Roche potential:

$$\phi = -\frac{GM_1}{\sqrt{(x-a_1)^2+y^2+z^2}} - \frac{GM_2}{\sqrt{(x+a_2)^2+y^2+z^2}} - \frac{1}{2}\Omega(x^2+y^2)$$

Substitution: $M = M_1 + M_2$, $q = \frac{M_1}{M_2}$, $A = a_1 + a_2$, $\Omega^2 A^3 = GM$ (Kepler's law)

$$\phi = -\frac{GM}{A}\Big(\frac{\frac{q}{q+1}}{\sqrt{(X-\frac{1}{q+1})^2+Y^2+Z^2}} + \frac{\frac{1}{q+1}}{\sqrt{(X+\frac{q}{q+1})^2+Y^2+Z^2}} + \frac{1}{2}(X^2+Y^2)\Big)$$

where $X = \frac{x}{A}$, $Y = \frac{y}{A}$, $Z = \frac{z}{A}$.

Dimensionless potential:

$$\widetilde{\phi} = \frac{\phi}{GM/A} = -\frac{\frac{q}{q+1}}{\sqrt{(X-\frac{1}{q+1})^2+Y^2+Z^2}} - \frac{\frac{1}{q+1}}{\sqrt{(X+\frac{q}{q+1})^2+Y^2+Z^2}} - \frac{1}{2}(X^2+Y^2)$$

To compute L_1, L_2, L_3 points should compare derivative of $\widetilde{\phi}$ with 0 (when $z = y = 0$).

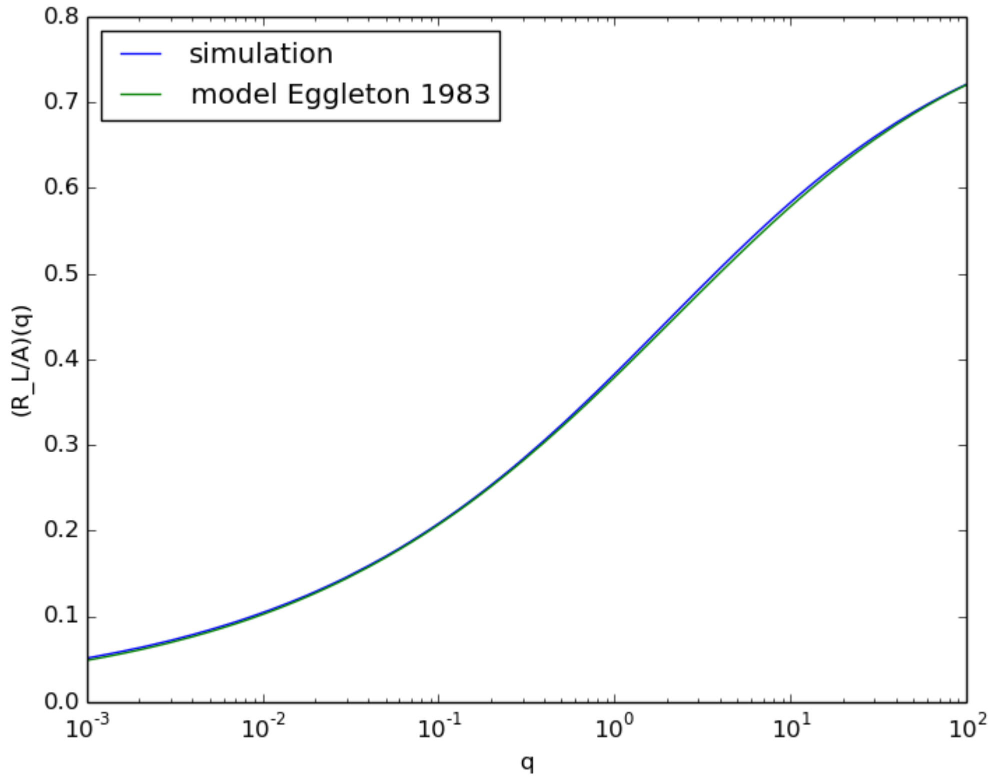
$$\frac{d\widetilde{\phi}(X,0,0)}{dX} = 0$$

$$sgn(X - \frac{1}{q+1})\frac{\frac{q}{q+1}}{(X-\frac{1}{q+1})^2} + sgn(X + \frac{q}{q+1})\frac{\frac{1}{q+1}}{(X+\frac{q}{q+1})^2} + X = 0$$

