First Session

Monday, February 6, 2023 9:05 AM

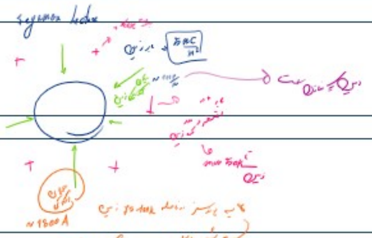
(22-32) - TA: Saturday 13-15

4pts → TA   midterm: 6pts   [7-8: 10 pts

02, 02, 7 [illegible]

Integration

Segment [illegible]



[illegible] 50 mA [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] × [illegible]

[illegible]

[illegible]

Tribo-electricity [illegible]

[illegible]

[illegible]

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

[illegible]

[illegible] q.nc

[illegible]

[illegible]

most Molecules tend to be bipolar

واندروالس

$$E = \left(\frac{q}{4\pi\varepsilon_0 (x-\frac{d}{2})^2} + \frac{-q}{4\pi\varepsilon_0 (x+\frac{d}{2})^2} \right) \hat{x}$$

if $x \ll 1$
$(1+x)^n \simeq 1+nx$

فرض $x \gg d$

$$\frac{q}{4\pi\varepsilon_0} \left[\frac{-1}{(x+\frac{d}{2})^2} + \frac{1}{(x-\frac{d}{2})^2} \right] \hat{x}$$

$$E = \frac{q}{4\pi\varepsilon_0 x^2} \left[\frac{-1}{(1+\frac{d}{2x})^2} + \frac{1}{(1-\frac{d}{2x})^2} \right] \hat{x}$$

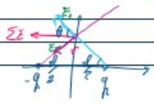
$$\Rightarrow \vec{E} = \frac{q}{4\pi\varepsilon_0 x^2} \left[-(1+\frac{d}{2x})^{-2} + (1-\frac{d}{2x})^{-2} \right]$$

$$= \frac{q}{4\pi\varepsilon_0 x^2} \Big[\underbrace{-(1-\frac{d}{x}) + (1+\frac{d}{x})}_{\frac{2d}{x}} \Big] = \boxed{\frac{qd}{2\pi\varepsilon_0 x^3}}$$

گشتاور دوقطبی ✓ $P = qd$

$$-q \xrightarrow{\vec{P}} +q$$

$$\boxed{\vec{E} = \frac{\vec{P}}{2\pi\varepsilon_0 x^2}}$$



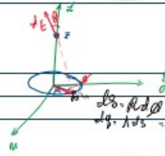
$$r_- = r_+ = \sqrt{\frac{d^2}{4} + r^2}$$

$$tg\theta = \frac{r}{\frac{d}{2}} = \boxed{\frac{2r}{d}}$$

$$\Sigma E_x = ? = -E_+ \cos\theta - E_- \cos\theta$$

$$E_x = \frac{-q\cos\theta}{4\pi\varepsilon_0} \left(\frac{1}{r_+^2} + \frac{1}{r_-^2} \right)$$

$$E_x = \frac{-2q\cos\theta}{4\pi\varepsilon_0 (r^2 + (\frac{d}{2})^2)}$$

$$\cos\theta = \frac{\frac{d}{2}}{\sqrt{r^2+(\frac{d}{2})^2}} = \frac{-qd\,(r^2+(\frac{d}{2})^2)^{-\frac{3}{2}}}{4\pi\varepsilon_0} = \frac{-qd}{4\pi\varepsilon_0 r^3}\left(1+\left(\frac{d}{2r}\right)^2\right)^{-\frac{3}{2}} = \frac{-qd}{4\pi\varepsilon_0 r^3} \quad \frac{-\vec{p}}{4\pi\varepsilon_0 r^3}$$

[illegible] → E ? [illegible]

dq → λ

$ds = R\,d\varphi$

$dq = \lambda ds = \lambda R\,d\varphi$

$\lambda = \frac{q}{2\pi R}$

$$dE = \frac{dq}{4\pi\varepsilon_0 (R^2+z^2)} = \frac{\lambda R\,d\varphi}{4\pi\varepsilon (R^2+z^2)}$$

$$dE_z = dE\cos\theta = \frac{\lambda R z}{4\pi\varepsilon_0}(R^2+z^2)^{-\frac{3}{2}}\,d\varphi$$

$$\cos\theta = \frac{z}{(R^2+z^2)^{\frac{1}{2}}}$$

$$E_z = \int_{\varphi=0}^{2\pi} \frac{\lambda R z}{4\pi\varepsilon_0}(R^2+z^2)^{-\frac{3}{2}}\,d\varphi = \frac{2\pi \lambda R\, z}{4\pi\varepsilon_0}(R^2+z^2)^{-\frac{3}{2}} \qquad \lambda = \frac{q}{2\pi R}$$

$$\boxed{= \frac{q z}{4\pi\varepsilon_0}(R^2+z^2)^{-\frac{3}{2}}}$$

$$\xrightarrow{z \gg R} = \frac{q}{4\pi\varepsilon_0 z^2}$$

$\lambda = \frac{q}{\frac{2\pi R}{3}}$

$dq = \lambda ds$

$\boxed{E = E_x}$

$$dE = \frac{dq}{4\pi\varepsilon_0 R^2} = \frac{\lambda\,ds}{4\pi\varepsilon_0 R^2} = \frac{\lambda R\,d\theta}{4\pi\varepsilon_0 R^2}$$

