Flexibility Versus Agency - A Model of Stage Financing *

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Abstract

I explore the optimal timing of financing for venture capital-backed companies. Raising a larger percentage of the total needed funds upfront presents operational efficiencies, lowering the cost of the project, but increases the scale of agency problems and sacrifices the option value of delay. The optimal financing path depends upon the relative magnitudes of efficiency gains from upfront financing and the value of waiting. Furthermore, I consider the effects of entrepreneurial optimism and find that the effect on staging depends on the precision of intermediate information.

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1. Introduction

The venture capital industry is characterized by a high risk of failure and a small possibility of extreme returns (Cochrane, 2005). Successful venture capital (VC) backed companies such as Apple, Google, Microsoft, and Facebook have grown to become some of the largest in the world. These are the outliers; many or most VC-backed companies fail or achieve only meager returns. The venture capital industry has developed several practices to cope with this risk including stage financing, convertible preferred stock, and the taking of board seats. This paper focuses specifically on stage financing. Investors do not generally provide all the capital required to complete a project upfront, but rather provide a small amount of financing initially in the expectation that further financing will be secured in the future, often from a new set of financiers. Despite the pervasiveness and importance of stage financing, the associated tradeoffs are not fully understood.

I model an entrepreneur who has an idea but requires outside financing to launch a viable business. She seeks financing from competitive and risk-neutral investors and has a choice over the timing of fundraising. The core tradeoff is that upfront financing creates operational efficiencies, but worsens agency problems and sacrifices option value. Operational efficiencies from upfront financing can arise in several ways. Raising more money initially provides a sense of legitimacy to the business, which can ease contracting with uninformed counterparties and lower costs. This is likely most relevant when hiring employees, where having more money in the bank can signal positive information to potential employees, allowing the firm to pay lower wages to employees in exchange for job security. In addition, contracts with suppliers and customers directly affect the cost of doing business and are facilitated more easily when the company is in a more certain financial position. Furthermore, as seeking financing is a time-consuming process, there are benefits to having more than the minimal needed capital on hand to quickly respond to shocks and changing market conditions.

On the other hand, raising more initial financing worsens agency problems. In particular, it is very costly to force the entrepreneur to return capital to investors, even when intermediate information indicates that abandonment is optimal. Indeed, when a valuable payoff is unlikely, the entrepreneur may choose to waste the remaining capital in a way that provides her utility, but offers no return to investors. The more money the entrepreneur raises, the more she can potentially

waste. This cost can be largely avoided by waiting until after intermediate information is revealed to provide the largest share of the capital. As the firm matures through additional rounds of venture financing, it is assumed to become more professionalized (Hellmann and Puri, 2002) and hence more transparent, thus reducing the potential for misuse of cash. Thus, the agency problem creates a real option associated with delaying fundraising.

The entrepreneur chooses a financing strategy to maximize her ex-ante expected payoff, weighing the costs and benefits discussed above. Her utility consists of two parts. If she chooses to not waste the money she has raised, she completes the project and receives the residual value after investors are paid. If she instead chooses to shirk and waste the capital, she receives utility equal to a fraction of the wasted capital. It is assumed that the entrepreneur holds all the bargaining power. Hence, when choosing a financing strategy, she internalizes the effect her choice has on the likelihood of each course of action and chooses the strategy that maximizes the expected sum of the two components.

The model provides several key results. When interim information is uninformative, the optimal financing strategy is to finance heavily upfront, as the real option associated with delay has little value. When information is very informative, the opposite is true, and financing occurs primarily at a later stage. In either case, the entrepreneur raises more initial financing than the amount that maximizes the project's NPV. Furthermore, the extent to which the entrepreneur can gain utility by misusing capital affects the optimal fundraising strategy in an intuitive way: the more the entrepreneur gains by misusing capital the less financing is raised initially.

A very natural extension to the core tradeoff concerns differences of beliefs between the entrepreneur and investors. When the entrepreneur is more optimistic about the prospects of the project than investors, she faces an additional tradeoff when making staging decisions. Because entrepreneurs waste capital when information indicates and insufficiently attractive return, optimists, who overestimate the expected return, entrepreneurs are less likely to misuse capital than their rational counterparts. This reduces a risk faced by investors. Thus, optimists can raise financing at better terms than rational entrepreneurs, encouraging early financing. However, optimists expect intermediate information to be positive on average. Hence, they expect to secure better terms when financing in the future, introducing a motive to delay. These forces interact with the core tradeoff in an interesting way. A low level of signal informativeness creates an environment

where optimists finance more heavily upfront than rational entrepreneurs, while a high level of signal informativeness causes optimists to delay financing.

This work relates to and extends several strands of literature. It is inspired by Hart and Moore (1994). As in their seminal work, an entrepreneur's human capital is inalienable—she cannot commit to fully provide effort in the future. This is a particularly important concern in the realm of high-growth entrepreneurship, where entrepreneurs are often of central importance, especially at an early stage (Bernstein et al., 2017). Nevertheless, this paper differs from Hart and Moore (1994) in important ways. Indeed, while that paper focuses on a setting of certainty, entrepreneurial finance is an area of extreme uncertainty. Hence, while they focus primarily on issues arising from the distribution of the payoff and the inability to commit over time, my focus is rather on the inability to commit to good behavior over the dimension of uncertainty—the entrepreneur may shirk when intermediate information is bad. As in many corporate finance settings, there is a cost generated when a manager (in this case entrepreneur) has access to cash (Jensen and Meckling, 1976), as the manager cannot be trusted to use available cash efficiently, thus this paper relates to the voluminous literature on free-cash flow problems. This research also draws on insights from the literature on financial contracting. Hart and Moore (1998) study debt as an instrument to incentivize an entrepreneur to pay out cash rather than divert, similar to the agency problem considered here. Nevertheless, the assumptions are different than those here and hence the problem considered is distinct from that considered here. Specifically, the diversion of cash flows in their paper is that the entrepreneur may steal cash flows generated from the business, while I consider the entrepreneur misusing the funds raised for investment. This is a distinct concern that must be considered on its own. Additionally, Aghion and Bolton (1992) model incomplete contracts between an investor and an entrepreneur. However, they are most concerned with security design and consider financing that occurs entirely upfront.

I add to the literature on the staging of venture capital investments. Previous literature has explored the staging decision largely from the perspective of the investor. Sahlman (1990) argues that the investors choose to stage investment in order to create option-like returns, but doesn't explicitly model the associated tradeoffs. Dahiya and Ray (2012) argue that staging allows investors to efficiently abandon low-performing projects, but don't consider an agency problem. Hence they study the optimal pattern of investment where the only concern is uncertainty about project

quality, but no specific incentives are required to induce any agents to take the preferred actions. Other papers explore how staging can limit misbehavior on the part of either the investor (Fluck et al., 2005) or the entrepreneur (Neher, 1999). Fluck et al. (2005) consider several tradeoffs simultaneously in a computational approach. They find significant value losses due to agency and information costs, but does not consider how investment should be spaced over time, instead they model these requirements as fixed parameters. Neher (1999) takes a different approach and considers the possibility of holdup by a valuable entrepreneur. In this setting the value of staged investment arises as the value of the idea becomes more tangible, and hence pledgable, over time, reducing the ability of the entrepreneur to hold up the existing investor. He finds that an increase in the tangibility of a project decreases the number of rounds of financing required and increases the likelihood that a project can be financed at all. Additionally, Admati and Pfleiderer (1994) explore the possibility of a hold-up on the part of the investor. Specifically when the time of refinancing occurs, the existing investor can hold up the entrepreneur by threating to not invest in the follow-on round, sending a negative signal to uninformed outside investors. This problem is alleviated by having an insider who maintains his stake in each round. Empirically Gompers (1995) examines staging and finds that the number of rounds increases with the scale of potential agency problems. This is consistent with the theory of Neher (1999), though the result is framed in the context of the real option to abandon rather than the hold-up problem Neher considers. Crucially, no prior paper, to the knowledge of the author, considers stage financing separately from staged investment. I study specifically stage financing, leaving the real investment fixed,[1] a departure from the prior literature which assumes that financing and investment are rigidly linked. While there is much to be learned from this approach, separating financing and investment allows me to study a novel tradeoff which arises very naturally, yet has not previously been considered.

The paper proceeds as follows. Section 2 introduces the model, which is then solved in Section 3. Differences of beliefs between the entrepreneur and investors are introduced and relevant results derived in Section 4. Section 5 concludes.

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[1]At least initially, initial financing decisions can have consequences for future real investment.

2. Model

The central tradeoff of the model is efficiency from upfront financing versus an inability to recover invested capital, even when the project is bad, due to an agency problem. Financing heavily upfront produces efficiency but increases the amount of money wasted in poor states of the world. The model is structured to give this central tradeoff preeminence.

The model begins with a wealthless entrepreneur who requires financing to complete a project. The gross payoff, $\tilde{R}$, is uncertain and takes two periods to reach fruition. Physical investment of $\underline{k}$ is required in the initial period, hence, the entrepreneur must finance at least $\underline{k}$ in period 0. If $\underline{k}$ is raised initially, the total capital required to complete the project is K. If more than $\underline{k}$ is raised, additional efficiencies are realized, reducing the total capital required to fund the project. Specifically, if k_0 is raised initially, the capital needed to complete the project is reduced by $f(k_0 - \underline{k})$, where f is increasing and concave, with $f(0) = 0$, resulting in a total capital requirement of $K - f(k_0 - \underline{k})$. This captures the aforementioned efficiency gains from upfront financing. Financing is secured selling equity. Specifically, k_0 is raised initially by selling equity stake γ_0. To complete the project, the remaining capital required, $K - f(k_0 - \underline{k}) - k_0$, is raised in the intermediate period from a new investor in exchange for equity of γ_1. The financing market is competitive, so financiers break even in expectation and all surplus goes to the entrepreneur.

