

294
HAND WRITTEN NOTES

OF

ELECTRONICS & COMMUNICATION
ENGINEERING

SUBJECT

ELECTROMAGNETIC THEORY

E.M.T

Q2.

$$W_t = 251 \text{ N}$$

$$d = 100 \text{ m}$$

$$A_{cr} = 5000 \times 10^{-4}$$

$$W_r = ?$$

294

$$W_r = \frac{251}{4\pi(100)^2} \times 5000 \times 10^{-4}$$

$$= 99 \times 10^{-6}$$

$$= 100 \mu \text{N}$$

Q3.

$$W_r = \frac{W_t G_t A_{cr}}{4\pi d^2}$$

$$W_r = \frac{800 \times 4 \times 10}{4\pi(1)^2} = \frac{1 \times 10 \times 1}{4\pi(1)^2} = \frac{10}{4\pi}$$

$$W_r = 0.8 \text{ N}$$

Q4

Q5

$$E_1 = \frac{d_1}{d_2}$$

$$E_2 = \frac{d_2}{d_1}$$

$$\frac{1}{E_2} = \frac{4 \times 10^3}{2 \times 10^3}$$

$$E_2 = \frac{1}{2} = 0.5 \text{ mV/m}$$

E.M.T 170

(2)

Friday

(3)

EMT

1. Static Electromagnetic fields - William Hayt & Rham Swies
 (theory) (Problems)
- IES = 30% - 40% Gate 10%
2. EM waves - Jordan Batmain }
 3. VI waves - John D Ryder }
 4. Guided waves - Jordan Batmain } 80% (gate) [15 question]
 5. Antennas - K.D. Prasad. (IES)

Static Electro-Magnetic fields:

Tough : coordinate systems
 Vector calculus

Easy: 1. Analogous E-H fields.

E - Electric field Intensity volt/m } strength of the
 H - Magnetic field intensity amp/m } field.

B - Magnetic flux density weber/m² } strength of the
 D - Electric flux density coulomb/m² } field.

$\epsilon = \epsilon_0 \epsilon_r$ = Permittivity of the medium

Physically meaning : It is the ability of the material to hold or to allow or to permit E field. Farad/m

$$C = \frac{Q}{\epsilon} A$$

The least value of ϵ is ϵ_0 because $\epsilon_r \geq 1$. So every medium in the world including vacuum permits electric field

Least Permittivity = Vacuum
Best Dielectric

(g)

$\mu = \mu_0 \mu_R$ = permeability of the medium

It is the ability of the material to hold or to allow or to permit H field.
unit :- Henry/m

$\mu = \mu_0$ when $\mu_R = 1$

$$\mu_R \geq 1$$

μ_0 = permeability of the vacuum.

$$C = \frac{Q}{V}$$

$$Coulomb = \frac{C}{m^2} \times \frac{V}{m}$$

$$D = \epsilon E$$

$$L = \frac{\Psi_m}{I}$$

$$\text{Henry} = \frac{\text{Henry} \times \text{amps}}{m^2 \cdot m}$$

$$B = \mu H$$

Law of Electric field - stationary point charge Q,

$$\begin{array}{lll} \text{Coulomb } Q & = \text{point} & \text{Ampere } I \\ \text{c/m} & \oint_L = \text{line} & \text{Amp} \end{array}$$

$$\begin{array}{lll} \text{c/m}^2 \quad \sigma_s & = \text{surface} & \text{Wb/m} \\ \sigma_n = \rho & = \text{volume} & F = \text{Wb/m} \end{array}$$

$$S_L = \frac{dq}{dl}$$

$$S_Y = \frac{dq}{dy}$$

$$S_Z = \frac{dq}{dz}$$

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cause: of Magnetic field - DC current I flowing in a line

$\bar{K} = \text{surface current Amp/m}$

$\bar{J} = \text{current in a solid conductor Amp/m}^2$

$I dl = \text{current element I flowing in a wire of zero length - (Amp-m)}$

$$Q = \text{colombs} = S_Y \cdot dl \Rightarrow \frac{\text{colombs}}{\text{m}}$$

$$f_S \cdot ds \Rightarrow \frac{\text{colombs}}{\text{m}^2}$$

$$f_Y \cdot dy \Rightarrow \frac{\text{colombs}}{\text{m}^3}$$

$$I dl = \text{Amp-m}$$

$$\bar{K} ds = \bar{J} dy$$

$$\bar{J} dy$$

Vector Calculus:

∇ is called as spatial derivative vector operator.

when operated on any quantity, gives an idea of the rate of change of the quantity in space.

It also specifies the direction in which the quantity is changing.

∇ can be operated on scalar quantities and it can be operated on vector quantity.

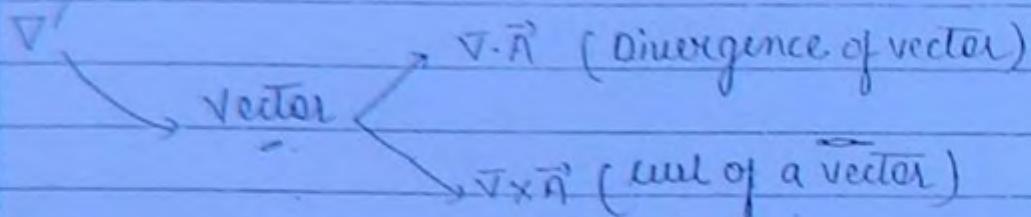
If a scalar quantity f is considered and ∇ is operated on it, it is called gradient of a scalar

eg: scalar $f = 4xy^2 - 4xz$
 vector $\vec{A} = 8xyz\hat{a}_x + 4x^2y\hat{a}_y + 4z\hat{a}_z \quad (6)$

If ∇ is operated on vector quantity then
 $\nabla \cdot \vec{A} \rightarrow$ Divergence of vector.

$\nabla \times \vec{A} \rightarrow$ curl of a vector.

Scalar $f \rightarrow \nabla f$ (gradient of a scalar)



Vector identity

1) $\nabla \times \nabla f = 0$ curl (Grad. of scalar) = 0 $(\nabla \cdot \nabla = 0)$

Ex. curl $\nabla \times \vec{A} = IV; \pi/2 \sin \theta$ when both lie along same $\theta = 90^\circ$.

2) $\nabla \cdot (\nabla \times \vec{A}) = 0$ Divergence (curl of vector) = 0
 (Orthogonal).

$\nabla \times \vec{A}$ result in a vector perpendicular to ∇ and \vec{A} .
 where the resultant vector is operated again with ∇ as dot product ans is 0.

$$A \times B = C \quad C \perp (A \& B)$$

$$A \cdot C = 0 \quad \left\{ |A||C| \cos 90^\circ = 0 \right\}$$

$$A \cdot (A \times B) = 0$$

3) $\nabla \times (\nabla \times \vec{A}) = \nabla (\nabla \cdot \vec{A}) - \vec{A} (\nabla \cdot \nabla)$

$$= \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

Summary:

1. $\nabla \times \nabla = 0$

2. $\nabla \cdot \nabla = \nabla^2 = (\text{scalar Laplacian operator})$

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Outflow & Divergence:

Consider a cause which have effects spread out from the cause. The effects are such that there outward dispersive and hence expand increasing their area of presence.

In the process as area increases the strength decreases. Hence strength is called as density as shown below.

$$\text{Area} \uparrow \times \text{Strength} \downarrow = \text{constt} \propto \text{cause.}$$

$$\begin{aligned}\text{Strength} &= \frac{\text{constt}}{\text{Area}} \\ &= \text{Density}\end{aligned}$$

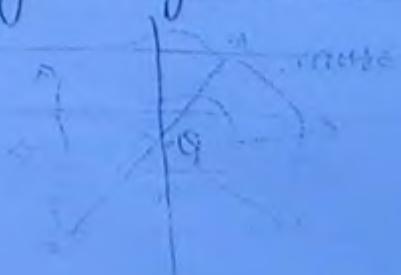
Cause $\rightarrow Q$ (charge)

Effect \rightarrow Electric flux Φ_E (outward from the cause)

Strength \rightarrow Electric flux density (D)

If a cause is a charge of Q coulombs then Φ_E is the effects because of the cause which are called as electric flux then strength is called as flux density D .

The total effects can always being analysed over a area completely enclosed the cause.



Hence strength x area over any closed surface should be equal to the cause.

(8)

Total effect

$$\oint_{\text{closed}} \mathbf{D} \cdot d\mathbf{s} = Q$$

cause

[Gauss Law]

Instead if an open circuit is considered the effects analysed partial effects and they are called as flux through the surface.

$$\int_{(\text{open})} \mathbf{D} \cdot d\mathbf{s} = \Psi_e = \text{const} \quad [\text{Gauss Law}]$$

Every closed surface has a volume

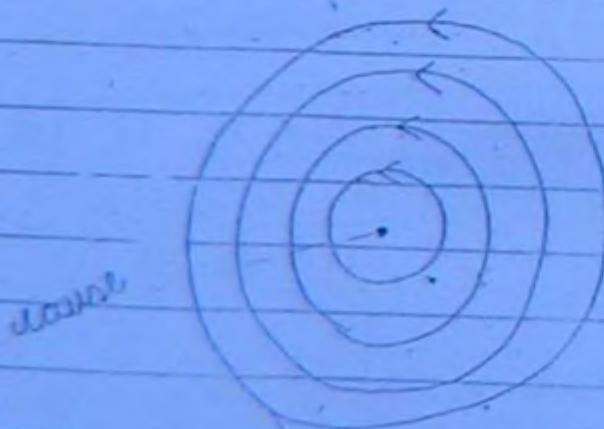
e.g. $4\pi r^2$ for a sphere

Integrate with r $\frac{4}{3}\pi r^3$

e.g. $\pi r^2 h$ for cylinder

$$\pi r^2 h$$

Circulation & curl



Consider a cause which has effects surrounding the cause, in the circulatory manner. The effect are such that the strength decreases as it take longer length of circulation. Hence strength is const per unit length.

(9)

$$\text{Strength} \downarrow \times \text{length} \uparrow = \text{const} \propto \text{cause}$$

$$\begin{aligned}\text{Strength of the effect} &= \text{const} / \text{length} \\ &= \text{Intensity (perm)}\end{aligned}$$

Cause - current I (Amp)

Effect around the cause - Magnetic field.

Strength of the effect - magnetic field intensity (H)

The cause is a current and the effect is circulatory magnetic field around the ^{cause} strength is magnetic field intensity H. Hence

$$\text{Strength} \times \text{length of circulation} = \text{const} \propto \text{cause}$$

$\oint H \cdot d\ell = I$	Ampere's law (Integral form)
closed	

Remark: closed line \rightarrow open surface

$$\text{eg. } 2\pi r \rightarrow \pi r^2 \text{ (circle)}$$

$$\text{eg. } 4a \rightarrow a^2 \text{ (square)}$$

Summary 1

Future density = C/m^2

$$\text{outflow} = \psi_e = \phi ; \text{ Electric flux} = \text{colombs.}$$

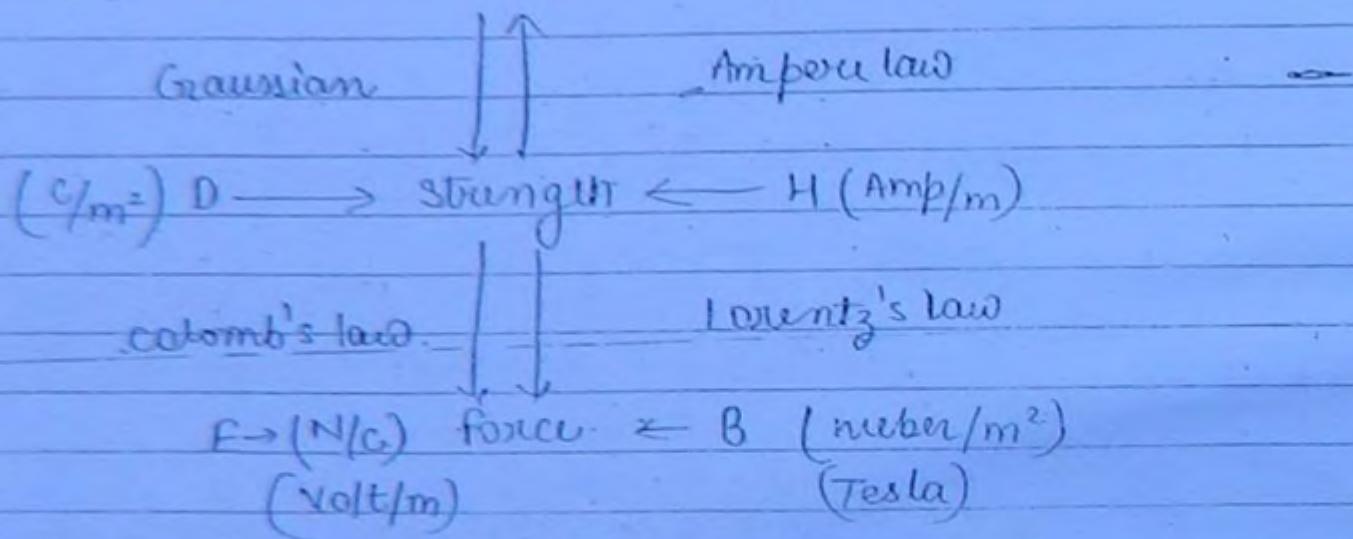
(15)

$$\text{circulation} = I \quad \text{magnetic field intensity}$$

$H = \text{amp}/\text{m}$

$\{\because \phi_H \cdot dL = I\}$

$$(c) Q \rightarrow \text{cause} \leftarrow I (\text{Amp})$$



Divergence & Stoke's Theorem

$$D \rightarrow \text{colomb}/\text{m}^2 = \text{outflow}/\text{area}$$

$$\nabla \cdot D \rightarrow \frac{1}{m} \times \frac{c}{m^2} = \text{Divergence} = \text{outflow}/\text{volume} = \frac{\text{charge}}{\text{volume}}$$

$$[\nabla \cdot D = \rho_v]$$

Divergence physically stands for the outward flowing ability at ~~area~~.

Strength of outflow at the cause which depends on the charge accumulation there. Hence charge density for

$$H \longrightarrow \text{Amp/m} = \text{circulation/length} . \quad (11)$$

$$\nabla \times H \longrightarrow \frac{1}{m} \times \frac{\text{Amp}}{m} = \text{curl} = \frac{\text{circulation}}{\text{area}} = \frac{\text{current}}{\text{area}} = \frac{\text{Amp}}{m^2}$$

$$\boxed{\nabla \times H = J}$$

closed surface $\xrightarrow{\nabla \cdot D}$ Volume
(vector) (scalar)

Surface is a vector quantity and flux density is also a vector quantity.

Volume is a scalar quantity which every closed surface has. hence divergence or dot product is a vector transformation giving a scalar quantity hence

$$\boxed{\nabla \cdot D = \oint_S} \quad \text{Gauss law in (point form)}$$

Closed line $\xrightarrow{\nabla \times H}$ open surface
(vector) (vector)

A line is a vector quantity and the enclosed surface by any line is also a vector quantity hence cross product is a vector transformation to another vector. Hence

$$\boxed{\nabla \times H = J} \quad \text{Ampere's law in point form.}$$

$$\oint D \cdot dL = \oint \mathbf{f}_v dv = \int (\nabla \cdot D) dv$$

$$\boxed{\oint D \cdot dS = \int (\nabla \cdot D) dv} \quad \text{Divergence theorem}$$

Practices:

1. $\int D \cdot d\ell = \int (\nabla \cdot D) dy \quad X$

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Because open surface don't have any volume.

2. $\oint D \cdot d\ell = \int (\nabla \times D) dy \quad X$

3. $\int D \cdot d\ell = \int D dy \quad X$

Stoke's theorem:

$$\oint H \cdot d\ell = I = \int J \cdot d\ell = \int (\nabla \times H) \cdot ds$$

$$\left[\oint H \cdot d\ell = \int (\nabla \times H) \cdot ds \right]$$

$$\int_H d\ell = \oint_H d\ell$$

Identify the wrong statement

1. $\int H \cdot d\ell = \oint (\nabla \times H) \cdot ds \quad X$

2. $\int H \cdot d\ell = \int (\nabla \times H) \cdot ds \quad X$

3. $\oint H \cdot d\ell = \int (\nabla \cdot H) ds \quad X$

Maxwell's equation for static E/H-fields.

Static

→ Time dependent $\rightarrow E(t)/H(t)$ (13)

→ Space dependent $\rightarrow E(x, y, z)$ or $H(x, y, z)$

E/H are the same at all the time.

E/H are not the same at all point in space.

If E/H are not changing with

Maxwell's first eqⁿ is as it is Gauss Law without modification

1. $\oint D \cdot d\ell = Q$ (integral form) $\nabla \cdot D = \rho_v$ (Point form)

Maxwell's fourth eqⁿ is Ampere's law without modification for static field

2. $\oint H \cdot d\ell = I$ $\nabla \times H = J$

(Integral form) (Point form)

These two laws define the basic nature of E and H field respectively

Maxwell's third and second eqⁿ define what is not the nature of E and H fields i.e. a divergent dispersive electric field cannot be axially & circulatory.

Maxwell's second eqⁿ states that

$$\nabla \times E = 0$$

Cross product and curl are always defined for intensity term (per meter length)

$\nabla \times D = 0$ and if ϵ is const throughout the medium
i.e. the medium is homogeneous and isotropic.
 $\nabla \times D = 0$ is also mathematically correct

(14)

2 Maxwell's second eqⁿ.

$$\oint E \cdot dL = 0$$

(Integral form)

(Apply Stoke's theorem)

$$\nabla \times E = 0$$

(Point form)
↓

Irrational nature

CURL (vector) = 0
vector \rightarrow Irrational.

3 Magnetic field is a circulatory field which has effect around the cause and hence cannot have divergent effect from the cause. Hence.

$$\nabla \cdot B = 0$$

Point form.

\rightarrow solenoidal nature

$$\nabla \cdot B = 0$$

$$\nabla \cdot (\mu H) = 0$$

$$\nabla \cdot H = 0$$

If $\mu = \text{const}$

= space independent, Homogeneous, Isotropic

Apply divergence theorem in integral form we get

$$\oint B \cdot dS = \int (\nabla \cdot B) dV = 0$$

$$\oint B \cdot dS = 0$$

Integral form

$$\nabla \cdot \vec{B} = 0$$

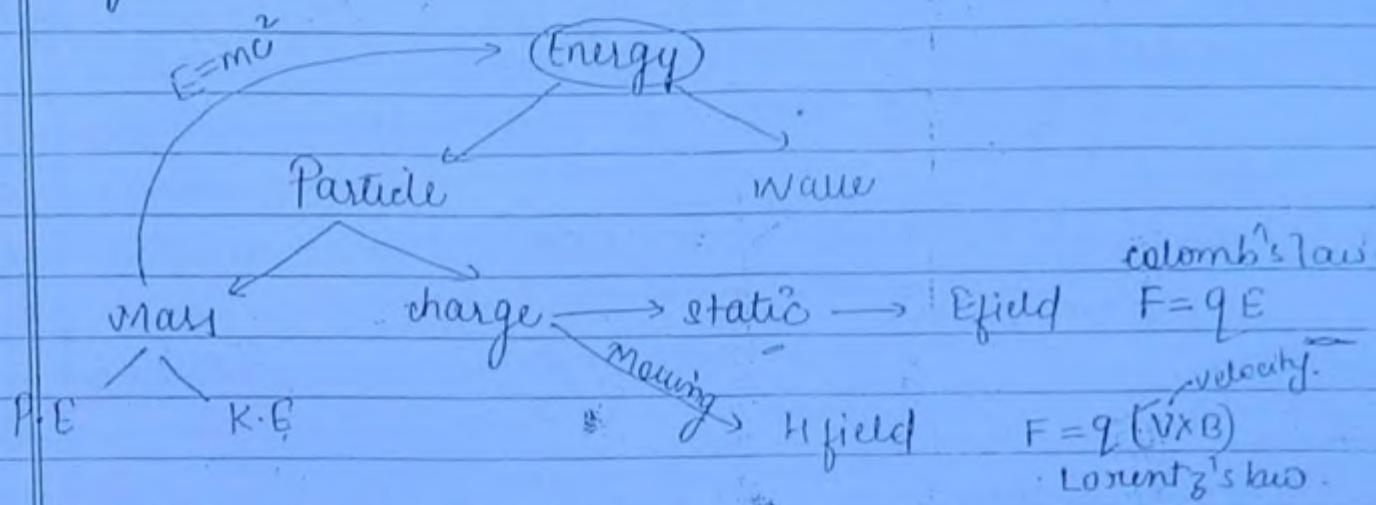
solenoidal nature

$$\text{Divergence (vector)} = 0$$

vector \rightarrow solenoid

(IS)

Physical interpretation of E/H



E field \rightarrow Energy format that is around a stationary charge and is felt by only other charge.

H field \rightarrow Energy format that is around a moving charge and is felt by only other moving charge.

Summary:

$q \rightarrow$ stationary \rightarrow E -field (static)
(Energy)

$q \rightarrow$ moving
without acceleration
with uniform velocity
linearly with time

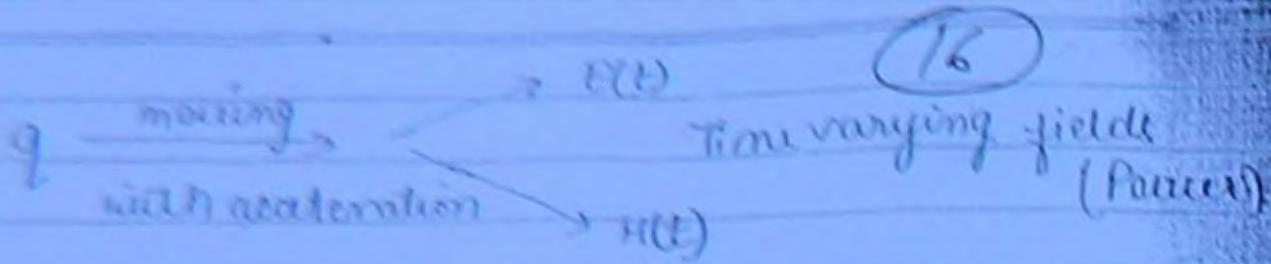
$$\frac{dq}{dt} = K = I$$

= NO. OF CHARGE

$$q = I t$$

H field (static)
(Energy)

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Energy is varied with time i.e. Power involved.

F

Voltage $\rightarrow E$ field
(charge accumulation)

Current $\rightarrow H$ field.
(charge flow)

Monday

Coordinate System:

It is a way of addressing point or locating point in a 3D space from a pre-defined reference.

System of class - Cartesian coordinate system

Reference \rightarrow 3 infinite mutually orthogonal planes.

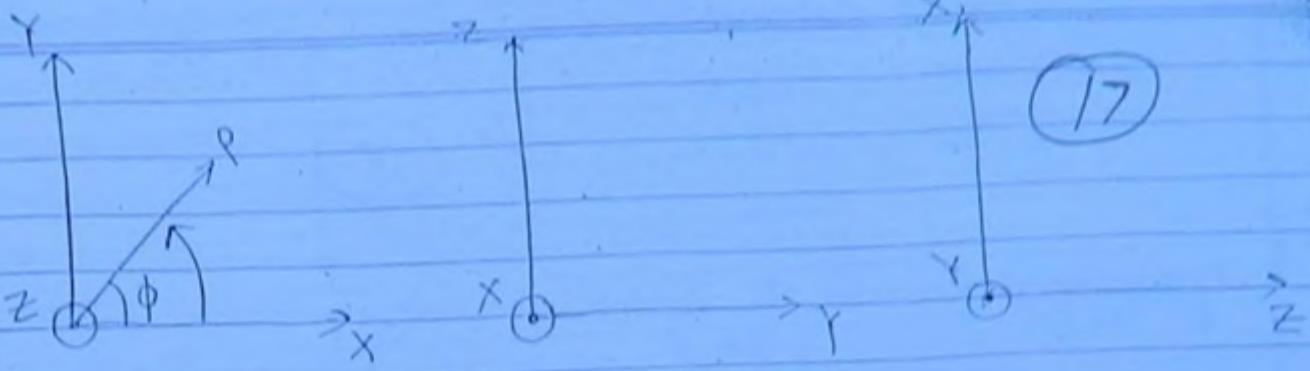
eg: refer to planar symmetry.

eg: - sheet of charge, UPW, Uniform Plane Wave, Rectangular waveguide.

Reference - XY, YZ, ZX plane.

Parameters - x, y, z
unit vectors

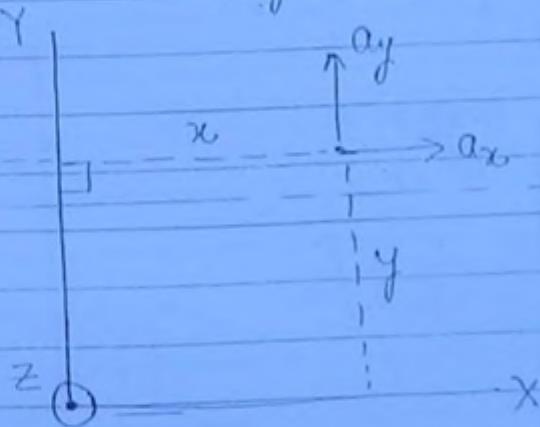
$$\hat{a}_x, \hat{a}_y, \hat{a}_z$$
$$(i, j, k)$$



Right handed system represents the increasing order of the parameters along the right hand fingers.

e.g. Angles:- anticlockwise - in cylindrical - around \hat{z} axis

Cartesian coordinate system:



x = The shortest distance or perpendicular distance from yz plane.

- Range $(-\infty, \infty)$
- It increases as we go along the x axis (It is called as α_x direction)

y = The shortest distance or perpendicular distance from zx plane

Range is $(-\infty, +\infty)$

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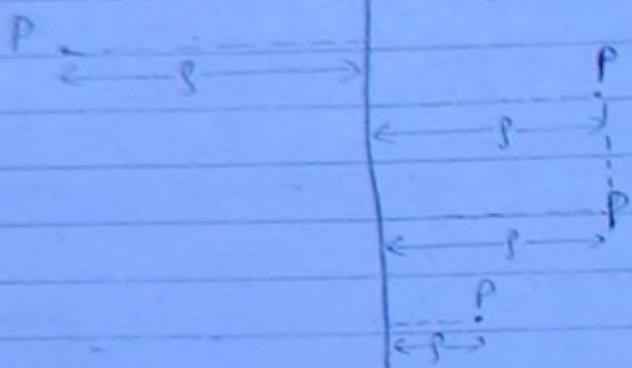
it increase as we go parallel to y axis (and)

z : the shortest distance or \perp distance from the xy plane

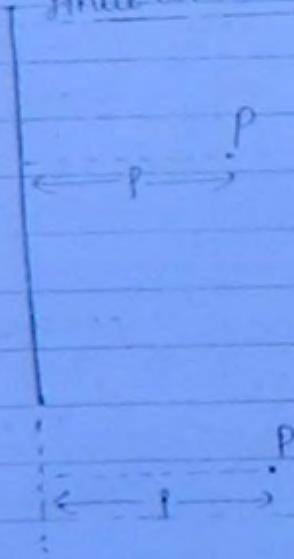
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Cylindrical Coordinate System

z axis



Finite line.



