## Analiza numeryczna

Wyklad 1. Dziwne rzeczy!

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## Przyklady

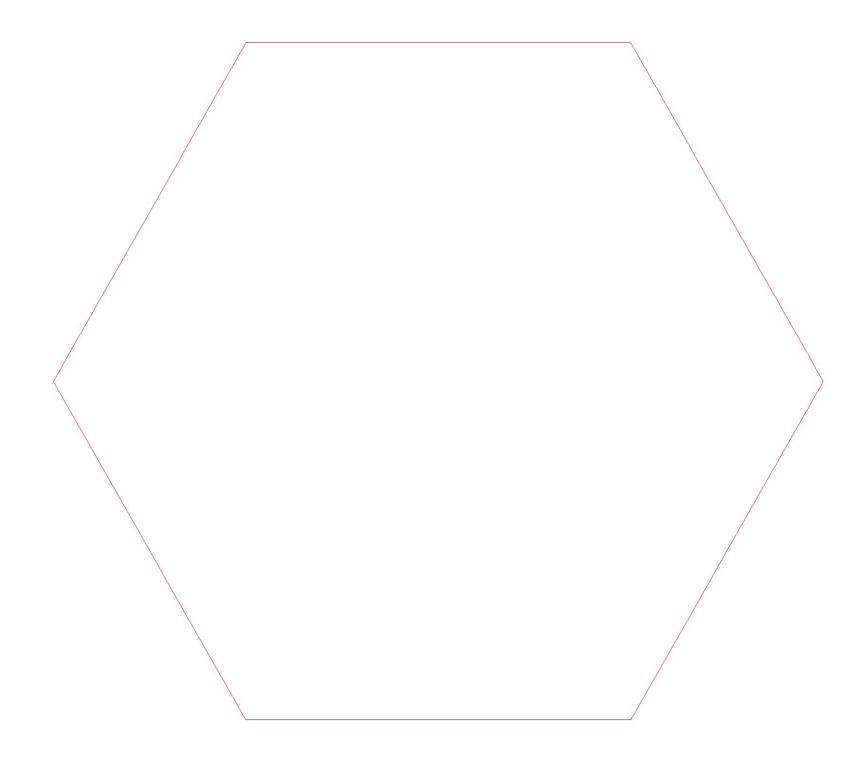
```
Przyklad 1. Co sie do licha dzieje...
  [ >
  >
  > restart;
  > Digits:=18:
  [ >
  > x:=1.0:
   > while 1<>1+x
       x := x/2.0
   > print(``);print(`========);print(``);
   > print(`Obliczona wartosc `=x);
   > print(``);print(`============);print(``);
                                                      _____
                                                          Obliczona wartosc = 0.346944695195361422 \cdot 10^{-17}
                                                      ______
  [ >
  | >
   > print(``);print(`========);print(``);
   > print(`Wartosc wyrazenia 1+x to`);
   > print(``);
   > 1.0+x;
   > print(``);
   > print(``);print(`=======);print(``);
```