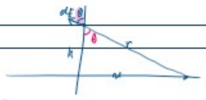
$$dE_x = \frac{\lambda\,d\theta}{4\pi\varepsilon_0 R}\cos\theta$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 R}\int_{-60}^{60} d\theta\cos\theta = \frac{3\sqrt{3}\,q}{2\pi R \times 4\pi\varepsilon_0}\ ?$$

$$dE = \frac{dq}{4\pi\varepsilon_0 (r-x)^2} = \frac{\lambda\,dx}{4\pi\varepsilon_0 (r-x)^2} \qquad u = r-x \qquad dx = -du$$

$$E = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{-\lambda\,du}{4\pi\varepsilon_0 u^2} = \frac{\lambda}{4\pi\varepsilon_0}\times\frac{1}{r-x}\Bigg|_{-\frac{L}{2}}^{\frac{L}{2}}$$

$$dE = \frac{dq}{4\pi\varepsilon_0 r^2} = \frac{\lambda\,dx}{4\pi\varepsilon_0 (h^2+x^2)}$$



$$dE_y = \frac{\lambda\,dx\,\cos\theta}{4\pi\varepsilon_0 (h^2+x^2)} \qquad \cos\theta \to \frac{h}{(x^2+h^2)^{\frac{1}{2}}}$$

$$E_y = \int_{-\infty}^{+\infty}\frac{\lambda h\,dx\,(x^2+h^2)^{-\frac{3}{2}}}{4\pi\varepsilon_0} = \frac{\lambda h}{4\pi\varepsilon_0}\int dx\,(x^2+h^2)^{-\frac{3}{2}} = \frac{\lambda}{h\,4\pi\varepsilon_0}\int_{-\infty}^{\infty}\left(\left(\frac{x}{h}\right)^2+1\right)^{-\frac{3}{2}} \ldots$$

θ [illegible]

$$dE_y = \frac{\lambda\,dx}{4\pi\varepsilon_0 r^2}\cos\theta$$

$$r = \frac{h}{\cos\theta}$$

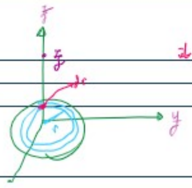
$$x = h\,tg\theta$$

$$dx = \frac{h\,d\theta}{\cos^2\theta}$$

$$\rightarrow dE_y = \frac{\lambda\,dx\cos\theta}{4\pi\varepsilon_0 h^2}\cos\theta$$

$$\Rightarrow dE_y = \frac{\lambda\cos\theta\,d\theta}{h\,4\pi\varepsilon_0}$$

$$E_y = \frac{\lambda}{h\,4\pi\varepsilon_0}\sin\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\lambda}{2\pi\varepsilon_0 h}$$

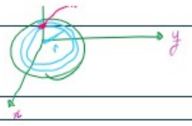


$$E_z = \frac{q z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$$ ← [illegible]

$$dE_z = \frac{z\,dq}{\ldots}$$

$$\frac{dq}{dA} = \delta$$

$A = \pi r^2$



$$dE_z = \frac{z\,dq}{4\pi\varepsilon(z^2+r^2)^{3/2}}$$

$$\frac{dq}{dA} = \sigma$$

$A = \pi r^2$

$dA = 2\pi r\,dr$

$dq = \sigma \times (2\pi r\,dr)$

$\Downarrow$

$$dE_z = \frac{\sigma z\,(2\pi r\,dr)}{4\pi\varepsilon_0 (z^2+r^2)^{3/2}}$$

$u = R^2 + z^2$

$(1+a)^n = 1 + na$

$a << 1$

$$E_z = \frac{\sigma z}{2\varepsilon_0}\int_{z^2}^{z^2+R^2} \frac{\frac{1}{2}\,du}{u^{3/2}} = \frac{\sigma z}{2\varepsilon_0}\,\frac{u^{-\frac{1}{2}}}{-\frac{1}{2}}\Bigg|_{z^2}^{R^2+z^2} = \frac{-\sigma z}{\varepsilon_0}\left(\frac{1}{\sqrt{R^2+z^2}} - \frac{1}{z}\right)$$

$$\Rightarrow = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{(z^2+R^2)^{\frac{1}{2}}}\right]$$

$$\rightarrow \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1 + \left(\frac{R^2}{z^2}\right)\right)^{-\frac{1}{2}}\right] = \frac{\sigma R^2}{4 z^2 \varepsilon_0} \xrightarrow{\times \frac{\pi}{\pi}} \frac{\sigma\,\pi R^2}{4 z^2 \pi \varepsilon_0} = \frac{q}{4\pi\varepsilon_0 z^2}$$

$$1 - 1 + \frac{R^2}{2z^2}$$

A q

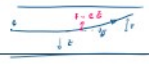
$dA = dr\,r\,d\phi$

$dq = \sigma\,dA = \sigma r\,dr\,d\phi$

$$dE = \frac{dq}{4\pi\varepsilon_0(r^2+z^2)}$$

$$\rightarrow dE_z = \frac{\sigma r\,dr\,d\phi}{4\pi\varepsilon_0(r^2+z^2)}\cos\theta \rightarrow \frac{z}{(z^2+R^2)^{\frac{1}{2}}}$$

$$\Rightarrow dE_z = \frac{\sigma z}{4\pi\varepsilon_0}\int_{r=0}^{R}\int_{\phi=0}^{2\pi} \frac{r\,dr\,d\phi}{(z^2+r^2)^{\frac{3}{2}}}$$

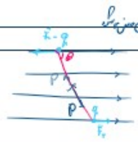


$x = v t$

$a = \frac{eE}{m}$

$y = \frac{1}{2}at^2$

$\rightarrow y = \frac{1}{2}\frac{qE}{m}\left(\frac{x}{v}\right)^2$



$$\tau = \left(\frac{\vec{d}}{2} \times \vec{F}\right) + \left(\frac{-\vec{d}}{2} \times \vec{F}\right)$$

qE $-qE$

$\tau = q\vec{d} \times \vec{E} = \vec{p} \times \vec{E}$

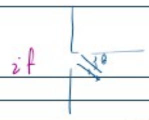
$\tau = pE\sin\theta$

$$\frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{k q_1 q_2}{r^2}$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

