Modelos y Optimización I Trabajo práctico: Problema Combinatorio

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TRABAJO PRÁCTICO

Problema Combinatorio

IngFraTech está evaluando ahora la ubicación de los datacenters necesarios para proveer los nuevos servicios.

Dado que la mayoría de sus clientes se encuentra en Estados Unidos, deciden ubicar allí los datacenter con la idea de hacer mínima la latencia entre los datacenters y los clientes. Como en un principio no saben de dónde serán sus clientes (es decir, a qué estado pertenecerán) la empresa plantea reducir la latencia global del servicio definiéndolo de la siguiente manera:

"La latencia global del sistema es la suma de las latencias de cada estado. El cálculo de latencia de cada estado se hace considerando la distancia con el datacenter más próximo, estimándose 1 ms de latencia por cada milla de distancia."

Actualmente cuenta con 2 datacenters, uno en Oregon y otro en Florida. Por motivos económicos la empresa no puede agregar más de 3 datacenters nuevos.

Dado que no esperan clientes de Hawái y Alaska, estos estados no son tenidos en cuenta para calcular la latencia global.

¿Qué es lo mejor que puede hacer IngFraTech?



Part I

Descripción de la situación problemática

Se trata de un problema de combinatoria, en el que habrá que incluir variables continuas y booleanas.

En esta instancia podemos afirmar que habrá que considerar una variable *latencia* que deberá ser una variable continua; así como una variable booleana que indique si un determinado datacenter se encuentra instalado en un estado específico.

Part II Objetivo

Determinar en qué estados van a estar los 3 nuevos datacenters (DB, DC, DD) durante un período de tiempo para minimizar la latencia global del sistema.

Part III

Hipótesis y supuestos

- Se instalarán los 3 datacenters puesto que cada datacenter agregado va a reducir la latencia.
- Se tomará un punto en cada estado. El mismo será referente para calcular las distancias entre estados y las respectivas latencias. No hay opción de instalar un datacenter en otro punto del estado que el mencionado.
- Para el cálculo se consideran únicamente las distancias, no la cantidad de usuarios por estado. O expresado de otra forma: para el modelo, la distribución de usuarios es uniforme a lo largo de todos los estados y en cada uno de los estados.
- No puede haber dos datacenters en un mismo estado.
- Los costos de instalación y mantenimiento de datacenters, así como cualquier otro costo que pueda implicar la instalación de los mismos, no serán tenidos en cuenta por el modelo.

Part IV

Definición de variables, incluyendo unidades

- L_i : variable continua que indica la latencia (en ms) correspondiente al estado i. $\forall i \in [1,48]$
- DC_i : variable continua que indica la distancia (en millas) del datacenter C al estado i. (ídem para datacenters D y E. DA_i y DB_i son datos conocidos). $\forall i \in [1, 48]$
- YA_i : variable bivalente que vale 0 cuando el estado i tiene la menor latencia. (ídem B, C, D, E)
- YCe_i : variable bivalente que vale 1 cuando el datacenter C está en el Estado i. (ídem D, E)

Constantes

- D_{ij} : distancia entre estado i y estado j.
- ullet M: valor mayor a cualquier distancia posible. Su valor se definirá al momento de pasar el modelo a software.

Part V

Modelo de Programación Lineal Continua

$$YAi + YB_i + YC_i + YD_i + YE_i = 1$$
 $\forall i \in [1, 48]$ (1)

1 Menor Distancia

Cada L_i tendrá como cota superior la distancia al datacenter más próximo, y como cota inferior esa misma distancia en el caso de que el estado i tenga la menor distancia.

 $M \to \infty$

$$DA_i - M * YA_i \le L_i \le DA_i, \quad \forall i \in [1, 48] \tag{2}$$

$$DB_i - M * YB_i \le L_i \le DB_i, \quad \forall i \in [1, 48]$$

$$DC_i - M * YC_i \le L_i \le DC_i, \quad \forall i \in [1, 48]$$

$$DD_i - M * YD_i \le L_i \le DD_i, \quad \forall i \in [1, 48]$$
(5)

$$DE_i - M * YE_i \le L_i \le DE_i, \quad \forall i \in [1, 48]$$

2 Asociación de Datacenter a estado

Se asegura de que cada datacenter pueda ser asignado únicamente a un estado.

$$\sum_{i=1}^{48} YCe_i = 1 \tag{7}$$

$$\sum_{i=1}^{48} YDe_i = 1 \tag{8}$$

$$\sum_{i=1}^{48} Y E e_i = 1 \tag{9}$$

3 Asociación de distancia de un estado al datacenter correspondiente

$$DC_{i} = \sum_{j=1}^{48} D_{ij} * YCe_{j} \qquad \forall i \in [1, 48]$$
 (10)

$$DD_i = \sum_{j=1}^{48} D_{ij} * YDe_j \qquad \forall i \in [1, 48]$$
 (11)

$$DE_i = \sum_{j=1}^{48} D_{ij} * YEe_j, \quad \forall i \in [1, 48]$$
 (12)

