This assignment is worth a total of 20 points. If you have questions or concerns regarding the course content, please email Leroy at *leroy_jia@brown.edu* or visit his office hours at 170 Hope St.

1 Linear ODEs

Consider the initial value problem (IVP)

$$y' = 3y + 1 \; , \; y(0) = 0$$

a) Plot a direction field for the equation (by hand or in MATLAB). Identify the solution to the IVP in the plot.

b) Assuming the solution is of the form $y(x) = Ae^{Bx} + C$, determine what the constants A, B, and C are.

c) Check your answer from the previous part by showing that your equation satisfies the ODE and the initial condition. Why are you guaranteed to have gotten the correct answer?

2 Tumor growth

The *Gompertz law* models the growth of cancerous tumors. The ODE looks like

$$N' = -aN \ln(bN)$$

where $N(t)$ is the number of cells in the tumor at time t, and a and b are parameters (positive constants). Even though the model is somewhat elementary, the predictions of this model agree suprisingly well with data on tumor growth (for large enough N).

a) What are the equilibrium points of the equation?

b) Classify the stability of each equilibrium point.

c) After a long time, how many cells will be in the tumor?

d) Interpret the constants a and b biologically. It may help to consider their dimensions.

3 Separable ODEs

A very basic class of ODEs that can be solved by hand without any necessary theory are the *separable equations*. They can be rearranged into a form

$$\frac{dy}{dx} = \frac{f(x)}{g(y)} \implies g(y)dy = f(x)dx$$

In other words, move everything with an x on one side and everything with a y on the other side (hence the term "separable"). The solution is then obtained by integration of both sides. Apply this technique to solve the following initial value problems (note that your answers may be not be explicit functions):

a) $y' = x^2/y$ with $y(1) = 1$

b) $y' = (1 - 2x)y^2$

c) $y' = xy^3/\sqrt{1+x^2}$

4 Backward Euler

Show (as we did in class for forward Euler) that backward Euler is $O(h^2)$ accurate locally and $O(h)$ globally.