

# A Method for the Analysis of the PA-X Agreement-Actor Signatories Dataset

Roy Gardner

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## 1 Motivation

To provide a useful abstraction from which to explore the agreements-actors signatory dataset.

## 2 Agreement-Actor Matrix

A binary-valued matrix is a matrix containing values from  $\{0, 1\}$  that can be used to represent binary relations between members of a pair of sets. For example, whether or not an actor is a signatory to a peace agreement.

For any binary-valued agreement-actor matrix there is an indexed set of agreements

$$A = \{a_1, a_2, \dots, a_M\} \quad (1)$$

where  $M$  is the total number of agreements, i.e.,  $M = |A|$ .

There is an indexed set of signatory actors

$$S = \{s_1, s_2, \dots, s_N\} \quad (2)$$

where  $N$  is the total number of actors, i.e.,  $N = |S|$ .

Each agreement  $a_i$  has a set of signatories

$$S_i \subset S \quad (3)$$

Each actor  $s_j$  has a set of agreements to which they are a signatory.

$$A_j \subset A \quad (4)$$

A binary-valued matrix  $U$  is generated with agreements in rows and actors in columns. Cells values are given by:

$$u_{ij} = \begin{cases} 0 & \text{if } s_j \notin S_i \\ 1 & \text{if } s_j \in S_i \end{cases} \quad (5)$$

### 2.1 Graphs

A binary-valued matrix describes an undirected bipartite graph where members of one set are connected to members of the other set, but where within-set connections do not exist.

A graph of the agreement-actor matrix of the Bosnia peace process is shown in Figure 1 of Appendix A.1.

A graph can be queried using depth-first search to show only the network of relations between a combination of selected actors and/or agreements. A search can be run against the binary matrix or against the graph if search is supported by the graph package.

A graph of a search against the Bosnia peace process is show in Figure 2 of Appendix A.2.

### 3 Agreement-Actor Co-occurrence Matrices

#### 3.1 Actor co-occurrence matrix

The actor co-occurrence matrices provides the number of agreements to which a pair of actors are co-signatories.

The actor co-occurrence matrix is given by

$$\mathbf{V} = \mathbf{U}^T \mathbf{U} \quad (6)$$

where  $\mathbf{V}$  is a symmetric matrix with the  $N$  actors of  $\mathbf{U}$  in both rows and columns. The diagonal of  $\mathbf{V}$  provides the columns marginal of  $\mathbf{U}$ .

A cell value contains the number of agreements to which a pair of actors are both signatories:

$$v_{ij} = |(A_i \cap A_j)| \quad (7)$$

i.e., the cardinality of the intersection of the agreement sets of the row actor  $s_i$  and the column actor  $s_j$ .

The set of indices  $I_{ij}$  of the agreements in a cell  $v_{ij}$  are the indices of non-zero values in the result of a bitwise AND between the  $i^{th}$  and  $j^{th}$  rows of  $\mathbf{U}^T$ :

$$\mathbf{x} = (\mathbf{U}^T)_i \wedge (\mathbf{U}^T)_j \quad (8)$$

$$n = \begin{cases} \in I_{ij} & \text{if } \mathbf{x}_n = 1 \\ \notin I_{ij} & \text{if } \mathbf{x}_n = 0 \end{cases} \quad (9)$$

#### 3.2 Agreement co-occurrence matrix

The agreement co-occurrence matrices provides the number of actors that are co-signatories to a pair of agreements.

The agreement co-occurrence matrix is given by

$$\mathbf{W} = \mathbf{U}\mathbf{U}^T \quad (10)$$

where  $\mathbf{W}$  is a symmetric matrix with the  $M$  agreements of  $\mathbf{U}$  in rows and columns. The diagonal of  $\mathbf{W}$  provides the rows marginal of  $\mathbf{U}$ .

A cell value contains the number of actors that are co-signatories to a pair of agreements:

$$w_{ij} = |(S_i \cap S_j)| \quad (11)$$

i.e., the cardinality of the intersection of the actor sets of the row agreement  $a_i$  and the column agreement  $a_j$ .

The set of indices  $I_{ij}$  of the actors in a cell  $w_{ij}$  are the indices of non-zero values in the result of a bitwise AND between the  $i^{th}$  and  $j^{th}$  rows of  $\mathbf{U}$ :

$$\mathbf{x} = (\mathbf{U})_i \wedge (\mathbf{U})_j \quad (12)$$

$$n = \begin{cases} \in I_{ij} & \text{if } \mathbf{x}_n = 1 \\ \notin I_{ij} & \text{if } \mathbf{x}_n = 0 \end{cases} \quad (13)$$

### 3.3 Graphs

An actor co-occurrence matrix can be represented as a graph where vertices are actors connected by edges with a weight given by the number of agreements to which the actors are co-signatories. The graph can be used to explore actor-actor relationships within a peace process including the existence of disjoint subgraphs. The interpretation of the graphical representation is being investigated. See Figure 3 in Appendix A.3 for an example from the Bosnia peace process.