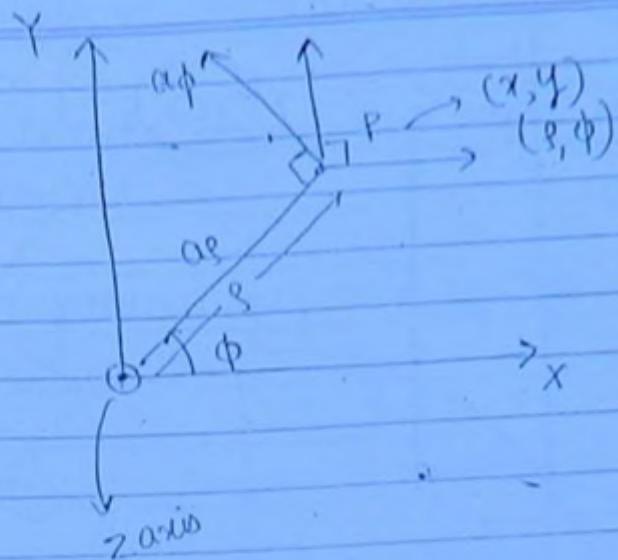
r = [shortest distance or radical distance of the point from the reference z axis]

All the points with the same r are on a concentric cylinder around the line

• Product → car

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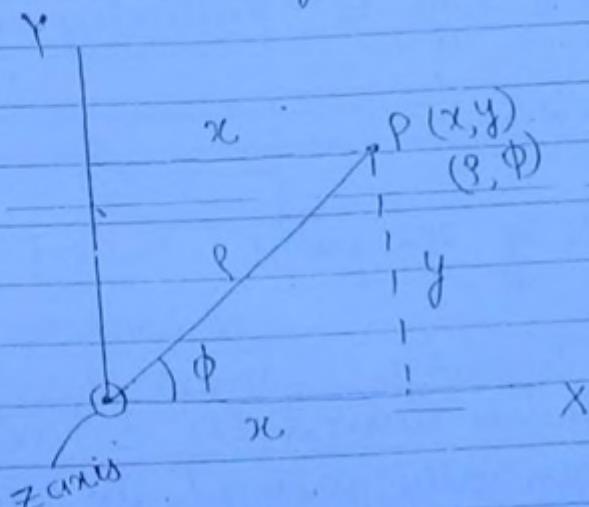
(19)

$$a_p \cdot a_\phi = 0$$

$$a_x \cdot a_y = 0$$

Point Transformation

Cartesian \longleftrightarrow cylindrical



$$x = r \cos \phi$$

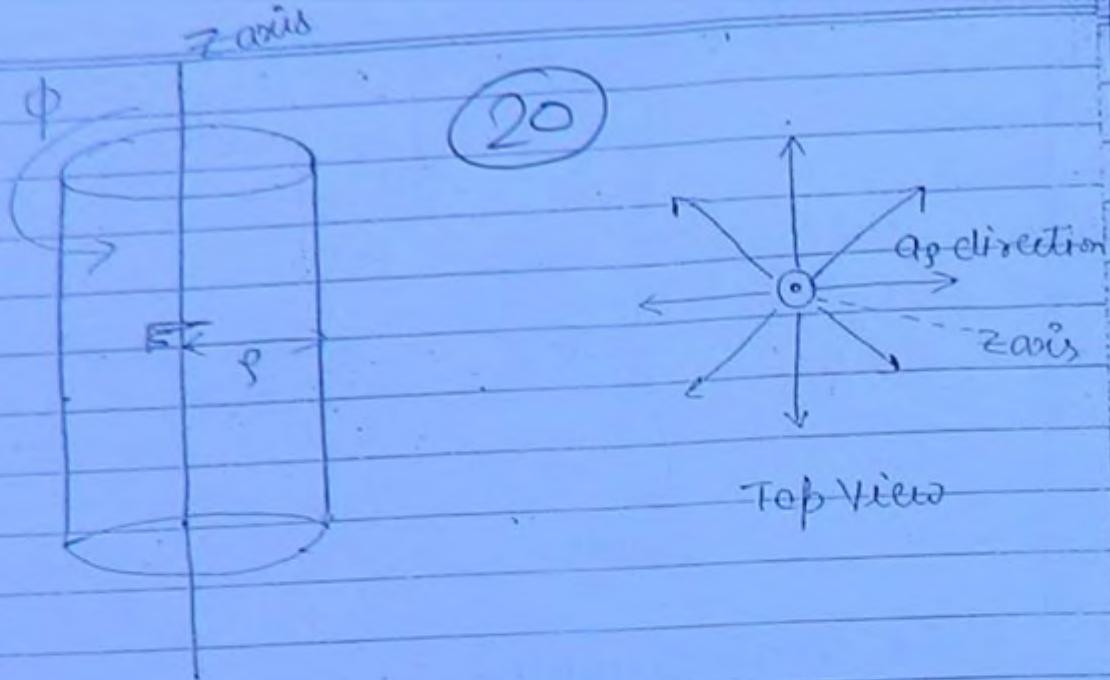
$$r = \sqrt{x^2 + y^2}$$

$$y = r \sin \phi$$

$$\phi = \tan^{-1}(y/x)$$

$$z = z$$

$$z = z$$

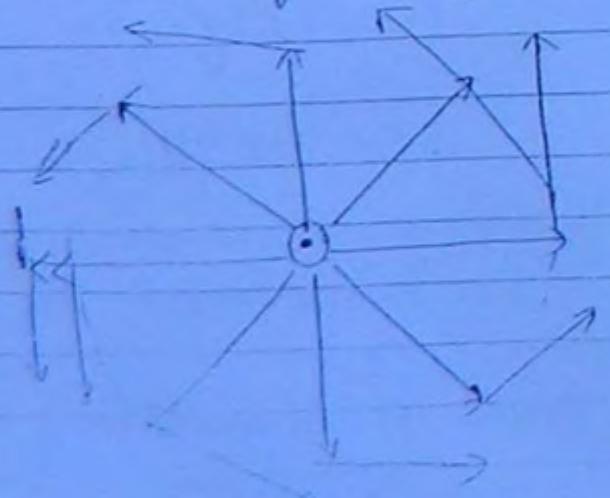


r increases radially outward from the z axis
 light from tubelight is radially out
 electric field from a line is radially out.

Range of r ($0, \infty$)

ϕ = orientation angle of the point around the reference z axis

ϕ increases anticlockwise around the z axis
 - It is also tangentially around the z axis



Range of ϕ

• ϕ is the orientation angle around the imaginary axis that locates a point on the σ circle.

• Different values of ϕ can be seen in anticlockwise direction for one given σ circle. Hence ϕ is out of the board, out of the paper and into the paper as shown. (21)

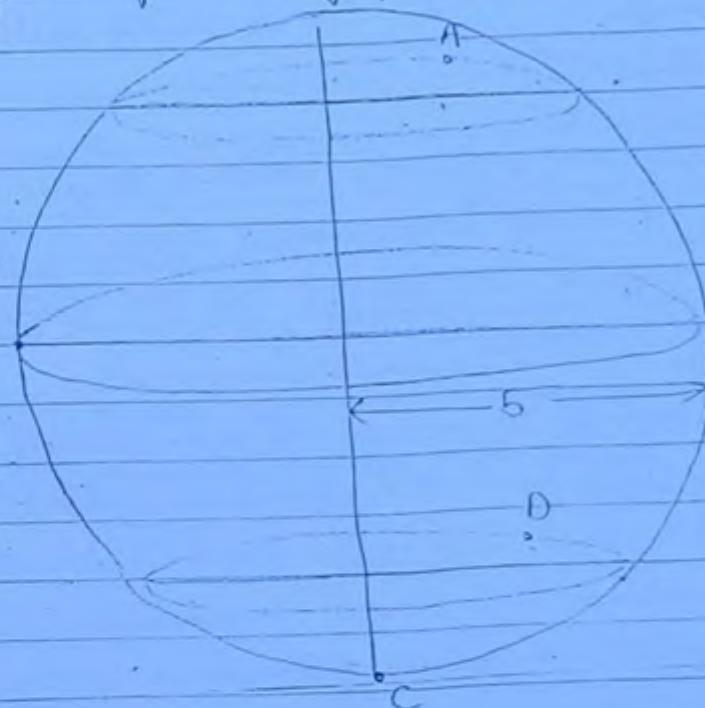
Q Identify the following points in spherical coordinate system:

$$A = (5, 30^\circ, \phi)$$

$$B = (5, 90^\circ, \phi)$$

$$C = (5, 120^\circ, \phi)$$

$$D = (5, 150^\circ, \phi)$$



- (i) $(5, 90^\circ, \phi)$
- (ii) $(5, 30^\circ, \phi)$
- (iii) $(5, 150^\circ, \phi)$
- (iv) $(5, 30^\circ, \phi)$
- (v) $(5, 120^\circ, \phi)$
- (vi) $(5, 150^\circ, \phi)$
- (vii) $(5, 90^\circ, \phi)$

Point Transformation
cylindrical \rightarrow spherical

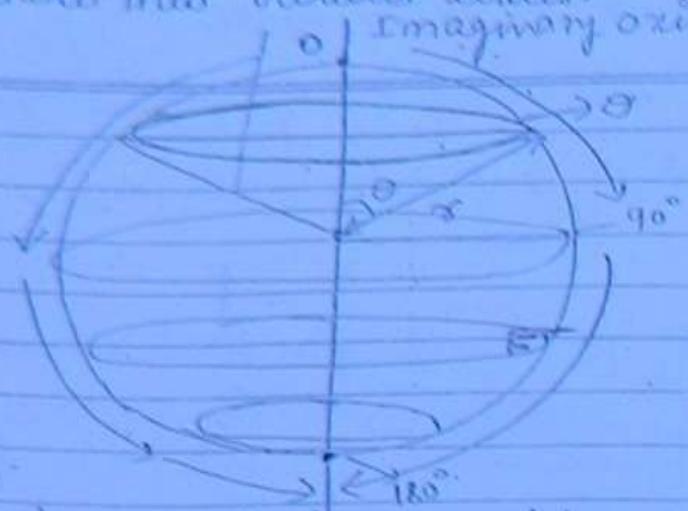
• For the point transformation imaginary axis in spherical coordinate system superimposed with z axis of cylindrical coordinates. Hence ϕ is common variable.

• Any point on the sphere is at any r distance from the z axis and at any height h along the z axis.

$$r = \sqrt{h^2 + r^2}$$

θ = It is an angle that distributes / identifies the sphere into various sides.

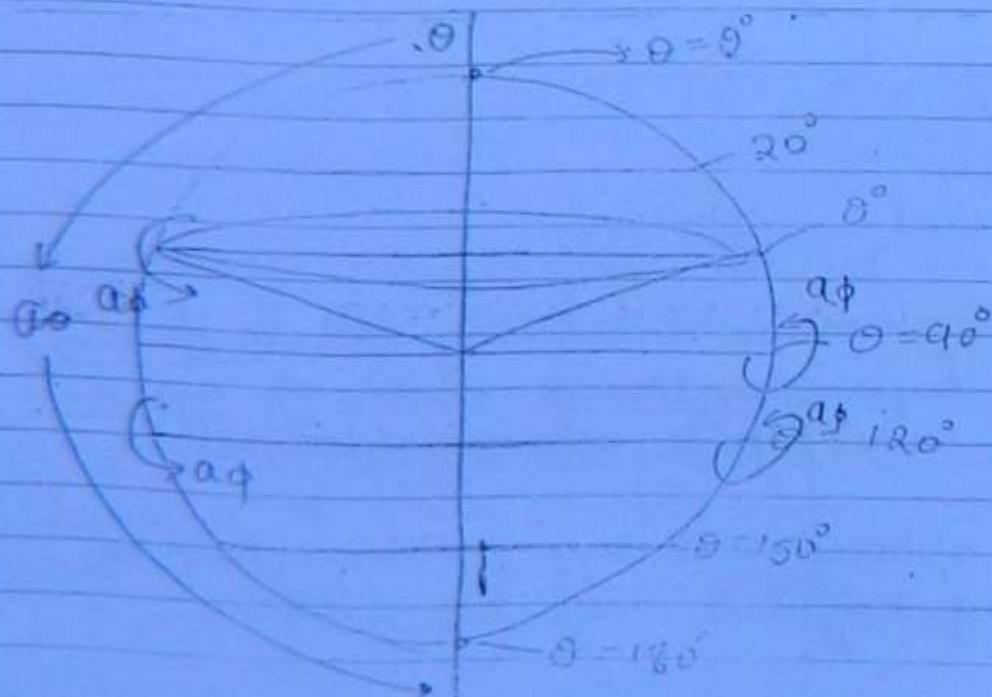
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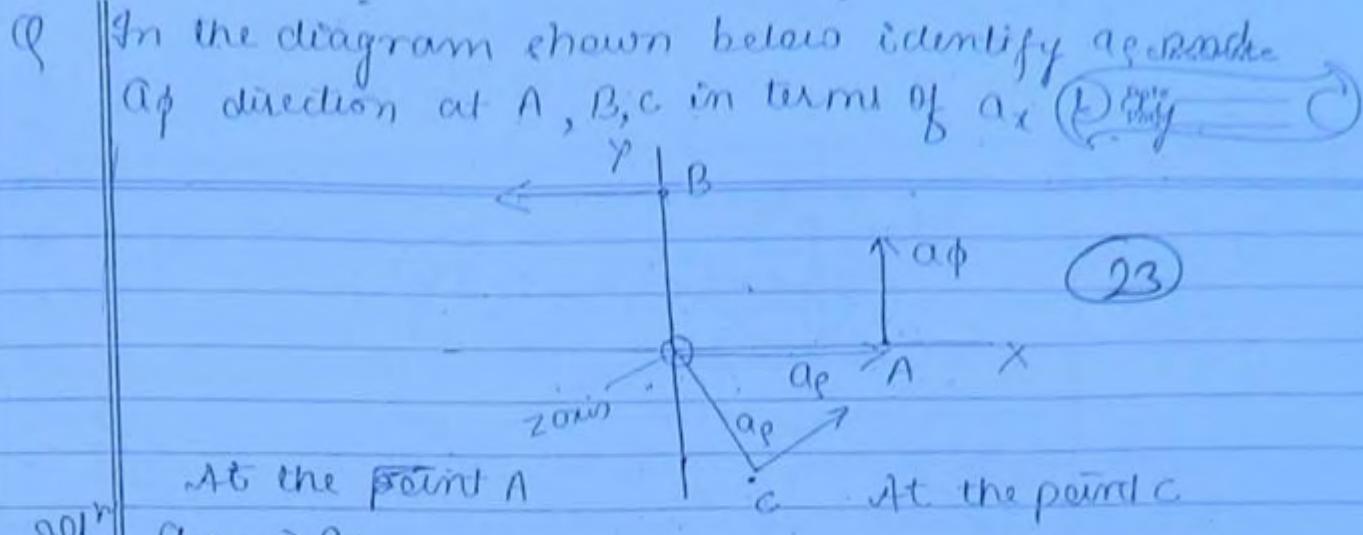
(22)

Identification of a circle on a sphere with a known angle θ rotate around the vertical imaginary axis of the sphere we obtained cone at its base on the sphere that circle is called as θ circle

θ = sphere λ, θ = circle, λ, θ, ϕ = point
The set of all possible circles on a sphere is complete in interval $\theta = [0, \pi]$

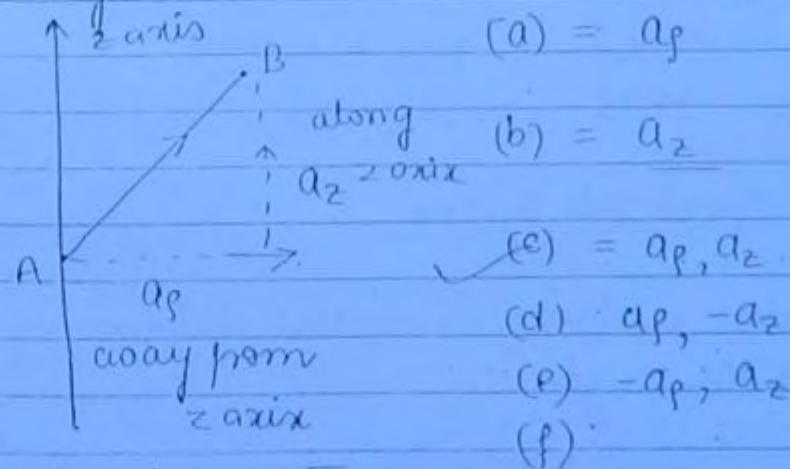


θ increases as we move on the sphere from its imaginary axis back to its imaginary axis at sphere

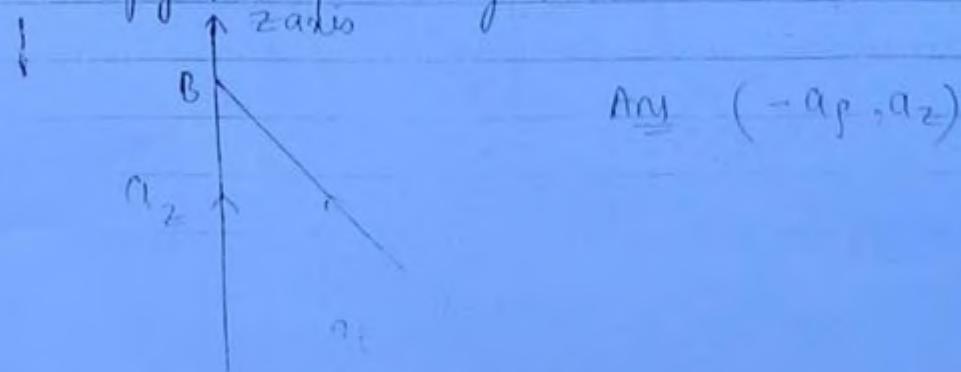


Any two a_x vectors always parallel to each other and horizontal but any two a_ϕ vectors are not necessarily parallel to each other.

Q Identify the following vector



Q Identify the following vector

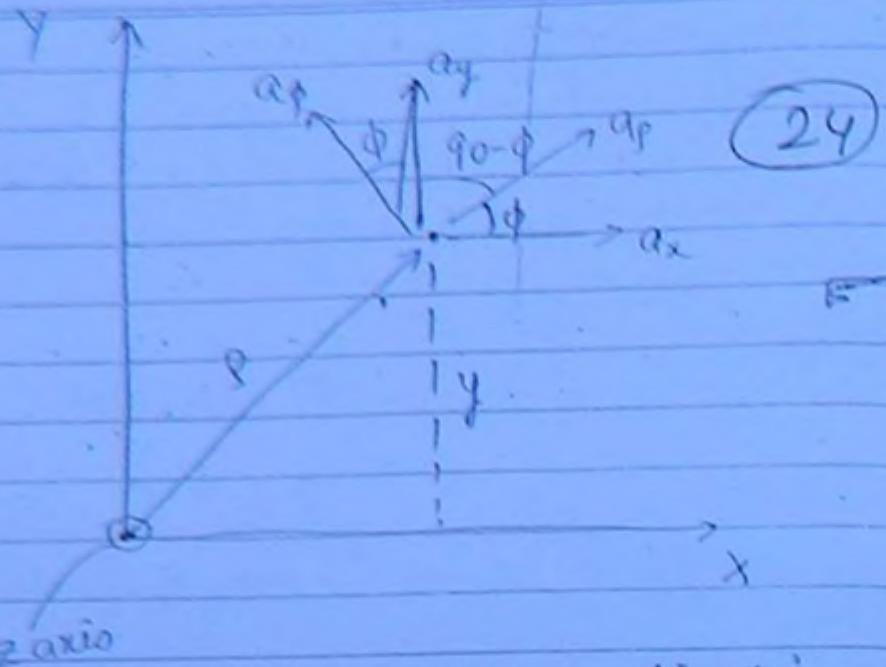


Summary :

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ϕ - angle of the point decides a_ϕ direction with respect to a_x and a_y .

$$a_x \cdot a_\phi = |a_x| |a_\phi| \cos \phi = \cos \phi$$

$$a_y \cdot a_\phi = \sin \phi \quad |a_y| |a_\phi| \cos(90^\circ - \phi)$$

$$a_y \cdot a_\phi = \cos \phi \quad |a_y| |a_\phi| \cos \phi$$

$$a_x \cdot a_\phi = -\sin \phi \quad |a_x| |a_\phi| \cos(90^\circ + \phi)$$

Spherical coordinate system:

r = shortest distance or radial distance of the point from the origin

If $r = \text{const}$, they are infinite points all located on a sphere concentric with the origin.

(iii) The direction has to obey the rule
 $E \times H =$ propagation
 direct \downarrow direction

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12 W.B (b) & (d) are wrong because it is not in
 the form of $25e^{j\omega t} \sin(\omega^2 t - y) a_2$

i) $\omega = 10^8 \quad \beta = 1 \quad \text{in} \quad E(y, t) = 25 \sin(10^8 t - y) a_2$
 $v_p' \leq 3 \times 10^8 \text{ m/sec}$

$$v_p' = \frac{\omega}{\beta} = 10^8 \quad \text{i.e. } \frac{3 \times 10^8}{\sqrt{\epsilon_r}} = v_p' \Rightarrow \sqrt{\epsilon_r} = \frac{3 \times 10^8}{10^8} = 3$$

$$\epsilon_r = 9$$

It is less than dielectric

(ii) $f = \frac{\omega}{2\pi} = \frac{10^8}{2\pi} \Rightarrow \lambda = 2\pi \quad 2\pi f = \omega$

(iii) H (vector)

$$H(y, t) = \frac{25}{\eta} \sin(10^8 t - y).$$

$$\eta = \frac{120\pi}{\sqrt{9}} = 40\pi$$

$$H(y, t) = \frac{25}{40\pi} \sin(10^8 t - y) a_x$$

$$a_2 \times ? = a_y$$

11) $H =$
 (a) what freq. is prop. constant for H .

b) correct

$$\beta = 2 = \frac{2\pi}{\lambda}$$

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$$\lambda = 3.14.$$

E

(c) correct

(d) wave propagation is defined only w.r.t to electric field.
situation but never H field

incorrect

w.f. $E = 50 \sin(10^8 t + kz) \hat{j} \text{ V/m}$

which is correct

(a) wrong

$$(b) f = \frac{3 \times 10^8}{\lambda} \Rightarrow \lambda = \frac{3 \times 10^8}{f} = \frac{3 \times 10^8}{10^7} \times \frac{2\pi}{\omega}$$

$$\Rightarrow \lambda = \frac{3 \times 10^8 \times 2\pi}{10^7} = \lambda = 30 \times 2\pi = 188.4 \text{ m}$$

correct

$$(c) K = \beta = \frac{2\pi}{\lambda} \Rightarrow \frac{1}{30} = \frac{2\pi}{\lambda} \text{ i.e. } 0.03$$

wrong

(d) wrong (because there is no exponential term only 50 is present so no attenuation)

w.f. $\beta = \frac{2\pi}{\lambda} \quad v_p = \frac{3 \times 10^8}{\sqrt{\epsilon_r}} \quad (\text{for lossless medium})$

$$v_p = \frac{3 \times 10^8}{\sqrt{31}} = \frac{3 \times 10^8}{\sqrt{9}} = \frac{10^8}{3}$$

$$v_p = \frac{10^8 \times 6.1}{10^9} = 6.1 \text{ m/s}$$

cylindrical coordinate system:

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Reference \rightarrow 1 infinite line

It has axial symmetry

e.g.: line charge, current carrying wire, cylindrical waveguide, coaxial cable.

Reference \rightarrow z axis

Parameter $\rightarrow r, \phi, z$

unit vectors $\rightarrow a_r, a_\phi, a_z$

Spherical Coordinate System:

Reference \rightarrow 1 point

Point Symmetry

e.g. of point symmetry: point charge, antenna, current element etc,

Reference - origin

Parameters $- r, \theta, \phi$

unit vectors $- a_r, a_\theta, a_\phi$

best eq is assume yourself a point and locate the position of stars.

All coordinate systems are assumed to be following unit orthogonal, orthonormal, right handed system.

orthogonal:

orthogonality: The dot product of any two different unit vectors of the same coordinate system is zero.

$$a_\phi \cdot a_\phi = 0$$

$$a_r \cdot a_\phi = 0$$

$$a_z \cdot a_\phi \neq 0$$

$$a_r \cdot a_r = 1$$

$$a_\phi \cdot a_\phi = 0$$

Orthonormality :

The cross (\times) product of any two different unit vector of the same coordinate system is always the third unit vector (obeying right hand rule).

$x \rightarrow y \rightarrow z$

$\beta \rightarrow \phi \rightarrow z$

$\lambda \rightarrow \theta \rightarrow \phi$

$$a_x \times a_y = a_z$$

$$a_\phi \times a_\beta = a_z$$

$$a_r \times a_\theta = a_\phi$$

$$a_y \times a_z = a_x$$

$$a_\beta \times a_\phi = a_\theta$$

$$a_\theta \times a_\phi = a_r$$

$$a_z \times a_x = a_y$$

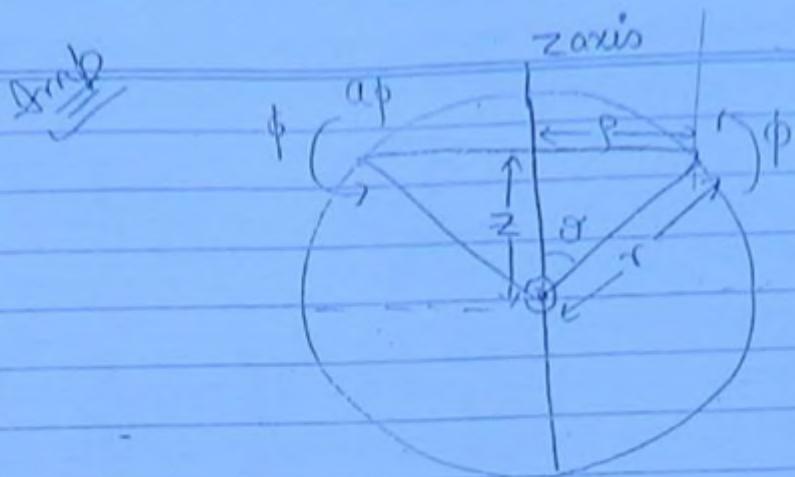
$$a_z \times a_\beta = a_\phi$$

$$a_\phi \times a_r = a_\theta$$

The cross (\times) product of any two similar unit vector of the same coordinate system is zero.

$$\text{--- } a_\beta \times a_\beta = 0$$

$$a_z \times a_z = 0$$



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conversion to spherical

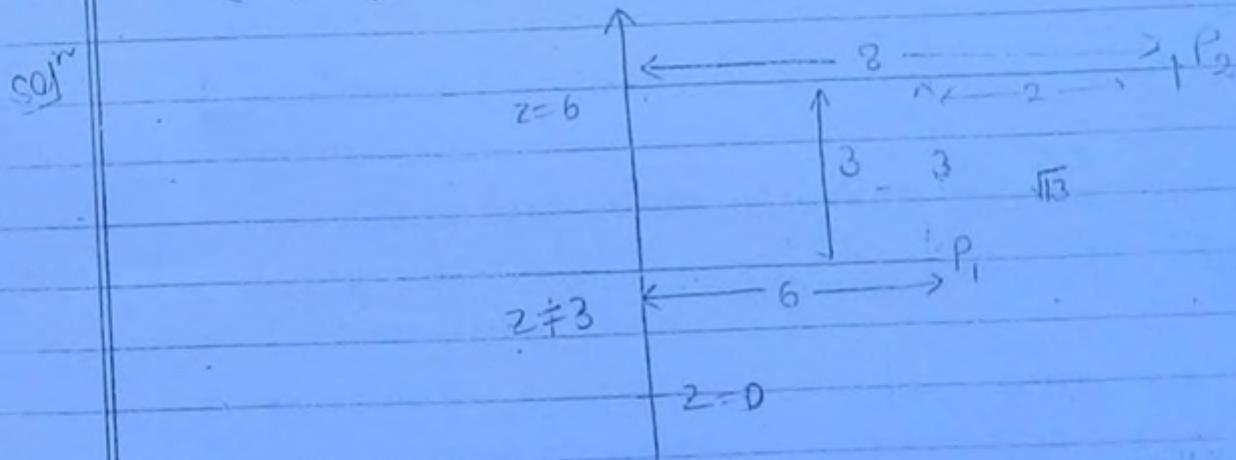
$$x = r \cos \phi \Rightarrow r \sin \theta \cos \phi$$

$$y = r \sin \phi \Rightarrow r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

- Q. calculate the distance b/w the two points because without using point transformation:

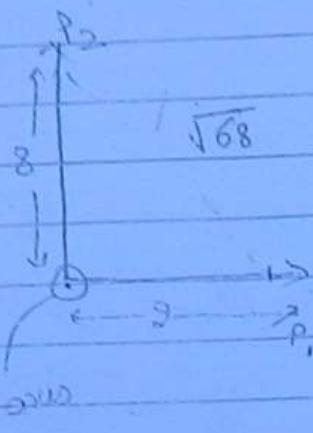
$$(5, \pi/2, 3) \text{ and } (8, \pi/2, 5)$$



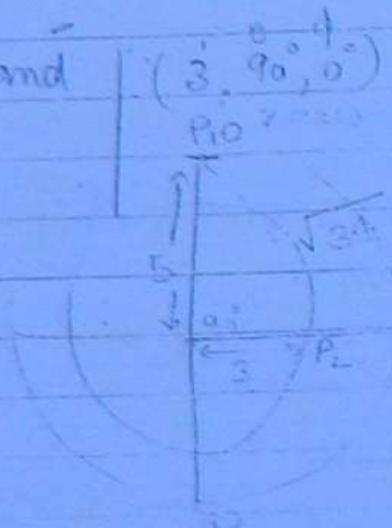
Q $(2, 0^\circ, 6)$ to $(6, 0^\circ, 2)$

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(c) $(2, 0, 5)$ to $(8, \sqrt{2}, 5)$



(d) $(5, 0, 0)$ and $(3, 90^\circ, 0)$



Q Identify the locus of the following description
 (a) Point (b) Line, (c) surface (d) Volume

(a) $x = 5$, for all y and z surface



(b) $\lambda = 5$, for all $\theta \in \phi$

surface \rightarrow closed. $r=5$ means on the surface

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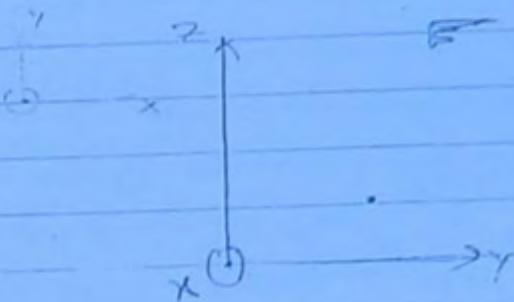
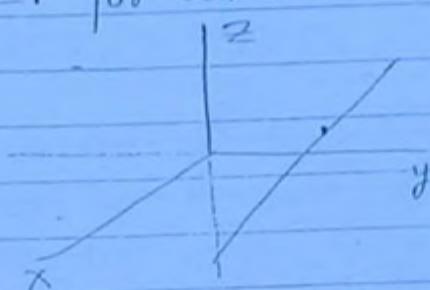
If $r \leq 5$ then volume

(c) $\delta = 5$, for all ϕ and z

surface \rightarrow cylindrical closed surface.

