choose to expand *stmt*.

Left Factoring

- $A \rightarrow \alpha\, \beta_1 \mid \alpha\, \beta_2$ are two A-productions.
- The input begins with a nonempty string derived from α.
- We do not know whether to expand A to $\alpha\, \beta_1$ or $\alpha\, \beta_2$.
- However, we may defer the decision by expanding A to α A′.
- Then, after seeing the input derived from we expand A_0 to β_1 or β_2.
- Left-factored, the original productions become,

$$A \rightarrow \alpha\, A'$$
$$A' \rightarrow \beta_1 \mid \beta_2$$

Left Factoring Algorithm

Input: Grammar G.
Output: An equivalent left-factored grammar.

Method:

- For each nonterminal A find the longest prefix common to two or more of its alternatives.
- If $\alpha \neq \epsilon$, replace all the A productions $A \rightarrow \alpha\,\beta_1 \mid \alpha\,\beta_2 \mid \ldots\ldots \mid \alpha\,\beta_n \mid \gamma$ where γ represents all alternatives that do not begin with α by

$$A \rightarrow \alpha\, A' \mid \gamma$$
$$A' \rightarrow \beta_1 \mid \beta_2 \mid \ldots\ldots \mid \beta_n$$

  where A' is a new nonterminal.
- Repeatedly apply this transformation until no two alternatives for a nonterminal have a common prefix.

Left Factoring Example

- The following grammar abstracts the dangling-else problem:

$$S \rightarrow iEtS \mid iEtSeS \mid a$$
$$E \rightarrow b$$

- Here i, t, and e stand for if, then and else, E and S for “expression” and “statement.”
- Left-factored, this grammar becomes:

$$S \rightarrow iEtSS' \mid a$$
$$S' \rightarrow eS \mid \epsilon$$
$$E \rightarrow b$$

- Thus, we may expand S to $iEtSS'$ on input i, and wait until $iEtS$ has been seen to decide whether to expand S' to eS or to ϵ.

Top-Down Parsing

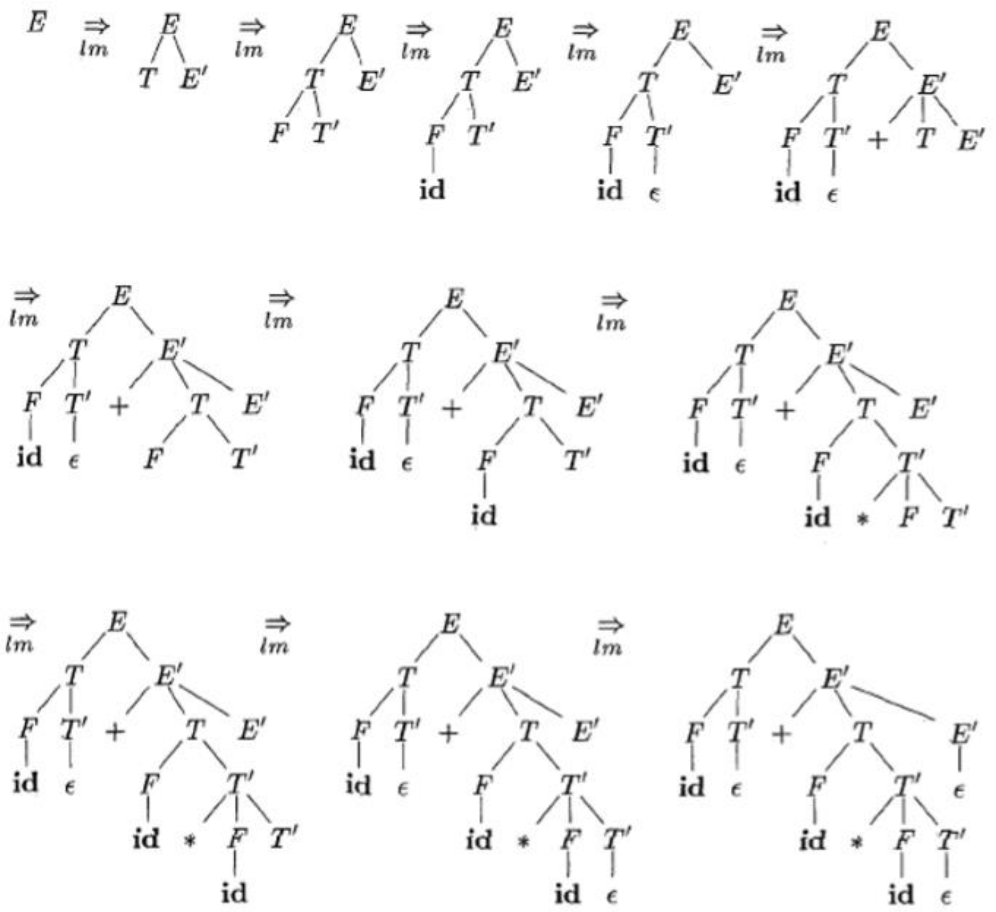
- Top-down parsing can be viewed as the problem of