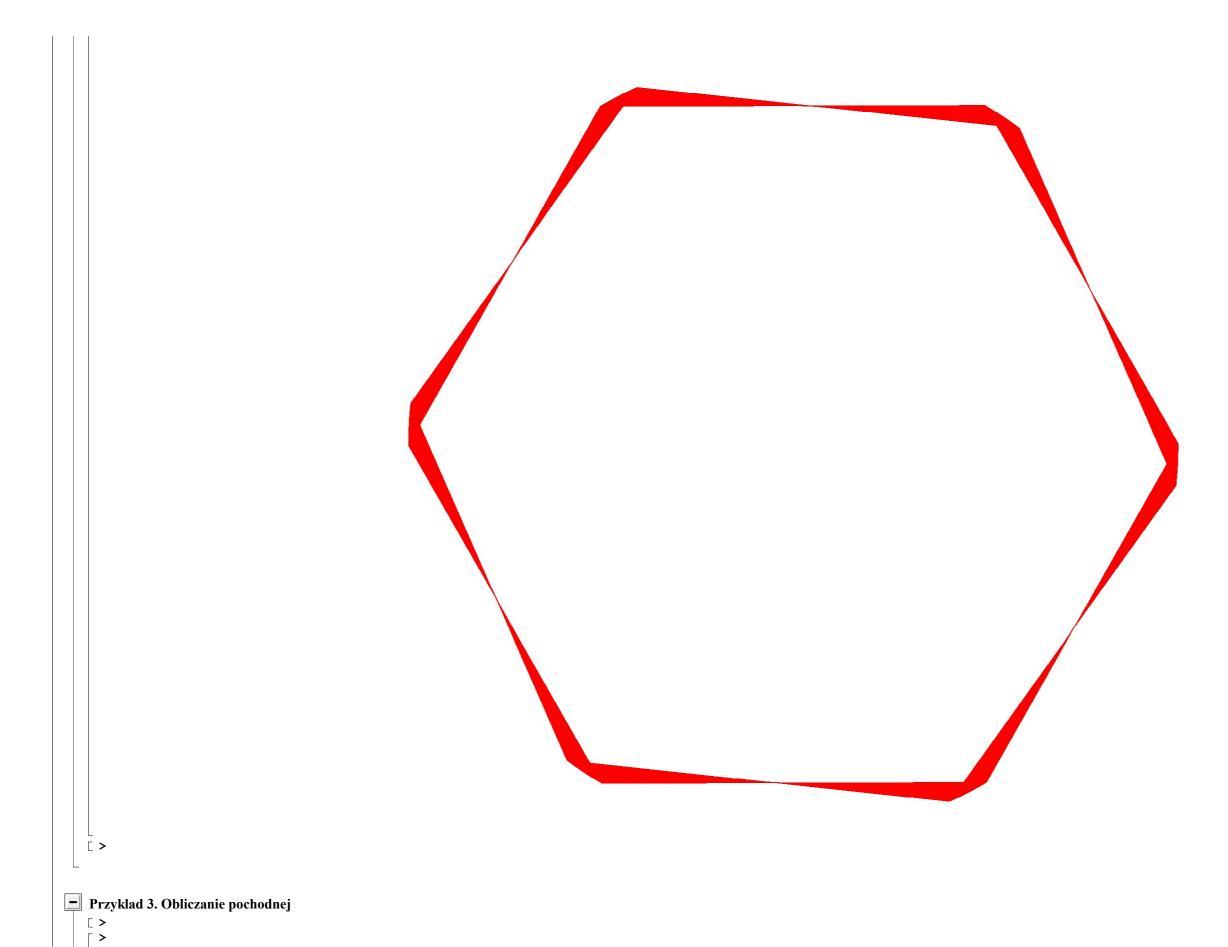
Wartosc wyrazenia 1+x to

1.0000000000000000000

[ > > print(``);print(`======);print(``); > print(`Czy rzeczywiscie 1+x = 1`); > print(``); > if 1+x=1 then print(`TAK!!!`) fi; > print(``);print(`======);print(``); Czy rzeczywiscie 1+x=1TAK!!! \_\_\_\_\_ [> > print(``);print(`======);print(``); > print(`Czy rzeczywiscie x <> 0`); > print(``); > if x<>0 then print(`TAK!!!`) fi; > print(``);print(`=======);print(``); Czy rzeczywiscie x <> 0TAK!!! \_\_\_\_\_ [ > > print(``);print(`=======);print(``); > print(`A ile wynosi x\*1/x ?`); > print(``); > w:=1/x: > x\*w; > print(``);print(`======);print(``);

