MEASURING WELFARE AND INEQUALITY WITH INCOMPLETE PRICE INFORMATION*

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We propose and implement a new approach that allows us to estimate income-specific changes in household welfare in contexts where well-measured prices are not available for important subsets of consumption. Using rich but widely available expenditure survey microdata, we show that we can recover income-specific equivalent and compensating variations from horizontal shifts in what we call "relative Engel curves"—as long as preferences fall within the broad quasi-separable class (Gorman 1970, 1976). Our approach is flexible enough to allow for nonparametric estimation at each point of the income distribution. We apply the methodology to estimate inflation and welfare changes in rural India between 1987 and 2000. Our estimates reveal that lower rates of inflation for the rich erased the real income convergence found in the existing literature that uses the subset of consumption with well-measured prices to calculate inflation. *JEL codes:* F63, O12, E31, D12.

I. Introduction

Measuring changes in household welfare is valuable in many contexts, to evaluate the effects of policies and to assess changes in well-being across time and space. With recent political upheaval and a renewed focus on inequality, there is increased urgency to capture not just average changes but the full distribution. Although we often have reliable data on changes in nominal income, measuring changes in the denominator of real

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income—the cost of living—requires detailed price information that are seldom, if ever, available.

A number of recent publications use rich consumption microdata to study income group–specific welfare changes: either under explicit nonhomothetic preferences, such as in Fajgelbaum and Khandelwal (2016), Handbury (2021), and Comin, Lashkari, and Mestieri (2021); or by allowing income groups to have different taste parameters as in Atkin, Faber, and Gonzalez-Navarro (2018), Jaravel (2019), and Argente and Lee (2021).[1] The dramatic increase in inflation experienced by many countries since 2021 has further increased interest in calculating income group–specific inflation rates (see Jaravel and Lashkari 2024; Baqaee, Burstein, and Koike-Mori 2024). Common to all these approaches is the requirement that the researcher has complete (quality- and variety-adjusted) price information.[2] Such detail is paramount for distributional analysis because we know that different income groups consume very different bundles.

While sufficiently rich data on consumption prices and quantities are available for some countries and for some components of household welfare—for example, U.S. retail consumption using scanner microdata covering roughly 10% of consumption, or developing country expenditure surveys on foods and fuels covering more than half of rural consumption—it is not feasible to collect such detailed data for the entire consumption basket. Accurately measuring prices, quality, and variety for services (e.g., housing, health care, and education) and differentiated manufactures (e.g., electronics) is particularly difficult. Even in the richest data environments, evaluating changes in welfare from observed price data typically still requires strong functional-form assumptions (e.g., quality-adjusting prices or accounting for variety gains).

We instead propose and implement a new approach that uses rich but widely available expenditure survey microdata—and in particular does not require observing reliable price data for all

1. Other recent work includes Borusyak and Jaravel (2018), Hottman and Monarch (2020), and Argente, Hsieh, and Lee (2023).

2. In subsequent work to ours, Baqaee, Burstein, and Koike-Mori (2024) extend their approach to account for missing price data. To do so, they make a stronger separability assumption on substitution patterns between groups of goods with and without price data compared to our quasi-separability assumption that we introduce shortly. For estimation, their method requires parametrization of substitution patterns across these two groups of goods (while ours, instead, requires parametrization of substitution patterns within the group of goods with price data).

consumption categories—to estimate changes in exact household price indices for the full consumption basket, as well as welfare, at every point of the income distribution. We apply this approach to quantify changes in welfare over time for Indian households at different levels of income.

Our analysis proceeds in three steps. First, we develop the theory behind our approach. In environments with incomplete price information, recovering changes in the full price index, and hence welfare, is not possible without restrictions on preferences. The cornerstone of our methodology is the insight that quasi-separable preferences, as defined by Gorman (1970), provide a natural and testable restriction that allows us to estimate income-specific welfare changes in the absence of complete price information. Quasi-separability requires that groups of goods or services G are separable in the expenditure function (not the utility function as under direct separability): $e(p,U) = \tilde{e}(e_G(p_G,U), p_{NG}, U)$, where p_G and p_{NG} denote the vector of prices for goods within and outside G, respectively, and U is household utility.[3] In our context, p_G is observable, but p_{NG} may not be. Although this property of quasi-separable demands makes them natural candidates for inferring welfare changes with incomplete price data, as we lay out below, we are not aware of prior work making such a connection (the primary use of these demands in the literature is to rationalize constructing a subgroup price index for group G).[4]

The power of this restriction on preferences is that, after conditioning on the observable prices p_G, horizontal shifts across time in what we call relative Engel curves—projections of relative expenditures $\frac{x_i}{x_G}$ for good i as a share of group G on log total per capita household expenditure, $\log y$—reveal the full price

3. Deaton and Muellbauer (1980) also refer to quasi-separability as implicit separability. Specific examples in this class include the popular nonhomothetic Constant Elasticity of Substitution (CES) preferences (e.g., Gorman 1965; Hanoch 1975; Comin, Lashkari, and Mestieri 2021), several variants of Price Independent Generalized Linearity (PIGL), Price Invariant Generalized Logarithmic (PIGLOG), and translog preferences (Deaton and Muellbauer 1980), and a class of Gorman preferences discussed in Fally (2022).

4. Blackorby and Russell (1978) show that we can construct a price subindex for group G if, and only if, we have quasi-separability in G, where such a subindex: (i) does not depend on outside-G prices, and (ii) can be combined with outside-G prices to construct the overall price index. Most applications further assume homothetic separability, where within-group expenditure shares are independent of income and utility (see Blackorby, Primont, and Russell 1978).

index that covers all household consumption. At the heart of this strong result is the fact that Hicksian relative expenditures are a function of the vector of within-G relative prices and the level of household utility but not prices outside of G, $\frac{x_i}{x_G} = H_{iG}(p_G, U)$, if and only if preferences are quasi-separable. Prices outside of G may affect total expenditures on group G in a fully flexible manner, and they may also affect relative expenditures within G, but this latter effect only operates through changes in utility.[5] Thus, as long as these demands are invertible, households at two points in time (or two households in different locations) with the same within-G relative expenditures have the same utility after conditioning on within-G prices. Comparing total nominal household expenditures across these two points in time (or space) reveals the full price index P that keeps utility fixed when the household's cost of living changes—and hence allows us to obtain money-metric welfare measures in the absence of well-measured prices outside of group G.

To use this insight to recover changes in welfare at each point of the income distribution—even when we do not observe households with the same relative expenditure at two points in time—we turn to relative Engel curves $E_{iG}^t(y^t)$ (period t projections of $\frac{x_i}{x_G}$ on $\log y$). Assume for now that within-G prices are unchanged, $p_G^0 = p_G^1$. Moving from Hicksian to Marshallian relative demand by substituting U for the indirect utility function $V(p^t, y^t)$, the price index $P^0(y^0)$ that keeps the utility of a period 0 household constant under the full vector of period 1 prices p^1 is implicitly defined by equalizing relative demands across the two periods:

$$\underbrace{H_{iG}\left(p_G^0, V\left(p^0, y^0\right)\right)}_{E_{iG}^0(y^0)} = \underbrace{H_{iG}\left(p_G^1, V\left(p^1, \frac{y^0}{P^0(y^0)}\right)\right)}_{E_{iG}^1\left(\frac{y^0}{P^0\left(y^0\right)}\right)}. \tag{1}$$

Thus, the log of this price index change, $\log P^0(y^0)$, is simply the horizontal distance (in $\log y$ space) between period 0 and period 1 relative Engel curves. If we relax the assumption that within-

5. Quasi-separability thus places weaker restrictions on demand than the more common assumption of homothetic separability of the expenditure function in p_G, where H_{iG} would be a function of p_G but not utility U (i.e., homothetic separability implies quasi-separability but not vice versa).

G prices are unchanged, we show that we simply need to adjust the latter curve to account for these price changes within G. It is then straightforward to recover changes in welfare for any household from the horizontal distance traveled between period 0 and 1 within-G relative expenditures, either traveling along period 0's relative Engel curve (to recover the equivalent variation, EV) or period 1's curve (to recover the compensating variation, CV).[6]

A strength of this approach comes from the fact that relative Engel curves can be estimated nonparametrically because quasi-separable demand can be of any rank (see Lewbel 1991) and so can accommodate arbitrarily nonlinear patterns of nonhomotheticity within G and without imposing cross-equation restrictions on goods outside of G. Thus, we can capture potentially complex patterns of inflation that favor certain parts of the income distribution.

We state our approach formally in a lemma and a proposition. Lemma 1 lays out the logic above when relative prices within group G are held fixed. Proposition 1 relaxes this assumption by using observed price changes within G to correct the welfare estimates, either to the first order or exactly under any specific demand structure within G. We argue that in most settings it is not possible to obtain reliable price data for large swaths of the services and manufacturing sectors, partly because of difficulties capturing quality and variety. Thus, Proposition 1 provides the minimal structure on preferences (quasi-separability) that allows us to uncover the full price index and welfare in such settings.[7]

In the second step, we form a bridge between the theoretical results and the empirical implementation by creating a manual for practitioners. Our estimation approach follows directly from our theory and uses expenditure survey microdata to estimate relative Engel curves for every location, every period, and every good inside a product group G. As quasi-separability places no restrictions on the shape of these curves, they can be estimated nonparametrically and horizontal shifts calculated (correcting

6. The data can come from repeated cross sections or true household panels. In the (more common) first case, our approach recovers welfare changes at each point of the income distribution. In the second case, our approach recovers welfare changes for each household.

7. Because price changes outside of G are unrestricted, we can accommodate arbitrary changes in quality and variety outside of G. We can also accommodate quality or variety changes inside G by adjusting the prices we use for our price correction using standard methods (see Section III.B).

for within-G price changes and taking averages across goods to guard against measurement error). A natural question in taking our approach to the data is how plausible are the assumptions behind our proposition, most notably the assumption of quasi-separability? We show that violations of quasi-separability from misclassifying which goods are and are not in the quasi-separable set G have to be systematically related to price and income elasticities to cause bias, and we provide expressions for the sign and magnitude of any bias. We also present several tests for quasi-separability using the available data. Beyond quasi-separability, we derive a set of testable requirements for unbiased identification: (i) on aggregating up to good-level data in settings where barcode-level data are available, (ii) on sample selection, (iii) on bias in the estimation of Engel curves, and (iv) on preference heterogeneity across households and over time.

In the final step, we implement our methodology using Indian expenditure survey microdata to quantify changes in rural welfare between 1987/88 and 1999/2000 at different points of the income distribution for every district in India.[8] We compare our estimates to the leading existing Indian CPI estimates that come from Deaton (2003b), who calculates standard Paasche and Laspeyres price index numbers using changes in prices of products in the household surveys with both quantity information and no evidence of multiple varieties in a given location. For poorer deciles of the income distribution, we find very similar levels of consumer price inflation. Given that the products Deaton deems to have reliable prices—foods and fuels—cover more than 80% of total outlays for poorer rural households, it is reassuring that our estimates of the full price index for these households are similar to Deaton's estimates of what is essentially a food and fuel price index (despite coming to this conclusion in different ways—we exploit shifts in relative Engel curves whereas Deaton uses observed price changes).

Looking across the income distribution, our estimates reveal that price inflation has been far from uniform, with significantly lower inflation rates for richer households—something that is

8. We focus on rural households because that has been the focus of the existing literature (e.g., the great Indian poverty debate, or Topalova 2010) and because well-measured food and fuel prices cover most of the consumption bundle for poor rural households, allowing us to validate our estimates against standard price indices for this group.

not apparent from calculating standard price indices, even when using income group– and district-specific expenditure weights, or from estimating nonhomothetic price indices using quadratic almost ideal demand system (AIDS) demand and goods with observable price data.[9] Thus, while estimates based on standard approaches designed for settings with complete price data suggest that India saw significant convergence between poor and rich households over this period, we find no convergence once we account for the income-specific inflation uncovered by our approach.

The most likely explanation for these findings is that higher-income Indian households disproportionately benefited from lower inflation in categories such as services and manufactures, where reliable price data are simply not available. This lower inflation is consistent with substantial increases in both the quality and variety of manufacturing products, and price declines, resulting from large reductions in tariff protection (see Goldberg et al. 2010); as well as rapid growth in the share of services in both GDP and employment over this period (Mukherjee 2015).[10] Standard approaches to price index estimation miss these patterns as these categories are either ignored entirely (as in Deaton 2003b) or included without any quality or variety correction (as in India's official CPI). Because wealthy households spend disproportionately on these categories and nonhomotheticities are most pronounced within them, difficulties in measuring service and manufacturing prices have the potential to substantially change the distribution of welfare changes as we find.

This analysis sheds new light on the great Indian poverty debate. Because India's 1999–2000 National Sample Survey (NSS) added an additional 7-day recall period for food products (which inflated answers to the consistently asked 30-day consumption questions and lowered poverty measures), there has been much disagreement on how much poverty changed over the reform

9. For the latter, see Almås and Kjelsrud (2017), who use the same National Sample Survey expenditure data but include two categories with poorly measured prices (clothing; bedding and footwear). In addition, as their method requires all prices, they assume that for miscellaneous nonfood—the large residual category for which prices are not available—all relative prices change by the ratio of the nonfood to food CPIs produced by the Indian government (CPIs that also struggle to account for changes in quality or variety).

10. Our finding may be driven in part by a surge in product innovation in these sectors that is disproportionately targeted at rich households, a mechanism Jaravel (2019) documents for the United States in the 2000s.

period.[11] As long as the additional recall period did not change relative budget shares within our groups of food products G, our approach remains unbiased. We show that this assumption holds by exploiting the fact that the 1998 "thin" survey round randomly assigned households to different recall periods. Thus, our approach provides a solution to the recall issues at the center of this debate. The Online Appendix also presents a second application of our methodology, revisiting Topalova's (2010) analysis of the local labor market effects of India's 1991 trade reforms and uncovering adverse effects of import competition across the full income distribution, including among the richest households.