The entrepreneur cannot commit to not waste the money provided for the business. She can choose to shirk and waste capital either immediately, or in the intermediate period before, and instead of, raising follow-on financing. The amount of utility that she can gain by wasting capital is uncertain. If she wastes capital equal to x, she obtains utility $\tilde{\theta}x$, where $\tilde{\theta}$ is revealed in the intermediate period. Her choice to continue or shirk is based purely on self-interest—she chooses the path that maximizes her expected utility. Her immediate choice of action can be forecasted with certainty by investors and represents a constraint on financing. The entrepreneur cannot in equilibrium raise a level of financing that will lead her to immediately waste it. Her choice in the intermediate period cannot be forecasted with certainty (except in edge cases) as it depends on the information realization of random variables. Her intermediate decision is based on two pieces of information revealed in the second period: a signal, $\tilde{S}$, about the quality of the project, and the amount of utility, $\tilde{\theta}$, she can derive from each dollar of remaining capital should she choose to

waste it. If the expected payoff of the project conditional on the signal is insufficiently attractive, the entrepreneur will choose to cease exerting effort on the project and instead misuse the capital to derive utility of $\tilde{\theta}(k_0 - \underline{k})$. If the entrepreneur instead chooses to waste capital immediately, she obtains $E[\tilde{\theta}]k_0$. I assume that the effort of the entrepreneur is essential to the success of the project. Thus, her shirking assures investors receive no payoff.

In the final period, the payoff of the project is realized and split between investors and the entrepreneur. The sequence of events is described in Figure 1.
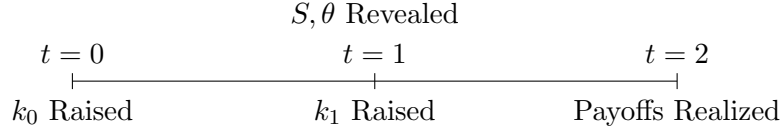

Fig. 1. Model Timeline

The entrepreneur chooses an initial level of financing, k_0^*, to maximize her ex-ante expected utility. As it is uncertain if the project will be completed, this can be written

$$E\left[\mathbb{1}_{\text{Complete}}(1 - \gamma_0)(1 - \tilde{\gamma}_1)\tilde{R} + (1 - \mathbb{1}_{\text{Complete}})\tilde{\theta}(k_0 - \underline{k})\right], \tag{1}$$

where $\mathbb{1}_{\text{Complete}}$ is an indicator function for the entrepreneur completing the project.

2.1. Discussion

This setup captures several key features of the venture capital market. There is uncertainty about the quality of the project, but potentially valuable information is revealed over time. Furthermore, investment is stage but financing and physical investment are not inseparably connected; money must be secured before it can be invested, but investment need not follow immediately. There is a choice of how much to raise today versus how much to leave for tomorrow. Most previous models of stage financing assume that there are distinct stages that have separate financing requirements. While simplifying the modeling, this approach ignores the entrepreneurs choice of financing strategy, which is given priority here.

A second realistic feature of this model is the central importance of the entrepreneur. In particular, the entrepreneur is essential to the success of the project, which pays off zero without

her human capital. Furthermore, the entrepreneur cannot commit to the project long term. If the information revealed about the project is not favorable, she may choose to stop exerting the requisite effort for the project to succeed. Instead, the entrepreneur remains with the firm and uses the capital in a way that provides her utility of $\theta \leq 1$ per dollar of remaining capital, but provides no payoff for investors and is socially wasteful. This is a reasonable characterization of the "death spiral" endured by VC-backed firms in their last days before failure. While investors would like to prevent this behavior, it is extremely costly to do so, leaving little recourse. I assume that when the entrepreneur shirks, she is able to obtain some level of utility from the remaining capital, for instance by spending it on lavish amenities, pursuing pet projects with little hope of success, or simply by delaying the inevitable end. While the assumption that investors receive nothing in these cases is somewhat stark, the results of the model would be qualitatively unchanged if instead investors recovered only a small amount when the entrepreneur misbehaves.

An important assumption is that the entrepreneur can commit to not waste capital after raising follow-on financing. That is, conditional on raising follow-on financing in the intermediate period, the entrepreneur completes the project and the final payoff is realized. This assumption is motivated by the findings of Hellmann and Puri (2002) that firms professionalize as they take VC funding. Hence, the assumption is that by raising second-round funding, the firm becomes transparent enough for the entrepreneur to commit to not wasting capital. In many ways, the continuation decision is similar to the canonical free cash flow problem, as too much cash on hand leads the manager, in this case, entrepreneur, to misuse it, either by inefficiently investing it, or using it for personal enrichment. However, the setting of entrepreneurship adds an element. In particular, entrepreneurship, as modeled here, has two components that are not always included in general models of free cash flow theory. These two are interconnected. First, financing is stage, with further rounds of financing expected in the near future, and related to the current project. Second, there is a lack of intermediate cash flows to pay the investor. Hence, the free cash flow problem does not arise as a result of cash flows from operations, but potentially as an ex-ante foreseeable problem arising from initial financing decisions. These setting-specific features make this problem particularly interesting.

Despite the many realistic features of the model, there are limitations and simplifications, as there are with all models. There are two important simplifications in the current model. The first

is the limitation to common stock. While convenient, this is a simplification of the securities used in reality, which are usually convertible preferred shares (Kaplan and Strömberg, 2003). However, the main intuition of the model is unchanged when more realistic securities are used, and the main results remain. However, the model becomes less tractable. The second important simplification is limiting the model to two periods, whereas in reality, projects continue for many stages and there is no certain end-date. This is not captured in the model, however, restricting to two periods allows the main features of the model to be presented with greater clarity. Extending the model to multiple periods is left to future research.

3.   Solution

The model is solved backward. In the final period there are no decisions to be made, so I begin in the intermediate period. After the signal, $\tilde{S}$ is revealed, all parties immediately and correctly update their prior distribution of the payoff $\tilde{R}$ with the new information using Bayes' rule. Simultaneously, the utility the entrepreneur can gain from each dollar of remaining capital, $\tilde{\theta}^2$, is revealed.

The entrepreneur's behavior in the intermediate period can be determined for any potential (S, θ) realization. If the entrepreneur initially raised k_0, the total amount needed to fund the project is $K - f(k_0 - \underline{k})$. Hence, $K - f(k_0 - \underline{k}) - k_0$ remains to be raised if the project is to be completed. As the funding market is competitive, the best price that the entrepreneur will be able to secure for her equity is one that ensures the investor breaks-even in expectation. Thus, the investor will require an equity stake, γ_1, of

$$\gamma_1 = \frac{K - f(k_0 - \underline{k}) - k_0}{E[\tilde{R}|S]}.$$

The expected payoff to the rational entrepreneur if the project is completed is then

$$(1 - \gamma_0)(1 - \gamma_1)E[\tilde{R}|S] = (1 - \gamma_0)\left[E[\tilde{R}|S] + k_0 + f(k_0 - \underline{k}) - K\right].$$

Consider the decision to continue the project or divert the remaining capital. There are two potential reasons a project may not receive second-period financing. The first is that after infor-

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[2]For any random variable, I use a tilde to denote the unknown value while no tilde denotes a realization.

mation revelation, financing is impossible to obtain. This is the case whenever the expected payoff of the firm is less than the remaining capital to be raised, that is

$$E[\tilde{R}|S] < K - f(k_0 - \underline{k}) - k_0. \tag{2}$$

Note that, whenever $k_0 > \underline{k}$, this decision does not correspond perfectly with the efficient decision rule which requires that a project is continued if and only if it is positive NPV to do so. The difference arises because, at the time of the continuation decision, only $\underline{k}$ has been spent, the remaining $k_0 - \underline{k}$ is in the possession of the firm. Hence, the money that remains to be invested is $K - f(k_0 - \underline{k}) - \underline{k}$. It is positive NPV to continue only when

$$E[\tilde{R}|S] > K - f(k_0 - \underline{k}) - \underline{k}. \tag{3}$$

The entrepreneur can raise financing for negative NPV continuation because she need not raise the entire amount that must be invested, only a portion of it. This is analogous to the well-studied free cash flow problem in corporate finance. As will become clear, the model will admit scenarios where a project is continued when it is negative NPV to do so, and scenarios where positive NPV continuations are not taken. The latter occurs because the entrepreneur can choose to misuse the capital on hand. This is the second reason for abandonment and occurs when it is possible to obtain financing, but it is not in the best interest of the entrepreneur to do so; she would rather shirk and benefit from the remaining capital. As the entrepreneur can commit to providing effort after raising follow-on financing, the entrepreneur will raise follow-on financing whenever

$$(1 - \gamma_0)\left[E[\tilde{R}|S] + k_0 + f(k_0 - \underline{k}) - K\right] < \theta(k_0 - \underline{k}). \tag{4}$$

While there are two potential reasons the entrepreneur could discontinue the project, only one will affect decision making, that is, only Condition 4 will enter into the decision making of either the entrepreneur or the investor, as shown in Claim 1.

Claim 1. *Whenever Condition 2 holds, Condition 4 holds as well. Thus, the shirking decision can be fully characterized by Condition 4.*

This is intuitive. The forced abandonment constraint holds when the expected surplus of the project, net of the needed investment, is negative. The voluntary abandonment constraint binds when entrepreneur's expected payout, a fraction of the total surplus net of needed investment, is smaller than the entrepreneur can gain from shirking. Hence, the latter is a more stringent condition and subsumes the former.

From Condition 4 it can be seen that the equity stake sold to the initial investor, γ_0, is key in determining the entrepreneur's behavior. When determining what equity stake to seek in exchange for the initial investment, the investor anticipates the future behavior of the entrepreneur. Because the financing environment is competitive, the equilibrium financing offer will allow the investor to break even in expectation. Let $\mathbb{C}_{(\gamma_0, k_0)}$ represent the set of S, θ pairs for which the entrepreneur will choose to raise k_1, conditional on γ_0 and k_0. This is the set for which

$$(1 - \gamma_0)\left[E[\tilde{R}|S] + k_0 + f(k_0 - \underline{k}) - K\right] < \theta(k_0 - \underline{k}). \tag{5}$$

Then, conditional on the entrepreneur requesting funding of k_0 initially, the investor will demand γ_0 such that

$$\gamma_0 \int \int \mathbb{1}_{\mathbb{C}_{(\gamma_0, k_0)}}\left(E[\tilde{R}|S] + k_0 + f(k_0 - \underline{k}) - K\right)dF(s)dG(\theta) = k_0, \tag{6}$$

where F and G represent the cumulative distribution functions of $\tilde{S}$ and $\tilde{\theta}$ respectively. This can be rewritten using the law of iterated expectations as

$$\gamma_0 P(\text{Continue}|k_0, \gamma_0)\left(E[R|\text{Continue}] + f(k_0 - \underline{k}) + k_0 - K\right) = k_0. \tag{7}$$

This representation makes clear the tradeoffs involved. For a given k_0, increasing γ_0 increases the investors percentage of the final cash flows, net of follow-on investment, but also increases the probability that the entrepreneur will choose to shirk in the intermediate period. Hence, in some cases, a solution to Equation 6 will not exist, even for a project that is positive NPV in the absence of an agency problem. In these cases it is impossible for the entrepreneur to secure financing of k_0.

