NYU, Tandon School of Engineering
Extended Bridge to CS – Winter 2022
Homework #6

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Question 5

Use the definition of Θ in order to show the following:

a. $5n^3 + 2n^2 + 3n = \theta(n^3)$

Step 1

- T(N) = O (f(n)) if and only if there are positive constants c and n_0 such that T(N) ≤ cf(N) when N ≥ n_0.
- In this case, $5n^3 + 2n^2 + 3n \leq 6n^3 \; for \; N \geq 3$. Thus, $5n^3 + 2n^2 + 3 = O(n^3)$.

Step 2

- T(N) = Ω (g(n)) if and only if there are positive constants c and n_0 such that T(N) ≥ cg(N) when N ≥ n_0.
- In this case, $5n^3 + 2n^2 + 3n \geq 5n^3 \; for \; N \geq 0$. Thus, $5n^3 + 2n^2 + 3 = \Omega(n^3)$.

Step 3

- T(N) = Θ (h(n)) if and only if T(N) = O(h(N)) and T(N) = Ω (N)).
- Since $5n^3 + 2n^2 + 3 = O(n^3)$ and $5n^3 + 2n^2 + 3 = \Omega(n^3)$, in this case, we can conclude that $5n^3 + 2n^2 + 3 = \Theta(n^3)$. ■

b. $\sqrt{(7n^2 + 2n - 8)} = \theta(n)$

Step 1

- T(N) = O (f(n)) if and only if there are positive constants c and n_0 such that T(N) ≤ cf(N) when N ≥ n_0.
- In this case, $\sqrt{(7n^2 + 2n - 8)} \leq 3n \; for$ N ≥ 0.936 (3d.p.)
- Thus, $\sqrt{(7n^2 + 2n - 8)} = O(n)$.

Step 2

- T(N) = Ω (g(n)) if and only if there are positive constants c and n_0 such that T(N) ≥ cg(N) when N ≥ n_0.
- In this case, $\sqrt{(7n^2 + 2n - 8)} \geq 2n \; for \; N \geq 1.333 \; (3 \; d.p.)$
- Thus, $\sqrt{(7n^2 + 2n - 8)} = \Omega(n)$.

Step 3

- T(N) = Θ (h(n)) if and only if T(N) = O(h(N)) and T(N) = Ω (N)).
- Since $\sqrt{(7n^2 + 2n - 8)} = O(n)$ and $\sqrt{(7n^2 + 2n - 8)} = \Omega(n)$, in this case, we can conclude that $\sqrt{(7n^2 + 2n - 8)} = \Theta(n)$. ■