Part VI

Funcional

$$Z(min) = \sum_{i=1}^{48} L_i \tag{13}$$

Part VII

Modelo y salida

```
# conjuntos
  \#set ESTADOS := \{1..48\};
2
   set ESTADOS;
   set s dimen 2;
   set s1;
   # Parametros (constantes)
   # Distancias entre ESTADOS
   param Dij{i in ESTADOS, j in ESTADOS};
10
  |\# DA i : variable continua que indica la distancia del datacenter A al estado
11
  # i. ( dem para datacenter B)
   param DAi{i in ESTADOS};
   param DBi{i in ESTADOS};
   #M: valor mayor a cualquier distancia posible.
   param M := 10000;
17
19
  # all distances
   table tab_distances IN "CSV" "distances.csv" : s <- [state1, state2], Dij~
  # distances from datacenters A and B, which correspond to states
  # of Florida and Oregon respectively.
   table tab_florida_distances IN "CSV" "florida_distances.csv" : ESTADOS <- [
       state2], DAi distance;
   table tab_oregon_distances IN "CSV" "oregon_distances.csv" : s1 <- [state2],
       DBi distance;
27
   # Variables
28
   # Li: variable continua que indica la latencia correspondiente al estado i
   var Li\{i in ESTADOS\} >= 0;
   # DC i : variable continua que indica la distancia del datacenter C
   # al estado i. Idem para D y E.
33
   var DCi\{i in ESTADOS\} >= 0;
   var DDi\{i in ESTADOS\} >= 0;
   var DEi\{i in ESTADOS\} >= 0;
   # YAi: variable bivalente que vale 0 cuando la distancia del Datacenter
38
   # A al Estado i es la menor respecto al resto de los Datacenters. ( de m
  # B, C, D, E)
   var YAi\{i in ESTADOS\} >= 0, binary;
   \mbox{var YBi} \{ \mbox{i in ESTADOS} \} >= \mbox{0} \,, \mbox{ binary} \,;
   var YCi\{i in ESTADOS\} >= 0, binary;
43
   var YDi{i in ESTADOS} >= 0, binary;
   var YEi{i in ESTADOS} >= 0, binary;
  #Y Ce i : variable bivalente que vale 1 cuando el datacenter C est en el
  # Estado i.
```

```
var YCei\{i in ESTADOS\} >= 0, binary;
49
        var YDei\{i in ESTADOS\} >= 0, binary;
50
        var YEei{i in ESTADOS} >= 0, binary;
51
52
       # Restricciones
53
       # Cada L i tendr como cota superior la distancia al datacenter m s prximo
54
                  , y
       # como cota inferior esa misma distancia en el caso de que el estado i tenga
55
                 lа
       # menor distancia.
56
        s.t. cota_sup_dcA{i in ESTADOS}: Li[i] <= DAi[i];
57
        s.t. cota_sup_dcB\{i in ESTADOS\}: Li[i] \ll DBi[i];
        s.t. cota_sup_dcC{i in ESTADOS}: Li[i] <= DCi[i];
        s.t. cota_sup_dcD{i in ESTADOS}: Li[i] <= DDi[i];
        s.t. cota_sup_dcE\{i in ESTADOS\}: Li[i] \le DEi[i];
61
62
63
        s.t. cota_inf_dcA\{i in ESTADOS\}: Li[i] >= DAi[i] - M * YAi[i];
        s.t. cota_inf_dcB\{i in ESTADOS\}: Li[i] >= DBi[i] - M * YBi[i];
64
        s.t. \ cota\_inf\_dcC\{i \ in \ ESTADOS\}: \ Li[i] >= DCi[i] - M*YCi[i];
        s.t. \ cota\_inf\_dcD \left\{i \ in \ ESTADOS \right\}: \ Li \left[i \right] >= DDi \left[i \right] - M * YDi \left[i \right];
66
        s.t. \cot_{i} \operatorname{inf_dcE}\{i \text{ in ESTADOS}\}: \operatorname{Li}[i] >= \operatorname{DEi}[i] - \operatorname{M} * \operatorname{YEi}[i];
       # sumatorioa de YXi debe ser 4
69
        s.t. total_datacenters{i in ESTADOS}: YAi[i] + YBi[i] + YCi[i] + YDi[i] + YEi[
                 i ] = 4;
       # Asociacion de datacenter a estado
72
        s.t. asoc_dcC: sum{i in ESTADOS} YCei[i] = 1;
73
        s.t. asoc_dcD: sum{i in ESTADOS} YDei[i] = 1;
74
        s.t. asoc_dcE: sum\{i \text{ in ESTADOS}\} YEei[i] = 1;
75
       # Asociaci n de distancia de un estado al datacenter correspondiente
77
        s.t. asoc_dist_dcC_estadoi\{i in ESTADOS\}: sum\{(i,j) in s\} Dij[i,j] * YCei[j] =
78
                    DCi[i];
        s.t. asoc_dist_dcD_estadoi{i in ESTADOS}: sum\{(i,j) \text{ in } s\} Dij[i,j] * YDei[j] =
79
                    DDi[i];
        s.t. \ asoc\_dist\_dcE\_estadoi\{i \ in \ ESTADOS\}: \ sum\{(i\,,j) \ in \ s\} \ Dij[i\,,j] \ * \ YEei[j] = (i,j) \ asoc\_dist\_dcE\_estadoi\{i \ in \ ESTADOS\}: \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ * \ YEei[j] = (i,j) \ asoc\_dist\_dcE\_estadoi\{i \ in \ ESTADOS\}: \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ * \ YEei[j] = (i,j) \ asoc\_dist\_dcE\_estadoi\{i \ in \ ESTADOS\}: \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ * \ YEei[j] = (i,j) \ asoc\_dist\_dcE\_estadoi\{i \ in \ ESTADOS\}: \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ * \ YEei[j] = (i,j) \ asoc\_dist\_dcE\_estadoi\{i \ in \ ESTADOS\}: \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ * \ YEei[j] = (i,j) \ asoc\_dist\_dcE\_estadoi\{i \ in \ ESTADOS\}: \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ * \ YEei[j] = (i,j) \ asoc\_dist\_dcE\_estadoi\{i \ in \ ESTADOS\}: \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ * \ YEei[j] = (i,j) \ asoc\_dist\_dcE\_estadoi\{i \ in \ ESTADOS\}: \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ * \ YEei[j] = (i,j) \ asoc\_dist\_dcE\_estadoi\{i \ in \ ESTADOS\}: \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ * \ YEei[j] = (i,j) \ asoc\_dist\_dcE\_estadoi\{i \ in \ ESTADOS\}: \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ * \ YEei[j] = (i,j) \ asoc\_dist\_dcE\_estadoi\{i \ in \ ESTADOS\}: \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ * \ YEei[j] = (i,j) \ asoc\_dist\_dcE\_estadoi\{i \ in \ ESTADOS\}: \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ * \ YEei[j] = (i,j) \ asoc\_dist\_dcE\_estadoi\{i \ in \ ESTADOS\}: \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ * \ YEei[j] = (i,j) \ asoc\_dist\_dcE\_estadoi[i,j] \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ sum\{(i,j) \ in \ s\} \ Dij[i,j] \ sum\{(i,j) \ in \ s\} \ sum\{(i,
80
                    DEi [ i ];
81
        # funcional
82
        minimize z: sum{i in ESTADOS} Li[i];
        end;
```