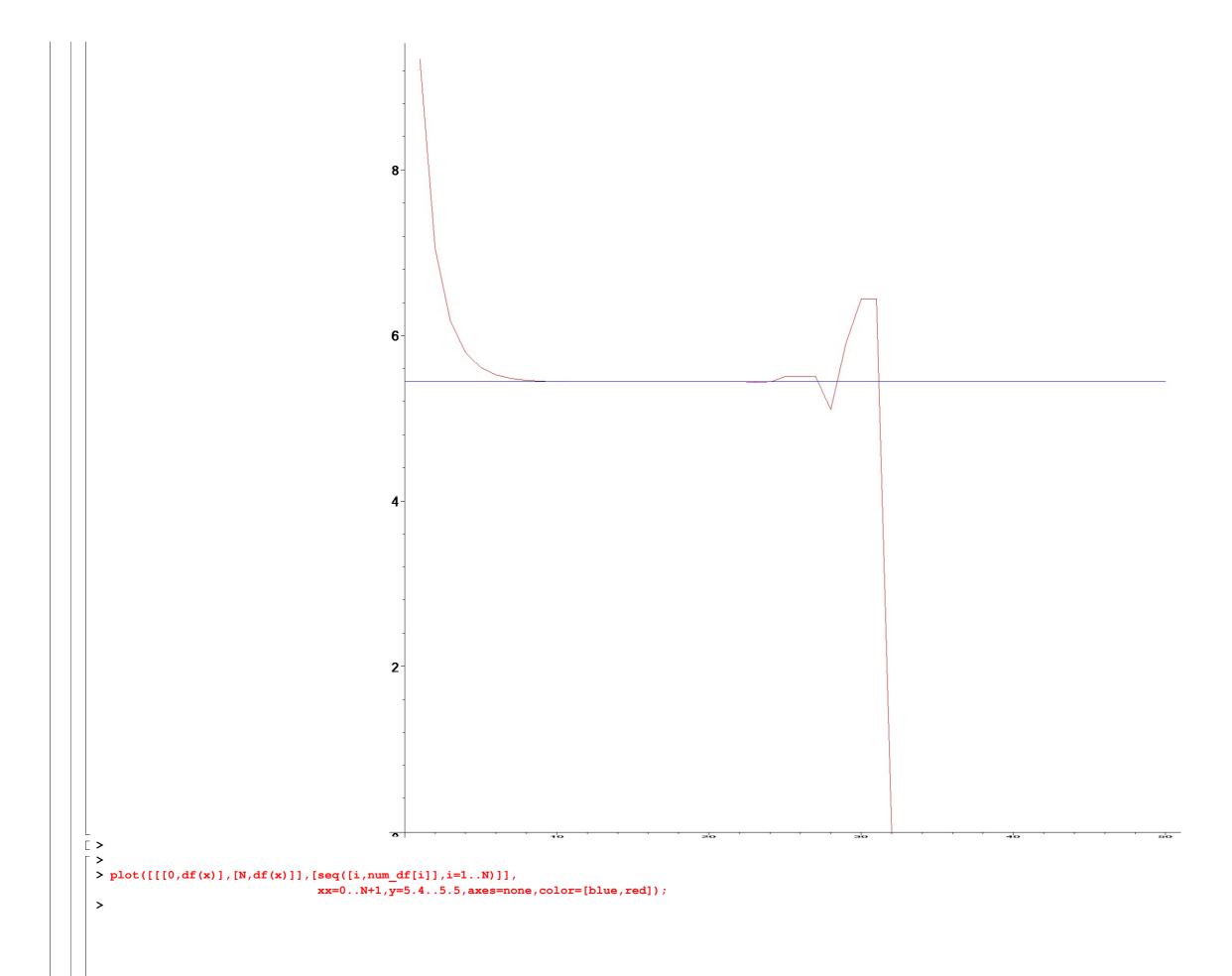
## Przyklad 2. Wielokąty foremne

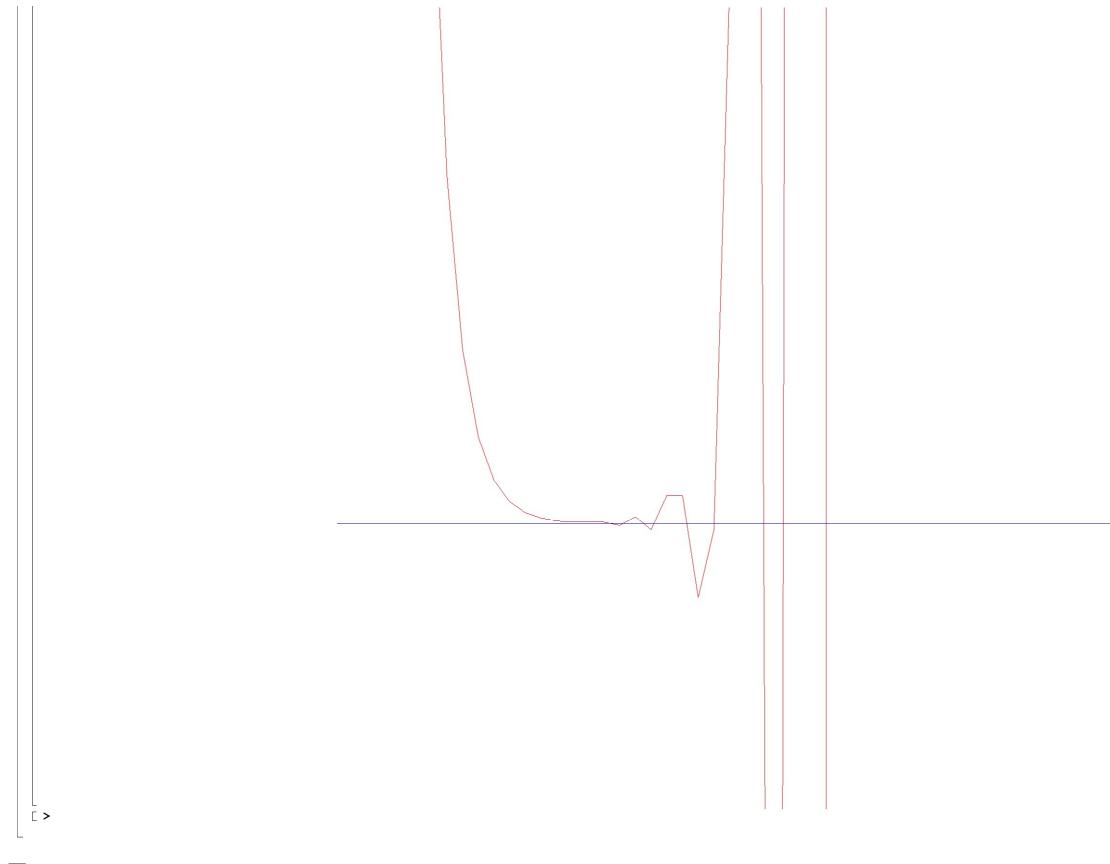




> restart; >

```
> print(``);print(`====
                                           ========:);print(``);
 > Diff(f(x),x)=Limit((f(x+h)-f(x))/h,h=0);
 > print(``);print(`===
                                                   === `);print(``);
                                                                 _____
                                                                           \frac{d}{dx}f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
[ >
 > print(``);print(`======
 > f:=x->exp(2*x);
 > df:=unapply(diff(f(x),x),x);
 > print(``);print(`========
                                      =======:);print(``);
 > print(`Niech x `=0.5);
 > print(`Wartosc pochodnej `=df(0.5));
 > print(``);print(`=======);print(``);
                                                                 _____
                                                                                f := x \to \mathbf{e}^{(2x)}
                                                                               df := x \to 2 e^{(2x)}
                                                                                Niech x = 0.5
                                                                          Wartosc\ pochodnej = 5.436563656
                                                                ______
[ >
 > N:=50: ### 5, 10, 15, 20, 30, 50
 > x:=0.5: h:=1.0:
 > for i from 1 to N
      h:=h/2;
      num_df[i] := (f(x+h)-f(x))/h;
 > plot([[[0,df(x)],[N,df(x)]],[seq([i,num_df[i]],i=1..N)]],color=[blue,red]);
```