If a solution to Equation 6 exists then then investors will be willing to provide k_0, receiving equity ownership γ_0 in return. If multiple solutions exist, the relevant solution is the infimum over

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the set of solutions. Over the set of values for which a solution exists, this defines a function $\gamma_0(k_0)$, providing the equity stake required for each level of capital raised.

Now consider the general optimization problem for the entrepreneur. She seeks to maximize her personal wealth. This can be written

$$\int \int (1 - \gamma_0(k_0)) \mathbb{1}_{\mathbb{C}_{(\gamma_0(k_0), k_0)}} \left(E[\tilde{R}|S] + k_0 + f(k_0 - \underline{k}) - K \right) + \left(1 - \mathbb{1}_{\mathbb{C}_{(\gamma_0(k_0), k_0)}} \right) \tilde{\theta}(k_0 - \underline{k}) dF(s) dG(\theta). \tag{8}$$

The first term is her share of firm value in the case that the firm is financed to completion, while the second is the amount gained by shirking and misusing remaining capital in the intermediate period. The entrepreneur chooses to raise k_0^*, the argument maximizing Expression 8.

3.1. A Functional Form

To clarify the exposition, I make some functional form assumptions. First, assume that the payoff, $\tilde{R}$, is log-normally distributed, as in Cochrane (2005). Specifically, let $\tilde{R} = e^{\tilde{X}}$, where $\tilde{X} \sim N(\mu, \sigma_x^2)$. Furthermore, in the intermediate period, the signal concerning project quality, $\tilde{S}$, is the realization of $\tilde{X}$, X, plus noise. That is, $\tilde{S} = X + \tilde{\epsilon}$, where $\tilde{\epsilon} \sim N(0, \sigma_\epsilon^2)$. Defining $\tau_x = \frac{1}{\sigma_x^2}$ and $\tau_\epsilon = \frac{1}{\sigma_\epsilon^2}$ allows Bayesian updating in the intermediate period to be accomplished in a simple way. Then

$$E\left[\tilde{X}|S\right] = \frac{\tau_x \mu + \tau_\epsilon S}{\tau_x + \tau_\epsilon} \tag{9}$$

$$Var\left(\tilde{X}|S\right) = \frac{1}{\tau_x + \tau_\epsilon} \tag{10}$$

$$E\left[\tilde{R}|S\right] = \exp\left[\frac{\tau_x \mu + \tau_\epsilon S + 1/2}{\tau_x + \tau_\epsilon}\right]. \tag{11}$$

The utility per dollar of available capital when shirking $\tilde{\theta}$ is also stochastic, and is uniformly distributed between $\underline{\theta}$ and $\bar{\theta}$. Finally, I choose a simple functional form for the efficiency of upfront financing, with $f(k_0 - \underline{k}) = a\sqrt{k_0 - \underline{k}}$. This form ensures the entrepreneur will not raise the minimal initial financing, as $f'(0) = \infty$, furthermore it is convenient and concave, as desired.

With these functional forms, given k_0 and g_1, the continuation region can be calculated explicitly.

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Recall that the entrepreneur will continue whenever

$$(1 - \gamma_0) \left[E[\tilde{R}|S] + k_0 + a\sqrt{k_0 - \underline{k}} - K \right] > \theta(k_0 - \underline{k})$$

With the assumed functional forms, this can be reduced, by algebraic manipulation, to two equivalent inequalities, each useful its own way. First, condition on a realization of $\tilde{\theta}$ and see that the project will be continued whenever the signal, $\tilde{S}$ is larger than a cutoff, C_θ, where

$$C_\theta = \frac{\sigma_x^2 + \sigma_\epsilon^2}{\sigma_x^2} \log \left(\frac{\theta(k_0 - \underline{k})}{1 - \gamma_0} + K - k_0 - a\sqrt{k_0 - \underline{k}} \right) - \frac{\sigma_\epsilon^2}{\sigma_x^2} \mu - \frac{\sigma_\epsilon^2}{2}. \tag{12}$$

Alternatively, conditioning on S, it becomes apparent that the entrepreneur will continue whenever $\tilde{\theta}$ is smaller than a cutoff M_s defined as

$$M_s = (1 - \gamma_0) \frac{1}{k_0 - \underline{k}} \left[e^{\frac{\sigma_x^2}{\sigma_x^2 + \sigma_\epsilon^2} \left(s + \frac{\sigma_\epsilon^2}{\sigma_x^2} \mu + \frac{\sigma_\epsilon^2}{2} \right)} + k_0 + a\sqrt{k_0 - \underline{k}} - K \right]. \tag{13}$$

The form of these cutoffs are intuitive. If the signal is very high, the entrepreneur is likely to continue, if the amount she can divert is very high, she is likely to abandon.

Given these functional forms, the model can be solved. First, I establish the benchmark, shown in Proposition 2.

Proposition 2. *In the absence of an agency problem, the optimal financing strategy, chosen by the entrepreneur, is to finance the project entirely upfront or not at all.*

The intuition here is straightforward. There is no cost to upfront financing in the absence of an agency problem, as capital will be returned to investors if intermediate information indicates that this is optimal. Thus, the entrepreneur should maximize the benefits upfront financing. Note that this does not mean raising K upfront, but rather k^U such that $k^U + f(k^U) = K$; because of the benefits of upfront financing less than K needs to be raised in total.

Now, the model, in general, is not solvable in closed form, nevertheless, much can be learned by considering various special cases of the model, which illustrate the key tradeoffs. To begin this exploration of the model, first shut down the variability of θ and the informativeness of the signal. That is, let $\underline{\theta} = \bar{\theta}$ and $\sigma_\epsilon^2 = \infty$. In this case, nothing is learned in the intermediate period,

hence, the behavior of the entrepreneur can be known with certainty beforehand. In particular, $E\left[\tilde{R}|S\right] = E\left[\tilde{R}\right]$ $\forall S$. Furthermore, because θ is known, the entrepreneur will never waste capital in the intermediate period. Instead, if she chooses to waste capital, she will waste it immediately. Hence, the the optimal financing strategy is to raise the maximum amount of financing such that the entrepreneur will not divert the capital in the immediately. This is the content of Proposition 3.

Proposition 3. *When θ is certain and known ex-ante, and no information is revealed in the interim ($\sigma_\epsilon^2 = \infty$), the optimal financing strategy is to finance projects with the largest $k_0 \in \left[\underline{k}, k^U\right]$ such that the entrepreneur will not abandon the project immediately.*

Because there is no information revelation, the entrepreneur will either continue or divert with probability 1. Clearly, no investor will finance an amount of capital guaranteeing abandonment, as the investment would be lost with certainty. Since providing additional capital upfront increases the benefit of shirking, providing too much will lead to abandonment. However, there are benefits to upfront financing, hence, it is optimal to provide as much initial financing as possible, conditional on guaranteed continuation.

While the above presents an informative special case, in reality, the ability to learn about the quality of the project over time is particularly important in VC-backed companies. Hence, consider how the previous result changes when the intermediate signal is informative, with $\sigma_\epsilon^2 < \infty$. To focus on one lever of the model, let θ be fixed and known ex-ante. In this case, Condition 12 implies that the project will be continued if and only if

$$\tilde{S} \geq \frac{\sigma_x^2 + \sigma_\epsilon^2}{\sigma_x^2} \log\left(\frac{\theta(k_0 - \underline{k})}{1 - \gamma_0} + K - k_0 - f(k_0 - \underline{k})\right) - \frac{\sigma_\epsilon^2}{\sigma_x^2}\mu - \frac{\sigma_\epsilon^2}{2}, \tag{14}$$

otherwise the remaining capital will be diverted by the entrepreneur. Because the signal is normally distributed, the probability of continuation will never be 0 or 1. It is useful to discuss how that choice of k_0 and the associated γ_0 affect the probability of continuation. The equity stake sold to the initial investor, γ_0, unambiguously increases the signal cutoff, making it more likely that the entrepreneur will choose to shirk. On the other hand, the direct effect of k_0 is ambiguous ex-ante, as there are two effects that work in opposite directions. First, the larger k_0, the lower the capital required to complete the project encouraging the entrepreneur to continue. Second, a larger k_0

Parameter	Value
μ	$\frac{3}{4}\log(2.2)$
σ_x	$\frac{1}{2}\log(2.2)$
σ_e	1
k	1.8
θ	0.8
a	0.3
$\underline{k}$	0.3

Table 1: Parameter Values

increases the benefit of shirking, increasing the allure of diverting capital. While it is instructive to consider these effects separately, in equilibrium, k_0 and γ_0 are linked by the investor's break-even condition, so the choice of k_0 will dictate the likelihood of continuation.

Defining C_{k_0,γ_0} as the minimum signal needed to induce continuation, the investor must choose γ_0 such that

$$\gamma_0 P(\tilde{S} > C_{k_0,\gamma_0})\left(E\left[\tilde{R}|\tilde{S} > C_{k_0,\gamma_0}\right] + k_0 + f(k_0 - \underline{k}) - K \right) = k_0. \tag{15}$$

There are two effects when setting γ_0. A higher γ_0 increases the investor's share of the proceeds, but reduces the total value of the project. A numerical example showing this is instructive. Table 1 provides a set of reasonable parameters for the model. The choice of μ and σ_x^2 ensure that the ex-ante expected payoff of the project is 2.2, larger than the maximum capital needed, so the project is positive NPV in the absence of an agency cost.

Figure 2 depicts the the two forces that affect the required size of γ_0. As can be seen from the left panel, an initial investment of $k_0 = 0.4$ can be financed, with the investor breaking even with an ownership percentage of either 44% or 94%. Clearly, the entrepreneur would choose, if she desired to raise $k_0 = 0.4$, to sell the smaller level of equity. The right panel shows a very different picture. There is no level of γ_0 such that an initial investment of 0.9 can be financed. This is due to agency costs. In the absence of these costs the entire project could, and indeed should, be financed entirely upfront. However, the temptation to misuse the capital raised is so large that, even a very attractive project such as this cannot raise even half the required capital upfront.

