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CSE 3723
Boolean Simplification

1. $a + \bar{a} = 1$ {5a} [∵ Laws of Complementarity $x + \bar{x} = 1$]
2. $a \cdot \bar{a} = 0$ {5b} [∵ Laws of complementarity $x \cdot \bar{x} = 0$]
3. $a + b \cdot \bar{b}$
$= a + 0$ {5b} [∵ Laws of complementarity $x \cdot \bar{x} = 0$]
$= a$ {1a} [∵ Identity Laws, $x + 0 = x$]

4. $a + \overline{b \cdot \bar{b}}$
$= a + \bar{b} + b$ {9a} [∵ De Morgan's law $\overline{x \cdot y} = \bar{x} + \bar{y}$]
$= a + 1$ {5a} [∵ Laws of complementarity $x + \bar{x} = 1$]
$= 1$ {2a} [∵ Laws of 0 and 1, $x + 1 = 1$]

5. $a + a \cdot b$
Using distributive laws, $x + (y \cdot z) = (x+y) \cdot (x+z)$
$= (a+a) \cdot (a+b)$ {8a}
Using idempotent laws, $x + x = x$
$= a \cdot (a+b)$ {3b}
Using simplification theorems, $x \cdot (x+y) = x$
$= a$ {10d}
Alternatively, apply the absorption law, $a + ab = a$

$\therefore a + a \cdot b = a$

6. $a + \bar{a} \cdot b$
$= (a + \bar{a}) \cdot (a+b)$ {8a} [∵ Distributive laws $x + (y \cdot z) = (x+y)(x+z)$]

$= 1 \cdot (a+b)$ {5a} [∵ Laws of complementarity, $x + \bar{x} = 1$]

$= a+b$ {1b} [∵ Identity laws $x \cdot 1 = x$]
Also, applying the absorption law, $\bar{a} \cdot b + a = b + a$

$\therefore a + \bar{a} \cdot b = a + b$

(7) $(a+b)\cdot\overline{(c\cdot d)} + (a+b)\cdot(c\cdot d)$

Using De Morgan Laws, $\overline{x\cdot y} = \bar{x} + \bar{y}$,

$= (a+b)(\bar{c}+\bar{d}) + (a+b)\cdot(c\cdot d)$ {9b}

Using distributive laws, $x\cdot(y+z) = x\cdot y + x\cdot z$.

$= a\cdot(\bar{c}+\bar{d}) + b(\bar{c}+\bar{d}) + (a+b)\cdot(c\cdot d)$ {8b}

Again, distributive laws,

$= a\bar{c} + a\bar{d} + b\bar{c} + b\bar{d} + (a+b)cd$ {8b}

$= a\bar{c} + a\bar{d} + b\bar{c} + b\bar{d} + cda + cdb$ {8b}

Applying the distributive law, $ab + ac = a(b+c)$

$= a\bar{d} + b\bar{c} + b\bar{d} + a(cd + \bar{c}) + cdb$ {8b}

Applying the absorption law, $ab + \bar{a} + b + \bar{a}$

$= a\bar{d} + b\bar{c} + b\bar{d} + a(d+\bar{c}) + cdb$

$= a\bar{d} + b\bar{d} + a(d+\bar{c}) + b(cd+\bar{c})$ [∵ Distributive]

$= a\bar{d} + b\bar{d} + a(d+\bar{c}) + b(d+\bar{c})$ [∵ Absorption law]

$= a\bar{d} + b\bar{d} + ad + a\bar{c} + b(d+\bar{c})$ [∵ Distributive]

$= a(\bar{d}+d) + b\bar{d} + a\bar{c} + b(d+\bar{c})$ [∵ $ab+ac = ab+c$]