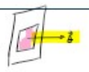
$dV = v\,dt\,A$

$\phi = \frac{dV}{dt} = \dot{V} = vA$



$\phi = \frac{dV}{dt} = \dot{V} = vA\cos\theta$

$\phi = \vec{v}\cdot\vec{A}$



$\phi = \int \vec{v}\cdot d\vec{A}$

$I = \frac{dP}{dA}$

$\int \vec{I}\cdot d\vec{A}$

$I = \frac{dP}{dA} \rightarrow \frac{\text{توان خروجی}}{\text{سطح}}$

شار نوری $\vec{\Phi} = I A$ ، کمیت

$\Phi = \int \vec{I}\cdot d\vec{A}$

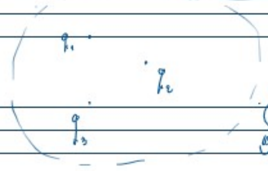
$4\pi a^2$

شار نوری : $\oint \vec{I}\cdot d\vec{A}$

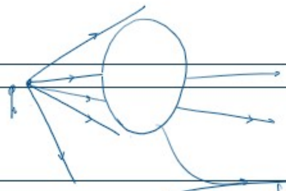
با تغییر حجم کره ← شار نوری ؟ ← ثابت خواهد بود

حتی اگر مکعب شود یعنی با تغییر شکل ، شار نوری ثابت خواهد بود

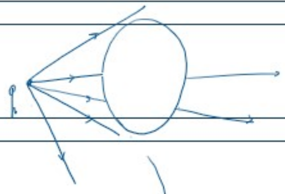
~~شار نوری وابسته به شکل~~



شار الکتریکی $= \oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0} = \frac{\sum q_i}{\varepsilon_0}$



اما



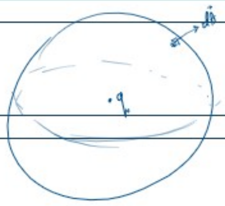
شار الکتریکی در داخل صفر

وقتی بار خارج دارد $= \oint \vec{E}\cdot d\vec{A} = 0$

شار در داخل غیر صفر

تنها وقتی غیر صفر که بار داخل سطح باشد

حلقه ← غیر صفر



$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$

$\cos\theta = 1$

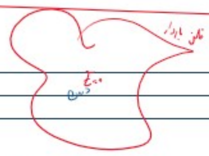
$\oint E\, dA$

میدان همه جا یکی $= E \oint dA$

$= EA = \boxed{E\, 4\pi a^2} = \frac{q}{\varepsilon_0}$

$E = \frac{q}{4\pi\varepsilon_0 a^2}$

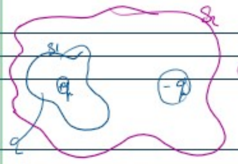
if داخل ← میدان صفر باشد = X



سطح گاوسی

$\oint \vec{E}\cdot d\vec{A} \xrightarrow{\text{میدان داخل}} \Phi = 0 = \frac{q}{\varepsilon_0} \rightarrow q_{\text{داخل}} = 0$

بار کاملاً روی

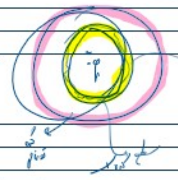


$$\oint \vec{E}\cdot d\vec{A} = 0$$

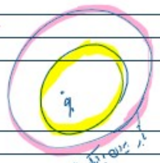
$$\sum q = 0$$

$$\oint_{S_1} \vec{E}\cdot d\vec{A} = \frac{+q}{\varepsilon_0}$$

$$\oint E\cdot dA = \frac{q}{\varepsilon_0}$$



$$\oint E\cdot dA = \frac{q_{in}}{\varepsilon_0} = 0$$



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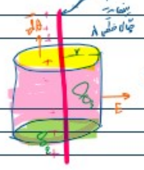


$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

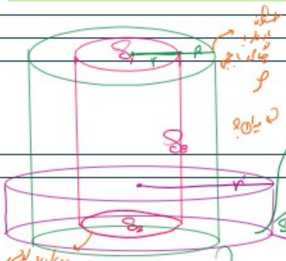
$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$E\cdot dA = E\,dA$$

$$\cancel{\int_{S_1}\vec{E}\cdot d\vec{A}} + \cancel{\int_{S_2}\vec{E}\cdot d\vec{A}} + \int_{S_3} E\cdot dA = E\int_{S_3} dA = E\,2\pi r h = \frac{\lambda h}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi r\varepsilon_0}$$

[illegible]



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon}$$

$$\int E\,dA$$

$$\cancel{\int_{S_1}\vec{E}\cdot d\vec{A}} + \cancel{\int_{S_2}\vec{E}\cdot d\vec{A}} + \int_{S_3}\vec{E}\cdot d\vec{A} = E\int_{S_3} dA$$

$$= E\,2\pi r h = \frac{q_{in}}{\varepsilon_0} = \frac{\pi r^2 h\rho}{\varepsilon_0}$$

$$\boxed{E = \frac{r\rho}{2\varepsilon_0}}$$

$r<R$

[illegible]

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$E\,2\pi r h = \frac{\pi R^2 h\rho}{\varepsilon_0}$$

$$E = \frac{\rho R^2}{2\varepsilon_0 r}$$

$r>R$

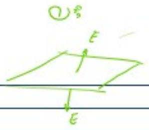
[illegible]

$$\rho(r) = ar$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{\int \rho(r)\,dr}{\varepsilon_0}$$

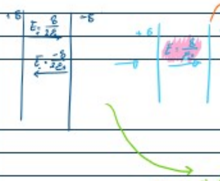
$$\oint \vec{E}\cdot d\vec{A} = \underbrace{\int_{S_1}\vec{E}\cdot d\vec{A}}_{EA} + \underbrace{\int_{S_2}\vec{E}\cdot d\vec{A}}_{EA} + \cancel{\int_{S_3}\vec{E}\cdot d\vec{A}} = 2EA = \frac{q_{in}}{\varepsilon_0}$$

σA



$$E = \frac{\sigma}{2\varepsilon_0}$$

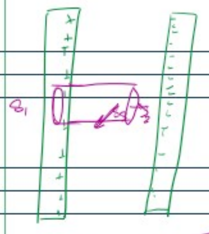
[illegible]



$\vec{E}_{in} = \frac{\sigma}{\varepsilon_0}$

$E = 0$ در خارج [illegible]