Consider the ex-ante expected utility of the entrepreneur. If the project is completed, the entrepreneur has claim to her portion of the proceeds; otherwise, she shirks and derives utility
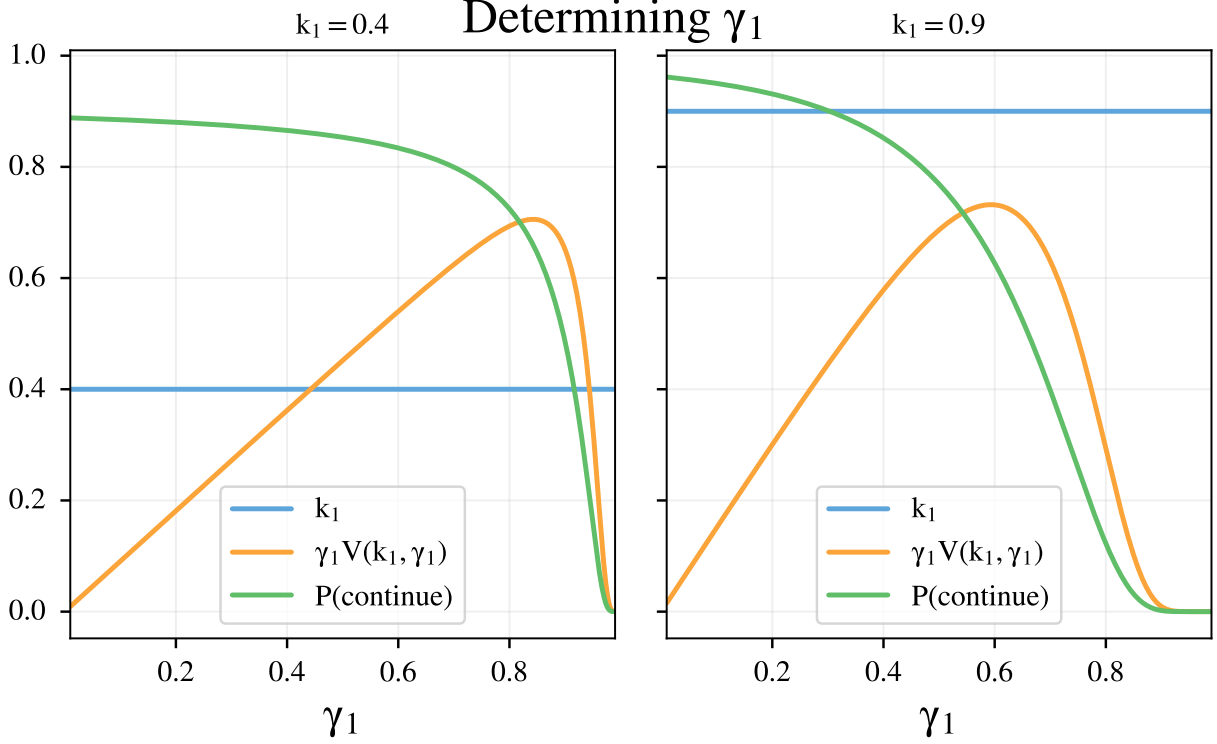
Fig. 2. Determination of γ_0

$\theta(k_0 - \underline{k})$ from the remaining capital. Hence, her objective function is

$$
P(\tilde{S} \geq C_{k_0,\gamma_0})(1 - \gamma_0)\left(E\left[\tilde{R}|\tilde{S} \geq C_{k_0,\gamma_0}\right] + k_0 + f(k_0 - \underline{k}) - K \right)
$$
$$
+ (1 - P(\tilde{S} \geq C_{k_0,\gamma_0}))[E\left[\tilde{\theta}|\tilde{S} \geq C_{k_0,\gamma_0}\right](k_0 - \underline{k}). \tag{16}
$$

Note that this consists of two parts. The first is the entrepreneurs fraction of firm-value; the second is her payoff when she shirks. Figure 3 shows the tradeoffs[3].

An important result can be seen from Figure 3.

Result 1. *The entrepreneurs preferred initial financing, k_0^*, is larger than the level maximizing the NPV of the project.*

The entrepreneur raises more financing upfront than maximizes the NPV of the underlying project. This result arises because the entrepreneur's expected utility consists of both the payoff

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[3]Because the marginal benefit of additional capital at $\underline{k}$ is infinite, there is a strong incentive to raise more than the minimum needed. Indeed, the equity that must be sold to raise $\underline{k}$ is actually higher than raising slightly more. However, this unpleasant effect only occurs for very small levels of k_0, and will never interfere with the optimization.
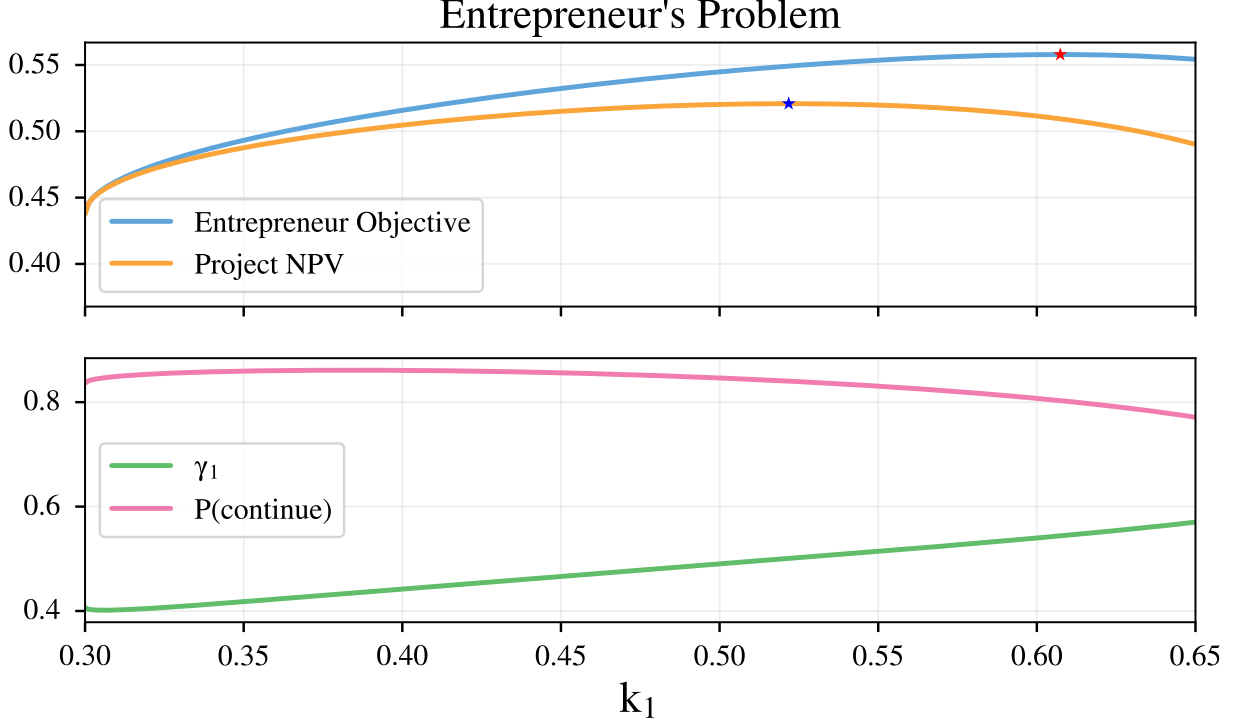
Fig. 3. Choosing k_0

of the project and the amount she wastes when the project is not completed. As the entrepreneur holds all of the surplus, she bears the cost that her choice of k_0 imposes on the project, as a result of her potential abandonment, ex-ante through the cost of capital. However, she is able to partially offset this through the utility she gains by wasting the remaining capital in poor states of the world. Hence, her choice of initial financing is higher than the firm-maximizing value. This result would not hold if her payoff upon shirking were not positively related to the initial capital raised.

A second key result relates to the informativeness of the signal, this is stated in Result 2, and illustrated in Figure 4.

Result 2. *When $\sigma_\epsilon^2 > \underline{\sigma_\epsilon^2}$, k_0^* is increasing in σ_ϵ^2.*

The entrepreneur finds it optimal to raise less initial financing when the signal is more informative. The intuition is that there is option value associated with stage financing. When investing only a fraction of the needed capital is raised, the option to abandon is preserved. As the entrepreneur holds all of the bargaining power, she captures this value ex-ante. Note however that this isn't a trivial result. While the entrepreneur captures the value of the option to abandon, by
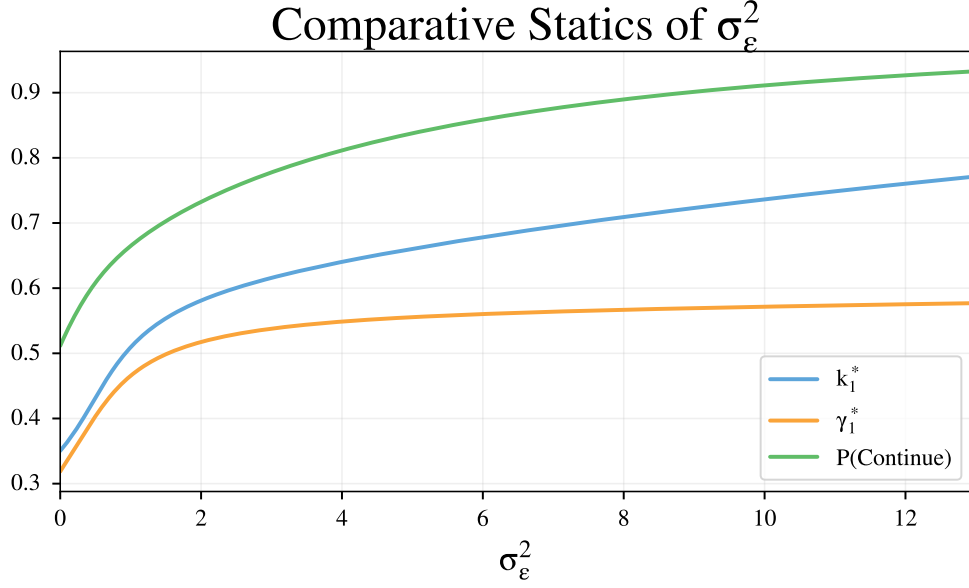
Fig. 4. Comparative Static of σ_ϵ^2

raising less she sacrifices the value of her put option associated with diverting the capital to private use.

