Supermarkets market in Spain

Superefficiency & Revenue Benchmark

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1 Motivation

Our activity is based on the Spanish food retailing market; we have decided to focus on this topic due to an article from 2018 that discussed an analysis report by the risk rating agency Moody's. The article argues that Mercadona has consolidated as the leader of the industry with a 24,1% market share Mercadona reached a 25% market share in 2021 as stated in Kantarworldpanel.

It also argues that the strategy that mercadona was following of dropping prices and remodelling their stores is harming companies like DIA that had to reduce their profit margins in order to be able to sustain their level of sales. Finally, the article ends up with Luis Osuna (CEO of Covirán) statating that the Spanish retailing market is very aggressive on their prices, so in his opinión those should be regulated by the administration, as if not this could end up with unfair competition.

This article encouraged us much more to study the Spanish food retailing market as we wanted to demsotrate that those firms that have the highest market share are not those that are the most efficient ones, as they obtain those high market shares by reducing its profit margin and not by allocating their ressources the most efficient way. This is cleary the case of Mercadona, that as it is not a publicly traded firm, can be engage in more aggresive policies that are going to harm their short-term margin, but that in the long term they will make them better off.

2 Announcement

In terms of difficulty the main problem of this exercise will be to rank supermarket firms based on the *Suppereficiency and unit revenue benchmark* concepts from a revenue efficiency perspective. To achieve this we will use real[1] data from 25 supermarkets operating in the Spanish supermarket market. The data is available to download in the appendix section.

[1] We researched information about 25 firms operating in the Spanish supermarket market and then adapted the data for this exercise. More information is given in the Appenddix.

2.1 Supermarkets market in Spain

The supermarkets market in Spain will be assumed to be a non-empty and finite set $S(C)$. By definition, $S(C)$ will be a n-tuple where n is the number of supermarkets in the supermarkets market or simply the length of $S(C)$ and C are the coordinates set of the supermarket market.

2.2 Coordinates set

The coordinates set of $S(C)$ will be a non-empty, unordered and finite set C. By definition, C will be of length n and formed by ordered pairs c. By definition, $c_i = \{c_{1i}, c_{2i}\}, \forall i \in n$.

2.3 Revenue efficient frontier

The revenue frontier will be an unordered and finite set R where $R \subset C$ and it will contain the most efficient firms for the specific situation market $S(C)$. The set R that contains the most efficient firms for the particular market being analyzed will be referred as a *layer*.

To solve this problem we basically need to solve the following optimization problem:

$max\ \theta$

Subject to:

$\theta \cdot y^{ot} \leq \Sigma_{i=1}^{k} \lambda_i^t \cdot y_i^t$

$x^{ot} \geq \Sigma_{i=1}^{k} \lambda_i^t \cdot x_i^t$

$\lambda \geq 0$

where:

$D_o^t(x^{ot}, y^{ot}) = 1/\theta$

Note that in our case since the firms we analyzed have more than one input and the inputs are in monetary terms and not unit terms, then $x_0 = \Sigma_{i=1}^{n} x_{0i}$ where n is the total amount of firms. But this does not affect the process of the optimization, therefore this exercise can be easily, and exactly, solved following the instructions of the course slides *2 Best Practice Frontiers* and *4 Introduction Benchmarking*.

Since the real data has some problems, as negative observations and non monotonic series, then we will assume a unique input to facilitate the exercise, so:

$x = \Sigma_{i=1}^{n} x_i = inputs$

and, to avoid having negative observations in some inputs, we will assume:

$x_i = [max(x_{i1}, 0), max(x_{i2}, 0), ..., max(x_{in}, 0)]$

3 Data

The data we used comes from three different sources, that include [1] where we extracted information about DIA S.A, [2], where we extracted information about the outputs and inputs of the supermarkets, [4], where we extracted information about Mercadona, and finally [3], where we obtained the prices at which each firm is selling their products from this link, since we could not obtain the real data of the selling price of the supermarkets. That is why we used the *ocu basket* of those supermarkets (in the case of Dia as there are 3 different types of supermarket we use the average of the 3).

All them are referenced with their respective publications details and links at the end of the document where you can find the reference section.

We gave the data a quantitative treatment so it can be used to analyze this exercise. The data, **DMUs.csv** has 11 columns, which are $firms$, y, $y1$, $y2$, x, $x1$, $x2$, $x3$, $x4$, $x5$ and r. We remarked this in the csv file, but we invented some variables so we can use them to solve the exercise. First we invented the column $r = N(\mu = 0.5, \sigma = 0.35)$, then using r we created the two columns $y1$ and $y2$ as follows $y1 = r \cdot y$ and $y2 = (1 - r) \cdot y$. Note $y1 + y2 = y$.

You can **download** the data here

Note y is the operating income, $y1$ the income generated by products bought to external, $y2$ income generated by own branded products, $x1$ procurement, $x2$ the personnel expenses, $x3$ other operating expenses, $x4$ amortization of fixed assets and $x5$ external financing. The table below just includes the firms, $y1$, $y2$ and x, but the original csv file contains all the other variables.

	Unnamed: 0	y1	y2	x
0	Alcampo SAU	1.931190e+09	1.418081e+09	2.967384e+09
1	Condis Supermercats SA	6.992635e+08	3.434796e+08	1.009214e+09
2	Bon preu SA	9.124458e+08	7.233480e+08	1.600846e+09
3	Centros Comerciales Carrefour SA	3.286869e+09	5.204139e+09	8.304552e+09
4	Mercadona SA	1.220157e+10	1.244806e+10	2.374652e+10
5	Lidl Supermercados SAU	2.137385e+08	2.294265e+08	4.229974e+09
6	Distribuidora Internacional De Alimentacion Sa	1.211749e+09	1.047290e+09	2.271638e+09
7	Consum S Coop V	1.616259e+09	1.474103e+09	3.004140e+09
8	AhorraMas SA	1.041988e+09	7.635710e+08	1.670954e+09
9	Dinosol Supermercados SL	5.496858e+08	6.372842e+08	1.172404e+09
10	Vego Supermercados SA	6.802396e+08	3.347404e+08	9.824750e+08
11	World Duty Free Group SA	6.439062e+07	1.416594e+08	5.711670e+08
12	Grup Supeco Maxor SL	4.401805e+08	3.920760e+08	8.120143e+08
13	Supercor SA	2.779243e+08	4.947315e+08	7.451254e+08
14	Grupo el Arbol Distribucion y Supermercados SA	3.834202e+08	3.987508e+08	8.126600e+08
15	Alimerka SA	4.637249e+08	2.525631e+08	6.963000e+08
16	Distribuciones Froiz SA	3.673044e+08	3.681870e+08	7.172577e+08
17	Semark AC Group SA	3.862484e+08	2.924520e+08	6.604337e+08
18	Supersol Spain SL	1.680837e+08	3.292053e+08	5.290710e+08
19	Supermercados Champion SA	2.827300e+08	2.287208e+08	4.824026e+08
20	Distribucion de Supermercados SL	1.960313e+08	3.459409e+08	5.143738e+08
21	Juan Fornes Fornes SA	9.480619e+07	2.140088e+08	3.062957e+08
22	Venta Peio SL	5.382969e+07	3.306247e+07	8.693541e+07