Problen	n: comb			
Rows:	116			
Column	`	nteger, 64 binary)	
Non-ze:		OPTIMAI		
Status Object		.681632 (MINimum)		
Object	IVC. Z = 1401	.001002 (11111111111)		
No.	Row name	Activity	Lower bound Upper bound	
1	cota_sup_dcA [Alabama] 223.749	223.749	
2	cota_sup_dcA [3090.64	
3	cota_sup_dcA [895.532	
4	cota_sup_dcA [4130.54	
5	cota_sup_dcA [2296.9	
6	cota_sup_dcA [1307.57	
7	cota_sup_dcA [-0	
8	cota_sup_dcA [4291.9	
9	cota_sup_dcB [Alabama] 223.749	4077.38	
10	cota_sup_dcB [Arizona] 817.982	1348.72	
11	cota_sup_dcB [Arkansas]	3417.83	
12	cota_sup_dcB [California] 415.951	415.951	
	cota_sup_dcB [0	2018.37	
	cota_sup_dcB[0	5596.86	
	cota_sup_dcB[0	4291.9	
	cota_sup_dcB[0	-0	
	cota_sup_dcC[-1853.4	-0	
	cota_sup_dcC[cota_sup_dcC[0	-0	
	cota_sup_dcC[-1408.12	-0	
	cota_sup_dcC[-1417.72	-0	
	cota_sup_dcC[0	-0	
	cota_sup_dcC[-3591.82	-0	
	cota_sup_dcC[-2296.9	-0	