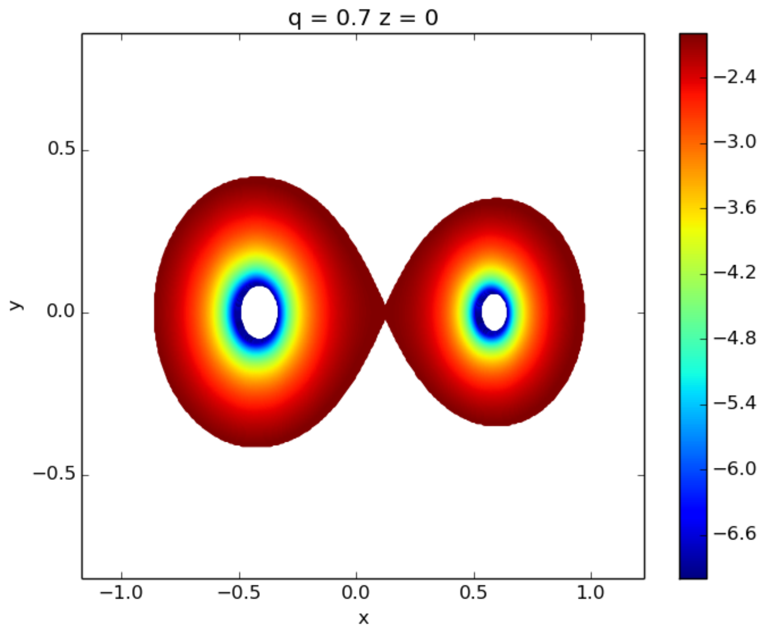
where $sgn()$ is signum funcion (because in $\widetilde{\phi}$ there are $\sqrt{(X-\frac{1}{q+1})}^2 = |X - \frac{1}{q+1}|$) In this paper derivative and L_i points was calculate by 'scipy' in python. Next step is numerical measure the volume. For each X between L_1 and L_3 points, for each Y from 0 to place where $\widetilde{\phi}(X,Y,Z) \leqslant \widetilde{\phi}(L_1,Y,Z)$ (the same for Z): step $V = V + dx + dy + dz$ when $\widetilde{\phi}(X,Y,Z) \leqslant \widetilde{\phi}(L_1,Y,Z)$. This result should be multiply by 4 because it is symmetrical by y- and z-axis. Radius of a sphere with the same volume as roche lobe is equal:

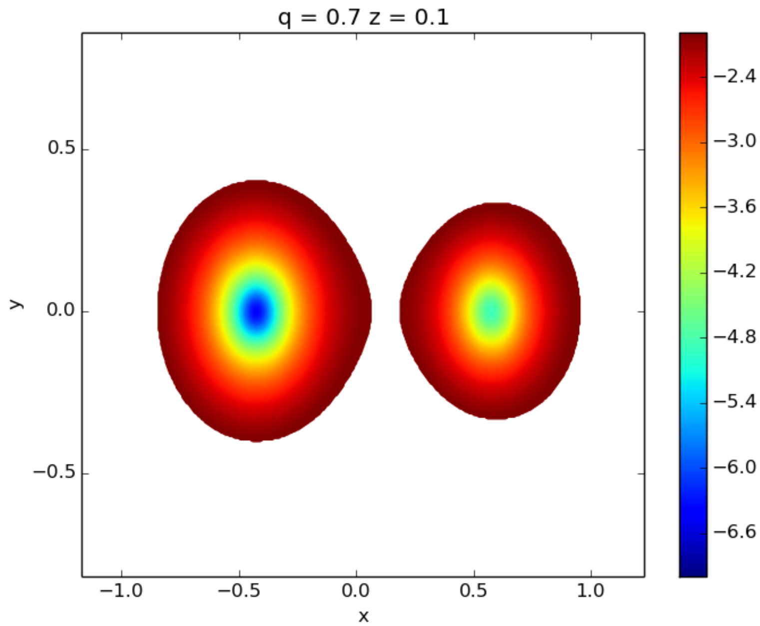
$$\frac{R_L}{A} = (\frac{3V}{4\pi})^{\frac{1}{3}}$$

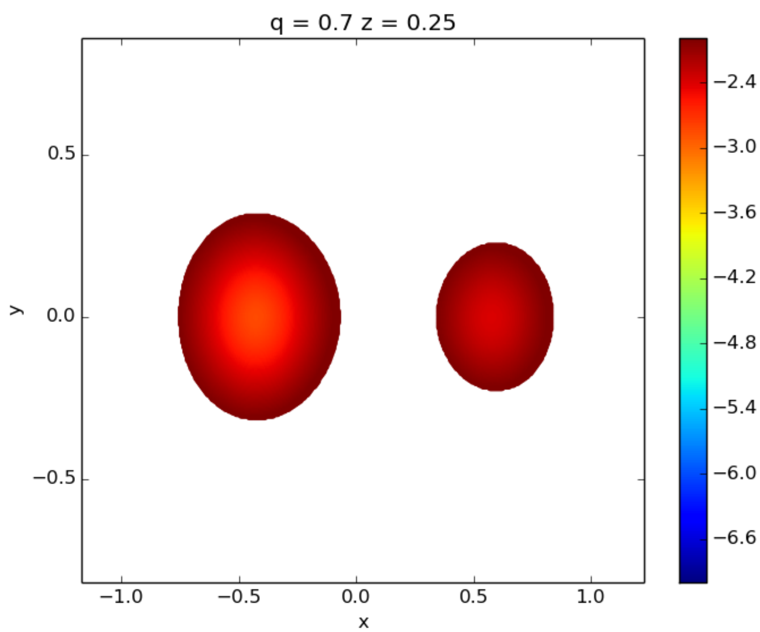
Figure presents relation between this value and q:



Example of section of roche lobe:




q = 0.7 z = 0.1
0.5
0.0
−0.5
y
−1.0
−0.5
0.0
0.5
1.0
x
−2.4
−3.0
−3.6
−4.2
−4.8
−5.4
−6.0
−6.6


q = 0.7 z = 0.25
0.5
0.0
−0.5
y
−1.0
−0.5
0.0
0.5
1.0
x
−2.4
−3.0
−3.6
−4.2
−4.8
−5.4
−6.0
−6.6