A Method for the Analysis of the PA-X Agreement-Actor Signatories Dataset

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Contents

1 Motivation

To provide a useful abstraction from which to explore the agreements-actors signatory dataset.

2 Agreement-Actor Matrix

A binary-valued matrix is a matrix containing values from $\{0, 1\}$ that can be used to represent binary relations between members of a pair of sets. For example, whether or not an actor is a signatory to a peace agreement.

For any binary-valued agreement-actor matrix there is an indexed set of agreements

$$A = \{a_1, a_2, \ldots, a_M\} \tag{1}$$

where M is the total number of agreements, i.e., $M = |A|$.

There is an indexed set of signatory actors

$$S = \{s_1, s_2, \ldots, s_N\} \tag{2}$$

where N is the total number of actors, i.e., $N = |S|$.

Each agreement a_i has a set of signatories

$$S_i \subset S \tag{3}$$

Each actor s_j has a set of agreements to which they are a signatory.

$$A_j \subset A \tag{4}$$

A binary-valued matrix $\boldsymbol{U}$ is generated with agreements in rows and actors in columns. Cells values are given by:

$$u_{ij} = \begin{cases} 0 & if \;\; s_j \notin S_i \\ 1 & if \;\; s_j \in S_i \end{cases} \tag{5}$$

2.1 Graphs

A binary-valued matrix describes an undirected bipartite graph where members of one set are connected to members of the other set, but where within-set connections do not exist.

A graph of the agreement-actor matrix of the Bosnia peace process is shown in Figure 1 of Appendix A.1.

A graph can be queried using depth-first search to show only the network of relations between a combination of selected actors and/or agreements. A search can be run against the binary matrix or against the graph if search is supported by the graph package.

A graph of a search against the Bosnia peace process is show in Figure 2 of Appendix A.2.

3 Agreement-Actor Co-occurrence Matrices

3.1 Actor co-occurrence matrix

The actor co-occurrence matrices provides the number of agreements to which a pair of actors are co-signatories.

The actor co-occurrence matrix is given by

$$\boldsymbol{V} = \boldsymbol{U}^{\boldsymbol{T}}\boldsymbol{U} \tag{6}$$

where $\boldsymbol{V}$ is a symmetric matrix with the N actors of $\boldsymbol{U}$ in both rows and columns. The diagonal of $\boldsymbol{V}$ provides the columns marginal of $\boldsymbol{U}$.

A cell value contains the number of agreements to which a pair of actors are both signatories:

$$v_{ij} = |(A_i \cap A_j)| \tag{7}$$

i.e., the cardinality of the intersection of the agreement sets of the row actor s_i and the column actor s_j.

The set of indices I_{ij} of the agreements in a cell v_{ij} are the indices of non-zero values in the result of a bitwise AND between the i^{th} and j^{th} rows of $\boldsymbol{U}^{\boldsymbol{T}}$:

$$\boldsymbol{x} = (\boldsymbol{U}^{\boldsymbol{T}})_i \wedge (\boldsymbol{U}^{\boldsymbol{T}})_j \tag{8}$$

$$n = \begin{cases} \in I_{ij} & if\ \boldsymbol{x}_n = 1 \\ \notin I_{ij} & f\ \boldsymbol{x}_n = 0 \end{cases} \tag{9}$$

3.2 Agreement co-occurrence matrix

The agreement co-occurrence matrices provides the number of actors that are co-signatories to a pair of agreements.

The agreement co-occurrence matrix is given by

$$\boldsymbol{W} = \boldsymbol{U}\boldsymbol{U}^{\boldsymbol{T}} \tag{10}$$

where $\boldsymbol{W}$ is a symmetric matrix with the M agreements of $\boldsymbol{U}$ in rows and columns. The diagonal of $\boldsymbol{W}$ provides the rows marginal of $\boldsymbol{U}$.

A cell value contains the number of actors that are co-signatories to a pair of agreements:

$$w_{ij} = |(S_i \cap S_j)| \tag{11}$$

i.e., the cardinality of the intersection of the actor sets of the row agreement a_i and the column agreement a_j.

The set of indices I_{ij} of the actors in a cell w_{ij} are the indices of non-zero values in the result of a bitwise AND between the i^{th} and j^{th} rows of $\boldsymbol{U}$:

$$\boldsymbol{x} = (\boldsymbol{U})_i \wedge (\boldsymbol{U})_j \tag{12}$$

$$n = \begin{cases} \in I_{ij} & if\ \boldsymbol{x}_n = 1 \\ \notin I_{ij} & f\ \boldsymbol{x}_n = 0 \end{cases} \tag{13}$$

3.3 Graphs

An actor co-occurrence matrix can be represented as a graph where vertices are actors connected by edges with a weight given by the number of agreements to which the actors are co-signatories. The graph can be used to explore actor-actor relationships within a peace process including the existence of disjoint subgraphs. The interpretation of the graphical representation is being investigated. See Figure 3 in Appendix A.3 for an example from the Bosnia peace process.