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-2018.37
                                                                             -0
57
58
         25 cota_sup_dcD [Alabama]
                                      -449.066
                                                                             -0
59
         26 cota_sup_dcD [Arizona]
60
                                      -1377.38
                                                                             -0
61
            cota_sup_dcD [Arkansas]
62
                                             0
                                                                             -0
63
            cota_sup_dcD [ California ]
64
                                      -2824.64
                                                                             -0
65
            cota_sup_dcD [Colorado]
66
                                       -1408.12
                                                                             -0
67
         30 cota_sup_dcD [Connecticut]
68
                                      -2183.71
                                                                             -0
69
            cota_sup_dcD [Florida]
70
                                      -895.532
                                                                             -0
71
            cota_sup_dcD [Oregon]
72
73
                                     -3417.83
                                                                             -0
         33 cota_sup_dcE[Alabama]
74
75
                                      -1295.76
                                                                             -0
         34 cota_sup_dcE[Arizona]
76
77
                                       -3550.92
                                                                             -0
         35 cota_sup_dcE[Arkansas]
78
                                      -2183.71
                                                                             -0
79
         36 cota_sup_dcE[California]
 80
                                       -5007.4
                                                                             -0
81
            cota_sup_dcE [Colorado]
82
                                       3591.82
                                                                             -0
83
            cota_sup_dcE [Connecticut]
84
                                                                             -0
85
         39 cota_sup_dcE[Florida]
86
                                      -1307.57
                                                                             -0
87
         40 cota_sup_dcE[Oregon]
88
                                     -5596.86
                                                                             -0
89
            cota_inf_dcA [Alabama]
90
                                      223.749
                                                       223.749
91
         42 cota_inf_dcA[Arizona]
92
                                         10818
                                                       3090.64
93
94
         43 cota_inf_dcA[Arkansas]
                                         10000
                                                       895.532
95
         44 cota_inf_dcA [California]
96
                                         10416
                                                       4130.54
97
         45 cota_inf_dcA[Colorado]
98
                                         10000
                                                        2296.9
99
         46 cota_inf_dcA [Connecticut]
100
                                         10000
                                                       1307.57
101
         47 cota_inf_dcA [Florida]
102
                                                            -0
103
         48 cota_inf_dcA[Oregon]
104
                                         10000
                                                        4291.9
105
         49 cota_inf_dcB[Alabama]
106
                                      10223.7
                                                       4077.38
107
         50 cota_inf_dcB[Arizona]
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                                         10818
109
                                                       1348.72
         51 cota_inf_dcB[Arkansas]
110
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111
                                                       3417.83
         52 cota_inf_dcB [California]
112
                                      415.951
                                                       415.951
113
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53 cota_inf_dcB[Colorado]
114
115
                                         10000
                                                       2018.37
         54 cota_inf_dcB [Connecticut]
116
                                         10000
                                                       5596.86
117
         55 cota_inf_dcB[Florida]
118
                                         10000
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119
            cota_inf_dcB [Oregon]
120
                                              0
                                                             -0
121
         57 cota_inf_dcC[Alabama]
122
                                        8146.6
                                                             -0
123
         58 cota_inf_dcC[Arizona]
124
                                              0
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125
            \cot a \inf_{-1} dc C [Arkansas]
126
                                       8591.88
                                                             -0
127
             cota_inf_dcC [ California ]
128
                                       8582.28
                                                             -0
129
         61 cota_inf_dcC[Colorado]
130
                                                             -0
131
132
         62 cota_inf_dcC[Connecticut]
                                       6408.18
                                                             -0
133
         63 cota_inf_dcC[Florida]
134
                                        7703.1
                                                             -0
135
         64 cota_inf_dcC[Oregon]
136
                                       7981.63
137
                                                             -0
             cota_inf_dcD [Alabama]
138
                                       9550.93
139
                                                             -0
         66 cota_inf_dcD [Arizona]
140
                                       8622.62
                                                             -0
141
         67 cota_inf_dcD [Arkansas]
142
                                                             -0
143
144
         68 cota_inf_dcD [California]
                                       7175.36
                                                             -0
145
             cota_inf_dcD [Colorado]
146
                                       8591.88
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147
             cota_inf_dcD [Connecticut]
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149
                                       7816.29
                                                             -0
            cota_inf_dcD [Florida]
150
                                       9104.47
151
                                                             -0
         72 cota_inf_dcD [Oregon]
152
153
                                       6582.17
                                                             -0
         73 cota_inf_dcE[Alabama]
154
                                       8704.24
                                                             -0
155
             cota_inf_dcE [Arizona]
156
                                       6449.08
                                                             -0
157
             cota_inf_dcE [Arkansas]
158
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159
             cota_inf_dcE [California]
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161
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             cota_inf_dcE [Colorado]
162
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163
         78 cota_inf_dcE[Connecticut]
164
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165
166
            cota_inf_dcE [Florida]
                                       8692.43
                                                             -0
167
         80 cota_inf_dcE[Oregon]
168
                                       4403.14
                                                             -0
169
170
         81 total_datacenters [Alabama]
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4
171
172
         82 total_datacenters [Arizona]
                                                             4
173
         83 total_datacenters [Arkansas]
174
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175
         84 total_datacenters [California]
176
177
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         85 total_datacenters[Colorado]
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179
         86 total_datacenters [Connecticut]
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         87 total_datacenters [Florida]
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183
            total_datacenters [Oregon]
184
                                             4
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185
         89
            asoc\_dcC
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186
187
         90 asoc_dcD
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         91 \ asoc\_dcE
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189
         92 asoc_dist_dcC_estadoi[Alabama]
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190
         93 asoc_dist_dcC_estadoi [Arizona]
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192
             asoc_dist_dcC_estadoi [Arkansas]
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            asoc_dist_dcC_estadoi [Florida]
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202
            asoc_dist_dcC_estadoi [Oregon]
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204
            asoc_dist_dcD_estadoi [Alabama]
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206
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        101 asoc_dist_dcD_estadoi[Arizona]
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        102 asoc_dist_dcD_estadoi[Arkansas]
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210
        103 asoc_dist_dcD_estadoi [California]
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        104 asoc_dist_dcD_estadoi [Colorado]
213
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        105 asoc_dist_dcD_estadoi [Connecticut]
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        107 asoc_dist_dcD_estadoi[Oregon]
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        108 asoc_dist_dcE_estadoi [Alabama]
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        109 asoc_dist_dcE_estadoi[Arizona]
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        110 asoc_dist_dcE_estadoi[Arkansas]
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226
        111 asoc_dist_dcE_estadoi [California]
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-0
228
        112 asoc_dist_dcE_estadoi [Colorado]
229
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230
        113 asoc_dist_dcE_estadoi [Connecticut]
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        114 asoc_dist_dcE_estadoi[Florida]
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        115 asoc_dist_dcE_estadoi[Oregon]
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236
                                       1457.68
        116 z
237
238
        No. Column name
                                                 Lower bound
                                                                  Upper bound
239
                                  Activity
240
          1 Li [Alabama]
                                                               0
                                       223.749
241
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          2 Li [Arizona]
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242
          3 Li [Arkansas]
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243
          4 Li [California]
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          5 Li [Colorado]
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          7 Li[Florida]
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250
          9 DCi [Alabama]
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251
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         10 DCi[Arizona]
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                                                               0
252
         11 DCi [Arkansas]
253
                                       1408.12
                                                               0
254
         12 DCi[California]
255
                                       1833.67
                                                               0
256
         13 DCi [Colorado]
257
                                              0
                                                               0
258
         14 DCi [Connecticut]
259
                                       3591.82
                                                               0
260
         15 DCi[Florida]
                                        2296.9
                                                               0
261
         16 DCi[Oregon]
                                       2018.37
                                                               0
262
         17 DDi [Alabama]
263
                                       672.815
                                                               0
         18 DDi [Arizona]
                                       2195.36
                                                               0
264
265
         19 DDi[Arkansas]
                                              0
                                                               0
266
         20 DDi[California]
267
                                                               0
                                       3240.59
268
         21 DDi [Colorado]
269
                                                               0
270
                                       1408.12
         22 DDi[Connecticut]
271
                                       2183.71
                                                               0
272
         23 DDi[Florida]
                                                               0
273
                                       895.532
         24 DDi Oregon
                                       3417.83
                                                               0
274
         25 DEi [Alabama]
                                                               0
275
                                       1519.51
         26 DEi[Arizona]
                                        4368.9
                                                               0
276
277
         27 DEi [Arkansas]
                                       2183.71
                                                               0
278
         28 DEi[California]
279
280
                                       5423.35
                                                               0
         29 DEi [Colorado]
281
                                                               0
282
                                       3591.82
         30 DEi [Connecticut]
283
284
```