While it is generally believed that the most important source of uncertainty in a venture context is that concerning the quality of the project, the model allows for another source of uncertainty that is, thus far, unexplored. In particular, uncertainty over the extent of the agency problem, that is, uncertainty over the value of θ. To explore this in the most illuminating way possible, it is helpful to focus on the case where $\sigma_\epsilon^2 = \infty$ so that nothing is learned in the interim, allowing the focus to be entirely on uncertainty about θ. In this case, the entrepreneur's expected utility from continuation is known in the intermediate period. It is simply

$$(1 - \gamma_0)(E[e^x] + k_0 + f(k_0 - \underline{k}) - K), \tag{17}$$

however, the value of shirking is not known. It is known that the investor will choose to shirk if the utility from doing so is sufficiently high. Indeed, taking the limit as σ_ϵ^2 approaches ∞ in Condition 13, the entrepreneur will choose to shirk whenever $\tilde{\theta} > M$, where

$$M = (1 - \gamma_0)\frac{1}{k_0 - \underline{k}}\left[e^{\frac{\sigma_x^2}{\sigma_x^2 + \sigma_\epsilon^2}\left(s + \frac{\sigma_\epsilon^2}{\sigma_x^2}\mu + \frac{\sigma_\epsilon^2}{2}\right)} + k_0 + f(k_0 - \underline{k}) - k\right]. \tag{18}$$

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Then, the investors break even condition is

$$\gamma_0 P(W < M)(E[e^x] + k_0 + f(k_0 - \underline{k}) - K) = k_0. \tag{19}$$

This can be solved in closed form. It is the case that

$$\gamma_0 = \frac{1}{2} + \frac{(1 - \pi)\bar{w} - \sqrt{(\pi v + (1 - \pi)\bar{w})^2 - 4\pi\bar{w}k_0}}{2\pi v}, \tag{20}$$

where $\bar{w} = \bar{\theta}(k_0 - \underline{k})$ and $v = E[e^x] + k_0 + f(k_0 - \underline{k}) - K$. A result can be derived concerning the uncertainty about the efficiency of shirking.

Result 3. *When θ is stochastic, k_0^* is lower than when θ is certain.*

This is often due to the fact that higher levels of capital can often not be funded when θ is stochastic. For instance, consider an entrepreneur seeking maximal financing with certain θ. There is no probability of abandonment. As a thought experiment, now consider if, rather than being fixed, the benefit of shirking is now uniformly distributed, centered at the previously fixed value. Rather than continuation happening with certainty, it now happens with a probability of only $\frac{1}{2}$, significantly reducing the value of the firm. Unless the project is particularly valuable, it is unlikely the project can even be financed to the previously optimal level, as the pledgable value is now just half what it was previously. For many parameters, the optimal level of capital is the largest that can guarantee continuation, which is increasing in $\underline{\theta}$.

These results show that uncertainty about the value of shirking and uncertainty about the product quality behave in a similar way, reducing the size of optimal initial fundraising. Importantly, the model also readily allows extensions incorporating other important factors that may also the timing of fundraising. The next section considers a particularly interesting additional factor that interplays in important ways with the staging decision: entrepreneurial optimism.

4. Entrepreneurial Optimism

Entrepreneurs voluntarily bear a large amount of risk when they start companies. However, the literature suggests that they are not compensated well for doing so (Hamilton, 2000). This finding

applies both to general entrepreneurship as well as to the specific setting venture capital (Hall and Woodward, 2010). Several explanations have been proposed as to why entrepreneurs choose this path despite the low premium. Candidate explanations include non-pecuniary benefits (Hurst and Pugsley, 2011), mismeasurement of returns Åstebro and Chen (2014), and optimism (Astebro et al., 2014). While empirical work has found it difficult to separate these explanations, it is well accepted that optimism provides at least a partial explanation (Astebro et al., 2014). Despite this, relatively little work has been undertaken to explore the further implications of optimism in the venture capital market. How do optimistic entrepreneurs choose to raise funding? Do they over-raise or under-raise? This model of stage financing is well suited to answer these questions.

Before directly discussing the incorporation of optimism into this research design, it is useful to clearly convey what is meant by the term. By "optimism", I mean that the entrepreneur's assessment of her project's payoff is higher than outside parties believe. It is not essential to take a stand on the source of these beliefs. It is unimportant to the modeling whether the source of the overestimation is due to the entrepreneur holding excessive beliefs of her own ability, the genius of her original idea, or simply a more general wide-ranging optimism. What is important is that her beliefs differ from those of outside investors and that the parties agree to disagree rather than update based on the other parties information.

With only slight modifications, the present model can address well differences of beliefs. Optimism is incorporated by introducing different perceived probability distributions held by the entrepreneur and the investor. The entrepreneur has a more rosy forecast than investors, in the sense of the monotone likelihood ratio property. She seeks to maximize her expected payoff under her probability distribution, subject to the investor at least breaking even under his assessment.

To make the intuition concrete, define the investor's prior probability distribution as $\mathbb{P}$, while the entrepreneur's prior is $\mathbb{Q}$. Let these be equivalent probability distributions where

$$\frac{f^{\mathbb{Q}}(x_2)}{f^{\mathbb{P}}(x_2)} > \frac{f^{\mathbb{Q}}(x_1)}{f^{\mathbb{P}}(x_1)} \ \forall x_2 > x_1. \tag{21}$$

Differences of beliefs are common knowledge. Both parties agree to disagree—their priors are not updated based on the other party's prior. However, upon receiving a the signal $\tilde{S}$ in the intermediate period, both the entrepreneur and the investor update their priors as proper Bayesians.

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Consider how the basic solution of Section 3 changes. As there is no decision-making in the final period, changes begin in the intermediate period. The entrepreneur will continue the project whenever her expected utility from doing so is greater than the utility she gains from diverting capital. The analog of Equation 5 suggests that this would be whenever

$$(1 - \gamma_0)\left[E^{\mathbb{Q}}[\tilde{R}|S] + k_0 + f(k_0 - \underline{k}) - K\right] \geq \theta(k_0 - \underline{k}). \tag{22}$$

However, this is not correct. Recall that Equation 5 was derived from the more primitive condition that the entrepreneur will continue whenever $(1 - \gamma_0)(1 - \gamma_1)E[\tilde{R}|S] \geq \theta(k_0 - \underline{k})$ and the fact that the equity in the second period would be fairly priced. However, with different priors, the entrepreneur does not believe that equity in the future will be fairly priced. In fact, in most cases the entrepreneur will believe that equity is always expensive[4]. The entrepreneur will continue whenever

$$(1 - \gamma_0)(1 - \gamma_1)E^{Q}[\tilde{R}|S] \geq \theta(k_0 - \underline{k}) \tag{23}$$

$$\iff (1 - \gamma_0)(1 - \gamma_1)\frac{E[\tilde{R}|S]}{E[\tilde{R}|S]}E^{Q}[\tilde{R}|S] \geq \theta(k_0 - \underline{k}) \tag{24}$$

$$\iff (1 - \gamma_0)(1 - \gamma_1)\frac{E^{Q}[\tilde{R}|S]}{E[\tilde{R}|S]}E[\tilde{R}|S] \geq \theta(k_0 - \underline{k}) \tag{25}$$

$$\iff (1 - \gamma_0)\frac{E^{Q}[\tilde{R}|S]}{E[\tilde{R}|S]}\left[E[\tilde{R}|S] + k_0 + f(k_0 - \underline{k}) - K\right] \geq \theta(k_0 - \underline{k}). \tag{26}$$

Expression 26 is particularly instructive. Whenever $\frac{E^{Q}[\tilde{R}|S]}{E[\tilde{R}|S]} > 1$, there is a set of (s, θ) pairs for which the optimist will continue and a rational entrepreneur will not. Note however, that when $\theta = 0$ the conditions decision rule is the same. In this case, the entrepreneur, no matter what the level of optimism, will continue if and only of she can obtain financing for her project. Hence, the decision rests with the follow-on investor, leading entrepreneurs of different beliefs to act identically.

Then, defining $\mathbb{C}_{(\gamma_0, k_0)}$ as the set for which the entrepreneur continues rather than stealing

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[4]Optimists expect to get better terms in the future, but realize they will not get what they consider fair pricing in the intermediate period

capital, the set for which inequality 26 holds, define the investor's problem as selecting γ_0 such that

$$\gamma_0 \int \int \mathbb{1}_{\mathbb{C}_{(\gamma_0,k_0)}}\left(E[\tilde{R}|S] + k_0 + f(k_0 - \underline{k}) - K \right)dF^P(s)dG(\theta) = k_0. \tag{27}$$

Note that both sets of beliefs enter into Equation 27. The entrepreneur's beliefs define the ex-post continuation region. There is no disagreement about this region—the set of (s, θ) pairs for which the entrepreneur will continue are common knowledge. However, the distribution of the underlying quality of the project, and hence of the signal, are a point of disagreement. When setting the price of capital the investors beliefs are used for the distribution of the signal and the distribution of the project quality, while the entrepreneur's beliefs enter into the definition of the continuation region. As before, this defines a function, $\gamma_0(k_0)$, that assigns the smallest possible γ_0 that satisfies Equation 27 for each choice of k_0. Note that common knowledge of beliefs is essential here, as the terms offered to the entrepreneur depend on her beliefs.

The entrepreneurs objective function is very similar to Expression 8, but with the entrepreneur's perceived distribution and the factor, $\frac{E^Q[\tilde{R}|S]}{E[\tilde{R}|S]}$, by which the entrepreneur overvalues continuation. This shown as Expression 28:

$$\int \int (1-\gamma_0)\mathbb{1}_{\mathbb{C}_{(\gamma_0,k_0)}}\frac{E^Q[\tilde{R}|S]}{E[\tilde{R}|S]}\left(E[\tilde{R}|S]+k_0+f(k_0-\underline{k})-K \right)+\left(1 - \mathbb{1}_{\mathbb{C}_{(\gamma_0,k_0)}} \right)\theta(k_0-\underline{k})dF^Q(s)dG(\theta). \tag{28}$$

Note particularly the blend of beliefs that goes into Expression 28. While the investor's belief determines the cost of capital, the distribution of the signal is taken according the entrepreneur's beliefs. This shows that there are two effects of entrepreneurial optimism, one ex-ante and one ex-post.