Apply the complement law, $a + \bar{a} = 1$

$= a\cdot 1 + b\bar{d} + a\bar{c} + b(d+\bar{c})$

Using identity law, $a\cdot 1 = a$

$= a + b\bar{d} + a\bar{c} + b(d+\bar{c})$

Apply the absorption law, $a + ab = a$

$= a + b\bar{d} + b(d+\bar{c})$

Apply the distributive law,

$= a + b\bar{d} + bd + b\bar{c}$ [∵ $ab+ac = ab+c$]

$= a + b(\bar{d}+d) + b\bar{c}$ [∵ complement law]

$= a + b\cdot 1 + b\bar{c}$ [∵ Identity law, $a\cdot 1 = a$]

$= a + b + b\bar{c}$

$= a + b$ [∵ Absorption law $a + ab = a$]

(8) $a \cdot b + a \cdot b \cdot c + a \cdot b \cdot c \cdot d$

Using simplification theorem, $x + x \cdot y = x$, we get

$= a \cdot b + a \cdot b \cdot c \cdot d \quad \{10c\}$

Again, using simplification theorem, $a \cdot b + a \cdot b \cdot c \cdot d = a \cdot b$

$x + x \cdot y = x$

$= a \cdot b \quad \{10c\}$

(9) $a(\bar{a} + b)$

Using distributive laws, $x \cdot (y + z) = x \cdot y + x \cdot z$

$= a \cdot \bar{a} + ab \quad \{8b\}$

Using laws of complementarity, $x \cdot \bar{x} = 0$

$= 0 + ab \quad \{5b\}$

Using identity laws, $x + 0 = x$

$= ab \quad \{1a\}$

(10) $a \cdot b + a \cdot \bar{b}$

Using distributive laws $a(b + \bar{b}) = a \cdot b + a \cdot \bar{b}$

$= a(b + \bar{b}) \quad \{8b\}$

Using laws of complementarity, $b + \bar{b} = 1$

$= a \cdot 1 \quad \{5a\}$

Applying the identity law, $a \cdot 1 = a$

$= a \quad \{1b\}$

(11) $\bar{a} + \bar{a} \cdot b$

Using simplification theorems, $x + x \cdot y = x$

$= \bar{a} \quad \{10c\}$

(12) $(a+\bar{b}) \cdot (a+b)$

Applying distributive laws, $x \cdot (y+z) = x \cdot y + x \cdot z$

$= a(a+b) + \bar{b}(a+b)$ {8b}

Applying distributive laws, $x \cdot (y+z) = x \cdot y + x \cdot z$

$= aa + ab + \bar{b}(a+b)$ {8b}

Applying idempotent laws, $x \cdot x = x$

$= a + ab + \bar{b}(a+b)$ {3b}

Applying simplification theorems, $x + x \cdot y = x$

$= a + \bar{b}(a+b)$ {10c}

Applying distributive laws, $x \cdot (y+z) = xy + xz$

$= a + a\bar{b} + b \cdot \bar{b}$ {8b}

Applying complement laws, $x \cdot \bar{x} = 0$

$= a + a\bar{b} + 0$ {5b}

Applying identity laws, $x + 0 = x$

$= a + a\bar{b}$ {1a}

Applying simplification theorem, $x + x \cdot y = x$

$= a$ {10c}

(13) $(\bar{a}+\bar{b}) \cdot (\bar{a}+b)$

$= \bar{a}(\bar{a}+b) + \bar{b}(\bar{a}+b)$ {8b}

$= \bar{a}\bar{a} + \bar{a}b + (\bar{a}+b)\bar{b}$ {8b}

$= \bar{a} + \bar{a}b + \bar{b}(\bar{a}+b)$ {3b}

$= \bar{a} + (\bar{a}+b)\bar{b}$ {10c}