  - constructing a parse tree for the input string,
  - starting from the root and
  - creating the nodes of the parse tree in preorder (depth-first).

- Equivalently, top-down parsing can be viewed as finding a leftmost derivation for an input string.

- The sequence of top-down parse trees for the input string $\mathbf{id} + \mathbf{id} * \mathbf{id}$ and grammar:

$$
\begin{array}{lcl}
E & \rightarrow & TE' \\
E' & \rightarrow & +TE' \mid \epsilon \\
T & \rightarrow & FT' \\
T' & \rightarrow & *FT' \mid \epsilon \\
F & \rightarrow & (E) \mid \mathbf{id}
\end{array}
$$



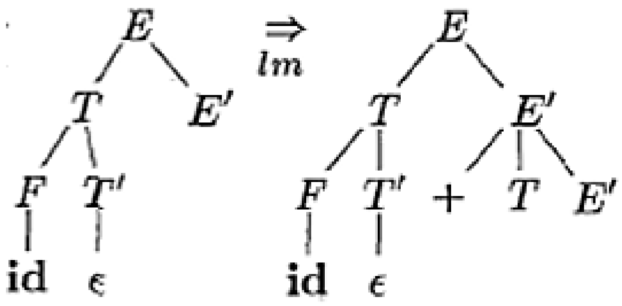
Top-down parse for **id** + **id** ∗ **id**

Top-Down Parsing

- At each step of a top-down parse, the key problem is that of determining the production to be applied for a nonterminal, say *A*.

- Once an *A*-production is chosen, the rest of the parsing process consists of "matching" the terminal symbols in the production body with the input string.

- Consider the top-down parse in figure.
- This constructs a tree with two nodes labeled E'.
- At the first E' node (in preorder), the production
  E' → +TE' is chosen.
- At the second E' node, the production E' → is chosen.
- A predictive parser can choose between E' -productions by looking at the next input symbol.
- The class of grammars for which we can construct predictive parsers looking k symbols ahead in the input is sometimes called the LL(k) class.



FIRST and FOLLOW

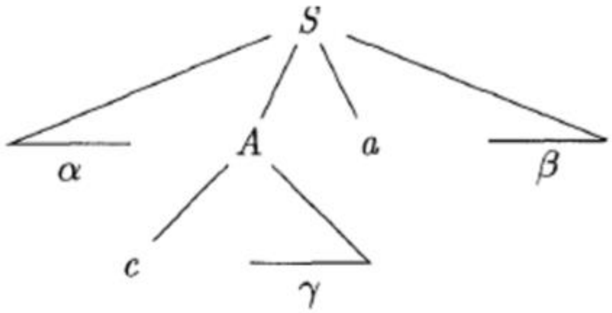
- The construction of both top-down and bottom-up parsers is aided by two functions, FIRST and FOLLOW, associated with a grammar G.
- During top-down parsing, FIRST and FOLLOW allow us to choose which production to apply, based on the next input symbol.
- During panic-mode error recovery, sets of tokens produced by FOLLOW can be used as synchronizing tokens.

FIRST

- Define FIRST(α), where α is any string of grammar symbols, to be the set of terminals that begin strings derived from α.

- If $\alpha \Rightarrow \epsilon$, then ϵ is also in FIRST(α).

- For example, in figure $A \Rightarrow c\gamma$, so c is in FIRST(A).