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(d) $y=2, z=1$ for all x



(e) $\delta = 5, z = 3$ for all ϕ

line = closed line

Summary:

- | | |
|----|--|
| 1. | <ul style="list-style-type: none">1 parameter - fixed2 parameters - variable |
| 2. | <ul style="list-style-type: none">1 parameter - variable2 parameters - fixed |
| 3. | <ul style="list-style-type: none">3 parameters - fixed4 parameters - variable |
- Vector
1. line definition (line is one dimensional)
2. surface definition. (surface is 2 dimensional)
3. point definition

Line, Surface, Volume Integrals

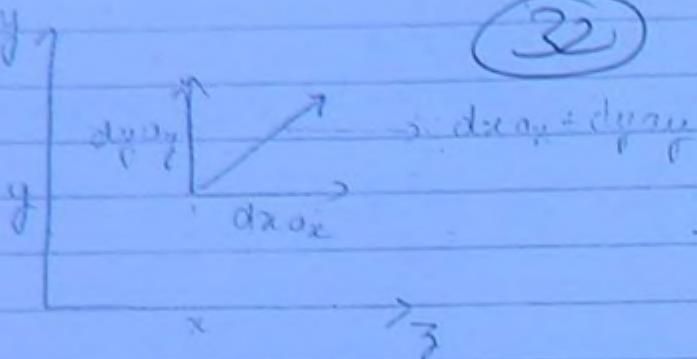
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- line element of zero length

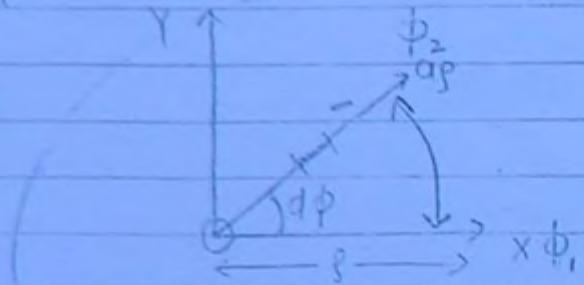
- its direction is in that direction in which the length changes.



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$$dL = dx a_x + dy a_y + dz a_z$$

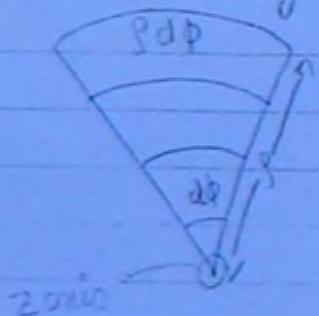
cylindrical



ϕ -direction movement or change in length is always a curvature

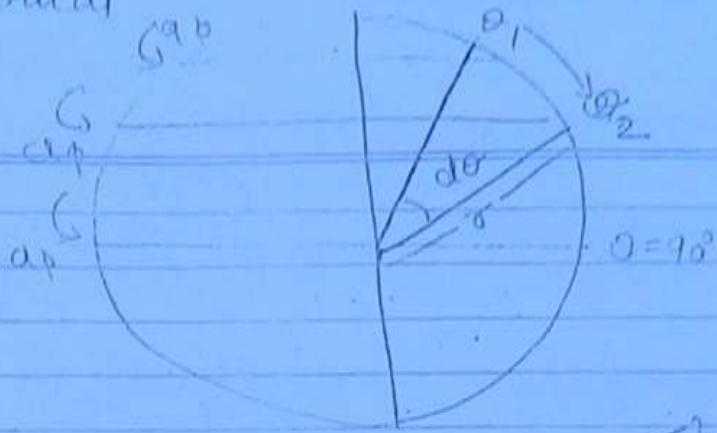
$$= dr a_r + r d\phi a_\phi + dz a_z$$

A curvature length is always dependent on curvature radius and angle of curvature



$r d\phi$ is always same

For spherical



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$$= dr \alpha_r + r d\theta \alpha_\theta + r \sin \theta d\phi \alpha_\phi$$

can be at

Movement in the ϕ direction calculate various height
on a sphere i.e. it can be a various. or conitt circles
The curvature at each circle is $r \sin \theta$ hence length
in ϕ direction is $r(\sin \theta) d\phi$

(c) cylinder has same curvature but sphere have different)

Parameters

Scaling factors

x	y	z	1	1	1
θ	ϕ	θ	1	r	1
λ	ϕ	ϕ	1	r	$r \sin \theta$
u	v	w	h_1	h_2	h_3

$$du = h_1 du \alpha_u + h_2 du \alpha_v + h_3 dw \alpha_w$$

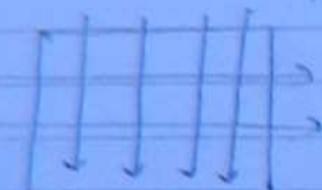
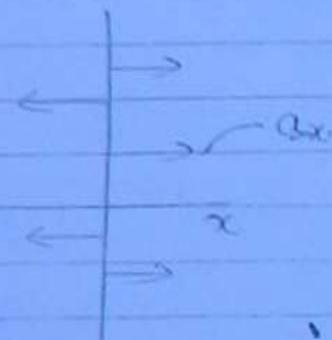
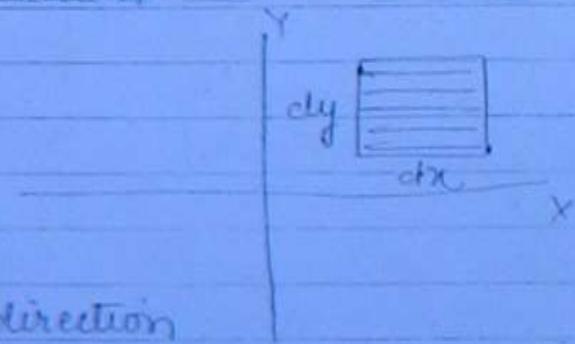
Sunday:

 \bar{ds}

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Surface direction is always normal to the plane of the surface.

Volume covering a surface, cuts normally to the surface

 $x = 5 = \text{constant}$  a_x direction a_z direction

XY Plane

 $z = 0$ surface $x, y \rightarrow \text{variable}$

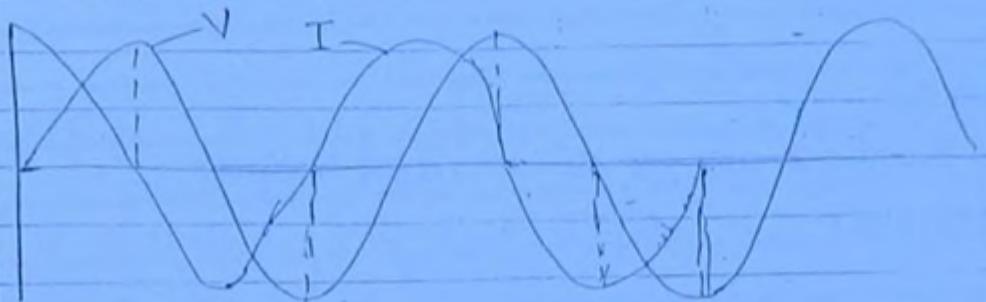
$$\bar{ds} = dy dz a_x + dz dx a_y + dx dy a_z$$

Parameter - fixed - constant

Position of the surface in the direction of the integrals
are the parameters

$$\begin{aligned} V &= -L \frac{dI}{dt} \\ I &= C \frac{dv}{dt} \end{aligned}$$

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1. The voltage V is initially held on to the plates of the capacitor which gradually discharges to zero through the inductor and hence accumulate on the other side due to the discharging current as $V = -L \frac{dI}{dt}$.

Process goes on and the voltage and the current sustain each other alternately as $I = C \frac{dv}{dt}$. Hence a

steady state oscillatory voltage and current exist without power dissipation.

P.V.T. $\int_0^T \sin^2 \omega t dt = 0$ (No power consumption)

Practical w/f.

we need always a feedback

The entire working of the ckt and steady state w/f is due to initial charge and discharge of the capacitor in the circuit and exponential gain of the

same function and hence v and T are said to be harmonic functions.

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$$\frac{e^{j\theta} + e^{-j\theta}}{2}$$

v/I Harmonic functions

Harmonic function

- All harmonic functions are 2 dimensional objects
- exist in 3 formats.
- There are

$$A \sin \theta \quad A \cos \theta \quad Ae^{j\theta}$$

A - dimension 1 - Amplitude

θ - dimension 2 - phase.

Properties of Harmonic funⁿ

1. θ - phase - should always linearly change with the variable

$\theta \propto t$ time Harmonic

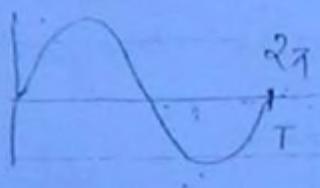
$$\theta = \omega t$$

$$\omega = \frac{\theta}{t}$$

$\frac{\partial \theta}{\partial t} = \omega$ = phase shift const per unit time

$\theta \propto z$ space Harmonics

$$\theta = \beta z$$



Phase shift w.r.t. unit length

Generalized formulae

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$$d\Phi = h_1 h_2 dV dW a_u + h_3 h_4 dW dU a_v + h_5 h_6 dU dV a_w$$

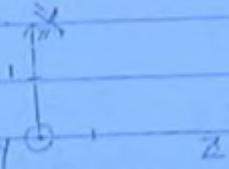
Given in electric field $\vec{D} = 4xy \hat{a}_x + 2yz^2 \hat{a}_y + 3zx^2 \hat{a}_z \text{ C/m}^2$
find the flux crossing the surface.

$$y=2, \quad 0 < x < 1$$

$$-2 < z < 2$$

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$$y=2,$$



$$4(x)(y)a_x + 2yz^2a_y + 3zx^2a_z$$

$$\psi = \int D \cdot dS = \iint [(4xy a_x) + (2yz^2 a_y) + (3zx^2 a_z)] [dx dz dy]$$

$$\text{as } y=2, \quad dS = dx dz dy$$

$$\psi = \int_{x=0}^1 \int_{z=-2}^2 8xy^2 [dx dz]$$

$$= y^2 \int_{x=0}^1 \left[\frac{x^2}{2} \right]_{-2}^2 = 2y^2 \left[\frac{1}{2} \right] [2+2] \\ = \frac{2y^2 \times 4}{2} = \frac{2 \times 4 \times 4}{2} \\ \Rightarrow 16$$

Given $\vec{D} = -xy \hat{a}_x$

find the flux ψ crossing the surface

$$y=2, \quad 0 < x < 1$$

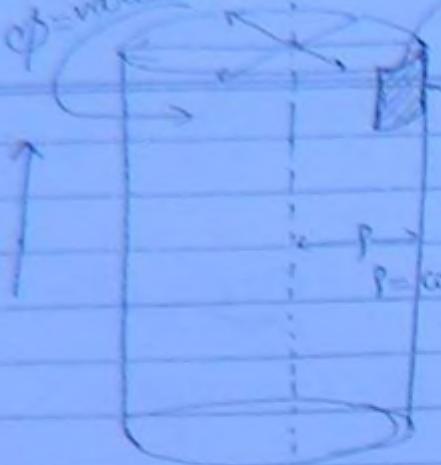
$$-2 < z < 2$$

$$dS = dx dz dy$$

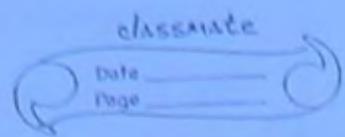
$$\psi = \int D \cdot dS = 0$$

$d\vec{r}$ in cylindrical
 ϕ -variable, z-axis. length = $\rho d\phi$

variable z



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x-axis

$\rho = \text{constt}$ surface (cylindrical)

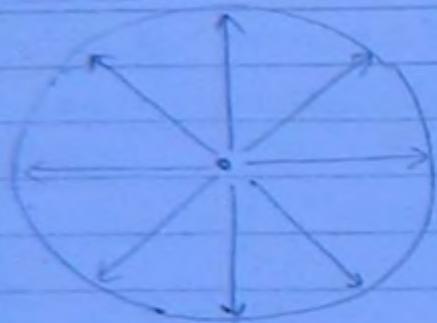
a_ρ direction anti the cylinder radially outwards.

$$d\vec{r} = \rho d\phi dz a_\rho + dz d\rho a_\phi + \rho d\rho d\phi a_z$$

$d\vec{s}$ in spherical coordinate sys.

$r = \text{constt}$

a_r = direction radially outward. hence a_r is normal



θ and $\phi \rightarrow$ dimension, length are
 $r d\theta, r \sin \theta d\phi$

$$d\vec{r} = r^2 \sin \theta d\theta d\phi a_r + r \sin \theta d\phi dz a_\theta + r dr d\theta a_\phi$$

spherical

$$S = \int d\vec{r} \Big|_{r=\text{constt}}$$

$$\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} r^2 \sin \theta d\theta d\phi$$

$$\text{If } D = 5(\theta-3)^2 a_p \text{ C/m}^2$$

Find the flux crossing the surface.

$$r = 4 \quad 0 < \phi < \pi \quad -5 < z < 5.$$

$$\psi = \int \mathbf{D} \cdot d\mathbf{S} = \int 5(\theta-3)^2 a_p [r d\phi dz a_p]$$

$$= \int 5(\theta-3)^2 r d\phi dz$$

$$= \int_0^\pi \int_{-5}^5 5(\theta-3)^2 r d\phi dz$$

$$= 5 \times 4(1)^2 \int_{\phi=0}^{\pi} \int_{z=-5}^5 d\phi dz$$

$$= 5 \times 4(1)^2 [\phi]_0^\pi [z]_{-5}^5$$

$$= 20 [\pi - 0] [5 + 5]$$

$$= 200\pi$$

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dV

volume is a scalar triple product of the lengths in 3 dimensions.

$$dV = dx dy dz$$

$$= r ds d\phi dz$$

$$= r^2 \sin\theta dr d\theta d\phi$$

Divergence, Curl, Gradient

$$\bar{A} = A_x a_u + A_y a_v + A_z a_w$$

$$\nabla \cdot \bar{A} = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u} (h_2 h_3 A_u) + \frac{\partial}{\partial v} (h_3 h_1 A_v) + \frac{\partial}{\partial w} (h_1 h_2 A_w) \right]$$

$$\nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

yo

$$\nabla \cdot \bar{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z} \quad [\text{cylindrical}]$$

$$= \frac{1}{r \sin \theta} \frac{\partial}{\partial r} (r^2 \sin \theta A_r) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (r \sin \theta A_\theta) \\ + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \phi} (r A_\phi)$$

$$\nabla \cdot \bar{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r^2 \sin \theta} \frac{\partial A_\phi}{\partial \phi} \quad [\text{spherical}]$$

End.

$$\nabla \times \bar{A} = \begin{vmatrix} h_1 a_u & h_2 a_v & h_3 a_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1 a_u & h_2 a_v & h_3 a_w \end{vmatrix}$$

Static Electric field:
Fundamental of colombs law behind it Gauss Law.

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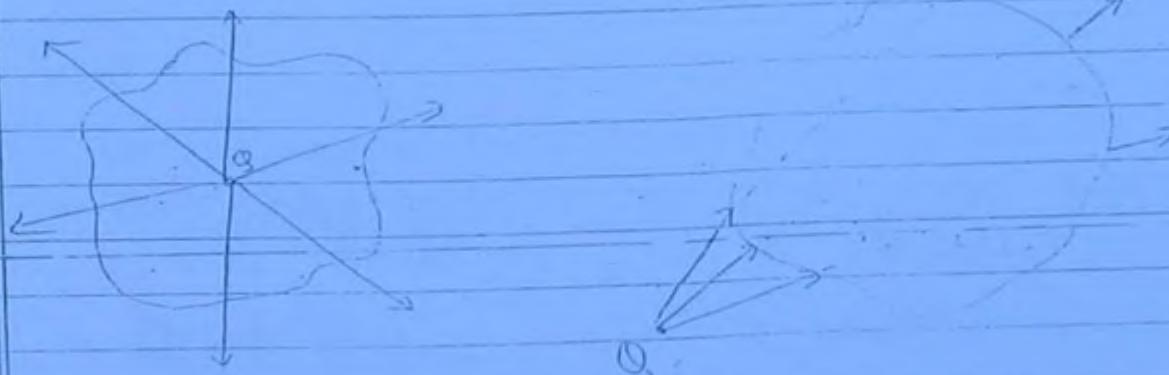
Gauss Law:

Statement: The (net) electric flux leaving any closed surface is always equal to the charge enclosed in that volume

The complete effects from any cause are analysed by considering an encapsulating surface i.e. closed surface. Hence.

$$\psi_{e(\text{total})} = Q$$

unit of electric flux is coulomb.



If the charge is inside the surface there are net flux lines crossing the surface outwardly.

If the same charge is outside flux entering the volume or surface should be equal to flux leaving.

Summary: charge - source/sink for flux lines

Note: Gauss law never define for an open surface i.e. for an open surface flux only out through it we cannot define entering/leaving flux

$$\oint D \cdot d\vec{s} = Q$$

integrated form of Gauss law.
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D = Flux density, displacement

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Strength of flux = Flux = $C = \frac{Q}{\text{area } m^2}$ Instantaneous flux at every point

$$\int D \cdot d\vec{l} = \frac{Q}{\epsilon_0}$$

Not gauss law.

$$\nabla \cdot D = \rho_v$$

point form of Gauss law.

$$\nabla \cdot D = \frac{\text{outflow}}{\text{volume}} = \text{flux density} = \frac{\text{charge}}{\text{volume}} = \rho_v$$

$$\frac{1}{m} \times \frac{C}{m^2} = \frac{C}{m^3}$$

Diverging ability always depends on charge density.

Gauss law: Application: Strength of the field due to a point charge Q

$$\oint D \cdot d\vec{s} = Q$$

$S = \text{const. spherical surface}$
Evident from the curv.



Strength of electric field decreases as we move away from the charge.

be used for any closed surface.

In this example we choose a symmetric spherical surface for applying Gauss law.

The choice of a sphere is because the surface is equidistant from the charge and hence strength is constant and hence the integration converges to multiplication

$$\mathbf{D}(\mathbf{r}) \cdot \mathbf{x} \text{ Area of the sphere.} = Q$$

$$\frac{\mathbf{D}(\mathbf{r})}{\text{C/m}^2} = \frac{Q}{4\pi r^2} \cdot \mathbf{a}_r$$

chosen surface is an $r = \text{const}$ sphere having \mathbf{a}_r direction so by logic \mathbf{D} also have same direction as $\mathbf{a}_r \cdot \mathbf{a}_r = 1$. Hence the field is radially outward and divergent from the cause or charge.

Colomb have a different measure of field strength which was in terms of force b/w charges per unit charge

$$E = \frac{F}{q}$$

He called it ^{as} intensity or electric field intensity with unit Newton/Coulomb.

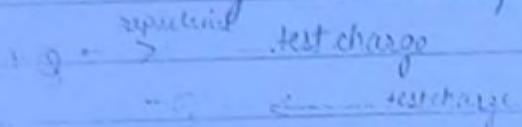
He also proved that charge having a mass should have force and hence in the field E and velocity

$$F = \frac{D}{E}$$

$$\text{since } E = \frac{q}{4\pi\epsilon_0 r^2} \cdot \hat{r}$$

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Ans 1 Electric field direction E is the direction of flux line, it is the direction of the repulsive force on a positive charge and hence it is always outward from a charge.



Static Magnetic fields.

Biot Savart's Law. [Ampere's law for current element]

Biot Savart's law derived from Ampere's law.

- applying that a small length ℓ carrying wire can be treated as the basic cause of magnetic fields

Ampere's law = Ampere's circuit law

Biot Savart's law states -

$I d\ell$ → small current element - vector quantity.
Amperes - cause.

H → magnetic field strength - field Intensity

- Effect.

~~Ampere's Law~~

$$H(r) = \frac{I d\ell \times \hat{r}}{4\pi r^3}$$

The strength expression very similar to the electric field and potential

but the direction is not as if in electric field.

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The direction of magnetic field is always current direction multiplied with radial direction to the point from the current.

current direction \times radial dirⁿ to point from the current

$$\text{Intensity } H(r) = \frac{\text{Amp} \cdot \text{m} \times \text{ar}}{\text{m}^2}$$
$$= \frac{\text{Amp}}{\text{m}^2}$$

Lorentz's basic force eqⁿ defines the field strength in magnetic field as flux density (weber) hence as shown below.

$$B = \frac{\bar{F}}{I dL} = \frac{\text{Force}}{\text{Basic cause}}$$

$$F = q(\vec{v} \times \vec{B}) \rightarrow \text{Lorentz's force eqn}$$

$$dF = dq \left(\frac{du}{dt} \times B \right) = I du \times B$$

q = charge

v = velocity of the moving charge

$\frac{du}{dt} = v_d$ = drift in a conductor of length l

Given $B(r) = uH$

$$\frac{F}{I dL} = \frac{\text{Newton}}{\text{Ampere}}$$

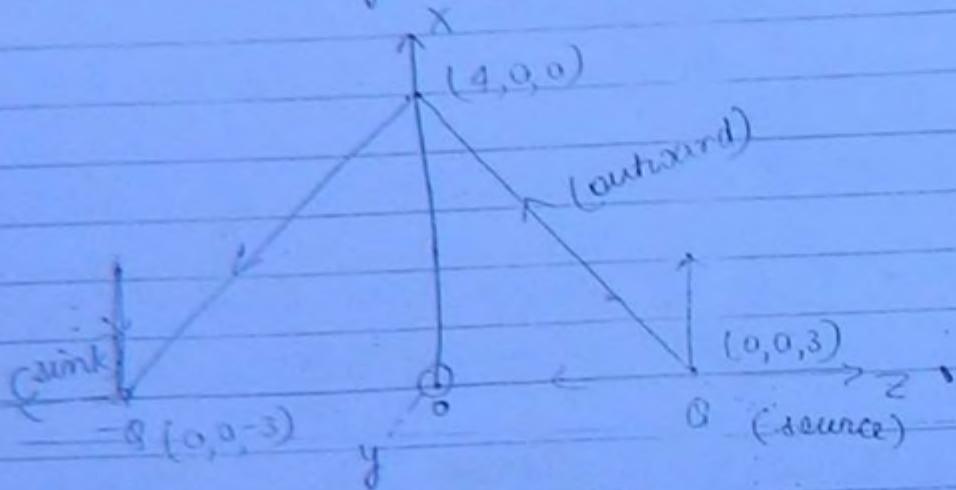
it relates current and force b/w currents

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B's direction is physically the direction which a moving charge tracing when it enters a magnetic field.

workbook:-

static electromagnetic field



$$r = \sqrt{4^2 + 0^2 + (-3)^2} = \sqrt{16+9} = 5$$

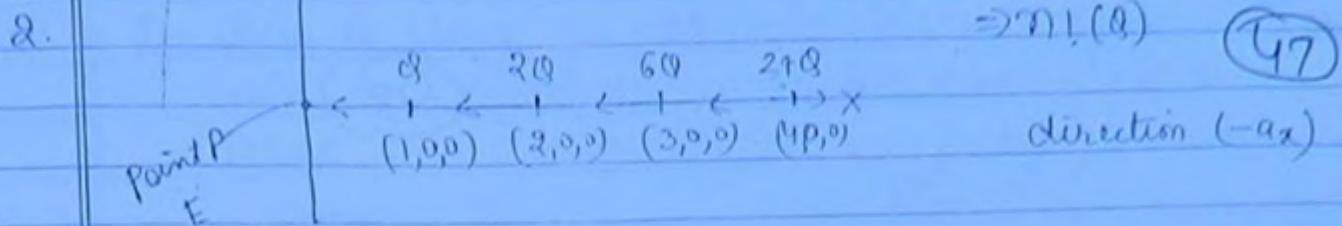
$$E_1 = \frac{q}{4\pi\epsilon_0 r^2} \left[\frac{4a_x - 3a_z}{\sqrt{4^2 + 3^2}} \right] \quad \left\{ a_r = \frac{r}{|r|} = \frac{\hat{r}}{|r|} \right\}$$

$$E_1 = \frac{q}{4\pi\epsilon_0 (5)^2} \left[\frac{4a_x - 3a_z}{\sqrt{4^2 + 3^2}} \right]$$

$$E_1 = a_x, -a_z$$
$$E_2 = -a_x, -a_z$$

$$E_2 = \frac{q}{4\pi\epsilon_0 r^2} \left[\frac{4a_x + 3a_z}{\sqrt{4^2 + 3^2}} \right]$$

$$r = \sqrt{4^2 + 0^2 + (-3)^2} = 5$$



$$E_T = \left(\frac{Q}{4\pi\epsilon_0(1)^2} + \frac{2Q}{4\pi\epsilon_0(2)^2} + \frac{6Q}{4\pi\epsilon_0(3)^2} + \frac{24Q}{4\pi\epsilon_0(4)^2} \right) (-ax)$$

$$E_T = \frac{Q}{4\pi\epsilon_0} \left[1 + \left(\frac{1}{2}\right) + \frac{2}{3} + \left(\frac{3}{2}\right) \right] (-ax)$$

minimum

as distance increases the charge is also increase.
 so 4th one has strongest charge and the 2nd one
 has the least charge contributed

4.

$$\begin{aligned} -5 < x < 5 \\ -5 < y < 5 \\ -5 < z < 5 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{cube}$$

$$K \text{ 8C} \rightarrow (4, 2, 3)$$

(inside) 8C $\rightarrow (2, -1, -3)$ Entering flux is dominating

$$(\text{outside}) 12C \rightarrow (-4, 0, 1)$$

-4C Any

5. center - origin

$$d_1 = \sqrt{4^2 + 2^2 + 3^2} = \sqrt{16 + 4 + 9} = \sqrt{29} > 6$$

$$d_2 = \sqrt{2^2 + (1)^2 + (0)^2} = \sqrt{4 + 1 + 0} = \sqrt{5} < 6 \rightarrow 8C \text{ (enclosed)}$$

$$d_3 = \sqrt{(4)^2 + (0)^2} = \sqrt{16 + 0} = \sqrt{16} = 4 < 6 \rightarrow 12C \text{ (enclosed)}$$

-4C Any

6

$$d_1 = \sqrt{2^2 + 4^2 + 1^2} > 6$$

$$d_2 = \sqrt{0^2 + 2^2 + 5^2} < 6 = 86$$

$$d_3 = \sqrt{6^2 + 3^2 + 2^2} > 6$$

(48)

86 Ans

E

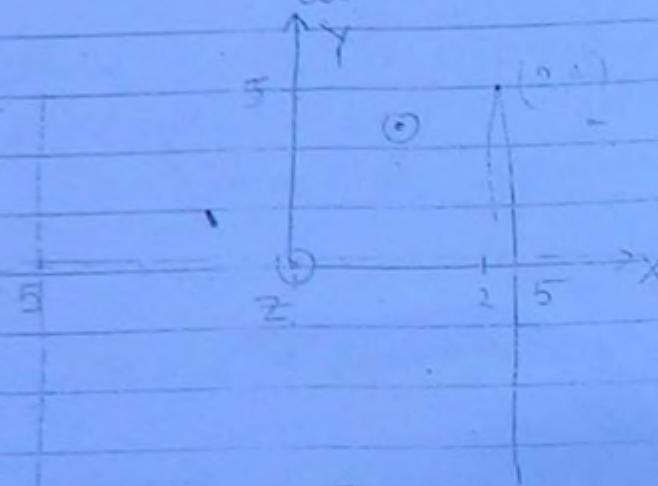
Exmple 20
 Bob calculate the net flux leaving the surface $-5 \leq z \leq 5$, $-5 \leq y \leq 5$, $-5 \leq z \leq 5$ due to a line charge $\rho_L = 10 \text{nC/m}$ located at $x=2$, $y=1$ for all z

sol

$$x=2, y=1$$

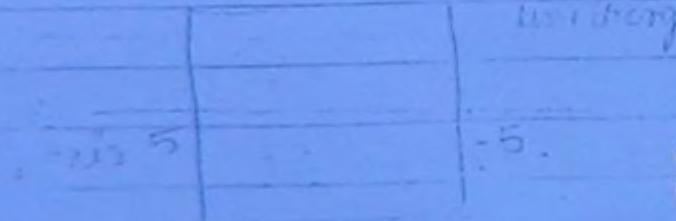
$$\vec{J}_L = \frac{dq}{dl}$$

$$\rho_L = 10 \text{nC/m}$$



flux leaving = charge enclosed = part of a line charge
 How much part of the charge is in the cube.

7th view



length of the line inside the cube = 10m

(-5 to 5 of z)

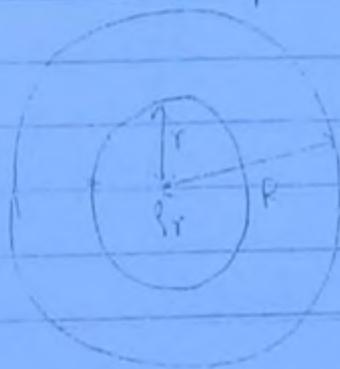
$$\phi = \rho_L \cdot l$$

$$10 \text{nC/m} \cdot 10 \text{m} = 100 \text{nC}$$

If line it along $y=x$ line in the $z=4$ plane 100F
 P hint = Repeat the question $z=0, y=x$ line
 Hint $z=b, y=x$ line 49

18. coulomb's law cannot apply because it is for point charge
 not for volume. we use $\oint \mathbf{D} \cdot d\mathbf{l} = Q$.
 E/D inside the charge.

The gaussian surface considered is concentric sphere of
 $r < R$ so that the strength on the surface same
 everywhere and hence loop $\oint \mathbf{D} \cdot d\mathbf{l} = Q$.