Przyklad 4. Związki rekurencyjne

```
> print(``);print(`=====
                                                -----:);print(``);
 > print(`Zwiazek rekurencyjny`);
 > print(``);
 > print(x[0]=1/3, ` ',x[1]=-1/9);
 > x[n+2]=8/3*x[n+1]+x[n];
 > print(``);print(`======
                                          =======);print(``);
 > print(`Rozwiazanie `);
  print(``);
 > print(x[n]=(-1)^n/3^(n+1));
 > print(``);print(`======);print(``);
                                                                                             Zwiazek rekurencyjny
                                                                             _____
                                                                                                Rozwiazanie
                                                                             _____
 > restart;
 > Digits:=18:
_ >
[ >
 > N:=100:
 > x[0]:=1.0/3.0:
 > x[1]:=-1.0/9.0:
 > for n from 2 to N
     do
       x[n] := 8*x[n-1]/3+x[n-2];
       printf("%45sx[%2d] = %3.18f\n","",n,x[n]);
        # b[n] := abs(x[n] - (-1)^n/3.0^(n+1))
     od:
 > print(``);print(``);
 > # plot([seq([i,b[i]],i=0..N)]);
                                        x[2] = .037037037037037037
                                        x[2] = .037037037037037037
x[3] = -.012345679012345679
x[4] = .004115226337448560
x[5] = -.001371742112482853
x[6] = .000457247370827618
x[7] = -.000152415790275872
x[8] = .000050805263425294
```

```
x[9] = -.000016935087808421
x[10] = .000005645029269506
x[11] = -.000001881676423071
x[12] = .000000627225474649
x[13] = -.000000209075157341
x[14] = .000000069691721738
x[15] = -.000000023230566040
x[16] = .0000000023230366040
x[16] = .000000007743545632
x[17] = -.0000000002581111021
x[18] = .000000000860582910
x[19] = -.000000000286223261
x[20] = .00000000097320881
x[21] = -.000000000026700912
x[22] = .000000000026118449
x[23] = .00000000042948285
x[24] = .000000000140647210
x[25] = .000000000418007511
x[26] = .000000001255333904
x[27] = .00000003765564589
x[28] = .000000011296839475
x[29] = .000000033890469856
x[30] = .000000101671425757
x[31] = .000000305014271873
x[32] = .000000915042817419
x[33] = .000002745128451657
x[34] = .000008235385355171
x[35] = .000024706156065446
x[36] = .000074118468196361
x[37] = .000222355404589075
x[38] = .000667066213767227
x[39] = .002001198641301681
x[40] = .006003595923905043
x[41] = .018010787771715130
x[42] = .054032363315145389
x[43] = .162097089945436168
x[44] = .486291269836308504
x[45] = 1.458873809508925510
x[46] = 4.376621428526776530
x[47] = 13.129864285580329600
x[48] = 39.389592856740988800
x[49] = 118.168778570222967000
x[50] = 354.506335710668901000
x[51] = 1063.519007132006700000
x[52] = 3190.557021396020100000
x[53] = 9571.671064188060300000
x[54] = 28715.013192564180900000
x[55] = 86145.039577692542700000
x[56] = 258435.118733077628000000
x[57] = 775305.356199232884000000
x[58] = 2325916.068597698650000000
x[59] = 6977748.205793095950000000
x[60] = 20933244.617379287800000000
x[61] = 62799733.852137863400000000
x[62] = 188399201.556413590000000000
x[63] = 565197604.669240770000000000
x[64] = 1695592814.00772231000000000
x[65] = 5086778442.02316693000000000
x[66] = 15260335326.06950080000000000
x[67] = 45781005978.20850240000000000
x[68] = 137343017934.625507000000000000
x[69] = 412029053803.87652100000000000
x[70] = 1236087161411.62957000000000000
x[71] = 3708261484234.888710000000000000
x[72] = 11124784452704.666100000000000000
x[73] = 33374353358113.998300000000000000
x[74] = 100123060074341.99500000000000000
x[77] = 2703322622007233.8600000000000000000
x[78] = 8109967866021701.58000000000000000
x[79] = 24329903598065104.800000000000000000
x[80] = 72989710794195314.400000000000000000
x[81] = 218969132382585943.000000000000000000
x[82] = 656907397147757829.000000000000000000
x[83] = 1970722191443273480.000000000000000000
x[84] = 5912166574329820440.000000000000000000
x[85] = 17736499722989461300.00000000000000000
x[86] = 53209499168968383900.00000000000000000
x[87] = 159628497506905151000.00000000000000000
x[88] = 478885492520715453000.00000000000000000
x[89] = 1436656477562146360000.00000000000000000
```

## - Przyklad 5. Perfidny wielomian

[ >

```
[ >
 > restart;
> Digits:=18:
_ >
[ >
 > print(``);print(`========
 > print(`Wzor I`);print(``);
 > f1:=x->(x-1)^8;
 > print(``);print(`====
                                 ======`);print(``);
 > print(`Wzor II`);print(``);
 > f2:=x->x^8-8*x^7+28*x^6-56*x^5+70*x^4-56*x^3+28*x^2-8*x+1;
 > print(``);print(`======);print(``);
 > print(`Wzor III`);print(``);
 > f3:=x->((((((((x-8)*x+28)*x-56)*x+70)*x-56)*x+28)*x-8)*x+1;
 > print(``);print(`==========);print(``);
```

