Due: Oct 14, 2022

Implementing a dynamic delta hedging strategy.

Delta-hedging is a hedging strategy that aims to replicate the value of a financial derivative, such as a Call option, written on a traded asset through dynamically buying (or selling) a proper number of shares of the underlying asset and borrowing from (or lending to) a bank.

Description of the delta-hedging process:

1. Let the hedging period be from a start-date t_0 to an end-date t_N. At start date t_0, assuming there is an initial cash position in the amount of \$0.

2. At t_0, we sell a European call option contract with expiration date T, strike price K. Assume the option contract is written on one share of stock and $t_0 < t_N \leq T$.

3. To hedge the short position in the European call, we decide to buy δ shares of the underlying stock at t_0, where $\delta = \frac{\partial V}{\partial S}$ is the rate of change of option value V with respect to changes in the underlying price S.

4. As δ changes during the hedging period, we need to re-balance our portfolio (buy/sell stocks) everyday to maintain a long position of δ_i shares of stock for each date t_i, $i = 1, 2, ..., N$. δ_i shall be calculated using implied volatility for each date.

5. For each date $t_i, i = 1, 2, ..., N$, calculate the cumulative hedging error till t_i:

$$HE_i = \delta_{i-1} S_i + B_{i-1} e^{r_{i-1}\Delta t} - V_i$$

   where $B_i = \delta_{i-1} S_i + B_{i-1} e^{r_{i-1}\delta t} - \delta_i S_i$ $(i \geq 1)$ and $B_0 = V_0 - \delta_0 S_0$. S_i, V_i, r_i respectively denote the stock price, option price, risk-free rate at time $t_i, i = 0, 1, ..., N$. Δt represents 1 business day, which is $\frac{1}{252}$ year.

6. For each date t_i, two types of profit-and-loss (PNL) of selling a Call option at time 0 are defined as follows. Assume no transaction costs.

   - PNL = Call option market quote at time 0 − Call option market quote at time t_i.
   - PNL with hedge = Cumulative hedging error.

Project tasks are:

1. Test your delta hedging implementation using the Black-Scholes model.

   - Use the following model to simulate the price series $\{S_0, S_{\Delta t}, S_{2\Delta t}, \cdots, S_T\}$ at N equally-spaced time points over time horizon $[0, T]$ where $\Delta t = \frac{T}{N}$:

   $$S_{t+\Delta t} = S_t + \mu S_t \Delta t + \sigma S_t \sqrt{\Delta t} Z_t,$$

     where $\{Z_t : t = 1, 2, \cdots, N\}$ are independent standard Normal random variables. Generate 1000 sample paths with the following set of parameters: $S_0 = 100$, $T = 0.4$, $\mu = 0.05$, $\sigma = 0.24$, $r = 0.025$, $N = 100$. Plot any 100 of these paths.
   - Apply Black-Scholes formula to obtain the option price series $\{C_0, C_{\Delta t}, C_{2\Delta t}, \cdots, C_T\}$ on each stock price sample path.
   - Assume $S_0 = 100$, $T = 0.4$, $\mu = 0.05$, $\sigma = 0.24$, $r = 0.025$, $N = 100$. Consider a European Call option on S_T with $K = 105$ and $T = 0.4$. Construct the delta-hedging portfolios for all N periods using delta obtained from the B-S model. Repeat this for all 1000 sample paths and report/plot the distribution of the hedging errors.

2. Use the real market data given in the project data files to test the validity of Black-Scholes model by using the Black-Scholes formula to construct the delta-hedging portfolio.

   (a) At each time t_i, calculate the total wealth if we sell a call without putting on any hedge.

   (b) Given t_0, t_N, T and K, the program should output a file "result.csv" containing stock price, option price, implied volatility, delta, hedging error, PNL, PNL with hedge.

   For example, if t_0=2011-07-05, t_N=2011-07-29, T=2011-09-17, K = 500, the output should be like:

date	S	V	...	PNL	PNL (with hedge)
2011-07-05	532.44	44.2			
2011-07-06	535.36	46.9			
...					
2011-07-28	610.94				
2011-07-29	603.69				

   (c) The following data files are provided. Data files should not be modified manually or by other tools.

   - "interest.csv" contains daily risk-free rates in 2011. When calculating implied volatility and delta for each date, use the risk-free rate of corresponding date.
   - "sec_GOOG.csv" contains adjusted closing stock prices in 2011. Assume there is no dividend.
   - "op_GOOG.csv" contains option prices data in 2011. Option price=(**best_bid**+**best_offer**)/2. **cp_flag** is C for call option, P for put option. **exdate** is expiration date. For option at date t_i, time to expiry = (number of business days between t_i and T)/252.

     Note: Feel free to modify the sample codes in the lab session to make it work. It is encouraged to design your own Option classes and you are required to use OOP or generic programming style to finish this project. Using only functions or trivial classes without any member variables are not acceptable.

Project deliverables:

1. Ready-to-compile codes in one zip file

2. A project report which contains the following components:

   - Problem addressed by the project and the model(s) used in the project
   - The structure of the model implementation: functionality of each code block or each code piece.
   - Unittest cases for the code
   - Outcome of the implementation, present discussion on whether the outcome solves the practical problem of hedging call/put options, and draw some conclusions.