Main Expression: (p→q)⊢(p→s)∨(r→q)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Line | Expression | Rule | First Segment | Second Segment | Third Segment |
| 1 | p→q | Data |  |  |  |
| 2 | p∨¬p | LEM |  |  |  |
| ┌--------------------------------------------------------------------------------------------------------------┐ | | | | | |
| 3 | p | Assumption |  |  |  |
| 4 | q | MP | 1 | 3 |  |
| ┌------------------------------------------------------------------------------------------------------┐ | | | | | |
| 5 | r | Assumption |  |  |  |
| 6 | q | Copy | 4 |  |  |
| └------------------------------------------------------------------------------------------------------┘ | | | | | |
| 7 | r→q | MP | 5 | 6 |  |
| 8 | (p→s)∨(r→q) | ∧e2 | 7 |  |  |
| └--------------------------------------------------------------------------------------------------------------┘ | | | | | |
| ┌--------------------------------------------------------------------------------------------------------------┐ | | | | | |
| 9 | ¬p | Assumption |  |  |  |
| ┌------------------------------------------------------------------------------------------------------┐ | | | | | |
| 10 | p | Assumption |  |  |  |
| 11 | ⊥ | ¬e | 9 | 10 |  |
| 12 | s | ⊥e | 11 |  |  |
| └------------------------------------------------------------------------------------------------------┘ | | | | | |
| 13 | p→s | →i | 10-12 |  |  |
| 14 | (p→s)∨(r→q) | ∨i1 | 13 |  |  |
| └--------------------------------------------------------------------------------------------------------------┘ | | | | | |
| 15 | (p→s)∨(r→q) | ∨e | 2 | 3-8 | 9-14 |