$\boxed{E=0}$



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$\vec{E} \perp d\vec{A}$

عمود

$EA = \frac{q_{in}}{\varepsilon_0}$

$$= \cancel{\int_{S_1} \vec{E}\cdot d\vec{A}}^{\,0} + \underbrace{\int_{S_2} E\cdot dA}_{EA} + \cancel{\int_{S_3} E\cdot dA}^{\,0} = \frac{q_{in}}{\varepsilon_0} = \boxed{\frac{\sigma}{\varepsilon_0}}$$

چون میدان در [illegible]

[illegible] نبود

σ [illegible] نبود

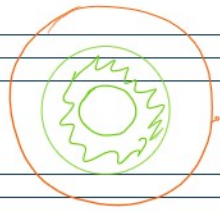
چون σ نصف در طرف راست (+) نصف در طرف چپ (−)

$$\frac{q_{in}}{\varepsilon_0} = \boxed{\frac{\sigma}{2\varepsilon_0}}$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$\oint (\vec{E}\cdot\hat{r})(d\vec{A}\cdot\hat{r}) = \oint E\,dA = \boxed{4\pi r^2 E} = \boxed{0}$$

$E = 0$



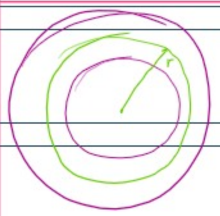
$\rightarrow \oint E\cdot dA = \frac{q_{in}}{\varepsilon_0}$

$$\Rightarrow E\,4\pi r^2 = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow \boxed{E = \frac{q_{in}}{4\pi r^2 \varepsilon_0}}$$

$q_{in} = \frac{4}{3}\pi(R_2^3 - R_1^3)\rho$

$$\rightarrow \boxed{E = \frac{(R_2^3 - R_1^3)\rho}{3r^2\varepsilon_0}}$$

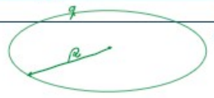


$$\oint E\cdot dA = \frac{q_{in}}{\varepsilon_0}$$

$$\rightarrow E\,4\pi r^2 = \frac{1}{\varepsilon_0}\left(\overbrace{\rho\left[\tfrac{4}{3}\pi(r^3 - R_1^3)\right]}^{q_{in}}\right)$$

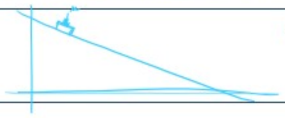
$$\boxed{E = \frac{r^3 - R_1^3}{3\varepsilon_0 r^2}}$$

[illegible] = [illegible]



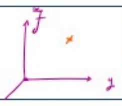
سطح گاوسی میگیریم
حالت بسته آوردیم
زمانی نگیم که [illegible]
چراکه تقارن [illegible]
[illegible]

$$\rightarrow \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$



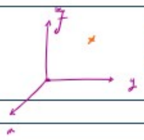
[illegible] = [illegible] (دیفرانسیلی)

(انرژی، تعمیم) = [illegible]



$$U(x,y,z) = U_0(x_0,y_0,z_0) + mgz$$

$$U(x,y,z) - U(x_0,y_0,z_0) = -W = -\int_{r_0}^{r} \vec{F}\cdot d\vec{s} = -\int_{r_0}^{r} (-mg\hat{z})\cdot d\vec{s}$$



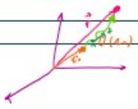
$$U(x,y,z) = U_0(x_0,y_0,z_0) + mgz$$

$$U(x,y,z) - U(x_0,y_0,z_0) = -W = -\int_{r_0}^{r} \vec{F}\cdot d\vec{s} = -\int_{r_0}^{r} (-mg\hat{z})\cdot ds$$

$Vq = W$

$\boxed{V = \frac{+W}{q}}$ $\boxed{-Vq = U}$

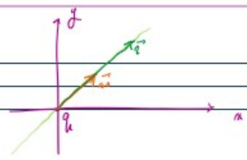
[illegible]



$\vec{E}(x,y,z)$

$$U(r) - U(r_0) = -W = -\int \vec{F}\cdot d\vec{s} = -q\int_{r_0} \vec{E}\cdot d\vec{s} \longrightarrow V(\vec{r}) - V(\vec{r}_0) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E}\cdot d\vec{s}$$

[illegible]



$$V(\vec{r}) - V(\vec{r}_0) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E}\cdot d\vec{s}$$

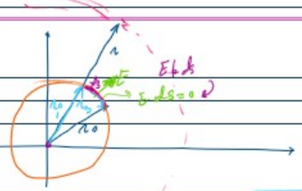
$$= -\int (E\cdot\hat{r})\cdot(dr\cdot\hat{r})$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$= -\int_{r_0}^{r} \frac{q}{4\pi\varepsilon_0 r^2}\,dr = \frac{-q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_0}^{r} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

$$\rightarrow (V(\vec{r}) - V(\vec{r}_0) = \frac{q}{4\pi\varepsilon_0 r} - \frac{q}{4\pi\varepsilon_0 r_0}$$

$$\xrightarrow[V(r_0)=0]{\text{if } r_0\to\infty} \boxed{V(r) = \frac{q}{4\pi\varepsilon_0 r}}$$



$$V = -\int_{r_0}^{r} \vec{E}\cdot d\vec{s} = -\int_{\vec{r}_0}^{\vec{r}} (E\hat{r})\cdot(dr\,\hat{r} + r\,d\phi\,\hat{\phi})$$

$r_0 = r_{0_1} + r_{0_2}$

$E \perp ds$

$\boxed{0 = E\cdot ds}$

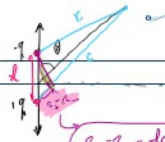
$$= -\int_{r_0}^{r} \vec{E}_{(r)}\,dr$$

$$\int_0^{r_0} E\cdot ds = \int_0^{r_1} E\cdot ds + \int_{r_{0_1}}^{r_{0_2}} E\cdot ds$$

[illegible]