$= \bar{a} + \bar{b}\bar{a} + b\bar{b}$ {8b}

$= \bar{a} + \bar{a}\bar{b} + 0$ {5b}

$= \bar{a} + \bar{a}\bar{b}$ {1a}

$= \bar{a}$ {10c}

(14). $(a+\bar{b}+c+\bar{d}).c$

Using the distributive laws, $x.(y+z) = x.y + x.z$

$= ac + \bar{b}c + cc + c\bar{d}$ {8b}

Using the idempotent laws, $x.x = x$

$= ac + \bar{b}c + c + c\bar{d}$ {3b}

$= c + ac + \bar{b}c + c\bar{d}$

Using the simplification theorem, $x + x.y = x$

$= c + \bar{b}c + c\bar{d}$ {10c}

Using the simplification theorem, $x + x.y = x$

$= c + c\bar{d}$ {10c}

Using the simplification theorem, $x + xy = x$

Using the

$= c$

(15). $\overline{(\bar{a}+\bar{a})}$

Using Demorgan's theorem, $\overline{x+y} = \bar{x}.\bar{y}$

$= \bar{\bar{a}}.\bar{\bar{a}}$ {9a}

Using involution (double negation) laws, $\bar{\bar{x}} = x$

$= a.a$ {4}

Using idempotent law, $x.x = x$

$= a$ {3a}

(16). $\overline{(a+\bar{a})}$

Applying DeMorgan's theorem, $\overline{x+y} = \bar{x}.\bar{y}$

$= \bar{a}.\bar{\bar{a}}$ {9a}

Applying involution laws, $\bar{\bar{x}} = x$

$= a.\bar{a}$ {4}

Applying laws of complementarity, $x.\bar{x} = 0$

$= 0$ {5b}

17. $a \cdot c + \bar{a} \cdot b + b \cdot c$

Using consensus theorem, $x \cdot y + y \cdot z + \bar{x} \cdot z = x \cdot y + \bar{x} \cdot z$

$= ac + \bar{a}b$ {11 a}

18. $a + (a \cdot \bar{b} \cdot c \cdot d)$

Using the simplification theorem, $a + a\bar{b}cd = a$

$= a$ {10 c} $[\because x + xy = x]$

19. $\bar{a} \cdot (\overline{a \cdot b \cdot c \cdot d})$

Applying DeMorgan Theorem, $\overline{x \cdot y} = \bar{x} + \bar{y}$

$= \bar{a}(\bar{a} + \bar{b} + \bar{c} + \bar{d})$ {9 b}

Applying distribution laws, $x \cdot (y + z) = xy + xz$

$= \bar{a}\bar{a} + \bar{a}\bar{b} + \bar{a}\bar{c} + \bar{a}\bar{d}$ {8 b}

Applying the idempotent laws, $x \cdot x = x$

$= \bar{a} + \bar{a}\bar{b} + \bar{a}\bar{c} + \bar{a}\bar{d}$ {3 b}

Applying the simplification theorem, $x + xy = x$

$= \bar{a} + \bar{a}\bar{c} + \bar{a}\bar{d}$ {10 c}

$= \bar{a} + \bar{a}\bar{d}$ {10 c}

$= \bar{a}$ {10 c}

20. $a + (\overline{a \cdot b}) + b$

Applying DeMorgan Theorem, $\overline{x \cdot y} = \bar{x} + \bar{y}$

$= a + (\bar{a} + \bar{b}) + b$ {9 b}

Applying laws of complement, $x + \bar{x} = 1$

$= 1 + \bar{b} + b$ {5 a}

Applying laws of 0 and 1, $x + 1 = 1$

$= 1 + b$ {2 a}

$= 1$ {2 a}

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(21) $a + \overline{a} \cdot \overline{b} + b$