In addition to the literatures mentioned already, our approach connects to a long-standing literature using traditional Engel curves and expenditure changes on income-elastic goods—typically foodstuffs—to recover unobserved changes in real income (e.g., Costa 2001; Hamilton 2001; Almås 2012; Young 2012; Nakamura, Steinsson, and Liu 2016). Hamilton's (2001) initial goal was to correct biases in the U.S. consumer price index (CPI) arising from difficulties in measuring quality-adjusted prices in consumption categories such as services and manufactures. We address a key shortcoming in this literature. Despite relying on the nonhomothetic AIDS demand system to generate nonhorizontal Engel curves, this approach recovers a single price index for all households and so is neither theory consistent nor suitable for distributional analysis. As shown in Almås, Beatty, and Crossley (2018), calculating income-specific price index changes under the existing Engel methodology reintroduces the need to observe the full vector of price changes. We propose a new approach that leverages the broad class of quasi-separable preferences to recover theory-consistent price index and welfare changes at any point of the income distribution when price information is incomplete.[12]

Finally, a recent literature uses barcode-level microdata for price index estimation, exploiting the granularity of these data to account for changes in product variety following Feenstra (1994).

11. See Deaton and Kozel (2005) for an overview. Deaton (2003a) calculates poverty by adjusting food expenditure using the initial mapping between food and fuels expenditure (which had no recall period added), implicitly assuming that relative prices of food and fuels did not change. Tarozzi (2007) explores a related approach.

12. In related work, Ligon (2019) shows how one can recover the marginal utility of expenditure from expenditure data by imposing demands that feature a constant Frisch elasticity for each good and assuming that unobserved price changes in the full consumption basket are orthogonal to these Frisch elasticities.

The second paragraph of the introduction cites those that calculate income group–specific price indices. Crawford and Neary (2023) extend this approach to product characteristic space. Redding and Weinstein (2020) show how to use CES preferences to account for changes in product quality when prices are observed. As we discuss in Section III.B, these recent advances complement our theoretical proposition by providing estimates of variety and quality-adjusted prices that can be used to correct for within-G relative price changes when products contain multiple varieties.

II. Theory

In this section we develop an approach to estimating income-specific changes in price indices and welfare that does not require reliable price data covering the full consumption basket. We first describe a data environment designed to mimic widely available household expenditure surveys. Next, we introduce our approach and establish the central role of quasi-separability in a simplified setting (Lemma 1) before proceeding to our main proposition.

II.A. Data Environment

Our starting point is an environment with information on total (nominal) household outlays per capita, y_h,[13] for different households h coupled with their per capita expenditures x_{hi} across the goods and services $i \in I$ that they consume (for readability we refer to them simply as goods). Well-measured prices p_i are available for some subset of goods G but not necessarily for the remaining goods NG. This data environment corresponds to expenditure survey data where either separate price surveys or unit values calculated from well-measured quantity data provide price information for some subset of goods, such as foodstuffs or fuels.

To match our empirical setting, we focus our discussion on inferring price index changes over time for households at a given percentile h of the income distribution in a particular location.[14] Inferring changes over time requires data for two time periods. In what follows, superscripts 0 and 1 indicate time periods and p is the full vector of consumption prices. Isomorphic results would hold across space if we replaced time periods by locations.

13. For readability, we also refer to total household outlays per capita as income.

14. If household panel data are available, we can infer price index changes for individual households.

II.B. Basic Approach and the Role of Quasi-Separability

In this environment, recovering changes in the full price index, and hence welfare, is challenging. As the following sections document—by focusing on relative expenditures within product groups where prices are well measured—quasi-separable preferences provide the minimal restrictions necessary to recover welfare changes and allow us to do so nonparametrically in this class of preferences.

Two definitions will be central. First, we define quasi-separable demand following Gorman's original formulation (1970, 1976).

DEFINITION. Preferences are quasi-separable in group G of goods if a household's expenditure function can be written as:

$$e(p, U_h) = \tilde{e}(e_G(p_G, U_h), p_{NG}, U_h), \tag{2}$$

where $e_G(p_G, U_h)$ is a scalar function of utility U_h and the vector of prices of goods $i \in G$, p_G, and is homogeneous of degree one in prices p_G.

Quasi-separability is separability in the expenditure function (rather than the utility function). Two features of these preferences merit discussion (see Lemma 2 in Online Appendix A.3 for proofs).[15]

First, preferences are quasi-separable if and only if relative expenditures on each good i within group G—$\frac{x_{ih}}{x_{Gh}}$ where x_{Gh} is total expenditure on group G—can be written as a compensated function $H_{iG}(p_G, U)$ of utility and within-G relative prices alone:

$$\frac{x_{ih}}{x_{Gh}} = H_{iG}(p_G, U_h) = \frac{\partial \log e_G(p_G, U_h)}{\partial \log p_i}. \tag{3}$$

Second, quasi-separability imposes no restrictions on substitution patterns between goods within G, or between goods outside of G, or between consumption aggregates for group G relative to NG, but limits substitution patterns between a good in G and a good in NG to operate through a common group-G aggregator (with the flexibility of that aggregator allowing the elasticity of substitution between $i \in G$ and $j \in NG$ to be pair specific). More precisely, preferences are quasi-separable if and only if we can

15. Lemma 2 combines existing results (see e.g., Blackorby, Primont, and Russell 1978) and provides a more direct proof.

define utility implicitly by $K(F_G(q_G, U_h), q_{NG}, U_h) = 1$, where q_G and q_{NG} denote vectors of consumption of goods in G and outside G, respectively, and the function $F_G(q_G, U_h)$ is homogeneous of degree one in q_G.

Several examples are instructive. The preferences used in Comin, Lashkari, and Mestieri (2021) and Matsuyama (2015), in which utility is implicitly defined by $\sum_i^N (\frac{q_i}{g_i(U)})^{\frac{\sigma-1}{\sigma}} = 1$, are quasi-separable in any arbitrary subset of goods. Translog (in expenditure functions), Exact Affine Stone Index (EASI), and PIGLOG demand systems also satisfy quasi-separability if there are no direct cross-price effects between goods within and outside of G. Beyond these special cases, we can construct highly flexible demand systems that allow for rich substitution effects within G (captured by function F_G) and between goods within and outside G (function K).

Turning to our second definition, we define what we call "relative Engel curves" as follows.

DEFINITION. Relative Engel curves, denoted by the function $E_{iG}^t(y_h) = \frac{x_{ih}}{x_{Gh}}$, describe how relative expenditure shares within G vary with total outlays per capita in period t (i.e., given the prevailing vector of prices p^t).

Note that since quasi-separable demand systems can have any rank in the sense of Lewbel (1991), they can accommodate arbitrarily nonlinear relative Engel curves and allow for nonparametric estimation, as we describe and implement in Sections III and IV.

Finally, we present our price index notation and define our two welfare metrics. $P^1(p^0, p^1, y_h^1)$ (or in more concise notation $P^1(y_h^1)$ or just P^1) is the exact price index change between period 0 and period 1 prices, holding utility at period 1's level (i.e., P^1 is defined implicitly by $V(p^1, y_h^1) = V(p^0, \frac{y_h^1}{P^1(y_h^1)})$ where V is the indirect utility function). In other words, the price index $P^1(y_h^1)$ converts the household's period 1 nominal income to the hypothetical level of income that would make them equally well off under period 0 prices. Analogously, we define $P^0(p^0, p^1, y_h^0)$ as the exact price index change between period 1 and period 0 prices holding utility at period 0's level (i.e., $V(p^0, y_h^0) = V(p^1, \frac{y_h^0}{P^0(y_h^0)})$).[16]

16. The two price indices mirror each other: $y_h^1 = \frac{y_h^0}{P^0(y_h^0)}$ implies $y_h^0 = \frac{y_h^1}{P^1(y_h^1)}$.

These two price indices are closely related to equivalent and compensating variations. $EV_h = e(p^0, U_h^1) - e(p^0, U_h^0) = \frac{y_h^1}{P^1(y_h^1)} - y_h^0$ is the amount of money that would bring a household in period 0 to their period 1 utility, and $CV_h = e(p^1, U_h^1) - e(p^1, U_h^0) = y_h^1 - \frac{y_h^0}{P^0(y_h^0)}$ is the amount of money taken away from a period 1 household to bring it back to its period 0 utility.

With these definitions in hand, we turn to our first result. Lemma 1 makes no assumptions on relative price changes outside of group G but fixes relative prices for goods within G. This assumption—which we relax below—is convenient to highlight the key role of quasi-separability in estimating welfare changes with incomplete price information for non-G goods.

LEMMA 1. Assume that relative prices within group G are unchanged (i.e., $p_i^1 = \lambda_G p_i^0$ for all $i \in G$ and for some $\lambda_G > 0$). If and only if preferences are quasi-separable in subset G:

i. The log price index change for a given income level in period 1, $\log P^1(y_h^1)$, or period 0, $\log P^0(y_h^0)$, is equal to the horizontal shift (in $\log y_h$ space) in the relative Engel curve of any good $i \in G$ at that income level, such that

$$E_{iG}^0\left(\frac{y_h^1}{P^1(y_h^1)}\right) = E_{iG}^1\left(y_h^1\right) \qquad \text{and} \qquad E_{iG}^1\left(\frac{y_h^0}{P^0(y_h^0)}\right) = E_{iG}^0\left(y_h^0\right).$$

ii. When the relative Engel curve for a good $i \in G$ is strictly monotonic in income y_h:
   a. $\log P^1(y_h^1)$ and $\log P^0(y_h^0)$ are uniquely identified by the horizontal shift in good i's relative Engel curve, as defined by the equalities above.
   b. EV and CV for a given income level are revealed by the horizontal distance between new and old expenditure shares along period 0's or period 1's relative Engel curve for good i, respectively, such that $E_{iG}^0(y_h^0 + EV_h) = \frac{x_{ih}^1}{x_{Gh}^1}$ and $E_{iG}^1(y_h^1 - CV_h) = \frac{x_{hi}^0}{x_{hG}^0}$.

Online Appendix A.1 provides the proofs.

Lemma 1 (i) states that the horizontal shift in relative Engel curves at any given point of the initial or final income distribution is equal to the change in the exact price index for that group. Whether we can use this result to infer changes in the price index by observing relative expenditure shares within G and total outlays depends on whether we can invert these relationships. If a

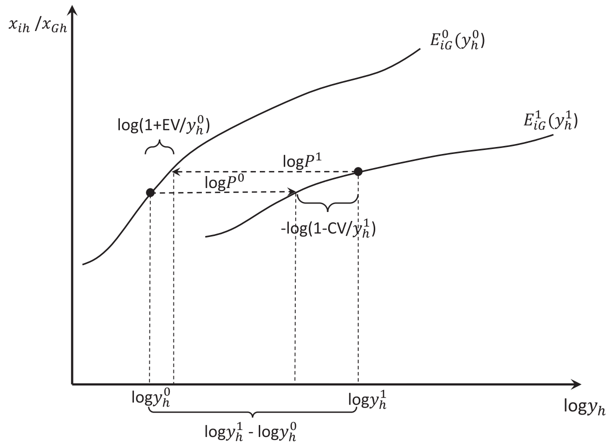

FIGURE I

Illustration of Lemma 1

The figure illustrates how price indices and welfare can be recovered from horizontal shifts in relative Engel curves (i.e., expenditure on good i as a share of total expenditure on group G plotted against log total outlays per capita) when relative prices within group G are unchanged but prices outside of G are unrestricted. Period 0 and period 1 relative Engel curves for good i are denoted by $E^0_{iG}(y^0_h)$ and $E^1_{iG}(y^1_h)$, respectively. See Section II for further discussion.

relative Engel curve is strictly monotonic, as assumed in Lemma 1 (ii), observing shifts for that single good is sufficient to infer price index and welfare changes. In contrast, if a relative Engel curve is flat (independent of income), a "horizontal shift" leaves the curve unchanged, and thus the shift is uninformative. In general, invertibility requires that the vector of relative expenditure shares $\boldsymbol{E}_{iG}$ across $i \in G$ is an injective function of income y_h (such that the vector of budget shares maps to a unique level of income). A sufficient condition for this invertibility requirement to hold is that at least one relative Engel curve $i \in G$ is strictly monotonic.

To aid intuition, Figure I graphically illustrates Lemma 1 (ii). Take as an example a household at percentile h with initial per capita outlays of y^0_h (the bottom-left dot in the figure). Because within-G relative prices are not changing, households with the same within-G budget shares must be equally well off (recall that quasi-separability implies that relative outlays depend only on within-G prices and utility, $\frac{x_{ih}}{x_{Gh}} = H_{iG}(p_G, U_h)$).[17] Thus, the horizontal distance (in $\log y_h$ space) between their initial position on the period 0 relative Engel curve and that same budget share on the period 1 relative Engel curve equals the log of the change in the price index P^0. The CV for this household is then revealed by the additional distance that must be traveled in $\log y_h$ space to go from the crossing point on the period 1 relative Engel curve to the actual within-G budget share of that household in period 1 (the upper-right dot). The same movements in reverse reveal P^1 and EV.

Since relative Engel curves are not parallel, the price index change P^0 and CV_h may vary with the household's position in the income distribution. Relatedly, P^1 and EV_h will not be identical to P^0 and CV_h if the household's utility differs in the two periods. Why are the curves not parallel? As relative prices within G are held fixed, it is changes in prices outside of group G (e.g., prices of manufactures and services) that rotate the curves apart when these goods are consumed disproportionately by richer (or poorer) households. By not placing restrictions on price changes outside of set G, income group–specific price indices can diverge, leading to nonparallel shifts in relative Engel curves.

To make these statements precise, we lay out several steps of Lemma 1's proof. To obtain $P^1(p^0, p^1, y_h^1)$, start with the period 1 relative budget share on period 1's relative Engel curve:

$$\begin{aligned} E_{iG}^1\left(y_h^1\right) = H_{iG}\left(p_G^1, U_h^1\right) &= H_{iG}\left(p_G^1, V\left(p^1, y_h^1\right)\right) \\ &= H_{iG}\left(p_G^0, V\left(p^1, y_h^1\right)\right) \\ &= H_{iG}\left(p_G^0, V\left(p^0, \frac{y_h^1}{P^1\left(p^0, p^1, y_h^1\right)}\right)\right) \\ &= E_{iG}^0\left(\frac{y_h^1}{P^1\left(p^0, p^1, y_h^1\right)}\right). \end{aligned}$$

The first line links this unobserved compensated Hicksian demand function to observed relative Engel curves by substituting in the indirect utility function $V(p, y)$ that connects total outlays and utility. Equality between the first and second line is an implication of the homogeneous price change $p_i^1 = \lambda_G p_i^0$ in group

17. Here we abstract from preference (taste) heterogeneity but discuss this possibility in Section III.B.

G.[18] Equality between the second and third lines follows from the definition of $P^1(p^0, p^1, y_h^1)$. The final line moves back to relative Engel curve functions. Thus, the difference between percentile h's total outlays in period 1 and the total outlays of a percentile in period 0 with the same relative budget share as h had in period 1 reveals the price index change $P^1(p^0, p^1, y_h^1)$. An analogous proof applies for $P^0(p^0, p^1, y_h^0)$.

