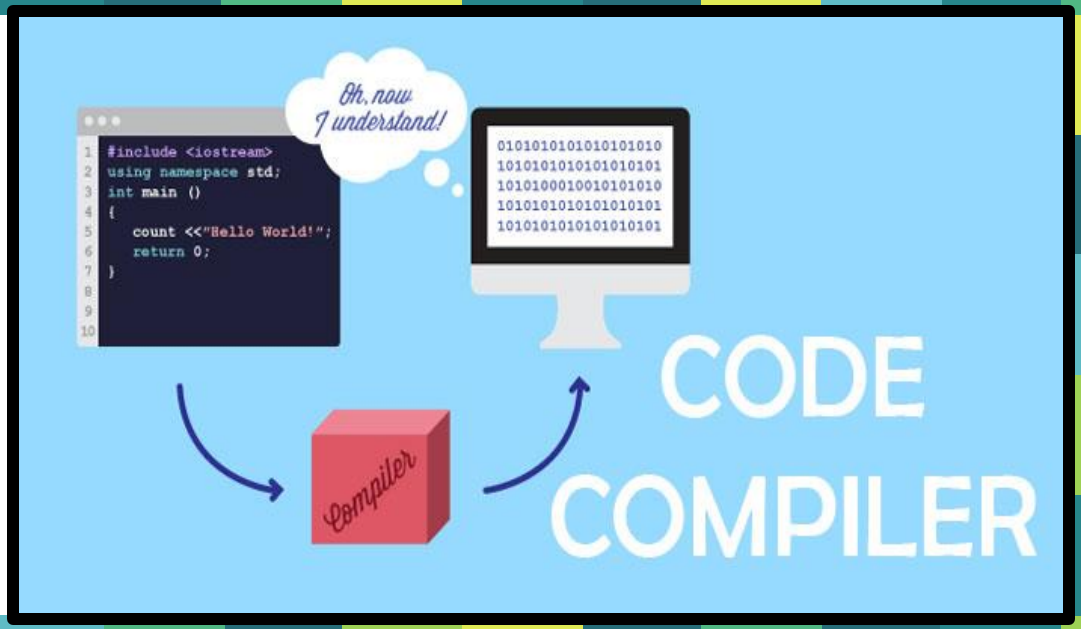
CSE- 303

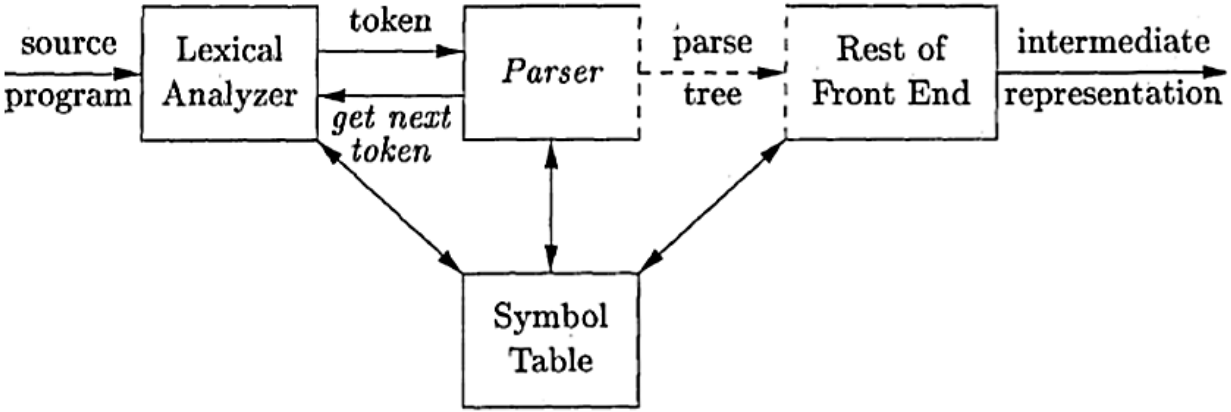
Compiler

Chapter - 4



The Role of the Parser

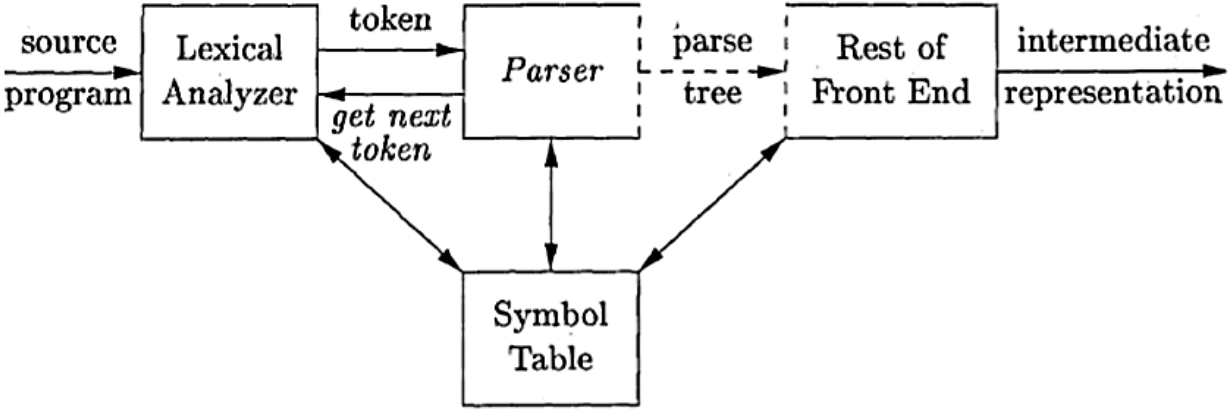
- In our compiler model, the parser obtains a string of tokens from the lexical analyzer.
- It then verifies that the string of token names can be generated by the grammar for the source language.



Position of parser in compiler model

The Role of the Parser

- We expect the parser
  - to report any syntax errors in an intelligible fashion and
  - to recover from commonly occurring errors to continue processing the remainder of the program.



Position of parser in compiler model

The Role of the Parser

- Conceptually, for well-formed programs, the parser constructs a parse tree and passes it to the rest of the compiler for further processing.
- In fact, the parse tree need not be constructed explicitly.
- Since checking and translation actions can be interspersed with parsing.
- Thus, the parser and the rest of the front end could well be implemented by a single module.

The Role of the Parser

- There are three general types of parsers for grammars:

1) **Universal:** can parse any type of grammar; not very efficient. Universal parsing methods are Cocke-Younger-Kasami algorithm and Earley's algorithm.
2) **Top-down:** build parse trees from the top (root) to the bottom (leaves); Ex: Recursive descent parsing, Backtracking
3) **Bottom-up:** start from the leaves and work their way up to the root

- The commonly used methods in compilers are top-down or bottom-up.
- In either case, the input to the parser is scanned from left to right, one symbol at a time.

The Role of the Parser

- The most efficient top-down and bottom-up methods work only for subclasses of grammars.
- But several of these classes, particularly, LL and LR grammars, are expressive enough to describe most of the syntactic constructs in modern programming languages.
- Parsers implemented by hand often use LL grammars. The predictive-parsing approach works for LL grammars.
- Parsers for the larger class of LR grammars are usually constructed using automated tools.

Syntax Error Handling

- If a compiler had to process only correct programs, its design and implementation would be simplified greatly.
- However, a compiler is expected to assist the programmer in locating and tracking down errors that inevitably creep into programs, despite the programmer's best efforts.
- Strikingly, few languages have been designed with error handling in mind, even though errors are so commonplace.
- Most programming language specifications do not describe how a compiler should respond to errors.
- Error handling is left to the compiler designer.
- Planning the error handling right from the start can both simplify the structure of a compiler and improve its handling of errors.

Syntax Error Handling

- Common programming errors can occur at many different levels.
- **Lexical errors** include misspellings of identifiers, keywords, or operators — e.g., the use of an identifier elipsesize instead of ellipsesize — and missing quotes around text intended as a string.

Syntax Error Handling

- Common programming errors can occur at many different levels.
- **Syntactic errors** include misplaced semicolons or extra or missing braces, that is, “{” or “}”. As another example, in C or Java, the appearance of a case statement without an enclosing switch is a syntactic error.

  However, this situation is usually allowed by the parser and caught later in the processing, as the compiler attempts to generate code.

Syntax Error Handling

- Common programming errors can occur at many different levels.
- **Semantic errors** include type mismatches between operators and operands. An example is a return statement in a Java method with result type void.

Syntax Error Handling

- Common programming errors can occur at many different levels.
- **Logical errors** can be anything from incorrect reasoning on the part of the programmer to the use in a C program of the assignment operator = instead of the comparison operator ==. The program containing = may be well formed; however, it may not reflect the programmer's intent.

Syntax Error Handling

- The precision of parsing methods allows syntactic errors to be detected very efficiently.
- Several parsing methods, such as the LL and LR methods, detect an error as soon as possible.
- That is, when the stream of tokens from the lexical analyzer cannot be parsed further according to the grammar for the language.
- More precisely, they have the viable-prefix property, meaning that they detect that an error has occurred as soon as they see a prefix of the input that cannot be completed to form a string in the language.

Syntax Error Handling

- The error handler in a parser has goals that are simple to state but challenging to realize:
- Report the presence of errors clearly and accurately.
- Recover from each error quickly enough to detect subsequent errors.
- Add minimal overhead to the processing of correct programs.

Syntax Error Handling

- Fortunately, common errors are simple ones.
- A relatively straightforward error-handling mechanism often suffices.
- How should an error handler report the presence of an error?
- At the very least, it must report the place in the source program where an error is detected.
- There is a good chance that the actual error occurred within the previous few tokens.
- A common strategy is to print the offending line with a pointer to the position at which an error is detected.

Writing a Grammar

- Grammars are capable of describing most, but not all, of the syntax of programming languages.
- For instance, the requirement that identifiers be declared before they are used, cannot be described by a context-free grammar.
- Therefore, the sequences of tokens accepted by a parser form a superset of the programming language.
- Subsequent phases of the compiler must analyze the output of the parser to ensure compliance with rules that are not checked by the parser.

Derivation

- A derivation is basically a sequence of production rules, in order to get the input string.
- During parsing, we take two decisions for some sentential form of input:

1) Deciding the non-terminal which is to be replaced.
2) Deciding the production rule, by which, the non-terminal will be replaced.