[illegible]

$$V = \frac{1}{4\pi\varepsilon_0}\sum \frac{q_i}{r_i}$$



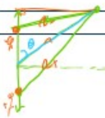
$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_+} - \frac{q}{r}\right)$$

$$\boxed{r_+ - r_- = d\cos\theta} = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_- - r_+}{\underbrace{r_+ r_-}_{\sim r^2}}\right) = \frac{-q\,d\cos\theta}{4\pi\varepsilon_0 r^2}$$

p

$$d \ll r \rightarrow \begin{cases} r_+ = r + \delta \\ r_- = r - \alpha \end{cases}$$

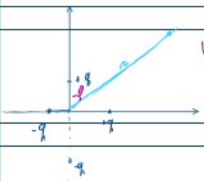
$$\boxed{V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}} \sim -p$$



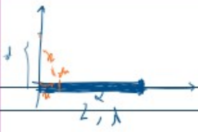
$$r_- = \sqrt{\left(r\cos\theta - \frac{d}{2}\right)^2 + (r\sin\theta)^2}$$

$$r_+ = \sqrt{\left(r\cos\theta + \frac{d}{2}\right)^2 + (r\sin\theta)^2}$$

$$\rightarrow \frac{q}{4\pi\varepsilon_0}\left(\frac{r_- - r_+}{r_+ r_-}\right) = \dots$$



$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} + \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^2}$$



$$\lambda\,dx = dq$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

$$\frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\,dx}{\sqrt{x^2+d^2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dx}{\sqrt{\frac{x^2}{d^2}+1}}$$

$x = \tan\theta$

$dx = \left(\frac{1}{\cos^2\theta}\right)$

$$\int \frac{\frac{1}{\cos^2\theta}\,d\theta}{\frac{1}{\cos\theta}} = 1$$

$$(1+x)^n = 1 + nx \quad (x \ll 1)$$

$$\boxed{\ln(1+x) \xrightarrow{x\ll 1} = x}$$

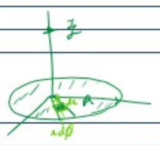
$= \lambda \int$

$$\int \frac{\frac{dx}{\cos^2\theta}}{\frac{1}{\cos\theta}} d\theta = 1$$

$$\ln(1+x) \xrightarrow{x \ll 1} = x$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int \frac{d\theta}{\cos\theta} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\sec\theta + \operatorname{tg}\theta\right)\Big|_0^{l} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{l+\sqrt{l^2+d^2}}{d}\right) \quad \sec\theta = \frac{\sqrt{d^2+x^2}}{d},\ \operatorname{tg}\theta = \frac{x}{d}$$

$$\frac{l}{d} = \frac{\lambda}{4\pi\varepsilon_0}\frac{l}{d} \xrightarrow{l\lambda = Q} \boxed{\frac{Q}{4\pi\varepsilon_0 d}}$$



$$dV = \frac{dq}{4\pi\varepsilon_0\sqrt{r^2+z^2}} \qquad \sigma\, dA = dq \qquad \sigma\, dr\, r\, d\phi = dq$$

$$q = \pi R^2 \sigma$$

$$V = \int \frac{\sigma\, dr\, r\, d\phi}{4\pi\varepsilon_0\sqrt{r^2+z^2}}$$

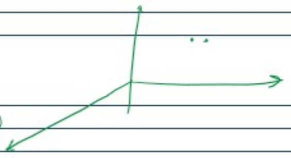
$$= \frac{\sigma}{4\pi\varepsilon_0}\int_{r=0}^{R}\int_{\phi=0}^{2\pi}\frac{r\,dr\,d\phi}{\sqrt{r^2+z^2}} = \frac{\sigma}{4\pi\varepsilon_0}\int_{r=0}^{R}\frac{r\,dr}{\sqrt{r^2+z^2}}\int_0^{2\pi} d\phi = \frac{\sigma}{2\varepsilon_0}\int \frac{\frac{1}{2}du}{\sqrt{u}}$$

$$r^2+z^2 = u \qquad \boxed{2r\,dr = du}$$

$$= \frac{\sigma}{2\varepsilon_0}\sqrt{r^2+z^2}\,\Big|_{z^2}^{R^2+z^2} = \frac{q}{2\pi\varepsilon_0 R^2}\left(\sqrt{R^2+z^2} - z\right) = \left(\frac{q}{2\pi\varepsilon_0 R^2}\right)\left(\frac{1}{2}\frac{R^2}{z^2}\right)z = \boxed{\frac{q}{4\pi\varepsilon_0 z}}$$

$$\xrightarrow{R \ll z} z\left(\sqrt{\left(\frac{R}{z}\right)^2+1} - 1\right) \qquad 1+\frac{1}{2}\frac{R^2}{z^2}$$

$$V_2 - V_1 = \int_1^2 \vec{E}\cdot\vec{ds}$$



$$V_{(x,y+dy,z)} - V_{(x,y,z)} = -\vec{E}\cdot\vec{ds} = \boxed{-E_y\, dy}$$

$$-\frac{\partial V}{\partial y} = E_y = \frac{V_{(x,y+dy,z)} - V_{(x,y,z)}}{dy}\hat{j}$$

$$E_x = -\frac{\partial V}{\partial x}\hat{i}$$

$$E_z = -\frac{\partial V}{\partial z}\hat{k}$$

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) = -\vec{\nabla} V$$

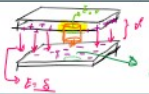
$$\rightarrow \boxed{\vec{E} = -\nabla V}$$

