Main Expression: ∃y¬P(y)⊢∃x(P(x)→∀yP(y))

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Line | Expression | Rule | First Segment | Second Segment | Third Segment |
| 1 | ∀yP(y)∨¬∀yP(y) | LEM |  |  |  |
| ┌--------------------------------------------------------------------------------------------------------------┐ | | | | | |
| 2 | ∀yP(y) | Assumption |  |  |  |
| ┌------------------------------------------------------------------------------------------------------┐ | | | | | |
| 3 | P(z) | Assumption |  |  |  |
| 4 | ∀yP(y) | Copy | 2 |  |  |
| └------------------------------------------------------------------------------------------------------┘ | | | | | |
| 5 | P(z)→∀yP(y) | →i | 3-4 |  |  |
| 6 | ∃x(P(x)→∀yP(y)) | ∃x i | 5 |  |  |
| └--------------------------------------------------------------------------------------------------------------┘ | | | | | |
| ┌--------------------------------------------------------------------------------------------------------------┐ | | | | | |
| 7 | ¬∀yP(y) | Assumption |  |  |  |
| 8 | ∃y¬P(y) | Data |  |  |  |
| ┌------------------------------------------------------------------------------------------------------┐ | | | | | |
| 9 | ¬P(Y0) | Assumption |  |  |  |
| 10 | Y0 | None |  |  |  |
| ┌----------------------------------------------------------------------------------------------┐ | | | | | |
| 11 | P(Y0) | Assumption |  |  |  |
| 12 | ⊥ | ¬e | 9 | 11 |  |
| 13 | ∀yP(y) | ⊥e | 12 |  |  |
| └----------------------------------------------------------------------------------------------┘ | | | | | |
| 14 | P(Y0)→∀yP(y) | →i | 11-13 |  |  |
| 15 | ∃x(P(x)→∀yP(y)) | ∃x i | 14 |  |  |
| └------------------------------------------------------------------------------------------------------┘ | | | | | |
| 16 | ∃x(P(x)→∀yP(y)) | ∃x e | 8 | 9-15 |  |
| └--------------------------------------------------------------------------------------------------------------┘ | | | | | |
| 17 | ∃x(P(x)→∀yP(y)) | ∨e | 1 | 2-6 | 7-16 |