Lemma 1 shows that quasi-separability is not only sufficient but a necessary condition to recover income-specific price indices and welfare from horizontal shifts in observed within-G outlays for arbitrary price realizations outside of G. Thus, in the absence of reliable price data outside of group G, quasi-separability provides the minimal restriction on preferences such that these unknown prices do not confound shifts in relative Engel curves.

Finally, an obvious question is why do we focus on relative Engel curves and whether alternative preferences could allow us to recover changes in the price index from shifts in traditional Engel curves (i.e., shares of total expenditure plotted against log total outlays). In Lemma 4 in Online Appendix A.5, we provide an impossibility result that no such approach is consistent with rational preferences while allowing for arbitrary changes in unobserved prices (if price changes are uniform, shifts in traditional Engel curves do recover price indices). These results connect to Almås, Beatty, and Crossley (2018), who show that the traditional Engel-curve methodology for recovering price indices under AIDS preferences (Hamilton 2001) requires information on all price changes to recover income-specific price indices. Shifting attention to relative Engel curves—and quasi-separable preferences—allows us to bypass these negative results.

II.C. Recovering Income-Specific Welfare Changes from Expenditure Survey Data

Even if preferences are quasi-separable, vertical shifts in relative Engel curves due to within-G relative price changes (assumed away in Lemma 1) can confound estimates of price index changes by raising or lowering relative expenditure shares conditional on utility. Our main proposition takes advantage of the fact that reliable price information may be available for some quasi-separable set of goods—but not for all consumption—to

18. Note that H_{iG} is homogeneous of degree zero in prices p_G^t within group G.

adjust relative Engel curves to account for these confounding vertical shifts and relax the assumption that relative prices in subset G are fixed.

PROPOSITION 1. If and only if preferences are quasi-separable in the subset G of goods:

i. The log price index change for a given income level in period 1, $\log P^1(y_h^1)$, is equal to the horizontal shift (in $\log y_h$ space) in the price-adjusted relative Engel curve of any good $i \in G$ at that income level, such that

$$E_{iG}^0\left(\frac{y_h^1}{P^1(y_h^1)}\right) = E_{iG}^1\left(y_h^1\right) \times \frac{H_{iG}\left(p_G^0, U_h^1\right)}{H_{iG}\left(p_G^1, U_h^1\right)}, \tag{4}$$

where $\frac{H_{iG}(p_G^1, U_h^1)}{H_{iG}(p_G^0, U_h^1)}$ is the change in expenditure shares induced by the change in (relative) prices within G evaluated along the indifference curve at period 1 utility, $U_h^1 = V(p^1, y_h^1)$.

ii. When the price-adjusted relative Engel curve for a good $i \in G$ is strictly monotonic in income y_h:

a. $\log P^1(y_h^1)$ is uniquely identified by the horizontal shift in good i's price-adjusted relative Engel curve, as defined by the equality above.

b. EV for a given income level is revealed by the horizontal distance between new and old expenditure shares along period 0's relative Engel curve for good i, such that

$$E_{iG}^0\left(y_h^0 + EV_h\right) = \frac{x_{ih}^1}{x_{Gh}^1} \times \frac{H_{iG}\left(p_G^0, U_h^1\right)}{H_{iG}\left(p_G^1, U_h^1\right)}.$$

Switching superscripts 0 and 1 provides the log price index change $\log P^0(y_h^0)$ and CV.

Online Appendix A.2 provides the proofs.[19]

This proposition shows that we can still infer changes in $\log P^1(y_h^1)$ from horizontal shifts in relative Engel curves but after first adjusting the period 1 curve by the term $\frac{H_{iG}(p_G^0, U_h^1)}{H_{iG}(p_G^1, U_h^1)}$, that is, the compensated shift in expenditure shares due to the change

19. Note that Proposition 1 also holds with an additive correction term, $+\left[H_{iG}(p_G^0, U_h^1) - H_{iG}(p_G^1, U_h^1)\right]$ instead of $\times\frac{H_{iG}(p_G^0, U_h^1)}{H_{iG}(p_G^1, U_h^1)}$, since $E_{iG}^1(y_h^1) = H_{iG}(p_G^1, U_h^1)$.

in within-G prices, with: $\log \frac{H_{iG}(p_G^1, U_h^1)}{H_{iG}(p_G^0, U_h^1)} = \sum_{j \in G} \int_{p_j^0}^{p_j^1} \frac{\partial \log H_{iG}}{\partial \log p_j} d \log p_j$. EV is then the additional horizontal distance traveled along the period 0 relative Engel curve to the period 0 expenditure share.

These adjustments require some knowledge of the within-group demand structure H_{iG} and within-group relative price changes. But crucially, they do not require information on the structure of preferences or prices for goods outside G. As long as there is a group G of goods for which preferences are quasi-separable and reliable price data are available, we can uncover changes in price indices and welfare.[20]

As described in Section III.A, we implement the price adjustment in Proposition 1 in several ways: in its exact form after specifying a range of different within-group demand structures H_{iG} and as a first-order approximation, evaluating elasticities in the base period. The latter approach brings two benefits. First, it does not require us to take a stand on the structure of within-group demand and second, it provides a natural and transparent two-step estimation strategy—first calculating horizontal shifts in the unadjusted relative Engel curves that present themselves directly in the data (as in Lemma 1), and then adding a correction term formed from local elasticities and observable within-G relative price changes.

To formalize this second approach, write equation (4) in logs and take a first-order approximation of changes in $\log H_{iG}$ due to relative price changes, holding utility fixed.[21] Subsequently inverting the relative Engel curve at $\frac{x_{ih}^1}{x_{Gh}^1}$, for any good $i \in G$ we

20. To be more precise, these vertical adjustments of relative Engel curves depend on compensated changes in expenditure shares within G, holding utility constant. One can infer compensated changes in within-group expenditures from a Slutsky-type decomposition involving slopes of relative Engel curves and uncompensated price elasticities of within-group expenditure shares (see the proof of Proposition 1 in Online Appendix A): $\frac{\partial \log H_{iG}}{\partial \log p_j} = \frac{\partial \log(\frac{x_i}{x_G})}{\partial \log p_j} + E_{jG} \frac{x_G}{y} \frac{\partial \log E_{iG}}{\partial \log y}$. Estimating these terms only requires information on household total outlays, expenditures on goods in group G, and prices of these goods.

21. That is, assuming that the vertical shifts in relative Engel curves due to within-G relative price changes are proportional to those price changes.

obtain:[22]

$$\log\left(y_h^1\right) - \log\left(E_{iG}^0\right)^{-1}\left(\frac{x_{ih}^1}{x_{Gh}^1}\right)$$
$$\approx \log\left(P^1\right) + \left(\beta_{ih}^0\right)^{-1}\log\frac{H_{iG}(p_G^0, U_h^1)}{H_{iG}(p_G^1, U_h^1)}, \tag{5}$$

where $\beta_{ih}^0 = \frac{\partial \log E_{iG}}{\partial \log y_h}$ denotes the slope of the relative Engel curve (i.e., the income elasticity) evaluated at income level $\frac{y_h^1}{P^1}$ and initial prices p^0. The left side of equation (5) is the horizontal shift in the price-unadjusted relative Engel curve as in Lemma 1. The first term on the right side of equation (5) is the price index change we are trying to estimate. The second term is the bias due to a potential confounder: the vertical shift in relative Engel curves due to relative price changes within G. Finally, using the local compensated cross-price elasticities of relative expenditures, $\sigma_{ijh} = \frac{\partial \log H_{iG}}{\partial \log p_j}$ with $\Sigma_{j\,\in\,G}\sigma_{ijh} = 0$, this vertical shift, again to the first order, can be rewritten as a function of observable relative price changes: $\log\frac{H_{iG}(p_G^0, U_h^1)}{H_{iG}(p_G^1, U_h^1)} \approx \sum_{j\in G}\sigma_{ijh}(\Delta\log p_j - \overline{\Delta\log p_G})$.

III. From Theory to Estimation: An Empirical Methodology

In this section, we build on the theoretical results to provide an empirical methodology for estimating price indices and welfare changes using household expenditure survey microdata with price information that covers only a subset of consumption. We then turn to identification and derive corollaries to Proposition 1 that define testable conditions for unbiased estimation. These results naturally suggest a number of validation exercises and robustness checks that we implement in our application in Section IV. Taken together, this section serves as a manual for practitioners to apply the methodology.

22. Symmetrically for P^0: $\log\left(y_h^0\right) - \log\left(E_{iG}^1\right)^{-1}\left(\frac{x_{ih}^0}{x_{Gh}^0}\right) \approx \log\left(P^0\right) + \left(\beta_{ih}^1\right)^{-1}\log\frac{H_{iG}(p_G^1, U_h^1)}{H_{iG}(p_G^0, U_h^1)}$.

III.A. Estimation Approach

Suppose that we want to estimate the welfare change between two periods for a specific percentile of the household income distribution in a particular location. First, focusing on goods within a quasi-separable group for which reliable price data are available, we use nonparametric methods to estimate flexible relative Engel curves separately for both periods. We can then recover changes in income-specific price indices and welfare from the horizontal shift in these curves at different points of the income distribution, either by adjusting relative Engel curves to account for within-G price changes or by adding a first-order correction term. In either case, combining estimates for multiple goods within G increases precision by allowing us to accommodate measurement error in the expenditure surveys. We now discuss each step.

1. *Estimating Horizontal Shifts in Relative Engel Curves.* We first describe the procedure for estimating shifts in (price-unadjusted) relative Engel curves from the raw household expenditure survey microdata in a given location. As we detail a little later, these estimates are direct inputs into our first-order approach to implement price corrections and the exact approach also builds on this procedure.

The first step is to estimate kernel-weighted local polynomial regressions of relative expenditure shares, $\frac{x^t_{ih'}}{x^t_{Gh'}}$, on log total outlays per capita, $\log y^t_{h'}$, for every good $i \in G$ and each period t, where $h^{'}$ indexes the individual households in the expenditure surveys. This provides estimates of $\frac{x^t_{ih}}{x^t_{Gh}}$ for any percentile h of households across the income distribution (where y_h is the predicted income for households at this percentile). We estimate these relative Engel curves at 101 points corresponding to percentiles 0 to 100 of the local income distribution.[23] Following Lemma 1 and Proposition 1, we restrict attention to goods where

23. We first smooth the distribution of local income using a local polynomial regression of log total outlays per capita on outlays rank divided by the number of households n (with a bandwidth equal to $\frac{101}{n-1}$) to obtain $\log y^t_h$ at the 101 percentiles. To obtain relative Engel curves, we use a bandwidth equal to one quarter of the range of the income distribution in a given market. In both cases we use an Epanechnikov kernel. Our application explores alternative bandwidth choices.

relative Engel curves are monotonic (ensuring that estimates of shifts are unique for each good).[24]

Abstracting from within-G relative price changes for now, consider estimating the log price index change for income percentile h in period 1, $\log P^1(p^0, p^1, y_h^1)$. The relative Engel curve for period 1 provides a point estimate of relative expenditures for households at this percentile of the initial income distribution, $\frac{x_{ih}^1}{x_{Gh}^1}$. The next step is to estimate the period 0 income level $\widehat{E_{iG}^{0}}^{-1}(\frac{x_{ih}^1}{x_{Gh}^1})$ associated with this relative expenditure share from the crossing point on the period 0 relative Engel curve. To do so, we find the crossing point $\widehat{\frac{x_{ih}^0}{x_{Gh}^0}}$ and take the corresponding income of this income percentile h, $\widehat{\log y_h^0}$.[25]

Given these estimates, the income percentile–specific price index change $\log P^1(p^0, p^1, y_h^1)$ is equal to the difference between $\log y_h^1$ (the period 1 level of income for percentile h) and the estimate of $\widehat{\log y_h^0}$—this is the horizontal shift labeled $\log P^1$ in Figure I. The welfare change for income group h, as measured by the EV, is recovered from the relationship $\log(1 + \frac{EV_h}{y_h^0}) = \widehat{\log y_h^0} - \log y_h^0$, where $\widehat{\log y_h^0}$ is the estimate of the period 0 income required to obtain period 1 utility and $\log y_h^0$ is the actual period 0 log income for percentile h. This expression recovers welfare changes for a hypothetical household that stays at the same point of the income distribution in both periods. If household panel data are available, we could recover welfare changes for a specific household using this methodology. To estimate the price index change holding utility at period 0's level, $\log P^0(p^0, p^1, y_h^0)$, one applies the same procedure in the opposite direction (and

24. As nonparametrically estimated Engel curves are often noisy at the extreme tails where there are few households across large ranges of outlays, we restrict attention to goods where relative Engel curves in both periods are monotonic between percentiles 5 and 95 and drop relative expenditure share estimates beyond those percentiles in cases where those tail portions are nonmonotonic (replacing those values with a linear extrapolation from the monotonic portion of the curve). To reduce noise in our estimates at the tails of the distribution, we linearly extrapolate the top and bottom three percentiles of all curves.

25. We take the two closest percentiles and linearly interpolate between them to obtain $\widehat{\log y_h^0}$.

recovering CV from $\log(1-\frac{CV_h}{y_h^1}) = \widehat{\log y_h^1} - \log y_h^1)$. Each good $i \in G$ provides a separate estimate for $\log P^0$, $\log P^1$, CV_h, and EV_h.

2. *Averaging Estimates across Goods.* Measurement error in expenditure surveys will bias estimates calculated using shifts in the relative Engel curve of any one specific good i. Averaging across multiple goods $i \in G$ at each percentile of the income distribution reduces such bias. Denote i.i.d. measurement error in percentile h expenditures by ϵ_{ih}: $x^*_{ih}(p, y_h) = x_{ih}(p, y_h)\epsilon_{ih}$ with $\epsilon_{ih} > 0$. Taking a first-order approximation as in equation (5) and averaging horizontal shifts across $i \in G$, we obtain the bias generated by such measurement error:

$$\frac{1}{G}\sum_{i\in G}\left(\log\left(y_h^1\right) - \log\left(E_{iG}^0\right)^{-1}\left(\frac{x_{ih}^{*1}}{x_{Gh}^{*1}}\right)\right)$$

$$\approx \log\left(P^1\right) - \frac{1}{G}\sum_{i\in G}\left(\left(\beta_{ih}^0\right)^{-1}\left(\Delta\log\epsilon_{ih} - \Delta\frac{1}{G}\sum_{i\in G}E_{iG}\log\epsilon_{ih}\right)\right). \quad (6)$$

Thus, averaging horizontal shifts over a large number of goods provides unbiased estimates—that is, the second term on the right side goes to zero—because the (demeaned) i.i.d. measurement error is uncorrelated with the slopes of relative Engel curves.[26] The exposition abstracts from changes in relative prices within G as we discuss price corrections next, but a similar logic applies to measurement error in prices.