4 Questions

4.1 Define the optimization problem, from a revenue perspective, for the market, taking into account definition *3.3. Remember to assume there is one single input for all the firms, which is* x *where* $x = x1 + x2 + x3 + x4$ and two outputs y_1 and y_2.

4.2 Draw the Data Envelopment Analysis (DEA) technology graph. Remember the vertical axes are y_2/x and the horizontal ones are $y1/x$.

4.3 Solve the optimization problem for all the firms. Explain what θ, D_o and $1/\theta$ represent in the DEA problem.

4.4 Which are the efficient and the inefficient food retailers of the market? Define them by calculating the output distance functions.

4.5 Rank the efficient retailers found in the previous exercise from the best to the worst.

4.6 Divide the Spanish food retailing market by layers and define the firms that belong to each one of them and calculate how many layers are possible in this market.

From now on we are just going to take into consideration the cases of Mercadona and Dia, as are the companies that the article was focusing on.

4.7 Calculate and compare the total costs of the inputs of both firms and their unit costs.

4.8 Calculate and compare the productivity of the inputs of both firms.

4.9 Assuming that DIA is a direct competitor of Mercadona, which strategy would you recommend them to use in order to try to surpass Mercadona or at least to reduce the difference between both firms. Explain it with the results obtained above.

4.10 Virtuous cycle

5 Solutions

5.1 Define the optimization problem, from a revenue perspective, for the market, taking into account definition *3.3*. *Remember to assume there is one single input for all the firms, which is* x *where* $x = x1 + x2 + x3 + x4$ and two outputs y_1 and y_2.

The maximization problem will be:

$max\ \theta$

Subject to:

$\theta \cdot y^{ot} \leq \Sigma_{i=1}^{k} \lambda_i^t \cdot y_i^t$

$x^{ot} \geq \Sigma_{i=1}^{k} \lambda_i^t \cdot x_i^t$

$\lambda \geq 0$

where:

$D_o^t(x^{ot}, y^{ot}) = 1/\theta$

Since we took real data, we encountered some problems in the optimization because some observations of the real inputs were negative. To solve this problem, we took an interesting approach by adding all the inputs and transforming them into one single vector of inputs, so:

$x = \Sigma_{i=1}^{n} x_i$

Then, we assumed for optimization purposes:

$x_i = [max(x_i 1, 0), max(x_{i2}, 0), ..., max(x_{in}, 0)]$

Finally, taking into account all the previous assumptions and modifications, the optimization problem will be:

$max\ \theta$

Subject to:

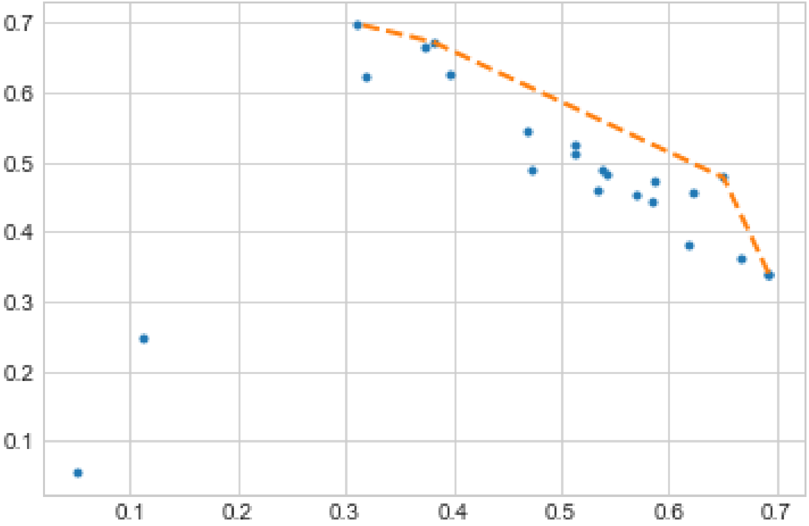
$\theta \cdot y_1 \leq \Sigma_{i=1}^{k} \lambda_i \cdot y_i$

$\theta \cdot y_2 \leq \Sigma_{i=1}^{k} \lambda_i \cdot y_i$

$x \geq \Sigma_{i=1}^{k} \lambda_i \cdot x_i$ where $x_i \geq 0$

$\lambda \geq 0$

5.2 Draw the Data Envelopment Analysis (DEA) technology graph. Remember the vertical axes are y_2/x and the horizontal ones are $y1/x$.