\_\_\_\_\_

Wzor I

$$fl := x \to (x-1)^8$$

-----

Wzor II

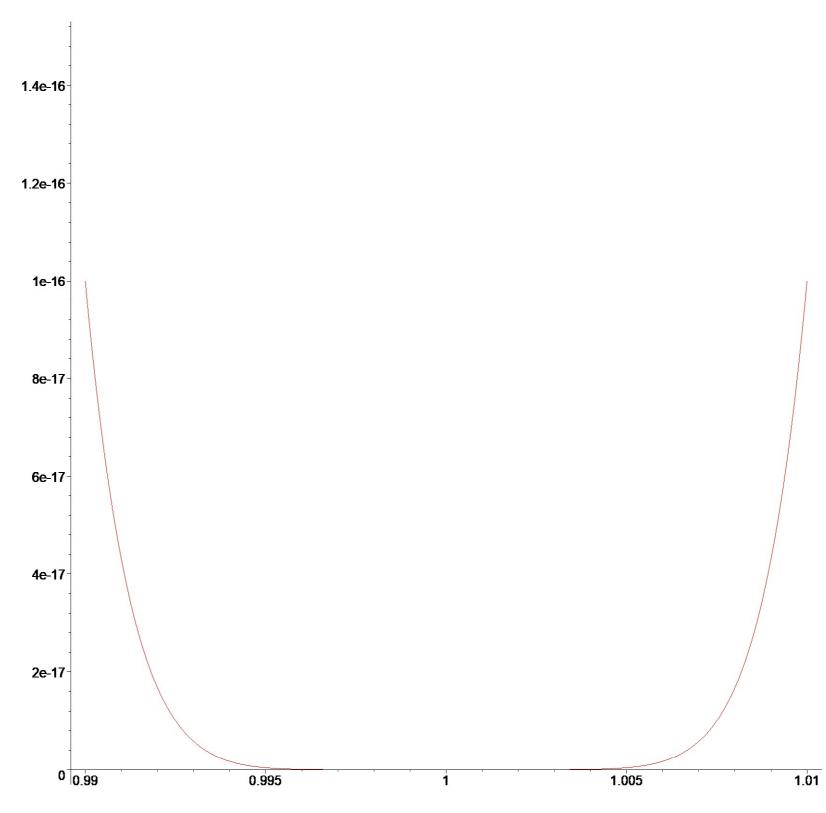
$$f2 := x \rightarrow x^8 - 8 x^7 + 28 x^6 - 56 x^5 + 70 x^4 - 56 x^3 + 28 x^2 - 8 x + 1$$