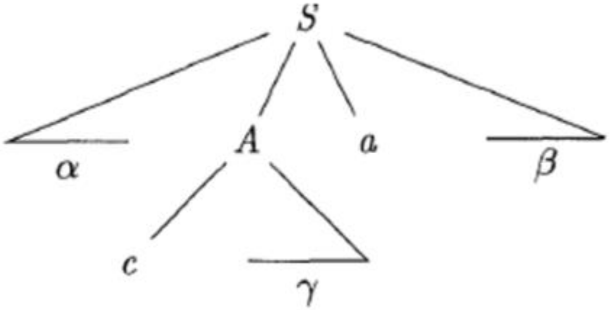
Terminal c is in FIRST(A) and a is in FOLLOW(A)

FIRST

- Let us see how FIRST can be used during predictive parsing.
- Consider two A-productions $A \rightarrow \alpha \mid \beta$, where FIRST(α) and FIRST(β) are disjoint sets.
- We can then choose between these A-productions by looking at the next input symbol a, since a can be in at most one of FIRST(α) and FIRST(β), not both.
- For instance, if a is in FIRST(β) choose the production $A \rightarrow \beta$.

FOLLOW

- Define FOLLOW(A), nonterminal A, to be the set of terminals a that can appear immediately to the right of A in some sentential form.
- That is, the set of terminals a such that there exists a derivation of the form $S \Rightarrow \alpha A a \beta$, for some α and β.
- Note that there may have been symbols between A and α, at some time during the derivation, but if so, they derived $\in$ and disappeared.

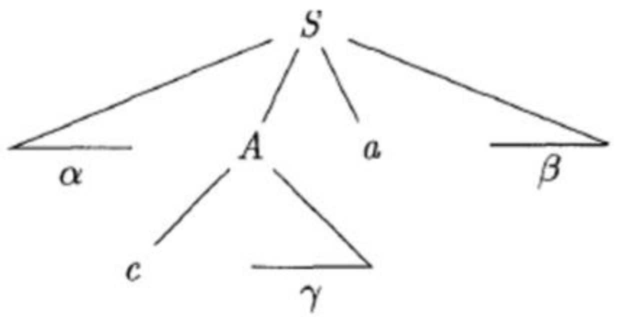


Terminal c is in FIRST(A) and a is in FOLLOW(A)

FOLLOW

- In addition, if *A* can be the rightmost symbol in some sentential form, then $ is in FOLLOW(A).

- Recall that $ is a special "endmarker" symbol that is assumed not to be a symbol of any grammar.



Terminal c is in FIRST(A) and a is in FOLLOW(A)

RULES TO COMPUTE FIRST

- To compute FIRST(X) for all grammar symbols X, apply the following rules until no more terminals or ∊ can be added to any FIRST set.

1. If X is terminal, then FIRST(X) is $\{X\}$.
2. If X is nonterminal and $X \rightarrow Y_1 Y_2 \ldots . Y_k$ is a production for some $k \geq 1$, then place a in FIRST(X) if for some i, a is in FIRST(Y_i), and ∊ is in all of FIRST(Y_1), ………, FIRST(Y_{i-1});
   If ∊ is in FIRST(Y_j) for all j = 1, 2, ….., k then add ∊ to FIRST(X).
3. If $X \rightarrow \epsilon$ is a production, then add ∊ to FIRST(X).

RULES TO COMPUTE FIRST

- Everything in FIRST(Y_1) is surely in FIRST(X).
- If Y_1, does not derive $\in$, then we add nothing more to FIRST(X), but if $Y_1 \overset{*}{\Rightarrow} \in$ then we add FIRST(Y_2) and so on.

- Now, we can compute FIRST for any string $X_1X_2 \ldots\ldots X_n$ as follows.

1. Add to FIRST($X_1X_2 \ldots\ldots X_n$) all the non-$\in$ symbols of FIRST(X_1).
2. Also add the non-$\in$ symbols of FIRST(X_2) if $\in$ is in FIRST(X_1), the non-$\in$ symbols of FIRST(X_3) if $\in$ is in both FIRST(X_1) and FIRST(X_2) and so on.
3. Finally, add $\in$ to FIRST($X_1X_2 \ldots\ldots X_n$) if, for all i, $\in$ is in FIRST(X_i).

RULES TO COMPUTE FOLLOW

- To compute FOLLOW(A) for all nonterminals A, apply the following rules until nothing can be added to any FOLLOW set.

1. Place \$ in FOLLOW(S), where S is the start symbol and \$ is the input right endmarker.
2. If there is a production $A \rightarrow \alpha B \beta$, then everything in FIRST(β) except ϵ is in FOLLOW(B).
3. If there is a production $A \rightarrow \alpha B$, or a production $A \rightarrow \alpha B \beta$ where FIRST(β) contains ϵ, then everything in FOLLOW(A) is in FOLLOW(B).