5.3 Solve the optimization problem for all the firms. Explain what θ, D_o and $1/\theta$ represent in the DEA problem.

	Unnamed: 0	y1	y2	x	D
0	Alcampo SAU	1.931190e+09	1.418081e+09	2.967384e+09	1.000000
21	Juan Fornes Fornes SA	9.480619e+07	2.140088e+08	3.062957e+08	1.000000
20	Distribucion de Supermercados SL	1.960313e+08	3.459409e+08	5.143738e+08	1.000000
1	Condis Supermercats SA	6.992635e+08	3.434796e+08	1.009214e+09	1.000000
10	Vego Supermercados SA	6.802396e+08	3.347404e+08	9.824750e+08	0.999506
13	Supercor SA	2.779243e+08	4.947315e+08	7.451254e+08	0.985765
15	Alimerka SA	4.637249e+08	2.525631e+08	6.963000e+08	0.974843
3	Centros Comerciales Carrefour SA	3.286869e+09	5.204139e+09	8.304552e+09	0.962761
8	AhorraMas SA	1.041988e+09	7.635710e+08	1.670954e+09	0.957820
19	Supermercados Champion SA	2.827300e+08	2.287208e+08	4.824026e+08	0.946738
4	Mercadona SA	1.220157e+10	1.244806e+10	2.374652e+10	0.944544
16	Distribuciones Froiz SA	3.673044e+08	3.681870e+08	7.172577e+08	0.931746
9	Dinosol Supermercados SL	5.496858e+08	6.372842e+08	1.172404e+09	0.930726
7	Consum S Coop V	1.616259e+09	1.474103e+09	3.004140e+09	0.927597
22	Venta Peio SL	5.382969e+07	3.306247e+07	8.693541e+07	0.922882
12	Grup Supeco Maxor SL	4.401805e+08	3.920760e+08	8.120143e+08	0.922420
17	Semark AC Group SA	3.862484e+08	2.924520e+08	6.604337e+08	0.912747
2	Bon preu SA	9.124458e+08	7.233480e+08	1.600846e+09	0.910961
18	Supersol Spain SL	1.680837e+08	3.292053e+08	5.290710e+08	0.909481
6	Distribuidora Internacional De Alimentacion Sa	1.211749e+09	1.047290e+09	2.271638e+09	0.892803
14	Grupo el Arbol Distribucion y Supermercados SA	3.834202e+08	3.987508e+08	8.126600e+08	0.877156
11	World Duty Free Group SA	6.439062e+07	1.416594e+08	5.711670e+08	0.356258
5	Lidl Supermercados SAU	2.137385e+08	2.294265e+08	4.229974e+09	0.095723

θ is the farrell technical efficiency measure, $D_o = 1/\theta$ and D_o is the output distance function.

Finally note this table was taken from the layer 1, computed with their respective instructions in the section 6.1.1 of the appendix

5.4 Which are the efficient and the inefficient food retailers of the market? Define them by calculating the output distance functions.

The efficient firms, as we calculated in the last question, are firms $0, 21, 20$ and 1. All the others are inefficient. We can see that by checking the value of D_o (the last column), so if the value is 1, the firm is efficient, otherwise the firm is inefficient.

5.5 Rank the efficient retailers found in the previous exercise from the best to the worst.

To rank the efficient retailers we created a algorithm that is able to compute the suppereffiency of a given DMU. This is discussed with a lot more of details in section 6.3 of the appendix but what we did basically is to use the following formula:

$y_1^o := o$ element of $\frac{y_1}{x}$. Note $y_1^o \subset y_1$ and $y_1^o \in \mathbb{R}$
$y_2^o := o$ element of $\frac{y_2}{x}$. Note $y_2^o \subset y_2$ and $y_2^o \in \mathbb{R}$
$s = 0.001$
$\upsilon_1^o := y_1^o \cdot s$ where $\upsilon_1^o \in \mathbb{R}$
$\upsilon_2^o := y_2^o \cdot s$ where $\upsilon_2^o \in \mathbb{R}$
$d_o := 1/\theta$
$S_o = \|(\upsilon_1^o, \upsilon_2^o)\|_2 \cdot \frac{1}{d_o} \cdot \|(y_1^o, y_2^o)\|_2$ where $S_o \in \mathbb{R}$

This was usefull to compute the following table:

	Unnamed: 0	y1	y2	x	D	S
0	Alcampo SAU	1.931190e+09	1.418081e+09	2.967384e+09	1.0	1.044612
21	Juan Fornes Fornes SA	9.480619e+07	2.140088e+08	3.062957e+08	1.0	1.038885
20	Distribucion de Supermercados SL	1.960313e+08	3.459409e+08	5.143738e+08	1.0	1.015350
1	Condis Supermercats SA	6.992635e+08	3.434796e+08	1.009214e+09	1.0	1.000731

Then it is easy to see that observation 0, which corresponds to Alcampo SAU, is the most efficient among the efficients and Condis Supermercats SA is the less efficient among the efficient supermarkets due to the S in the S column.

5.6 Divide the Spanish food retailing market by layers and define the firms that belong to each one of them and calculate how many layers are possible in this market.

To compute first the first that belong to each layer, we created an algorithm that first computes the D_o value for each DMU and then removes all the DMU'S with the condition of D_o. Doing so we discovered there were possible 8 layers. This is again discussed further in appendix 6.1.2.

5.6.1 Layers 1, 2, 3, 4, 5, 6, 7 and 8 respectively

Note how we remove on every layer the rows that have a $D = 1$, and then we compute again the minimzation. Finally it is relevant to observe how the values are ordered by D so we can easily see on the table how the most efficient firms are at the top and the less efficient at the bottom of the table on each layer.

	Unnamed: 0	y1	y2	x	D
0	Alcampo SAU	1.931190e+09	1.418081e+09	2.967384e+09	1.000000
21	Juan Fornes Fornes SA	9.480619e+07	2.140088e+08	3.062957e+08	1.000000
20	Distribucion de Supermercados SL	1.960313e+08	3.459409e+08	5.143738e+08	1.000000
1	Condis Supermercats SA	6.992635e+08	3.434796e+08	1.009214e+09	1.000000
10	Vego Supermercados SA	6.802396e+08	3.347404e+08	9.824750e+08	0.999506
13	Supercor SA	2.779243e+08	4.947315e+08	7.451254e+08	0.985765
15	Alimerka SA	4.637249e+08	2.525631e+08	6.963000e+08	0.974843
3	Centros Comerciales Carrefour SA	3.286869e+09	5.204139e+09	8.304552e+09	0.962761
8	AhorraMas SA	1.041988e+09	7.635710e+08	1.670954e+09	0.957820
19	Supermercados Champion SA	2.827300e+08	2.287208e+08	4.824026e+08	0.946738
4	Mercadona SA	1.220157e+10	1.244806e+10	2.374652e+10	0.944544
16	Distribuciones Froiz SA	3.673044e+08	3.681870e+08	7.172577e+08	0.931746
9	Dinosol Supermercados SL	5.496858e+08	6.372842e+08	1.172404e+09	0.930726
7	Consum S Coop V	1.616259e+09	1.474103e+09	3.004140e+09	0.927597
22	Venta Peio SL	5.382969e+07	3.306247e+07	8.693541e+07	0.922882
12	Grup Supeco Maxor SL	4.401805e+08	3.920760e+08	8.120143e+08	0.922420
17	Semark AC Group SA	3.862484e+08	2.924520e+08	6.604337e+08	0.912747
2	Bon preu SA	9.124458e+08	7.233480e+08	1.600846e+09	0.910961
18	Supersol Spain SL	1.680837e+08	3.292053e+08	5.290710e+08	0.909481
6	Distribuidora Internacional De Alimentacion Sa	1.211749e+09	1.047290e+09	2.271638e+09	0.892803
14	Grupo el Arbol Distribucion y Supermercados SA	3.834202e+08	3.987508e+08	8.126600e+08	0.877156
11	World Duty Free Group SA	6.439062e+07	1.416594e+08	5.711670e+08	0.356258
5	Lidl Supermercados SAU	2.137385e+08	2.294265e+08	4.229974e+09	0.095723