\_\_\_\_\_

Wzor III

$$f3 := x \to ((((((((x-8)x+28)x-56)x+70)x-56)x+28)x-8)x+1$$

\_\_\_\_\_



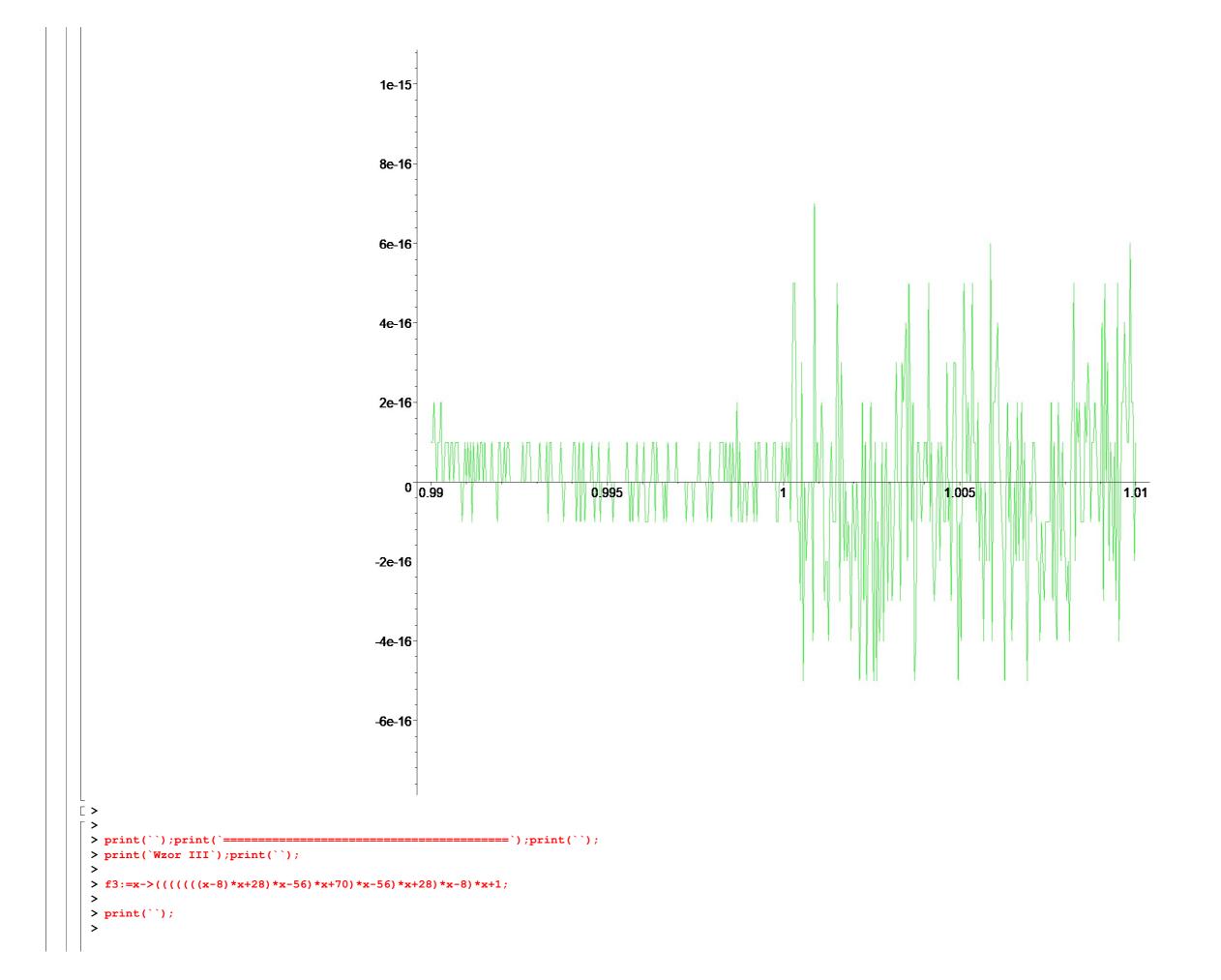
```
> 
> print(``);print(`==========);print(``);
> print(`Wzor II`);print(``);
> 
> f2:=x->x^8-8*x^7+28*x^6-56*x^5+70*x^4-56*x^3+28*x^2-8*x+1;
> 
> print(``);
> 
> print(``);
>
```

```
> n:=500:
> w2:=[seq([0.99+i/n*0.02,f2(0.99+i/n*0.02)],i=0..n)]:
>
> plot([w2],color=[green]);
>
```

\_\_\_\_\_

Wzor II

 $f2 := x \to x^8 - 8x^7 + 28x^6 - 56x^5 + 70x^4 - 56x^3 + 28x^2 - 8x + 1$ 



```
> n:=500:
> w3:=[seq([0.99+i/n*0.02,f3(0.99+i/n*0.02)],i=0..n)]:
>
> plot([w3],color=[blue]);
>
```

\_\_\_\_\_

Wzor III

$$f3 := x \to ((((((((x-8)x+28)x-56)x+70)x-56)x+28)x-8)x+1$$

```
2e-16-
                     1e-16-
                      0 0.99
                                                             1.005
                                                                         1.01
                                    0.995
                     -1e-16
```

```
> restart;
 > print(``);print(`======);print(``);
 > print(`Rownanie kwadratowe`);
 > print(``);
 > a*x^2+b*x+c=0;
 > print(``);print(`=======);print(``);
 > print(`Rozwiazanie`);
 > print(``);
 > r := [solve(a*x^2+b*x+c,x)]:
 > 'x[1]'=r[1];
> 'x[2]'=r[2];
 > print(``);print(`======);print(``);
                                                             Rownanie kwadratowe
                                                               ax^2 + bx + c = 0
                                                   _____
                                                               Rozwiazanie
                                                   _____
[ >
┌ >
 > restart;
 > Digits:=18:
>
[>
>
 > print(``);print(`========);print(``);
 > a:=1.0;
 > b:=10^9;
 > c:=1.0;
 > print(``);print(`==========);print(``);
  'f(x)'=a*x^2+b*x+c; 
 f:=x-a*x^2+b*x+c:
 > print(``);print(`======);print(``);
 > x[1]:=(-b+sqrt(b^2-4*a*c))/(2*a);
 > x[2]:=(-b-sqrt(b^2-4*a*c))/(2*a);
```

```
> print(``);print(`=========
                     -----`);print(``);
                                              _____
                                                           a := 1.0
                                                         b := 1000000000
                                                           c := 1.0
                                                     f(x) = 1.0 x^2 + 1000000000 x + 1.0
                                                     ______
[ >
> print(``);print(`===========);print(``);
> print(`Wartosc f(x[1])`);
> print(``);
> f(x[1]);
                    -----`);print(``);
 > print(``);print(`=========
                                                         Wartosc f(x[1])
                                                          0.1 \, 10^{-17}
[ >
> print(``);print(`=======);print(``);
> print(`Wartosc f(x[2])`);
> print(``);
> f(x[2]);
> print(``);print(`======
                     -----`);print(``);
                                              _____
                                                         Wartosc f(x[2])
```