Using the simplification theorem, $x \cdot \overline{y} + y = x + y$

$= a + \overline{b} + b$ {10f}

Using the laws of complement, $x + \overline{x} = 1$

$= a + 1$ {5a}

Using the laws of 0 and 1, $x + 1 = 1$

$= 1$ {2a}

(22) $a \cdot (a + ab)$

Using the simplification theorem, $a + ab = a$

$= a \cdot a$ {10c} $[\because x + xy = x]$

Using the idempotent laws, $x \cdot x = x$

$= a$ {3b}

(23) $a \cdot (a + b + c + d + e)$

Using the distribution laws,

$= aa + ab + ac + ad + ae$ {8b}

Using the idempotent laws, $x \cdot x = x$

$= a + ab + ac + ad + ae$ {3b}

Using the simplification theorem, $a + ab = a$

$= a + ac + ad + ae$ {10c}

$= a + ad + ae$ {10c}

$= a + ae$ {10c}

$= a$ {10c}

24) $(a+c)\cdot(\bar{a}+b)\cdot(b+c)$

Applying the laws of distribution,

$= a(\bar{a}+b)(b+c) + c(\bar{a}+b)(b+c)$ {8b}

$= (a\bar{a}+ab)(b+c) + \bar{a}c(b+c) + cb(b+c)$ {8b}

$= a\bar{a}(b+c) + ab(b+c) + \bar{a}c(b+c) + cb(b+c)$ {8b}

Apply the complement, $x\cdot\bar{x} = 0$

$= 0 + ab(b+c) + \bar{a}c(b+c) + cb(b+c)$ {5b}

Applying the identity law, $a+0=a$

$= ab(b+c) + \bar{a}c(b+c) + cb(b+c)$ {10a}

Applying distributive laws,

$= abb + abc + \bar{a}c(b+c) + cb(b+c)$ {8b}

Applying the idempotent law, $x\cdot x = x$

$= ab + abc + \bar{a}c(b+c) + cb(b+c)$ {3b}

Applying the simplification theorem, $x+xy=x$

$= ab + \bar{a}c(b+c) + cb(b+c)$ {10c}

Applying distributive laws,

$= ab + \bar{a}cb + \bar{a}cc + cbb + ccb$ {8b}

Applying the idempotent law,

$= ab + \bar{a}cb + \bar{a}c + cb + cb$ {3b}

Applying the simplification theorem,

$= ab + \bar{a}c + cb + cb$ {10c}

Applying the idempotent law, $x+x = x$

$= ab + \bar{a}c + cb$ {3a}

Applying consensus theorem.

$= ab + \bar{a}c$ {11a}

$\therefore (a+c)\cdot(\bar{a}+b)\cdot(b+c) = ab+\bar{a}c$.

(25) $\bar{a} + \bar{b} + a \cdot b \cdot \bar{c}$

Using the simplification theorem, $x \cdot \bar{y} + y = x +$

$ab + \bar{a} = b + \bar{a}$

$= \bar{b} + \underline{ab\bar{c}} + \bar{a}$ {10 f}

$= \bar{a} + \bar{b} + b\bar{c}$

Using the simplification theorem,

$= \bar{a} + \bar{b} + \bar{c}$ {10f}

(26) $a \cdot b \cdot (\bar{a} + \bar{b} + c)$

Using the distribution laws, $x \cdot (y+z) = xy$

$= ab\bar{a} + ab\bar{b} + abc$ {8b}

$= a\bar{a}b + ab\bar{b} + abc$

Using the complement laws, $x \cdot \bar{x} = 0$

$= 0 + ab\bar{b} + abc$ {5b}

Using the identity laws, $x + 0 = x$

$= ab\bar{b} + abc$ {1a}

Using the complement laws, $x \cdot \bar{x} =$

$= 0 + abc$ {5b}

Using the identity laws

$= abc$ {1a}

(27) $\bar{a} \cdot (a \cdot b + b \cdot c + \bar{a} \cdot c)$

$= \bar{a}ab + \bar{a}bc + \bar{a}\bar{a}c$ {8b}

$= \bar{a}ab + \bar{a}bc + \bar{a}c$ {3b, $x \cdot x = x$}

$= 0 + \bar{a}bc + \bar{a}c$ {5b}

$= \bar{a}bc + \bar{a}c$ {1a}

$= \bar{a}c$ {10a}

(28) $\bar{c} \cdot (a \cdot b + b \cdot c + \bar{a} \cdot c)$

$= \bar{c}ab + \bar{c}bc + \bar{c}\bar{a}c$ [∵ Distribution, 8b]