	Unnamed: 0	y1	y2	x	D
10	Vego Supermercados SA	6.802396e+08	3.347404e+08	9.824750e+08	1.000000
13	Supercor SA	2.779243e+08	4.947315e+08	7.451254e+08	1.000000
8	AhorraMas SA	1.041988e+09	7.635710e+08	1.670954e+09	1.000000
19	Supermercados Champion SA	2.827300e+08	2.287208e+08	4.824026e+08	0.985788
15	Alimerka SA	4.637249e+08	2.525631e+08	6.963000e+08	0.985048
3	Centros Comerciales Carrefour SA	3.286869e+09	5.204139e+09	8.304552e+09	0.981006
4	Mercadona SA	1.220157e+10	1.244806e+10	2.374652e+10	0.975902
16	Distribuciones Froiz SA	3.673044e+08	3.681870e+08	7.172577e+08	0.963239
7	Consum S Coop V	1.616259e+09	1.474103e+09	3.004140e+09	0.961974
9	Dinosol Supermercados SL	5.496858e+08	6.372842e+08	1.172404e+09	0.957610
12	Grup Supeco Maxor SL	4.401805e+08	3.920760e+08	8.120143e+08	0.957364
17	Semark AC Group SA	3.862484e+08	2.924520e+08	6.604337e+08	0.952518
2	Bon preu SA	9.124458e+08	7.233480e+08	1.600846e+09	0.949183
22	Venta Peio SL	5.382969e+07	3.306247e+07	8.693541e+07	0.944346
18	Supersol Spain SL	1.680837e+08	3.292053e+08	5.290710e+08	0.937158
6	Distribuidora Internacional De Alimentacion Sa	1.211749e+09	1.047290e+09	2.271638e+09	0.927563
14	Grupo el Arbol Distribucion y Supermercados SA	3.834202e+08	3.987508e+08	8.126600e+08	0.905702
11	World Duty Free Group SA	6.439062e+07	1.416594e+08	5.711670e+08	0.373544
5	Lidl Supermercados SAU	2.137385e+08	2.294265e+08	4.229974e+09	0.098735

	Unnamed: 0	y1	y2	x	D
15	Alimerka SA	4.637249e+08	2.525631e+08	6.963000e+08	1.000000
19	Supermercados Champion SA	2.827300e+08	2.287208e+08	4.824026e+08	1.000000
3	Centros Comerciales Carrefour SA	3.286869e+09	5.204139e+09	8.304552e+09	1.000000
18	Supersol Spain SL	1.680837e+08	3.292053e+08	5.290710e+08	0.992934
4	Mercadona SA	1.220157e+10	1.244806e+10	2.374652e+10	0.991689
16	Distribuciones Froiz SA	3.673044e+08	3.681870e+08	7.172577e+08	0.978694
7	Consum S Coop V	1.616259e+09	1.474103e+09	3.004140e+09	0.976720
17	Semark AC Group SA	3.862484e+08	2.924520e+08	6.604337e+08	0.974408
9	Dinosol Supermercados SL	5.496858e+08	6.372842e+08	1.172404e+09	0.974015
12	Grup Supeco Maxor SL	4.401805e+08	3.920760e+08	8.120143e+08	0.971867
2	Bon preu SA	9.124458e+08	7.233480e+08	1.600846e+09	0.965356
22	Venta Peio SL	5.382969e+07	3.306247e+07	8.693541e+07	0.963095
6	Distribuidora Internacional De Alimentacion Sa	1.211749e+09	1.047290e+09	2.271638e+09	0.941402
14	Grupo el Arbol Distribucion y Supermercados SA	3.834202e+08	3.987508e+08	8.126600e+08	0.920485
11	World Duty Free Group SA	6.439062e+07	1.416594e+08	5.711670e+08	0.395776
5	Lidl Supermercados SAU	2.137385e+08	2.294265e+08	4.229974e+09	0.100370

	Unnamed: 0	y1	y2	x	D
17	Semark AC Group SA	3.862484e+08	2.924520e+08	6.604337e+08	1.000000
18	Supersol Spain SL	1.680837e+08	3.292053e+08	5.290710e+08	1.000000
4	Mercadona SA	1.220157e+10	1.244806e+10	2.374652e+10	1.000000
22	Venta Peio SL	5.382969e+07	3.306247e+07	8.693541e+07	1.000000
9	Dinosol Supermercados SL	5.496858e+08	6.372842e+08	1.172404e+09	0.996015
7	Consum S Coop V	1.616259e+09	1.474103e+09	3.004140e+09	0.994791
2	Bon preu SA	9.124458e+08	7.233480e+08	1.600846e+09	0.992814
12	Grup Supeco Maxor SL	4.401805e+08	3.920760e+08	8.120143e+08	0.991937
16	Distribuciones Froiz SA	3.673044e+08	3.681870e+08	7.172577e+08	0.988444
6	Distribuidora Internacional De Alimentacion Sa	1.211749e+09	1.047290e+09	2.271638e+09	0.963422
14	Grupo el Arbol Distribucion y Supermercados SA	3.834202e+08	3.987508e+08	8.126600e+08	0.930178
11	World Duty Free Group SA	6.439062e+07	1.416594e+08	5.711670e+08	0.398593
5	Lidl Supermercados SAU	2.137385e+08	2.294265e+08	4.229974e+09	0.101781

	Unnamed: 0	y1	y2	x	D
2	Bon preu SA	9.124458e+08	7.233480e+08	1.600846e+09	1.000000
9	Dinosol Supermercados SL	5.496858e+08	6.372842e+08	1.172404e+09	1.000000
7	Consum S Coop V	1.616259e+09	1.474103e+09	3.004140e+09	1.000000
16	Distribuciones Froiz SA	3.673044e+08	3.681870e+08	7.172577e+08	1.000000
12	Grup Supeco Maxor SL	4.401805e+08	3.920760e+08	8.120143e+08	0.997468
6	Distribuidora Internacional De Alimentacion Sa	1.211749e+09	1.047290e+09	2.271638e+09	0.969211
14	Grupo el Arbol Distribucion y Supermercados SA	3.834202e+08	3.987508e+08	8.126600e+08	0.941675
11	World Duty Free Group SA	6.439062e+07	1.416594e+08	5.711670e+08	0.456275
5	Lidl Supermercados SAU	2.137385e+08	2.294265e+08	4.229974e+09	0.102788

