Proportional, Integral, Derivative

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Abstract

This article presents an in-depth exploration of the classic PID (Proportional-Integral-Derivative) control mechanism and its implementation using C++ as the core language with R as the testing engine. The PID control function is mathematically defined in terms of control-measurement error and is widely utilized in real-world industrial analysis and control applications.

The article delves into the C++ implementation of the PID controller, providing a detailed analysis of the core computation. Furthermore, it discusses the incorporation of precision matching for scalar and factor types, offering insights into the efficiency and precision considerations in the implementation.

Overall, this article serves as a comprehensive guide to understanding the PID control mechanism and its practical application in the C++ programming language, along with its integration with R for testing purposes.

Keywords: C++, R, PID

PID is a classic control mechanism. It controls an output based on periodic measurements using the error term, its integral and its derivative. The technique is a common, even *ubiquitous*, method for real-world industrial analysis and control.

Mathematically, the control function $u(t)$ in terms of control-measurement error ϵ amounts to:

$$u(t) = K_p\epsilon(t) + K_i \int \epsilon(\tau)d\tau + K_d \frac{d\epsilon(t)}{dt}$$

This article applies C++ as the core PID implementation with use of R as the testing engine.

1. C++ Implementation

See the core computation in the listing below.

```
const scalar_type p = control_ - measure_;
const scalar_type i = i_ + p;
const scalar_type d = p - p_;
out_ = kp_ * p + ki_ * i + kd_ * d;
p_ = p;
i_ = i;
```

The implementation uses trailing underscore to mark PID class members. This helps to differentiate member variable access from stack or register variable access. The computation benefits from this notation as well; the

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trailing underscore matches the *prime* in (p', i', d') whose values persist in volatile memory between computation cycles. The output calculation derives from the current monotonic cycle's proportion, integral and derivative. The order of declarations reflects an input-compute-output approach. The compiler may decide to rearrange the order for optimisation purposes.

The `const` declarations are computationally redundant but semantically useful. The initial statements load the core's register set with the new monotonic cycle's (p, i, d) triplet. The latest output value derives from these. The fourth statement relies on operator precedence; multiplication precedes addition. Finally, the cycle persists (p', i') for the next monotonic cycle.

The PID derivative term deserves some comment. The output does not divide by time despite its delta time denominator. The denominator factors in the integral factor. Divide the factor by delta time. This obviates a divide operation, an expensive machine operation in cycle times. The integral factor pre-divides by dt.

1.1. Full Class

The full `pid::controller` class listing appears below.

```
#pragma once

namespace pid {
template <typename T, typename U = T> class controller {
public:
  typedef T scalar_type;
  typedef U factor_type;

  controller(factor_type kp, factor_type ki, factor_type kd)
      : kp_(kp), ki_(ki), kd_(kd),
        // proportional, integral and derivative terms
        p_(0), i_(0), d_(0),
        // control point, measure point, output of PID controller
        control_(0), measure_(0), out_(0) {}

  void set_measure(scalar_type measure) { measure_ = measure; }
  scalar_type measure() const { return measure_; }

  void set_control(scalar_type control) { control_ = control; }
  scalar_type control() const { return control_; }

  //! \brief Resets the controller.
  //! \details Resetting the PID controller restarts the proportional error as
  //! the difference between the new control and the existing measure. The
  //! integral and derivative reset to zero. Always reset after setting a new
  //! control.
  void reset() {
    p_ = control_ - measure_;
    i_ = 0;
    d_ = 0;
  }

  void monotonic() {
    const scalar_type p = control_ - measure_;
    const scalar_type i = i_ + p;
    const scalar_type d = p - p_;
```

```
    out_ = kp_ * p + ki_ * i + kd_ * d;
    p_ = p;
    i_ = i;
    d_ = d;
  }

  scalar_type out() const { return out_; }

  // general accessors

  void set_proportional_factor(factor_type kp) { kp_ = kp; }
  factor_type proportional_factor() const { return kp_; }
  scalar_type proportional() const { return p_; }

  void set_integral_factor(factor_type ki) { ki_ = ki; }
  factor_type integral_factor() const { return ki_; }
  scalar_type integral() const { return i_; }

  void set_derivative_factor(factor_type kd) { kd_ = kd; }
  factor_type derivative_factor() const { return kd_; }
  scalar_type derivative() const { return d_; }

private:
  factor_type kp_, ki_, kd_;
  scalar_type p_, i_, d_;
  scalar_type control_, measure_, out_;
};
} // namespace pid
```

The C++ code defines two types: one for the scalars and another for the factors. This allows for higher precision when computing the output. For example, assuming that the platform supports both 32- and 64-bit floating-point numbers, the application may choose to apply double precision for the computation, but cast to single-precision floats for the persistent scalar terms. Many embedded platforms will support only 32-bit single-precision floating-point arithmetic. The C++ implementation therefore defaults to factor precision matching scalar precision.

The full implementation additionally persists the *derivative* term in volatile memory. This is not strictly necessary. The derivative becomes redundant once applied to the output. Its value may prove useful for diagnostics.