- Two ways of derivation:

1) Left-most Derivation: input is scanned and replaced from left to right
2) Right-most Derivation: scan and replace the input with production rules, from right to left

Derivation

- **Input**: id + id * id
- **Production rules**:

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow id$

Left-most Derivation

$E \rightarrow E * E$

$E \rightarrow E + E * E$

$E \rightarrow id + E * E$

$E \rightarrow id + id * E$

$E \rightarrow id + id * id$

Right-most Derivation

$E \rightarrow E + E$

$E \rightarrow E + E * E$

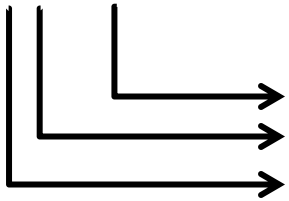
$E \rightarrow E + E * id$

$E \rightarrow E + id * id$

$E \rightarrow id + id * id$

LL(k) Grammar

LL (1) Grammar



Read one symbol at a time
Leftmost Derivation
Left to Right Scanning

What about LL (k) Grammar?
What about LR (1) Grammar?

*****Determine if a grammar is LL (1)*****

LL(1) grammar is not left-recursive!!! Then check for the other three conditions…

Representative Grammars

Ambiguity: A grammar G is said to be ambiguous if it has more than one parse tree for at least one string.

- The following grammar treats + and $*$ alike.

$$E \rightarrow E + E \mid E * E \mid (E) \mid \textbf{id}$$

- So it is useful for illustrating techniques for handling ambiguities during parsing.
- Here, E represents expressions of all types.
- This grammar permits more than one parse tree for expressions like a + b $*$ c.

Representative Grammars

- Constructs that begin with keywords like while or int, are relatively easy to parse.
- We therefore concentrate on expressions, which present more of challenge, because of the associativity and precedence of operators.

- **Associativity:** If an operand has operators on both sides, the side on which the operator takes this operand is decided by the associativity of those ope-rators.
  - Left-associative: +, -, *, / ; Ex: (id + id) + id
  - Right-associative: Exponentiation; Ex: id ^ (id ^ id)

- **Precedence:** If two different operators share a common operand, the precedence of operators decides which will take the operand.
  - Precedence Sequence: * / have higher precedence than + - ; Ex: 2+(3*4)

Representative Grammars

- Associativity and precedence are captured in the following grammar.
- *E* represents expressions consisting of terms separated by + signs.
- *T* represents terms consisting of factors separated by * signs.
- *F* represents factors that can be either parenthesized expressions or identifiers:

$$
\begin{aligned}
E &\rightarrow E + T \mid T \\
T &\rightarrow T * F \mid F \\
F &\rightarrow (E) \mid \mathbf{id}
\end{aligned}
$$

Representative Grammars

- The above grammar belongs to the class of LR grammars that are suitable for bottom-up parsing.
- This grammar can be adapted to handle additional operators and additional levels of precedence.
- However, it cannot be used for top-down parsing because it is left recursive.

$$
\begin{aligned}
E &\rightarrow E + T \mid T \\
T &\rightarrow T * F \mid F \\
F &\rightarrow (E) \mid \mathbf{id}
\end{aligned}
$$

Representative Grammars

- The following non-left-recursive variant of the expression grammar will be used for top-down parsing:

$$
\begin{aligned}
E &\rightarrow TE' \\
E' &\rightarrow +TE' \mid \epsilon \\
T &\rightarrow FT' \\
T' &\rightarrow *FT' \mid \epsilon \\
F &\rightarrow (\ E\ ) \mid \textbf{id}
\end{aligned}
$$

Elimination of Left Recursion

- A grammar is left recursive if it has a nonterminal A such that there is a derivation $A \overset{+}{\Rightarrow} Aa$ for some string a.
- Top-down parsing methods cannot handle left-recursive grammars, so a transformation that eliminates left recursion is needed.
- In simple left recursion there was one production of the form $A \rightarrow A\alpha$.
- In general case, left recursion:

$A \rightarrow Bac$

$B \rightarrow Ccd$

$C \rightarrow Abd$

Elimination of Left Recursion

- Left-recursive pair of productions $A \rightarrow A\alpha \mid \beta$ can be replaced by the non-left-recursive productions

$$A \rightarrow \beta A'$$
$$A' \rightarrow \alpha A' \mid \epsilon$$

without changing the set of strings derivable from A.

- This rule by itself suffices in many grammars.

Elimination of Left Recursion

$$A \rightarrow A\alpha \mid \beta$$

to be replaced by

$$\begin{aligned} A &\rightarrow \beta A' \\ A' &\rightarrow \alpha A' \mid \epsilon \end{aligned}$$

- Grammar for arithmetic expressions,

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T * F \mid F \\ F &\rightarrow (E) \mid \mathbf{id} \end{aligned}$$

- Eliminating the immediate left recursions we obtain,

$$\begin{aligned} E &\rightarrow TE' \\ E' &\rightarrow +TE' \mid \epsilon \\ T &\rightarrow FT' \\ T' &\rightarrow *FT' \mid \epsilon \\ F &\rightarrow (E) \mid \mathbf{id} \end{aligned}$$

Elimination of Left Recursion

- No matter how many A-productions there are, we can eliminate immediate left recursion from them.
- First, we group the A-productions as,

$$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid A\alpha_3 \mid \ldots \mid A\alpha_m \mid \beta_1 \mid \beta_2 \mid \beta_3 \mid \ldots \mid \beta_n$$

  where no β_i, begins with an A.
- Then, we replace the A-productions by,

$$A \rightarrow \beta_1 A_0 \mid \beta_2 A_0 \mid \beta_3 A_0 \mid \ldots \mid \beta_n A_0$$
$$A_0 \rightarrow \alpha_1 A_0 \mid \alpha_2 A_0 \mid \alpha_3 A_0 \mid \ldots \mid \alpha_m A_0 \mid$$

- It does not eliminate left recursion involving derivations of two or more steps.

Elimination of Left Recursion

- Consider the grammar,

$$S \rightarrow Aa \mid b$$
$$A \rightarrow Ac \mid Sd \mid \epsilon$$

- The nonterminal S is left-recursive because **S ⇒ Aa ⇒ Sda**, but it is not immediately left recursive.

- In such cases, we need to use the algorithm shown next.

Algorithm

Input: Grammar G with no cycles or ∈-productions.
Output: An equivalent grammar with no left recursion.
METHOD: Apply the algorithm to G. Note that the resulting non-left-recursive grammar may have ∈-productions.

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) **for** (each i from 1 to n) {
3) **for** (each j from 1 to $i-1$) {
4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

Algorithm

- We eliminate left-recursion in three steps.
  - eliminate $\in$-productions
  - eliminate cycles ($A \stackrel{+}{\Rightarrow} A$)
  - eliminate left-recursion

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) **for** (each i from 1 to n) {
3) **for** (each j from 1 to $i-1$) {
4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

Algorithm

- Non-terminal *A* is nullable if $A \overset{*}{\Rightarrow} \in$

$$S \rightarrow XZ$$
$$X \rightarrow aXb \mid \epsilon$$
$$Z \rightarrow aZ \mid ZX \mid \epsilon$$

Find out the nullable non-terminals in the above grammar!

- Grammar is with cycles if $A \overset{+}{\Rightarrow} A$

$$S \rightarrow X \mid Xb \mid SS$$
$$X \rightarrow S \mid a$$

Find out the non-terminals which form cycle!

Algorithm

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) **for** (each i from 1 to n) {
3) **for** (each j from 1 to $i-1$) {
4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

- In the first iteration for $i = 1$, the outer for-loop of lines (2) through (7) eliminates any immediate left recursion among A_1-productions.
- Any remaining A_1-productions of the form $A_1 \rightarrow A_l\alpha$ must therefore have $l > 1$

Algorithm

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) **for** (each i from 1 to n) {
3) **for** (each j from 1 to $i-1$) {
4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

- After the 1st iteration of the outer for-loop, all nonterminals A_k , where k < i, are “cleaned”.
- That is, any production $A_k \rightarrow A_l\alpha$, must have l > k.

Algorithm

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) **for** (each i from 1 to n) {
3) **for** (each j from 1 to $i-1$) {
4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

- As a result, on the i^{th} iteration, the inner loop of lines (3) through (5) progressively raises the lower limit in any production $A_i \rightarrow A_m\alpha$, until we have $m \geq I$
- Then, eliminating immediate left recursion for the A_i productions at line (6) forces m to be greater than i.

Algorithm- Example

- We apply the procedure to grammar,

$$S \rightarrow Aa \mid b$$
$$A \rightarrow Ac \mid Sd \mid \epsilon$$

- Technically, the algorithm is not guaranteed to work, because of the ϵ-production.
- But in this case the production $A \rightarrow \epsilon$ turns out to be harmless.
- To remove A_j from the right-hand side of the A_i production $A_i \rightarrow A_j \gamma$ replace $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \delta_3\gamma \mid \ldots\ldots\ldots\ldots \mid \delta_k\gamma$
  where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \delta_3 \mid \ldots\ldots\ldots\ldots \mid \delta_k$ and $\delta_i \neq \epsilon$

Algorithm- Example

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.