EXAMPLE

Grammar,

$$
\begin{aligned}
E &\rightarrow TE' \\
E' &\rightarrow +TE' \mid \epsilon \\
T &\rightarrow FT' \\
T' &\rightarrow *FT' \mid \epsilon \\
F &\rightarrow (E) \mid \mathbf{id}
\end{aligned}
$$

FIRST(E) = FIRST(T) = FIRST(F) = { (, id }
FIRST(E') = { +, ∈}
FIRST(T') = {*, ∈}

EXAMPLE

Try to solve it!!!

$S \rightarrow ACB \mid Cbb \mid Ba$
$A \rightarrow da \mid BC$
$B \rightarrow g \mid \epsilon$
$C \rightarrow h \mid \epsilon$

FIRST(S) = { d, g, h, ∈ , b, a }
FIRST(A) = { d, g, h, ∈}
FIRST(B) = { g, ∈}
FIRST(C) = { h, ∈}

EXAMPLE

- Grammar:

$$E \rightarrow TE' \qquad T \rightarrow FT' \qquad F \rightarrow (E) \mid \textbf{id}$$
$$E' \rightarrow +TE' \mid \epsilon \qquad T' \rightarrow *FT' \mid \epsilon$$

- Computation of FOLLOW:

FOLLOW(E)	FOLLOW(E')	FOLLOW(T)	FOLLOW(T')	FOLLOW(F)

Initially all sets are empty

Put \$ in FOLLOW(E) by rule (1) (Place \$ in FOLLOW(S), where S is the start symbol and \$ is the input right endmarker)

\$				

$$
\begin{array}{lllllllll}
E & \rightarrow & TE' & T & \rightarrow & FT' & F & \rightarrow & (E) \mid \textbf{id} \\
E' & \rightarrow & +TE' \mid \epsilon & T' & \rightarrow & *FT' \mid \epsilon & & &
\end{array}
$$

$\text{FIRST}(E) = \text{FIRST}(T) = \text{FIRST}(F) = \{(, \textbf{id}\}$, $\text{FIRST}(E') = \{+, \epsilon\}$,
$\text{FIRST}(T') = \{*, \epsilon\}$

By rule (2) (If there is a production $A \rightarrow \alpha B\beta$, then everything in FIRST(β) except for ϵ is placed in FOLLOW(B)) applied to,

$E \rightarrow TE'$: FIRST(E') except ϵ i.e. $\{+\}$ are in FOLLOW(T)

$E' \rightarrow +TE'$: FIRST(E') except ϵ i.e. $\{+\}$ are in FOLLOW(T)

$T \rightarrow FT'$: FIRST(T') except ϵ i.e. $\{*\}$ are in FOLLOW(F)

$T \rightarrow *FT'$: FIRST(T') except ϵ i.e. $\{*\}$ are in FOLLOW(F)

$F \rightarrow (E)$: FIRST()) i.e. $\{)\}$ are in FOLLOW(E)

FOLLOW(E)	FOLLOW(E')	FOLLOW(T)	FOLLOW(T')	FOLLOW(F)
\$,)		+		*

Rule (2) is not applicable any more since it depends only on FIRST, which are now stable sets.

Application of rule (3) *(If there is a production $A \rightarrow \alpha B$, or a production $A \rightarrow \alpha B \beta$ where FIRST(β) contains ϵ (i.e., $\beta \overset{*}{\Rightarrow} \epsilon$), then everything in FOLLOW(A) is in FOLLOW(B))*

$$E \rightarrow TE' \qquad T \rightarrow FT' \qquad F \rightarrow (E) \mid \mathbf{id}$$
$$E' \rightarrow +TE' \mid \epsilon \qquad T' \rightarrow *FT' \mid \epsilon$$

FOLLOW(E)	FOLLOW(E')	FOLLOW(T)	FOLLOW(T')	FOLLOW(F)
\$,)		$+$		$*$

$E \rightarrow TE'$: Everything in FOLLOW(E) are in FOLLOW(E')

FOLLOW(E)	FOLLOW(E')	FOLLOW(T)	FOLLOW(T')	FOLLOW(F)
\$,)	\$,)	$+$		$*$

$E' \rightarrow +TE'$ (also $\epsilon \in$ FIRST(E')): Everything in FOLLOW(E') are in FOLLOW(T)

