Main Expression: (f(x,y) = f(y,x)) ∨ R(z,z) , z = f(x,y), Q(z,z) ⊢ R(f(x,y),z) ∨ Q(f(x,y), f(y,x))

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Line | Expression | Rule | First Segment | Second Segment | Third Segment |
| 1 | Q(z,z) | Data |  |  |  |
| 2 | z=f(x,y) | Data |  |  |  |
| 3 | (f(x,y)=f(y,x))∨R(z,z) | Data |  |  |  |
| ┌--------------------------------------------------------------------------------------------------------------┐ | | | | | |
| 4 | f(x,y)=f(y,x) | Assumption |  |  |  |
| 5 | Q(f(x,y),f(x,y)) | =e | 1 | 2 |  |
| 6 | Q(f(x,y),f(y,x)) | =e | 4 | 5 |  |
| 7 | R(f(x,y),z)∨Q(f(x,y),f(y,x)) | ∨i2 | 6 |  |  |
| └--------------------------------------------------------------------------------------------------------------┘ | | | | | |
| ┌--------------------------------------------------------------------------------------------------------------┐ | | | | | |
| 8 | R(z,z) | Assumption |  |  |  |
| 9 | R(f(x,y),z) | =e | 2 | 8 |  |
| 10 | R(f(x,y),z)∨Q(f(x,y),f(y,x)) | ∨i1 | 9 |  |  |
| └--------------------------------------------------------------------------------------------------------------┘ | | | | | |
| 11 | R(f(x,y),z)∨Q(f(x,y),f(y,x)) | ∨e | 3 | 4-7 | 8-10 |