$= \bar{c}ab + 0 + \bar{c}\bar{a}c$ [∵ Laws of complimentarity, 5b]

$= \bar{c}ab + \bar{c}\bar{a}c$ [∵ Identity law, $x+0=x$, 1a]

$= \bar{c}ab + 0$ [∵ 5b]

$= \bar{c}ab$ [∵ 1a]

(29) $b \cdot (a + \bar{b} \cdot c + c) + \bar{a} \cdot c$

$= b(a+c) + \bar{a}c$ {10c, $x+xy=x$}

$= ba + bc + \bar{a}c$ {8b, distribution}

$= ba + \bar{a}c$ {11a, Consensus theorem, $x \cdot y + y \cdot z + \bar{x} \cdot z = xy + \bar{x}z$}

(30) $a \cdot c + c \cdot (\bar{a} + a \cdot c)$

$= ac + c(\bar{a} + c)$ {∵ $xy + \bar{x} = y + \bar{x}$}

$= ac + \bar{a}c + cc$ {∵ 8b}

$= ac + \bar{a}c + c$ {∵ 3b} {6a}

$= \bar{a}c + c$ {∵ 10c} {6a}

$= c$ {∵ 10c}

(31) $\overline{(a \cdot b)} \cdot \overline{(b \cdot c)} \cdot (a + \bar{c})$

Using De Morgan's theorem,

$= (\bar{a} + \bar{b})(\bar{b} + \bar{c})(a + \bar{c})$ {9b}

$= \bar{a}(\bar{b} + \bar{c})(a + \bar{c}) + \bar{b}(\bar{b} + \bar{c})(a + \bar{c})$ {8b}

$= \bar{a}\bar{b}(a + \bar{c}) + \bar{a}\bar{c}(a + \bar{c}) + \bar{b}(\bar{b} + \bar{c})(a + \bar{c})$ {8b}

(31) contd>

$= \bar{a}\bar{b}a + \bar{a}\bar{b}\bar{c} + \bar{a}\bar{c}(a+\bar{c}) + \bar{b}(\bar{b}+\bar{c})(a+\bar{c})$ {8b}

$\downarrow$

$\bar{a}a\bar{b}$ {6b}

$\downarrow$

$= 0 + \bar{a}\bar{b}\bar{c} + \bar{a}a\bar{c} + \bar{a}\bar{c}\bar{c} + (\bar{b}\bar{b} + \bar{b}\bar{c})(a+\bar{c})$ {5b, 8b}

$= \bar{a}\bar{b}\bar{c} + \bar{a}a\bar{c} + \bar{a}\bar{c}\bar{c} + \bar{b}\bar{b}(a+\bar{c}) + \bar{b}\bar{c}(a+\bar{c})$

{1a, 8b}

$= \bar{a}\bar{c} + \bar{b}\bar{b}(a+\bar{c}) + \bar{b}\bar{c}(a+\bar{c})$ {10c}

$= \bar{a}\bar{c} + \bar{b}(a+\bar{c}) + \bar{b}\bar{c}(a+\bar{c})$ {3b}

$= \bar{a}\bar{c} + \bar{b}(a+\bar{c})$ {10c}

$= \bar{a}\bar{c} + a\bar{b} + \bar{b}\bar{c}$ {8b}

$= \bar{a}\bar{c} + a\bar{b}$ {11a}

(33) $b\bar{c}\bar{d} + a\cdot b\cdot\bar{c} + a\cdot\bar{c}\cdot d + a\cdot\bar{b}\cdot d + \bar{a}b\cdot\bar{d}$