```
31 DEi[Florida]
                                         1307.57
                                                                  0
285
          32 DEi [Oregon]
                                         5596.86
                                                                  0
286
          33 YAi [Alabama]
                                                0
                                                                  0
                                                                                   1
287
          34 YAi[Arizona]
                                                1
                                                                  0
288
          35 YAi[Arkansas]
289
                                                1
                                                                  0
290
          36 YAi [California]
291
                                                1
                                                                  0
                                                                                   1
292
          37 YAi [Colorado]
293
                                                                  0
294
                                                1
          38 YAi [Connecticut]
295
                                                                  0
296
          39 YAi[Florida]
                                                0
                                                                  0
                                                                                   1
297
                                                                  0
298
          40 YAi [Oregon]
                                                1
                                                                                   1
          41 YBi [Alabama]
                                                                  0
                                                1
                                                                                   1
299
          42 YBi [Arizona]
                                                                  0
300
301
          43 YBi[Arkansas]
                                                                  0
302
303
          44 YBi[California]
                                                0
                                                                  0
304
305
          45 YBi [Colorado]
                                                                  0
                                                1
306
          46 YBi [Connecticut]
307
308
                                                1
                                                                  0
                                                                                   1
          47 YBi[Florida]
                                                                  0
                                                                                   1
                                                1
309
          48 YBi [Oregon]
310
                                                0
                                                                  0
                                                                                   1
          49 YCi [Alabama]
                                                                  0
                                                1
                                                                                   1
311
          50 YCi[Arizona]
                                                0
                                                                  0
312
          51 YCi [Arkansas]
313
                                                                  0
314
315
          52 YCi [California]
                                                                  0
316
                                                1
                                                                                   1
          53 YCi [Colorado]
317
                                                                  0
                                                0
318
                                                                                   1
          54 YCi [Connecticut]
319
                                                                  0
320
          55 YCi[Florida]
                                                1
                                                                  0
                                                                                   1
321
322
          56 YCi [Oregon]
                                                1
                                                                  0
                                                                                   1
          57 YDi[Alabama]
                                                                  0
                                                1
                                                                                   1
323
          58 YDi[Arizona]
                                                                  0
324
          59 YDi[Arkansas]
325
                                                0
                                                                  0
326
          60 YDi[California]
327
                                                                  0
                                                                                   1
                                                1
328
          61 YDi [Colorado]
329
                                                                  0
330
                                                                                   1
          62 YDi [Connecticut]
331
                                                                  0
332
                                                1
                                                                                   1
                                                                  0
          63 YDi[Florida]
                                                1
                                                                                   1
333
          64 YDi [Oregon]
                                                1
                                                                  0
                                                                                   1
334
          65 YEi [Alabama]
                                                                  0
335
                                                1
                                                                                   1
          66 YEi[Arizona]
                                                                  0
                                                1
                                                                                   1
336
337
          67 YEi [Arkansas]
                                                                  0
                                                1
                                                                                   1
338
          68 YEi [California]
339
                                                                  0
                                                                                   1
340
341
          69 YEi [Colorado]
```

```
0
342
343
          70 YEi [Connecticut]
                                                  0
                                                                    0
                                                                                      1
344
          71 YEi[Florida]
                                                                    0
345
          72 YEi Oregon
                                                                    0
                                                                                      1
                                                  1
^{346}
          73 YCei [Alabama]
347
                                                                    0
                                                  0
                                                                                      1
348
          74 YCei [Arizona]
349
                                                                    0
350
                                                  0
          75 YCei [Arkansas]
351
                                                  0
                                                                    0
352
          76 YCei[California]
353
                                                  0
                                                                    0
354
          77 YCei [Colorado]
355
                                                                    0
                                                  1
                                                                                      1
356
          78 YCei [Connecticut]
357
                                                                    0
358
                                                  0
                                                                                      1
          79 YCei[Florida]
359
360
                                                  0
                                                                    0
                                                                                      1
          80 YCei[Oregon] *
                                                  0
                                                                    0
                                                                                      1
361
362
          81 YDei [Alabama]
                                                  0
                                                                    0
                                                                                      1
363
          82 YDei[Arizona]
364
                                                                    0
365
                                                  0
                                                                                      1
          83 YDei [Arkansas]
366
                                                                    0
                                                                                      1
367
                                                  1
          84 YDei[California]
368
                                                                    0
369
          85 YDei [Colorado]
370
                                                  0
                                                                    0
371
372
          86 YDei [Connecticut]
                                                  0
                                                                    0
                                                                                      1
373
          87 YDei[Florida]
374
                                                  0
                                                                    0
375
                                                                                      1
          88 YDei [Oregon]
                                                  0
                                                                    0
376
          89 YEei [Alabama]
377
                                                                    0
                                                  0
                                                                                      1
378
379
          90 YEei [Arizona]
                                                  0
                                                                    0
                                                                                      1
380
          91 YEei [Arkansas]
381
                                                                    0
                                                  0
382
          92 YEei [California]
383
                                                  0
                                                                    0
384
                                                                                      1
          93 YEei [Colorado]
385
                                                  0
                                                                    0
386
          94 YEei [Connecticut]
387
                                                                    0
388
          95 YEei [Florida]
389
                                                  0
                                                                    0
390
                                                                                      1
          96 YEei[Oregon] *
391
                                                  0
                                                                    0
392
     Integer feasibility conditions:
393
394
     KKT.PE: max.abs.err = 1.82e-12 on row 74
395
               \mathrm{max.rel.err} \ = \ 8.41\mathrm{e}{-17} \ \mathrm{on} \ \mathrm{row} \ 74
396
               High quality
397
398
```