میدان الکتریکی عمود بر سطوح هم پتانسیل

$$q \propto V \qquad q = CV$$

$$\sigma = \frac{q}{A}$$

$$F = \frac{C}{V} \qquad \mu F,\ nF \qquad 10^{-6},\ 10^{-9} F$$



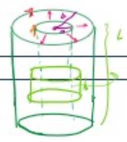
$$E = \frac{\sigma}{\varepsilon}$$

$$EA = \frac{q_{in}}{\varepsilon_0} \rightarrow E = \frac{q_{in}}{\varepsilon_0 A} = \frac{\sigma}{\varepsilon}$$

$$V = \int \vec{E}\cdot\vec{ds} = E\cdot d = \frac{qd}{A\varepsilon_0} \longrightarrow q = \frac{\varepsilon_0 A}{d}V$$

$$C = \frac{\varepsilon_0 A}{d}$$

خازن استوانه‌ای



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$E\underbrace{A}_{2\pi r L} = \frac{q_{in}}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{q}{2\pi\varepsilon_0 L r}}$$

$$V = \int \vec{E}\cdot d\vec{s} = \int_{r=a}^{b} E(r)\,dr = \int_{r=a}^{b}\frac{q}{2\pi\varepsilon_0 L r}dr = \frac{q}{2\pi\varepsilon_0 L}\ln r\Big|_a^b = \boxed{\frac{q}{2\pi\varepsilon_0}\ln\frac{b}{a} = V}$$

$$\boxed{C = \frac{2\pi\varepsilon_0}{\ln\frac{b}{a}}}$$

$$E_{(r)} = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$V = \int_a^b \frac{q}{4\pi\varepsilon_0 r^2}dr = \frac{-q}{4\pi\varepsilon_0 r} = \boxed{\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = V}$$

$$\downarrow$$

$$\boxed{C = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}}$$

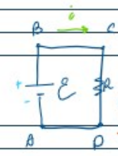
$$\downarrow$$

$$b \gg a$$

$$\boxed{\frac{1}{a}-\frac{1}{b}}$$

$b \gg a$, $b \to \infty \to C = 4\pi\varepsilon_0 a$

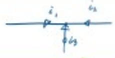
$b - a \ll a \to C = \frac{4\pi\varepsilon_0}{\frac{d}{a^2}} = \frac{4\pi\varepsilon_0 a^2}{d} = \frac{A\varepsilon_0}{d}$



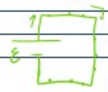
$dW = 0 = dq\,V_{AB} + dq\,V_{BC} + dq\,V_{CD} + dq\,V_{DA} \to \underbrace{V_{AB}}_{-\varepsilon} + V_{BC} + \underbrace{V_{CD}}_{RI} + V_{DA} = 0 \to i = \frac{\varepsilon}{R}$

$-Ri$

$\underbrace{V_{AB}}_{-\varepsilon} = V_A - V_B$



$i_1 + i_2 + i_3 = 0$

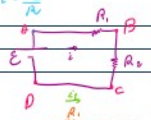


① KVL [illegible]

② KCL [illegible]

$-\varepsilon - iR = 0$

$i = \frac{-\varepsilon}{R}$



$-\varepsilon + R_1 i + R_2 i = 0$

$\to i_1 = i_2 + i_3$ KCL

$-\varepsilon + i_1R_1 + i_2R_2 = 0$

$i_2R_2 - R_3i_3 = 0$

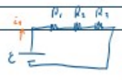
$i_1 = i_2 + \frac{R_2}{R_3} i_2 \to i_2 = \frac{i_1}{(1+\frac{R_2}{R_3})}$

$R_1 i_1 + R_2 i_2 = \varepsilon$

$\to R_1 i_1 + R_2 R_3 \frac{i_1}{R_3 + R_2} = \varepsilon$

$\to (R_1 + \frac{R_2R_3}{R_2+R_3})\, i_1 = \varepsilon$

$\to i_1 = \frac{\varepsilon}{R_1 + \frac{R_2R_3}{R_2+R_3}}$



$-\varepsilon + iR_1 + iR_2 + iR_3 = 0$

$\to \frac{\varepsilon}{\sum_{n=1}^{3} R_n} = i$

$i_4 = i_1 + i_2 + i_3$

$-\varepsilon + i_3R_3 = 0$

$i_1R_1 - i_2R_2 = 0$

$i_2R_2 - i_3R_3 = 0$

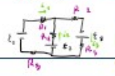
$i_1R_1 = i_2R_2 = i_3R_3 = \varepsilon$

$\to i_n = \frac{\varepsilon}{R_n}$

$\to i_4 = \varepsilon \sum_{n=1}^{3} \frac{1}{R_n}$

R_{total} [illegible] $< R_i$

R_{total} [illegible] $> R_i$



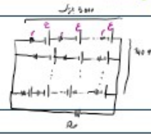
$i_1 = i_2 + i_3$

$\varepsilon_2 - \varepsilon_1 + R_1 i_1 - R_2 i_3 + R_5 i_1 = 0$

[illegible]

[illegible]

$\frac{\varepsilon}{100}$



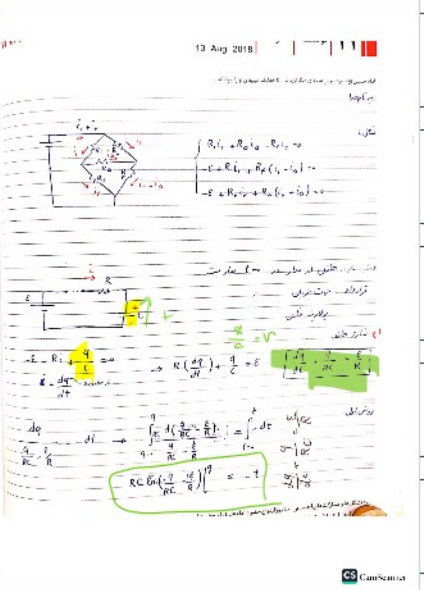
=

$V_A = V_B$

≡

≡

$i = \frac{5000\,\varepsilon}{R_w + \frac{5000}{100} r}$



13 Aug 2018

$R_1 i_1 + R_G i_G - R_2 i_2 = 0$

$-\varepsilon + R_1 i_1 + R_4 (i_1 - i_G) = 0$

$-\varepsilon + R_2 i_2 + R_3 (i_2 + i_G) = 0$

$\frac{q}{C} = V$

$-\varepsilon + Ri + \frac{q}{C} = 0$

$i = \frac{dq}{dt}$

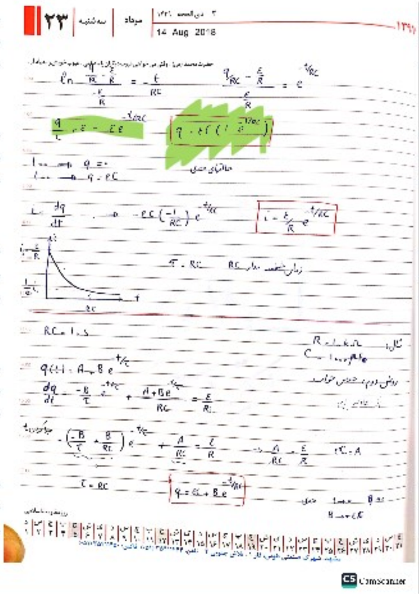
$\to R\left(\frac{dq}{dt}\right) + \frac{q}{C} = \varepsilon$