	Unnamed: 0	y1	y2	x	D
12	Grup Supeco Maxor SL	4.401805e+08	3.920760e+08	8.120143e+08	1.000000
14	Grupo el Arbol Distribucion y Supermercados SA	3.834202e+08	3.987508e+08	8.126600e+08	1.000000
6	Distribuidora Internacional De Alimentacion Sa	1.211749e+09	1.047290e+09	2.271638e+09	0.984025
11	World Duty Free Group SA	6.439062e+07	1.416594e+08	5.711670e+08	0.505463
5	Lidl Supermercados SAU	2.137385e+08	2.294265e+08	4.229974e+09	0.110538

	Unnamed: 0	y1	y2	x	D
6	Distribuidora Internacional De Alimentacion Sa	1.211749e+09	1.047290e+09	2.271638e+09	1.000000
11	World Duty Free Group SA	6.439062e+07	1.416594e+08	5.711670e+08	0.537965
5	Lidl Supermercados SAU	2.137385e+08	2.294265e+08	4.229974e+09	0.117646

	Unnamed: 0	y1	y2	x	D
11	World Duty Free Group SA	64390625.0	141659375.0	5.711670e+08	1.000000
5	Lidl Supermercados SAU	213738479.5	229426520.5	4.229974e+09	0.448214

From now on we are just going to take into consideration the cases of Mercadona and Dia (Distribuidora Internacional De Alimentación SA), as are the companies that the article was focusing on.

Note y is the operating income, $x1$ procurement, xl personnel expenses, $x2$ other operating expenses, $x3$ amortization of fixed assets, $x4$ external financing and xk capital.

	Unnamed: 0	Mercadona SA	DIA SA
0	Revenues (€)	1.080000e+02	1.110000e+02
1	Y (quantity)	2.282373e+08	2.035170e+07
2	X1 (quantity)	2.282373e+08	2.035170e+07
3	w1 (€)	7.905974e+01	9.665771e+01
4	Xl (nº workers)	9.800000e+04	2.520900e+04
5	wl (€)	3.292965e+04	6.184299e+02
6	X2 (quantity)	2.328024e+08	2.578085e+07
7	w2 (€)	7.650000e+00	1.039000e+01
8	X3 (quantity)	6.974858e+09	7.792800e+07
9	w3 (€)	8.816222e-02	2.699415e-01
10	X4 (€)	7.792212e+07	2.514875e+08
11	w4 (€)	1.016130e+00	1.016130e+00
12	Xk (quantity of capital)	1.592100e+07	6.678000e+07
13	wk (€)	5.400000e-02	4.710000e-02

5.7 Calculate and compare the total costs of the inputs of both firms and their unit costs.

	Unnamed: 0	Mercadona SA	DIA SA
0	Input 1 cost	1.804438e+10	1.967149e+09
1	Labour cost	3.227106e+09	1.559000e+07
2	Input 2 cost	1.780938e+09	2.678630e+08
3	Input 3 costs	6.149190e+08	2.103600e+07
4	Input 4 costs	7.917900e+07	2.555440e+08
5	Cost of capital	8.597340e+05	3.145338e+06
6	Total cost	2.374738e+10	2.530327e+09
7	Unit cost	1.040469e+02	1.243300e+02
8	Unit revenue - Unit cost	3.953102e+00	-1.333001e+01

We can observe that Mercadona has a higher total cost than DIA, there are just to inputs that are higher for Dia than for Mercadona, those are the cost of input 4 and the cost of capital. This last one can be easily explained as

Mercadona is not publicly traded, so they can not benefit from the advantage of being able to raise capital as DIA does. Apart from that, we can observe a lower unit cost for Mercadona, this is due to the fact that the amount of sales is much higher for Mercadona than for DIA. It is worth mentioning the difference between both, as each unit they sale is 20,28 euros more costly for DIA. Finally, we want to add that Mercadona are earning a profit of 3,95 euros per unit sold, while DIA is incurring a loss of 13,33 euros per unit sold. In our opinion, this per unit losses can be explained by the huge cost of input 4 (the financial cost of the firm), as in the studied period it increases a lot the per unit cost, but in the future this cost will probably be banished or at least reduced. This financing cost can have positive effects in the future, like decreasing some cost due to an increase of an input productivity or to boost the amount of sales. This cost in the present future is positive if the decrease in the per unit cost of the firm in the future is higher than the harm that are suffering in the current period.

5.8 Calculate and compare the productivity of the inputs of both firms.

	Unnamed: 0	Mercadona SA	DIA SA	A-B	Difference %
0	Input 1 productivity	1.000000	1.000000	0.000000	0.000000
1	Labour productivity	2328.952098	807.318922	1521.633176	-0.653355
2	Input 2 productivity	0.980391	0.789412	0.190979	-0.194799
3	Input 3 productivity	0.032723	0.261160	-0.228437	6.980975
4	Input 4 productivity	2.929044	0.080925	2.848119	-0.972371
5	Capital productivity	14.335614	0.304757	14.030856	-0.978741

To begin with, we can see that both firms have an input 1 productivity of 1, this is due to the assumption that for producing one unit of Y, it requires a unit of procurement (x1). Mercadona has a labour productivity 65% higher than DIA, this means that each worker in Mercadona produce 1521,63 units of Y more than one from DIA. Input 2 difference is not that notable, being the 19% higher for Mercadona (even though is not that notable than the difference of the other productivity, it is a big difference). The input 3 productivity is the only one that is better for DIA than for Mercadona, and it has a difference of the 698%. Finally, Mercadona has a higher input 4 and capital productivity, almost doubling the one of DIA, being 97 and 98% respectively. We can explain that with the reasons that we mentioned in the previous exercise, the first one being that DIA has a really high amount of financing in this period, but this high amount of financing it is probably a thing of the studied period and not a thing that is sustained in the majority of their periods. Apart from that, having this high amount of financing makes them be less productive in the current period as the benefits (like an increase in Y) might come in future periods (the benefits that can be achieved with this financing are not received in the moment the financing is done, but in the future). And the fact that DIA is a publicly traded firm, so they have much more capital and then the amount of Y they produce for each unit of capital is inferior (as Mercadona has a higher Y than DIA and has really few capital).