```
399 KKT.PB: max.abs.err = 0.00e+00 on row 0
400 max.rel.err = 0.00e+00 on row 0
401 High quality
402
403 End of output
```

Part VIII

Conclusiones

Se decidió realizar la corrida sobre los siguientes estados:

Alabama, Arizona, Arkansas, California, Colorado, Connecticut, Florida, Oregon

Fuera de Florida (Datacenter A) y Oregon (Datacenter B), las 3 nuevas datacenters son las siguientes

Datacenter C: Colorado (77 YCei[Colorado] 1) Datacenter D: Arkansas (83 YDei[Arkansas] 1) Datacenter E: Connecticut (94 YEei[Connecticut] 1)

La latencia global del sistema es: 1457.681632

Las latencias de cada estado son las siguientes:

Li[Alabama] 223.749 Li[Arizona] 817.982

Li[Arkansas] 0

Li[California] 415.951

Li[Colorado] 0

Li[Connecticut] 0

Li[Florida] 0

Li[Oregon] 0

Esto tiene sentido pues solo Alabama, Arizona y California no tienen un datacenter en el estado.

El datacenter C de Colorado es el mas cercano a Arizona (la distancia del datacenter al estado es la misma que su latencia mínima):

DCi[Arizona] 817.982

El datacenter A de Florida es el mas cercano a Alabama (define la cota inferior a la latencia del estado)

cota_inf_dcA[Alabama] 223.749

El datacenter B de Oregon es el mas cercano a California (define la cota inferior a la latencia del estado)

cota_inf_dcB[California] 415.951

Part IX

Heurística

```
compilar: g++ -std=c++11 heuristic.cpp -o heuristic
correr: cat distances.csv | ./heuristic
```

```
heuristic.cpp
   #include <iostream>
1
2
   #include <map>
   #include <cfloat>
3
   void get_problem_values(int &num_of_states, int &num_of_datacenters){
        std::cout << "Insert_number_of_states" << std::endl;</pre>
6
        std::cin >> num_of_states;
        std::cout << "Insert_number_of_datacenters" << std::endl;
8
        std::cin >> num_of_datacenters;
10
11
    void fill_distances_matrix(int num_of_states,
12
                                  std::map<int, std::map<int, double> > &distances) {
13
        for(int i = 1; i \le num_of_states; ++i){
14
             \label{eq:for_int} \mbox{for} \left( \mbox{int} \ j \ = \ i \ + \ 1; \ j <= \ \mbox{num\_of\_states} \, ; \ +\!\!\!\! + \!\!\! j \, \right) \{
15
                 double current_distance;
16
                 std::cout << "insert_distance_between_state_"
17
                            << i << "_and_" << j << std::endl;</pre>
18
                 std::cin >> current_distance;
                 distances[i][j] = current\_distance;
20
                 distances [j][i] = current_distance;
21
22
             /st distance from a state to itself is 0 st/
23
24
             distances[i][i] = 0;
25
26
27
    double calculate_current_latency(int num_of_states,
28
                                         const std::map<int, int> &datacenters.
29
                                         const std::map<int, std::map<int, double> >
30
                                         &distances){
31
        double total\_latency = 0;
32
        /* for each state, check the minimum distance to each datacenter */
33
        for(int i = 1; i \le num\_of\_states; ++i){
34
             double this_state_lat = DBLMAX;
35
             for(auto it = datacenters.begin(); it != datacenters.end(); ++it){
36
                 double read_lat = distances.at(i).at(it->second);
37
                 if (read_lat < this_state_lat){</pre>
38
                      this_state_lat = read_lat;
39
40
41
             total_latency += this_state_lat;
42
43
        return total_latency;
44
45
46
   void manage_manually_allocated_dataceneters (
```