$\boxed{\frac{dq}{dt} + \frac{q}{RC} = \frac{\varepsilon}{R}}$

$\frac{dq}{\frac{q}{RC} - \frac{\varepsilon}{R}} = -dt \to \int_{q=0}^{q} \frac{d\left(\frac{q}{RC} - \frac{\varepsilon}{R}\right)}{\frac{q}{RC} - \frac{\varepsilon}{R}} = -\int_{t=0}^{t} dt$

$\boxed{RC \ln\left(\frac{q}{RC} - \frac{\varepsilon}{R}\right)\Big|_0^q = -t}$

CamScanner



۲۳ سه‌شنبه مرداد ۳ ذی‌الحجه ۱۴۳۹
14 Aug 2018

$$\ln \frac{\frac{q}{RC}-\frac{\varepsilon}{R}}{-\frac{\varepsilon}{R}} = -\frac{t}{RC} \qquad \frac{\frac{q}{RC}-\frac{\varepsilon}{R}}{-\frac{\varepsilon}{R}} = e^{-t/RC}$$

$$\frac{q}{C}-\varepsilon=-\varepsilon e^{-t/RC} \qquad \boxed{q=\varepsilon C(1-e^{-t/RC})}$$

حالت‌های حدی

$t=0 \to q=0$

$t=\infty \to q=\varepsilon C$

$$i=\frac{dq}{dt} \to -\varepsilon C\left(\frac{-1}{RC}\right)e^{-t/RC} \qquad \boxed{i=\frac{\varepsilon}{R}e^{-t/RC}}$$

$i_0=\frac{\varepsilon}{R}$, $\frac{1}{e}i_0$, RC, t

$\tau=RC$ RC ثابت زمانی مدار

مثال $R=1\,k\Omega$, $C=1000\,\mu F$

$RC=1\,s$

روش دوم حل معادله

$$q(t)=A+Be^{-t/\tau}$$

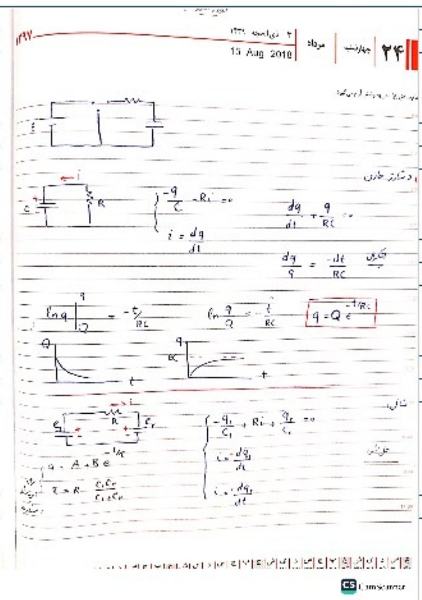
$$\frac{dq}{dt}=-\frac{B}{\tau}e^{-t/\tau}+\frac{A+Be^{-t/\tau}}{RC}=\frac{\varepsilon}{R}$$

$$\left(\frac{-B}{\tau}+\frac{B}{RC}\right)e^{-t/\tau}+\frac{A}{RC}=\frac{\varepsilon}{R} \to \frac{A}{RC}=\frac{\varepsilon}{R} \qquad \varepsilon C=A$$

$$\tau=RC \qquad \boxed{q=\varepsilon C+Be^{-t/RC}}$$

$B=-\varepsilon C$

CamScanner



۲۴ چهارشنبه مرداد ۴ ذی‌الحجه ۱۴۳۹
15 Aug 2018

دشارژ خازن

$$\begin{cases}-\frac{q}{C}-Ri=0\\ i=\frac{dq}{dt}\end{cases} \qquad \frac{dq}{dt}+\frac{q}{RC}=0$$

$$\frac{dq}{q}=-\frac{dt}{RC}$$

$$\ln q\Big|_Q^q=-\frac{t}{RC} \qquad \ln\frac{q}{Q}=-\frac{t}{RC} \qquad \boxed{q=Qe^{-t/RC}}$$

Q, t, q, RC, t

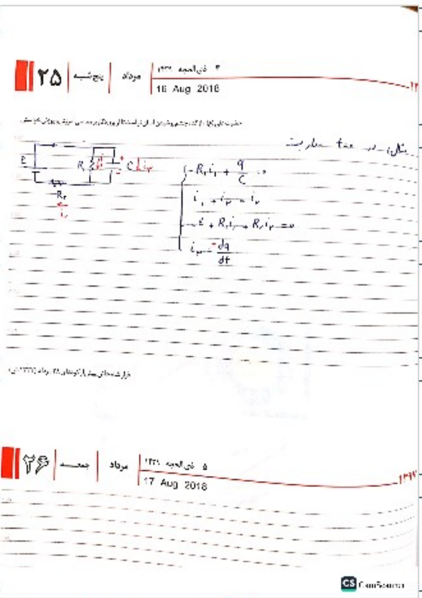
مثال

$$\begin{cases}-\frac{q_1}{C_1}+Ri+\frac{q_2}{C_2}=0\\ i=-\frac{dq_1}{dt}\\ i=\frac{dq_2}{dt}\end{cases}$$

$$q=A+Be^{-t/\tau}$$

$$\tau=R\frac{C_1C_2}{C_1+C_2}$$

CamScanner



۲۵ پنجشنبه مرداد ۴ ذی‌الحجه ۱۴۳۹
16 Aug 2018

مثال

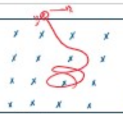
$$\begin{cases}-R_1i_1+\frac{q}{C}=0\\ i_1+i_2=i_3\\ -\varepsilon+R_1i_1+R_2i_2=0\\ i_2=-\frac{dq}{dt}\end{cases}$$