FOLLOW(E)	FOLLOW(E')	FOLLOW(T)	FOLLOW(T')	FOLLOW(F)
\$,)	\$,)	+, \$,)		$*$

$T \rightarrow FT'$: Everything in FOLLOW(T) are in FOLLOW(T')

FOLLOW(E)	FOLLOW(E')	FOLLOW(T)	FOLLOW(T')	FOLLOW(F)
\$,)	\$,)	+, \$,)	+, \$,)	$*$

Application of rule (3) — continued *(If there is a production $A \to \alpha B$, or a production $A \to \alpha B \beta$ where FIRST(β) contains ϵ (i.e., $\beta \overset{*}{\Rightarrow} \epsilon$), then everything in FOLLOW(A) is in FOLLOW(B))*

$$
\begin{array}{lllllllll}
E & \to & TE' & T & \to & FT' & F & \to & (E) \mid \textbf{id} \\
E' & \to & +TE' \mid \epsilon & T' & \to & *FT' \mid \epsilon & & &
\end{array}
$$

FOLLOW(E)	FOLLOW(E')	FOLLOW(T)	FOLLOW(T')	FOLLOW(F)
\$,)	\$,)	+, \$,)	+, \$,)	*

$T' \to *FT'$ (also $\epsilon \in$ FIRST(T')): Everything in FOLLOW(T') are in FOLLOW(F)

\$,)	\$,)	+, \$,)	+, \$,)	*, +, \$,)

We can try applying Rule (3) again, but will find that the sets have stabilized (nothing can be added to any FOLLOW set).

LL(1) Grammars

- Predictive parsers, that is, recursive-descent parsers needing no backtracking, can be constructed for a class of grammars called LL(1).
- The first “L” in LL(1) stands for scanning the input from left to right.
- The second “L” for producing a leftmost derivation.
- And the “1” for using one input symbol of lookahead at each step to make parsing action decisions.
- The class of LL(1) grammars is rich enough to cover most programming constructs.
- Although care is needed in writing a suitable grammar for the source language.
- For example, no left-recursive or ambiguous grammar can be LL(1).

LL(1) Grammars

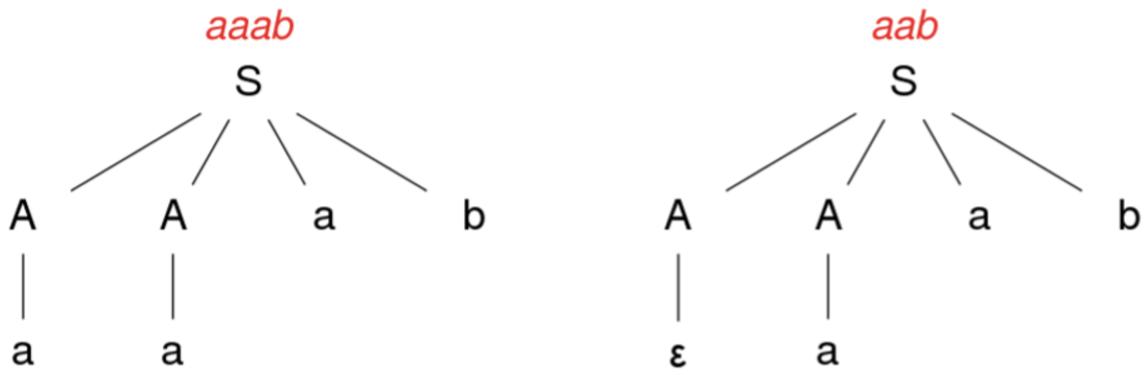
- A grammar G is LL(1) if and only if whenever $A \rightarrow \alpha \mid \beta$ are two distinct productions of G the following conditions hold:

1. For no terminal a do both α and β derive strings beginning with a.
2. At most one of α and β can derive the empty string.
3. If $\beta \overset{*}{\Rightarrow} \in$, then α does not derive any string beginning with a terminal in FOLLOW(A). Likewise, $\alpha \overset{*}{\Rightarrow} \in$, then β does not derive any string beginning with terminal in FOLLOW(A).

- The first two conditions are equivalent to the statement that FIRST(α) and FIRST(β) are disjoint sets.
- The third condition is equivalent to stating that if $\in$ is in FIRST(β), then FIRST(α) and FOLLOW(A) are disjoint sets, and likewise if $\in$ is in FIRST(α).