Left-Recursive Grammar

$$S \rightarrow Aa \mid b$$
$$A \rightarrow Ac \mid Sd \mid \epsilon$$

- We order the non-terminals *S, A*
- *A1 = S*, *A2 = A*

Algorithm- Example

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) **for** (each i from 1 to n) {
3) **for** (each j from 1 to $i - 1$) {
4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

- $i = 1$, $A_1 = S$
- $j = 1$ to $j = i - 1 = 1 - 1 = 0$, the loop is not entered
- There is no immediate left recursion among the S-productions, so nothing happens for the case $i = 1$. ($A_1 = S$)

Algorithm- Example

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) **for** (each i from 1 to n) {
3) **for** (each j from 1 to $i - 1$) {
4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

- $i = 2$, $A_2 = A$
- $j = 1$ to $j = i - 1 = 2-1 = 1$, *the loop is entered*

Algorithm- Example

4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions

Left-Recursive Grammar

$$S \rightarrow Aa \mid b$$
$$A \rightarrow Ac \mid Sd \mid \epsilon$$

- *i = 2, A2 = A, j = 1, A1 = S*
- *We need to*
  - put productions of the form $S \rightarrow \delta 1 \mid \delta 2 \mid \delta 3 \ldots\ldots\ldots\ldots \mid \delta k$
  - in productions of the form $A \rightarrow S\gamma$
- Production(s) with *S* at the left-hand-side, $S \rightarrow Aa \mid b$
- Productions(s) with A at the left side and right side beginning with *S* is (are), $A \rightarrow Sd$

Algorithm- Example

4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions

Left-Recursive Grammar

$$S \rightarrow Aa \mid b$$
$$A \rightarrow Ac \mid Sd \mid \epsilon$$

- $S \rightarrow Aa \mid b$ to be put in A, $A \rightarrow Sd$
- We substitute $S \rightarrow Aa \mid b$ in $A \rightarrow Sd$ to get the following A-productions,

$$\mathbf{A \rightarrow Aad \mid bd}$$

Algorithm- Example

6) eliminate the immediate left recursion among the A_i-productions;

- *All $A_i = A_2 = A$-productions together,*

  $\boldsymbol{A \rightarrow Ac \mid Aad \mid bd \mid \in}$

- Eliminating the immediate left recursion among the A-productions yields the following,

  $\boldsymbol{A \rightarrow bdA' \mid A'}$

  $\boldsymbol{A' \rightarrow cA' \mid adA' \mid \in}$

Algorithm- Example

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) **for** (each i from 1 to n) {
3) **for** (each j from 1 to $i - 1$) {
4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

- *i has attained the value of $n = 2$ and the loops are no more entered.*

Algorithm- Example

Left-Recursive Grammar

$$S \rightarrow Aa \mid b$$
$$A \rightarrow Ac \mid Sd \mid \epsilon$$

- Put together we get the following non-left-recursive grammar,

$$S \rightarrow Aa \mid b$$
$$A \rightarrow bdA' \mid A'$$
$$A' \rightarrow cA' \mid adA' \mid \epsilon$$

1) arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$.
2) **for** (each i from 1 to n) {
3) **for** (each j from 1 to $i-1$) {
4) replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all the current A_j-productions
5) }
6) eliminate the immediate left recursion among the A_i-productions;
7) }

Conceptual Technique Summary (AGAIN)

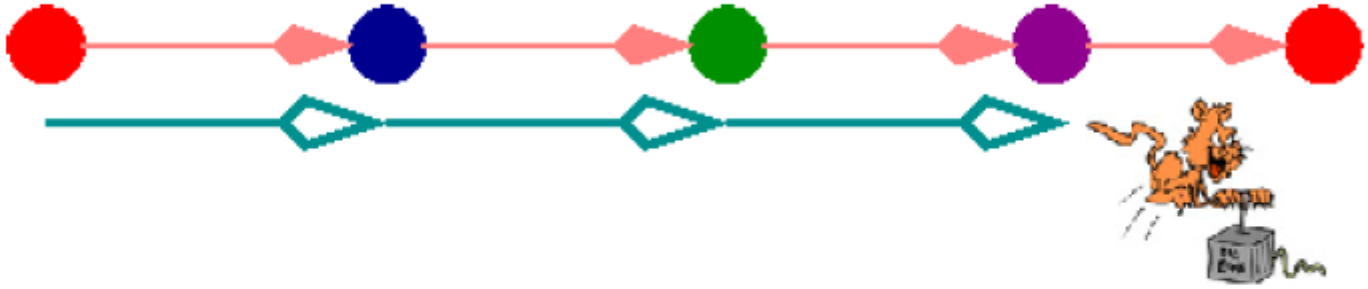
- Put some order in the nonterminals.
- Start by making first nonterminal productions left-recursion-free.
- Put the first nonterminal left-recursion-free productions into those of the second one.
- Now make the productions of second nonterminal left-recursion-free.
- Thus keep on growing the set of left-recursion-free productions.

Left Factoring

- Left factoring is a grammar transformation that is useful for producing a grammar suitable for predictive parsing.
- The basic idea is that sometimes it is not clear which of two alternative productions to use to expand a nonterminal A.
- We may be able to rewrite the *A*-productions to defer the decision until we have seen enough of the input to make the right choice.

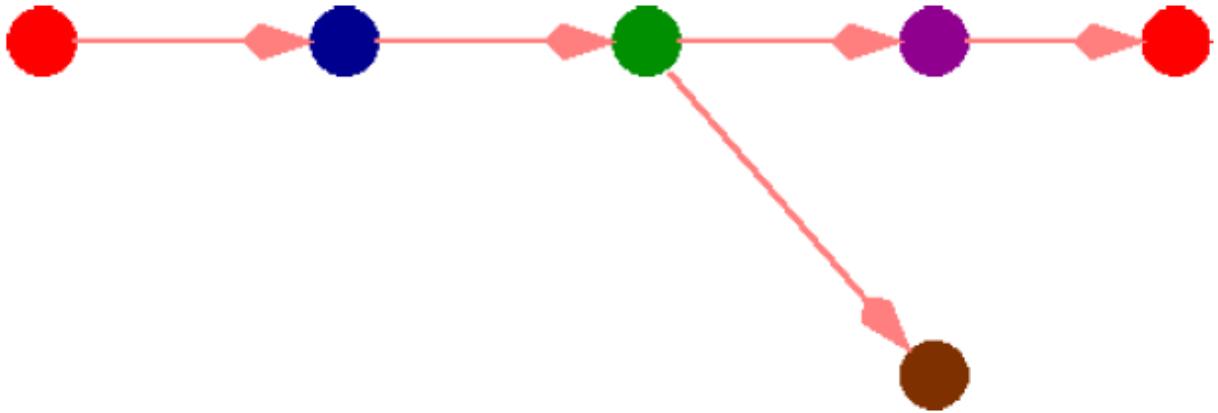
Left Factoring

Road Direction: *Red* → *Blue* → *Green* → *Brown*



Left Factoring

Defer the decision until we have seen enough of the input to make the right choice.



Left Factoring

- We have the two productions,

$$\begin{aligned} stmt &\rightarrow \textbf{if } expr \textbf{ then } stmt \textbf{ else } stmt \\ &\mid \textbf{if } expr \textbf{ then } stmt \end{aligned}$$

- On seeing the input token if, we cannot immediately tell which production to choose to expand *stmt*.

Left Factoring

- $A \rightarrow \alpha \beta_1 \mid \alpha \beta_2$ are two A-productions.
- The input begins with a nonempty string derived from α.
- We do not know whether to expand A to $\alpha \beta_1$ or $\alpha \beta_2$.
- However, we may defer the decision by expanding A to α A′.
- Then, after seeing the input derived from we expand A_0 to β_1 or β_2.
- Left-factored, the original productions become,

$$A \rightarrow \alpha A'$$
$$A' \rightarrow \beta_1 \mid \beta_2$$

Left Factoring Algorithm

Input: Grammar G.
Output: An equivalent left-factored grammar.

Method:

- For each nonterminal A find the longest prefix common to two or more of its alternatives.
- If $\alpha \neq \in$, replace all the A productions $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \ldots\ldots\ldots \mid \alpha\beta_n \mid \gamma$ where γ represents all alternatives that do not begin with α by

  $A \rightarrow \alpha A' \mid \gamma$

  $A' \rightarrow \beta_1 \mid \beta_2 \mid \ldots\ldots \mid \beta_n$

  where A' is a new nonterminal.
- Repeatedly apply this transformation until no two alternatives for a nonterminal have a common prefix.

Left Factoring Example

- The following grammar abstracts the dangling-else problem:

  $S \rightarrow iEtS \mid iEtSeS \mid a$

  $E \rightarrow b$

- Here i, t, and e stand for if, then and else, E and S for “expression” and “statement.”
- Left-factored, this grammar becomes:

  $S \rightarrow iEtSS' \mid a$

  $S' \rightarrow eS \mid \in$

  $E \rightarrow b$

- Thus, we may expand S to $iEtSS'$ on input i, and wait until $iEtS$ has been seen to decide whether to expand S' to eS or to $\in$.

Top-Down Parsing

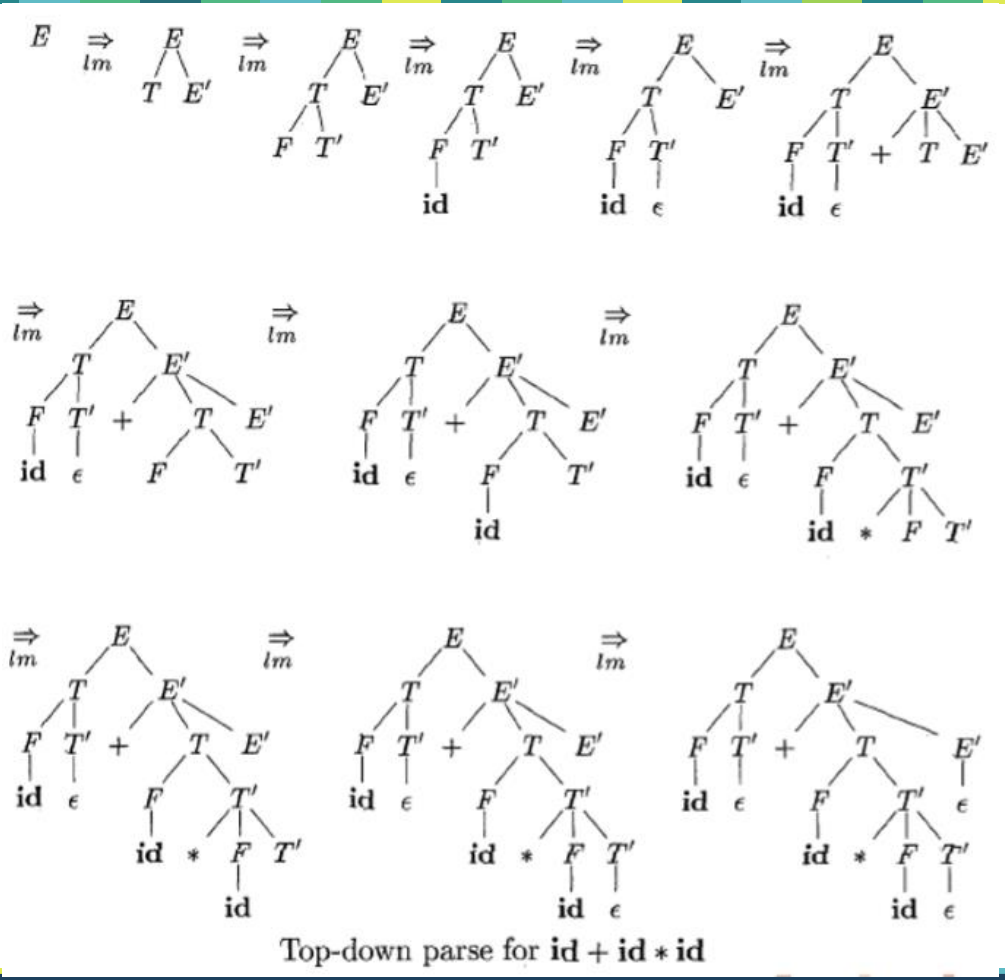
- Top-down parsing can be viewed as the problem of

  - constructing a parse tree for the input string,
  - starting from the root and
  - creating the nodes of the parse tree in preorder (depth-first).

