ENME 303 LAB

Week 7: Matrices III

Nameless Lab

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I. Matrix Transpose
II. Matrix Inverse
III. Determinant of Matrix
IV. Major Determinant Properties

I. Matrix Transpose

The transpose operator switches the columns and rows in a matrix

$$A = \begin{bmatrix} 1 \\ 7 \\ 3 \end{bmatrix} \longrightarrow A^T = \begin{bmatrix} 1 & 7 & 3 \end{bmatrix}$$

In Matlab we can use built-ins:

B = A.'
B = transpose (A)

I. Matrix Transpose

We can also take the conjugate transpose, which is just a transpose matrix with its complex conjugates as elements

$$X = \begin{bmatrix} 1+2i \\ 7 \\ 3-7i \end{bmatrix} \longrightarrow X^T = \begin{bmatrix} 1-2i & 7+0i & 3+7i \end{bmatrix}$$

Those built-ins in Matlab are:

B = A’
B = ctranspose (A)

I. Matrix Transpose

```
%% Exercise: Matrix Transpose

A = [1 2 3 4+4i; 5 6 7 8; 9 10+9i 11 12];

%Normal transpose

B= transpose (A)

C = A.'

%Conjugate transpose

D=ctranspose(A)

E=A'
```

```
A =

   1.0000 + 0.0000i   2.0000 + 0.0000i   3.0000 + 0.0000i   4.0000 + 4.0000i
   5.0000 + 0.0000i   6.0000 + 0.0000i   7.0000 + 0.0000i   8.0000 + 0.0000i
   9.0000 + 0.0000i  10.0000 + 9.0000i  11.0000 + 0.0000i  12.0000 + 0.0000i

C =

   1.0000 + 0.0000i   5.0000 + 0.0000i   9.0000 + 0.0000i
   2.0000 + 0.0000i   6.0000 + 0.0000i  10.0000 + 0.0000i
   3.0000 + 0.0000i   7.0000 + 0.0000i  11.0000 + 0.0000i
   4.0000 + 0.0000i   8.0000 + 0.0000i  12.0000 + 0.0000i

E =

   1.0000 + 0.0000i   5.0000 + 0.0000i   9.0000 + 0.0000i
   2.0000 + 0.0000i   6.0000 + 0.0000i  10.0000 - 9.0000i
   3.0000 + 0.0000i   7.0000 + 0.0000i  11.0000 + 0.0000i
   4.0000 - 4.0000i   8.0000 + 0.0000i  12.0000 + 0.0000i
```

II. Matrix Inverse

The inverse of a matrix A denoted by A^{-1} such that the following relationship is true:

$$AA^{-1} = \mathbf{I}_{n\times n} = A^{-1}A$$

Where I is the identity matrix

In Matlab, we can calculate the inverse of a matrix A using built-in:

inv(A)

Where A is a square matrix. If A is badly scaled or nearly singular, then the inv calculation loses numerical accuracy. Use round() to increase accuracy and avoid errors.

II. Matrix Inverse

```
%% Exercise: Matrix Inverse

A = [1 0 2; -1 5 0; 0 3 -9]

Ainv= inv(A)

%Check if relationship AA^(-1)=I holds

Check = A*Ainv
```

```
A =

     1     0     2
    -1     5     0
     0     3    -9
```

```
Ainv =

    0.8824   -0.1176    0.1961
    0.1765    0.1765    0.0392
    0.0588    0.0588   -0.0980
```

```
Check =

    1.0000         0         0
    0.0000    1.0000         0
         0         0    1.0000
```

III. Determinant of a Matrix

In Matlab we can calculate the determinant of any square matrix A using the following built-in:

det(A)

```
%% Exercise: Matrix Determinant

A = [1 -2 4; -5 2 0; 1 0 3]

det(A)
```

```
A =

     1    -2     4
    -5     2     0
     1     0     3
```

```
ans =

   -32
```

IV. Major Determinant Properties

The determinant of a product of two matrices A and B is the product of their determinants, that is:

$$\det(AB) = \det(A)\det(B)$$

The above theorem says that the determinant is a multiplicative function

IV. Major Determinant Properties

```
%% Exercise Determinant

A = [3 2 1; 1 1 5;6 7 7];

B = [8 1 1; 6 4 2;3 2 5];

round(det(A*B))

det(A)*det(B)

if isequal(round(det(A*B)),det(A)*det(B))

    fprintf("Theorem proved\n")

end
```

```
ans =

      -3848


ans =

      -3848

Theorem proved
```

IV. Major Determinant Properties

If we multiply a scalar, c, by an nxn matrix A, then the determinant will change by a factor of c^n

$$\det(cA) = c^n * \det(A)$$

```
A = [3 2 1; 1 1 5;6 7 7];
C = 2;

[numrow, numcol]=size(A);

LHS=round(det(A*C))
RHS=round(det(A)*(C^numrow))

if isequal(LHS,RHS)
    fprintf("Theorem proved\n")
end
```

```
LHS =

  -296


RHS =

  -296

Theorem proved
```

Lab HW 7 Exercise 1

Write a script that asks the user to input a nxn matrix and returns its inverse.

THEOREM 8.5: Let A be a square matrix. Then the following are equivalent:

(i) A is invertible; that is, A has an inverse A^{-1}.

(ii) $AX = 0$ has only the zero solution.

(iii) The determinant of A is not zero; that is, $\det(A) \neq 0$.

```
% For matrix to be invertible, it must:
%1) Be square
%2) det ~= 0

X = input('Enter your matrix X (in brackets): \n')

[num_row, num_col]=size(X);
```

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