Non-LL(1) Grammar Example

$$
\begin{aligned}
S &\rightarrow AAab \mid BBba \\
A &\rightarrow a \mid \epsilon \\
B &\rightarrow b \mid \epsilon
\end{aligned}
$$



- Now, when expanding the first *A*, we can not decide which of the two production rules to be used.
- Both give us the promise of an a.

LL(1) Grammars

- Predictive parsers can be constructed for LL(1) grammars since the proper production to apply for a nonterminal can be selected by looking only at the current input symbol.
- Flow-of-control constructs, with their distinguishing key-words, generally satisfy the LL(1) constraints.
- For instance, if we have the productions,

$$\begin{array}{lcl} stmt & \rightarrow & \textbf{if}\ (\ expr\ )\ stmt\ \textbf{else}\ stmt \\ & | & \textbf{while}\ (\ expr\ )\ stmt \\ & | & \{\ stmt\_list\ \} \end{array}$$

then the keywords **if**, **while**, and the symbol **{** tell us which alternative is the one that could possibly succeed if we are to find a statement.

LL(1) Grammars

- The next algorithm collects the information from FIRST and FOLLOW sets into a predictive parsing table $M[A, a]$, a two dimensional array, where A is a nonterminal, and a is a terminal or the symbol \$, the input endmarker.
- The idea behind the algorithm is the following:

- Suppose $A \rightarrow \alpha$ is a production with a in FIRST(α).
- Then, the parser will expand A by α when the current input symbol is a.
- The only complication occurs when $\alpha = \epsilon$ or $\alpha \stackrel{*}{\Rightarrow} \epsilon$.
- In this case, we should again expand A by α if the current input symbol is in FOLLOW(A), or if the \$ on the input has been reached and \$ is in FOLLOW(A).

Algorithm for Construction of Predictive Parsing Table

INPUT: Grammar G.

OUTPUT: Parsing table M.

METHOD: For each production $A \to \alpha$ of the grammar, do the following:

1. For each terminal a in FIRST(A), add $A \to \alpha$ to $M[A, a]$.
2. If ϵ is in FIRST(α), then for each terminal b in FOLLOW(A), add $A \to \alpha$ to $M[A, b]$.
   If ϵ is in FIRST(α) and \$ is in FOLLOW(A), add $A \to \alpha$ to $M[A, \$]$ as well.

- If, after performing the above, there is no production at all in $M[A, a]$, then set $M[A, a]$ to **error** (which we normally represent by an empty entry in the table).

EXAMPLE

- For the expression grammar below,

$$E \rightarrow TE' \qquad T \rightarrow FT' \qquad F \rightarrow (E) \mid \mathbf{id}$$
$$E' \rightarrow +TE' \mid \epsilon \qquad T' \rightarrow *FT' \mid \epsilon$$

the algorithm produces the parsing table in figure.

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \mathbf{id}$			$F \rightarrow (E)$		

- Blanks are error entries.
- Nonblanks indicate a production with which to expand a nonterminal.

For each production $A \to \alpha$ of the grammar, do the following:

1. For each terminal a in FIRST(A), add $A \to \alpha$ to $M[A, a]$.
2. If ϵ is in FIRST(α), then for each terminal b in FOLLOW(A), add $A \to \alpha$ to $M[A, b]$. If ϵ is in FIRST(α) and \$ is in FOLLOW(A), add $A \to \alpha$ to $M[A, \$]$ as well.

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \textbf{id}$			$F \to (E)$		

- Consider production $E \to TE'$.
- Since

$$\text{FIRST}(TE') = \text{FIRST}(T) = \{(, \textbf{id}\}$$

this production is added to $M[E, (]$ and $M[E, \textbf{id}]$.

- Production $E' \to +TE'$ is added to $M[E', +]$ since $\text{FIRST}(+TE') = \{+\}$.
- Since $\text{FOLLOW}(E') = \{), \$\}$, production $E' \to \epsilon$ is added to $M[E',)]$ and $M[E', \$]$.

Algorithm for Construction of Predictive Parsing Table

- The aforementioned algorithm can be applied to any grammar G to produce a parsing table *M*.
- For every LL(1) grammar, each parsing-table entry uniquely identifies a production or signals an error.
- For some grammars, however, M may have some entries that are multiply defined.
- For example, if G is left-recursive or ambiguous, then M will have at least one multiply defined entry.
- Although left-recursion elimination and left factoring are easy to do, there are some grammars for which no amount of alteration will produce an LL(1) grammar.
- The language in the following example has no LL(1) grammar at all.