- Equivalently, top-down parsing can be viewed as finding a leftmost derivation for an input string.

- The sequence of top-down parse trees for the input id + id $*$ id and grammar:

$$
\begin{aligned}
E &\rightarrow TE' \\
E' &\rightarrow +TE' \mid \epsilon \\
T &\rightarrow FT' \\
T' &\rightarrow *FT' \mid \epsilon \\
F &\rightarrow (E) \mid \mathbf{id}
\end{aligned}
$$

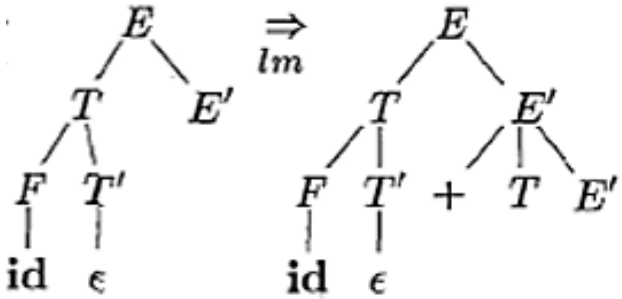

E ⇒lm T E′ ⇒lm F T′ E′ ⇒lm id T′ E′ ⇒lm id ϵ E′ ⇒lm id ϵ + T E′
⇒lm id ϵ + F T′ E′ ⇒lm id ϵ + id T′ E′
id ϵ + id * F T′
⇒lm id ϵ + id * id T′ ⇒lm id ϵ + id * id ϵ
Top-down parse for id + id * id

Top-Down Parsing

- At each step of a top-down parse, the key problem is that of determining the production to be applied for a nonterminal, say *A*.

- Once an *A*-production is chosen, the rest of the parsing process consists of "matching" the terminal symbols in the production body with the input string.

- Consider the top-down parse in figure.
- This constructs a tree with two nodes labeled E'.
- At the first E' node (in preorder), the production E' → +TE' is chosen.
- At the second E' node, the production E' → is chosen.
- A predictive parser can choose between E' -productions by looking at the next input symbol.
- The class of grammars for which we can construct predictive parsers looking k symbols ahead in the input is sometimes called the LL(k) class.

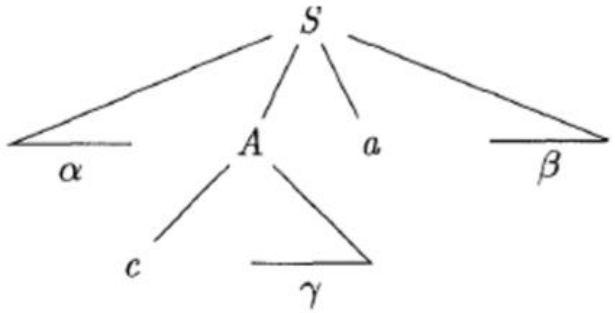


FIRST and FOLLOW

- The construction of both top-down and bottom-up parsers is aided by two functions, FIRST and FOLLOW, associated with a grammar G.
- During top-down parsing, FIRST and FOLLOW allow us to choose which production to apply, based on the next input symbol.
- During panic-mode error recovery, sets of tokens produced by FOLLOW can be used as synchronizing tokens.

FIRST

- Define FIRST(α), where α is any string of grammar symbols, to be the set of terminals that begin strings derived from α.
- If $\alpha \Rightarrow \in$, then $\in$ is also in FIRST(α).
- For example, in figure $A \Rightarrow c\gamma$, so c is in FIRST(A).



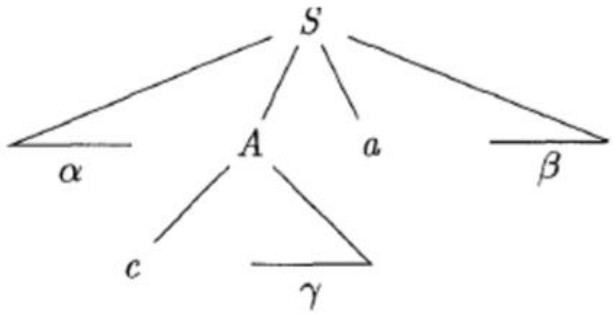
Terminal c is in FIRST(A) and a is in FOLLOW(A)

FIRST

- Let us see how FIRST can be used during predictive parsing.
- Consider two *A*-productions $A \rightarrow \alpha \mid \beta$, where FIRST(α) and FIRST(β) are disjoint sets.
- We can then choose between these *A*-productions by looking at the next input symbol a, since a can be in at most one of FIRST(α) and FIRST(β), not both.
- For instance, if a is in FIRST(β) choose the production $A \rightarrow \beta$.

FOLLOW

- Define FOLLOW(A), nonterminal A, to be the set of terminals a that can appear immediately to the right of A in some sentential form.
- That is, the set of terminals a such that there exists a derivation of the form $S \Rightarrow \alpha A a \beta$, for some α and β.
- Note that there may have been symbols between A and α, at some time during the derivation, but if so, they derived $\in$ and disappeared.

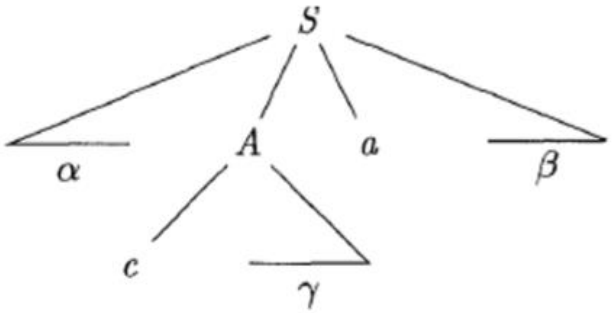


Terminal c is in FIRST(A) and a is in FOLLOW(A)

FOLLOW

- In addition, if *A* can be the rightmost symbol in some sentential form, then *$* is in FOLLOW(A).

- Recall that *$* is a special “endmarker” symbol that is assumed not to be a symbol of any grammar.



Terminal c is in FIRST(A) and a is in FOLLOW(A)

RULES TO COMPUTE FIRST

- To compute FIRST(X) for all grammar symbols X, apply the following rules until no more terminals or $\in$ can be added to any FIRST set.

1. If X is terminal, then FIRST(X) is {X}.
2. If X is nonterminal and $X \rightarrow Y_1 Y_2 \ldots Y_k$ is a production for some $k \geq 1$, then place a in FIRST(X) if for some i, a is in FIRST(Y_i), and $\in$ is in all of FIRST(Y_1), ……, FIRST(Y_{i-1});
   If $\in$ is in FIRST(Y_j) for all j = 1, 2, ….., k then add $\in$ to FIRST(X).
3. If $X \rightarrow \in$ is a production, then add $\in$ to FIRST(X).

RULES TO COMPUTE FIRST

- Everything in FIRST(Y_1) is surely in FIRST(X).
- If Y_1, does not derive $\in$, then we add nothing more to FIRST(X), but if $Y_1 \stackrel{*}{\Rightarrow} \in$ then we add FIRST(Y_2) and so on.

- Now, we can compute FIRST for any string $X_1X_2 \ldots\ldots X_n$ as follows.

1. Add to FIRST($X_1X_2 \ldots\ldots X_n$) all the non-$\in$ symbols of FIRST(X_1).
2. Also add the non-$\in$ symbols of FIRST(X_2) if $\in$ is in FIRST(X_1), the non-$\in$ symbols of FIRST(X_3) if $\in$ is in both FIRST(X_1) and FIRST(X_2) and so on.
3. Finally, add $\in$ to FIRST($X_1X_2 \ldots\ldots X_n$) if, for all i, $\in$ is in FIRST(X_i).

RULES TO COMPUTE FOLLOW

- To compute FOLLOW(*A*) for all nonterminals *A*, apply the following rules until nothing can be added to any FOLLOW set.

1. Place $ in FOLLOW(*S*), where *S* is the start symbol and $ is the input right endmarker.
2. If there is a production $A \rightarrow \alpha B \beta$, then everything in FIRST(β) except $\in$ is in FOLLOW(*B*).
3. If there is a production $A \rightarrow \alpha B$, or a production $A \rightarrow \alpha B \beta$ where FIRST(β) contains $\in$, then everything in FOLLOW(*A*) is in FOLLOW(*B*).

EXAMPLE

Grammar,

$$
\begin{aligned}
E &\rightarrow TE' \\
E' &\rightarrow +TE' \mid \epsilon \\
T &\rightarrow FT' \\
T' &\rightarrow *FT' \mid \epsilon \\
F &\rightarrow (E) \mid \textbf{id}
\end{aligned}
$$

FIRST(E) = FIRST(T) = FIRST(F) = { (, id }
FIRST(E') = { +, ∈}
FIRST(T') = {∗, ∈}

EXAMPLE

Try to solve it!!!

$S \rightarrow ACB \mid Cbb \mid Ba$
$A \rightarrow da \mid BC$
$B \rightarrow g \mid \in$
$C \rightarrow h \mid \in$

FIRST(*S*) = { d, g, h, ∈ , b, a }
FIRST(A) = { d, g, h, ∈}
FIRST(*B*) = { g, ∈}
FIRST(*C*) = { h, ∈}

EXAMPLE

- Grammar:

$$E \rightarrow TE' \qquad T \rightarrow FT' \qquad F \rightarrow (E) \mid \mathbf{id}$$
$$E' \rightarrow +TE' \mid \epsilon \qquad T' \rightarrow *FT' \mid \epsilon$$

- Computation of FOLLOW:

FOLLOW(E)	FOLLOW(E')	FOLLOW(T)	FOLLOW(T')	FOLLOW(F)

Initially all sets are empty

Put \$ in FOLLOW(E) by rule (1) (Place \$ in FOLLOW(S), where S is the start symbol and \$ is the input right endmarker)

\$				

$$E \rightarrow TE' \qquad T \rightarrow FT' \qquad F \rightarrow (E) \mid \mathbf{id}$$
$$E' \rightarrow +TE' \mid \epsilon \qquad T' \rightarrow *FT' \mid \epsilon$$

$\text{FIRST}(E) = \text{FIRST}(T) = \text{FIRST}(F) = \{(, \mathbf{id}\}, \text{FIRST}(E') = \{+, \epsilon\},$
$\text{FIRST}(T') = \{*, \epsilon\}$

By rule (2) (If there is a production $A \rightarrow \alpha B \beta$, then everything in FIRST(β) except for ϵ is placed in FOLLOW(B)) applied to,

- $E \rightarrow TE'$: FIRST(E') except ϵ i.e. $\{+\}$ are in FOLLOW(T)
- $E' \rightarrow +TE'$: FIRST(E') except ϵ i.e. $\{+\}$ are in FOLLOW(T)
- $T \rightarrow FT'$: FIRST(T') except ϵ i.e. $\{*\}$ are in FOLLOW(F)
- $T \rightarrow *FT'$: FIRST(T') except ϵ i.e. $\{*\}$ are in FOLLOW(F)
- $F \rightarrow (E)$: FIRST()) i.e. $\{)\}$ are in FOLLOW(E)

FOLLOW(E)	FOLLOW(E')	FOLLOW(T)	FOLLOW(T')	FOLLOW(F)
\$,)		+		*

Rule (2) is not applicable any more since it depends only on FIRST, which are now stable sets.