3. *Price Corrections.* Proposition 1 shows how to correct the price index estimates—derived solely from horizontal shifts in relative Engel curves—when relative prices are changing within group G. The first-order approach adds a price correction term composed of local elasticities and observable price changes to the average horizontal shift. The exact approach uses knowledge of the shape of function $H_{iG}(p_G, U)$ to adjust relative Engel curves before calculating horizontal shifts. We discuss the two procedures in turn.

i. *First-Order Price Correction.* Equation (5) provides an estimate of $\log P^1$ as a function of the horizontal shift in good

26. Ultimately, we will use the median as an unbiased estimate of the mean since not all goods $i \in G$ have overlapping relative Engel curves for a particular percentile (see Section III.B).

i's relative Engel curve and a first-order correction for vertical shifts in i expenditure due to relative price changes. Substituting $\log \frac{H_{iG}(p_G^0, U_h^1)}{H_{iG}(p_G^1, U_h^1)} \approx \sum_{j\in G} \sigma_{ijh}(\Delta \log p_j - \overline{\Delta \log p_G})$, averaging estimates across multiple goods $i \in G$ as above, and rearranging, we obtain:

$$\log\left(P^1\right) \approx \frac{1}{G}\sum_{i\in G}\left(\log\left(y_h^1\right) - \log\left(E_{iG}^0\right)^{-1}\left(\frac{x_{ih}^1}{x_{Gh}^1}\right)\right)$$

$$(7) \qquad -\frac{1}{G}\sum_{i\in G}\left(\left(\beta_{ih}^0\right)^{-1}\left(\sum_{j\in G}\sigma_{ijh}\left(\Delta \log p_j - \overline{\Delta \log p_G}\right)\right)\right).$$

The left side is the price index we are trying to estimate. The first term on the right is the average estimate of horizontal shifts in relative Engel curves across multiple goods $i \in G$. The final term captures the bias: the covariance between price-induced vertical shifts across $i \in G$ and the slopes of relative Engel curves at a given income level. Combining price changes for $i \in G$ as well as local income and price elasticities, β_{ih}^0 and σ_{ijh}, this covariance term corrects for such bias to the first order. If relative price changes are only weakly related to within-G income elasticities, the bias from averaging multiple estimates of price index and welfare changes will tend to cancel out and the size of the correction will be small.

To implement this correction, we estimate the slope-to-price-change correlation term. The price changes are observed, local income elasticities, β_{ih}^0, come from local slopes of the relative Engel curves estimated above and, in principle, the full set of local cross-price relative expenditure elasticities σ_{ijh} can be estimated using price variation within group G.

If we assume a constant elasticity of substitution σ_G within group G, the (percentile-specific) bias correction term takes the simple form:

$$(8) \qquad \frac{1}{G}\sum_{i\in G}\left(\beta_{ih}^0\right)^{-1}\sigma_G\left(\Delta \log p_i - \overline{\Delta \log p_G}\right).$$

As noted, the correction term is small if relative price changes are weakly correlated with slopes of relative Engel curves, but also if within-G elasticities are small or if within-G price changes are similar.

ii. *Exact Price Correction*. To provide an exact correction, recall from Proposition 1 that we must adjust one of the period's relative Engel curves to account for within-G relative price changes and then calculate horizontal shifts using this adjusted curve. Thus, we proceed as above, but modifying the appropriate relative Engel curve before calculating horizontal differences for each good $i \in G$ and then averaging.

First, we propose two practical specifications that only require estimating a single elasticity parameter. One is to specify a constant (compensated) elasticity of substitution between goods in group G, with an expenditure function that satisfies:[27]

$$e(p, U_h) = \tilde{e}\left(\left(\sum_{j\in G} A_j(U)p_j^{1-\sigma_G}\right)^{\frac{1}{1-\sigma_G}}, p_{NG}, U_h\right). \tag{9}$$

With such preferences, relative expenditures within G are given by $H_{iG}(p_G, U) = \frac{A_i(U)p_i^{1-\sigma_G}}{\sum_{j\in G} A_j(U)p_j^{1-\sigma_G}}$. This generalizes the preferences in Hanoch (1975) and Comin, Lashkari, and Mestieri (2021) by allowing for flexible substitution patterns outside of group G. The required adjustment due to confounding within-G relative price changes then takes the form:

$$\begin{aligned} &\log H_{iG}\left(p_G^1, U_h^1\right) - \log H_{iG}\left(p_G^0, U_h^1\right) \\ &= (1-\sigma_G)\left[\Delta \log p_i - \overline{\Delta \log p_G}\right] \end{aligned} \tag{10}$$

where $\overline{\Delta \log p_G} = \log[\sum_{j\in G}(\frac{p_j^1}{p_j^0})^{1-\sigma_G}(\frac{x_{jh}^1}{x_{Gh}^1})]^{\frac{1}{1-\sigma_G}}$ is a CES index of relative price changes. With an estimate of the elasticity of substitution σ_G between goods of group G (which can be estimated using prices and expenditures on goods $i \in G$), we have a simple-to-compute multiplicative adjustment term.

To account for richer patterns of substitution, we can increase the number of nests in this constant-elasticity structure to allow own- and cross-price elasticities to differ across subgroups of goods. Consider a partition of group $G = g_1 \cup g_2 \cup \ldots$ and a

27. The corresponding utility function can be implicitly defined as: $K(\sum_{j\in G} A_j(U)^{\frac{1}{\sigma_G}} q_j^{\frac{\sigma_G}{\sigma_G - 1}}, q_{NG}, U_h) = 1$.

within-group expenditure function:

(11)
$$e(p, U_h) = \tilde{e}\left(\left(\sum_g \left(\sum_{j\in g} A_j(U) p_j^{1-\sigma_g}\right)^{\frac{1-\eta_G}{1-\sigma_g}}\right)^{\frac{1}{1-\eta_G}}, p_{NG}, U_h\right).$$

Adjustments for within-G relative price changes are now given by:

$$\log H_{iG}(p_G^1, U_h^1) - \log H_{iG}(p_G^0, U_h^1)$$
$$= (1-\sigma_g)\left[\Delta \log p_i - \overline{\Delta \log p_g}\right]$$
(12)
$$+ (1-\eta_G)\left[\overline{\Delta \log p_g} - \overline{\Delta \log p_G}\right],$$

where $\overline{\Delta \log p_g} = \log[\sum_{j\in g}(\frac{p_j^1}{p_j^0})^{1-\sigma_g}(\frac{x_{jh}^1}{x_{gh}^1})]^{\frac{1}{1-\sigma_g}}$ is the price index change for subgroup $g \subset G$ and $\overline{\Delta \log p_G} = \log[\sum_{j\in G} e^{(1-\eta_G)\overline{\Delta \log p_g}}(\frac{x_{gh}^1}{x_{Gh}^1})]^{\frac{1}{1-\eta_G}}$ is the overall price index change for group G.[28]

Alternatively, recall from note 19 that the correction term in Proposition 1 can also be written in an additive form. Specifying that semi-elasticities ξ_G within group G are constant akin to EASI demands (Lewbel and Pendakur 2009) provides an additive adjustment expressed in levels rather than logs of expenditure that is again simple to compute:

(13) $$H_{iG}(p_G^1, U_h^1) - H_{iG}(p_G^0, U_h^1) = -\xi_G \times \left[\Delta \log p_i - \overline{\Delta \log p_G}\right].$$

For additional flexibility, these semi-elasticities ξ_i can also be good-specific (see Online Appendix A.6):

(14)
$$H_{iG}\left(p_G^1, U_h^1\right) - H_{iG}\left(p_G^0, U_h^1\right) = -\xi_i \times \left[\Delta \log p_i - \frac{\sum_{j\in G}\xi_j \Delta \log p_j}{\sum_{k\in G}\xi_k}\right].$$

28. This more flexible structure allows for heterogeneous own-price elasticities and a more complex cross-price substitution matrix. For each good i, the own-price elasticity for relative expenditures is an i-specific weighted average between the subgroup-specific parameter σ_g and the upper-tier elasticity η_G (as opposed to a single parameter σ_G in the one-layer case). In subgroup g, cross-price elasticities are also good-specific and differ from cross-price elasticities with goods in subgroups outside of g. See Online Appendix A.6 for a description of the full substitution matrix.

III.B. Identification and Validation

In this subsection, we derive a number of corollaries and tests related to unbiased identification when taking Proposition 1 to the data. Specifically, we derive an expression for the potential bias from violations of quasi-separability, construct several tests of the quasi-separability assumption, and discuss how to deal with other potential biases due to data aggregation, omitted variables in Engel curve estimation, sample selection issues, and taste heterogeneity.

1. *Quasi-Separability and Misspecification.*

i. *Bias from Violations of Quasi-Separability*. Although our main propositions assume that preferences are quasi-separable in group G, violations of this assumption only induce bias in our welfare estimates if they are systematically related to price elasticities and slopes of relative Engel curves. Here we make this statement precise by solving for the first-order bias.

Suppose we misclassfiy a good i that truly belongs in G as a non-G (NG) good (i.e., we omit a good that belongs within quasi-separable group G). Alternatively, suppose we falsely classify a NG good j as part of G. In both cases, price changes outside of what we believe to be the G group can directly affect within-G relative outlays (rather than only affect relative outlays through utility as would be true if goods were correctly classified into quasi-separable groups).

COROLLARY 1. To the first order, the bias from taking an average over estimates from all goods i that we believe to be in G (potentially including misclassified goods) is equal to:

$$\frac{1}{G}\sum_{i\in G}\log\left(E_{iG}^{0}\right)^{-1}\left(\frac{x_{ih}^{1}}{x_{Gh}^{1}}\right)-\log\left(\frac{y_{h}^{1}}{P^{1}}\right)\approx\frac{1}{G}\sum_{i\in G}\left(\beta_{ih}^{0}\right)^{-1}$$

$$\times\sum_{k\in NG}\left(\Delta\log p_{k}-\overline{\Delta\log p_{G}}\right)\left.\frac{\partial\log\left(\frac{x_{i}}{x_{G}}\right)}{\partial\log p_{k}}\right|_{U},\tag{15}$$

where k denotes the goods we believe to be in NG.

For correctly classified goods, $\frac{\partial \log(\frac{x_i}{x_G})}{\partial \log p_k} |_U = \frac{\partial \log H_{iG}}{\partial \log p_k} = 0$ and there is no bias.[29] If good $k^{'} \in NG$ is actually a G good, $\frac{\partial \log(\frac{x_i}{x_G})}{\partial \log p_{k'}} |_{U} \neq 0$ for some is. If good $i^{'} \in G$ is actually a NG good, $\frac{\partial \log(\frac{x_{i'}}{x_G})}{\partial \log p_k} |_{U} \neq 0$ for some ks.

Averaging across multiple i estimates, these violations of quasi-separability only generate bias if the direction and magnitude of the confounding (compensated) cross-price effects from unobserved NG price changes $(\sum_{k\in NG}(\Delta \log p_k - \overline{\Delta \log p_G}) \frac{\partial \log(x_i/x_G)}{\partial \log p_k} |_U)$ are systematically related to the slopes of relative Engel curves (β^0_{ih}) for the goods within G. In addition, the bias will be small if most goods are correctly classified, if price changes are similar for G and NG goods, or if compensated cross-price elasticities are small. In our application, this result motivates both averaging over multiple i estimates and exploring the sensitivity of our estimates to alternative classifications of goods into quasi-separable nests G.

ii. *Testing for Quasi-Separability with Outside Price Data.* We now present a direct test of quasi-separability that relies on the key property that expenditure shares within group G, $\frac{x_i}{x_G}$, can be expressed as a function $H_{iG}(U, p_G)$ of utility and relative prices within group G—that is, within-G expenditure shares do not depend on outside prices after conditioning on these variables. This property is a necessary and sufficient condition for quasi-separability:

COROLLARY QS1. Preferences are quasi-separable in group G if and only if compensated expenditure shares for good i within group G do not depend on outside prices p_j for any $j \notin G$:

$$\left.\frac{\partial \log\left(\frac{x_i}{x_G}\right)}{\partial \log p_j}\right|_U = 0.$$

This corollary is a direct consequence of Lemma 2 in Online Appendix A.3 (discussed in Section II.B).

29. Equation (15) abstracts from relative price changes within G (or assumes they all equal $\Delta\log p_G$) since, as we describe, these relative price changes can be observed and corrected for.

We can also derive a test based on uncompensated rather than compensated demand, providing an alternative characterization of quasi-separability if one cannot condition on utility. A necessary and sufficient condition for quasi-separability in G is that the uncompensated price effect of each good j outside G on the relative expenditure share of good i within G is given by:

$$\left.\frac{\partial \log\left(\frac{x_i}{x_G}\right)}{\partial \log p_j}\right|_y = -\frac{x_j}{y}\,\frac{\partial \log E_{iG}}{\partial \log y}, \tag{16}$$

where $\frac{\partial \log E_{iG}}{\partial \log y}$ is the slope of the relative Engel curve for good $i \in G$, and $\frac{x_j}{y}$ the overall expenditure share on good $j \notin G$. The proof (see Online Appendix A.6) relies on Roy's identity linking changes in utility to changes in income and prices.[30]

iii. *Testing for Quasi-Separability with Outside Expenditures Data*. The tests described above require price information for goods outside group G to test that preferences are quasi-separable in group G. We argue that reliable price data may not exist for large parts of consumption. Thus, we propose a further characterization that relies only on prices for goods within group G.

COROLLARY QS2. Preferences are quasi-separable in group G if and only if the elasticity of expenditures for any good $j \notin G$ with the price of good $i \in G$ is proportional to the share of good i in group G expenditures, that is,

$$\frac{\partial \log x_j}{\partial \log p_i} = \frac{x_i}{x_G} \times \gamma_G$$

for any $j \notin G$ and $i \in G$, where $\gamma_G = \sum_{k \in G} \frac{\partial \log x_j}{\partial \log p_k}$ is common across all goods i in group G.