EXAMPLE

- The following grammar, which abstracts the dangling-else problem, is repeated here:
- The grammar,

$$
\begin{aligned}
S &\rightarrow iEtSS' \mid a \\
S' &\rightarrow eS \mid \epsilon \\
E &\rightarrow b
\end{aligned}
$$

- The grammar is ambiguous.
- On input e, it will not be clear which alternative for S0 should be chosen.

EXAMPLE

$S \rightarrow iEtSS' \mid a$

$S' \rightarrow eS \mid \epsilon$

$E \rightarrow b$

NON - TERMINAL	INPUT SYMBOL					
	a	b	e	i	t	$\$$
S	$S \rightarrow a$			$S \rightarrow iEtSS'$		
S'			$S' \rightarrow \epsilon$ $S' \rightarrow eS$			$S' \rightarrow \epsilon$
E		$E \rightarrow b$				

- The entry for $M[S', e]$ contains both $S' \rightarrow eS$ and $S' \rightarrow \epsilon$, since FOLLOW(S') = {e, $}
- The grammar is ambiguous and the ambiguity is manifested by a choice in what production to use when an e (**else**) is seen.

EXAMPLE

$$S \rightarrow iEtSS' \mid a$$
$$S' \rightarrow eS \mid \epsilon$$
$$E \rightarrow b$$

NON - TERMINAL	INPUT SYMBOL					
	a	b	e	i	t	\$
S	$S \rightarrow a$			$S \rightarrow iEtSS'$		
S'			$S' \rightarrow \epsilon$ $S' \rightarrow eS$			$S' \rightarrow \epsilon$
E		$E \rightarrow b$				

- We can resolve the ambiguity if we choose $S' \rightarrow eS$
- This choice corresponds to associating **else**'s with the closest previous **then**'s.
- Note that the choice $S' \rightarrow \epsilon$ would prevent e from ever being put on the stack or removed from the input, and is therefore surely wrong.

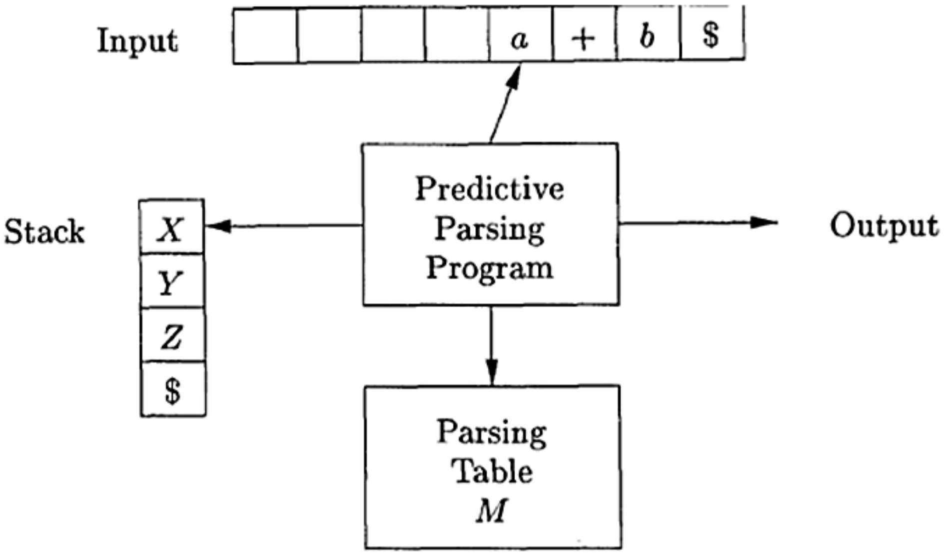
Nonrecursive Predictive Parsing

- A nonrecursive predictive parser can be built by maintaining a stack explicitly, rather than implicitly via recursive calls.
- The parser mimics a leftmost derivation.
- If w is the input that has been matched so far, then the stack holds a sequence of grammar symbols α such that