```
int num_of_states, int &num_of_manually_allocated_datacenters,
48
              std::map<int, int> &datacenters
49
              std::map<int, std::map<int, double>> &distances){
50
         std::cout << "How_many_datacenters_are_manually_allocated?" << std::endl;
51
         std::cin >> num_of_manually_allocated_datacenters;
52
53
         for (int i = 1; i <= num_of_manually_allocated_datacenters; ++i){
              int state;
55
              std::cout << "Insert_number_of_state_where_datacenter_" << i
56
                         << "_should_be." << std::endl;</pre>
57
              std::cin >> state;
58
              datacenters [i] = state;
59
              /* display current latency */
60
              std::cout << "latency_after_datacenter_" << i
61
                         << "_was_allocated:_"
62
                          << calculate_current_latency(num_of_states, datacenters,</pre>
63
64
                                                             distances) << std::endl;
         }
65
66
67
    bool state_already_has_datacenter(int state,
68
                                             const std::map<int, int> &datacenters){
69
         for (auto it = datacenters.begin(); it != datacenters.end(); ++it){
70
71
              if (it->second == state) {
                  return true;
72
73
74
75
         return false;
76
77
    int locate_new_datacenter(int num_of_datacenter,
78
                                   \mathbf{int} \ \mathtt{num\_of\_states} \ , \ \mathtt{std} :: \mathtt{map} \!\! < \!\! \mathbf{int} \ , \ \mathbf{int} \!\! > \mathtt{\&datacenters} \ ,
79
                                   {f const} std::map<{f int}, std::map<{f int}, {f double}>>
80
81
                                   &distances){
         /* returns number of state which minimizes latency */
82
         double current_lat = DBLMAX;
         int state_that_minimizes_lat;
84
         for(int i = 1; i \le num_of_states; ++i){
              if (!state_already_has_datacenter(i, datacenters)){
86
                   datacenters [num_of_datacenter] = i;
87
                   if (calculate_current_latency(num_of_states, datacenters,
                       distances)
                       < current_lat){
                       state\_that\_minimizes\_lat = i;
90
91
              }
92
93
         return state_that_minimizes_lat;
94
95
96
    \mathbf{void} \;\; \mathbf{locate\_remaining\_datacenters} \; (\mathbf{int} \;\; \mathbf{num\_of\_states} \;, \;\; \mathbf{int} \;\; \mathbf{num\_of\_datacenters} \;, \;\;
97
                                             int num_of_manually_allocated_datacenters,
98
99
                                             std::map<int, int> &datacenters
                                             std::map < int, std::map < int, double > >
100
                                             &distances){
         for (int i = num_of_manually_allocated_datacenters + 1;
102
              i <= num_of_datacenters; ++i){
103
```

```
datacenters[i] = locate_new_datacenter(i, num_of_states,
104
                                                          datacenters, distances);
105
             /* display current latency */
106
             std::cout << "latency_after_datacenter_" << i << "_located:_" <<
107
                        calculate_current_latency(
108
                                 num_of_states , datacenters , distances ) << std::endl;</pre>
109
110
111
112
113
    void inform_datacenter_positions(const std::map<int, int> &datacenters){
         for (auto it = datacenters.begin(); it != datacenters.end(); ++it){
    std::cout << "datacenter\t" << it->first <<</pre>
114
115
                        "\tlocated_in_state\t" << it->second << std::endl;
116
117
    }
118
119
120
    int main(){
         int num_of_states;
121
         int num_of_datacenters;
122
123
         /* A map is used to store de distance matrix.
          * States are identified with numbers from 1 to n,
125
          st where n is the number of states to consider. st/
126
127
         std::map<int, std::map<int, double> > distances;
128
         /* A map is used to link each datacenter to a state.
129
          *K = datacenter.
130
          * V = state. */
131
         std::map<int, int> datacenters;
132
133
         get_problem_values(num_of_states, num_of_datacenters);
134
135
         fill_distances_matrix(num_of_states, distances);
136
137
         int num_of_manually_allocated_datacenters;
138
139
         manage_manually_allocated_dataceneters(
                  num_of_states , num_of_manually_allocated_datacenters , datacenters ,
140
141
                  distances);
142
         locate_remaining_datacenters(num_of_states, num_of_datacenters,
143
144
                                          {\tt num\_of\_manually\_allocated\_datacenters}\ ,
                                          datacenters, distances);
145
146
         /* inform results */
147
         std::cout << "***_Final_datacenter_positions_****" << std::endl;
148
149
         inform_datacenter_positions (datacenters);
         return 0;
150
```

distances.csv

```
48
2 5
3 2868.1784177465
4 672.8151726255
5 3910.7411305068
6 2077.1452377083
7 1519.5077579655
```

```
1199.4986222794
8
    223.7493394677
   210.4252102536
10
   3324.1942219342
11
   375.2257089259
12
    56.4357710091
13
   813.7155779585
   1044.7398456498
15
    165.0340299937\\
    540.4773059684
17
    1847.1499908138
18
   1091.0322829094\\
19
   1703.4547303348
20
   212.816842606
    753.2459183061
22
    436.760545128\\
23
24
   656.2792698469
   2852.0588142381\\
    1153.1140642009
   3719.6061504499
27
   1647.0895439219
   1286.3997375109\\
29
    2188.3611696487
30
    1398.2602236424\\
    850.7371193588
32
33
   1599.6208068671\\
   372.2093675161
34
    1251.1843173346
35
    4077.3821775963
36
    1051.0725631423
37
    1658.9964185501\\
   583.4074521691
39
    1556.4006060984\\
40
    61.6660433356
41
    1275.3164983935
42
43
   2844.2698934059
    1535.3683688696
44
    983.0281549644
    4058.9796061032
46
47
    520.6289399976
   346.722732452
48
   2055.7517642547
49
   2195.3633128392
    1074.6564829771
51
    817.9816362195
52
   4368.8983013915\\
53
    4058.8562357351\\
   3090.6378199187
    3078.2955554998
56
    652.9574510614
   2493.1423425089\\
58
    2880.6046484923
59
   2056.6878450176\\
    1825.8003590143
61
   3024.0858609986
   2327.8933566076
63
  4682.3639246028
```