## 4 Process-Actor Matrix

The methods described above can be applied to peace process-actor matrices.

There is an indexed set of processes

$$P = \{p_1, p_2, \dots, p_M\} \quad (14)$$

where  $M$  is the total number of peace processes, i.e.,  $M = |P|$ .

There is an indexed set of signatory actors

$$S = \{s_1, s_2, \dots, s_N\} \quad (15)$$

where  $N$  is the total number of actors, i.e.,  $N = |S|$ .

Each process  $p_i$  has a set of signatories

$$S_i \subset S \quad (16)$$

Each actor  $s_j$  has a set of processes in which they are a signatory to at least one agreement.

$$P_j \subset P \quad (17)$$

A binary-valued matrix  $\mathbf{C}$  is generated with agreements in rows and actors in columns. Cells values are given by:

$$c_{ij} = \begin{cases} 0 & \text{if } s_j \notin S_i \\ 1 & \text{if } s_j \in S_i \end{cases} \quad (18)$$

## A Visualisations

### A.1 Graph of agreement-actor matrix

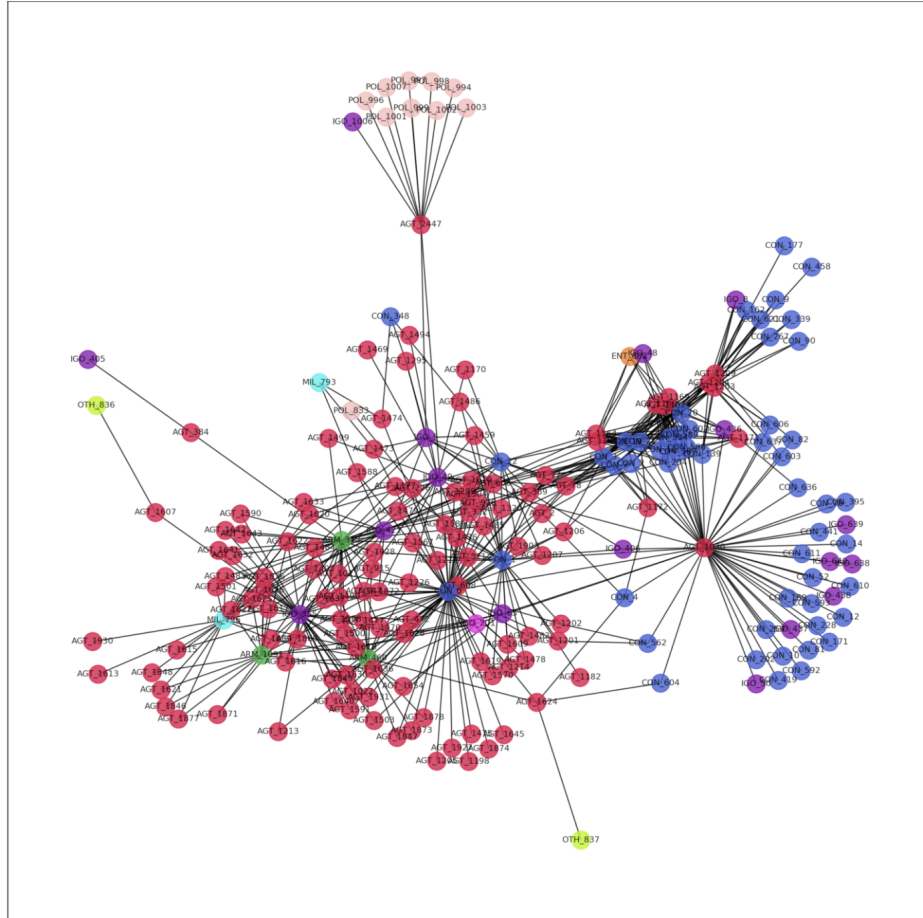


Figure 1: A graph of the binary-valued agreement-actor matrix for the Bosnia peace process. Vertices in red are agreements. Actors are colour coded by type, for example, countries in blue.

