Mass-radius relation in quark stars

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ABSTRACT

This is a report on mass-radius relation in quark stars. Numerical calculations were performed with the assumed equation of state for quark matter. In paper used the Tolman-Oppenheimer-Volkoff (TOV) equation and the mass continuity equation. Differential equations are solved using the classical Runge-Kutta method (RK4).

Subject headings: quark star, mass-radius relation, TOV

1. Introduction

Quark star is a hypothetical type of compact object consisting of quark matter. It is a ultra-density star (about 10^{14}-10^{18} gm/cm^3) and it may be an intermediate state between neutron stars and black holes. Their size can be in the range $5-10$ kilometres which is less than neutron star. The comparison between quark and neutron star is shown in Figure 1. The great problem in astrophysics is to find out how looks the equation of state for this matter. We assumed that the equation of state is presented as (also in Figure 2):

$$P = \begin{cases} a(\rho - \rho_0) & \text{when } \rho < \rho_c \\ a(\rho_c - \rho_0) + b(\rho - \rho_c) & \text{when } \rho > \rho_c, \end{cases} \tag{1}$$

where:

$a = 0.2c^2$

$b = 0.3c^2$

$c = 29979245800$ [cm/s]

$\rho_0 = 10^{14}$ [g/cm3]

$\rho_c = 8 \cdot 10^{14}$ [g/cm3]

This paper presents numerically calculated mass-radius relation for quark star with above EoS. In section 2 shown assumptions and method how this relation can be calculated. In section 3 and 4 presented results and conclusion about it.

2. Theoretical assumptions and algorithm

For calculation used two differential equations: TOV and mass continuity. Tolman-Oppenheimer-Volkoff (TOV) equation describe the stucture of a spherically

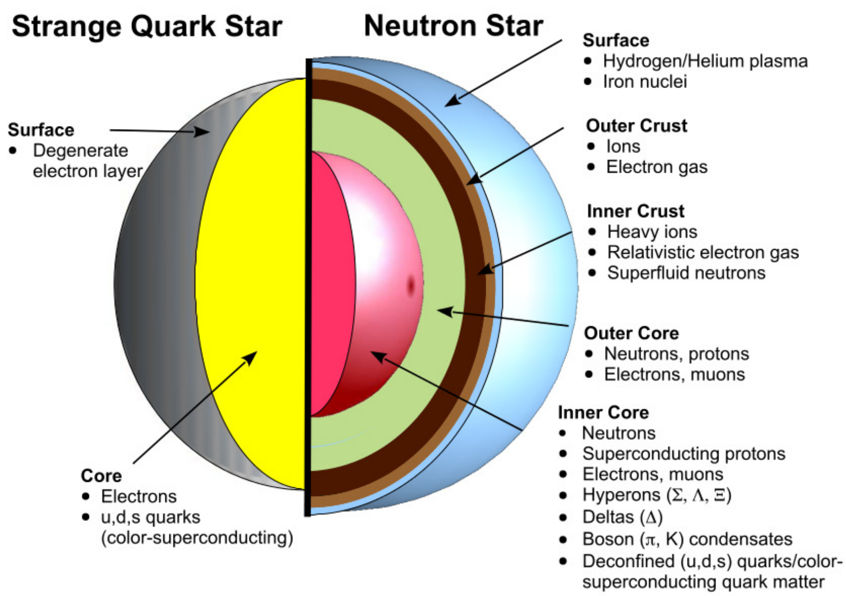

Fig. 1.— The comparison between quark and neutron star (http://astrobites.org/).

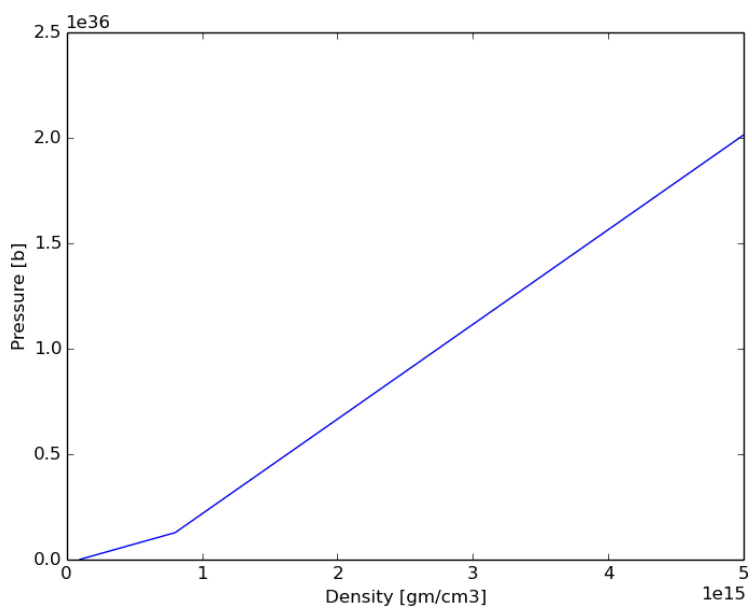

Fig. 2.— The equation of state for quark matter.