Application of rule (3) *(If there is a production $A \to \alpha B$, or a production $A \to \alpha B\beta$ where FIRST(β) contains ϵ (i.e., $\beta \stackrel{*}{\Rightarrow} \epsilon$), then everything in FOLLOW(A) is in FOLLOW(B))*

$E \to TE'$ $\quad T \to FT'$ $\quad F \to (E) \mid \mathbf{id}$

$E' \to +TE' \mid \epsilon$ $\quad T' \to *FT' \mid \epsilon$

FOLLOW(E)	FOLLOW(E')	FOLLOW(T)	FOLLOW(T')	FOLLOW(F)
\$,)		+		*

$E \to TE'$: Everything in FOLLOW(E) are in FOLLOW(E')

\$,)	\$,)	+		*

$E' \to +TE'$ (also $\epsilon \in$ FIRST(E')): Everything in FOLLOW(E') are in FOLLOW(T)

\$,)	\$,)	+, \$,)		*

$T \to FT'$: Everything in FOLLOW(T) are in FOLLOW(T')

\$,)	\$,)	+, \$,)	+, \$,)	*

Application of rule (3) — continued *(If there is a production $A \to \alpha B$, or a production $A \to \alpha B\beta$ where FIRST(β) contains ϵ (i.e., $\beta \stackrel{*}{\Rightarrow} \epsilon$), then everything in FOLLOW(A) is in FOLLOW(B))*

$$E \to TE' \qquad T \to FT' \qquad F \to (E) \mid \mathbf{id}$$
$$E' \to +TE' \mid \epsilon \qquad T' \to *FT' \mid \epsilon$$

FOLLOW(E)	FOLLOW(E')	FOLLOW(T)	FOLLOW(T')	FOLLOW(F)
\$,)	\$,)	+, \$,)	+, \$,)	*

$T' \to *FT'$ (also $\epsilon \in$ FIRST(T')): Everything in FOLLOW(T') are in FOLLOW(F)

\$,)	\$,)	+, \$,)	+, \$,)	*, +, \$,)

We can try applying Rule (3) again, but will find that the sets have stabilized (nothing can be added to any FOLLOW set).

LL(1) Grammars

- Predictive parsers, that is, recursive-descent parsers needing no backtracking, can be constructed for a class of grammars called LL(1).
- The first “L” in LL(1) stands for scanning the input from left to right.
- The second “L” for producing a leftmost derivation.
- And the “1” for using one input symbol of lookahead at each step to make parsing action decisions.
- The class of LL(1) grammars is rich enough to cover most programming constructs.
- Although care is needed in writing a suitable grammar for the source language.
- For example, no left-recursive or ambiguous grammar can be LL(1).

LL(1) Grammars

- A grammar G is LL(1) if and only if whenever $A \rightarrow \alpha \mid \beta$ are two distinct productions of G the following conditions hold:

1. For no terminal a do both α and β derive strings beginning with a.
2. At most one of α and β can derive the empty string.
3. If $\beta \overset{*}{\Rightarrow} \in$, then α does not derive any string beginning with a terminal in FOLLOW(A). Likewise, $\alpha \overset{*}{\Rightarrow} \in$, then β does not derive any string beginning with terminal in FOLLOW(A).

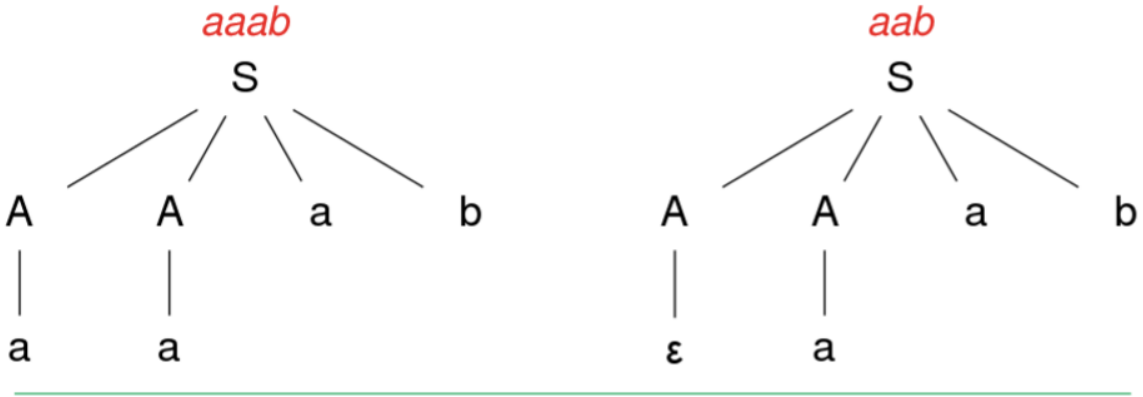
- The first two conditions are equivalent to the statement that FIRST(α) and FIRST(β) are disjoint sets.
- The third condition is equivalent to stating that if $\in$ is in FIRST(β), then FIRST(α) and FOLLOW(A) are disjoint sets, and likewise if $\in$ is in FIRST(α).

Non-LL(1) Grammar Example

$S \rightarrow AAab \mid BBba$

$A \rightarrow a \mid \epsilon$

$B \rightarrow b \mid \epsilon$



- Now, when expanding the first *A*, we can not decide which of the two production rules to be used.
- Both give us the promise of an a.

LL(1) Grammars

- Predictive parsers can be constructed for LL(1) grammars since the proper production to apply for a nonterminal can be selected by looking only at the current input symbol.
- Flow-of-control constructs, with their distinguishing key-words, generally satisfy the LL(1) constraints.
- For instance, if we have the productions,

$$
\begin{aligned}
stmt \rightarrow\ & \textbf{if}\ (\ expr\ )\ stmt\ \textbf{else}\ stmt \\
\mid\ & \textbf{while}\ (\ expr\ )\ stmt \\
\mid\ & \{\ stmt\_list\ \}
\end{aligned}
$$

then the keywords **if**, **while**, and the symbol **{** tell us which alternative is the one that could possibly succeed if we are to find a statement.

LL(1) Grammars

- The next algorithm collects the information from FIRST and FOLLOW sets into a predictive parsing table $M[A, a]$, a two dimensional array, where A is a nonterminal, and a is a terminal or the symbol $, the input endmarker.
- The idea behind the algorithm is the following:

- Suppose $A \rightarrow \alpha$ is a production with a in FIRST(α).
- Then, the parser will expand A by α when the current input symbol is a.
- The only complication occurs when $\alpha = \in$ or $\alpha \overset{*}{\Rightarrow} \in$.
- In this case, we should again expand A by α if the current input symbol is in FOLLOW(A), or if the $ on the input has been reached and $ is in FOLLOW(A).

Algorithm for Construction of Predictive Parsing Table

INPUT: Grammar G.

OUTPUT: Parsing table M.

METHOD: For each production $A \to \alpha$ of the grammar, do the following:

1. For each terminal a in FIRST(A), add $A \to \alpha$ to $M[A, a]$.
2. If ϵ is in FIRST(α), then for each terminal b in FOLLOW(A), add $A \to \alpha$ to $M[A, b]$.
   If ϵ is in FIRST(α) and \$ is in FOLLOW(A), add $A \to \alpha$ to $M[A, \$]$ as well.

- If, after performing the above, there is no production at all in $M[A, a]$, then set $M[A, a]$ to **error** (which we normally represent by an empty entry in the table).

EXAMPLE

- For the expression grammar below,

$$E \rightarrow TE' \qquad T \rightarrow FT' \qquad F \rightarrow (E) \mid \mathbf{id}$$
$$E' \rightarrow +TE' \mid \epsilon \qquad T' \rightarrow *FT' \mid \epsilon$$

the algorithm produces the parsing table in figure.

NON-TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \mathbf{id}$			$F \rightarrow (E)$		

- Blanks are error entries.
- Nonblanks indicate a production with which to expand a nonterminal.

For each production $A \to \alpha$ of the grammar, do the following:

1. For each terminal a in FIRST(A), add $A \to \alpha$ to $M[A, a]$.
2. If ϵ is in FIRST(α), then for each terminal b in FOLLOW(A), add $A \to \alpha$ to $M[A, b]$. If ϵ is in FIRST(α) and \$ is in FOLLOW(A), add $A \to \alpha$ to $M[A, \$]$ as well.

NON-TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to$ **id**			$F \to (E)$		

- Consider production $E \to TE'$.
- Since

$$\text{FIRST}(TE') = \text{FIRST}(T) = \{(, \textbf{id}\}$$

this production is added to $M[E, (]$ and $M[E, \textbf{id}]$.

- Production $E' \to +TE'$ is added to $M[E', +]$ since FIRST$(+TE') = \{+\}$.
- Since FOLLOW$(E') = \{), \$\}$, production $E' \to \epsilon$ is added to $M[E',)]$ and $M[E', \$]$.

Algorithm for Construction of Predictive Parsing Table

- The aforementioned algorithm can be applied to any grammar **G** to produce a parsing table ***M***.
- For every LL(1) grammar, each parsing-table entry uniquely identifies a production or signals an error.
- For some grammars, however, ***M*** may have some entries that are multiply defined.
- For example, if **G** is left-recursive or ambiguous, then ***M*** will have at least one multiply defined entry.
- Although left-recursion elimination and left factoring are easy to do, there are some grammars for which no amount of alteration will produce an LL(1) grammar.
- The language in the following example has no LL(1) grammar at all.