۲۶ جمعه مرداد ۵ ذی‌الحجه ۱۴۳۹
17 Aug 2018

CamScanner

$$m\frac{v^2}{r}=|q|vB \to r=\frac{m}{Bq}v$$

$$\omega=2\pi f=\frac{v}{r}=\frac{Bq}{m}$$



$dF=dq\,v\times B$

$$dF=\boxed{dq\frac{dl}{dt}}B$$

$dF=i\,dl\times B$

$B=-B_0\hat{j}$

$$F=\int dF=-2\int_0^{\frac{\pi}{2}}B_0 i\,dl\sin\theta=-2B_0iR\cos\theta\Big|_0^{\frac{\pi}{2}}=+2B_0iR$$

$$d\vec F=i\,d\vec l\times\vec B$$

$$\vec B=-B\hat j$$

$$L=Rd\theta\sin\theta\hat j+Rd\theta\cos\theta\hat i$$

$$\vec{dl}=Rd\theta\sin\theta\hat i-Rd\theta\cos\theta\hat j$$

$$\vec F=\int d\vec F=\int i(Rd\theta(\sin\theta\hat i-\cos\theta\hat j))\times(-B\hat j)$$

$$\int -RBi\,d\theta(-\sin\theta\hat j-\cos\theta\hat k)=2iRB\hat j$$

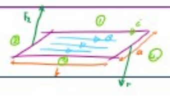
(Ampere...) [illegible]

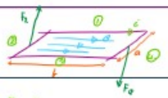
$2R$

$F=2B_0iR\hat j \leftarrow l=2R\hat i$

$$\vec{dF}=i\,d\vec l\times\vec B$$

$$\vec F=i[l_2-l_1]\times B$$



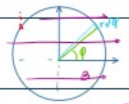


$F_1 = F_3 = 0$

$F_2 = i(-a\hat{y})\times(B\hat{x}) = iaB\hat{z}$

$F_4 = i(a\hat{y})\times(B\hat{x}) = -iaB\hat{z}$

$\sum F_{net} = 0$, $\sum M = iaB\times b = iabB$

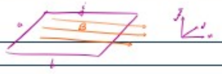


$d\vec{F} = i\,d\vec{l}\times\vec{B}$

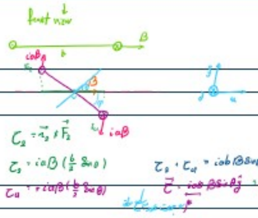
$= i(a\,d\varphi)B\sin(\frac{\pi}{2}+\varphi) = iaB\cos\varphi\,d\varphi$

$d\vec{\tau} = \vec{a}\times d\vec{F} = (a\cos\varphi)iaB\cos\varphi\,d\varphi$

$\tau = \int d\vec{\tau} = \int_0^{2\pi} ia^2B\cos^2\varphi\,d\varphi = \underbrace{i\pi a^2}_{A}B = iAB$ [illegible]



دوقطبی الکتریکی: 2 بار کنار هم
دوقطبی مغناطیسی: [illegible]



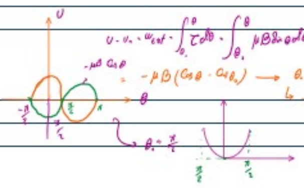
$\tau_2 = \vec{r}_2\times\vec{F}_2$

$\tau_3 = iaB(\frac{b}{2}\sin\theta)$

$\tau_4 = iaB(\frac{b}{2}\sin\theta)$

$\tau_3+\tau_4 = iabB\sin\theta$

$\vec{\tau} = \underbrace{iab}B\sin\theta\hat{j} = \vec{\mu}\times\vec{B}$



$U - U_0 = -W_{ext} = \int_{\theta_0}^{\theta}\tau\,d\theta = \int_{\theta_0}^{\theta}\mu B\sin\theta\,d\theta$

$= -\mu B(\cos\theta - \cos\theta_0) \longrightarrow \theta_0 = \frac{\pi}{2} \rightarrow U_0 = 0$

$\hookrightarrow -\mu B\cos\theta$

$\theta_0 = \frac{\pi}{2}$

$\mu = NiA$ [illegible]

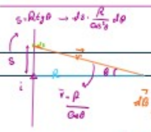
[illegible]

[illegible]

$d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\vec{r}}{r^3} = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$

$\mu_0 =$ [illegible] $4\pi\times10^{-7}\,\frac{T\,m}{A}$

$s = R\,tg\theta \rightarrow ds = \frac{R}{\cos^2\theta}d\theta$



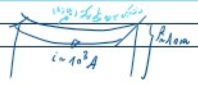
$r = \frac{R}{\cos\theta}$

$\frac{\mu_0 i}{4\pi}\frac{R}{R^2} \approx \frac{\mu_0 i}{2\pi R}$

$d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\vec{r}}{r^3}$

$dB = \frac{\mu_0 i}{4\pi}\frac{ds\,R}{r^3}\hat{j}$

$B = \int dB = \frac{\mu_0 i}{4\pi}R\int\frac{ds}{r^3} = \frac{\mu_0 i}{R4\pi}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = \frac{\mu_0 i}{2\pi R}$



$B = \frac{4\pi\times10^{-7}\times10^3}{2\pi\,10} = 2\times10^{-5}\,T \approx 10^{-1}\,gauss$

مثال: [illegible]

$d\vec{B} = dB\hat{j}$

$dB = \frac{\mu_0 i}{4\pi}\frac{ds\,R}{R^3}$

$B = \frac{S\mu_0 i}{4\pi R^2} = \frac{R\alpha\,\mu_0 i}{4\pi R^2} = \frac{\mu_0 i\alpha}{4\pi R} \rightarrow$ [illegible] $= \frac{\mu_0 i}{2R}$

[illegible]