In other words, this corollary states that the effect of prices of goods within G on expenditures for outside goods are fully captured by $\overline{\Delta \log p_G} = \sum_i (\frac{x_i}{x_G}) \Delta \log p_i$, the change in the relative expenditure share weighted log price change across goods in G (with

30. Given that these local slopes (for each i in a given period, market, and income level) require imprecise nonparametric estimation, we prefer to test the null of zero effects for outside-G price changes as in Corollary QS1.

coefficient γ_G).[31] The proof (see Online Appendix A.6) exploits Slutsky symmetry under rational preferences, which implies that the compensated price effects are symmetric, $\frac{\partial x_j}{\partial \log p_i}\mid_U = \frac{\partial x_i}{\partial \log p_j}\mid_U$, for any pair of goods i and j. This symmetry property allows us to rely on outside expenditures and prices within group G (instead of within-group expenditure and outside prices). As information on expenditures outside G is generally easier to obtain than prices, the data requirements for this test are more easily met.

2. *Aggregation across Varieties of a Good.* Researchers often estimate Engel curves for a broadly defined good (indexed here by g) that itself contains many varieties (the i's in our exposition up to now, e.g., different types, preparations, brands, sizes, or flavors), either because that is the level the data are collected at or because specific varieties are not consumed widely enough given the number of households sampled. Fortunately, Lemma 1 and Proposition 1 can also be applied to aggregates of varieties rather than individual varieties, even if demands for those varieties are nonhomothetic in g.

COROLLARY 2. Suppose that G in our exposition can be partitioned into subgroups of goods: $G = g1 \cup g2 \cup g3$ ... (e.g., salt, milk, lentils, etc.). Denote by $E_{g,G}$ the expenditure share on subgroup g within group G. Under the assumptions of Lemma 1:

$$E^1_{g,G}(y^1_h) = E^0_{g,G}\left(\frac{y^1_h}{P^1(y^1_h)}\right) \text{ and } E^0_{g,G}(y^0_h) = E^1_{g,G}\left(\frac{y^0_h}{P^0(y^0_h)}\right).$$

In other words, the key equivalence continues to hold if we treat the subgroups g as products (instead of the individual varieties i). Furthermore, under the assumption that prices across the i's in each subgroup g can be aggregated into price indices, $P_g(p_g, U)$, we can apply Proposition 1 and the price adjustment corollaries above to correct for relative price changes, but now

31. Thus, if and only if preferences are quasi-separable, the equality $\Delta \log x_j = \gamma_G \overline{\Delta \log p_G}$ must hold to the first order when prices of goods $i \in G$ change. Note also that this result holds for both uncompensated and compensated price effects (i.e., controlling for utility), since the difference between the two is proportional to the expenditure share of good i.

using subgroup price indices $P_g(p_g, U)$ instead of individual prices p_i.[32]

Several remarks are in order. First, note that these subgroup price indices can be nonhomothetic: relative consumption within subgroup g can vary with utility U (and thus income); the rich and poor can even consume distinct varieties. Second, aggregation can accommodate differences in shopping amenities and store-level price differences (modeled as store-specific varieties). Third, aggregation can accommodate new and disappearing varieties in subgroup g using existing methods. For example, if a popular new variety of salt appeared between periods 0 and 1, this would lower the salt price index $P_g(p_g, U)$. If g is in the NG group, then no correction is necessary, with the reduction in the salt price index raising utility, altering within-G expenditure shares, and lowering the full price index $P^1(y_h^1)$. If g is in the quasi-separable group G, we would either need to calculate the change in the salt price index (e.g., using the share of salt expenditure spent on the new variety and the within-salt elasticity of substitution as in Feenstra 1994) and correct for it using one of our price correction approaches or assume that the mismeasured or omitted relative price changes satisfy an orthogonality condition similar to expression (15). Finally, a more practical consideration that favors aggregation is that relative Engel curves for subgroup g may be strictly monotonic while consumption of specific varieties within g are zero (and thus relative Engel curves are flat) for some locations, periods, and/or ranges of income.

Taken together, these aggregation results are particularly valuable when attempting to estimate price indices and welfare from highly disaggregated data that are only available for some subset of consumption G—most prominently, barcode-level retail scanner data.

3. *Bias in Engel Curve Estimation.* Omitted variables can bias estimates of relative Engel curves just as they can traditional Engel curves—biased in the sense that the estimated curve does not provide a causal estimate of how consumption patterns change with exogenous changes in income. One source of such bias is if rich and poor households (along the x-axis) pay

32. For example, the price aggregates derived in Redding and Weinstein (2020) could be used for $P_g(p_g, U)$ if we assume that within-g preferences have their CES structure.

different prices for the same goods (with relative expenditures on the y-axis).[33] An important example in the Indian context is the Public Distribution System (PDS), which provides poor households with subsidized staples. However, even if the curves themselves are not causally identified, our method still uncovers unbiased estimates of price index and welfare changes, as long as the price vector faced by households is a function of real income.

To see this point, recall that the horizontal shifts in relative Engel curves recover the price index from the change in nominal income required to hold utility at either its initial (P^0) or final (P^1) level. Thus, even if price vectors differ with real income, we are correctly comparing the change in the price index holding utility fixed. Returning to the PDS example, eligibility criteria are indeed based on a utility metric rather than just nominal income—specifically, households below the poverty line are eligible, with the poverty line based on real needs. Therefore, when moving horizontally between period 0 and period 1 relative Engel curves, PDS eligibility does not change. A household initially at a utility level below (above) the PDS cutoff will be eligible (ineligible) in both periods at the utility level used to construct P^0. A similar logic applies to price differences emanating from variation in store or product availability, as long as store entry and stocking are functions of real income (which many models of retail would predict). A later section addresses the closely related topic of taste differences correlated with income.

As the foregoing discussion makes clear, our method does not in general require that estimates of relative Engel curves are causally identified. However, concerns remain if the relationship between real income and the price vector is not stable across the two periods (in which case horizontal shifts will not hold utility constant over time). These remaining concerns can be addressed either by controlling for the location or household characteristics at the root of the price differences when estimating relative Engel curves or by estimating curves separately for these different types of location or households (as we do in our application).

33. Related to unobserved price heterogeneity, choke prices may result in the rich and poor being exposed to different sets of price changes. Zero consumption is admissible with quasi-separability. However, relative Engel curves will be flat at low or high income levels, violating the strict monotonicity restriction we use for estimation. Thus, our methodology would discard such products from the set we average over to obtain the estimated price index change.

A second concern frequently discussed when estimating Engel curves is idiosyncratic (i.e., household-level) measurement error in expenditures. Because total outlays are simply the sum of expenditures, there will be correlated measurement error in the dependent variable (expenditure shares) and independent variable (total outlays per capita). Estimating relative Engel curves poses a similar problem, although potentially less severe since measurement error in expenditures outside of G (which does not appear in the denominator of relative shares) will simply attenuate the coefficient on total outlays per capita.[34] In either case, a similar logic applies to that discussed already, with causal identification of relative Engel curves not a necessary requirement for unbiased price index estimates. Specifically, if the distribution of measurement error is common across survey rounds (e.g., due to similar survey designs and implementation), the size of the horizontal shift remains unaffected as with the heterogeneous price example.

Finally, note that the discussion relates to using a single good's relative Engel curve for estimation. Since we average horizontal shifts across goods $i \in G$, as we describe in Section III.A, any i-specific bias in the estimation of relative outlays as a function of incomes would also have to be systematically related to the slopes of the relative Engel curves across the $i \in G$ in order to bias the welfare estimates.

4. *Unobserved Welfare Changes (Sample Selection).* Not all levels of household utility in period 0 are necessarily observed in period 1 and vice versa. For example, when evaluating price index changes P^0 for poor households in period 0, there may be no equally poor households in period 1 if there is real income growth (and similarly when evaluating P^1 for rich households in period 1). This means that Engel curves may not always overlap in budget share space for all income percentiles and gives rise to sample selection concerns, especially at the tails.

These selection issues take two forms: missing goods and missing markets. Recall from Section III.A that averaging multiple price index estimates (one for each good for which we can measure the horizontal shift in its relative Engel curve) can

34. A direct solution would be to instrument for total outlays per capita, with outside-G total outlays per capita, which would address any remaining concerns regarding correlated measurement error.

potentially eliminate bias from measurement error in expenditures or prices in the G group. However, in the presence of such shocks, averaging over the subset of goods for which there is overlap in relative Engel curves at a given percentile h generates potential biases because overlapping and nonoverlapping goods experienced different shocks. This is particularly problematic at the tails of the distribution. For example, suppose real income grew and there is no true overlap when estimating P^0 for the poorest households. Any overlapping goods we do observe must have experienced large vertical shocks to relative Engel curves such that the resulting price index estimate makes the poorest period 1 households appear to have real incomes similar to the poorest in period 0.

To address such sample selection concerns, we exploit the fact that we observe whether a particular good has no overlap at a particular income percentile and if so, whether the missing estimate is censored from above or below (which depends on the sign of the slope of the relative Engel curve). Combining this information with the assumption that the distribution of price index estimates across different goods in G is symmetric for a given income percentile allows us to consistently estimate the price index change.

To implement this correction, we order the observed (i.e., overlapping goods) and unobserved (i.e., nonoverlapping goods) price index estimates and take the median (which is an unbiased estimate of the mean).[35] In the rare cases where the median is unobserved due to most estimates being censored, we require a stronger assumption: that the distribution of price index estimates across different goods in G is uniform for a given income percentile. That allows us to solve for the mean as long as at least two goods overlap (see Sarhan 1955). As we will discuss, the symmetry assumption alone proves sufficient to solve selection issues in our Indian application.

35. We rank estimates, placing unobserved estimates below the lowest or above the highest observed estimate depending on whether they were censored from below (e.g., when calculating P^0 for poor households or P^1 for rich households) or above (e.g., when calculating P^0 for rich households and P^1 for poor households). For example, if a relative Engel curve for some good i is upward sloping and the period 0 relative budget share for a particular income percentile is lower than any point on the period 1 curve, there is no equivalently poor household in period 1. This implies that the missing estimate of the price index change for this percentile must be smaller than the lowest estimate obtained from other goods in G where we do observe overlap at this income percentile.

A different type of sample selection arises if, for a particular market, we don't observe any goods for which relative Engel curves overlap for a given percentile. In this case, we face a market-level sample selection issue when aggregating across markets. For example, if real incomes grew, there may be missing markets among poor percentiles for P^0 and rich percentiles for P^1. In practice, we find that almost no markets are missing after we implement the good-level selection correction (i.e., we observe overlap in strictly monotonic relative Engel curves for at least two goods for close to every decile-market pair in our sample). Therefore, the good-level selection correction is sufficient to solve market-level selection issues. Were it not, we could apply existing two-step Heckman selection corrections or make assumptions on the distribution of estimates across markets to recover the missing markets for a given percentile h.

5. *Taste Heterogeneity.* Finally, we consider four concerns related to taste differences.

i. *Statistical Demand.* While taste heterogeneity correlated with incomes or price changes poses challenges to estimation that we discuss next, even random preference heterogeneity across households requires additional assumptions when moving from the theory in Proposition 1 to estimation. The demand patterns that we estimate in the data (for a given level of household income) are "statistical" in the sense of Lewbel (2001), that is, a conditional expectation that may not be rational and quasi-separable even if (heterogeneous) individual preferences satisfy these conditions. Extending the approach of Lewbel (2001) to quasi-separability, we provide conditions such that these statistical demands satisfy both Slutsky conditions (i.e., are "integrable") and equation (16), which holds if and only if preferences are quasi-separable.

COROLLARY. Suppose that demand $x_i(y, p, z)$ for each individual indexed by z is both rational and quasi-separable in group G of goods. Statistical demand $X_i = E[x_i \mid y, p, x_G]$ is integrable and quasi-separable in G if and only if the conditional covariance matrix L between expenditure and income effects, that is, a matrix with elements $L_{ij} = Cov(x_j\,,\, \frac{\partial x_i}{\partial \log y} \mid y, p, x_G)$, is symmetric with zero off-diagonal blocks in $(i,\ j) \in G \times NG$ and $(i,\ j) \in NG \times G$ (sufficient and necessary conditions). If, in addition, matrix L is semi-definite negative, statistical demand is both rational and quasi-separable (sufficient conditions).

Symmetry and semi-definite negativity of matrix L are conditions already laid out in Lewbel (2001). The additional restrictions on the $G \times NG$ and $NG \times G$ blocks ensure quasi-separability. Online Appendix A.6 provides proofs and further discussion.

ii. *Taste Heterogeneity Correlated with Income.* We already discussed omitted variable bias in the estimation of relative Engel curves due to heterogenous price vectors in Section III.B. Similar issues arise if taste differences are correlated with income in the cross section. For example, households with different levels of education or family composition may value certain goods more and have different average incomes. As was the case for price heterogeneity by income, and with the same caveats, these relationships do not necessarily bias our estimates of horizontal shifts in relative Engel curves even if they confound attempts to estimate causal relationships between consumption patterns and income.

Specifically, if these taste differences are, directly or indirectly, functions of real income—for example, richer households may acquire more education or have more children, thereby changing their tastes—traveling horizontally between relative Engel curves at a given initial (or final) level of utility holds tastes constant. As before, any remaining bias can be addressed directly by controlling for household characteristics when estimating relative Engel curves or by estimating curves separately for different types of household (we pursue both in our application).

iii. *Taste Heterogeneity Correlated with Price Changes.* A different challenge arises if tastes differ in an income percentile and those taste differences correlate with relative price changes across goods. In this scenario, the price index and welfare changes for a given income percentile will differ by household type. More precisely, Online Appendix A.6 shows that our method will, to the first order, yield a weighted average of price index changes: $\widetilde{P}^1(y_h^1) \approx \sum_z w_z^1(y_h^1) P_z^1(y_h^1)$ with weights given by the relative

Engel slopes of household type z: $w_z \equiv \frac{\sum_i \frac{\beta^1_{i,z}}{\beta^1_i}}{\sum_{z'} \sum_i \frac{\beta^1_{i,z'}}{\beta^1_i}}$. If, instead, one wants to obtain the welfare change for a particular household type, such as households with large family sizes, we can carry out our procedure just for those households.

iv. *Taste Changes over Time.* The final set of issues arise when household tastes change over time. Such taste changes are problematic if they are systematically related to differences in slopes of relative Engel curves across goods. To be precise, we can derive an orthogonality condition analogous to the orthogonality condition for measurement error in expenditures in equation (6). Denoting taste shocks—shifts in within-G budget shares conditional on prices and income—by $\Delta \log \alpha_{ih}$ and abstracting from within-G relative price changes, we obtain the bias on $\log P^1$:[36]

$$\frac{1}{G} \sum_{i \in G} \left(\log \left(y^1_h \right) - \log \left(E^0_{iG} \right)^{-1} \left(\frac{x^1_{ih}}{x^1_{Gh}} \right) \right)$$
$$\approx \log \left(P^1 \right) - \frac{1}{G} \sum_{i \in G} \left(\left(\beta^0_{ih} \right)^{-1} \Delta \log \alpha_{ih} \right). \tag{17}$$

If taste shocks across i in subset G are orthogonal to the local slope of i's relative Engel curve in period 0 (or period 1 to identify P^0), the bias averages to zero across goods.