2. C++ with R Wrapper

Import the wrapper class for R as follows. R has a canny ability to dynamically compile and run chunks of C++. It works well.

```
#include <Rcpp.h>

using namespace Rcpp;

#include "pid_controller.hpp"

typedef pid::controller<double> PIDController;

RCPP_MODULE(mod_pid) {
```

```
  class_<PIDController>("PIDController")
    .constructor<PIDController::scalar_type, PIDController::scalar_type, PIDController::scalar_type>()
    .property("measure", &PIDController::measure, &PIDController::set_measure)
    .property("control", &PIDController::control, &PIDController::set_control)
    .method("reset", &PIDController::reset)
    .method("monotonic", &PIDController::monotonic)
    .property("out", &PIDController::out)
    .property("proportional", &PIDController::proportional)
    .property("integral", &PIDController::integral)
    .property("derivative", &PIDController::derivative);
}

```

Sourcing the C++ compiles the code, building and loading a library. The Rcpp library synthesises an S4 class.

```
require(Rcpp)
```

```
Loading required package: Rcpp
```

```
# Compile and link the C++ code.
Rcpp::sourceCpp("pid_controller.cpp")

# Show the class wrapper.
PIDController
```

```
C++ class 'PIDController' <0000016625fffca0>
Constructors:
    PIDController(double, double, double)

Fields:
    double control
    double derivative [readonly]
    double integral [readonly]
    double measure
    double out [readonly]
    double proportional [readonly]

Methods:
     void monotonic()

     void reset()
```

```
# Create a new PID controller instance.
# Examine its type and structure.
pid <- new(PIDController, 0.1, 0.01, 0.01)
typeof(pid)
```

```
[1] "S4"
```

```
str(pid)
```

```
Reference class 'Rcpp_PIDController' [package ".GlobalEnv"] with 6 fields
 $ control     : num 0
 $ derivative  : num 0
```

```
 $ integral    : num 0
 $ measure     : num 0
 $ out         : num 0
 $ proportional: num 0
 and 18 methods, of which 4 are  possibly relevant:
   finalize, initialize, monotonic, reset
```

3. Testing

How does it work? First set up the control point. Feed in the measurement samples while applying the `monotonic` method until the output matches the control. The name implies that the control hardware runs it periodically at a real-time fixed rate. No faster, no slower. The application is *real-time*.

What does the simulation need for testing purposes? Effectively, the PID controller is a generator function.

3.1. PID Properties

You can access the names of the PID controller fields using the following expression. It uses R's "currying" pipe operator `|>` for nested function-calling clarity.

```
PIDController$fields() |>
  names()
```

```
[1] "control"      "derivative"   "integral"     "measure"      "out"
[6] "proportional"
```

The following expression answers a named numerical vector for a given PID controller. It extracts all the PID controller fields using the property accessors.

```
pid$control <- 10
PIDController$fields() |>
  names() |>
  vapply(\(x, y) y[[x]], numeric(1L), y = pid)
```

```
     control   derivative     integral      measure          out proportional
          10            0            0            0            0            0
```

3.2. PID Generator for Simulation

The listing below defines a `pid_gen` function that wraps a PID controller within a co-routine generator.

```
# Using R's co-routines.
require(coro)
```

```
Loading required package: coro
```

```
#' Compiles a PID co-routine.
#' @param hysteresis Optional hysteresis function.
#' Its result becomes the next PID measurement.
pid_gen <- \(control, measure,
             kp = 0.1, ki = 0.01, kd = 0.05,
             hysteresis = \(pid) pid$measure + pid$out)
  coro::gen({
    pid <- new(PIDController, kp, ki, kd)
    pid$control <- control
    pid$measure <- measure
```

```
    pid$reset()
    repeat {
      PIDController$fields() |>
        names() |>
        vapply(\(x, y) y[[x]], numeric(1L), y = pid) |>
        coro::yield()
      pid$monotonic()
      pid$measure <- do.call(hysteresis, list(pid))
    }
  })

#' Bind generated rows.
#' Useful convenience function. Collects then applies data frame row binding.
rbind_gen <- \(...)
  coro::collect(...) |>
    do.call(what = rbind)
```

The default hysteresis function simply adds the output to the current measure for the next monotonic cycle. This is a gross simplification. In real life, in practice, the lag between output and measurable effect carries additional hysteresis.