The ex-ante effect of entrepreneurial optimism is that optimists find equity expensive, and expect better terms in the future. Specifically,

$$E^{\mathbb{Q}}\left[E^{\mathbb{P}}\left[\tilde{R}|\tilde{S} \right] \right] > E^{\mathbb{P}}\left[\tilde{R} \right]. \tag{29}$$

That is, because the optimist has a more favorable distribution of project quality than the investor; she expects better signals on average. Hence, she expects that the signals in the interim will support

her belief, and, on average, she will get better terms by delaying the bulk of financing until the intermediate period, introducing a strong motive to delay.

The ex-post effect is that the set of (s, θ) pairs for which the optimist continues the project is a superset of the corresponding set for rational entrepreneurs, so optimists will behave better ex-post. Of course, this is priced in ex-ante, so optimists can secure better terms for their first-round equity than rational entrepreneurs.

The two effects of optimism interact with the staging decision in a very interesting way. The ex-ante effect makes the entrepreneur want to raise less financing initially, as they expect good signals on average that will allow them to get better terms in the future. However, the good ex-post behavior induced by optimism lets the optimistic entrepreneur raise initial financing at better terms than a rational entrepreneur, reducing the cost to obtain the benefits of upfront financing and encouraging the optimist to raise more initially. The following subsection digs deeper into this tradeoff.

4.1.   Functional Form

The ways in which optimism affects the model become even clearer when considering a functional form. To this end, I adapt the functional form of Section 3.1 to allow for heterogeneous beliefs. The only modification needed to allow for rich differences of beliefs is to allow the investor and the entrepreneur to have different prior values for the mean of $\tilde{X}$. Let the investor believe that $\tilde{X} \sim N(\mu, \sigma_x^2)$ while the entrepreneur believes $\tilde{X} \sim N(\mu_q, \sigma_x^2)$, with $\mu_q > \mu$. Then, the perceived distributions of the payoff $\tilde{R}$ satisfy the monotone likelihood ratio property.

From Equation 26 it can be seen that the entrepreneur will continue whenever

$$(1 - \gamma_0) \frac{E^Q[\tilde{R}|S]}{E[\tilde{R}|S]} \left[E[\tilde{R}|S] + k_0 + f(k_0 - \underline{k}) - K \right] \geq \theta(k_0 - \underline{k}). \tag{30}$$

A benefit of the log-normal setting is that the overvaluation fraction, $\frac{E^Q[\tilde{R}|S]}{E[\tilde{R}|S]}$, is constant over $\tilde{S}$.

Conditional on any signal S,

$$\frac{E^Q[\tilde{R}|S]}{E[\tilde{R}|S]} = \frac{e^{\frac{\tau_x \mu_q + \tau_\epsilon S + 1/2}{\tau_x + \tau_\epsilon}}}{e^{\frac{\tau_x \mu + \tau_\epsilon S + 1/2}{\tau_x + \tau_\epsilon}}} \tag{31}$$

$$= \frac{e^{\frac{\tau_x \mu_q}{\tau_x + \tau_\epsilon}}}{e^{\frac{\tau_x \mu}{\tau_x + \tau_\epsilon}}} \tag{32}$$

$$= e^{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_x^2}(\mu_q - \mu)}. \tag{33}$$

Then, as before, the continuation region can be expressed in two different ways. First, by conditioning on the realized value of θ, the entrepreneur will continue whenever

$$\tilde{S} \geq \frac{\sigma_x^2 + \sigma_\epsilon^2}{\sigma_x^2} \log\left(e^{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_x^2}(\mu - \mu_q)} \frac{\theta(k_0 - \underline{k})}{1 - \gamma_0} + K - k_0 - f(k_0 - \underline{k})\right) - \frac{\sigma_\epsilon^2}{\sigma_x^2}\mu - \frac{\sigma_\epsilon^2}{2}. \tag{34}$$

Alternatively, by conditioning on the signal, it can be seen that the entrepreneur will continue whenever

$$\tilde{\theta} \leq \frac{(1 - \gamma_0)\left[e^{\frac{\sigma_x^2}{\sigma_\epsilon^2 + \sigma_x^2}(s + \frac{\sigma_\epsilon^2}{\sigma_x^2}\mu + \frac{\sigma_\epsilon^2}{2})} + k_0 + f(k_0 - \underline{k}) - K\right]}{e^{\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_x^2}(\mu - \mu_q)}(k_0 - \underline{k})}. \tag{35}$$

Note that both expressions are pure generalizations of their rational counterparts, and reduce to Expressions 12 and 13, respectively, when $\mu_q = \mu$. Then, calculating γ_0 and the entrepreneur's objective function is just a case of plugging these expressions into Equation 27 and Expression 28, respectively.

It is most illustrative to consider edge cases of the model to see the different mechanisms at work. To this end, first consider the edge case where $\sigma_\epsilon^2 = 0$, so that the project's payoff is revealed with certainty in the intermediate period. This edge case isolates the ex-ante effect of optimism: the entrepreneur expects to obtain better terms in the future, as she expects the signal will reveal that the project is good. Because the value is revealed with certainty in the intermediate period, optimism does not induce better behavior on the part of the entrepreneur. She will continue if the actual payoff from doing so is large enough; her prior plays no role in the calculation. For ease of exposition let $\tilde{\theta}$ be fixed as well. In this edge case, the optimistic entrepreneur will raise less than a rational entrepreneur would, as in Result 4.

Result 4. *The entrepreneur's optimal initial fundraising level, k_0^*, is decreasing in μ_q when $\sigma_\epsilon^2 = 0$.*

The intuition is that the more optimistic the entrepreneur, holding the investor's beliefs fixed, the more she expects that terms tomorrow will be better than today, providing the incentive to delay financing, despite there being some cost do doing so due to the forgone benefits of upfront financing.

It is worth restating the content of Result 1, stating that the rational entrepreneur raises more capital upfront than the level that maximizes the NPV of the project. Optimism can in some cases temper this problem, resulting in a chosen level of capital closer to that maximizing firm value. In the current case, where $\sigma_\epsilon^2 = 0$, the level of capital maximizing firm-value is independent of the entrepreneur's level of optimism, as this has no effect on ex-post decision making. This, combined with Results 1 and 4 indicate that, at least for moderate levels of optimism, a more appropriate level of capital is chosen. However, extreme optimism can lead the entrepreneur to raise less than the level which maximizes the project NPV. This is illustrated in Figure 5. The parameters used
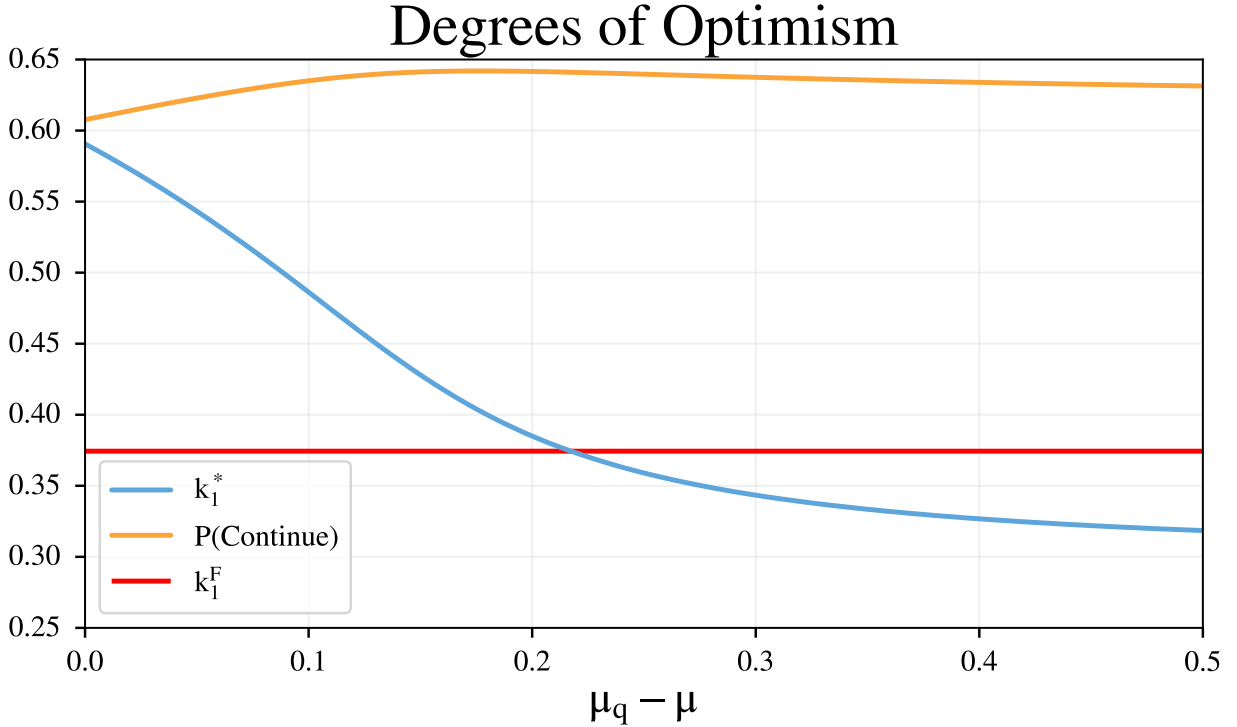

Fig. 5. Optimal k_0 with Optimism

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to generate this figure are identical to those of Table 1, with μ_q added and varied along the x-axis. This interplay with optimism leads the entrepreneur to raise less capital the more optimistic she is.

The other edge case to consider is when $\sigma_\epsilon^2 = \infty$, so that nothing is learned about the project in the intermediate period, only the entrepreneur's ability to steal, $\tilde{\theta}$ is revealed. The interaction with optimism has a very different effect here. In this edge case, the ex-ante effect of optimism is shut down, leaving only the ex-post effect. To be clear, the entrepreneur still believes that the equity she sells is underpriced, however, she does not believe she will get better terms by waiting. Because the signal in the intermediate period is completely uninformative, both the investor and the entrepreneur continue to believe their prior in the intermediate period. Nevertheless, because the entrepreneur has inflated beliefs, there are more values of θ for which she will choose to continue than if she were not an optimist. Thus, she behaves better ex-post, and will hence be able to sell her equity for a higher value ex-ante than she could if she were rational.