\_\_\_\_\_

+ Przyklad 7. Prawo laczności

[ >

[ **>** 

```
Przyklad 8. Czy bankowcy mogą przypadkiem oszukać?
```

```
> restart;
>
> print(``);print(`========");print(``);
> print(`Kapitalizacja odsetek`);
> print(``);
> W[n]=W[0]*(1+r/n)^n;
> print(``);print(`=====");print(``);
>
```

-----

Kapitalizacja odsetek

$$\mathbf{W}_n = \mathbf{W}_0 \left( 1 + \frac{r}{n} \right)^n$$

\_\_\_\_\_

Digits := 8

```
> print(`Oczywiscie im czesciej w roku naliczane sa odsetki tym wiecej zarabiamy`);
 > print(``);
 > seq(printf("%37s n = %2d. Na koncie mamy %f zlotych.\n","",n,W(Wklad,r,n)),n=1..12);
 > print(``);print(`=======);print(``);
                                                                             _____
                                                                                               Wklad := 1000
                                                                                               r := 0.0333
                                                                           Oczywiscie im czesciej w roku naliczane sa odsetki tym wiecej zarabiamy
                                  n = 1. Na koncie mamy 1033.300000 zlotych.
                                  n = 2. Na koncie mamy 1033.577200 zlotych.
                                  n = 3. Na koncie mamy 1033.671000 zlotych.
                                  n = 4. Na koncie mamy 1033.718200 zlotych.
                                  n = 5. Na koncie mamy 1033.746600 zlotych.
n = 6. Na koncie mamy 1033.765400 zlotych.
                                  n = 7. Na koncie mamy 1033.778700 zlotych.
                                  n = 8. Na koncie mamy 1033.789100 zlotych.
                                  n = 9. Na koncie mamy 1033.797100 zlotych.
                                  n = 10. Na koncie mamy 1033.803500 zlotych.
                                  n = 11. Na koncie mamy 1033.809000 zlotych.
                                  n = 12. Na koncie mamy 1033.813000 zlotych.
[ >
 > print(``);print(`======);print(``);
 > print(`Im czesciej w roku naliczane sa odsetki tym wiecej zarabiamy,`);
 > print(`ale czy na pewno tak jest???`);
 > print(``);
                                                                              Im czesciej w roku naliczane sa odsetki tym wiecej zarabiamy,
                                                                                          ale czy na pewno tak jest???
[ >
 > n:=365: ## Każdego dnia
 > printf("%33s n = %9d. Na koncie mamy %f zlotych.\n","",n,W(Wklad,r,n));
 > print(``);
 > n:=24*365: ## Co godzinę
 > printf("%33s n = 24*365. Na koncie mamy %f zlotych.\n","",W(Wklad,r,n));
 > print(``);
 > n:=30*24*365: ## Co dwie minuty
 > printf("%33s n = 30*24*365. Na koncie mamy %f zlotych.\n","",W(Wklad,r,n));
 > n:=2*60*24*365: ## Co 30 sekund
```

```
> printf("%33s n = 2*60*24*365. Na koncie mamy %f zlotych.\n","",W(Wklad,r,n));
> print(``);print(`===========);print(``);
> n = 365. Na koncie mamy 1033.846700 zlotych.

n = 24*365. Na koncie mamy 1033.817700 zlotych.

n = 30*24*365. Na koncie mamy 1026.280000 zlotych.

n = 2*60*24*365. Na koncie mamy 1000.000000 zlotych.
```

+ Przyklad 9. Szereg Taylora funkcji f(x)=ln(x)

+ Przyklad 10. Obliczanie wartości funkcji cos(x)