4 Process-Actor Matrix

The methods described above can be applied to peace process-actor matrices.

There is an indexed set of processes

$$P = \{p_1, p_2, \ldots, p_M\} \tag{14}$$

where M is the total number of peace processes, i.e., $M = |P|$.

There is an indexed set of signatory actors

$$S = \{s_1, s_2, \ldots, s_N\} \tag{15}$$

where N is the total number of actors, i.e., $N = |S|$.

Each process p_i has a set of signatories

$$S_i \subset S \tag{16}$$

Each actor s_j has a set of processes in which they are a signatory to at least one agreement.

$$P_j \subset P \tag{17}$$

A binary-valued matrix $\boldsymbol{C}$ is generated with agreements in rows and actors in columns. Cells values are given by:

$$c_{ij} = \begin{cases} 0 & if \;\; s_j \notin S_i \\ 1 & if \;\; s_j \in S_i \end{cases} \tag{18}$$

A Visualisations

A.1 Graph of agreement-actor matrix

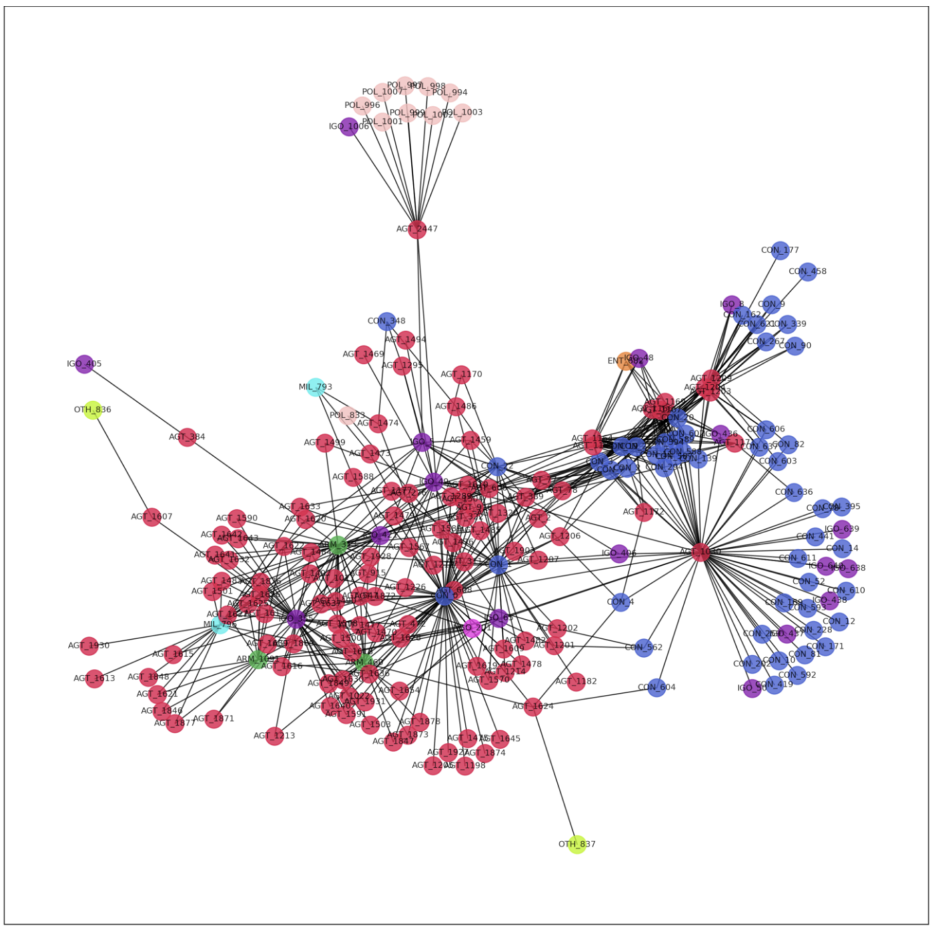

Figure 1: A graph of the binary-valued agreement-actor matrix for the Bosnia peace process. Vertices in red are agreements. Actors are colour coded by type, for example, countries in blue.