$$S \underset{lm}{\overset{*}{\Rightarrow}} w\alpha$$

Nonrecursive Predictive Parsing

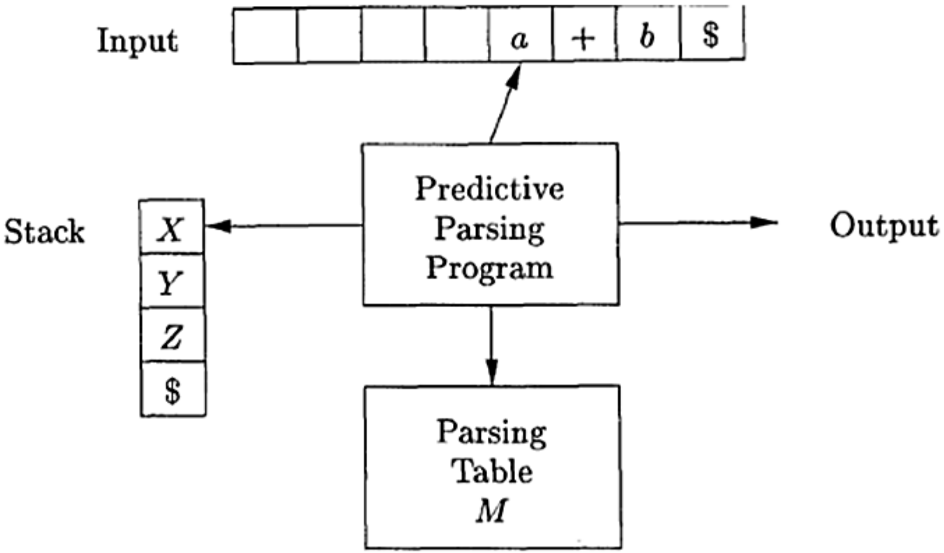
- The table-driven parser in figure has an input buffer, a stack containing a sequence of grammar symbols, a parsing table constructed by algorithm, and an output stream.
- The input buffer contains the string to be parsed, followed by the endmarker $.
- We reuse the symbol $ to mark the bottom of the stack, which initially contains the start symbol of the grammar on top of $.



Model of a table-driven predictive parser

Nonrecursive Predictive Parsing

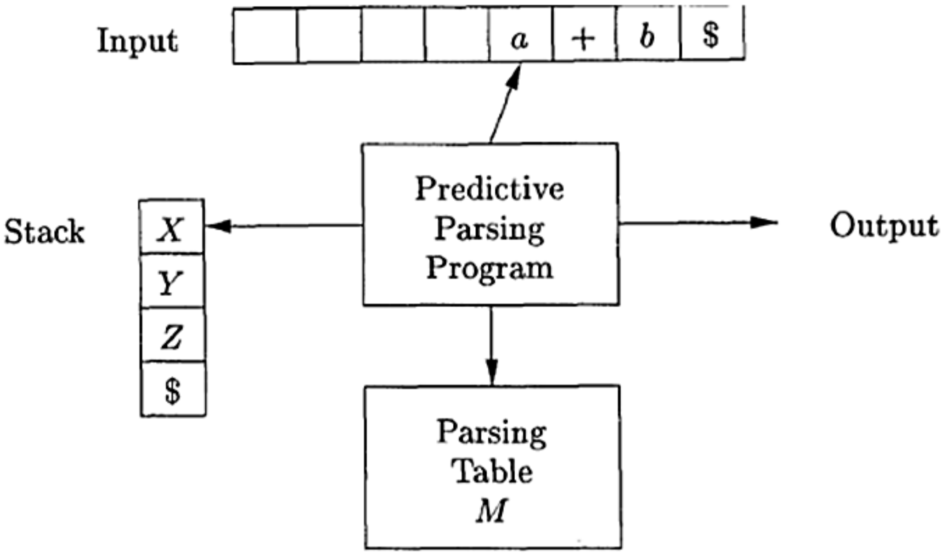
- The parser is controlled by a program that considers X, the symbol on top of the stack, and a, the current input symbol.
- If X is a nonterminal, the parser chooses an X-production by consulting entry $M[X,a]$ of the parsing table M.
- Additional code could be executed here, for example, code to construct a node in a parse tree.
- Otherwise, it checks for a match between the terminal X and current input symbol a.



Model of a table-driven predictive parser

Nonrecursive Predictive Parsing

- The behavior of the parser can be described in terms of its configurations, which give the stack contents and the remaining input.
- The next algorithm describes how configurations are manipulated.



Model of a table-driven predictive parser

Thank You