```
3951.0378063587
65
    4550.2158738859\\
    3054.42205873
67
    2118.4265973449\\
68
    2431.4866187941\\
69
    2212.1368745122
70
    547.8271383216
71
    1720.8529029881
72
    897.0547741451
    4489.1977165938
74
    4142.2868098361\\
75
    683.9429951775
76
    4243.9969329672
77
    3717.1929493347
    1335.176983076
79
    3228.9139729094
80
81
    1617.4340416483\\
    1348.7170068791
82
    3907.1834067229
    4507.7632004432
84
    3451.3284563862
    1342.8449481975\\
86
    2811.7489989948
87
    1594.7492824989
    304.6040016489
89
    4372.3388621472\\
    3846.5508340495
91
    1383.7138875033
92
    3382.3197560766
93
    2522.0070767663
94
    851.0210240327
    3240.5930582521
96
    1408.1193462358\\
    2183.7128516647\\
98
    1867.8921975774
99
    895.5322444057
    882.9858748922
101
    2661.6694918504
    297.9545644178
103
    686.9378827219
104
    148.5196443215\\
105
    374.724802599
106
    830.0793441766
    133.1107231847
108
    2505.9449694443
109
    1759.60566892
110
    2366.8704387846\\
111
    863.818072177
    100.1013188851
113
    236.2140420465\\
    23.0538798924\\
115
    2196.2325277927
116
117
    485.2828349181
    3050.1034594321
118
    2308.321592842\\
    1953.2153440986
120
   1516.0107427025
```

```
2060.4684236256\\
122
123
     1522.5705106878\\
    949.2174855858
124
    1036.6914655118\\
125
    578.6759461004
126
    3417.8319636982
127
    1717.5448495975
    2323.2052980925
129
    1256.0544480086\\
130
    894.8381828103
131
     616.765049642
132
    603.6759142219\\
133
    2176.7297626343
134
    2193.9499122674
     1653.3308235565
136
     3404.6887339159
137
138
    1189.1402961561
    327.0826055126
139
    1388.6180685584
     1833.6679973809
141
142
    5423.3472152311
    5108.4715702844
143
    4130.5385102755\\
144
145
    4121.1002845619\\
     641.4336791534
146
    3538.0528310694
    3927.4529733155
148
    3097.1937543928
149
    2866.5128820872
150
     4070.3712586562
151
    3373.7037732339
152
    5741.7610750756
153
     5000.1586332735
154
     5605.9389092081
155
     4104.165847916
156
157
    3157.4969452398
    3476.3045536175
158
159
    3255.7477674731
    1122.1693629777
160
     2757.6338063377
161
    193.7582962079\\
162
    5546.2706377884
163
    5193.7171224844
    1728.3472481221\\
165
     5299.3955074081
166
167
    4761.4435405175
    2328.5909304151\\
168
    4277.281128777\\
     2662.8775648463
170
171
     415.9506567564
    4958.0687708492
172
     5562.7014963965
173
174
     4493.5222460133\\
     2358.3271992773
175
    3856.5024348272
    2649.0475568585
177
   1070.3124627028
```

```
5430.5327664793
179
    4893.4060165068
    526.5427934998
181
    4429.5152900794\\
182
    3567.4224858431
183
    1856.2968125313
184
    3591.8183125852
    3275.4722802973
186
    2296.901617194\\
187
    2287.4751061824
188
    1259.8310612474\\
189
    1705.0400877621\\
    2094.5210942818
191
    1263.817123616
    1033.5395038933
193
    2237.2042155199\\
194
195
    1541.1679976617
    3913.0019859645
196
    3167.0941116809
    3774.899349793
198
    2271.9304862058\\
    1323.9084208644\\
200
    1643.3413275397
201
    1422.7850433852
    818.3775515832
203
    924.1301791049
    1642.4800997284
205
    3716.0608213891
206
    3361.172953584
207
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Part X

Resultados de Heurística

Se corrió la heurística con los 8 estados y se consiguieron los siguientes resultados:

```
Insert number of state where datacenter 1 should be. latency after datacenter 1 was allocated: 16236.8

Insert number of state where datacenter 2 should be. latency after datacenter 2 was allocated: 6209.89

latency after datacenter 3 located: 4902.32

latency after datacenter 4 located: 2353.21

latency after datacenter 5 located: 1937.26

*** Final datacenter positions ****

datacenter 1 located in state 7

datacenter 2 located in state 8

datacenter 3 located in state 6

datacenter 4 located in state 5

datacenter 5 located in state 4
```

Los datacenters seleccionador quedaron en los estados California Colorado y Conetticut (4, 5 y 6) dejando la latencia en 1937.26 Este resultado es peor comparado con el modelo de GLPK que dejó la latencia en 1457

Se corrió la heurística con los 48 estados y se consiguieron los siguientes resultados:

```
Insert number of state where datacenter 1 should be. latency after datacenter 1 was allocated: 68827.2

Insert number of state where datacenter 2 should be. latency after datacenter 2 was allocated: 44071.9 latency after datacenter 3 located: 32303.7 latency after datacenter 4 located: 27360.3 latency after datacenter 5 located: 22764.1

*** Final datacenter positions **** datacenter 1 located in state 8 datacenter 2 located in state 35 datacenter 3 located in state 48 datacenter 4 located in state 47 datacenter 5 located in state 46
```

Los datacenters seleccionador quedaron en los estados Wyoming, Wisconsin y west virginia dejando la latencia en 22764.1