5.9 Assuming that DIA is a direct competitor of Mercadona, who would benefit if they engage in a price war? Which strategy would you recommend them to use in order to try to surpass Mercadona or at least to reduce the difference between both firms? Explain it with the results obtained above.

A price war would be very harmful for DIA, as their unitary cost is much higher than the one of Mercadona (If this was not a real case but a case study, we would assume that if the unit cost is higher than the sell price the firm would have to exit the market as they are having looses, that's the case of DIA). We would recommend DIA to focus on a couple things. The first one would be to try to increase the amount of sales, and the second one is to try to increase the labour productivity as it is much lower than the one of Mercadona. All this could be achieved by a proper allocation of the resources obtained by input 4. They could spend this money on a proper training for their workers in order to have a better and much qualified set of professionals (the amount of Y that each worker produces would be bigger), or by engaging on marketing campaigns that would boost their amount of sales. Finally, we want to add that if those measures are achieved, and there is not another period with financial costs that are that high, the unit cost of the firm will be reduced. With per low unit costs they can engage into a price war by lowering their markup (and so reduce prices), so they can gain additional market share and their profits can get boosted. This strategies will probably not make them be better off than Mercadona, but they would help to reduce the difference with them and surpass other firms from the market (the market was defined in the first questions of our activity) that are not that much of a monster as Mercadona is in terms of sales and market share.

5.10 Explain the Virtuous cycle of Mercadona.

Mercadona has a high demand of goods (both from external brands and produced for their brand Hacendado, that normally it is outsourced but labeled with their brand). This high volume of demand allows them to have high bargaining power with suppliers, this bargaining power lowers their fixed costs. As they have lower costs, it allows Mercadona to lower their prices and still be profitable. As the price is lower and demand is downward sloping, a higher volume of products is demanded to Mercadona (sales increase). As sales increase, it also increases the demand of Mercadona for goods to the firms that supply them... And the cycle is restarted.

6 Appendix

6.1 Data manipulation

To handle all the data manipulation we used the programming language Python. We will do an special emphasis in this section, since we think it is worth to know all the steps we followed to get the results exposed in this exercise. Even though, we did the exercise completely in Python, it can be replicated using any other software, as for instance Onfront.

Before anything else the Python libraries we will use are Pandas, Numpy, Matplotlib and Scipy.

```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import linprog
from scipy.spatial import ConvexHull
```

Listing 1: Python libraries used

Then we proceeded to read the data from the csv file. Since the original data contains 5 inputs, we created a new column x which is the result of the sum of all the input costs, so $x = x1 + x2 + x3 + x4 + x5$. We also converted the *y1* and *y2* variables to float types (decimals) so we can do mathematical operations with them.

```python
data = pd.read_csv(’DMUs.csv’)
data = data.iloc[0:23]
data.y1 = data.y1.astype(float)
data.y2 = data.y2.astype(float)
data[’x’] = data.x1 + data.x2 + data.x3 + data.x4 + data.x5
```

Listing 2: Read data and add new column x

Now, to compute the Farrel Tecnical Efficiency θ we modelled the mathematical optimization and encapsulated that optimization into a function named $farrell$ which takes four arguments: *obs*, *y1*, *y2* and *x*. These arguments respectively stand for the observation selected to compute the Farrel output with, the output of product 1, output of product 2 and total input used.

```python
def farrell(obs, y1, y2, x):
    obj = [-1] + len(x.values) * [0]

    lhs_ineq = [np.append(y1.values[obs], -1 * y1.values),  # y1 constraint left side
        np.append(y2.values[obs], -1 * y2.values), # y2 constraint left side
        np.append(0, x.values)]  # x constraint left side

    rhs_ineq = [0,  # y1 constraint right side
        0,  # y2 constraint right side
        x.values[obs]]  # x1 constraint right side

    opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,
          method="revised simplex")

    return opt
```

Listing 3: Creation of the Farrell Technical Efficiency computer function

6.1.1 Layer 1

At this point, we are already able to compute the layers based on the $D_o = 1/\theta$ criterion. To compute the first layer, we will loop through all the firms and compute D_o for every firm so we can see which are initially on the frontier and which not. Note the firms in the frontier will have a $D_o = 1$ for that particular layer. To do so, we will create a new column D where $D = D_o = 1/\theta$. Finally, note that aside from computing the D values for all the firms, we also sorted them by the D value.

```
frontier1 = pd.DataFrame(data[['Unnamed: 0', 'y1', 'y2', 'x']])
frontier1D = []
for firm in range(frontier1.shape[0]):
    frontier1D.append(1/farrell(firm, frontier1.y1, frontier1.y2,
    frontier1.x).x[0])
frontier1['D'] = frontier1D
frontier1.sort_values(by='D', ascending=False, inplace=True) # Sort
print(frontier1.to_latex()) # Remove the to_latex method
```

Listing 4: Compute D for all the firms and sort them by the D value

	Unnamed: 0	y1	y2	x	D
0	Alcampo SAU	1.931190e+09	1.418081e+09	2.959410e+09	1.000000
21	Juan Fornes Fornes SA	9.480619e+07	2.140088e+08	3.062303e+08	1.000000
1	Condis Supermercats SA	6.992635e+08	3.434796e+08	1.009214e+09	1.000000
20	Distribucion de Supermercados SL	1.960313e+08	3.459409e+08	5.143738e+08	1.000000
10	Vego Supermercados SA	6.802396e+08	3.347404e+08	9.822220e+08	0.999753
13	Supercor SA	2.779243e+08	4.947315e+08	7.451254e+08	0.985757
15	Alimerka SA	4.637249e+08	2.525631e+08	6.957530e+08	0.975004
3	Centros Comerciales Carrefour SA	3.286869e+09	5.204139e+09	8.304552e+09	0.962474
8	AhorraMas SA	1.041988e+09	7.635710e+08	1.670954e+09	0.955262
19	Supermercados Champion SA	2.827300e+08	2.287208e+08	4.822501e+08	0.944792
4	Mercadona SA	1.220157e+10	1.244806e+10	2.374652e+10	0.943011
16	Distribuciones Froiz SA	3.673044e+08	3.681870e+08	7.172577e+08	0.930181
9	Dinosol Supermercados SL	5.496858e+08	6.372842e+08	1.172404e+09	0.929589
7	Consum S Coop V	1.616259e+09	1.474103e+09	3.004140e+09	0.925759
22	Venta Peio SL	5.382969e+07	3.306247e+07	8.693541e+07	0.921588
12	Grup Supeco Maxor SL	4.401805e+08	3.920760e+08	8.120143e+08	0.920521
2	Bon preu SA	9.124458e+08	7.233480e+08	1.589306e+09	0.915339
17	Semark AC Group SA	3.862484e+08	2.924520e+08	6.604337e+08	0.910386
18	Supersol Spain SL	1.680837e+08	3.292053e+08	5.290710e+08	0.909395
6	Distribuidora Internacional De Alimentacion Sa	1.211749e+09	1.047290e+09	2.271638e+09	0.890879
14	Grupo el Arbol Distribucion y Supermercados SA	3.834202e+08	3.987508e+08	8.126600e+08	0.875786
11	World Duty Free Group SA	6.439062e+07	1.416594e+08	5.248480e+08	0.387624
5	Lidl Supermercados SAU	2.137385e+08	2.294265e+08	4.229974e+09	0.095583