D-area = charge

$$\frac{\rho_1 \times 4\pi r^3}{3} \text{ C/m}^3 \times m^3 = C$$

$$D(r) = \frac{\text{charge inside}}{\text{Area}} = \frac{\rho_1 \frac{4\pi r^3}{3}}{4\pi r^2} = \frac{\rho_1 r}{3}$$

$$E(r) = \frac{\rho_1 r}{3\epsilon}$$

$$E(R/2) = \frac{\rho_1 R}{6\epsilon}$$

E at the centre is perfectly zero and maximum
 on the surface



6. $\frac{q}{4\pi R}$

(56)

$$\oint \mathbf{D} \cdot d\mathbf{l} = Q$$

$$D(r) = \frac{\text{charge inside}}{\text{area}} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2}$$

$$= \frac{\frac{1}{3}\pi r^3}{r^2} = \frac{\frac{1}{3}\pi r^3}{3r^2}$$

$$E(r) = \frac{\rho_v r^3}{3r^2 \epsilon}$$

$$E(2R) = \frac{\rho_v (2R)^3}{3r^2 \epsilon} = \frac{\rho_v 8R}{3\epsilon}$$

$$E(2R) = \frac{\rho_v R}{12\epsilon}$$

$$D(r) = \frac{\rho_v r^3}{3r^2}$$

$$E(r) = \frac{\rho_v (R)^3}{3r^2 \epsilon}$$

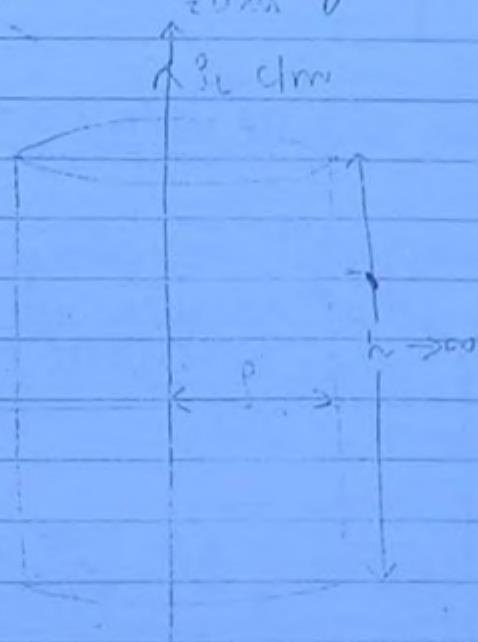
$$E(2R) = \frac{\rho_v R^3}{3\epsilon (2R)^2} = \frac{\rho_v R^3}{12\epsilon R^2} \Rightarrow \frac{\rho_v R}{12\epsilon}$$

Line charges & I carrying wires

(57)

Gauss Law Application 2: Strength E due to an infinite length line charge.

The application of gauss law involves choosing an cylindrical surface. If $\oint D \cdot d\ell = \text{constt}$ value. The surface have equidistant nature from the charge and hence D is constt everywhere.



$$\oint D \cdot d\ell = Q$$

$D(p) \times \text{area} = \text{charge enclosed}$

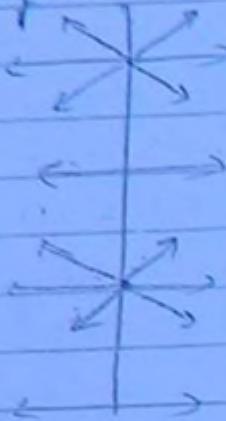
$$D(p) = \frac{\rho_L h}{2\pi p} = \frac{\rho_L}{2\pi p}$$

The closed surface is a curved surface area hence the flux will pass through the surface with zero top and bottom components considered.

As the air surface is $\mu = \text{const}$ surface $d\mathbf{s}$ is as directed
hence by logic \mathbf{D} is as directed.

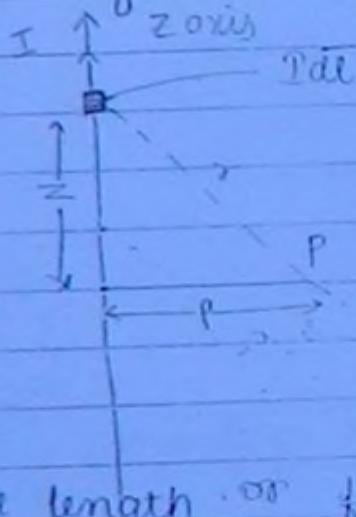
(52)

\mathbf{a}_p = radially outward from the line



Strength H due to an infinite length I carrying wire

Consider a current carrying wire along the z axis.
Let us calculate the field strength at r distance from it



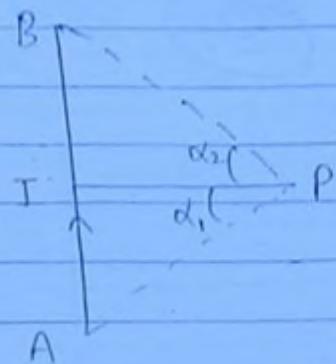
For any infinite length or finite length.

Identify a small incremental length element dx ,
such that $|dx| \rightarrow 0$

Find the incremental strength dH due to this length.

$$dH = \frac{\mu_0 I}{2\pi r} \times a_r$$

3. When applied for a finite length current carrying wire



(S3)

$$H = \frac{I}{4\pi r} (\sin \alpha_1 + \sin \alpha_2) a_\phi$$

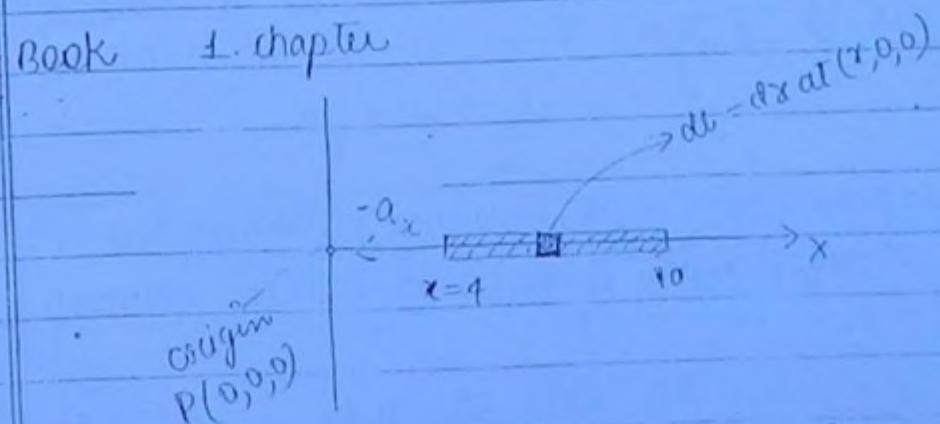
If E is to be calculated in line charge in Electrostatics

$$E(r) = \frac{\beta_L}{2\pi \epsilon_0 r} a_\phi = \frac{D(r)}{\epsilon}$$

If B is to be calculated in I carrying wire in magnetostatics:

$$B(r) = \mu_0 H = \frac{\mu_0 I}{2\pi r} a_\phi$$

Work Book 4. chapter



Lit small length dx on this wire.

find def due to this length using $\frac{dq}{dx}$.

$$\Phi = \oint_L d\mathbf{x} \cdot$$

$$\mathbf{S} = \mathbf{x}$$

$$a_x = -ax$$

(54)

$$dE = \frac{\rho_L dx}{4\pi\epsilon_0 x^2} (-ax)$$

$$E = \int_{x=4}^{10} \frac{\rho_L dx}{4\pi\epsilon_0 x^2} (-ax)$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_4^{10} \frac{1}{x^2} dx (-ax)$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \left[-x^{-2+1} \right]_4^{10} \Rightarrow \frac{\rho_L}{4\pi\epsilon_0} \left[-\left[\frac{1}{10} - \frac{1}{4} \right] \right].$$

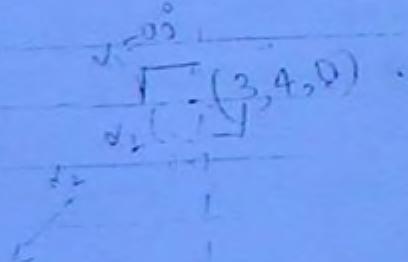
$$\Rightarrow \frac{\rho_L}{4\pi\epsilon_0} \left[-\frac{1}{10} + \frac{1}{4} \right] \Rightarrow \frac{\rho_L}{4\pi\epsilon_0} \left[\frac{-4+10}{10 \times 4} \right] \Rightarrow \frac{\rho_L}{4\pi\epsilon_0} \left(\frac{6}{40} \right) (-a_x)$$

Ans 3, 4

- (1) Volume is same then no effect on flux Φ_E
(2) gauss law. the flux leaving the surface is equal to the source i.e charge.

Ans 5

8A



(Q) Repeat the same quest if the current was flowing on the entire y-axis.
 your v

SS

(3, 4, 0)

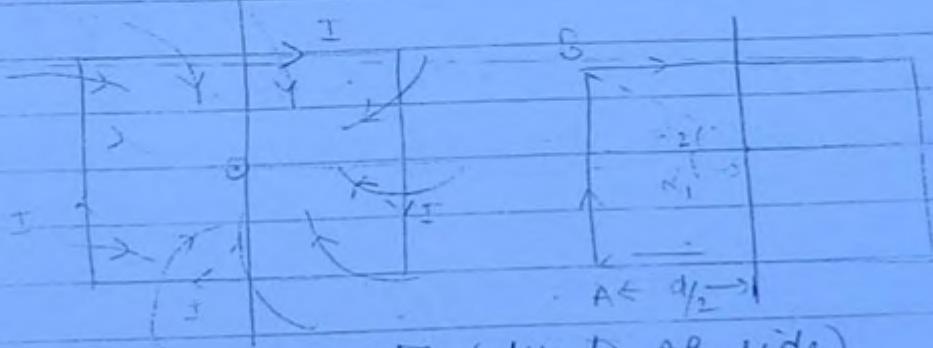
r = 3

For infinite line

$$H = \frac{I}{2\pi r} = \frac{8}{2\pi(3)} a_z = 1.33 a_z$$

The field in L shape is stronger and in infinite line is weak as in L shape the current come close.)

W.B
12



H direction at the centre (due to AB side)

$$a_y \times a_z = -a_z$$

Note

for a symmetric current distribution magnetic field at the geometric centre is always maximum.

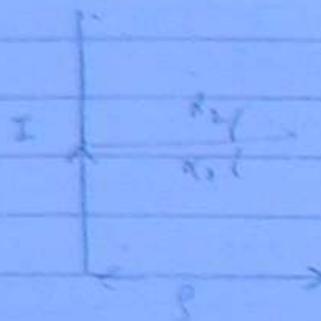
H magnitude due to side AB

$$H = \frac{I}{4\pi r} (\sin d_1 + \sin d_2)$$

$$\frac{I}{4\pi r} (\sin 45^\circ + \sin 75^\circ)$$

Break the L-shape wire into two part and then calculate the H for y-axis and H for x-axis separately and then take the vector sum.

(S6)



$$H = \frac{I}{4\pi R} (\sin \alpha_1 + \sin \alpha_2) \hat{\alpha}_\phi$$

$$(y\text{-axis}) = \frac{8}{4\pi(3)} (\sin 90^\circ + \sin \alpha_2) \hat{\alpha}_\phi$$

$$(y\text{-axis}) = \frac{8}{4\pi(3)} \left(1 + \frac{4}{5} \right) \hat{\alpha}_\phi$$

direction current direction \times radial dirⁿ to the point from the current

$$(-\hat{\alpha}_y \times \hat{\alpha}_x) = \hat{\alpha}_z$$

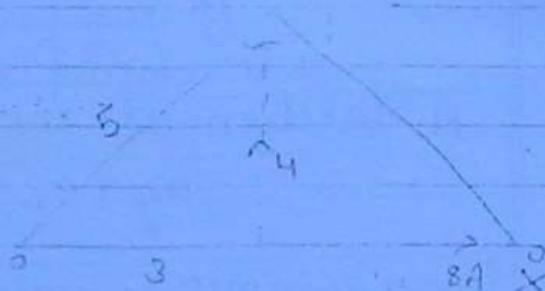
$x \rightarrow y \rightarrow z$

$$(x\text{-axis}) = \frac{8}{4\pi(4)} (\sin \alpha_1 + \sin \alpha_2)$$

$$= \frac{8}{16\pi} \left(1 + \frac{3}{5} \right)$$

$$= \frac{8}{16\pi} \frac{(8)}{5}$$

$$= \frac{64}{16\pi 5} = \frac{8}{2 \times 6.25} = \frac{8}{10\pi} = \frac{4}{5\pi} \hat{\alpha}_z$$



$x \rightarrow y \rightarrow z$

$\alpha_1 = \alpha_2$

$$\frac{\pi}{4\pi(d/2)} \left(\frac{I}{\sqrt{2}} + \frac{I}{\sqrt{2}} \right) (-a_2)$$

H at the centre, totally due to 4 sides

(57)

$$H = \frac{4\pi I}{4\pi d/2} \left(\frac{2}{\sqrt{2}} \right) (-a_2)$$

$$= \frac{2\pi I}{\pi d} \left(\frac{2}{\sqrt{2}} \right) (-a_2)$$

$$= \frac{4\pi I}{\sqrt{2}\pi d} (-a_2)$$

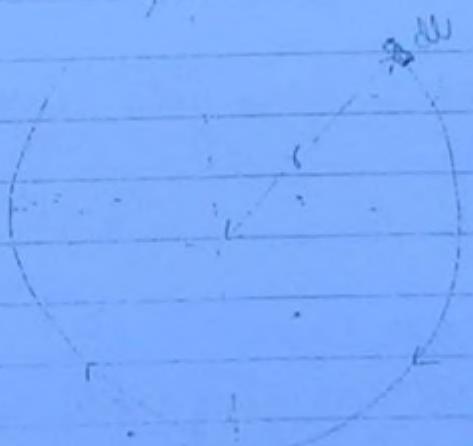
$$= \frac{2\sqrt{2}\pi I}{\sqrt{2}\pi d} (-a_2)$$

$$= \frac{2\sqrt{2} I}{\pi d} (-a_2)$$

$$H = \frac{2(\times 1.41)}{3.14} \frac{I}{d} = \frac{0.9 I}{d}$$

Q

Repeat the same question for circle - find the H at the centre . O



using Biot Savart's law.

$$dH = \frac{Idl \times a_r}{4\pi\mu_0 r^2}$$

$$dI = s d\phi$$

$$H = \frac{I}{4\pi} \int \frac{s d\phi}{s^2}$$

(58)

$$H = \frac{I}{4\pi} \int \frac{d\phi}{s}$$

$$H = \frac{I}{4\pi r} \int_0^{2\pi} d\phi$$

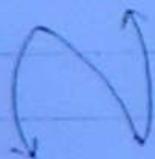
$$H = \frac{I}{4\pi r} (2\pi) \Rightarrow \frac{I}{2r} = \frac{I}{d}$$

clockwise current - H field direction into the paper.
All current carrying wires that are closed (square, circle) and have a finite area of enclosed are referred as magnetic dipole

clockwise current I flows side - South pole



anticlockwise current I flows side North pole



Atom is a example of dipole.
dipole \rightarrow closed current carrying wire.

$$\begin{aligned}\tan^{-1}\infty &= \pi/2 \\ \tan^{-1}(-\infty) &= -\pi/2 \\ \tan^{-1}(0) &= 0 \\ \tan(\pi/2) &\text{ undefined}\end{aligned}$$

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using Biot Savart's law.

$$dH = \frac{\mathbb{I} dz a_z \times (\vec{a}_r)}{4\pi (\rho^2 + z^2)}$$

(59)

$$|\vec{a}_r| = \sqrt{\rho^2 + z^2}$$

$$a_r = \hat{a}_r = \bar{a}$$

$$\hat{a}_r = \frac{\bar{a}}{|z|} = \frac{(\rho a_\phi - z a_z)}{\sqrt{\rho^2 + z^2}}$$

$$dH = \frac{\mathbb{I} dz a_z}{4\pi (\rho^2 + z^2)} \times \frac{(\rho a_\phi - z a_z)}{\sqrt{\rho^2 + z^2}}$$

The total strength H is

$$H = \int_{z=-\infty}^{\infty} dH = \int \frac{\mathbb{I} dz}{4\pi (\rho^2 + z^2)^{3/2}} a_z \times (\rho a_\phi - z a_z)$$

$$H = \int \frac{\mathbb{I} dz}{4\pi (\rho^2 + z^2)^{3/2}} a_\phi$$

$$= \frac{\mathbb{I} \cdot \mathbb{I}}{4\pi} \int_{-\infty}^{\infty} \frac{dz}{(\rho^2 + z^2)^{3/2}}$$

$$\text{put } z = \rho \tan \theta$$

$$dz = \rho \sec^2 \theta d\theta$$

$$f^2 + f^2 \tan^2 \theta = f^2 \sec^2 \theta$$

(6)

$$H = \int dH$$

$$= \frac{I \cdot \beta}{4\pi} \int_{-\pi/2}^{\pi/2} f \frac{\sec^2 \theta}{f^3 \sec^3 \theta} d\theta$$

$$= \frac{I \cdot \beta}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{1}{\beta^2 \sec \theta} d\theta$$

$$= \frac{I \cdot \beta}{4\pi \beta^2} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta$$

$$= \frac{I}{4\pi \beta} \left[\sin \theta \right]_{-\pi/2}^{\pi/2}$$

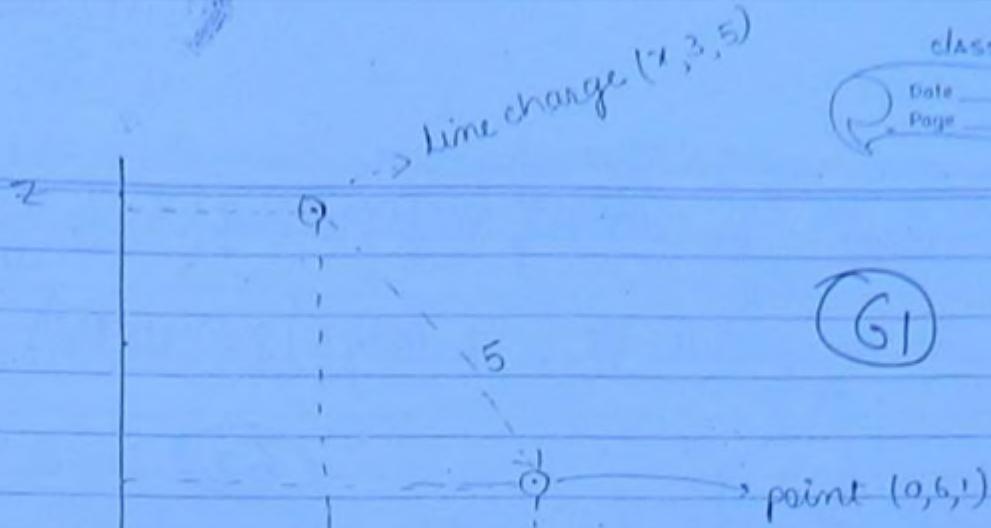
$$= \frac{I}{4\pi \beta} [\sin \frac{\pi}{2} - \sin (-\frac{\pi}{2})]$$

$$= \frac{I}{4\pi \beta} [1 - (-1)] \Rightarrow \frac{I \times 2}{4\pi \beta} \Rightarrow \frac{I}{2\pi \beta}$$

$$H = \frac{I}{2\pi \beta} a_\phi$$

1) a_ϕ - The H direction which is $(a_z \times a_\phi)$ is mandatory around the current.

2) the expression is similar to $D = \frac{f_1}{2\pi \beta} a_\phi$



$$E = \frac{q_L}{2\pi\epsilon_0} \frac{a_p}{r}$$

$$E \propto \frac{1}{r}$$

\oint between $(0, 6, 1)$ & $(1, 3, 5)$

$$r = \sqrt{3^2 + 4^2} = 5$$

$$E = ? \text{ at } (5, 6, 1)$$

$$E \propto \frac{1}{r}$$

$r = 5$ in both cases

E is the same.

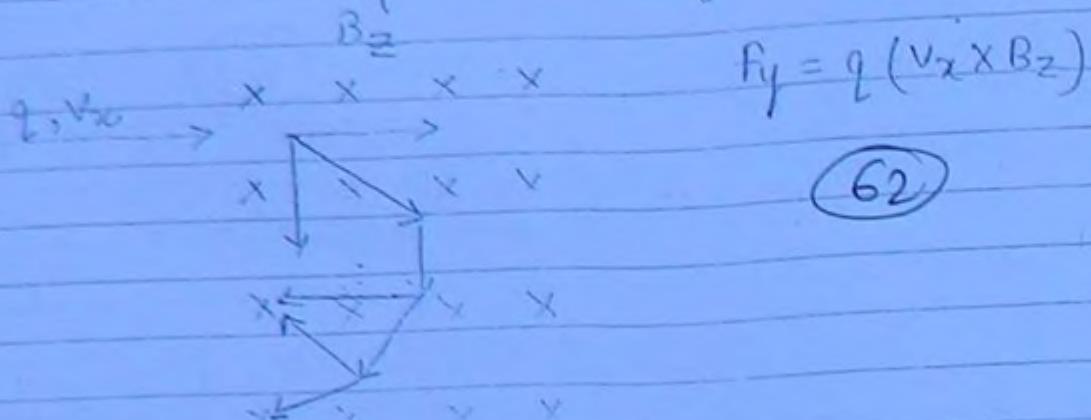
Note: If x axis is defined as $(0, 0, z)$

Any point (x, y, z) had a radial distance

$$r = \sqrt{x^2 + y^2}$$

which is independent of z

Maxwell's III Eqⁿ - closed surface integral of B.



1. The nature of magnetic field line is always to form closed loop around the current as seen in H due to a lone current carrying wire (ap direction)
2. Any phenomena that is circulatory or closed never has a distinct starting/ending point

There are no source/sink points for magnetic flux lines

$$\nabla \cdot B = 0$$

Because:

Hence divergence of magnetic flux density is zero everywhere because divergence needs a distinct start or an end

Maxwell's IIIrd Eq in point form.

$$\boxed{\nabla \cdot B = 0}$$

Mathematically this property is called as solenoidal nature of B fields

9) $\vec{Q} = 0$ - no charge

9) $-1 Q - 0$ (monopole & dipole in a pair)

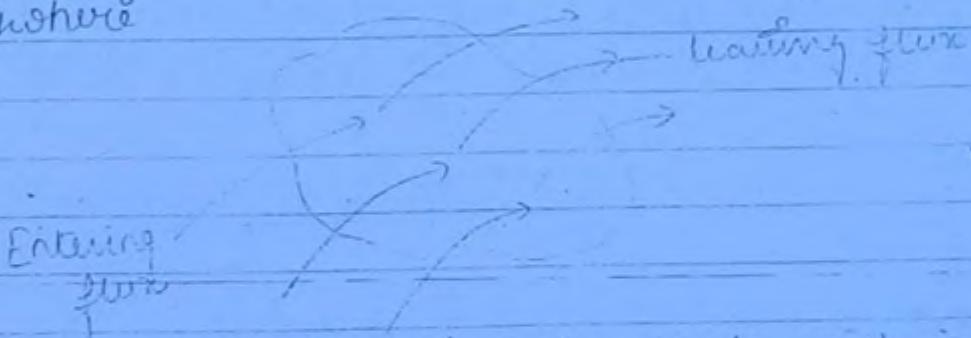
Applying divergence theorem for the point form. (63)

$$\oint \vec{B} \cdot d\vec{s} = \int (\nabla \cdot \vec{B}) \cdot d\vec{v} \quad [\text{Divergence theorem}]$$

$$\therefore \nabla \cdot \vec{B} = 0$$

$$\left[\oint \vec{B} \cdot d\vec{s} = 0 \right] \text{ Maxwell's III Eq^n - Integral form.}$$

Magnetic Monopoles don't exist because entering flux into any closed surface ^{should} always be equal to leaving flux as flux cannot start from anywhere or terminate anywhere.



Hence we conclude that only dipoles exist in magnetic fields i.e. the basic cause of magnetic fields is a current I which flows only in closed circuits. It flows only when both the polarities (dipoles) exist.

Summary:- E-fields vs H-fields

1. Q - Basic cause in E-fields
- scalar quantity

1. i - Point current - in H-fields
- vector quantity

2. ρ_i - line charge density

$i \rightarrow$ flux and line

3. ϵ_0 - medium's capacity
 $\mu_0 = 1$ in free space

ϵ_r - relative dielectric constant
 μ_r - relative magnetic permeability

4. D is called as flux density in E fields 64 B is called as flux density in magnetic field.

It is always a measure of strength in terms of charge.
It is always independent of E .

It is always a measure of strength - force.
It is always \propto dependent.

E-field intensity in E fields. $H \rightarrow$ field intensity in magnetic fields
 It is always a measure of strength - force.
 It is always \propto dependent.

It is always a measure of strength in terms of current (cause).
 It is always independent of w .

5. $F \propto E \propto \frac{1}{E (10^2)}$

Strongest
E field is one of the strongest force

$$F \propto B \times w (157)$$

Weakest
B field is one of the weakest force in nature

Potential, Gradient, closed line Integral of E

Potential:

A scalar measure of field strength of E field in terms of the energy at a point or in terms of work done to reach the point.

It's work done to reach the point from a reference point.

$$E = \frac{F}{Q}$$

$$V = \frac{W}{Q}$$

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$$V = \frac{W}{Q} = \frac{\text{Joule}}{\text{coulomb}} = \text{volt}$$

(65)

Note if work done by the charge is the measure of potential and never work done on the charge.

Work = force · displacement

$$W = F \cdot l$$

$$dW = F \cdot dl$$

$$dW = -Q E \cdot dl$$

Note work is done by the charge only when it goes against the field (against the repulsive force) (hence -ve sign)

$$W = - \int Q \cdot E \cdot dl$$

$$W = -Q \int E \cdot dl$$

$$V = \frac{W}{Q} = - \int E \cdot dl$$

$V \rightarrow$ Potential function of space but it is a scalar function.

$$E = u_1 \hat{i} + u_2 \hat{j} + u_3 \hat{k}$$

\rightarrow It is similar to intensity function which is an vector function

If the potential is evaluated b/w two distinct point with reference at B then V_{AB} is called potential difference with b/w A & B

at A

(Potential difference) $V_{AB} = \int_{\text{surf } B}^A E \cdot d\ell$ (Potential difference b/w A & B)

(66)

\rightarrow If surf B is assumed to be zero value; then
 $V_A = \text{absolute potential w.r.t B.}$

e.g. Ground is taken zero in most electric ckt's
Infinite distance is zero potential.

14/3/11

Thursday

$$V = - \int \vec{E} \cdot d\vec{\ell}$$

$$= - \int \frac{Q}{4\pi\epsilon_0 r^2} ar dr$$

when the field intensity is radially directed the potential calculation is simplified when $d\ell = dr ar$

$$V = - \int \frac{Q}{4\pi\epsilon_0 r^2} dr$$

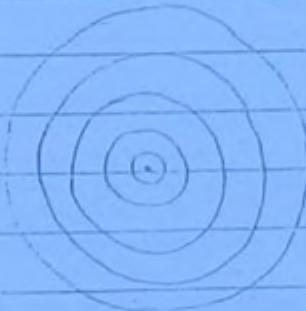
$$= - \frac{Q}{4\pi\epsilon_0} \int \frac{dr}{r^2}$$

$V = \frac{Q}{4\pi\epsilon_0 r}$

Note: If E decreases at $\frac{1}{r^2}$ V decreases at $\frac{1}{r}$
 $E\left(\frac{1}{r^2}\right) \rightarrow V\left(\frac{1}{r}\right)$

If $r = \text{const}$ then $V = \text{const}$ the locus of all these points forms a sphere around the charge. When the potential is const this is called an equipotential surface. The family of equipotential surfaces graphically represent the change in potential.