In this case, the entrepreneur will continue whenever

$$\tilde{\theta} \leq \frac{(1 - \gamma_0)\left[e^{\mu + \frac{\sigma_x^2}{2}} + k_0 + f(k_0 - \underline{k}) - K\right]}{e^{(\mu - \mu_q)}(k_0 - \underline{k})}. \tag{36}$$

Then, defining $\rho = e^{\mu - \mu_q}$, this can be plugged into Equation 27 to obtain

$$\gamma_0 = \frac{1}{2} + \frac{(1 - \pi)\bar{w} - \sqrt{(\pi v + (1 - \pi)\rho\bar{w})^2 - 4\pi\rho\bar{w}k_0}}{2\pi\rho v}, \tag{37}$$

where $\bar{w} = \bar{\theta}(k_0 - \underline{k})$ and $v = E[e^x] + k_0 + f(k_0 - \underline{k}) - K$. From here it can be shown that the optimist will raise more than the rational entrepreneur, as stated in Result 5.

Result 5. *The optimal level of initial capital, k_0^* is increasing in the entrepreneurs perceived μ_q, holding μ fixed.*

The intuition is that the optimist gets better terms, pushing her to raise more capital, and does not receive any benefit from waiting to receive better terms in the future, and so maximizes her utility by raising a greater proportion of funds upfront, harvesting the benefits of upfront financing.

While the intuition is clearest when focusing on the edge cases, the results interpolate between the edge cases in a smooth way. When σ_ϵ^2 is large, the ex-post effect dominates k_0^* is increasing in μ_q.

Parameter	Value
μ	$\frac{3}{4}\log(2.2)$
μ	$\frac{3}{4}\log(2.2) + 0.2$
σ_x	$\frac{1}{2}\log(2.2)$
k	1.8
$\bar{\theta}$	0.9
$\underline{\theta}$	0.7
δ	0.3
$\underline{k}$	0.3

Table 2: Parameter Values

On the other hand, when σ_ϵ^2 is large, the ex-ante effect dominates and the opposite is true. This is illustrated in Figure 6, which is constructed using the parameters in Table 2, with σ_ϵ^2 varying across the x-axis. Much can be learned from this plot. On the left, the signal is particularly informative,
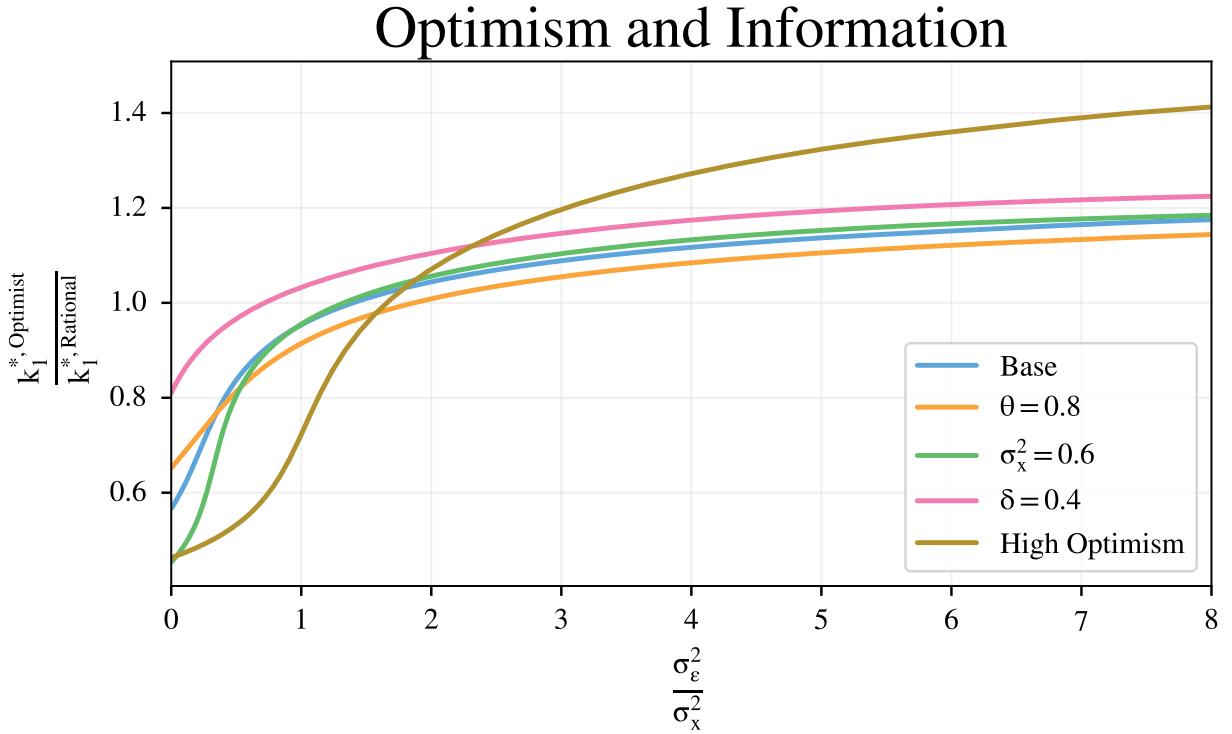

Fig. 6. Effect of Optimism By Signal Informativeness

with σ_ϵ^2 very small. In these cases, the optimist raises less than the rational entrepreneur, as the ex-ante effect dominates. This begins to change as the signal becomes less and less informative and the ex-post effect begins to dominate. The base parameters are shown in the blue line. In the others, I vary one parameter at a time to show the effect on the tradeoff. In the orange line

I shut down the variability of $\tilde{\theta}$ and the results are changed only very slightly. In the green line I increase the variance of $\tilde{X}$[5] and show with a larger variance of the project payoff the ex-ante effect of optimism is enlarged but the ex-post effect remains largely the same. In the pink line, I increase the benefit of upfront financing and illustrate that this increase the benefit of upfront financing. The optimist responds more to this than the rational entrepreneur, however, this does not vary over the informativeness of the signal. Finally, I increase the disagreement between the entrepreneur and the investors and show that both the ex-ante and ex-post effect of optimism are exaggerated. Overall, Figure 6 illustrates that the role of σ_ϵ^2 in moderating which effect of optimism dominates is a general feature of the model and not the result of any specific parameters.

5.   Conclusion

This paper explores the determinants of the staging decision in venture capital. Specifically, the staging of financing is given priority over the staging of physical investment. The central tradeoff is efficiency gains from upfront financing versus the entrepreneur's ability to misuse the capital when the return is likely to be poor. The staging decision is shown to be primarily driven by the level of informativeness of the signal in the intermediate period, as a more informative signal increases the option value of waiting, affecting the balance between the benefits of upfront financing and the agency problems associated.

When optimism on the part of the entrepreneur is included, additional insights are revealed. Optimistic entrepreneurs face a tradeoff when deciding how to finance their ventures. On the one hand, they are confident of the value of their venture and hence can be trusted to abandon less frequently, reducing the cost of upfront financing. On the other hand, they expect to get better terms in the future, providing an incentive to delay. The former effect is shown to dominate when intermediate information is uninformative, while the latter dominates when the signal is informative. While optimism is difficult to measure if a suitable proxy can be determined this would be a particularly interesting prediction to take to the data, potentially exploiting differences between industries to capture differences in signal informativeness.

While this work raises and answers important questions regarding stage financing and opti-

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[5]I adjust μ_q and μ properly so that the expected value of $\tilde{R}$ is not changed.

mism, there are further questions to be explored. How do the staging of financing and the staging of investment interact? Do optimistic entrepreneurs take different actions beyond the financing decision? How do they respond to acquisition offers, and what is the ex-ante effect of this on the staging decision. These and other questions are left to future research.

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Appendix: Proofs

Claim 1. *Whenever Condition 2 holds, Condition 4 holds as well. Thus, the shirking decision can be fully characterized by Condition 4.*

Proof. The entrepreneur will choose to abandon whenever

$$(1 - \gamma_0)\left[E[\tilde{R}|S] + k_0 + f(k_0 - \underline{k}) - K \right] < \theta(k_0 - \underline{k}).$$

This can be rewritten as

$$E[\tilde{R}|S] + k_0 + f(k_0 - \underline{k}) - K < \frac{\theta(k_0 - \underline{k})}{1 - \gamma_0}.$$

The entrepreneur will not be able to raise the required capital whenever

$$E[\tilde{R}|S] + k_0 + f(k_0 - \underline{k}) - K < 0.$$

Then, given that $\frac{\theta(k_0 - \underline{k})}{1 - \gamma_0} \geq 0$, it is the case that

$$E[\tilde{R}|S] + k_0 + f(k_0 - \underline{k}) - K < 0 \implies E[\tilde{R}|S] + k_0 + f(k_0 - \underline{k}) - K < \frac{\theta(k_0 - \underline{k})}{1 - \gamma_0},$$

establishing the desired result. $\square$

Proposition 2. *In the absence of an agency problem, the optimal financing strategy, chosen by the entrepreneur, is to finance the project entirely upfront or not at all.*

Proof. The proof is trivial. In the absence of an agency problem, the optimal decision is made in the intermediate period. If it is positive NPV to continue the project, it is continued, otherwise, the money is returned. Hence, there is no cost to upfront financing, while there is a benefit. Since $f(k_0 - \underline{k})$ is increasing in k, the total value of the project in this instance is increasing in k_0. Hence, the optimal financing strategy is to fully finance the project upfront. $\square$

Proposition 3. *When θ is certain and known ex-ante, and no information is revealed in the interim ($\sigma_\epsilon^2 = \infty$), the optimal financing strategy is to finance projects with the largest $k_0 \in \left[\underline{k}, k^U \right]$*

such that the entrepreneur will not abandon the project immediately.