6.1.2 Layer 2

To continue, we will remove the firms with a $D = 1$ on the last layer so we can continue calculating the super efficiency. We will repeat this process successively on every layer until the computation of the last layer. Since this is pure repetition, we encapsulated this process into a function named *layercomputer* that basically computes the layer n where n is the specific layer being analyzed among all the possible layers based on the original data.

```
def layercomputer(layer_number, data):
    frontier = data
    for i in range(layer_number - 1):
        frontier = pd.DataFrame(frontier[frontier.apply(lambda x:
    np.abs(x['D'] - 1) >= 0.0001, axis=1)])
        frontierD = []
        for firm in range(frontier.shape[0]):
            frontierD.append(1/farrell(firm, frontier.y1, frontier.
    y2, frontier.x).x[0])
        frontier[’D’] = frontierD
        frontier.sort_values(by=’D’, ascending=False, inplace=True)
      # Sort
    return frontier

print(layercomputer(1, frontier1).to_latex())
```

Listing 5: Creation of the function able to compute the layer for any iteration

	Unnamed: 0	y1	y2	x	D
0	Alcampo SAU	1.931190e+09	1.418081e+09	2.967384e+09	1.000000
21	Juan Fornes Fornes SA	9.480619e+07	2.140088e+08	3.062957e+08	1.000000
20	Distribucion de Supermercados SL	1.960313e+08	3.459409e+08	5.143738e+08	1.000000
1	Condis Supermercats SA	6.992635e+08	3.434796e+08	1.009214e+09	1.000000
10	Vego Supermercados SA	6.802396e+08	3.347404e+08	9.824750e+08	0.999506
13	Supercor SA	2.779243e+08	4.947315e+08	7.451254e+08	0.985765
15	Alimerka SA	4.637249e+08	2.525631e+08	6.963000e+08	0.974843
3	Centros Comerciales Carrefour SA	3.286869e+09	5.204139e+09	8.304552e+09	0.962761
8	AhorraMas SA	1.041988e+09	7.635710e+08	1.670954e+09	0.957820
19	Supermercados Champion SA	2.827300e+08	2.287208e+08	4.824026e+08	0.946738
4	Mercadona SA	1.220157e+10	1.244806e+10	2.374652e+10	0.944544
16	Distribuciones Froiz SA	3.673044e+08	3.681870e+08	7.172577e+08	0.931746
9	Dinosol Supermercados SL	5.496858e+08	6.372842e+08	1.172404e+09	0.930726
7	Consum S Coop V	1.616259e+09	1.474103e+09	3.004140e+09	0.927597
22	Venta Peio SL	5.382969e+07	3.306247e+07	8.693541e+07	0.922882
12	Grup Supeco Maxor SL	4.401805e+08	3.920760e+08	8.120143e+08	0.922420
17	Semark AC Group SA	3.862484e+08	2.924520e+08	6.604337e+08	0.912747
2	Bon preu SA	9.124458e+08	7.233480e+08	1.600846e+09	0.910961
18	Supersol Spain SL	1.680837e+08	3.292053e+08	5.290710e+08	0.909481
6	Distribuidora Internacional De Alimentacion Sa	1.211749e+09	1.047290e+09	2.271638e+09	0.892803
14	Grupo el Arbol Distribucion y Supermercados SA	3.834202e+08	3.987508e+08	8.126600e+08	0.877156
11	World Duty Free Group SA	6.439062e+07	1.416594e+08	5.711670e+08	0.356258
5	Lidl Supermercados SAU	2.137385e+08	2.294265e+08	4.229974e+09	0.095723

Now that we encapsulated the layercomputer, we just need to provide the layer number to know all the firms that belong to that specific layer

6.1.3 Layer 3

	Unnamed: 0	y1	y2	x	D
15	Alimerka SA	4.637249e+08	2.525631e+08	6.963000e+08	1.000000
19	Supermercados Champion SA	2.827300e+08	2.287208e+08	4.824026e+08	1.000000
3	Centros Comerciales Carrefour SA	3.286869e+09	5.204139e+09	8.304552e+09	1.000000
18	Supersol Spain SL	1.680837e+08	3.292053e+08	5.290710e+08	0.992934
4	Mercadona SA	1.220157e+10	1.244806e+10	2.374652e+10	0.991689
16	Distribuciones Froiz SA	3.673044e+08	3.681870e+08	7.172577e+08	0.978694
7	Consum S Coop V	1.616259e+09	1.474103e+09	3.004140e+09	0.976720
17	Semark AC Group SA	3.862484e+08	2.924520e+08	6.604337e+08	0.974408
9	Dinosol Supermercados SL	5.496858e+08	6.372842e+08	1.172404e+09	0.974015
12	Grup Supeco Maxor SL	4.401805e+08	3.920760e+08	8.120143e+08	0.971867
2	Bon preu SA	9.124458e+08	7.233480e+08	1.600846e+09	0.965356
22	Venta Peio SL	5.382969e+07	3.306247e+07	8.693541e+07	0.963095
6	Distribuidora Internacional De Alimentacion Sa	1.211749e+09	1.047290e+09	2.271638e+09	0.941402
14	Grupo el Arbol Distribucion y Supermercados SA	3.834202e+08	3.987508e+08	8.126600e+08	0.920485
11	World Duty Free Group SA	6.439062e+07	1.416594e+08	5.711670e+08	0.395776
5	Lidl Supermercados SAU	2.137385e+08	2.294265e+08	4.229974e+09	0.100370

6.1.4 Layer 8

	Unnamed: 0	y1	y2	x	D
5	Lidl Supermercados SAU	213738479.5	229426520.5	4.229974e+09	1.0

As you see, the last layer is layer 8, which contains the specific firm *Lidl Supermercados SAU*.