EXAMPLE

- The following grammar, which abstracts the dangling-else problem, is repeated here:
- The grammar,

$$
\begin{aligned}
S &\rightarrow iEtSS' \mid a \\
S' &\rightarrow eS \mid \epsilon \\
E &\rightarrow b
\end{aligned}
$$

- The grammar is ambiguous.
- On input e, it will not be clear which alternative for S' should be chosen.

EXAMPLE

$$S \rightarrow iEtSS' \mid a$$
$$S' \rightarrow eS \mid \epsilon$$
$$E \rightarrow b$$

NON - TERMINAL	INPUT SYMBOL					
	a	b	e	i	t	$\$$
S	$S \rightarrow a$			$S \rightarrow iEtSS'$		
S'			$S' \rightarrow \epsilon$ $S' \rightarrow eS$			$S' \rightarrow \epsilon$
E		$E \rightarrow b$				

- The entry for $M[S', e]$ contains both $S' \rightarrow eS$ and $S' \rightarrow \in$, since FOLLOW(S') = {e, $}
- The grammar is ambiguous and the ambiguity is manifested by a choice in what production to use when an e (**else**) is seen.

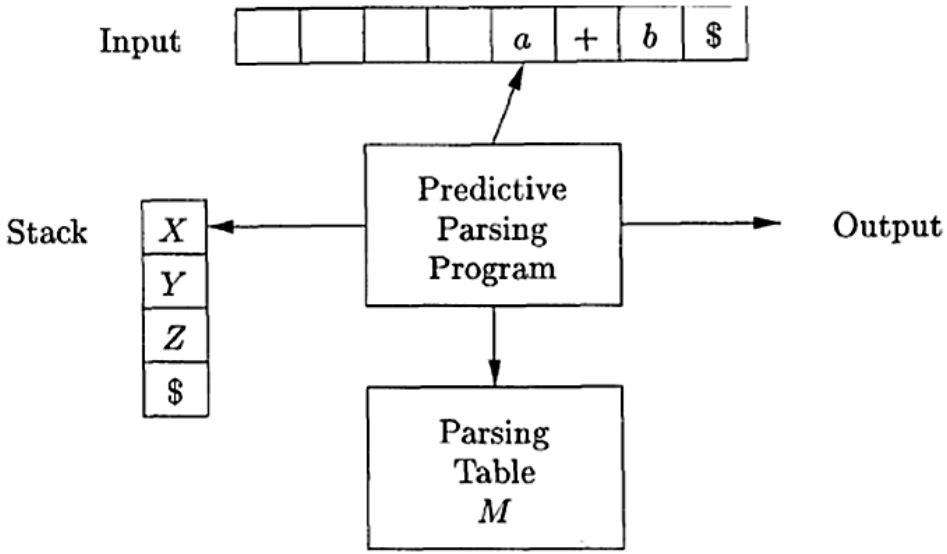
Nonrecursive Predictive Parsing

- A **nonrecursive predictive parser** can be built by maintaining a stack explicitly, rather than implicitly via recursive calls.
- The parser mimics a leftmost derivation.
- If w is the input that has been matched so far, then the stack holds a sequence of grammar symbols α such that

$$S \overset{*}{\underset{lm}{\Rightarrow}} w\alpha$$

Nonrecursive Predictive Parsing

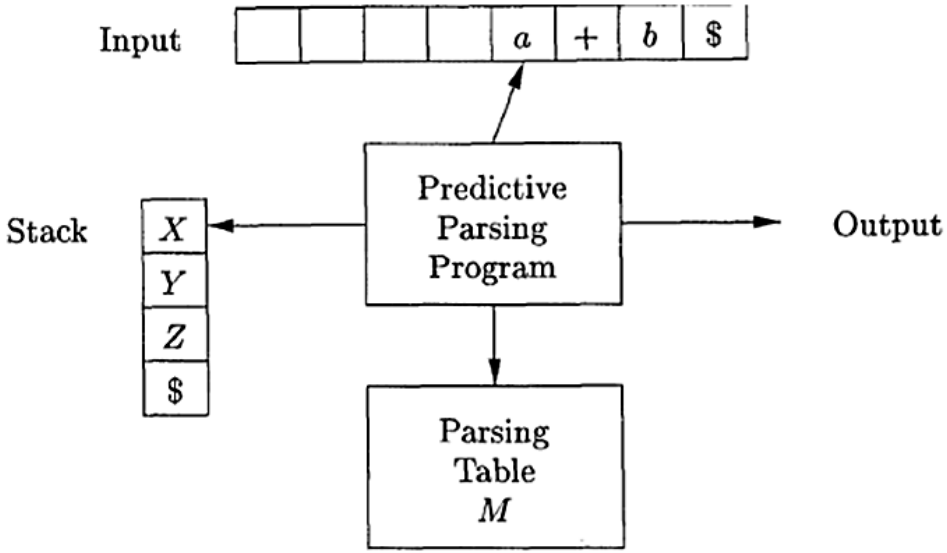
- The table-driven parser in figure has an input buffer, a stack containing a sequence of grammar symbols, a parsing table constructed by algorithm, and an output stream.
- The input buffer contains the string to be parsed, followed by the endmarker $.
- We reuse the symbol $ to mark the bottom of the stack, which initially contains the start symbol of the grammar on top of $.



Model of a table-driven predictive parser

Nonrecursive Predictive Parsing

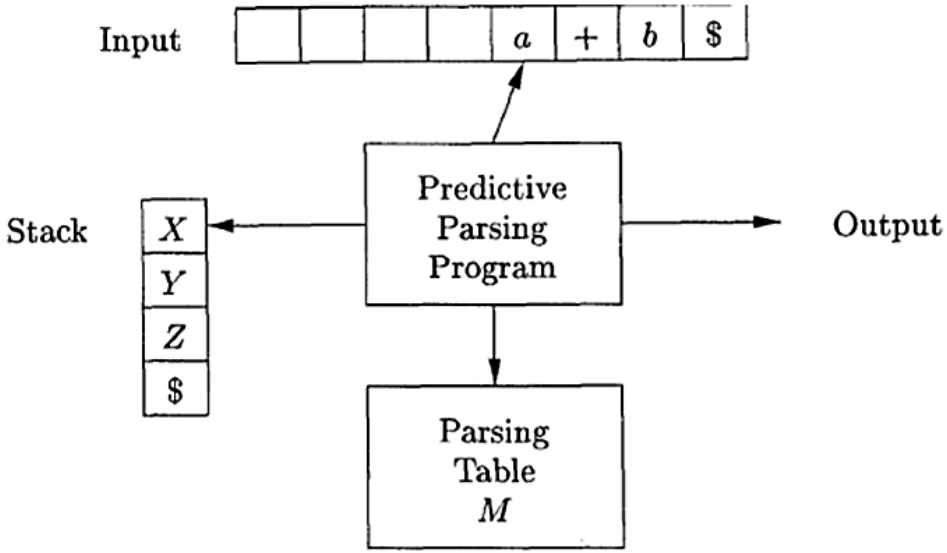
- The parser is controlled by a program that considers X, the symbol on top of the stack, and a, the current input symbol.
- If X is a nonterminal, the parser chooses an X-production by consulting entry ***M*[*X*,*a*]** of the parsing table ***M***.
- Otherwise, it checks for a match between the terminal X and current input symbol a.



Model of a table-driven predictive parser

Nonrecursive Predictive Parsing

- Additional code could be executed here, for example, code to construct a node in a parse tree.
- The behavior of the parser can be described in terms of its configurations, which give the stack contents and the remaining input.
- The next algorithm describes how configurations are manipulated.



Model of a table-driven predictive parser

Nonrecursive Predictive Parsing

INPUT. A string w and a parsing table M for grammar G.

OUTPUT.
- If w is in $L(G)$, a leftmost derivation of w;
- otherwise, an error indication.

METHOD.
- Initially, the parser is in a configuration in which it has $\$S$ on the stack with S, the start symbol of G on top, and w in the input buffer.
- The program that utilizes the predictive parsing table M to produce a parse for the input is shown here.

Nonrecursive Predictive Parsing

set *ip* to point to the first symbol of w;
set X to the top stack symbol;
while ($X \neq \$$) { /* stack is not empty */
    if (X is a) pop the stack and advance *ip*;
    else if (X is a terminal) *error*();
    else if ($M[X, a]$ is an error entry) *error*();
    else if ($M[X, a] = X \rightarrow Y_1 Y_2 \cdots Y_k$) {
        output the production $X \rightarrow Y_1 Y_2 \cdots Y_k$;
        pop the stack;
        push $Y_k, Y_{k-1}, \ldots, Y_1$ onto the stack, with Y_1 on top;
    }
    set X to the top stack symbol;
}

Predictive parsing algorithm

Example

- We consider grammar:

$$
\begin{aligned}
E &\rightarrow TE' \\
E' &\rightarrow +TE' \mid \epsilon \\
T &\rightarrow FT' \\
T' &\rightarrow *FT' \mid \epsilon \\
F &\rightarrow (E) \mid \textbf{id}
\end{aligned}
$$

- We have already seen its parsing table.

