

Subject: CSE-407, Applied Statistics and Queuing Theory

Total: 2.00 hours
Section A : 1.00 hour

Full Marks: 180
Section A : 90

INSTRUCTIONS:

- a. Use **SEPARATE** answer scripts for each section.
- b. **Question – 1 and Question – 4 (Viva Voce)** in **Section A** are compulsory.
- c. Answer any **OTHER ONE** question from this section (**From Q - 2 & Q - 3**).
- d. Figures in the margin indicate full marks.
- e. Assume reasonable data if necessary.
- f. **Symbols** used have their usual meanings.

SECTION-A

Question – 1 (Compulsory)

- a. "The probability of the union of n events equals the sum of the probabilities of these events taken one at a time minus the sum of the probabilities of these events taken two at a time plus the sum of the probabilities of these events taken three at a time, and so on." – Justify the statement with proper theorem. 15
- b. In this global pandemic, laboratory X has started COVID-19 test. The test is 96% efficient to detect COVID-19. However, the test also yields a "false-positive" result for 1.5% of the healthy persons tested. (That is if a healthy person is tested, then, with probability 0.015, the test result will imply he has the disease.) If 0.9% of the population actually has COVID-19 virus, what is the probability a person has the disease given that his test result is positive? 09
- c. Shahriar, Samia and Sujana are three friends. They went to a gaming zone and the game was target shooting together. They shoot one target at the same time. Suppose, Shahriar hits the target with the probability 0.7 whereas Samia independently hits the target with probability 0.2 and Sujana independently hits the target with probability 0.2 6+6=12
 - i. Given that exactly one shot hit the target, what is the probability that it was Sujana's shot?
 - ii. Given that the target is hit, what is the probability that Samia hit it?

Question – 2

- a. Let, $X \sim \text{Uniform}(-\frac{\pi}{2}, \pi)$ and $Y = \sin(X)$. Find $f_Y(y)$ for the given uniform random variable. 13
- b. For n independent trials each of which results in a "success" with probability P and in "failure" with probability $1-P$ are to be performed. If X represents the number of success and X is said to be a binomial random variable with parameters (n, p) . Show that according to the binomial theorem, the sum of the probability is 1. 13
- c. Consider yourself in a situation that an airplane engine will fail when in flight with probability $1-P$ independently from engine to engine; suppose that the airplane will make a successful flight if at least 50 percent of its engines remain operative. For what value of P you will choose a four-engine plane for your flight over a two-engine plane. 10

Question – 3

- a. X is a random variable with PDF given by

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$$f_X(x) = \begin{cases} Cx^2 & |x| \leq 1 \\ 0 & \text{Otherwise} \end{cases}$$

- i. Find the constant C.
 - ii. Find EX and $\text{Var}(X)$.
 - iii. Find $P(X \geq \frac{1}{2})$
- b. Derive the Bayes' Formula for two mutually exclusive events E and F. **10**
- c. You along with your 3 friends went to a party. At the party you played a game where you need to throw your hat at the center of the room. Four of you throw the hat and mixed up them and put them under four black boxes. Then each of you randomly select a hat. What is the probability that none of the 4 of you select your own hat. **11**

Question – 4 Viva Voce (Compulsory)**18**