Main Expression: p∨q,¬q∨r⊢p∨r

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Line | Expression | Rule | First Segment | Second Segment | Third Segment |
| 1 | p∨q | Data |  |  |  |
| 2 | ¬q∨r | Data |  |  |  |
| 3 | p∨q⊢¬p→q | Proven i |  |  |  |
| 4 | ¬p→q | Proven e | 3 | 1 |  |
| 5 | ¬q∨r⊢q→r | Proven i |  |  |  |
| 6 | q→r | Proven e | 5 | 2 |  |
| ┌--------------------------------------------------------------------------------------------------------------┐ | | | | | |
| 7 | p | Assumption |  |  |  |
| 8 | p∨r | ∨i1 | 7 |  |  |
| └--------------------------------------------------------------------------------------------------------------┘ | | | | | |
| ┌--------------------------------------------------------------------------------------------------------------┐ | | | | | |
| 9 | q | Assumption |  |  |  |
| 10 | r | MP | 6 | 9 |  |
| 11 | p∨r | ∨i2 | 10 |  |  |
| └--------------------------------------------------------------------------------------------------------------┘ | | | | | |
| 12 | p∨r | ∨e | 1 | 7-8 | 9-11 |
| ┌--------------------------------------------------------------------------------------------------------------┐ | | | | | |
| 13 | ¬q | Assumption |  |  |  |
| 14 | ¬¬p | MT | 4 | 13 |  |
| 15 | p | ¬¬e | 14 |  |  |
| 16 | p∨r | ∨i1 | 15 |  |  |
| └--------------------------------------------------------------------------------------------------------------┘ | | | | | |
| ┌--------------------------------------------------------------------------------------------------------------┐ | | | | | |
| 17 | r | Assumption |  |  |  |
| 18 | p∨r | ∨i2 | 17 |  |  |
| └--------------------------------------------------------------------------------------------------------------┘ | | | | | |
| 19 | p∨r | ∨e | 2 | 13-16 | 17-18 |