Unfortunately, such a condition is not generaly testable. To see this, note that knowledge of expenditure and price changes within G and the shape of within-G preferences—that is, the moments and parameters that allow estimation of taste shocks under separable but homothetic preferences—are insufficient in our context where preferences are nonhomothetic. In such settings, to estimate taste shocks we must also net out the changes in within-G relative expenditures due to changes in real income, which would require observing the full vector of price changes—data we argue are absent in most if not all empirical contexts.[37] One scenario that may violate this orthogonality condition is if

36. To ensure shares sum to unity within G, we assume that these taste shocks sum to zero. Such shocks can be defined, for example, in terms of price shifters as in Redding and Weinstein (2020).

37. It is precisely because these data are not typically available that we require our methodology that attributes changes in within-G demands (conditional

household types have different tastes and there are compositional changes over time (e.g., increases in education). We can (and do) address this concern explicitly by separately estimating and comparing price index changes for different household types.

IV. Application: Rural Indian Welfare 1987–2000

We apply our methodology to estimate changes in rural Indian welfare between 1987 and 2000. This exercise not only serves as a proof of concept but is also of independent interest given the large literature—and large amount of disagreement—regarding how real incomes changed over this period of major economic reforms, particularly for India's almost 250 million rural poor at or below the poverty line (a literature dubbed the "great Indian poverty debate" by Deaton and Kozel 2005).

IV.A. Data

Following Deaton and Kozel (2005), we draw on rural households in two of India's "thick" NSS survey rounds covering 1987/88 (43rd round) and 1999/2000 (55th round).[38] Each round provides detailed expenditure data on approximately 80,000 households residing in more than 400 Indian districts. Households are asked about their expenditures on 310 goods and services in each survey round. Examples include wheat, turmeric, washing soap, and diesel. The sum of all expenditures over 30 days provides our measure of total household outlays. Given limited saving in India, this will closely approximate nominal income (and even more closely permanent income). As noted previously, we use the word "outlays" interchangeably with "income". The surveys also contain basic household characteristics, district of residence, and survey weights that we use to make the sample nationally representative.

We use these data to estimate changes in household price indices and welfare for rural Indians between 1987 and 2000. We do this for nine income deciles (i.e., percentiles 10, 20, ..., 90) in each district. Given the need to nonparametrically estimate

on within-G prices) to changes in the price index. Taste shocks across goods in G are thus not separately identified when allowing for nonhomotheticity.

38. As we discuss later, focusing on rural areas also allows us to validate our estimates since well-measured food and fuel prices cover most of the consumption bundle for poor rural households.

relative Engel curves, we restrict attention to the 249 districts where we observe at least 100 households in both survey rounds. (As we show, results are not sensitive to this restriction.)

To obtain the subset of goods with reliable price data, we mimic the approach of Deaton and Tarozzi (2005), who carefully analyze NSS expenditure surveys to identify the subset of goods for which prices can be measured using unit values (i.e., expenditures divided by quantities) and the resulting prices are robust to concerns about unobserved product quality or variety.

Online Appendix B describes their procedure in detail, as well as data-cleaning procedures to remove obvious price outliers. Here, we briefly summarize their methodology to identify goods with reliable price information. First, they exclude all goods and services categories where quantity data are not recorded. Next, they exclude the clothing and footwear categories for which quantity data exist (e.g., two pairs of leather boots/shoes) but where product descriptions are too broad and styles too numerous to generate reliable unit values. The remaining goods are all food and fuel products. Third, they discard any goods where the variation in prices in localities suggests that these products likely contain multiple varieties or quality levels; either because there is strong evidence of multimodal price distributions (e.g., liquid petroleum gas) or because of the combination of high price dispersion and broad product descriptions (e.g., "other milk products"). Finally, they discard goods where changes in the unit of measurement over rounds make temporal comparisons impossible.

These restrictions leave us with a sample of 132 food and fuel goods for which we have unit values and where issues related to multiple quality levels are minimized. To alleviate the remaining concern of measurement error when using unit values, we again follow Deaton and Tarozzi (2005) and use the median unit value from each district and survey round (our market and period unit, respectively) as our price measures. We echo these authors in arguing that the combination of these procedures provides reliable price data for this subset of goods.

The final column of Table I lists these 132 goods that cover, on average, 75% of household consumption in our sample.[39] As

39. As the survey questionnaires change slightly over time, we aggregate a small number of goods to the most disaggregate classification reported consistently across rounds. In three cases we must combine purchases made at a dis-

TABLE I

PRODUCT GROUPINGS

3 G groups	8 G groups	34 g goods	Disaggregated NSS survey items included in the g goods
Raw food products	Cereals	Cereals—rice	Rice; chira; khoi, lawa; muri; other rice products
Raw food products	Cereals	Cereals—wheat	Wheat, atta, wheat/atta PDS; maida; suji, rawa; sewai (noodles); bread (bakery)
Raw food products	Cereals	Cereals—coarse	Jowar, jowar products; bajra, bajra products; maize, maize products; barley, barley products; small millets, small millets products; ragi, ragi products
Raw food products	Gram and pulses	Gram	Gram (full grain/whole); gram products
Raw food products	Gram and pulses	Pulses—besan, moong	Besan; moong; soyabean; other pulse products
Raw food products	Gram and pulses	Pulses—urd, masur	Urd; masur; arhar (tur); khasari; peas (dry); gram (split); other pulses
Raw food products	Meat, fish, and eggs	Meat	Goat meat, mutton; beef, buffalo meat; pork; poultry
Raw food products	Meat, fish, and eggs	Fish, prawn	Fish, prawn
Raw food products	Meat, fish, and eggs	Eggs	Eggs, egg products
Raw food products	Fruits and vegetables	Vegetable—root vegetables	Potato; arum; radish; carrot; turnip; beet; sweet potato; onion; other root vegetables
Raw food products	Fruits and vegetables	Vegetable—gourds	Pumpkin; gourd; bitter gourd; cucumber; parwal/patal; jhinga/torai; snake gourd; other gourds
Raw food products	Fruits and vegetables	Vegetable—leafy vegetables	Cauliflower; cabbage; brinjal; lady's finger; french beans, barbati; tomato; palak/other leafy vegetables
Raw food products	Fruits and vegetables	Vegetable—other vegetables	Peas (fresh); chilli (green); capsicum; plantain (green); jackfruit (green)
Raw food products	Fruits and vegetables	Premium Fruits	Apple; grapes; leechi; orange/mausani; pineapple; pears (naspati); mango; watermelon
Raw food products	Fruits and vegetables	Other fresh fruits	Banana; jackfruit; singara; papaya; kharbooza; berries
Raw food products	Fruits and vegetables	Dry fruits and nuts	Coconut (copra); groundnut; dates; cashew nut; walnut; raisin (kishmish, monacca, etc.)
Other food products	Dairy products and edible oils	Ghee	Ghee; butter
Other food products	Dairy products and edible oils	Milk	Milk (liquid)

TABLE I

(CONTINUED)

3 G groups	8 G groups	34 g goods	Disaggregated NSS survey items included in the g goods
Other food products	Dairy products and edible oils	Other milk products	Milk (condensed/powder); curd; baby food
Other food products	Dairy products and edible oils	Vanaspati, margarine	Vanaspati, margarine
Other food products	Dairy products and edible oils	Edible oils	Ground nut oil; mustard oil; coconut oil; other edible oils
Other food products	Sugar, salt, and spices	Sugar	Sugar; gur; sugar candy (misri); sugar (other sources); honey
Other food products	Sugar, salt, and spices	Salt	Salt
Other food products	Sugar, salt, and spices	Spices	Turmeric; black pepper; dry chillies; garlic; tamarind; ginger; curry powder
Other food products	Refreshments and intoxicants	Beverages	Tea (leaf); coffee (cups); coconut (green)
Other food products	Refreshments and intoxicants	Processed food	Cooked meals; pickles; sauce; jam, jelly
Other food products	Refreshments and intoxicants	Pan	Pan (finished); supari; lime; katha
Other food products	Refreshments and intoxicants	Tobacco	Bidi; cigarettes; leaf tobacco; snuff
Other food products	Refreshments and intoxicants	Intoxicants	Country liquor; beer; foreign liquor or refined liquor
Fuels	Fuels	Coke, coal, charcoal	Coke; coal; charcoal
Fuels	Fuels	Kerosene	Kerosene
Fuels	Fuels	Firewood and chips	Firewood and chips
Fuels	Fuels	Electricity	Electricity
Fuels	Fuels	Matches	Matches

Notes. This table details the classification of disaggregated NSS items (fourth column) into various levels of aggregation: the 34 g goods used in our baseline analysis (third column); the 8 groups that form the basis of the alternative G groupings we explore in Section IV.B (second column); and the 3 G groups each g good is assigned to in our baseline analysis (first column). Different disaggregated NSS items in the fourth column are separated by a semicolon. NSS items exclude those dropped by Deaton (2003b) (see Online Appendix B). Some NSS items were not consistently classified over rounds. Specifically: (Concorded) Rice uses individual items from R43 {Rice; Paddy} and R55 {Rice; Rice PDS}. (Concorded) Wheat uses R43 {Wheat, Atta} and R55 {Wheat, Atta PDS; Wheat, Atta other sources}. (Concorded) Jowar and Jowar products uses R43 {Jowar; Jowar products} and R55 {Jowar, Jowar products}. (Concorded) Bajra and Bajra products uses R43 {Bajra; Bajra products} and R55 {Bajra, Bajra products}. (Concorded) Maize and Maize products uses R43 {Maize; Maize products} and R55 {Maize, Maize products}. (Concorded) Barley and Barley products uses R43 {Barley; Barley products} and R55 {Barley, Barley products}. (Concorded) Small millets and Small millets products uses R43 {Small millets; Small millets products} and R55 {Small millets, Small millets products}. (Concorded) Ragi and Ragi products uses R43 {Ragi; Ragi products} and R55 {Ragi, Ragi products}. (Concorded) Beef, buffalo meat uses R43 {Beef; Buffalo meat} and R55 {Beef, Buffalo meat}. (Concorded) Goat, mutton uses R43 {Goat; Mutton} and R55 {Goat, Mutton}. (Concorded) Fish, Prawn uses R43 {Fish fresh; Fish dry} and R55 {Fish, prawn}. (Concorded) Eggs, Egg products uses R43 {Eggs; Egg products} and R55 {Eggs}. Vegetable—Gourds includes R43 {Papaya (green)} and R55 {Other gourds}. Vegetable—leafy vegetables includes R43 {Palak; Other leafy vegetables} and R55 {Palak, other leafy vegetables}. (Concorded) Vanaspati, margarine uses R43 {Vanaspati; Margarine} and R55 {Vanaspati, margarine}. (Concorded) Edible oils includes R43 {Linseed oil, Palm oil, Refined oil, Gingelly (til) oil, Rapeseed oil} and R55 {Edible oils (other)}. (Concorded) Sugar uses R43 {Sugar (crystal)} and R55 {Sugar PDS; sugar (other sources)}. (Concorded) Salt uses R43 {Sea salt; other salt} and R55 {Salt}.

we emphasize throughout the article, this subset of goods with reliable price data is crucial for our estimation because it allows us to implement Proposition 1 and compute exact or first-order price corrections, as well as test for and assess potential bias from violations of quasi-separability.

Finally, we note that in the 55th round, the surveys included a 7-day recall period for all food products (in addition to the standard 30-day recall period asked in the 43rd round). Although we only use the responses to the 30-day recall questions, Deaton (2003a, 2003b) and others show that households inflated their 30-day reports to be consistent with their 7-day ones. Thus, this "recall bias" raises reported total outlays (the numerator for evaluating changes in real incomes) even using the 30-day recall data and is at the center of the great Indian poverty debate. In Section IV.C, we show that our approach is robust to this recall bias as relative consumption patterns within product groupings are unaffected by the additional 7-day recall question.

IV.B. Product Aggregation and Product Groups

To reduce measurement error when estimating relative Engel curves for rarely consumed items, we aggregate these 132 food and fuel items with well-measured prices to the second-lowest level of aggregation in the NSS surveys. This aggregation yields 34 goods indexed by g (listed in the third column of Table I). The results in Section III.B prove that such an aggregation is admissible and that we can implement price corrections, as long as we can measure price indices $P_g(p_g, U)$ for these 34 goods. We use a Stone price index to construct such indices (Online Appendix Table C.2 reports descriptive statistics for these price changes).[40] This aggregation dramatically reduces the share of empty product-by-period-by-market cells (from 50% to less than 15% as shown

count through India's PDS (available to households below the poverty line) and those bought at regular markets. Online Appendix B discusses these aggregations and Section III.B explains how our methodology accommodates price vectors that vary with real income.

40. Specifically, we aggregate the observed log price changes for the 132 items i to 34 goods g using survey-weighted mean initial expenditure shares across the $i \in g$ as weights. We compute price changes for each i from changes in district median unit values as described in Online Appendix B. When unit values are observed in the district for one but not the other period, we replace i's missing price change with the state-level change.