Proof. There is no stochastic element, so it is known with certainty that the entrepreneur will either continue or abandon in the intermediate period. Furthermore, if the entrepreneur were to prefer abandon in the intermediate period, she would choose to abandon immediately, as $\theta k_0 > \theta(k_0 - \underline{k})$. Then, we must have abandonment happen either immediately or not at all. Clearly, the optimal k_0 cannot be one that guarantees abandonment, as the entrepreneur would be unable to finance such a k_0, as the investor will always receive a payoff of zero. Thus, the probability of continuation at the optimal k_0 is 1. Then, the entrepreneur's utility is

$$(1 - \gamma_0)\left(E\left[\tilde{R}\right] + f(k_0 - \underline{k}) + k_0 - K \right). \tag{38}$$

Furthermore, because γ_0 ensures the initial investors stake is worth k_0. Hence, the entrepreneur's utility can be simplified to

$$E\left[\tilde{R}\right] + f(k_0 - \underline{k}) + k_0 - K - k_0 = E\left[\tilde{R}\right] + f(k_0 - \underline{k}) - K. \tag{39}$$

Then, the fact that f is increasing implies that the entrepreneur's utility is maximized when k_0 as large as possible, conditional on the probability of continuation being 1. If the probability of continuation is 1 at k^U, then this is the optimal k_0. If not, the optimal is the largest financeable k_0, as desired. $\square$

Result 1. *The entrepreneurs preferred initial financing, k_0^*, is larger than the level maximizing the NPV of the project.*

Proof. The project NPV is

$$P(\tilde{S} \geq C_{k_0,\gamma_0})\left(E\left[\tilde{R} | \tilde{S} \geq C_{k_0,\gamma_0}\right] + k_0 + f(k_0 - \underline{k}) - K \right) - k_0, \tag{40}$$

while the expected utility of the entrepreneur is

$$
P(\tilde{S} \geq C_{k_0,\gamma_0})(1 - \gamma_0)\left(E\left[\tilde{R}|\tilde{S} \geq C_{k_0,\gamma_0}\right] + k_0 + f(k_0 - \underline{k}) - K \right)
$$
$$
+ (1 - P(\tilde{S} \geq C_{k_0,\gamma_0}))E\left[\tilde{\theta}|\tilde{S} \geq C_{k_0,\gamma_0}\right](k_0 - \underline{k})
$$

which can be rewritten

$$
P(\tilde{S} \geq C_{k_0,\gamma_0})\left(E\left[\tilde{R}|\tilde{S} \geq C_{k_0,\gamma_0}\right] + k_0 + f(k_0 - \underline{k}) - K \right)
$$
$$
+ (1 - P(\tilde{S} \geq C_{k_0,\gamma_0}))E\left[\tilde{\theta}|\tilde{S} \geq C_{k_0,\gamma_0}\right](k_0 - \underline{k}) - k_0.
$$

Then, firm value is maximized when the first order condition to Equation 40 equals 0. The first derivative of the entrepreneur's utility is

$$
P(\tilde{S} \geq C_{k_0,\gamma_0})\left[E_{k_0}\left[\tilde{R}|\tilde{S} \geq C_{k_0,\gamma_0}\right] + 1 + f'(k_0 - \underline{k}) \right]
$$
$$
+P_{k_0}(\tilde{S} \geq C_{k_0,\gamma_0})\left(E\left[\tilde{R}|\tilde{S} \geq C_{k_0,\gamma_0}\right] + k_0 + f(k_0 - \underline{k}) - K \right) - 1
$$
$$
+(1 - P(\tilde{S} \geq C_{k_0,\gamma_0}))\left[E\left[\tilde{\theta}|\tilde{S} \geq C_{k_0,\gamma_0}\right] + E_{k_0}\left[\tilde{\theta}|\tilde{S} \geq C_{k_0,\gamma_0}\right](k_0 - \underline{k}) \right]
$$
$$
- P_{k_0}(\tilde{S} \geq C_{k_0,\gamma_0})E\left[\tilde{\theta}|\tilde{S} \geq C_{k_0,\gamma_0}\right](k_0 - \underline{k}).
$$

Now, at k_0^P, the value that maximizes the project NPV, this reduces to

$$
(1 - P(\tilde{S} \geq C_{k_0,\gamma_0}))\left[E\left[\tilde{\theta}|\tilde{S} \geq C_{k_0,\gamma_0}\right] + E_{k_0}\left[\tilde{\theta}|\tilde{S} \geq C_{k_0,\gamma_0}\right](k_0 - \underline{k}) \right]
$$
$$
- P_{k_0}(\tilde{S} \geq C_{k_0,\gamma_0})E\left[\tilde{\theta}|\tilde{S} \geq C_{k_0,\gamma_0}\right](k_0 - \underline{k}).
$$

Now, consider the sign of this. Consider two mutually exclusive and exhaustive cases. First, assume $C_{k_0^P,\gamma_0}$ in increasing (or constant) in k_0. Then, this can be rewritten

$$
E_{k_0}\left[\mathbb{1}_{\tilde{S} \geq C_{k_0,\gamma_0}}\right](k_0 - \underline{k}) + E\left[\mathbb{1}_{\tilde{S} \geq C_{k_0,\gamma_0}}\right],
$$

Which is positive. Now, consider the alternative case, where $C_{k_0^P,\gamma_0}$ in decreasing in k_0. Note that

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we have

$$(1 - P(\tilde{S} \geq C_{k_0,\gamma_0})) \left[E \left[\tilde{\theta} | \tilde{S} \geq C_{k_0,\gamma_0} \right] \right]$$

$$- P_{k_0}(\tilde{S} \geq C_{k_0,\gamma_0}) E \left[\tilde{\theta} | \tilde{S} \geq C_{k_0,\gamma_0} \right] (k_0 - \underline{k})$$

$$+ (1 - P(\tilde{S} \geq C_{k_0,\gamma_0})) E_{k_0} \left[\tilde{\theta} | \tilde{S} \geq C_{k_0,\gamma_0} \right] (k_0 - \underline{k}).$$

The final term is unambiguously positive. Then, if the first two can be shown to be positive, it will be sure that the sum is positive. The first two can be rewritten

$$(1 - P(\tilde{S} \geq C_{k_0^P,\gamma_0}))[E \left[\tilde{\theta} | \tilde{S} \geq C_{k_0,\gamma_0} \right] - P_{k_0^P}(\tilde{S} \geq C_{k_0^P,\gamma_0})[E \left[\tilde{\theta} | \tilde{S} \geq C_{k_0,\gamma_0} \right] (k_0^P - \underline{k}) > 0 \qquad (41)$$

$$\iff (1 - P(\tilde{S} \geq C_{k_0^P,\gamma_0})) - P_{k_0^P}(\tilde{S} \geq C_{k_0^P,\gamma_0})(k_0^P - \underline{k}) > 0 \qquad (42)$$

$$\iff (1 - P(\tilde{S} \geq C_{k_0^P,\gamma_0})) > P_{k_0^P}(\tilde{S} \geq C_{k_0^P,\gamma_0})(k_0^P - \underline{k}). \qquad (43)$$

Now, consider two mutually exclusive and exhaustive cases. First, assume $C_{k_0^P,\gamma_0}$ in increasing (or constant) in k_0. Then, $P_{k_0^P}(\tilde{S} \geq C_{k_0^P,\gamma_0}) < 0$, so Inequality 43 must hold, as the left is positive, while the right is non-positive. Now, consider the alternative case, where $C_{k_0^P,\gamma_0}$ in decreasing in k_0. Then, note that at k_0^P, we have

$$P(\tilde{S} \geq C_{k_0,\gamma_0}) \left[E_{k_0} \left[\tilde{R} | \tilde{S} \geq C_{k_0,\gamma_0} \right] + 1 + f'(k_0 - \underline{k}) \right] + P_{k_0}(\tilde{S} \geq C_{k_0,\gamma_0}) \left(E \left[\tilde{R} | \tilde{S} \geq C_{k_0,\gamma_0} \right] + k_0 + f(k_0 - \underline{k}) - K \right) = 1,$$

which can be rearranged to

$$P_{k_0}(\tilde{S} \geq C_{k_0,\gamma_0}) \left(E \left[\tilde{R} | \tilde{S} \geq C_{k_0,\gamma_0} \right] + k_0 + f(k_0 - \underline{k}) - K \right) = 1 - P(\tilde{S} \geq C_{k_0,\gamma_0}) \left[E_{k_0} \left[\tilde{R} | \tilde{S} \geq C_{k_0,\gamma_0} \right] + 1 + f'(k_0 - \underline{k}) \right].$$

Now, we have that $E_{k_0} \left[\tilde{R} | \tilde{S} \geq C_{k_0,\gamma_0} \right]$, so,

$$P_{k_0}(\tilde{S} \geq C_{k_0,\gamma_0}) \left(E \left[\tilde{R} | \tilde{S} \geq C_{k_0,\gamma_0} \right] + k_0 + f(k_0 - \underline{k}) - K \right) < 1.$$

Then,

$$1 - P(\tilde{S} \geq C_{k_0^P, \gamma_0}) > 1 - P(\tilde{S} \geq C_{k_0^P, \gamma_0}) \left[E_{k_0} \left[\tilde{R} | \tilde{S} \geq C_{k_0, \gamma_0} \right] + 1 + f'(k_0 - \underline{k}) \right]$$

$$= P_{k_0}(\tilde{S} \geq C_{k_0, \gamma_0}) \left(E \left[\tilde{R} | \tilde{S} \geq C_{k_0, \gamma_0} \right] + k_0^P + f(k_0 - \underline{k}) - K \right)$$

$$> P_{k_0}(\tilde{S} \geq C_{k_0, \gamma_0})(k_0^P - \underline{k}).$$

The final inequality holds because

$$E \left[\tilde{R} | \tilde{S} \geq C_{k_0, \gamma_0} \right] + k_0^P + f(k_0 - \underline{k}) - K > k_0^P - \underline{k}$$

$$\iff E \left[\tilde{R} | \tilde{S} \geq C_{k_0, \gamma_0} \right] + k_0^P + f(k_0 - \underline{k}) - K > k_0^P - \underline{k}$$

$$\iff E \left[\tilde{R} | \tilde{S} \geq C_{k_0, \gamma_0} \right] + \underline{k} + f(k_0 - \underline{k}) - K > 0.$$

Then, in all cases, the derivative of the entrepreneur's utility at the NPV maximizing value of k_0 is positive, establishing the result. □