A.2 Graph of agreement-actor matrix search results

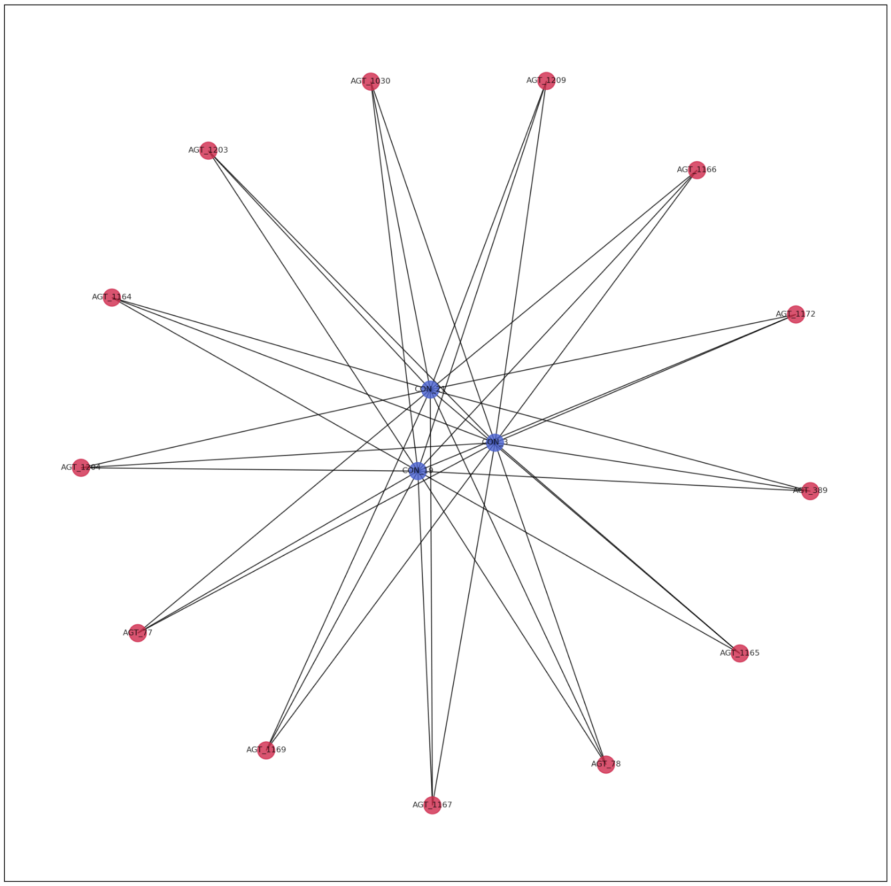

Figure 2: A graph of the results of a depth-first search for the actors Russia, UK, and USA in the graph of the Bosnia peace process (see Figure 1 above). The results show the agreements (in red) to which all the selected countries (in blue) are signatories.

A.3 Graph of actor co-occurrence matrix

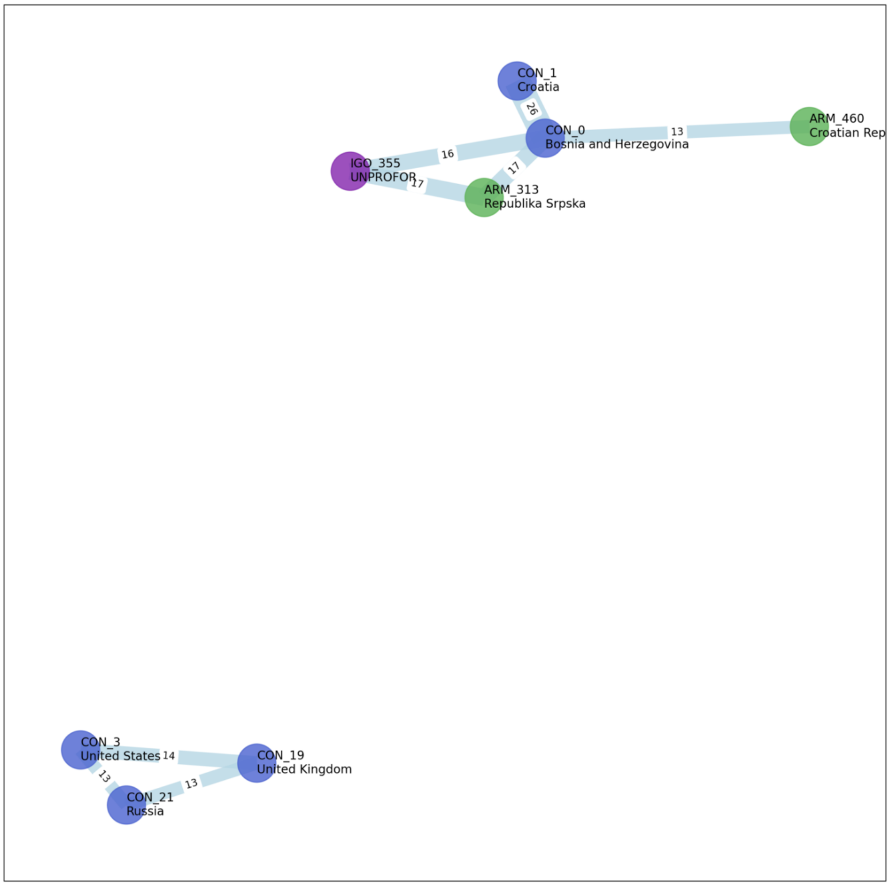

Figure 3: A graph of the actor co-occurrence matrix from the Bosnia peace process. To qualify for inclusion in the graph a pair of actors must be co-signatories to 13 or more agreements. Two disjoint subgraphs are visible. Vertex colour codes for actor type: blue = countries, purple = IGO, green = armed group.