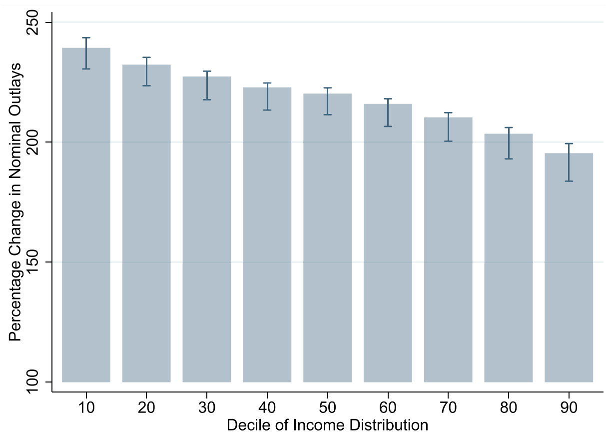

FIGURE II

Rural Indian Growth in Nominal Income 1987/88–1999/2000

The figure shows the percentage change in rural total outlays per capita between 1987/88 and 1999/2000 for each decile of the local per capita outlay distribution (averaged across districts using population weights). Bootstrapped confidence intervals are based on sampling with replacement 1,000 times from the distribution of households in each district-survey round and plotting the 2.5% and 97.5% envelope of nominal income estimates at each decile. See Section IV.C for further discussion.

in Online Appendix Figure C.1), and moving to the next highest level of aggregation (8 goods) provides little additional benefit.[41]

We divide these 34 aggregate g goods into three broader consumption groups shown in the first column of Table I: raw foodstuffs (e.g., rice, leafy vegetables), other food products (e.g., milk, edible oils), and fuels (e.g., firewood, kerosene). In our baseline estimation, we assume these three groups each form a quasi-separable G group, with all remaining goods (e.g., processed food, manufactures, and services) excluded as part of the NG group. We combine estimates from goods in all three G groups by taking medians following the discussion in Section III.B.[42]

41. Online Appendix Figure C.4 reports qualitatively similar inflation estimates using these alternate levels of aggregation.

42. In principle, comparing estimates obtained from different G groups provides an overidentification test (price index estimates from different G groups should be identical if there is no misclassification of goods into quasi-separable groups and orthogonality conditions on measurement error and taste shocks are satisfied). However, given the limited number of products in our setting (recall we have about 11 goods in each of the three G groups and for a given market-decile

As we describe shortly, Figure VI explores robustness across 108 perturbations of sensible G groupings, including a single G group.

IV.C. Changes in Indian Price Indices and Welfare over Time

Before describing the results of our approach and comparing them to estimates derived from existing Indian CPI statistics, we summarize the changes in nominal income between 1987 and 2000. Figure II plots growth rates in total household outlays per capita for each decile of the local income distribution (using population-weighted averages of log changes across all 249 rural districts).[43] Nominal income growth exceeded 200% and there is a clear and strong pattern of convergence over this 13-year period, with outlays per capita rising substantially faster for the poor than for the rich. Our nonhomothetic price indices allow us to determine whether this nominal income convergence translated into convergence in standards of living.

Figure III presents our price index estimates using the methodology outlined in Section III.A (from hereon the AFFG Price Index after the authors' initials).[44] The left panel presents our estimates absent any within-G price corrections—that is, simply utilizing the horizontal shifts in relative Engel curves for goods in our three G groups. As before, we plot population-weighted averages across districts by decile. The remaining panels of Figure III apply the two variants of Proposition 1 described in Section III.A that draw on the well-measured price changes we have for goods in our food and fuels G groups to account for potentially confounding within-G relative price changes. The middle panel displays the first-order price correction where we assume a common elasticity of substitution of $\sigma_G = 0.7$ based on averages from Cornelsen et al.'s (2015) systematic review of food price elasticities in low-income countries that uses similar levels of aggregation to our 34 goods. The right panel plots the

not all goods have both strictly monotonic and overlapping relative Engel curves), these conditions are unlikely to be satisfied without pooling the estimates.

43. For each decile, we report percentage changes for incomes, price indices, and welfare calculated by exponentiating the population-weighted mean of district-level log changes between 1987 and 2000.

44. As an example of the horizontal shifts in relative Engel curves we use to obtain our price index estimates, Online Appendix Figure C.2 plots relative Engel curves in 1987/88 and 1999/2000 for one g good, salt, as a share of the G group "other food products" for the largest districts in the north, east, south, and west of India.

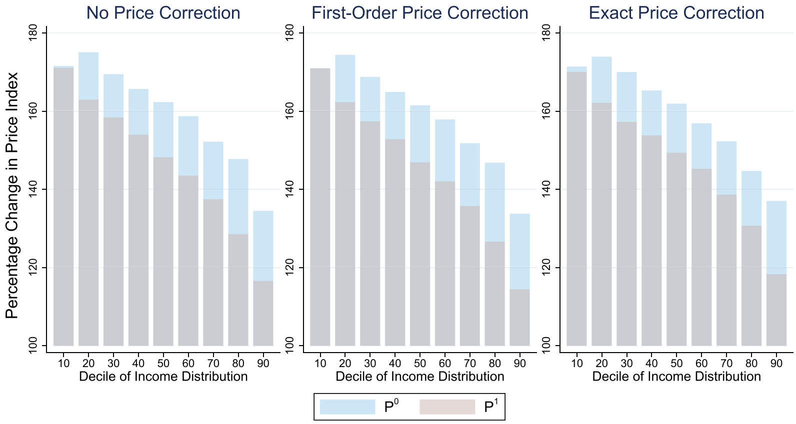

FIGURE III

Rural Indian Cost of Living Inflation 1987/88–1999/2000: AFFG Price Index with No Price Correction, First-Order Price Correction, and Exact Price Correction

The figure shows the percentage change in the rural AFFG price index between 1987/88 and 1999/2000 for each decile of the local per capita outlay distribution (averaged across districts using population weights). Panels show estimates both with and without corrections to account for relative price changes within G groups. The left panel reports the uncorrected price index change. The middle panel applies the first-order price correction and the right panel applies the exact correction, both described in Proposition 1 and Section III.A, using $\sigma_G = 0.7$. See Section IV.C for further discussion.

exact price correction using the isoelastic correction (nonhomothetic CES) in equation (10) with the same elasticity assumption.

The first thing to notice is that the estimated inflation rates across deciles change very little after adjusting for relative price changes within G groups using either the first-order or exact approach. This is not simply the result of assuming a single elasticity that limits patterns of cross-price substitution. Online Appendix Figure C.3 presents exact corrections using the more flexible multinest nonhomothetic CES demands in equation (11), calibrated using two different sets of price elasticities from the literature.[45] Under all three parameterizations,

45. Specifically, rather than a single elasticity governing own- and cross-price elasticities, we now have nine parameters governing these elasticities, as described in Section III.A. We use Kumar et al.’s (2011) demand system estimates for food items in India that are calculated using NSS data and a meta-analysis of price elasticity estimates for various commodities by Fally and Sayre (2018). The resulting parameter values are presented in Online Appendix Table C.3.

estimates are almost identical to the uncorrected price index for all income deciles. Recall from equation (8) that to the first order, our estimates are unbiased if within-G price changes are uncorrelated with slopes of relative Engel curves. Thus, the fact the estimates change little with our price corrections implies that relative price changes in our three food and fuel G groups are either small or only weakly related to income elasticities in our context. To streamline the exposition given these results, we focus our remaining analysis on the no price correction approach (labeled AFFG NPC price index). In all cases, we draw similar conclusions using the first-order or exact price correction estimates.

Before discussing magnitudes and differences in inflation across deciles of the income distribution, it is instructive to plot our AFFG approach alongside the leading existing CPI estimates for rural India. The left panel of Figure IV repeats our AFFG NPC price index. The middle panel plots Paasche and Laspeyres price index estimates using the methodology of Deaton (2003b) that draws on observed price changes weighted by average district-level expenditure shares for the 132 food and fuels items where price data are deemed reliable.[46] Mechanically, these price indices do not vary across the income distribution. The right panel of Figure IV relaxes this homotheticity by using district-decile-specific expenditure shares when calculating Paasche and Laspeyres price indices. We obtain bootstrapped confidence intervals for all three indices by sampling with replacement 1,000 times from the distribution of households in each district-survey round and plotting the 2.5% and 97.5% envelope of price index estimates at each decile (bootstrapping over the entire procedure in the case of the AFFG price index, including the nonparametric estimation of relative Engel curves).

Two main findings emerge. First, our AFFG approach generates broadly similar estimates of Indian consumer price inflation among low-income deciles compared to existing CPI estimates that are based on changes in observed food and fuel prices. Specifically, all three approaches predict price rises of between 160% and 180% for the poorest deciles. Since food and fuels

46. As before, price changes are computed from changes in district median unit values for the 132 items. We calculate Laspeyres and Paasche price indices using survey-weighted mean expenditure shares at the district level (thus the index is democratic not plutocratic). We replace missing district-level price changes with state-level ones.

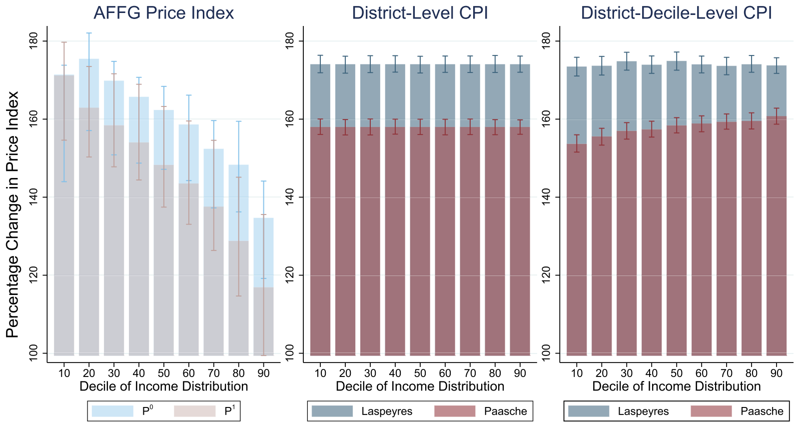

FIGURE IV

Rural Indian Cost of Living Inflation 1987/88–1999/2000: Comparison to Existing CPI Estimates

The figure shows the percentage change in the rural price index between 1987/88 and 1999/2000 for each decile of the local per capita outlay distribution (averaged across districts using population weights). The left panel plots our AFFG NPC price index changes estimated from horizontal shifts in relative Engel curves. The middle panel plots price index changes using Laspeyres and Paasche district-level CPIs calculated using price changes for food and fuels following Deaton (2003b). The right panel repeats the middle panel but uses district-income-decile-specific budget shares to calculate the Laspeyres and Paasche indices. Bootstrapped confidence intervals are based on sampling with replacement 1,000 times from the distribution of households in each district-survey round and plotting the 2.5% and 97.5% envelope of price index estimates at each decile. See Section IV.C for further discussion.

represent a sizable fraction of rural household consumption for poor households in India (more than 80% for the poorest decile, averaging across both survey rounds), this finding is reassuring—particularly because we are comparing a standard price index that explicitly uses observed price changes to our approach that exploits very different variation coming from horizontal shifts in relative Engel curves.

Second, we estimate that cost of living inflation has been substantially higher for poor households compared with the rich, the opposite of what one would infer from the food and fuel Paasche and Laspeyres indices, which are slightly pro poor. Figure V combines the estimated changes in nominal incomes and price indices to obtain welfare changes (EV and CV in our approach, and real income for the standard CPI approach). The income-specific

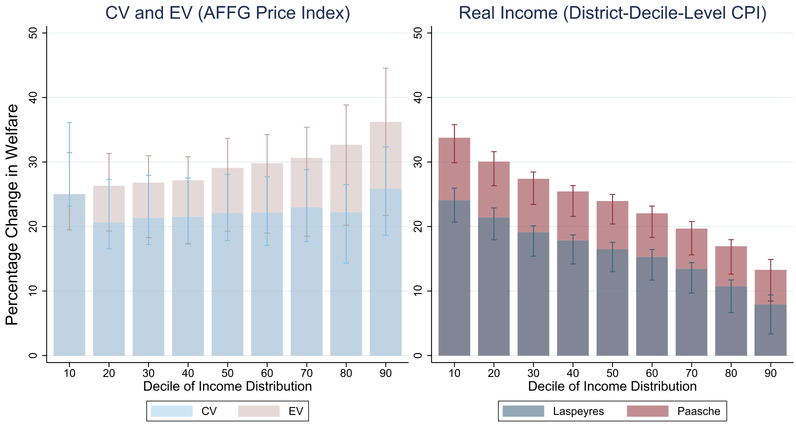

FIGURE V

Rural Indian Welfare Growth 1987/88–1999/2000

The figure shows the percentage change in rural welfare between 1987/88 and 1999/2000 for each decile of the local per capita outlay distribution (averaged across districts using population weights). The left panel plots the percentage change in both equivalent and compensating variation estimated from outlay changes and horizontal shifts in relative Engel curves (the AFFG NPC price index). The right panel plots the percentage change in real income calculated by deflating per capita outlay changes by Laspeyres and Paasche price index changes (using price changes for food and fuels and district-income-decile-specific budget shares). Bootstrapped confidence intervals are based on sampling with replacement 1,000 times from the distribution of households in each district-survey round and plotting the 2.5% and 97.5% envelope of price index estimates at each decile. See Section IV.C for further discussion.

inflation rates estimated using the AFFG approach eliminate any convergence in welfare between the rich and poor over this period. In fact, if anything, welfare grew more for rich households. This finding contrasts starkly with the changes in real income calculated using food and fuel Paasche and Lespeyres indices which slightly magnify the already substantial convergence seen in nominal incomes. This result also stands in contrast to Almås and Kjelsrud (2017), who estimate nonhomothetic price indices using a quadratic AIDS demand system that does not impose quasi-separability but requires knowledge of price changes for the full consumption basket, including manufactures and services.[47] They find that inflation was pro poor over 1993–2005.

47. See note 9 for a description of how Almås and Kjelsrud (2017) use India's nonfood CPI to navigate the lack of well-measured price data for categories beyond food and fuels.

Why are our price index estimates lower for richer households? The most likely explanation is that high-income households disproportionately benefited from price drops, new varieties, and quality increases in consumption categories where price measurement is challenging. In particular, the rich spent a large and increasing share of their budget on durables such as manufactures and on services. These are exactly the categories for which unobserved quality differences make price data unreliable and are omitted in Deaton's CPI approach, which only covers well-measured food and fuels, and are crudely captured, if at all, by the government nonfood CPI in the Almås and Kjelsrud (2017) approach. Lower inflation in these specific categories is consistent with the fact that the Indian trade reforms were centered on manufacturing intermediates, which substantially raised the quality and variety of Indian manufactures (Goldberg et al. 2010); and that there was a dramatic increase in share of services in GDP over the reform period (Mukherjee 2015). Online Appendix D discusses this explanation further and contains four pieces of corroborating evidence: that expenditure shares were greater for the rich in these categories, that government-measured inflation in these categories was lower, that relative Engel curves are steepest in these categories, and that these categories saw the most new product entry based on the Indian Prowess microdata used by Goldberg et al. (2010) and others.