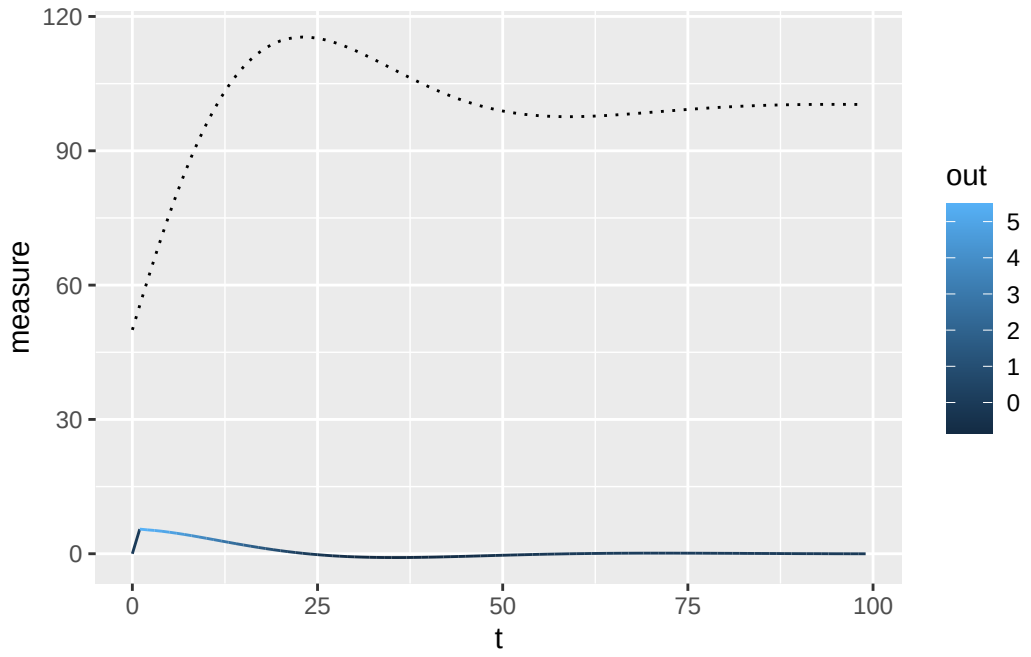
3.3. Measure and Output

Use the generator simulation to plot measurements side-by-side with PID controller output.

```
library(ggplot2)

df <- pid_gen(100, 50, ki = 0.01, kd = 0.01) |>
  rbind_gen(n = 100L)
df <- cbind(df, t = 1:nrow(df) - 1L)

ggplot(df, aes(x = t)) +
  geom_line(aes(y = measure), linetype = "dotted") +
  geom_line(aes(y = out, colour = out))
```

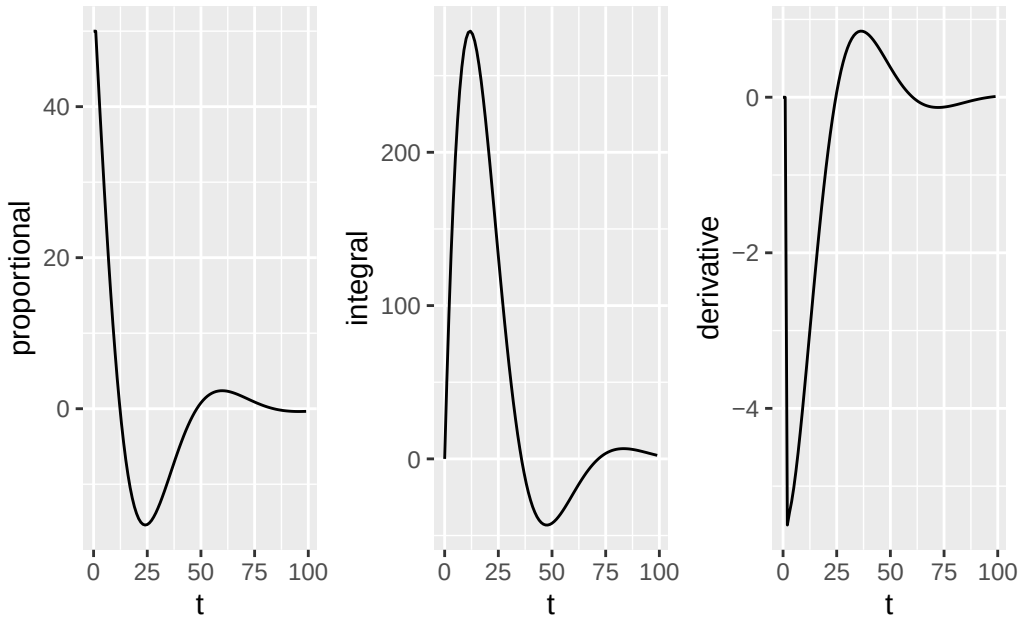


3.4. Proportion, Integral, Derivative

Plot the (p', i', d') terms retained by the PID controller in-between computation cycles.

```
library(patchwork)

(ggplot(df, aes(t, proportional)) + geom_line()) +
  (ggplot(df, aes(t, integral)) + geom_line()) +
  (ggplot(df, aes(t, derivative)) + geom_line())
```



4. Conclusions

Importantly, the PID output operation runs monotonically. Control and measure events may trigger asynchronously and outside the same monotonic process. This is important because the measurements may have a sampling rate that differs from the output's control rate. CAN-based signals have such limitations. PID control operates statefully; its output at time t depends on not only the control and measurement signals but also the previously latched measurements—hysteresis.

The output is **not** the measurement. Its dimensions are the same but the interpretation may not directly add to the next measurement. Interpret the output as a sign and magnitude applied to the output mechanism to move the next measurement towards the control goal. For example, the actual output could be valves that energise and move the measurement indirectly. The sign tells the control logic which valves to open. The magnitude informs the logic of how much energy to apply.

The integral term continuously accumulates. Depending on the application, this may not prove to be ultimately desirable, although it tends to zero by definition.