(67)



if $r = \text{const}$
concentric sphere

Potential of a line charge

$$V = - \int \vec{E} \cdot d\vec{r}$$

$$= - \int \frac{\rho_L}{2\pi\epsilon_0 r} dr \cdot d\phi \cdot d\phi$$

{ Always use natural log
not a

$$V = - \frac{\rho_L}{2\pi\epsilon_0} \int \frac{dr}{r} \ln \left(\frac{R}{r} \right)$$

$$\left\{ - \int \frac{dr}{r} = \ln \left(\frac{1}{r} \right) \right\}$$

1. $E \left(\frac{1}{r} \right) \rightarrow \text{vector}$

$V \left\{ \ln \left(\frac{1}{r} \right) \right\} \rightarrow \text{scalar}$

If $\phi = \text{const}$ then $V = \text{const}$ then we get concentric cylinders.

$$-\frac{1}{2} \left(\frac{x^{3/2}}{3/2} \right) \Big|_0^4 - 4 \left[\frac{y^3}{3} \right]_0^1$$

$$\Rightarrow -\frac{1}{2} \left[\frac{x^{3/2}}{3/2} \right]_0^4 - 4 \left[\frac{1}{3} \right]$$

$$\Rightarrow -\frac{1}{2} \left[\frac{x^2}{3} \right] \left[(4)^{3/2} \right] - \frac{4}{3}$$

$$\Rightarrow -\frac{1}{3} \left[4^{3/2} \right] - \frac{4}{3}$$

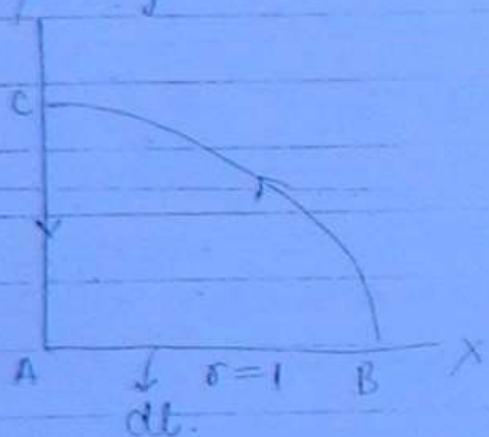
$$\Rightarrow -\frac{8}{3} - \frac{4}{3} \Rightarrow -4$$

(Q8)

Q 81 W.B

$$\bar{A} = 2\rho \cos\phi \hat{a}_\rho$$

$$\oint \bar{A} \cdot d\bar{l} = ?$$



$$\oint \bar{A} \cdot d\bar{l} = 3 \text{ lines}$$

$$\oint \bar{A} \cdot d\bar{l} = \int_A^B \bar{A} \cdot d\bar{l} + \int_B^C \bar{A} \cdot d\bar{l} + \int_C^A \bar{n} \cdot d\bar{l}$$

$$(1) \quad \int_A^B \bar{A} \cdot d\bar{l} = \int_{\rho=0}^1 2\rho d\rho = 1$$

$$d\bar{l} = d\rho \hat{a}_\rho$$

$$\phi = 0^\circ$$

$$\bar{A} = 2\rho d\rho \hat{a}_\rho$$

2. B to C

$$d\ell = \rho d\phi a_\phi$$

$$\bar{A} = 2\rho \cos\phi a_S$$

$$\oint_B^C \bar{A} \cdot d\ell = 0$$

(69)

3. C to A

$$d\ell = d\rho a_\rho$$

$$\phi = 90^\circ$$

$$\oint_B^C \bar{A} \cdot d\ell = \int_B^C 2\rho \cos\phi$$

$$= 0.$$

Potential Gradient

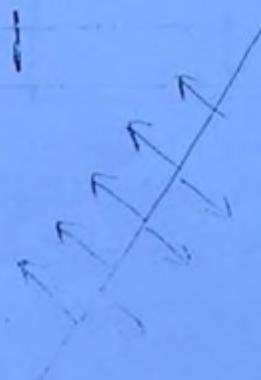
Scalar $\mathbf{g^n}$ of a surface \rightarrow gradient vector direction of the surface

In maths the gradient is used to find direction vector of any scalar surface $\mathbf{g^n}$. i.e. gradient is used to find a normal vector everywhere given to the surface.

eg. Linear surface

$$f = 3x - 4y - 8z = 100$$

$$\nabla f = 3a_x + 4a_y - 8a_z$$



Non linear.

$$g = 4x^2y - 8xz = 100$$

$$\nabla g = (8xy - 8z)a_x$$

$$+ 4x^2ay - 8xa_z$$



$$V = - \int E \cdot d\vec{r}$$

$$dV = - \vec{E} \cdot d\vec{r}$$

$$dV = - |E| \cdot du \cos \theta$$

$$\frac{dV}{du} = - E \cos \theta$$

If $\theta = 90^\circ$ i.e. the change of potential per unit length is analytical orthogonal to or \perp to the electric field direction. the potential has the same value. Hence the locus of all the points \perp to E field constitutes equi-potential surface.

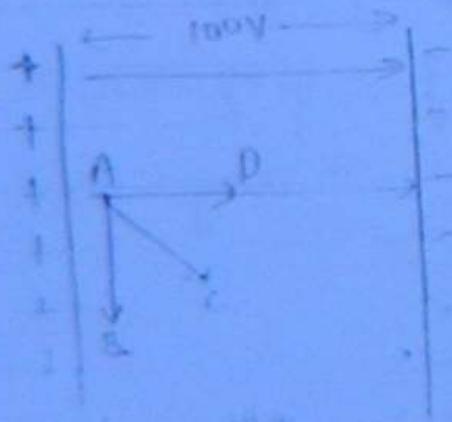
$$\theta = 90^\circ$$

$$V = \text{const}$$

$$\text{If } \theta = 0^\circ / 180^\circ$$

$$\left. \frac{dV}{du} \right|_{\max} = |E|$$

The magnitude of the E field intensity is the maximum rate of change of potential per unit length.



Closed

$$\theta = 0^\circ$$

$$\frac{dV}{dt} \Big|_{\text{max}} = -E$$

(7)

The direction of E field intensity is the direction in which potential decreases at a maximum rate.

Hence every scalar can have a vector defined from a unique direction of change by maximum and the rate of change by maximum. This is called as gradient operation. If the vector is γ the scalar is V .

then

$$E = -\nabla V$$

Volts
metre

Potential gradient means E field intensity (E)

Potential Eqn $\xrightarrow{\text{Gradient}}$ Vector Intensity

unit of electric field intensity (E) is Volts
metre

Formulae for gradient operation

If V = scalar function of space

$$V(u, v, w)$$

$$\nabla \cdot \gamma = \left[\frac{1}{h_1} \frac{\partial V}{\partial u} a_u + \frac{1}{h_2} \frac{\partial V}{\partial v} a_v + \frac{1}{h_3} \frac{\partial V}{\partial w} a_w \right]$$

Given the potential fun $V = a_1(x^2 - y^2)$ for all z ,
find the Eqn of the rectangular surface passing
through the pt. (3, 2).

Setⁿ

(3,1,1)

(22)

Eqn of equipotential surface is

$$V = 25(x^2 - y^2) = k$$

(as voltage is const on equipotential surface)

$$25(x^2 - y^2) = k$$

$$25(x^2 - y^2) = k \text{ at } (3,1,1)$$

$$25(9-1) = k \Rightarrow 200$$

$$x^2 - y^2 = k'$$

$$25(x^2 - y^2) = 200 \Rightarrow x^2 - y^2 = 8$$

The potential funⁿ given in the question is ⁱⁿ itself equi-potential surface definition

given $V = \frac{4 \cos \theta}{r^2}$. Find \vec{E} at $(2, \pi/2, \pi/2)$

$$V = \frac{4 \cos \theta}{r^2}$$

$$\nabla \cdot V = \frac{1}{h_1} \left(\frac{\partial}{\partial r} \left(\frac{4 \cos \theta}{r^2} \right) \right) a_r + \frac{1}{h_2} \left(\frac{\partial}{\partial \theta} \left(\frac{4 \cos \theta}{r^2} \right) \right) a_\theta \\ + \frac{1}{h_3}$$

$$\nabla \cdot V = \frac{4}{r} \cos \theta \left(\frac{1}{r^2} \right) (-2) + \frac{4}{r^3} (-\sin \theta) a_\theta$$

$$= -8 \cos \theta \left(\frac{1}{r^3} \right) + \frac{4}{r^3} (-\sin \theta) \quad |$$

$$= -8 \cos \pi/2 \left(\frac{1}{8} \right) + \frac{4}{8} (-\sin \pi/2) \quad |$$

$$= \frac{4}{8} (-1) = -0.5$$

Closed line integral of E - Maxwell's II Eqⁿ

$$\int \mathbf{E} \cdot d\mathbf{l} = \text{potential}$$

(73)

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

F

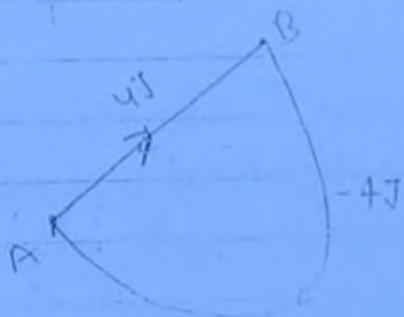
- Potential at a point in space is always unique at a point of time
- Potential cannot be a multivalued function
- The work done in moving a charge in any closed loop is zero, i.e. in a closed loop we sometime acquire energy sometimes loss energy, such that energy is conserved.

Hence E field is a conservative field.

E-field lines never forms closed loop, the lines are always outwardly divergent from a charge.

E field is an irrotational vector

$$\nabla \times \mathbf{E} = 0$$



Work done in moving a charge b/w two points is independent of path of consideration

Note: $\oint \mathbf{E} \cdot d\mathbf{l} = 0$ - Maxwell's 2nd Eqⁿ in integral form

but not $\int \mathbf{E} \cdot d\mathbf{l} = \nabla$

(74)

similarly $\nabla \times \mathbf{E} = 0$ — Maxwell's II eqn in point form.

but not $\mathbf{E} = -\nabla V$

Note: To identify whether a given vector field is a valid \mathbf{E} or \mathbf{H} put $\nabla \times \mathbf{E} = 0$ = valid Electric (\mathbf{E}) field.

put $\nabla \cdot \mathbf{B} = 0$ = valid Magnetic field.

2 -- In static E/H only.

Potential, Vector Potential, Maxwell IV Eqⁿ (Ampere's law)

Potential in Magnetic fields expressed as a scalar quantity is called as MMF (magneto motive force)

$$V_m = \int \mathbf{H} \cdot d\mathbf{l} \quad (\text{ampere})$$

$$\mathbf{H} = \nabla V_m$$

Its unit is ampere but it is more similar to current as it is equal to current when analysed for a closed path. its current flows only in closed circuits hence

$$\oint \mathbf{H} \cdot d\mathbf{l} = I$$

Ampere's law in integral form.

Maxwell IV Eqⁿ in integral form.

$$\text{but not } V_H = \int H \cdot dL$$

(75)

Statement of Ampere's Law:

The circulation of magnetic field intensity in any closed loop is always equal to the current flowing through the surface enclosed.

→ circulation means the effects which are around the current.

→ current means the cause of the effects

$$\nabla \times H = \text{curl of } H = \frac{\text{circulation}}{\text{area}} = \frac{\text{current}}{\text{area}} = J \text{ A/m}^2$$

$$\boxed{\nabla \times H = J}$$

Ampere's law in point form

Maxwell's IV Eqⁿ in point form

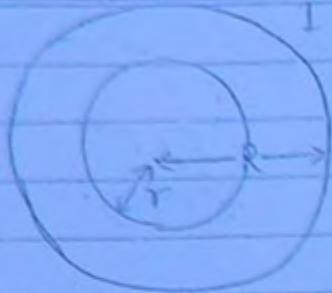
e.g.:



$$\oint H \cdot dL = I$$
$$H(B) = \frac{I}{2\pi R}$$

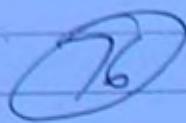
As the length of the circulation increases the strength of the effect it reduces. If the circulation is in a length of $2\pi R$ strength is $\frac{I}{2\pi R}$

26.W.E



given $r < R$

$$H(r) = ?$$



case I

$$r < R$$

To apply amper's law consider a circular line concentric and symmetric with the current, so the strength (H) is constt everywhere.

$$\oint H \cdot d\ell = I$$

$H(r) \times \text{length of circulation} = \text{current in the area}$

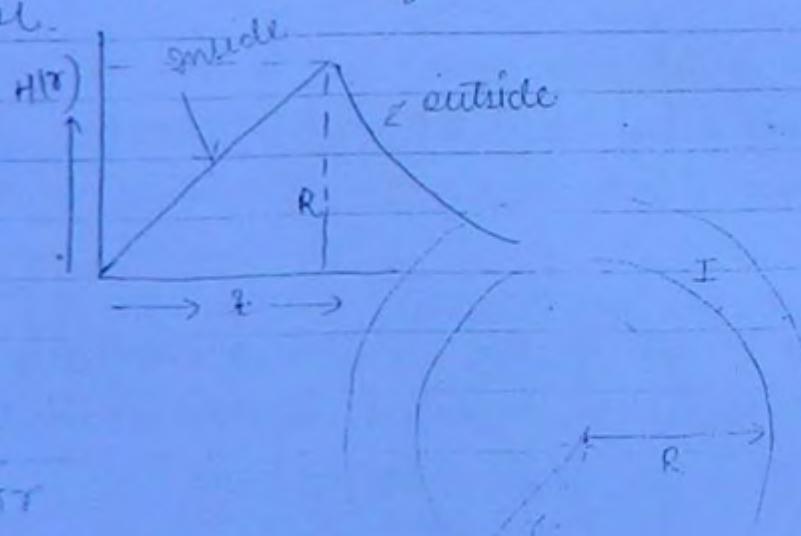
current density $\rightarrow \left(\frac{I}{\pi R^2}\right) \pi r^2 = \text{current encircling the area formed by the closed line}$

Total area

$$H(r) = \frac{I}{\pi R^2} \times \pi r^2$$

$$= \frac{Ir^2}{2\pi R^2 r} \Rightarrow \frac{Ir}{2\pi R^2}$$

notes: the field is zero at the centre of the conductor and max^m of the conductor.



case II

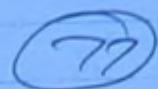
$$r = R$$

$$H(r) = \frac{I}{2\pi r}$$

22 W.B

A zephuse has the same geometry as that of point and hence a potential can be calculated on the same basis as.

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

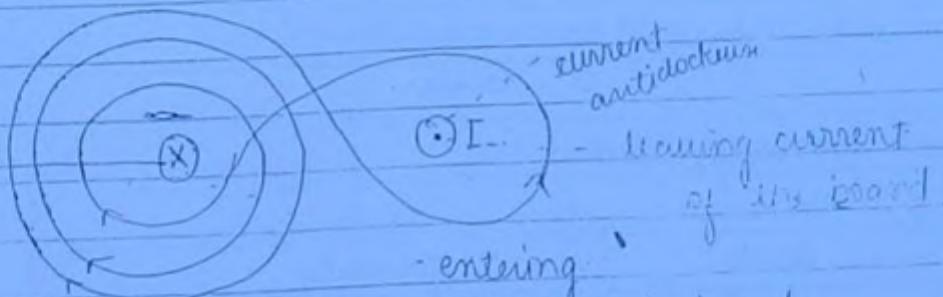


F

$$V = \frac{2 \times 10^8}{4\pi\epsilon_0 (10 \text{ cm})} \Rightarrow \frac{2 \times 10^8}{4\pi \times}$$

25 N.B

current
clockwise



$$\mathcal{T} + \mathcal{T} + \mathcal{T} - (-I) = 4I$$

Vector Magnetic Potential \bar{A}

Failure of MMF in certain regions

$$H = \nabla V_m$$

$$\begin{aligned} E &= -\nabla V \\ \nabla \times E &= 0 \\ \nabla \times (-\nabla V) &= 0 \end{aligned}$$

$$\nabla \times H = J$$

$$\nabla \times (\nabla V_m) = 0 = J$$

Hence $H = \nabla V_m$ definition is correct and exist only in those regions where there is no current density, i.e. in free space or certain free-regions only but not inside conductor.

How we define potential in magnetic field as
you know which is only in free space and not in
any conductor.

If the curl of vector potential is \vec{B} , as divergence of \vec{B} is 0 nowhere this definition satisfied for any point in magnetic fields

(78)

$$\vec{B} = \nabla \times \vec{A}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot (\nabla \cdot \vec{A}) = 0$$

$$\text{Div} \cdot (\text{curl of vector}) = 0$$

\vec{A} has the unit of -weber
meter

From faraday's law.

$$\frac{\text{weber}}{\text{second}} = \frac{\text{volts}}{} = \frac{\text{Joule}}{\text{colombs}}$$

$$\therefore \text{volts} = \frac{w}{q} = \frac{\text{Joule}}{\text{colombs}}$$

$$\text{weber} = \frac{\text{Joule} \times \text{sec}}{\text{colombs}} \Rightarrow \frac{\text{Joule}}{\text{colombs}} \frac{1}{\text{sec}}$$

$$\text{weber} = \frac{\text{Joule}}{\text{Amp}}$$

$$\frac{\text{weber}}{\text{meter}} = \frac{\text{Joule}}{\text{Amp} \cdot \text{meter}}$$

$$\vec{A} = \frac{w}{\text{potential}}$$

hence work or energy measured per basic cause of field signified by the basic unit of $\text{Amp} \cdot \text{meter}$

current element $\vec{D}\vec{dl}$ is a vector quantity

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{I\vec{dl}}{r\vec{dl}}$$

classmate

Date _____

Page _____

Note 1

$$\text{If } \vec{E} = \frac{F}{q} \quad \vec{B} = \frac{F}{I\vec{dl}}$$

$$\text{If } \vec{V} = \frac{W}{q} \quad \vec{A} = \frac{W}{I\vec{dl}}$$

$$P = \nabla V \quad B = \nabla \times A$$

$$\text{If } V = \frac{q}{4\pi\epsilon_0 r} - \text{ point}$$

(79)

Expression of A is obtained by duality of expression of V .

$$\boxed{\vec{A} = \frac{\mu_0 I\vec{dl}}{4\pi r}} \quad \text{point current element.}$$

The direction of \vec{A} is always the current direction itself

The closed line integral of vector p-magnetic potential

$$\oint A \cdot dl = \frac{\text{weber}}{\text{metre}} \times \text{metre} = \text{weber.}$$

Magnetic flux crossing the open surface formed by the closed line

$$\oint A \cdot dl = \int (\nabla \times A) \cdot ds$$

$$= \int B \cdot ds$$

max. air flux.

unit: weber or tesla

Note:

$$\oint \mathbf{B} \cdot d\mathbf{l} = 0 = \text{Entering flux}$$

$$= \text{Leaving flux}$$

Maxwell III Eqⁿ

(8)

$$\text{But } \oint \mathbf{B} \cdot d\mathbf{l} = \Psi_m = \text{cutting flux}$$

this is net Maxwell III Eqⁿ

15/3/11

Friday

Laplace / Poisson's Equation -

The most common form of charge is a volume charge moving inside materials. Hence $\rho_v = ne$ for most applications where $n = \text{no. of carriers per unit volume}$
 $e = \text{charge of the carrier}$

Poisson Eqⁿ relates in potential developed around any volume charge.

$$\nabla \rightarrow \rho_v ($$

$$\nabla \cdot D = \rho_v$$

$$D = \epsilon E$$

$$E = -\nabla V$$

$$\nabla \cdot (\epsilon E) = \rho_v$$

$$\nabla \cdot (\epsilon (-\nabla V)) = \rho_v$$

$\epsilon = \text{permittivity}$ (depends on material) $\approx 10^{-12} \text{ F/m}$

$$\nabla = -\frac{6r^5}{\epsilon_0}$$

(81)

$$\begin{aligned}
 \nabla^2 V &= \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial r} \left(h_2 h_3 \frac{\partial}{\partial r} \left(\frac{-6r^5}{\epsilon_0} \right) \right) \right] \\
 &= \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial r} \left(r^2 \sin\theta \frac{\partial}{\partial r} \left(\frac{-6r^5}{\epsilon_0} \right) \right) \\
 &= \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial r} \left[\frac{r^2 \sin\theta (-6 \times 5 r^4)}{\epsilon_0} \right] \\
 &= \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial r} \left[\frac{-30 r^6 \sin\theta}{\epsilon_0} \right] \\
 &= \frac{1}{r^2 \sin\theta} \left(\frac{-30 \sin\theta}{\epsilon_0} \right) \frac{\partial}{\partial r} [r^6] \\
 &= \frac{1}{r^2 \sin\theta} \left(\frac{-30 \sin\theta}{\epsilon_0} \right) 6r^5 \\
 &= \frac{-180 r^3}{\epsilon_0}
 \end{aligned}$$

$$\text{as } \nabla^2 V = -\frac{\rho_V}{\epsilon}$$

$$\nabla^2 V = -\frac{180 r^3}{\epsilon_0} = -\frac{\rho_V}{\epsilon}$$

$$\rho_V = 180 r^3$$

$$\begin{aligned}
 \text{Sip 2 } Q &= \int \rho_V dy \\
 &= \int_0^1 \int_{\pi/2}^{2\pi} \int_0^{2r} 180 r^3 \lambda^2 \sin\theta dr d\theta d\phi
 \end{aligned}$$

$$180 \left[\frac{8\pi^2}{6} \right]^7 \left[-\cos \theta \right]^7 \left[\theta \right]^{2x}$$

$$180 \times \frac{1}{6} (2\pi) = 120\pi \quad \textcircled{Q2}$$

Alternate Methode

$$\mathbf{D} \leftarrow \mathbf{E} = -\nabla V$$



$$\oint \mathbf{D} \cdot d\mathbf{l} = Q$$

31 w.b

$$\nabla^2 V = -\frac{\delta V}{\epsilon}$$

given E & V are zero } Initial condition
 ↓ ↓
 function zero }

$$2. \sqrt{V'} = E = \frac{dy}{dp}$$

$$\frac{1}{\delta} \frac{\partial}{\partial p} \left(\delta \frac{\partial V}{\partial p} \right) = + \frac{10^{-8} (1+10s)}{36\pi \times 10^9}$$

$$\frac{1}{\delta} \frac{\partial}{\partial p} \left(\delta \frac{\partial V}{\partial p} \right) = 360\pi (1+10s)$$

$$\delta \frac{\partial V}{\partial p} = \int 360\pi (\delta + 10\delta^2) d\delta$$

$$\delta \cdot \frac{\partial V}{\partial p} = 360\pi \left(\frac{\delta^2}{2} + \frac{10\delta^3}{3} \right)$$

$$V = \int_{\delta=0}^{\delta=5cm} 360\pi \left(\frac{\delta}{2} + \frac{10\delta^2}{3} \right) d\delta$$

$$V = 360 \pi \left[\frac{(\rho^2 + 10\rho^3)}{4} \right] \quad \begin{array}{c} 5\text{cm} \\ \rho = 2\text{cm} \end{array}$$

(8)

$$V = 360 \pi \left[\frac{\rho^2 + 10\rho^3}{4} \right] \frac{5 \times 10^{-2}}{2 \times 10^{-2}}$$

$$V =$$

$$32. \quad \nabla^2 V = -\frac{\rho_0}{\epsilon} \quad V = [20x^3 + 10y^4]$$

$$\begin{aligned} \nabla^2 V &= \frac{\partial}{\partial x} \left(\frac{\partial V}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\partial V}{\partial y} \right) \\ &= \frac{\partial}{\partial x} [60x^2] + \frac{\partial}{\partial y} [40y^3] \end{aligned}$$

$$\Rightarrow (120x + 120y)_{(2,0)} = -\frac{\rho_0}{\epsilon_0}$$

$$\Rightarrow 240 = -\frac{\rho_0}{\epsilon_0} \Rightarrow -240\epsilon_0 = \rho_0 \quad \text{Ans}$$

34.

$\phi \rightarrow$ 1 dimensional fm - Laplace

II derivative of $\phi = 0$

$$\nabla^2 \phi = 0$$

$\phi \rightarrow$ linear fun

$\phi \rightarrow$ changes at constt. rate

$$y = mx + c$$

$$dy = m$$

{ A linear funⁿ has constt. in first derivative and it never has a second derivative.

$$(\text{charge per unit length}) \frac{\phi_2 - \phi_1}{d} = \frac{\phi_3 - \phi_2}{2d}$$

$$\frac{\phi_2 - \phi_1}{d} = \frac{\phi_3 - \phi_2}{2}$$

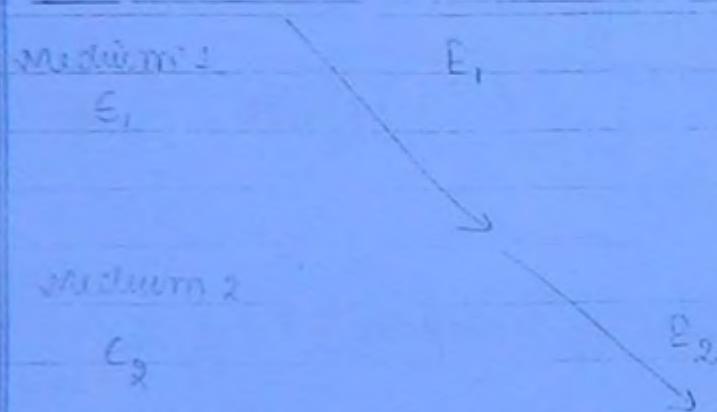
(84)

$$2\phi_2 - 2\phi_1 = \phi_3 - \phi_2 \Rightarrow \frac{\phi_2 + \phi_1}{\phi_3} \quad \phi_2 = \frac{2\phi_1 + \phi_3}{3}$$

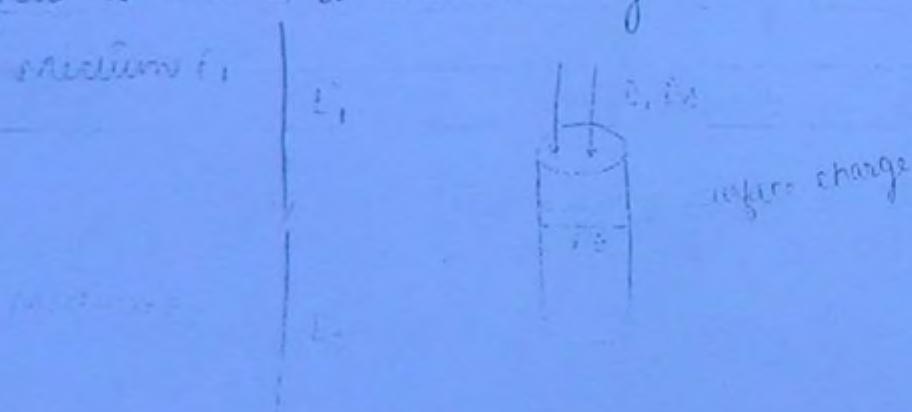
cont

~~QMB~~ Boundary conditions (vacuum & dielectric)

- If a field is known in one medium and the field is to be calculated in the adjacent medium we use boundary conditions.
- Boundary condition can be defined for only two types of fields for normal and tangential directions only.



case I E field is normal to the boundary



$\epsilon = \text{const}$

$$\nabla^2 V = -\frac{\rho_V}{\epsilon}$$

Poisson's Eqⁿ.

(P)

If $\rho_V = 0$ charge free regions.

e.g. free space

$$\nabla^2 V = 0$$

Laplace Eqⁿ

∇^2 - scalar Laplacian operator

Note 1: Laplace and Poisson's are second order differential Eqⁿ
but don't have two solutions we always have a unique
solution. This is called as Uniqueness Theorem.

Voltage at a point is unique it cannot have multiple
values. This is the physical meaning of uniqueness
theorem.

Laplace and Poisson in magnetic fields.

In magnetic field the same relationship exist b/w.
vector potential \vec{A} and current density J

$$\nabla \cdot D = \rho_V \rightarrow \nabla \times H = J$$

$$D = \epsilon E \rightarrow B = \mu H$$

$$E = \nabla V \rightarrow B = \nabla \times A$$

$$\nabla \times (\nabla \times A) = J$$

$$\nabla \cdot B = 0$$

$$\nabla \times (\nabla \times A) = uJ$$

$$\nabla(\nabla \cdot A) - \nabla^2 A = uJ$$

$\left\{ (\nabla \cdot A) = 0$ at magnetic field doesn't have divergence
it has only curly nature)

$$\nabla^2 A = -uJ$$

(86)

$$\nabla^2 A = 0$$

If $J = 0$ current free region

Formula for Laplace operation on scalar V .

If $V = V(u, v, w)$ = scalar function of space

$$\nabla \cdot (\nabla V) = \nabla^2 V$$

vector scalar

Dimensions of vector gives scalar (∇^2)

$$\nabla^2 V = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u} \left(\frac{h_2 h_3 \partial V}{h_1 \partial u} \right) + \frac{\partial}{\partial v} \left(\frac{h_3 h_1 \partial V}{h_2 \partial v} \right) + \frac{\partial}{\partial w} \left(\frac{h_1 h_2 \partial V}{h_3 \partial w} \right) \right]$$

$$g = \int g_v dv \quad \text{step 2.}$$

$$\nabla^2 V = -\frac{g_v}{\epsilon} \quad \text{step 1}$$

$v(r)$ only so it reduces the calculation

Page

consider a cylinder symmetrically in both the medium and use the of $\oint D \cdot d\ell = Q$ (87)

if there is no charge enclosed in the cylinder entering flux equals to the leaving flux because cylinder is consist of material and atoms has no charge inside it is electrically neutral.

$$\text{so. } D_1 \Delta S = D_2 \Delta S$$

$$D_1 = D_2$$

$$\epsilon E_1 = \epsilon E_2$$

If the Boundary has surface charge $\sigma_s \text{ C/m}^2$

$$D_2 \cdot \Delta S = D_1 \cdot \Delta S + \sigma_s \cdot \Delta S$$

$$(D_2 - D_1) = \sigma_s$$

in words statement of Boundary Condition

the normal component of flux density are continuous on either sides if there is no surface charge else discontinuous (not equal) (some gap) by amount equal to the surface charge density on the boundary

$$D_{n1} = D_{n2}$$

$$\text{or. } D_{n2} - D_{n1} = \sigma_s$$

assuming that n is for normal

ii) E field is tangential to the boundary.

medium 1

medium 2

ϵ_1

E_1

$\Delta x \rightarrow$



88

using Maxwell eqn (II) $\oint E \cdot dL = 0$

Take a closed line symmetrical in both the media that has Δx length.

$$\oint E \cdot dL = 0$$

$$E_1 \Delta x - E_2 \Delta x = 0$$

$$E_1 = E_2$$

Statement:

The tangential components of electric field intensity are always continuous.