6.2 Data frontier with the convex hull algorithm

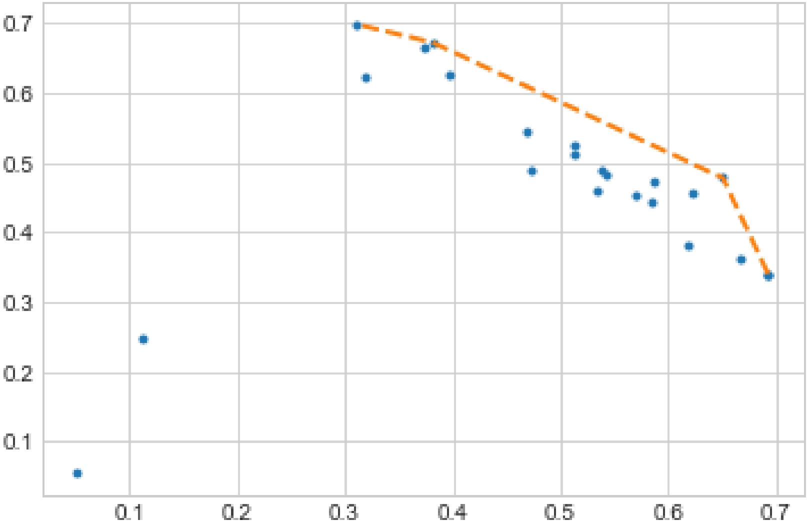
As a matter of curiosity we have also computed the first layer using the mathematical concept of the convex hull. However, this is just a curiosity, since it is not useful to know the Farrell θ nor the distance from the point to the frontier.

```
rng = np.random.default_rng()
# 30 random points in 2-D
points = np.array([[xx, yy] for xx, yy in zip(data.y1/data.x, data.
    y2/data.x)])
hull = ConvexHull(points)

plt.plot(points[:, 0], points[:, 1], '.')


# the list that contains the frontier is hull.vertices
plt.plot(points[hull.vertices[1:-1], 0],
         points[hull.vertices[1:-1], 1], '--', lw=2)
plt.show()
```

Listing 6: Computation of layer 1 using the convex hull algorithm and its visualization



Note this graph was used also to answer the question 5.2

6.3 Superefficiency

To compute the suppereficiency we applied a pretty interesting approach, which was in part based in the slides of the *Introduction: Benchmarking* presentation. This because the slides just illustrate how is it computed, but it does not give any formula nor step to do it.

To calculate the suppereficiency S_o of an obsevation o we need to do the following:

Note that to compute d_o it is necessary to do it with υ_1^o and υ_2^o instead of y_1^o and y_2^o.

$y_1^o := o$ element of $\frac{y_1}{x}$. Note $y_1^o \subset \frac{y_1}{x}$ and $y_1^o \in \mathbb{R}$

$y_2^o := o$ element of $\frac{y_2}{x}$. Note $y_2^o \subset \frac{y_2}{x}$ and $y_2^o \in \mathbb{R}$

$s = 0.001$

$\upsilon_1^o := y_1^o \cdot s$ where $\upsilon_1^o \in \mathbb{R}$

$\upsilon_2^o := y_2^o \cdot s$ where $\upsilon_2^o \in \mathbb{R}$

$d_o := 1/\theta$

$S_o = \|(\upsilon_1^o, \upsilon_2^o)\|_2 \cdot \frac{1}{d_o} \cdot \|(y_1^o, y_2^o)\|_2$ where $S_o \in \mathbb{R}$

Then, we encapsulated this math computation into a function named *superefficiency* in Python, so we can compare all the efficient points quickly.

```python
def supperefficiency(obs, base_data, smallness=0.001):

    def distance(x, y):
        return (x**2 + y**2)**0.5

    data_not_obs = base_data.copy()
    # efficients = base_data[base_data.D == 1]

    data_not_obs.at[obs, 'y1'] = base_data.loc[obs, 'y1'] * smallness
    data_not_obs.at[obs, 'y2'] = base_data.loc[obs, 'y2'] * smallness

    end_point = distance(
        base_data.loc[obs, 'y1']/base_data.loc[obs, 'x'],
        base_data.loc[obs, 'y2']/base_data.loc[obs, 'x']
        )

    d_o = 1/farrell(obs, data_not_obs.y1, data_not_obs.y2, data_not_obs.x).x[0]

    start_point = distance(
        data_not_obs.loc[obs, 'y1']/data_not_obs.loc[obs, 'x'],
        data_not_obs.loc[obs, 'y2']/data_not_obs.loc[obs, 'x']
    )

    intersection_point = start_point / d_o

    supper_efficiency = end_point / intersection_point

    return supper_efficiency
```

Listing 7: Computation of super-efficiency and its encapsulation into a function

Then, to calculate, the most efficient points of the first layer we just need to apply the superefficiency computation to the observations who have got a $D_o = 1$ in the computation of the fist layer.

```python
supperefficient_data = frontier1[frontier1.D == 1].copy()
S = []
for index in frontier1[frontier1.D == 1].index:
    S.append(supperefficiency(index, data[['Unnamed: 0', 'y1', 'y2', 'x']]))
```

```python
supperefficient_data['S'] = S
supperefficient_data.sort_values(by='S', ascending=False, inplace=True)  # Sort
print(supperefficient_data.to_latex())
```

Listing 8: Superefficiency table for the fist layer

	Unnamed: 0	y1	y2	x	D	S
0	Alcampo SAU	1.931190e+09	1.418081e+09	2.967384e+09	1.0	1.044612
21	Juan Fornes Fornes SA	9.480619e+07	2.140088e+08	3.062957e+08	1.0	1.038885
20	Distribucion de Supermercados SL	1.960313e+08	3.459409e+08	5.143738e+08	1.0	1.015350
1	Condis Supermercats SA	6.992635e+08	3.434796e+08	1.009214e+09	1.0	1.000731

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