symmetric body of isotropic material which is in static gravitational equilibrium (Oppenheimer&Volkoff,1939). This equation is

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}\left(1+\frac{P(r)}{\rho(r)c^2}\right)\left(1+\frac{4\pi r^3 P(r)}{M(r)c^2}\right)\left(1-\frac{2GM(r)}{rc^2}\right)^{-1}, \tag{2}$$

and mass continuity equation is

$$\frac{dM}{dr} = 4\pi\rho(r)r^2. \tag{3}$$

First step to do is insert equation 1 into TOV equation. For different ρ we obtain (left side of TOV equation)

$$\frac{dP}{dr} = \begin{cases} a\frac{d\rho}{dr} & \text{when } \rho < \rho_c \\ b\frac{d\rho}{dr} & \text{when } \rho > \rho_c, \end{cases} \tag{4}$$

Finally, there are two differential equations to solve at the same time

$$\frac{d\rho}{dr} = \begin{cases} -\frac{GM(r)\rho(r)}{ar^2}\left(1+\frac{P(r)}{\rho(r)c^2}\right)\left(1+\frac{4\pi r^3 P(r)}{M(r)c^2}\right) & \text{when } \rho < \rho_c \\ -\frac{GM(r)\rho(r)}{br^2}\left(1+\frac{P(r)}{\rho(r)c^2}\right)\left(1+\frac{4\pi r^3 P(r)}{M(r)c^2}\right) & \text{when } \rho > \rho_c, \end{cases} \tag{5}$$

$$\frac{dM}{dr} = 4\pi\rho(r)r^2. \tag{6}$$

To solve equation above used the classical Runge-Kutta method where for equation of the form $y' = f(x,y)$ the iteration process is: $k_1 = hf(x_n, y_n)$,
$k_2 = hf(x_n + h/2, y_n + k_1/2)$,
$k_3 = hf(x_n + h/2, y_n + k_2/2)$,
$k_4 = hf(x_n + h, y_n + k_3)$,
$y_{n+1} = y_n + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4)$,
$x_{n+1} = x_n + h$.
Algotithm starts at $r = 0$, $\rho = \rho_{central}$ (input data), and ends when $P(\rho) = 0$. Output data are radius $R(\rho_{central})$ and mass $M(\rho_{central})$.

3. Results

Calculation were made for $\rho_{central}$ in range from ρ_0 to $10^4\rho_0$. Results are presented in figure 3. The maximum value of mass for quark stars is (2.87 ± 0.01) M_{Sun}.

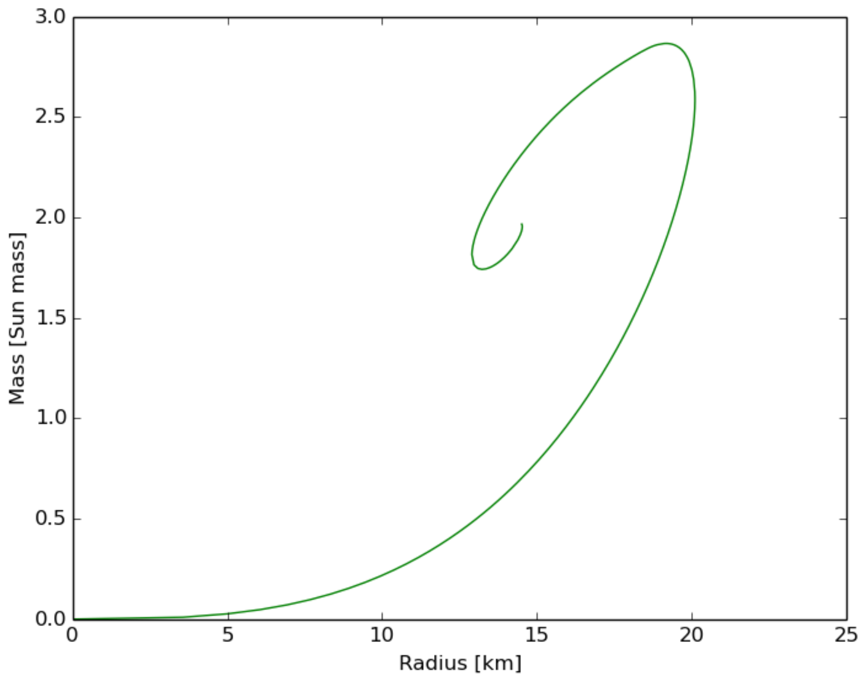

Fig. 3.— Mass-radius relation for quark stars.

4. Summary and conclusions

These results show that for a given EoS exists well-defined relation between mass and radius. Moreover, there are maximum value of mass and radius for this type of compact star. However, there are no lower limit for this matter which means that could exist small objects consisting of quark matter. In literature it is called strangelet (Farhi&Jaffe,1984). The existence of such objects could explain high-mass neutron star observations.

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