WB

$$\chi < 0$$

$$\epsilon_1 = 1.5\epsilon_0$$

$$E_1 = 2ax - 3ay + a_z$$

$$E_{n_1} = -2ax$$

$$E_{t_2} = -3ay + a_z$$

$\chi = 0$

$\chi > 0$

$$\epsilon_2 = 2.5\epsilon_0$$

$$E_{t_2} = -3ay + a_z$$

$$\therefore |E_{t_1} - E_{t_2}|$$

$\chi = \text{constant surface}$.
 Normal is always $\frac{\partial \chi}{\partial x}$ (89) { $\frac{\partial \chi}{\partial x} = 0$
 direction " " $\frac{\partial \chi}{\partial x}$ Take gradient)

$$D_{n_1} = \epsilon_0 \rightarrow D_{n_2} = 3\epsilon_0 a_x \\ = 1.5\epsilon_0 2a_x \\ = 3\epsilon_0 a_x$$

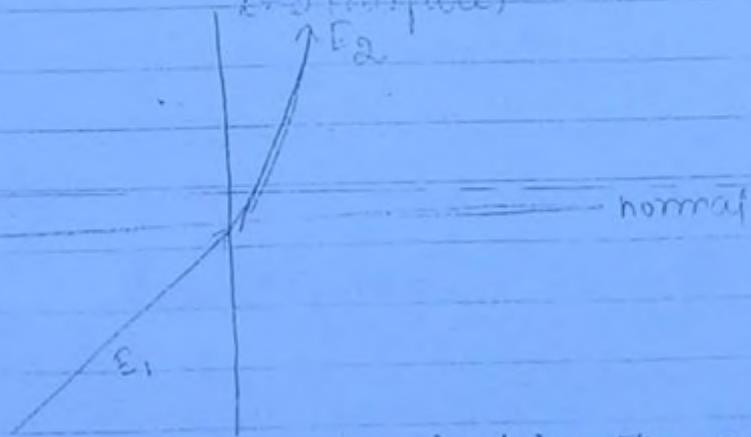
$$E_{n_2} = D_{n_2} = \frac{3\epsilon_0 a_x}{2.5\epsilon_0} = 1.2a_x$$

$$E_g = 1.2a_x - 3a_y + a_z \quad \text{Ans}$$

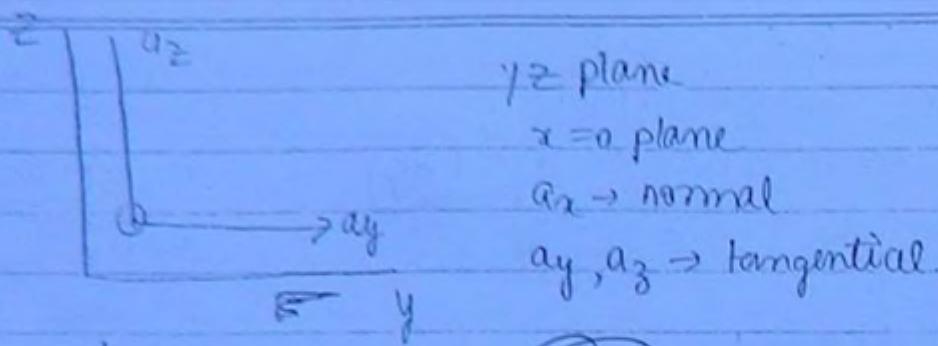
$$D_1 = \epsilon_0 (3a_x - 4.5a_y + 1.5a_z) \quad D_2 = \epsilon_0 (3a_x - 7.5a_y + 2.5a_z)$$

zero (surface)

Note 3:



As seen in the diagram the field in the second medium shifting away from the normal which can be understood the deviate normal components in E_2 . It can also be understood that the field is shifting towards the boundary in case tangential component as seen in D_2 .

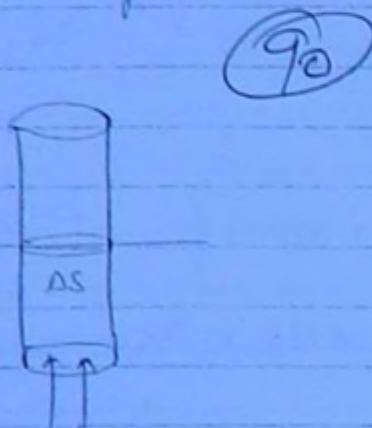


37. W.B.

$$\epsilon = 1$$

$$E_2 = a_2$$

$$f_d = 2a_2$$



$$2 \cdot 2\epsilon_0 \cdot dS = \text{Entering}$$

$$1 \cdot 1\epsilon_0 \cdot dS = \text{leaving}$$

$$f_s \cdot dS = -36_0 \cdot dS$$

$$f_s = -3\epsilon_0$$

sink at the boundary

Extension : Magnetic Boundary Conditions

1. $B_{n1} = B_{n2}$ Apply Maxwell III Eqⁿ

Always $\oint B \cdot dS = 0$

2. $H_{t1} = H_{t2}$ Apply Maxwell IV Eqⁿ

net always $\oint H \cdot dI = I$

$$H_{t1} = H_{t2}, I$$

\bar{K} = surface current density Atm^2/m

(91)

Note: D and ρ_c

have same units C/m^2 \bar{H} and \bar{K} have the same units.

A/m

38 W.B

$z=0$

$z < 0$

$z > 0$

$$u_{R1} = 2$$

$$u_{R2} = 1$$

$$\bar{B}_1 = 1.2 \bar{a}_x + 0.8 \bar{a}_y + 0.4 \bar{a}_z$$

$$\bar{B}_2$$

$$B_{n1} = 0.4 \bar{a}_z$$

$$H_2 = ?$$

$$\therefore z=0$$

$$B_{n2} = 0.4 \bar{a}_z$$

$$B_t = 1.2 \bar{a}_x + 0.8 \bar{a}_y$$

$$H_{t1} = \frac{B_t}{u_{R1}}$$

$$H_{t2} = \frac{1}{\mu_0} (0.6 \bar{a}_x + 0.4 \bar{a}_y)$$

$$= \frac{1}{2\mu_0} (1.2 \bar{a}_x + 0.8 \bar{a}_y)$$

$$H_{n2} = \frac{1}{\mu_0} (0.4 \bar{a}_z)$$

$$= \frac{1}{\mu_0} (0.6 \bar{a}_x + 0.4 \bar{a}_y)$$

39 W.B

air

$$|E| = 900 \text{ V/m}$$

z_0

$\epsilon = ?$ (dielectric

car is tangential as it is adjacent

sim is taken as normal.

$$E_{\text{fr}} = 400 \cos 30^\circ = 400 \times \frac{\sqrt{3}}{2} = 200\sqrt{3} = E_{\text{fr}}$$

$$E_{\text{fr}} = 400 \sin 30^\circ = 200$$

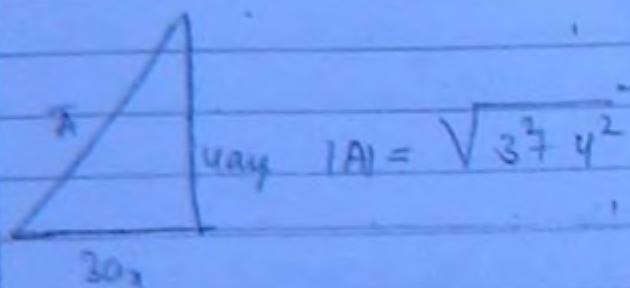
(92)

$$D_{\text{fr}} = 200 \epsilon_0 = D_{\text{fr}}$$

$$E_{\text{fr}} = \frac{200 \epsilon_0}{\epsilon_r}$$

$$E_{\text{fr}} = \frac{200 \epsilon_0}{20 \epsilon_0} = 10$$

$$(\text{magnitude}) E_2 = \sqrt{(200\sqrt{3})^2 + (10)^2} \approx 200\sqrt{3}$$



Energy Density in a electric field.

Consider a system of n discrete point charges having a electric field E and the total energy W_E .

The total energy can be equal to the energy expended in assembling the charges in their positions.

Total energy in field = Energy expended in assembling the charges

It involves bringing a charge against the repulsive force of the quanta already assembled charge.

$$W_1 = 0$$

$$W_2 = -Q_2 V_{21}$$

$$W_3 = -Q_3 V_{31} - Q_3 V_{32}$$

$$W_4 = -Q_4 V_{41} - Q_4 V_{42} - Q_4 V_{43}$$

$$\vdots$$

$$W_n = -Q_n V_{n1} - Q_n V_{n2} - \dots - Q_n V_{n(n-1)}$$

V_{21} = potential of 2.

due to charge 1

93

The total energy W_E is the sum of all the energy

The subscript can be interchange without changing the meaning. Hence

$$Q_2 \cdot V_{21} = Q_2 \cdot Q_1 \\ 4\pi\epsilon_0 r_{21}$$

$$Q_1 \cdot V_{12} = Q_1 \cdot Q_2 \\ 4\pi\epsilon_0 r_{12}$$

$$W_E = \left\{ \begin{array}{l} W_1 = 0 \\ W_2 = -Q_1 V_{12} \\ W_3 = -Q_1 V_{13} - Q_2 V_{23} \\ W_4 = -Q_1 V_{14} - Q_2 V_{24} - Q_3 V_{34} \\ \vdots \\ W_n = -Q_1 V_{1n} - \dots - Q_{n-1} V_{n-1n} \end{array} \right.$$

$$\delta W_E = -Q_1 V_1 - Q_2 V_2 - Q_3 V_3 - \dots - Q_n V_n$$

total
energy

$$W_E = -\frac{1}{2} \sum_{i=1}^n Q_i V_i$$

for continuous charges & E yields

$$W_E = -\frac{1}{2} \int \rho_V V dV \quad \left\{ \begin{array}{l} Q = \rho_V dV \end{array} \right.$$

$$= -\frac{1}{2} \int (\nabla \cdot D) V dV$$

$$W_E = \frac{1}{2} \int D (\nabla \cdot \mathbf{E}) dV$$

$$W_E = \int \frac{1}{2} (D \cdot E) dV$$

(94)

$$\frac{dW_E}{dV} = \frac{1}{2} D \cdot E$$

$$\frac{dW_E}{dV} = \frac{1}{2} \epsilon E^2$$

$\frac{dW_E}{dV}$ = Strength of the energy at every point in the
E field.

$$\frac{dW_E}{dV} = \frac{1}{2} \epsilon E^2$$

Note: $D \cdot E = \frac{\text{Joule}}{\text{m}^3} = \frac{\text{Newton} \times \text{metre}}{\text{m}^3} = \frac{\text{Newton}}{\text{m}^2}$

$$\frac{N}{C} \cdot \frac{C}{m^2} \Rightarrow \frac{\text{Newton}}{m^2} = \text{Pressure.}$$

2. $W_E = \frac{1}{2} \epsilon N^2$ is similar to $\frac{1}{2} \epsilon E^2$

Extension:- $\frac{dW_H}{dV} = \frac{1}{2} \mu H^2$

$$= \frac{1}{2} B \cdot H$$

It is similar to $\frac{1}{2} L I^2$

Ohm's Law and Continuity Eqⁿ.

$$I = \frac{Q}{t} = \frac{Ne}{t} = \frac{NeVA}{RA}$$

(95)

$$J = neAV$$

$$I = \frac{dQ}{dt} = \frac{d}{dt} \int \rho_v dv = \int \frac{\partial \rho_v}{\partial t} dv$$

ρ = continuous volume charge movement

the current can be considered as the current crossing the closed surface formed by the volume charge. Hence in this case

$$I = \oint J \cdot ds$$

Note: Generally $\oint J \cdot ds = 0$ (loop integral) means law of conservation of charge because entering charge is equalizing the leaving charge.

Apply divergence theorem

$$\oint J \cdot ds = \int (\nabla \cdot J) dv$$

Hence by comparison,

$\nabla \cdot J = \frac{\partial \rho_v}{\partial t}$	continuity eq ⁿ :
---	------------------------------

This is called as continuity eqⁿ which is the definition of current in field theory.

- outflow of current depends on movement of volume charge density

$\frac{\partial J}{\partial x}$

Apply for flow.

$$\nabla \cdot J = \frac{\partial \delta_V}{\partial t}$$

$$\nabla \cdot J = \frac{\partial J}{\partial x} = \frac{\partial \delta_V}{\partial t}$$

$$\frac{\partial J}{\partial x} = \delta_V \frac{\partial x}{\partial t}$$

$$\left. \begin{array}{l} \therefore \frac{\partial x}{\partial t} = v_d \\ \text{drift velocity} \end{array} \right\}$$

$$J = \delta_V v_d$$

$$v_d \propto E$$

$$v_d = \mu E$$

mobility (ability to move)

$$J = \delta_V \mu E$$

in point form

$J = \sigma E$ Ohm's law & continuity Eq'n in

here $\sigma = \delta_V \mu$ conductivity

Hence conductivity is ability to allow current in to the medium. It is the product of availability and ability to move.

$\sigma \rightarrow$ very good conductor.

$$\frac{J}{\sigma} = E$$

$$\frac{J}{\infty} = E$$

$$E = 0$$

- E field cannot exist inside a good conductor
- only flux exist but not accumulation of charge - hence only current exist but not E field.

$$\nabla \cdot \mathbf{D} = \rho$$

97

Every conductor is an equipotential surface
Potential diff. b/w any 2 points is zero.

for conductor surface at $\sigma = \infty$ on the surface

1. E along the surface = $E_{tan} = 0 = E_t$
(Electric field never be parallel) to the conductor surface
2. D_{normal} to the surface = $D_{normal} \neq 0$

$$D_n = \rho_s$$

case 2 $\epsilon = \infty$ very good dielectric

$$\mathbf{J} = \sigma \mathbf{E} \Rightarrow \mathbf{J} = \sigma \times \mathbf{E}$$

$$\mathbf{J} = 0$$

- E field can exist inside a good dielectric
- not flux exist but accumulation exists hence only E field exist

$$\nabla \cdot \mathbf{J} = \frac{\partial \rho}{\partial t}$$

$$\mathbf{J} = \sigma \mathbf{E}$$

$$\nabla \cdot (\sigma \mathbf{E}) = \frac{\partial \rho}{\partial t}$$

$$\sigma \cdot \nabla \cdot \left(\frac{D}{\epsilon} \right) = \frac{\partial \beta_V}{\partial t}$$

$$\boxed{\frac{\partial \beta_V}{\partial t} = \frac{\sigma}{\epsilon} \cdot f_V}$$

98

The solution of the Eqⁿ is - an exponentially decaying function (derivative back the same function is exponential) hence

$$f_V(t) = f_{V_0} \cdot e^{-\frac{t\sigma}{\epsilon}}$$

The Eqⁿ shows the any charge placed in any medium exponentially spread into the medium with the time constt depending on (ϵ/σ)

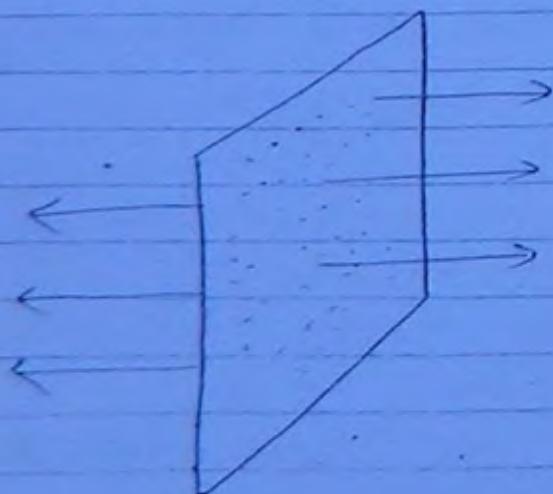
This ~~con~~ time constt is called as relaxation time or average spreading time -

$$\boxed{\frac{\epsilon}{\sigma} = \text{Relaxation time}}$$

If $\sigma = \infty$, good conductor $\frac{\epsilon}{\sigma} = 0$

Sheet charges of ρ_s C/m² & uniform fields

$$E \left(\frac{1}{\sigma} \right)$$



$$D = \frac{\rho_s}{2}$$

$$E \left(\frac{1}{\sigma} \right)$$

E (uniform)

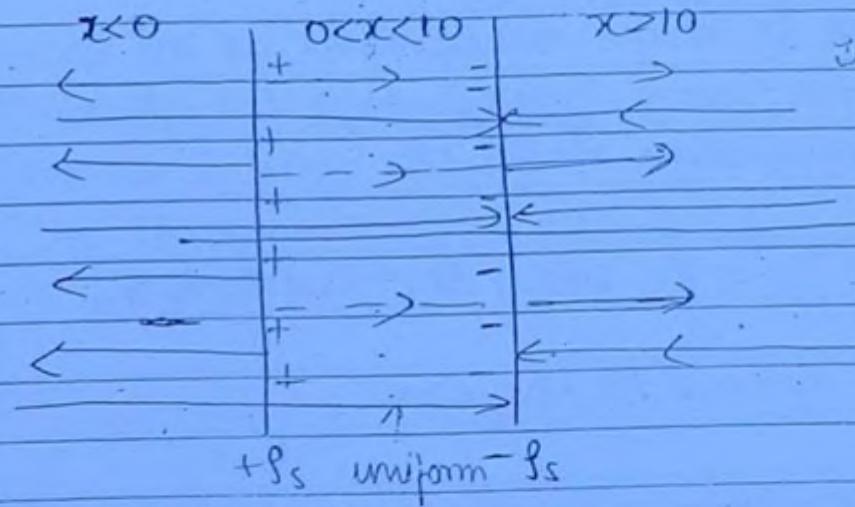
$E(x)$

$$\Sigma d\bar{U} = \bar{K} ds = \bar{J} dw$$

$$H = \frac{K}{2} x a_N$$

99

$$D = \frac{\rho_s}{2} a_N$$



$$E_1 = \left| \frac{\rho_s}{2\epsilon_0} \right| (-ax)$$

$$E_2 = \left| \frac{\rho_s}{2\epsilon_0} \right| (ax)$$

left side $E=0$

Right side two fields because of two sheets.

$$E_1 = \left| \frac{\rho_s}{2\epsilon_0} \right| ax$$

$$E_2 = \left| \frac{\rho_s}{2\epsilon_0} \right| (-ax)$$

$E_0 = 0$ on right all 0.
field in b/w.

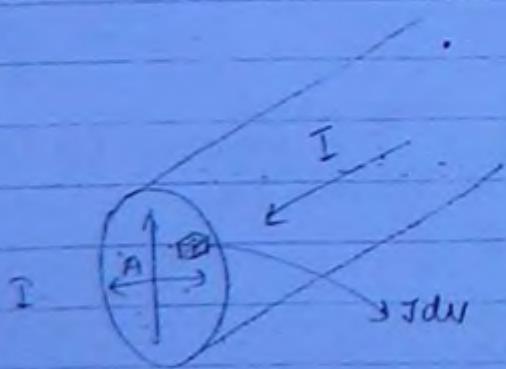
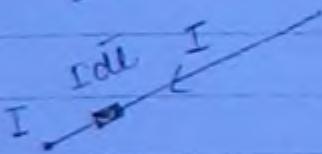
$$E = \frac{\rho_s}{\epsilon_0} ax$$

- consider an infinite sheet of charge density σ C/m² the field can be found Normal to the sheet and only to the right or left of the sheet. TOP
- The sheet is ^{being} infinitely large the lines are not divergent but are parallel to themselves.
- Hence the strength is same everywhere as the spacing b/w the lines is the same everywhere the flux density is same everywhere Hence the field is a uniform field.

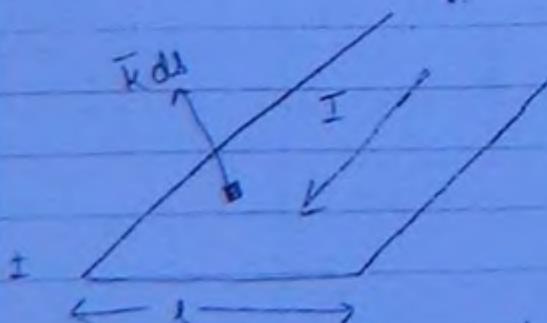
Hence the density $D = \frac{\sigma}{2}$ with the direction a_N

where a_N = unit normal to the sheet.

sheet of currents \mathbf{A} A/m



$$J = \frac{I}{A} = \frac{Ampere}{m^2}$$



$$K = \frac{I}{e} = \frac{Amp}{meter}$$

Two infinite sheets of charge density equivalent opposite nature have field only b/w the sheets and cancel out everywhere else

(10)

N.B

3 infinite sheet

$18 \text{ nC/m}^2 \pm 0_2$ at $x=4$

$9 \text{ nC/m}^2 \pm 0_2$ at $y=3$

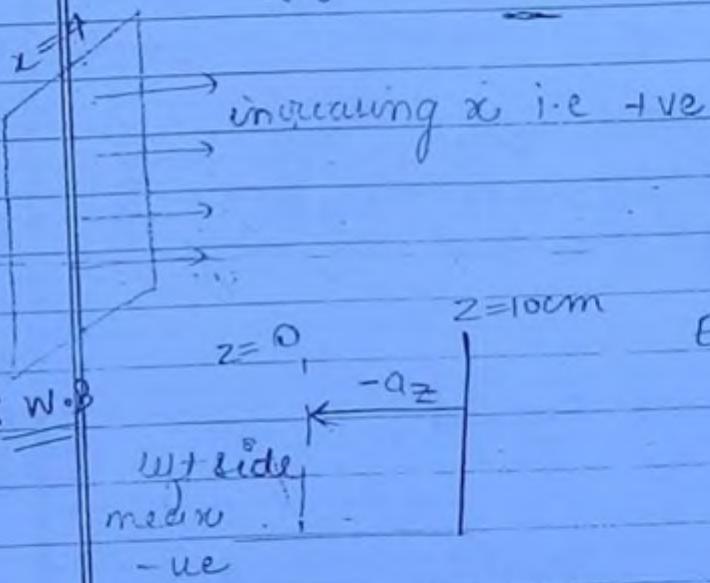
$-24 \text{ nC/m}^2 \pm 0_2$ at $z=0$

F

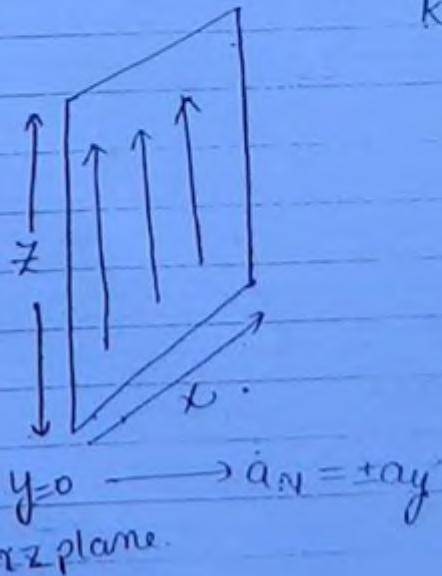
$$E = E_1 \hat{a}_x + E_2 \hat{a}_y + E_3 \hat{a}_z$$

$$E = \frac{D}{\epsilon} = \frac{\rho_s}{2\epsilon} \hat{a}_N$$

$$E_1 = \frac{18 \text{ a}_x}{2\epsilon_0} \quad E_2 = \frac{+9}{2\epsilon_0} \text{ a}_y \quad E_3 = \frac{-24}{2\epsilon_0} \text{ a}_z$$



$$E = \frac{\rho_s}{2\epsilon} = \frac{20 \times 10^9}{2 \times 1} = -360 \text{ a}_z \text{ V/m}$$



$$\bar{k} = 30 \hat{k} = 30 \text{ a}_z \text{ mA/m}$$

$$H \text{ at } (1, 20, -2) = ?$$

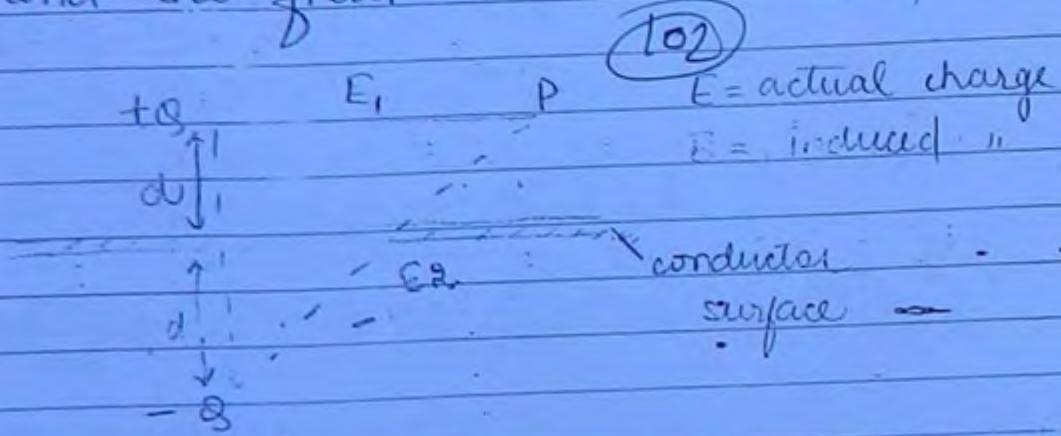
$$H = \frac{\bar{k} \times a_N}{2}$$

$$= \frac{30 \times a_N}{2} \text{ current direction}$$

$$= 15 (\hat{a}_z \times \hat{a}_y)$$

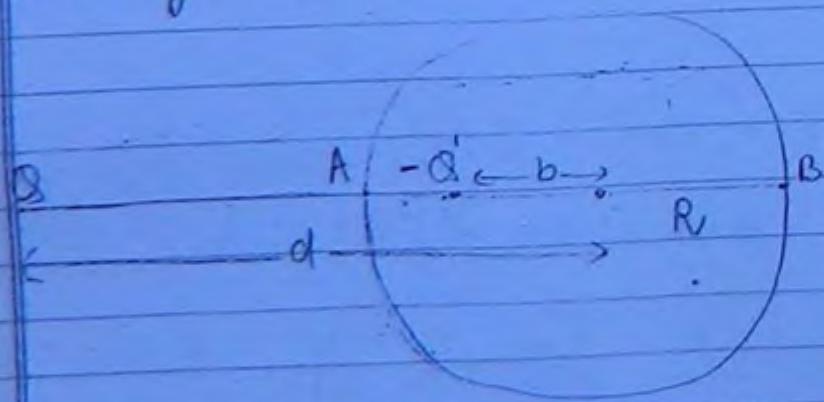
$$= -15 \text{ a}_x$$

when charge is brought to the near a conductor surface. The charges inside a conductor are rearranged due to the field of the charge that is the charges are displaced. Hence the final field anywhere is sum of the field of the actual charge and the field due to this induced charge.



This induced charge is represented with a image charge following the dynamics of a mirror. Since on a flat conductor the charges are equal the image has opposite sign.

Q.W. 9 Image should always be with coaxial.



Let induced charge = image charge = $-Q'$
let the image be at 'b' distance

Every conductor is a equipotential surface

$$\text{In } V_A = \frac{Q}{4\pi\epsilon_0(d-R)} - \frac{Q'}{4\pi\epsilon_0(R-b)} = 0$$

(103)

Let find potential at B also so that eqn become simple.

$$V_B = \frac{Q}{4\pi\epsilon_0(d+R)} - \frac{Q'}{4\pi\epsilon_0(R+b)} = 0$$

Potential at A and B are zero because the sphere is given a grounded sphere.

$$\begin{matrix} Q = d-R \\ Q' = R-b \end{matrix} = d+R = R+b$$

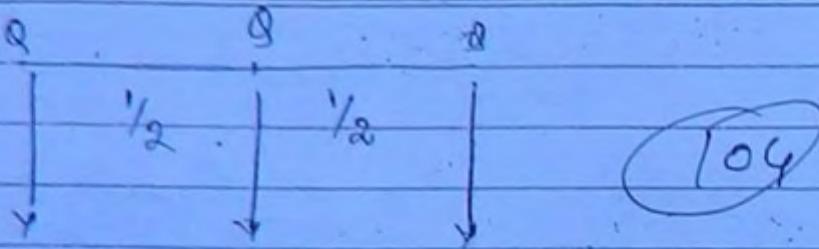
~~29 v.B~~ $W_E = \frac{-1}{2} \sum_{i=1}^n Q_i V_i$

$$W_E = \frac{-1}{2} \sum Q_i \frac{Q}{4\pi\epsilon_0 r}$$

$W_E \propto \frac{1}{r}$

$$\frac{W_1}{W_2} = \frac{1}{2}$$

method



$$W_1 = 0$$

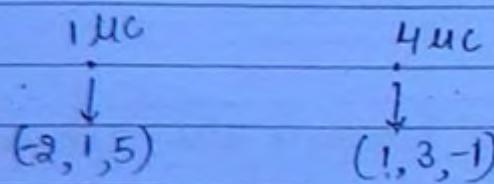
$$W_2 = \frac{Q \cdot Q}{4\pi\epsilon_0(\frac{1}{2})}$$

$$W_3 = \frac{Q \cdot Q}{4\pi\epsilon_0(1)} + \frac{Q \cdot Q}{4\pi\epsilon_0(\frac{1}{2})}$$

$$W_E = \frac{5Q^2}{4\pi\epsilon_0}$$

$$W_E = \frac{5Q^2}{8\pi\epsilon_0}$$

for W.E



$$W_1 = 0$$

$$W_2 = \frac{-Q_1 \cdot Q_2}{4\pi\epsilon_0 r}$$

$$W_2 = \frac{1 \times 4 \times 10^{-12}}{4\pi\epsilon_0}$$

$$\sqrt{3^2 + 2^2 + 6^2}$$

$$W_E = W_1 + W_2 = 5.14 \text{ mJ}$$

We can also use $W_E = \frac{1}{2} \sum_{i=1}^2 Q_i V_i$

now $D_n = \rho_s$ as the surface charge density is

$$\rho_s = 2 \sqrt{(1)^2 + (V_3)^2}$$

$$2\sqrt{4}$$

$$\rho_s = 2 \times 2$$

$$\rho_s = 9$$

$$\oint \mathbf{D} \cdot d\mathbf{l} = 0$$

i) $\oint Q = 0$

$$\oint \mathbf{B} \cdot d\mathbf{l} = 0$$

(105)

ii) $+Q, -Q$.

magnetic dipole in magnetic field.

Whenever a current flows in a closed loop it is treated as magnetic dipole.