$= b\bar{c}\bar{d} + a(b\bar{c} + \bar{b}d + \bar{c}d) + \bar{a}b\bar{d}$ {8b}

Consensus theorem,

{11 a} $xy + yz + \bar{x}z = xy + \bar{x}z$

Here, $x = b$, $y = c'$, $z = d$

$= b\bar{c}\bar{d} + a(b\bar{c} + \bar{b}d) + \bar{a}b\bar{d}$ {11a}

$= b\bar{c}\bar{d} + ab\bar{c} + a\bar{b}d + \bar{a}b\bar{d}$ {8b}

$= b(\bar{c}\bar{d} + a\bar{c} + \bar{a}\bar{d}) + a\bar{b}d$ {8b} {6a}

$= b(a\bar{c} + \bar{a}\bar{d} + \bar{c}\bar{d}) + a\bar{b}d$ {6a}

{11a}

$= b(a\bar{c} + \bar{a}\bar{d}) + a\bar{b}d$ $\searrow$ here $x = a$, $y = \bar{c}$, $z = \bar{d}$

$= ab\bar{c} + \bar{a}b\bar{d} + a\bar{b}d$

$= \bar{a}b\bar{d} + ab\bar{c} + a\bar{b}d$

(32) $(b+c+d)(a+b+c)(\bar{a}+c+d)(\bar{b}+\bar{c}+\bar{d})$

$\boxed{= (a'bc' + b'c + cd' + ac'd)}$ Ans

$= (b+c+d)(b'+c'+d')(a+b+c)(a'+c+d)$ {6b}

Dual form: $(x+y)(x'+z)(y+z) = (x+y)(x'+z)$

$= (b'+c+d')(a+b+c)(a'+c+d)$ {11b}

We deleted $(b+c+d)$ and kept $(a+b+c)(a'+c+d)$

$= (ab' + ac' + ad' + bb' + bc' + bd' + cb' + cc' + cd')$ {8b}

$(a'+c+d)$

$= (ab' + ac' + ad' + 0 + bc' + bd' + cb' + 0 + cd')(a'+c+d)$

{5b} {1a}

$= (ab' + ac' + ad' + bc' + bd' + cb' + cd')(a'+c+d)$

$= (aa'b' + aa'c' + aa'd' + a'bc' + a'bd' + a'cb' + a'cd' + acb' + acc' + acd' + bcc' + bcd' + ccb' + ccd' + ab'd + ac'd + add' + bc'd + bd'd + cb'd + cd'd)$ {8b}

$= 0 + 0 + 0 + a'bc' + a'bd' + a'cb' + a'cd' + acb' + 0 + acd' + 0 + bcd' + cb' + cd' + ab'd + ac'd + 0 + bc'd + 0 + cb'd + 0$ {5b}

$= a'bc' + a'bd' + a'cb' + a'cd' + acb' + acd' + bcd' + cb' + cd' + ab'd + ac'd + bc'd + cb'd$ {1a}

$= b'c + cb'd + cd' + bcd' + a'bc' + a'bd' + a'cb' + a'cd' + acb' + acd' + ab'd + ac'd + bc'd$ {6b}

$= b'c + cd' + a'bc' + ac'd + a'bd' + a'cb' + acb' + acd' + ab'd$ {10a}

$= b'c + ab'c + cd' + acd' + a'bc' + ac'd + a'bd' + a'b'c + ab'd$ {6b}

$= b'c + cd' + a'bc' + ac'd + a'bd' + a'b'c + ab'd$ {10a}

$= b'c + cd' + a'bc' + a'b'c + ac'd + a'bd' + ab'd$ {6b}

$= b'c + cd' + a'bc' + ac'd$ {11 a}

$\therefore (b+c+d)(a+b+c)(\bar{a}+c+d)(\bar{b}+\bar{c}+\bar{d})$

$= b'c + cd' + a'bc' + ac'd$