Beyond accounting for inflation in hard-to-measure categories, our methodology is also immune to the concerns that lie at the center of the great Indian poverty debate. Recall that the 1999–2000 NSS added a 7-day recall period for food expenditures, which inflated answers to the consistently-asked 30-day recall questions. The most influential solution, that of Deaton (2003a), adjusts food expenditure using the mapping between food and fuels expenditure (for which no additional recall period was added) from earlier rounds. That solution requires that relative price of food and fuels did not change. In contrast, our welfare estimates are robust to the additional recall period as long as it did not change relative consumption shares in a given food or fuel group G. This condition is testable using the thin NSS round 54 (1998) where, to test proposed changes to the surveys, households were randomly assigned to different recall periods. Consistent with our claim, Online Appendix Table C.1 shows that the choice of recall period did not affect relative consumption shares within our G

groups.[48] Thus, our finding of no convergence in real incomes has the potential to inform and revise the conclusions of the great Indian poverty debate summarized in Deaton and Kozel (2005).

IV.D. Validation Results

We perform a number of validation exercises that follow from our corollaries in Section III.B, as well as reporting several additional context-specific robustness checks.

1. *Quasi-Separability and Misclassification.* We first investigate bias from potential violations of quasi-separability due to misclassifying products into G groups. To this end, we re-estimate our price indices for each decile and market across 108 sensible splits of our g goods into plausibly quasi-separable groupings G.[49] Figure VI presents the estimation results for each decile, plotting our baseline point estimate on top of the mean and the 2.5th–97.5th percentile range of point estimates from the 108 sensible G groupings. Reassuringly, our baseline is close to the mean for every decile of the income distribution. The 2.5th–97.5th percentile ranges are also reasonably tight—suggesting that the conditions under which misclassification bias is small (equation (15)) are met in our setting.

Next we present our two preferred tests of quasi-separability from Section III, one using proxies for price changes outside of G (Corollary QS1) and one using outside-G expenditures (Corollary QS2). In the first test, for each good $i \in G$, we regress log changes of within-G relative expenditure shares on within-G log price changes and controls for changes in household welfare: our estimates of CV and EV and total outlays per capita. We further include log changes of outside-G price indices and test whether

48. In addition, Online Appendix Figure C.5 shows similar patterns of pro rich inflation between the 1987/88 and 1994/95 survey rounds, when the questionnaire was unchanged.

49. As shown in Table I, the 34 g products fall into three high-level groups (raw food, other food, and fuel) and 8 subgroups in those. To discipline plausibly quasi-separable nests G, we impose that a g can only be bundled together with other g's in the same high-level group. In addition, different gs in one of the eight subgroups cannot be grouped into more than one G (as they are likely closely related). With these restrictions, we generate 105 possible ways of allocating gs into G groups based on tuples: that is, $(2^4 - 1) \times (2^3 - 1) \times 1 = 105$. Finally, we add: only one G group across all 34 products, two G groups (food and fuel), and eight G groups (one for each subgroup above).

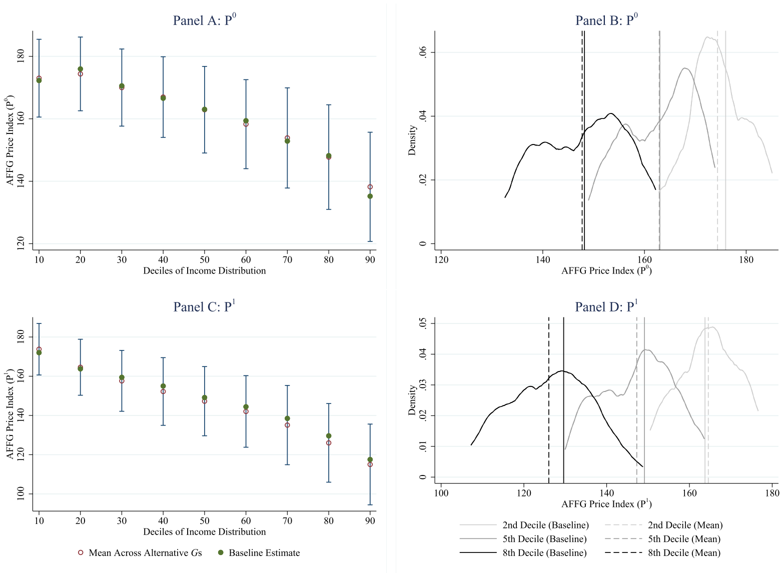

FIGURE VI

AFFG Price Index Changes across Alternative G Groupings

The figure reports AFFG NPC price index changes by decile of the local per capita outlay distribution for each of 108 alternative classifications of goods into plausibly quasi-separable groupings G. Our baseline classification of three quasi-separable groups is one of the 108 classifications, and we indicate our baseline estimates in all panels. The two left panels depict for each decile the mean and the 2.5% and 97.5% envelope of point estimates across the 108 alternative groupings (Panel A for $P^0$and Panel B for P^1). The two right panels depict the distribution of these estimates for the 2nd, 5th, and 8th deciles of the local per capita outlay distribution (Panel B for $P^0$and Panel D for P^1). See Section IV.D for further discussion.

they affect within-G relative expenditures (they should not if preferences are quasi-separable in group G). For outside-G price information, we make use of available (but imperfect) state-level price indices computed for rural households by the Indian Labor Bureau for two categories: (i) clothing, bedding, and footwear, and (ii) miscellaneous (which includes all services and durable manufactures).

This specification generates $I - 1$ regression equations for our three G groups (31 in total). To obtain the correct error distribution, we first randomly draw the two outside-G price changes 500 times and perform a joint test that these (fictitious) outside-G price changes have a zero coefficient in every equation. We compute the test statistic for joint significance of changes

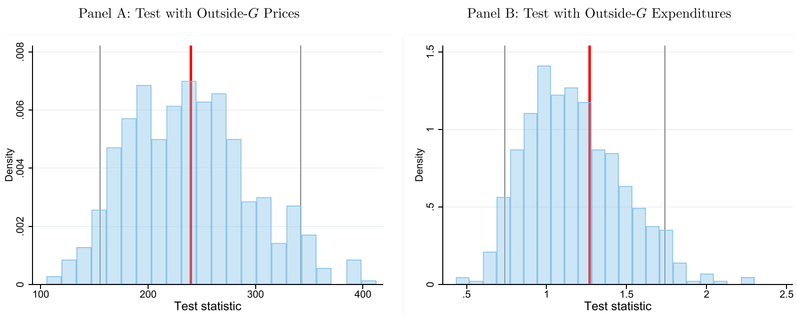

FIGURE VII

Quasi-Separability Tests

The figure reports two tests of quasi-separability described in Section III.B. The test in Panel A uses price changes outside of G (Corollary QS1), and the test in Panel B relies on outside-G expenditures instead (Corollary QS2). Vertical thin gray lines show 95% confidence intervals. Vertical heavy red lines show χ^2 (Panel A) and F (Panel B) statistics obtained from sample data, respectively. Blue bars plot the empirical distribution of QS test statistics obtained from 500 independently drawn random price data sets from a normal distribution with mean and variance identical to that of the distribution of price variation in the data. Permutation test p-values are .44 and .33 for the tests in Panels A and B, respectively. The Panel A test statistic is obtained from regressing the change in the log relative expenditure share of good $i \in G$ on within-G log price changes, changes in utility ($\log(1 + \frac{EV_h}{y_h})$, and $\log(1 - \frac{CV_h}{y_h})$, that is, the horizontal distances illustrated in Figure I, and log expenditures per capita) as well as two proxies for outside-G price changes from India's state-level CPI: (i) clothing, bedding, and footwear, and (ii) miscellaneous goods. The χ^2 statistic is obtained for the joint test that the coefficients on both outside goods prices in each of the 31 regressions are zero. The Panel B test statistic is obtained from regressing the change of log expenditures on all outside-G goods on a within-G Stone price index (relative expenditure share weighted log price changes for goods in G), changes in within-G log prices, and changes in household welfare as above. The F-statistic is obtained from the joint test that coefficients on all $i \in G$ within-G log prices are equal to zero. In both panels we cluster standard errors at the market level, use survey weights, and perform the test at the median decile of households in each market.

in the actual outside-G price indices and compare it to the χ^2 test distribution from our random draws.[50] Figure VII, Panel A overlays the value of the test statistic (the thick red line) on top of the statistic's empirical distribution calculated above, with further details of the test provided in the table notes. Reassuringly,

50. For both tests, we cluster standard errors at the market level, use survey weights, and perform the test at the median decile of households using cross-market variation in price and expenditure changes across survey rounds.

we cannot reject the null that preferences are quasi-separable between goods in and outside our G groups, with a p-value of .44.

A natural limitation of the previous test is that it requires data on prices for non-G categories such as manufactures and services, yet our methodology is motivated by the difficulty in reliably measuring prices for these sectors. Our second test (Corollary QS2) does not require such price information. Instead, we flip the previous test and ask whether changes in outside-G log expenditures (the sum of outlays spent on outside-G goods) respond to within-G log price changes once we control for relative expenditure share weighted changes in within-G log prices and the same controls for changes in utility as before (they should not if preferences are quasi-separable in group G). As before, we obtain the statistic's empirical distribution by randomly drawing the 34 within-G log price changes 500 times and testing for their joint significance (conditional on the within-G price index and utility controls). Figure VII, Panel B marks the value of the F-test statistic with a thick red line laid on top of the distribution of the test statistic. As with our first test, we cannot reject the null that preferences are quasi-separable between goods in and outside our G groups, with a p-value of .33.

2. Sample Selection Issues. As described in Section III, our baseline estimates address sample selection issues due to nonoverlapping relative Engel curves by ranking both missing and non-missing estimates and taking the median under the assumption of a uniform distribution of estimates across $g \in G$. Online Appendix Figures C.6–C.8 illustrate and assess these sample selection issues. The left panel of Online Appendix Figure C.6 presents price index estimates that do not correct for non-overlap issues and simply average over non-missing goods. As anticipated, the biggest discrepancies with our baseline (the right panel) occur for P^0 among the poorest deciles and P^1 among the richest deciles. These are the households where we would expect no true overlap in a growing economy.[51]

The middle panel of Online Appendix Figure C.6 implements only the first step of our selection correction, applying symmetry but not uniformity. This step alone eliminates almost all the

51. Online Appendix Figure C.7 illustrates this fact by showing the frequency of nonoverlapping estimates by decile, broken out by type of non-overlap (censored from above or below) that we use to rank missing estimates.

discrepancy between P^0 and P^1 due to sample selection issues and generates very similar estimates to our uniformity baseline (right panel). However, by only imposing symmetry, we lose any market-decile pairs for which the median ranked good has no overlap. As shown in Online Appendix Figure C.8, a substantial number of pairs are missing when only imposing symmetry (particularly for P^0 since the distribution of log total outlays per capita is right skewed). However, we obtain estimates for essentially all market-deciles once uniformity is imposed, so market-level selection issues do not arise under our baseline specification.

3. Taste Heterogeneity and Taste Changes. We investigate concerns that our estimates may be affected by taste heterogeneity across households or taste changes over time (see Section III.B). Online Appendix Figure C.9 recalculates price indices using nonparametric relative Engel curves that condition on a set of linear controls for household characteristics.[52] Reassuringly, results change little, suggesting that systematic bias in estimates of cross-sectional Engel curves is unlikely to be driving our findings.

Online Appendix Figure C.10 corroborates this finding by presenting separate price index estimates for different types of rural households; small versus large households, high versus low education, young versus old, and literate versus illiterate (with the last three comparisons based on characteristics of the household head). Recall from Section III.B that these exercises are informative on a number of fronts. First, by estimating Engel curves separately across demographic groups, we limit potential bias in estimates of cross-sectional Engel curves. Second, we can explore to what extent different types of household experienced different inflation rates, both on average and by income decile, as a result of taste heterogeneity. Third, we can address concerns that the composition of household types may have changed over time, biasing our estimates if taste heterogeneity across types is systematically related to slopes of relative Engel curves (e.g.,

52. In particular, for each good and market (pooling across both periods) we estimate coefficients on the following controls: a scheduled caste dummy, a literacy of household head dummy, log of household size, and the share of children in the household. We then use relative Engel curves for each good-period-market evaluated at the controls' market-level median (i.e., holding demographic characteristics fixed across periods).

if average education or household size changed over time and educated or large households have different tastes). The fact that the price index estimates show very similar patterns for different household types provides reassurance that taste heterogeneity and taste changes (at least those due to compositional changes) are not driving our findings.

4. Additional Robustness Checks. We report several additional robustness checks. Online Appendix Figure C.11 presents results for alternative bandwidth choices when nonparametrically estimating relative Engel curves and for alternative strategies to deal with noise at the tails. Online Appendix Figure C.12 reports results without restricting attention to markets with at least 100 household observations in both survey rounds. Reassuringly, results are qualitatively similar to our baseline estimates in both cases. Online Appendix E assesses our methodology via a Monte Carlo simulation. We generate a fictitious second-period data set with the same number of households and statistical error in relative Engel curves as in our actual sample but fixing inflation to be a step function that declines by income decile. Simulating our methodology over 250 error draws, we find that the truth lies in the 95th percentile envelope of estimates for all deciles, although the addition of measurement error slightly attenuates the slope of the mean estimates with respect to income.

5. Application to India's 1991 Trade Reforms. Online Appendix F uses our methodology to revisit the effect of India's 1991 trade reforms on the welfare of rural households in India. Closely following Topalova (2010)—but replacing her outcomes (district-level rural poverty rates and per capita outlays) with our welfare estimates—we find that the adverse effects of import competition on local labor markets are borne by households across the income distribution, including by rural households in the richest income deciles.

V. Conclusion

Measuring changes in household welfare and the distribution of those changes is challenging and typically requires the researcher to observe the full vector of quality- and variety-adjusted price changes—an incredibly difficult task for categories such as manufacturing and services. In this article, we propose and implement a new approach that only requires reliable price information for some quasi-separable subset of products G. Horizontal shifts in relative Engel curves in this group—adjusted for within-G price changes—reveal changes in household price indices and welfare across the income distribution.

We apply this new method to measure changes in household welfare in rural India. We find that consumer price inflation was substantially higher for poor households than rich, essentially eliminating the convergence seen in nominal incomes. This finding is missed by standard price indices using the subset of consumption where prices are well measured.

Beyond providing a deeper understanding of India's economic reforms, we believe our methodology is widely applicable in the many settings where expenditure survey data are available or can be easily collected. Given the increasing availability of survey microdata over time and across space, and the growing interest in distributional analysis, the usefulness of such an approach is only likely to grow.

Supplementary Material

An Online Appendix for this article can be found at *The Quarterly Journal of Economics* online.

Data Availability

The data underlying this article are available in the Harvard Dataverse, https://doi.org/10.7910/DVN/MSQE6U (Atkin et al. 2023).

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