- finite area.

$$\text{Dipole moment } m = I \times \text{area} = I \cdot A$$

Electric dipole in electric field

Whenever two charges of equal and opposite are separated by a finite distance

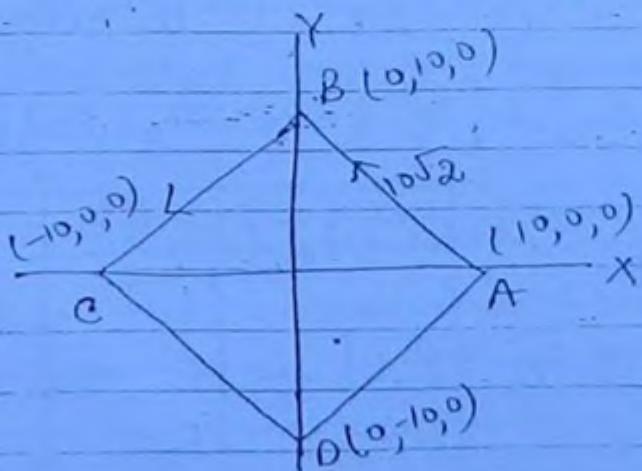
Electric-dipole

finite distance

$$\text{Dipole moment } p = q d$$

The importance of dipole moment is it inside the torque or the moment of the dipole in an external field such that magnetic torque is equal $= T = M \times B$

Electric torque is equal $T = P \times E$



$$M = I \times \text{Area}$$

$$= 0.01 \times (10\sqrt{2})^2$$

$$= 2$$

- M's direction is the direction of the area surface of the loop

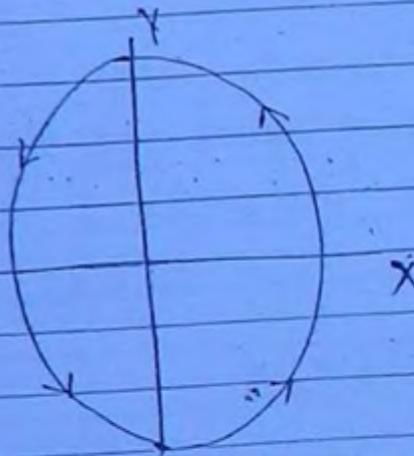
Surface $z = 0$, xy plane

Direction $= \pm a_z$

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current is anticlockwise as per RHS thumb direction
 M direction $(+a_x - a_y)$ $a_m + 2a_z$

45 W.B.



$$\text{Magnetic dipole moment (Torque)} = I \times A$$

$$M = I \times (\pi r^2)$$

$$= (0.1) \times \pi \times (10^{-3})^2 a_z$$

$$\text{Torque} = M \times B$$

$$= [(0.1) \times \pi \times (10^{-3})^2 a_z] \times [10^{-5} (2a_z - 2a_y + a_x)]$$

$$= [10^{-7} \pi a_z] \times [10^{-5} (2a_z - 2a_y + a_x)]$$

$$= [2 \times 10^{-12} \pi a_y - 2 \times 10^{-12} \pi a_x]$$

$$= 2 \times 10^{-12} \pi [a_y + a_x]$$

capacitors & Inductors:

(107)

- Capacitors -

Ability to hold E field confining it into a small region

$$C = \text{Farads} = \frac{\oint D \cdot ds}{\oint E \cdot dl} = \epsilon_0 \frac{\oint E \cdot dl}{\oint D \cdot dl} = \frac{Q}{V}$$

- It is always measured in terms of charge utilized and the potential developed by this charge. Because potential is accumulation and hence the measure of holding ability.

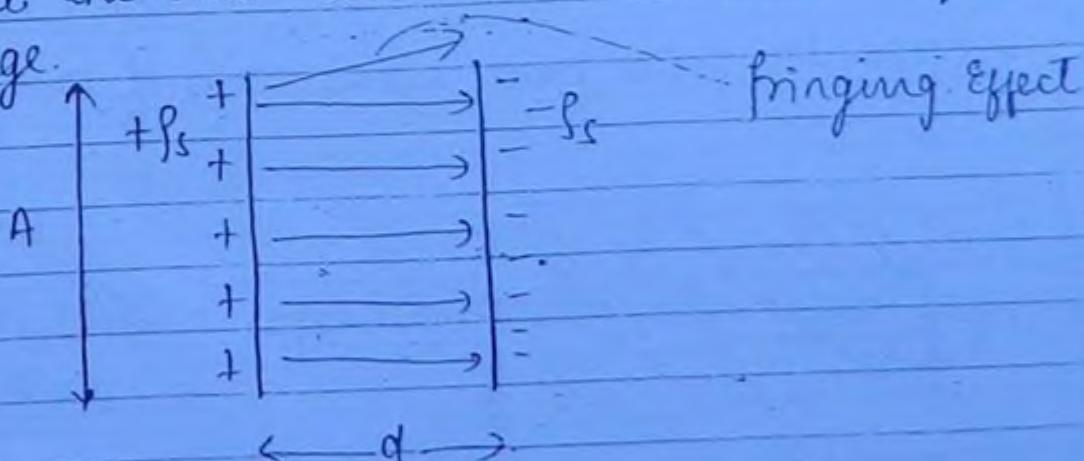
- The best examples of capacitor are geometry involving
 - e.g. Parallel plates
 - concentric cylinders
 - concentric spheres

Parallel Plate capacitors.

Two sheet charge of Area A

separation d ($A \gg d$)

since the sheet can be considered an infinite sheet of charge.



$$C = \frac{Q}{V} \quad Q = σ_s A$$

$$V = Ed = \frac{\rho_s d}{\epsilon}$$

so

$C = \frac{\epsilon A}{d}$

(108)

$$C = \epsilon \int E \cdot ds$$

$\int E \cdot dl$

$E = \text{const.}$ for uniform field.

$$C = \frac{\epsilon EA}{E \cdot d} = \frac{\epsilon A}{d}$$

Total energy held by the field

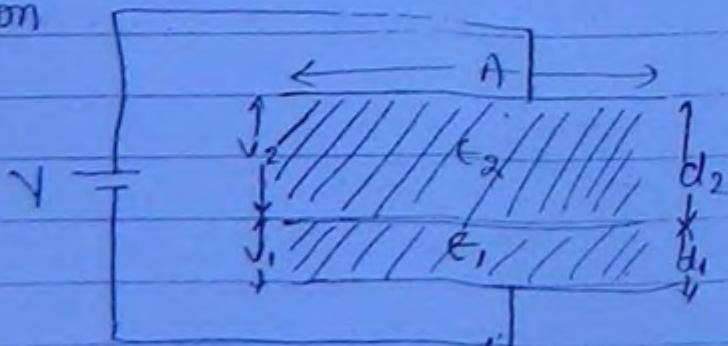
$$W_E = \left(\frac{1}{2} \epsilon E^2 \right) (Ad)$$

$$= \frac{1}{2} \frac{\epsilon A}{d} (Ed)^2$$

$$W_E = \frac{1}{2} CV^2$$

multiple dielectric in parallel plate capacitor

- i) Dielectric & capacitor plates have equal area of cross section



C_1 and $C_2 \rightarrow$ series

$$C_1 = \frac{\epsilon_1 A}{d_1} \quad C_2 = \frac{\epsilon_2 A}{d_2}$$

(109)

Voltage divider b/w the dielectrics.

$$V_1 + V_2 = V$$

$$C_{eq} = \frac{1}{C_1} + \frac{1}{C_2} \quad (\text{in series})$$

$$= \frac{C_1 + C_2}{C_1 C_2}$$

$$C_{eq} \Rightarrow \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{eq} = \frac{\epsilon_1 A}{d_1} + \frac{\epsilon_2 A}{d_2}$$

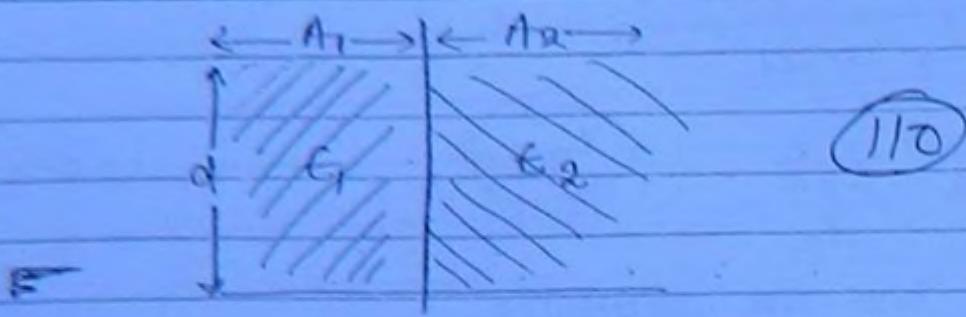
$$\frac{\epsilon_1 A}{d_1} + \frac{\epsilon_2 A}{d_2}$$

$C_1 V_1 = C_2 V_2$ (as charge is common and voltage divider in series).

$$\frac{V_1}{V_2} = \frac{C_2}{C_1}$$

$$\frac{V_1}{V_2} = \frac{\epsilon_2 \cdot d_1}{d_2 \cdot \epsilon_1}$$

Ex(i) Dielectrics & capacitor plates have equal separation / thickness

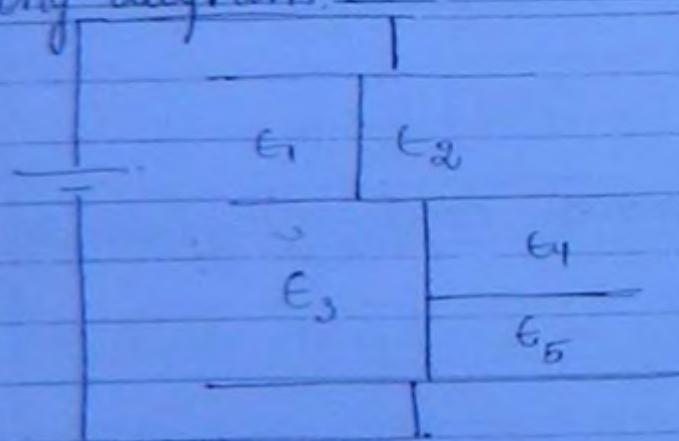


$C_1 C_2 \rightarrow$ are in shunt parallel.

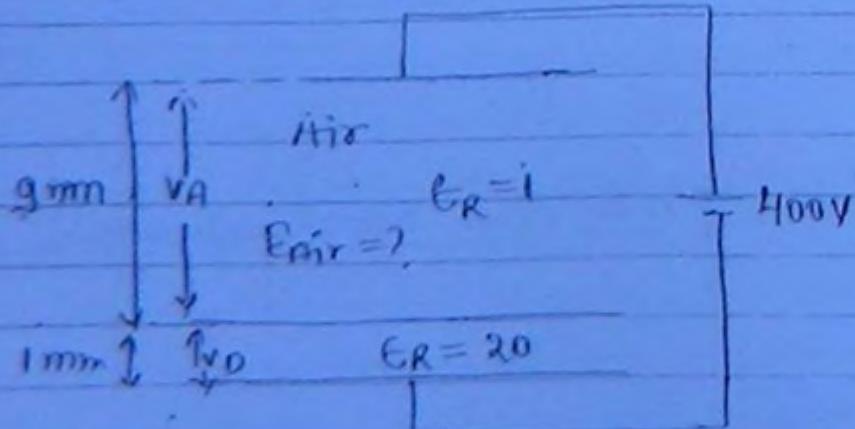
Voltage applied to plates = voltage b/w the dielectrics

$$C_{eq} = C_1 + C_2 \dots \\ = \frac{\epsilon_1 A_1}{d} + \frac{\epsilon_2 A_2}{d}$$

Identify the equivalent capacitance in the following diagram:



$$C_{eq} = (C_1 \text{ parallel } C_2) \text{ series } (C_3 \text{ parallel } (C_4 \text{ series } C_5))$$



$$V_A + V_D = 400$$

\ 1

$$\frac{V_A}{V_D} = \frac{20}{1} \times 9$$

(11)

$$\frac{V_A}{V_D} = 180 \frac{V_D}{V_D}$$

$$180V_D + V_D = 400$$

$$181V_D = 400$$

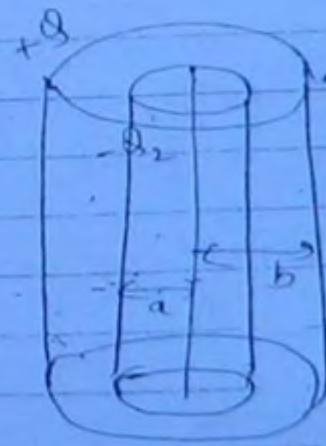
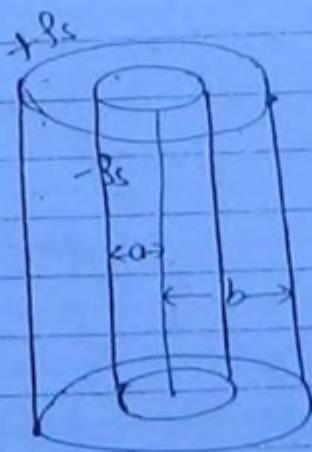
$$V_D = 2.21$$

$$V_A + 2.21 = 400$$

$$V_A = 397.8 \text{ V}$$

$$E = \frac{V_A}{d} = \frac{397.8 \text{ V}}{9 \text{ mm}} \approx 44 \frac{\text{KV}}{\text{m}}$$

concentric cylinders:



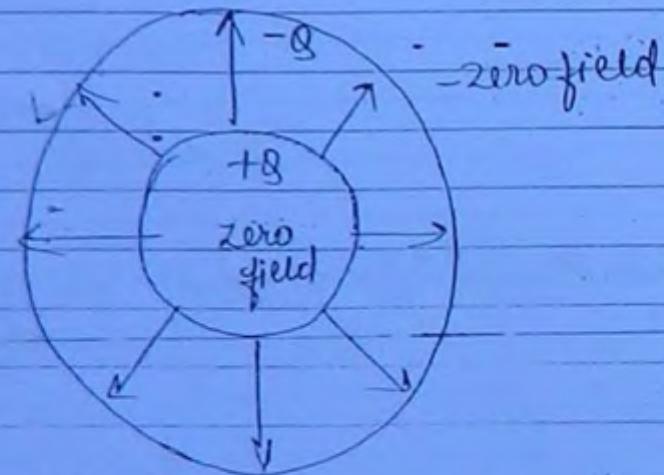
- (i) If two concentric cylinders have equal and opposite charge densities that means the charges are unequal on their surfaces. Hence a field or flux leaving exist outside the cylinder also.

- ii If two concentric cylinders have equal and opposite charge the densities are unequally spread but net flux outside of cylinder is zero. Hence the field is confined b/w the cylinders

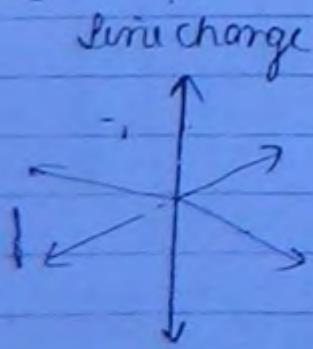
112

$$f_{S_1} = Q \\ 2\pi b h$$

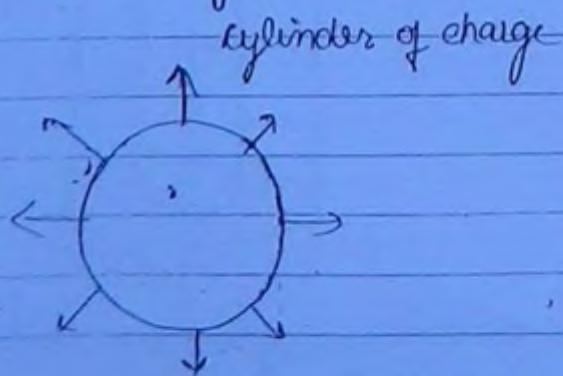
$$f_{S_2} = -\frac{Q}{2\pi a h}$$



A charge on the cylindrical surface has the same radial field, divergent from the surface similar to that of a field from a line charge.



$$E \propto \frac{1}{r}$$



$$E \propto \frac{1}{r}$$

Inductors:

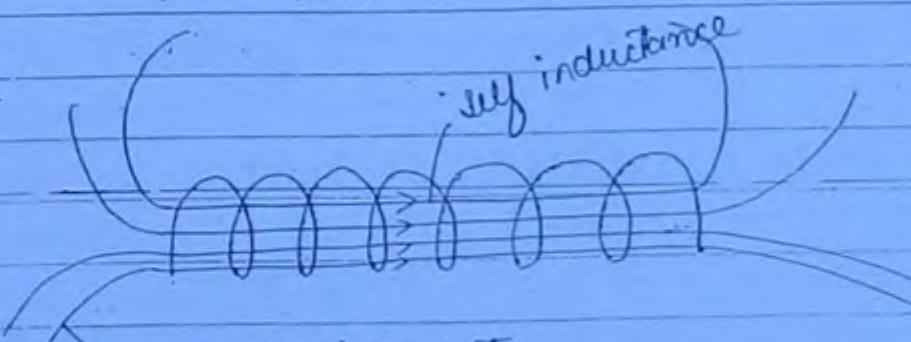
- Ability to hold magnetic field (H) confined into a small region is called inductance .

(T13)

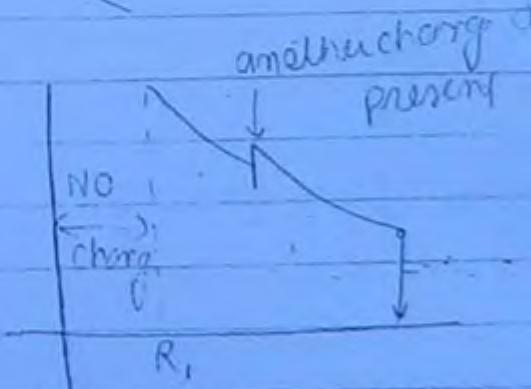
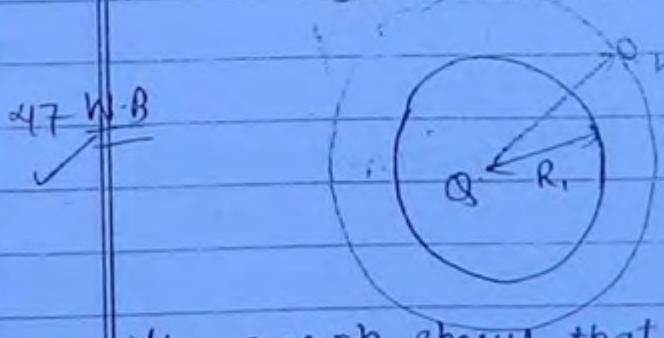
$$\text{Henry } L = \frac{\int B \cdot ds}{\oint H \cdot dl} = \frac{\mu \int H \cdot ds}{\oint H \cdot dl} = \frac{\Psi_m - \text{flux}}{I - \text{current}}$$

Inductance is a measure of the confined flux and the current utilized for this confinement.

- Geometries :- Solenoids
 - concentric cylinders
 - Toroids.
- All the have some or other circular dielectric



mutual inductance



- The graph shows that the field is zero upto R , distance from where there is no charge enclosed upto R , so that charge is hollow sphere of charge Q , and radius R ,

$$J \cdot A = \frac{\text{Amp} \times \text{weber}}{\text{m}^2 \text{ m}}$$

$$\text{m Weber} = \frac{\text{Joule}}{\text{m}} \quad \frac{\text{Amp-m}}{\text{Amp-m}}$$

$$\text{weber} = \frac{W}{m}$$

(114)

$$J.A = \frac{\text{Amp}}{m^2} \times \frac{\text{weber}}{m} = \frac{\text{Amp}}{m^2} \times \frac{\text{Joule}}{\text{Amp} \cdot \text{m}} = \frac{\text{Joule}}{m^3}$$

Summary 1

Scalar function $\xrightarrow{\nabla\text{-gradient}}$ Vector
Intensity per m

e.g. voltage $\xrightarrow{\nabla \cdot V}$ $E \frac{\text{volt}}{\text{m}}$

vector function $\xrightarrow{\nabla \times \text{curl}}$ vector function
Intensity per m \rightarrow Density per m^3 .

e.g. Intensity

Vector function \rightarrow Scalar function

Density \rightarrow (per m^3) volume

C/m^2 flux density $\xrightarrow{\nabla \cdot D}$ $f_V (\text{C/m}^3)$

summary 2.

Maxwell's Equations:

Integral form

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Point form

Not Maxwell's Eqn.

open integral.

$$1. \oint D \cdot d\ell = Q$$

$$1. \nabla \cdot D = \rho_v$$

$$1. \int D \cdot ds = \psi_e = \text{coulomb}$$

$$2. \oint E \cdot d\ell = 0$$

$$2. \nabla \times E = 0$$

$$\rightarrow 2. - \int E \cdot dl = V = \text{volts}$$

$$3. \oint B \cdot d\ell = 0$$

$$3. \nabla \cdot B = 0$$

$$3. \int B \cdot dl = \psi_m = \text{weber}$$

$$4. \oint H \cdot d\ell = I$$

$$4. \nabla \times H = J$$

$$4. \int H \cdot dl = V_m = \text{amps}$$

Note: - Rate of change of magnetic flux with time is EMF
voltage.

Rate of change of electric flux with the time is
current.

Weber = volts [Faraday's Law]
sec

Coulomb = amps.
sec

Chapter II.

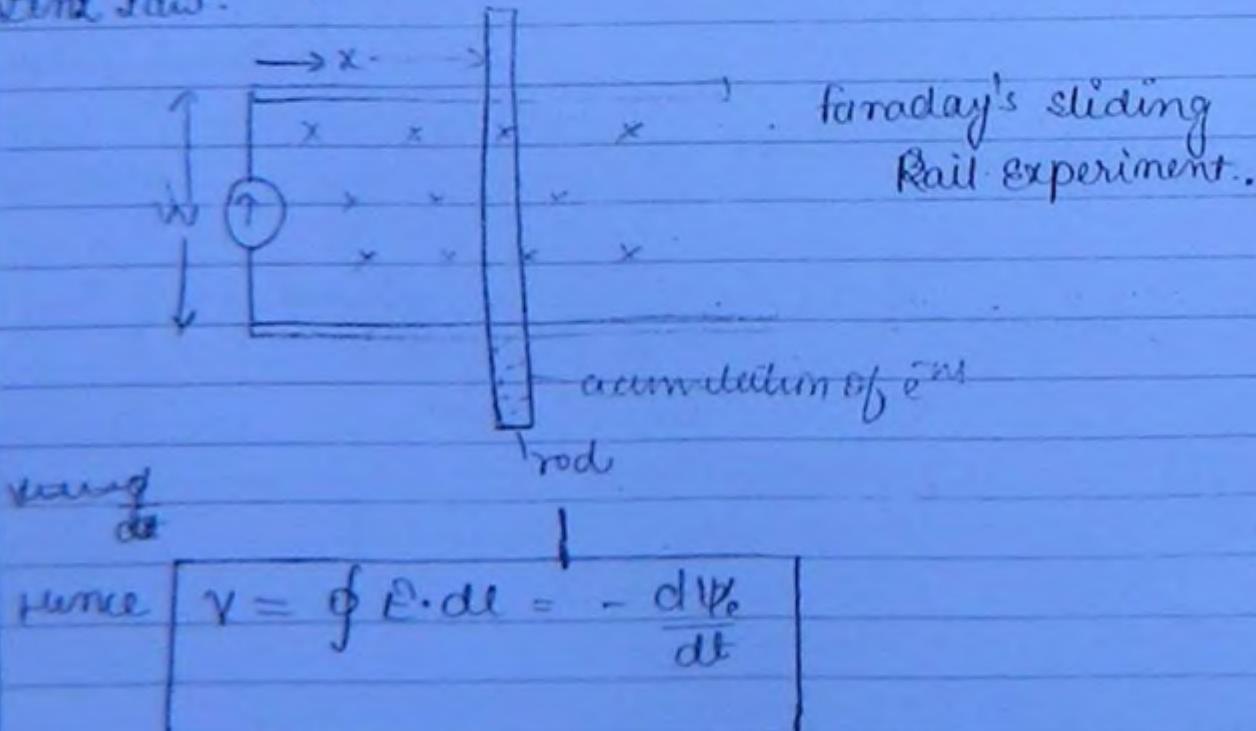
Time varying fields & Maxwell's eqⁿ. 116

- Maxwell's 1st and 3rd eqⁿ i.e. surface integrals are unmodified and are consistent for time varying fields also. However the line integral are modified.

Faraday's Law and Maxwell's II Eqⁿ

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0 \quad \nabla \times \mathbf{E} = 0$$

Faraday's law states that the EMF is produced even in a closed conductor and the magnetic flux through the area of the conductor changes with the time. Faraday voltage opposes the changing flux - this is Lenz law.



$$V = -\frac{d}{dt} (B \cdot A) \quad (F_y = q(V_x B_z))$$

(117)

$$= -B \cdot W \frac{dx}{dt}$$

$$V = -B \cdot W \cdot V_{ds}$$

$$= \oint E \cdot dl = -\frac{d}{dt} \int B \cdot dl$$

$$(\nabla \times E) \cdot dl = \oint E \cdot dl = \int -\frac{\partial B}{\partial t} \cdot dl$$

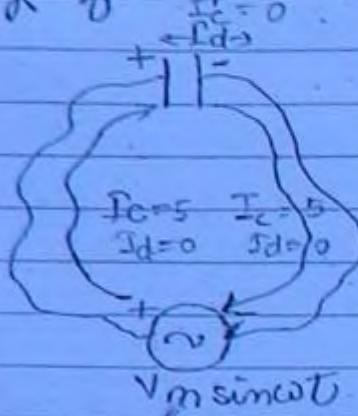
$$\oint E \cdot dl = \int -\frac{\partial B}{\partial t} \cdot ds \quad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

Hence modified

Potential at a point is unique at a time but it can change at various time. Hence the modification

Inconsistency of Ampere's Law / Maxwell's III Eqn.



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F

$$I = \frac{cdv}{dt}$$

$$I = c \cdot w \cdot V_m \sin(\omega t + 90^\circ)$$

$$V_m \sin \omega t = \left(\frac{1}{j\omega c} \right) I$$

$$\{ e^{j90^\circ} = j \}$$

$$V \equiv IX$$

It is clear from the eqn that a current flows in the wire and on the place of the capacitor but there is no current b/w capacitor plates hence ampere's law fails to explain the closed nature of current in the circuit.

Maxwell added a term I_d or J_d which is said to flow b/w the capacitor plates making the circuit closed

Applying continuity Eqn.

$$\nabla \cdot J_d = \frac{\partial \Phi_B}{\partial t}$$

$$\nabla \cdot J_d = \frac{\partial}{\partial t} (\nabla \cdot D)$$

$$\nabla \cdot J_d = \nabla \cdot \frac{\partial D}{\partial t}$$

$$\frac{520}{\sqrt{2}} = 230V$$

$$\oint H \cdot dL = I_c + I_d$$

$$\nabla \times H = J_c + J_d$$

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$$J_d = \int \frac{\partial D}{\partial t} \cdot dL$$

$$J_d = \frac{\partial D}{\partial t}$$

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The term J_d is called as displacement current density and it accepted as a current format but it is not due to a moving electron ~~it~~ is due to a time varying electric flux line.

Physical interpretation of flow in a capacitor:

When a time varying voltage is given to the capacitor the plates are alternately charge and discharge in accordance to the polarity changes. This is itself a continuous process and hence the wires always have a J_c or P_c .

Between the plates of the capacitor there is an electric flux changing due to charge and discharge of the plates. This is a form of current this is J_d or I_d .

Mathematically

$$|J_c| = |J_d|$$

In this example

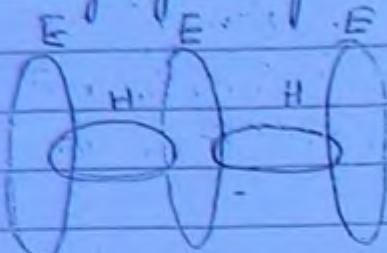
Summary:

$$1. \nabla \times H = \frac{\partial D}{\partial t} + J_c$$

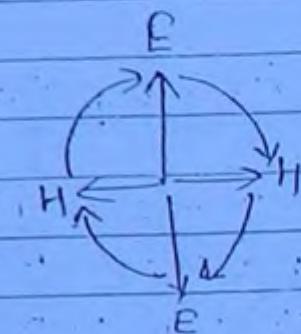
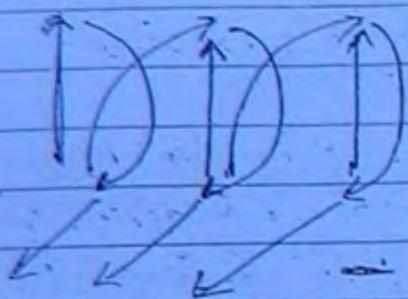
$$= \epsilon \frac{\partial E}{\partial t} + \sigma E \quad (\text{Time varying electric field})$$

$$2. \nabla \times E = - \frac{\partial B}{\partial t} = - \frac{u \partial H}{\partial t}$$

Time varying electric field produces a orthogonal space varying magnetic field, and vice versa



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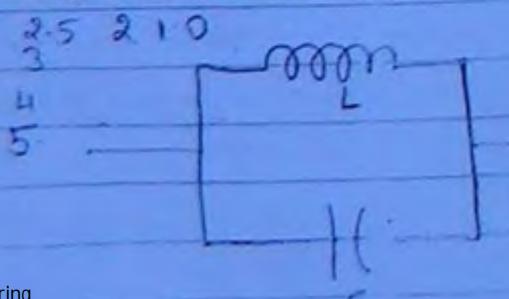


Electric field or accumulation is a form of energy which gives rise to flow, or magnetic field which is also a form of energy. Thus to support each other and hence the energy flows continually like a wave.