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \textbf{id}$			$F \rightarrow (E)$		

On input **id** + **id** * **id**, the nonrecursive predictive parser algorithm makes the sequence of moves,

MATCHED	STACK	INPUT	ACTION
	$E\$$	**id** + **id** * **id**$	
	$TE'\$$	**id** + **id** * **id**$	output $E \to TE'$
	$FT'E'\$$	**id** + **id** * **id**$	output $T \to FT'$
	id $T'E'\$$	**id** + **id** * **id**$	output $F \to$ **id**
id	$T'E'\$$	+ **id** * **id**$	match **id**
id	$E'\$$	+ **id** * **id**$	output $T' \to \epsilon$
id	$+ TE'\$$	+ **id** * **id**$	output $E' \to + TE'$
id +	$TE'\$$	**id** * **id**$	match +
id +	$FT'E'\$$	**id** * **id**$	output $T \to FT'$
id +	**id** $T'E'\$$	**id** * **id**$	output $F \to$ **id**
id + **id**	$T'E'\$$	* **id**$	match **id**
id + **id**	$* FT'E'\$$	* **id**$	output $T' \to * FT'$
id + **id** *	$FT'E'\$$	**id**$	match *
id + **id** *	**id** $T'E'\$$	**id**$	output $F \to$ **id**
id + **id** * **id**	$T'E'\$$	$	match **id**
id + **id** * **id**	$E'\$$	$	output $T' \to \epsilon$
id + **id** * **id**	$\$$	$	output $E' \to \epsilon$

Moves made by a predictive parser on input **id** + **id** * **id**

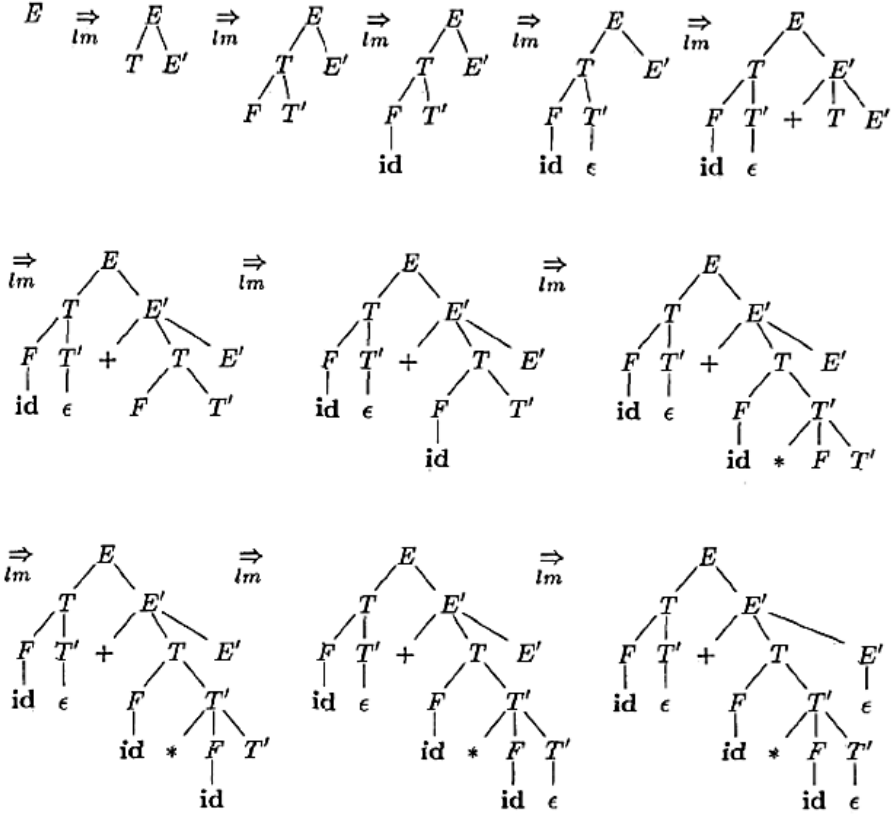
These moves correspond to a leftmost derivation,

$$E \underset{lm}{\Rightarrow} TE' \underset{lm}{\Rightarrow} FT'E' \underset{lm}{\Rightarrow} \mathbf{id}\,T'E' \underset{lm}{\Rightarrow} \mathbf{id}E' \underset{lm}{\Rightarrow} \mathbf{id} + TE' \underset{lm}{\Rightarrow} \cdots$$

MATCHED	STACK	INPUT	ACTION
	$E\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	
	$TE'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	output $E \to TE'$
	$FT'E'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	output $T \to FT'$
	$\mathbf{id}\,T'E'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	output $F \to \mathbf{id}$
$\mathbf{id}$	$T'E'\$$	$+\,\mathbf{id} * \mathbf{id}\$$	match $\mathbf{id}$
$\mathbf{id}$	$E'\$$	$+\,\mathbf{id} * \mathbf{id}\$$	output $T' \to \epsilon$
$\mathbf{id}$	$+\,TE'\$$	$+\,\mathbf{id} * \mathbf{id}\$$	output $E' \to +\,TE'$
$\mathbf{id} +$	$TE'\$$	$\mathbf{id} * \mathbf{id}\$$	match $+$
$\mathbf{id} +$	$FT'E'\$$	$\mathbf{id} * \mathbf{id}\$$	output $T \to FT'$
$\mathbf{id} +$	$\mathbf{id}\,T'E'\$$	$\mathbf{id} * \mathbf{id}\$$	output $F \to \mathbf{id}$
$\mathbf{id} + \mathbf{id}$	$T'E'\$$	$* \mathbf{id}\$$	match $\mathbf{id}$
$\mathbf{id} + \mathbf{id}$	$*\,FT'E'\$$	$* \mathbf{id}\$$	output $T' \to *\,FT'$
$\mathbf{id} + \mathbf{id} *$	$FT'E'\$$	$\mathbf{id}\$$	match $*$
$\mathbf{id} + \mathbf{id} *$	$\mathbf{id}\,T'E'\$$	$\mathbf{id}\$$	output $F \to \mathbf{id}$
$\mathbf{id} + \mathbf{id} * \mathbf{id}$	$T'E'\$$	$\$$	match $\mathbf{id}$
$\mathbf{id} + \mathbf{id} * \mathbf{id}$	$E'\$$	$\$$	output $T' \to \epsilon$
$\mathbf{id} + \mathbf{id} * \mathbf{id}$	$\$$	$\$$	output $E' \to \epsilon$

Moves made by a predictive parser on input $\mathbf{id} + \mathbf{id} * \mathbf{id}$

$$E \underset{lm}{\Rightarrow} TE' \underset{lm}{\Rightarrow} FT'E' \underset{lm}{\Rightarrow} \mathbf{id}\,T'E' \underset{lm}{\Rightarrow} \mathbf{id}\,E' \underset{lm}{\Rightarrow} \mathbf{id} + TE' \underset{lm}{\Rightarrow} \cdots$$



Top-down parse for $\mathbf{id} + \mathbf{id} * \mathbf{id}$

Nonrecursive Predictive Parsing

- Note that the sentential forms in this derivation correspond to the input that has already been matched (in column MATCHED) followed by the stack contents.
- The matched input is shown only to highlight the correspondence.
- For the same reason, the top of the stack is to the left.
- When we consider bottom-up parsing, it will be more natural to show the top of the stack to the right.
- The input pointer points to the leftmost symbol of the string in the INPUT column.

Example

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \mathbf{id}$			$F \to (E)$		

STACK	INPUT	ACTION
$E\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	
$TE'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	output $E \to TE'$

Example

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \mathbf{id}$			$F \to (E)$		

STACK	INPUT	ACTION
$TE'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	
$FT'E'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	output $T \to FT'$

Example

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \mathbf{id}$			$F \to (E)$		

STACK	INPUT	ACTION
$FT'E'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	
$\mathbf{id}\,T'E'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	output $F \to \mathbf{id}$

Example

STACK	INPUT	ACTION
id $T'E'$\$	**id** + **id** * **id**\$	match **id**

Both are terminals and match. So, popped from the stack and input pointer advanced

$T'E'$\$	+**id** * **id**\$

Example

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \mathbf{id}$			$F \to (E)$		

STACK	INPUT	ACTION
$T'E'\$$	$+\mathbf{id} * \mathbf{id}\$$	
$E'\$$	$+\mathbf{id} * \mathbf{id}\$$	output $T' \to \epsilon$

Example

- Finally…..

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \mathbf{id}$			$F \to (E)$		

STACK	INPUT	ACTION
$E'\$$	$\$$	
↑	↑	
$\$$	$\$$	output $E' \to \epsilon$

Example

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \mathbf{id}$			$F \to (E)$		

STACK	INPUT	ACTION
$T'E'\$$	$+\mathbf{id} * \mathbf{id}\$$	
$E'\$$	$+\mathbf{id} * \mathbf{id}\$$	output $T' \to \epsilon$

Example

- At the end,

STACK	INPUT	ACTION
$ ↑	$ ↑	

Both are $, the parser halts and announces successful completion of parsing.

- For a leftmost derivation the production rules in the ACTION column (outputs only) are to be used from top to bottom

$$E \xRightarrow[E \to TE']{} TE'$$

$$\xRightarrow[T \to FT']{} FT'E'$$

$$\xRightarrow[F \to \mathbf{id}]{} \mathbf{id}\,T'E'$$

$$\xRightarrow[T' \to \epsilon]{} \mathbf{id}\,E'$$

$$\xRightarrow[E' \to +TE']{} \mathbf{id} + TE'$$

$$\xRightarrow[T \to FT']{} \mathbf{id} + FT'E'$$

$$\xRightarrow[F \to \mathbf{id}]{} \mathbf{id} + \mathbf{id}\,T'E'$$

JT	ACTION
$*\,\mathbf{id}\$$	
$*\,\mathbf{id}\$$	output $E \to TE'$
$*\,\mathbf{id}\$$	output $T \to FT'$
$*\,\mathbf{id}\$$	output $F \to \mathbf{id}$
$*\,\mathbf{id}\$$	match **id**
$*\,\mathbf{id}\$$	output $T' \to \epsilon$
$*\,\mathbf{id}\$$	output $E' \to +\,TE'$
$*\,\mathbf{id}\$$	match $+$
$*\,\mathbf{id}\$$	output $T \to FT'$
$*\,\mathbf{id}\$$	output $F \to \mathbf{id}$
$*\,\mathbf{id}\$$	match **id**
$*\,\mathbf{id}\$$	output $T' \to *\,FT'$
$\mathbf{id}\$$	match $*$
$\mathbf{id}\$$	output $F \to \mathbf{id}$
$\$$	match **id**
$\$$	output $T' \to \epsilon$
$\$$	output $E' \to \epsilon$

$$E \xRightarrow[E \to TE']{} \ldots\ldots$$

$$\xRightarrow[F \to \mathbf{id}]{} \mathbf{id} + \mathbf{id}\, T'E'$$

$$\xRightarrow[T' \to *FT']{} \mathbf{id} + \mathbf{id} * FT'E'$$

$$\xRightarrow[F \to \mathbf{id}]{} \mathbf{id} + \mathbf{id} * \mathbf{id}\, T'E'$$