## A.2 Graph of agreement-actor matrix search results

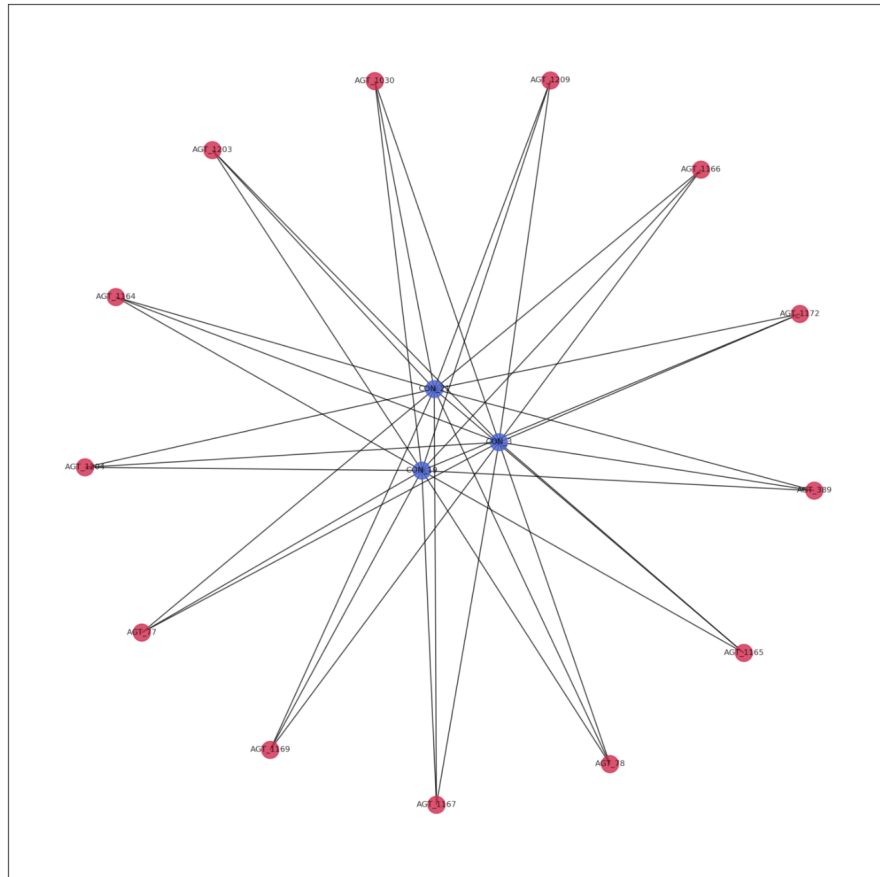


Figure 2: A graph of the results of a depth-first search for the actors Russia, UK, and USA in the graph of the Bosnia peace process (see Figure 1 above). The results show the agreements (in red) to which all the selected countries (in blue) are signatories.

### A.3 Graph of actor co-occurrence matrix

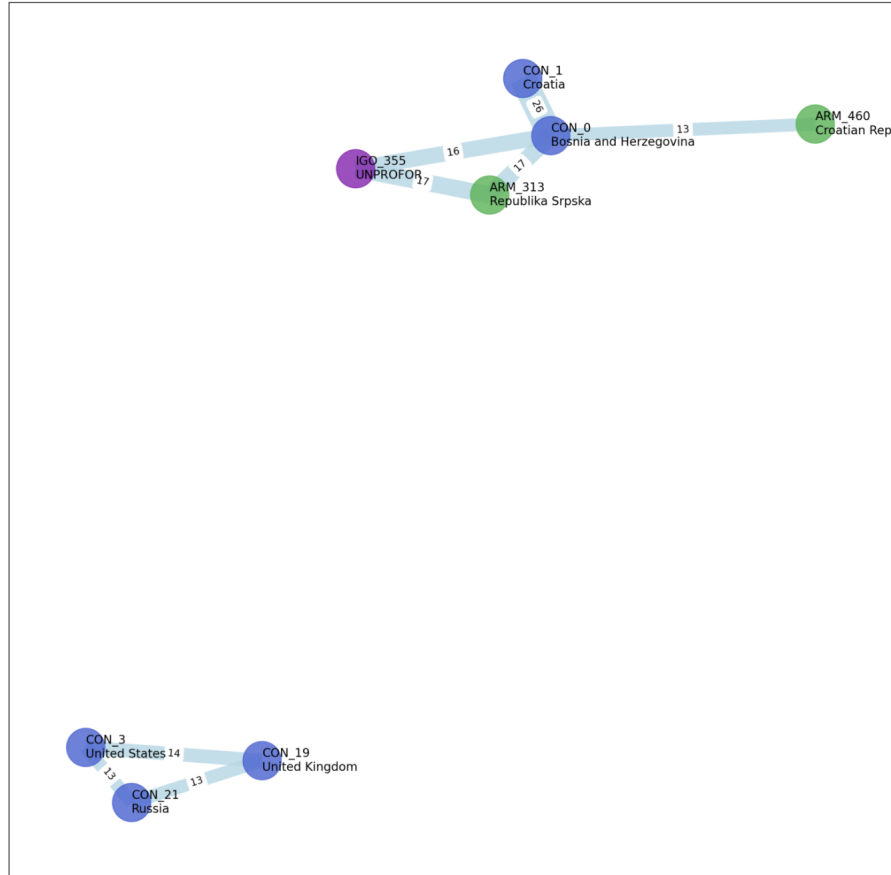


Figure 3: A graph of the actor co-occurrence matrix from the Bosnia peace process. To qualify for inclusion in the graph a pair of actors must be co-signatories to 13 or more agreements. Two disjoint subgraphs are visible. Vertex colour codes for actor type: blue = countries, purple = IGO, green = armed group.