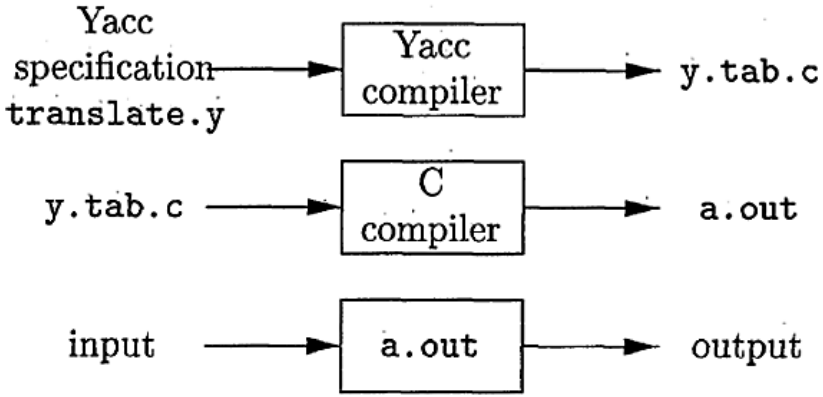
$$\xRightarrow[T' \to \epsilon]{} \mathbf{id} + \mathbf{id} * \mathbf{id}\, E'$$

$$\xRightarrow[E' \to \epsilon]{} \mathbf{id} + \mathbf{id} * \mathbf{id}$$

JT	ACTION
$*$ id\$	
$*$ id\$	output $E \to TE'$
$*$ id\$	output $T \to FT'$
$*$ id\$	output $F \to$ id
$*$ id\$	match id
$*$ id\$	output $T' \to \epsilon$
$*$ id\$	output $E' \to + TE'$
$*$ id\$	match $+$
$*$ id\$	output $T \to FT'$
$*$ id\$	output $F \to$ id
$*$ id\$	match id
$*$ id\$	output $T' \to * FT'$
id\$	match $*$
id\$	output $F \to$ id
\$	match id
\$	output $T' \to \epsilon$
\$	output $E' \to \epsilon$

Parser Generator

- We show how a parser generator can be used to facilitate the construction of the front end of a compiler.
- Yacc stands for “yet another compiler-compiler,”
- Yacc is available as a command on the UNIX system, and has been used to help implement many production compilers.
- A translator can be constructed using Yacc in the manner illustrated in figure.



Creating an input/output translator with Yacc

Parser Generator

- First, a file, say translate.y, containing a Yacc specification of the translator is prepared.
- The UNIX system command yacc translate.y transforms the file translate.y into a C program called y.tab.c.
- The program y.tab.c is a representation of a parser written in C, along with other C routines that the user may have prepared.
- By compiling y.tab.c along with the ly library that contains the LR parsing program using the command we obtain the desired object program a.out that performs the translation specified by the original Yacc program.
- If other procedures are needed, they can be compiled or loaded with y.tab.c, just as with any C program.

Parser Generator

- A Yacc source program has three parts:

```
declarations
%%
translation rules
%%
supporting C routines
```

Example

- To illustrate how to prepare a Yacc source program, let us construct a simple desk calculator that reads an arithmetic expression, evaluates it, and then prints its numeric value.
- We shall build the desk calculator starting with the with the following grammar for arithmetic expressions:

$$
\begin{array}{lcl}
E & \rightarrow & E + T \mid T \\
T & \rightarrow & T * F \mid F \\
F & \rightarrow & (E) \mid \textbf{digit}
\end{array}
$$

- The token digit is a single digit between 0 and 9.

Fig: A Yacc desk calculator program derived from this grammar is shown in figure.

```
%{
#include <ctype.h>
%}

%token DIGIT

%%
line   : expr '\n'          { printf("%d\n", $1); }
       ;
expr   : expr '+' term      { $$ = $1 + $3; }
       | term
       ;
term   : term '*' factor    { $$ = $1 * $3; }
       | factor
       ;
factor : '(' expr ')'       { $$ = $2; }
       | DIGIT
       ;
%%
yylex() {
    int c;
    c = getchar();
    if (isdigit(c)) {
        yylval = c-'0';
        return DIGIT;
    }
    return c;
}
```

Yacc specification of a simple desk calculator

The Declaration Part

- There are two sections in the declarations part of a Yacc program.
- Both are optional.
- In the first section, we put ordinary C declarations, delimited by %{and %}.
- Here we place declarations of any temporaries used by the translation rules or procedures of the second and third sections.
- This section contains only the include-statement **#include <ctype.h>** that causes the C preprocessor to include the standard header file **<ctype.h>** that contains the predicate isdigit.

```
%{
#include <ctype.h>
%}

%token DIGIT
```

The Declaration Part

- Also in the declarations part are declarations of grammar tokens.
- The statement **%token DIGIT** declares DIGIT to be a token.
- Tokens declared in this section can then be used in the second and third parts of the Yacc specification.
- If Lex is used to create the lexical analyzer that passes token to the Yacc parser, then these token declarations are also made available to the analyzer generated by Lex.

```
%{
#include <ctype.h>
%}

%token DIGIT
```

The Translation Rules Part

In the part of the Yacc specification after the first %% pair, we put the translation rules.

- Each rule consists of a grammar production and the associated semantic action.
- In a Yacc production, unquoted strings of letters and digits not declared to be tokens are taken to be nonterminals.
- A quoted single character, e.g. **'c'**, is taken to be the terminal symbol c, as well as the integer code for the token represented by that character (i.e., Lex would return the character code for 'c' to the parser, as an integer).

```
%%
line   : expr '\n'          { printf("%d\n", $1); }
       ;
expr   : expr '+' term      { $$ = $1 + $3; }
       | term
       ;
term   : term '*' factor    { $$ = $1 * $3; }
       | factor
       ;
factor : '(' expr ')'       { $$ = $2; }
       | DIGIT
       ;
```

The Translation Rules Part

- A set of productions that we have been writing:

$$\langle \text{head} \rangle \rightarrow \langle \text{body} \rangle_1 \mid \langle \text{body} \rangle_2 \mid \cdots \mid \langle \text{body} \rangle_n$$

would be written in Yacc as

```
<head>  :  <body>1      { <semantic action>1 }
        |  <body>2      { <semantic action>2 }
           ...
        |  <body>n      { <semantic action>n }
        ;
```

The Translation Rules Part

- Alternative bodies can be separated by a vertical bar, and a semicolon follows each head with its alternatives and their semantic actions.
- The first head is taken to be the start symbol.
- A Yacc semantic action is a sequence of C statements.
- In a semantic action, the symbol $$ refers to the attribute value associated with the nonterminal of the head.
- While $i refers to the value associated with the ith grammar symbol (terminal or nonterminal) of the body.
- The semantic action is performed whenever we reduce by the associated production, so normally the semantic action computes a value for $$ in terms of the $i's.

```
%%
line   : expr '\n'          { printf("%d\n", $1); }
       ;
expr   : expr '+' term      { $$ = $1 + $3; }
       | term
       ;
term   : term '*' factor    { $$ = $1 * $3; }
       | factor
       ;
factor : '(' expr ')'       { $$ = $2; }
       | DIGIT
       ;
```

The Translation Rules Part

- While $i refers to the value associated with the ith grammar symbol (terminal or nonterminal) of the body.
- In the Yacc specification, we have written the two E-productions and their associated semantic actions.

$$
\begin{array}{lcl}
E & \rightarrow & E + T \mid T \\
T & \rightarrow & T * F \mid F \\
F & \rightarrow & (E) \mid \text{digit}
\end{array}
$$

```
%%
line   : expr '\n'          { printf("%d\n", $1); }
       ;
expr   : expr '+' term      { $$ = $1 + $3; }
       | term
       ;
term   : term '*' factor    { $$ = $1 * $3; }
       | factor
       ;
factor : '(' expr ')'       { $$ = $2; }
       | DIGIT
       ;
```

- Note that the nonterminal term in the first production is the third grammar symbol of the body, while + is the second.

The Translation Rules Part

- The semantic action associated with the first production adds the value of the expr and the term of the body and assigns the result as the value for the nonterminal expr of the head.
- We have omitted the semantic action for the second production altogether, since copying the value is the default action for productions with a single grammar symbol in the body.
- In general, { $$ = $1; } is the default semantic action.
- Notice that we have added a new starting production line : expr '\n' { printf ("\%d\n", $1); } to the Yacc specification.
- This production says that an input to the desk calculator is to be an expression followed by a newline character.
- The semantic action associated with this production prints the decimal value of the expression followed by a newline character.

The Supporting C-Routines Part

- The third part of a Yacc specification consists of supporting C-routines.
- A lexical analyzer by the name yylex() must be provided.
- Using Lex to produce yylex() is a common choice.
- Other procedures such as error recovery routines may be added as necessary.

```
%%
yylex() {
    int c;
    c = getchar();
    if (isdigit(c)) {
        yylval = c-'0';
        return DIGIT;
    }
    return c;
}
```

The Supporting C-Routines Part

- The lexical analyzer yylex() produces tokens consisting of a token name and its associated attribute value.
- If a token name such as DIGIT is returned, the token name must be declared in the first section of the Yacc specification.
- The attribute value associated with a token is communicated to the parser through a Yacc-defined variable yylval.

```
%%
yylex() {
    int c;
    c = getchar();
    if (isdigit(c)) {
        yylval = c-'0';
        return DIGIT;
    }
    return c;
}
```

The Supporting C-Routines Part

- The lexical analyzer here is very crude.
- It reads input characters one at a time using the C-function getchar().
- If the character is a digit, the value of the digit is stored in the variable yylval, and the token name DIGIT is returned.
- Otherwise, the character itself is returned as the token name.
- Lex was designed to produce lexical analyzers that could be used with Yacc.
- The Lex library ll will provide a driver program named yylex(), the name required by Yacc for its lexical analyzer.

```
%%
yylex() {
    int c;
    c = getchar();
    if (isdigit(c)) {
        yylval = c-'0';
        return DIGIT;
    }
    return c;
}
```

Creating Yacc Lexical Analyzers with Lex

- If Lex is used to produce the lexical analyzer, we replace the routine yylex() in the third part of the Yacc specification by the statement #include "lex.yy.c" and we have each Lex action return a terminal known to Yacc.

- By using the #include "lex.yy.c" statement, the program yylex has access to Yacc's names for tokens, since the Lex output file is compiled as part of the Yacc output file y.tab.c.

Creating Yacc Lexical Analyzers with Lex

- Under the UNIX system, if the Lex specification is in the file first.l and the Yacc specification in second.y, we can say

```
lex first.l
yacc second.y
cc y.tab.c -ly -ll
```

to obtain the desired translator.

- The Lex specification in figure can be used in place of the lexical analyzer for Yacc.
- The last pattern, meaning "any character," must be written \n | . since the dot in Lex matches any character except newline.

```
number    [0-9]+\e.?|[0-9]*\e.[0-9]+
%%
[ ]       { /* skip blanks */ }
{number}  { sscanf(yytext, "%lf", &yylval);
                  return NUMBER; }
\n|.      { return yytext[0]; }
```

Lex specification for yylex()

Thank You