For the propagation of energy in E and H format σ , ϵ , and μ called as material constt. or permitting abilities are very crucial

$$\sigma = \frac{v}{m}, \quad \epsilon = \frac{F}{m}, \quad \mu = \frac{H}{m}$$

Ideal oscillator circuit: (Ideal LC circuit as an oscillator)



2.5 3 4 5
2
0

2. Each derivative of a harmonic is back again the same function orthogonally shifted i.e by 90° . such that II^{nd} derivative is back the same function with negation with -ve sign.

(121)

$$A \sin \omega t \xrightarrow{\text{I}^{\text{st} \text{ den}}} \omega A \sin(\omega t + 90^\circ) \xrightarrow{\text{II}^{\text{nd}}} \overline{\omega} A \sin(\omega t + 180^\circ) \xrightarrow{\text{III}^{\text{rd}}} \omega^3 A \sin(\omega t + 270^\circ)$$

$$\text{Acos}\omega t \xrightarrow{\text{I}^{\text{st} \text{ den}}} \omega A \cos(\omega t + 90^\circ) \xrightarrow{\text{II}^{\text{nd}}} \omega^2 A \cos(\omega t + 180^\circ) \xrightarrow{\text{III}^{\text{rd}}} \omega^3 A \cos(\omega t + 270^\circ)$$

$$j\omega t \xrightarrow{\text{Ae}} \omega A e^{j(\omega t + 90^\circ)} \xrightarrow{j(\omega t + 180^\circ)} \omega^2 A e^{j(\omega t + 180^\circ)} \xrightarrow{j(\omega t + 270^\circ)} \omega^3 A e^{j(\omega t + 270^\circ)}$$

$$j = e^{j90^\circ} \quad -1 = e^{j180^\circ} \quad -j = e^{j270^\circ}$$

The discussed ckt excellently obeys the harmonic anal^{ly}
as shown

$$v = -L \frac{dI}{dt}$$

$$I = C \frac{dv}{dt}$$

$$v = -L \frac{d}{dt} \left(C \frac{dv}{dt} \right) \Rightarrow -LC \frac{d^2v}{dt^2}$$

$$\frac{d^2v}{dt^2} = \left(-\frac{1}{LC} \right) v$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

Charging and discharging rate of the capacitor decides the frequency of the oscillation. Hence

$$w = \frac{1}{\sqrt{LC}}$$

(12)

$$\text{obj: } \nabla \times H = \epsilon \frac{\partial E}{\partial t}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

These are called as EM wave eqⁿ. which are second order differential eqⁿ of time and space. Hence EM waves are harmonic of time and space. at the second order derivative in space this is same as second order derivative in time.

$$\nabla \times (\nabla \times H) = \epsilon \frac{\partial}{\partial t} (\nabla \times E)$$

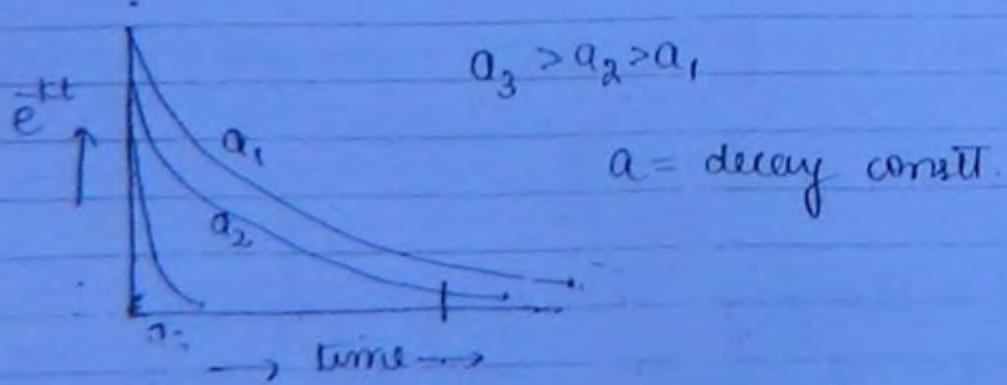
$$-\nabla^2 H = -\mu \epsilon \frac{\partial^2 H}{\partial t^2}$$

$$\boxed{\nabla^2 H = \mu \epsilon \frac{\partial^2 H}{\partial t^2}}$$

$$\boxed{-\nabla^2 E = \mu \epsilon \frac{\partial^2 E}{\partial t^2}}$$

Types of exponential function (e^{-kt} vs t)

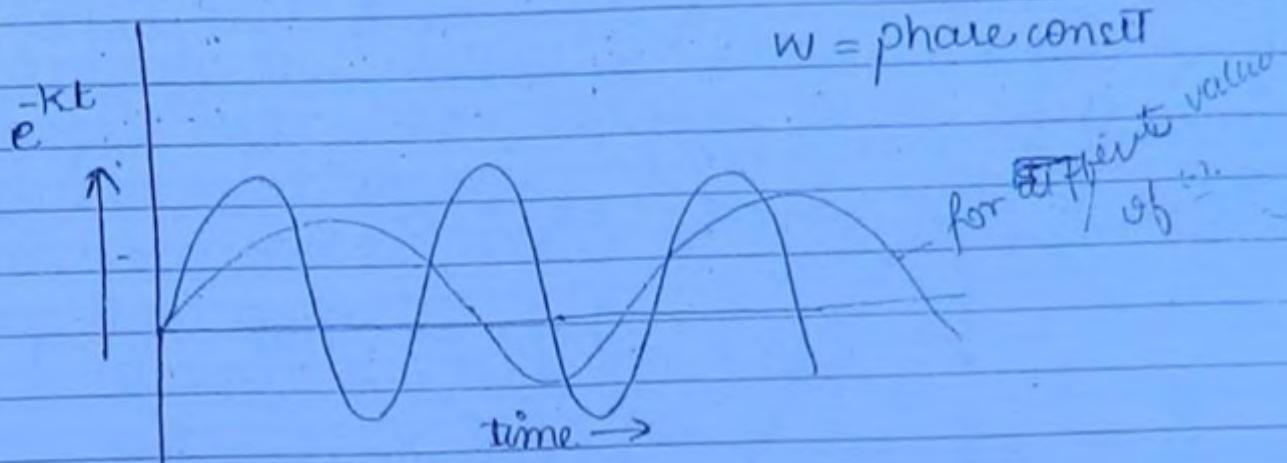
case i) $k = a$ - +ve real no.



case(i) $K = j\omega$ = any imaginary no.

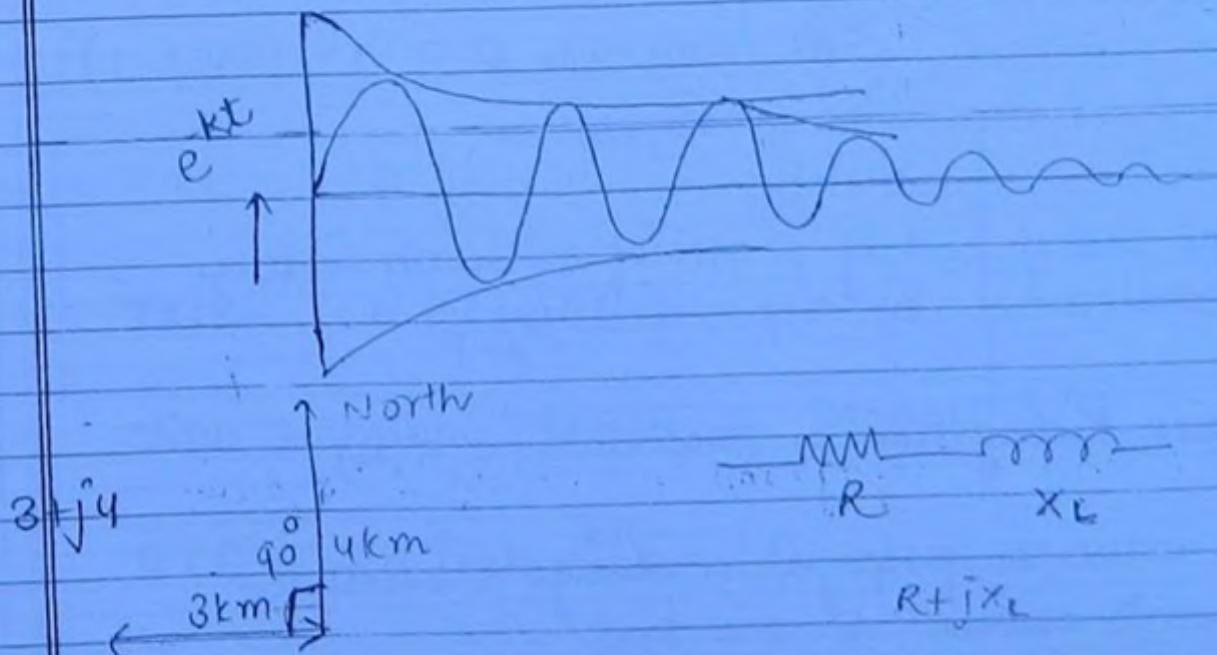
123

ω = phase const



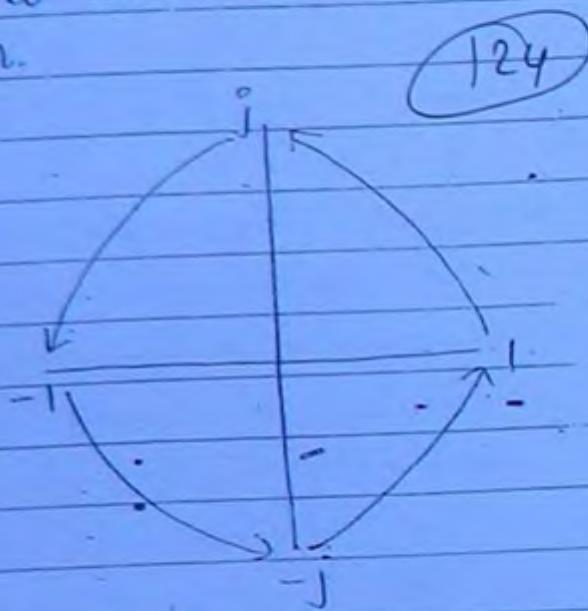
case(ii) $K = a + j\omega$ = any complex no.

$$e^{-kt} = (e^{-at})(e^{j\omega t})$$



$$V = I |Z|$$
$$I (R+jX_L)$$

j always represents shifting factor b/w two independent domains or b/w two orthogonal dimensions of any phenomenon. If a dimension is multiplied by j it will translate to the second orthogonal dimension.



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$$i \times j = 1 | 90^\circ = j$$

$$j \times j = -1 = 1 | 180^\circ$$

$$-1 \times i = -j = 1 | 270^\circ$$

$$-j \times i = 1 | 360^\circ = 1 | 0^\circ$$

e.g. i) North and South journey

ii) Resistance and Reactance in the circuit

If current being out of phase of 90° with voltage then inductance is represented by j

EM wave Propagation in Materials (σ, ϵ, μ)

Every EM wave to propagation always needs a harmonic source at one end.

$$\left. \begin{array}{l} E_s = E_0 e^{j\omega t} \\ H_s = H_0 e^{j\omega t} \end{array} \right\} w = \text{frequency of the source}$$

$$\nabla \times E = -\frac{\partial H}{\partial t}$$

Sound waves are particle waves

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$$\nabla \times H = \frac{\partial E}{\partial t} + \sigma E$$

(125)

✓ 1

As seen in the Maxwell's eqⁿ every EM wave to propagate needs a time harmonic source which is E_s and H_s as shown.

The source can be crystal oscillator, x-ray gun, or any light source

$H_2 - KH_2 \rightarrow$ Audio freq. waves

$MH_2 \rightarrow$ Radio freq. "

$GH_2 - 10^2 \rightarrow$ Microwaves

$10^{15} - 10^{18} \rightarrow$ Light

$10^{20} - 10^{22} \rightarrow$ X-rays, Gamma rays.

$10^{25} \rightarrow$ Cosmic rays.

Put source E_s into Maxwell's eqⁿ.

- q^o phase shift

$$\nabla \times E = -u j \omega H e^{j \omega t} = -j \omega u H \quad (1)$$

$$\nabla \times H = \sigma E + \epsilon j \omega E = (\sigma + j \omega \epsilon) E \quad (2)$$

Time Harmonic format of Maxwell eqⁿ.

$$\nabla \times E = -u j \omega H e^{j \omega t} = -j \omega u H$$

$$H = \frac{\nabla \times E}{-j \omega u}$$

Put H in (2) eqⁿ

$$\nabla \times \left(\frac{\nabla \times E}{-j \omega u} \right) = \sigma E + \epsilon j \omega E$$

When medium is considered as source free medium or charge free medium or Homogeneous then
 $\nabla \cdot E = 0$ or $\nabla \cdot H = 0$ always.

$\nabla^2 E = j\omega\mu(\sigma + j\omega\epsilon)E$ $\nabla^2 H = j\omega\mu(\sigma + j\omega\epsilon)H$	126 } Helmholtz's Eqn
---	--

The two eqns. have second order derivative with space giving the same function again i.e. E and H are Harmonic in space and the eqns are called as Helmholtz's Eqn.

uniform plane wave

Let $j\omega\mu(\sigma + j\omega\epsilon) = \gamma^2$

$\nabla \rightarrow \frac{\partial}{\partial z}$ only.

This called as uniform plane wave assumed to be from a uniform planar source.

Let propagation be assumed in z direction only.

$E \rightarrow a_x$ only.

$H \rightarrow a_y$ only.

$$\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0 \quad \text{--- (3)}$$

$$\frac{\partial^2 H_y}{\partial z^2} - \gamma^2 H_y = 0 \quad \text{--- (4)}$$

Propagation eqns
for a plane wave.

Solution are - ^{right}

$$E(z)_x = (c_1 e^{-\gamma z} + c_2 e^{\gamma z}) a_x$$

$$H(z)_y = (c_3 e^{-\gamma z} + c_4 e^{\gamma z}) a_y$$

(27)

$$\gamma = \sqrt{j\omega\mu_0(\epsilon_r - j\omega\epsilon_0)}$$

The two solns because there are two possibilities of wave travelling in $+z$ direction from the planar source.

Therefore $e^{-\gamma z}$ is the solution of the decaying exponential for +ve z and hence assuming our material to the right of the source we choose this soln.

$$E(t) = e^{j\omega t} \quad E(z) = e^{-\gamma z}$$

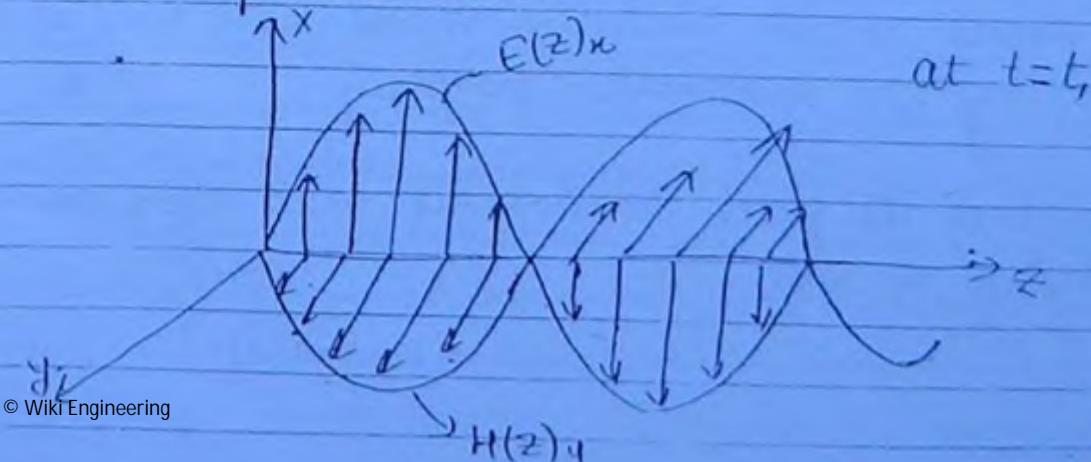
Hence the final wave solution is the product soln of time and space harmonics of E and H .

$$E(z,t)_x = E_0 e^{-\gamma z} e^{j\omega t} a_x$$

final wave solution

$$H(z,t)_y = H_0 e^{-\gamma z} e^{j\omega t} a_y$$

EM wave Representation:



at $t = t_2$ the wave shifts and propagate further.

Propagation const (γ)

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It is a constt of the exponential funⁿ that decides the shape of the exponential funⁿ as x increases. Hence γ decides the course of propagation in the medium.

$$\text{Let } \gamma = \sqrt{j\omega u(\sigma + j\omega e)} = \alpha + j\beta.$$

Let y any two dimensional funⁿ

$$E(z,t)_x = (E_0 e^{-\alpha z}) e^{j(\omega t - \beta z)} \quad \left. \begin{array}{l} \text{amplitude} \\ \text{Harmonic} \end{array} \right\} \text{field due}$$

$$H(z,t)_y = (H_0 e^{-\alpha z}) (e^{j(\omega t - \beta z)})_{ay} \quad \left. \begin{array}{l} \text{phase} \\ \text{Harmonic} \end{array} \right\}$$

- Every electromagnetic (Em) wave have amplitude which exponentially decays at a rate hence it called as attenuation const.

d = attenuation constt per meter

- It has a phase which linearly varies with time and space following the harmonic property.
 - Every wave has its field components with their directions obeying a rule.

$$\frac{E}{\text{direction}} \times \frac{H}{\text{direction}} = \text{Propagation direction}$$

(129)

Intrinsic wave Impedance (η_i)

$$\rightarrow \text{Effect } \gamma = \alpha + j\beta$$

Volt E \longleftrightarrow H current

$$\eta = \frac{E}{H}$$

$$\text{cause} = \eta = R + jX$$

When E is converted into H and H to E the rate of transformation or the slope of transformation is called as η . In the process of transformation there is loss in amplitude and a change in phase. Hence η has resistance and reactance. Hence every medium has η

Resistance \rightarrow attenuation

Reactance \rightarrow phase.

$a_x \ a_y$

$$\nabla \times E = -j\omega \mu H$$

$$\frac{\partial E}{\partial z} = -j\omega \mu H$$

$$-\gamma E = -j\omega \mu H$$

$$\frac{E}{H} = \frac{j\omega \mu}{\sqrt{j\omega \mu (\sigma + j\omega \epsilon)}}$$

$$n = \sqrt{j\omega u} / (\sigma + j\omega t)$$

(130)

W-B

Chapter 2
100 turns — $(t^3 - \alpha t) = \Psi_m$ m nuber

$$V = - \frac{d}{dt} (\Psi_m)$$

$$= - \frac{d}{dt} (100 \times (t^3 - \alpha t) \times 10^{-3})$$

$$V = -0.1 (3t^2 - 2) \Big|_{t=2}$$

$$V = -1 V$$

W-B
B — uniform \rightarrow (decrease linearly)
 ↓ ↓
 with space with time

$$V = - \frac{d}{dt} (BA) = -A(-k) = DC \text{ voltage}$$

decreasing means slope is -ve means dc voltage.

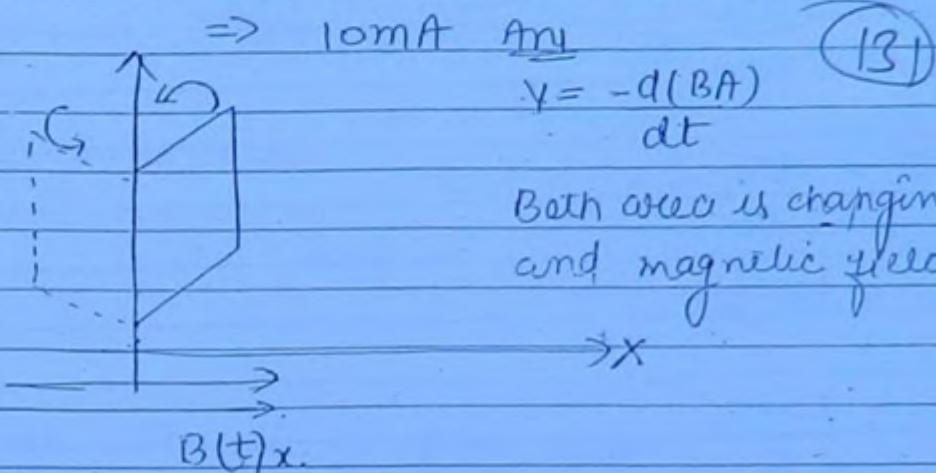
$$\begin{cases} V = I \left(\frac{1}{j\omega c} \right) \\ |I| = \omega c \cdot V \\ = \omega \frac{EA}{d} V \end{cases}$$

$$\begin{cases} I_A = \frac{\partial E}{\partial t} \\ I_d = \frac{\partial E}{\partial t} \cdot A \\ = j\omega E \cdot \frac{EA}{d} \\ = j\omega \frac{EA \cdot V}{d} \end{cases}$$

$$I_c = I_d$$

$$c = 8.71$$

$$I = \frac{W \cdot A \cdot v}{d} \Rightarrow \frac{2 \times \pi \times 3.6 \times 10^9 \times 1}{36 \pi \times 10^9} \times \frac{10^{-4}}{10^{-3}} \times 0.5$$



80 m $\xrightarrow{\text{medium}} 1 \text{ times initial value}$

$$|E(z, t)| = E_0 e^{-\alpha z} = |E(z)|$$

$$z = l = 80$$

$$E(\frac{l}{\alpha}) = \frac{E_0}{e} = 0.37 E_0 \quad \left\{ \alpha = \frac{1}{80} \right\}$$

$$\beta = \frac{2\pi}{\lambda} = \frac{\pi}{80} \Rightarrow \frac{\pi/6}{80} = \frac{\pi}{120} \text{ m}^{-1}$$

$$\gamma = \alpha + j\beta \Rightarrow \frac{1}{80} + j\frac{\pi}{120}$$

$$3/2 \left\{ \frac{2}{\alpha} \left\{ \begin{array}{l} \frac{1}{2} \downarrow \frac{1}{e} \\ \frac{37}{2} \\ \frac{1}{2} \downarrow \frac{1}{e} \end{array} \right\} \frac{1}{e^2} \right\} \frac{1}{e^3}$$

$\frac{13}{2} \downarrow \frac{1}{e}$
 $\cdot 4$

4 W.F.

$$100 \rightarrow 20 \rightarrow 5m$$

$$|E| = E_0 e^{-\alpha z}$$

$$20 = 100 e^{-\alpha(5)}$$

(132)

$$e^{5\alpha} = \frac{100}{20} = 5$$

$$5\alpha = \ln(5)$$

$$\alpha = \frac{\ln(5)}{5}$$

$$40 = 100 e^{-\alpha^2}$$

$$e^{\alpha^2} = \frac{100}{40} = 2.5$$

$$\alpha^2 = \ln\left(\frac{5}{2}\right)$$

$$z = \frac{5 \ln(5/2)}{\ln(5)} = 2.8466$$

Note: If $\left(\frac{P_1}{P_2}\right) = \text{Power Ratio}$

$$10 \log\left(\frac{P_1}{P_2}\right) = \left(\frac{P_1}{P_2}\right) \text{ dB.}$$

$$\ln\left(\frac{P_1}{P_2}\right) = \left(\frac{P_1}{P_2}\right) \text{ Nepers.}$$

Wednesday.

EM wave propagation in free space ($\sigma=0$, $\epsilon=\epsilon_0$, $\mu=\mu_0$)

$$\epsilon_R = \mu_R = 1$$

$$\gamma = \sqrt{j\omega\mu(\sigma+j\omega\epsilon)} = j\omega\sqrt{\mu_0\epsilon_0}$$

(133)

$$\Rightarrow \alpha + j\beta = j\omega\sqrt{\mu_0\epsilon_0}$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

EM waves never attenuate in free space.

They propagate with phase shift only.

They are permitting abilities even for free space for:
E/H Hence propagation.

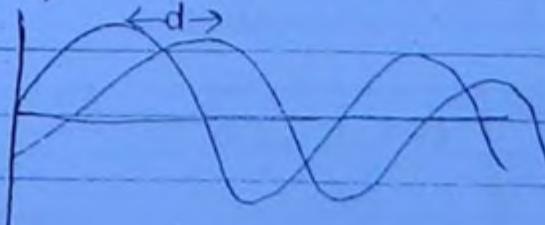
$$n = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \text{real no.}$$
$$= \frac{E}{H}$$

E and H are orthogonal in space i.e. E_x & E_y But are in phase in time

$$n = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{\frac{1}{36\pi \times 10^9}}} = \sqrt{4\pi \times 36\pi \times 10^2} = 120\pi = 377\Omega$$

v_p = Phase velocity

Phase velocity defines the distance in ^{an} phase point move for a given time elapsed.



$$\frac{d}{t} = \frac{\lambda}{T} = \alpha f$$

134

$$\lambda \rightarrow 2\pi \rightarrow T$$

$$V_p = \frac{\omega d}{2\pi T} = \frac{\omega}{\beta}$$

$$V_p = \frac{\omega}{\omega \sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \Rightarrow \frac{1}{\sqrt{4\pi \times 10^{-7} \times 1 / 36\pi \times 10^{-9}}} \Rightarrow 3 \times 10^8 \text{ m/s}$$

Ques (ii) EM wave propagation in Ideal dielectrics / lossless dielectrics / perfect conductors ($\sigma = 0$, $\epsilon = \epsilon_0 \epsilon_R$, $\mu = \mu_0$ ($\epsilon_R \gg 1$))

most dielectrics have $\epsilon_R = 1$

It is rare to have a magnetic with dielectric

$$\gamma = \sqrt{j\omega \mu (\sigma + j\omega \epsilon)} = j\omega \sqrt{\mu_0 \epsilon_0 \epsilon_R}$$

$$\Rightarrow \alpha + j\beta = j\omega \sqrt{\mu_0 \epsilon_0 \epsilon_R}$$

$$\sigma = 0$$

$$\eta = \sqrt{\frac{j\omega \mu}{\sigma + j\omega \epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_R}} = \frac{1}{\sqrt{\epsilon_R}} \text{ no.}$$

$$= \frac{B}{H}$$

$$\eta = \sqrt{\frac{4 \times 10^{-7}}{\frac{1}{36\pi \times 10^{-9}}}} = \sqrt{\frac{4\pi \times 36\pi \times 10^{-2}}{\sqrt{\epsilon_R}}}$$

$$= \frac{120\pi}{\sqrt{\epsilon_r}} = \frac{377\Omega}{\sqrt{\epsilon_r}}$$

135

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega \sqrt{\mu_0 \epsilon_0} \epsilon_r}$$

chapter - 2.

W.B

$$\mathbf{J}_c = \nabla \times \mathbf{H}$$

$$\mathbf{J}_d = \epsilon \frac{\partial \mathbf{E}}{\partial t}$$

Magnetic current is due to \mathbf{J}_b = Bound current density.

It exists whenever a material is magnetized. $\left\{ \begin{array}{l} \mathbf{J}_b = \text{Bound current density.} \\ \text{- Ferromagnetic} \\ \text{- spinning/revolving electrons} \end{array} \right.$

W.B (i) Propagation direction

$$\sin(\omega t - \beta z) \quad e^{-kz}$$

$$e^{j(\omega t - \beta z)}$$

+z

$$\sin(\omega t + 4x)$$



x direction ($-a_x$)

(ii) $\beta = 4$ as $E(x, t) = 25 \sin(\omega t + 4x) a_y$

$$\beta = 4 = 2\lambda = \frac{\lambda}{\alpha}$$

$$\lambda = \frac{\pi}{2}$$

(136)

(iii) $f = \frac{3 \times 10^8}{\lambda} = 600 \text{ MHz}$ ($f = \frac{c}{\lambda}$)

F

(iv) $H(x,t) \Rightarrow \nabla \times E = -\mu \frac{\partial H}{\partial t}$

$$H = -\frac{1}{\mu} \int \nabla \times E dt$$

$$H(x,t) = \left(\frac{25}{180\pi} \right) \sin(\omega t + 4x) (-a_z)$$

↓ ↓ ↓
① ② ③

$$a_y \times (2) = (-a_x)$$

$$-a_z$$

for a given field component of a wave the other field component is calculated with three rules.

i) Amplitude should obey the rule

$$\frac{E}{H} = \eta$$

i.e. $\frac{|E|}{|H|} = |\eta|$

ii) The phase should be the same in the same harmonic with free space or ideal dielectric

$$18 \text{ W.B} \quad \eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \times 2}{\epsilon_0 \times 8}} = \frac{120\pi}{2} = 188.4$$

(137)

22 W.B

$H(2, t)$

$$\frac{E}{H} = 120\pi$$

$$E(z, t) = (3.77) \cos((4 \times 10^7 t - \beta z) \text{ rad}) \quad E = 120\pi \times 10$$

21 W.B

λ — free space — 2 cm

λ — dielectric — 1 cm

$$\epsilon_R = ?$$

$$\beta = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_R} = \frac{2\pi}{\lambda}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{\sqrt{\epsilon_0 \epsilon_R}}{\sqrt{\epsilon_0}}$$

$$\lambda \times \frac{1}{\sqrt{\epsilon_R}}$$

$$\frac{2}{1} = \sqrt{\epsilon_R}$$

$$\epsilon_R = 4.$$

$$\frac{\lambda_1}{\lambda_2} = \frac{2}{1} = \sqrt{\frac{\epsilon_0 \epsilon_R}{\epsilon_0}}$$

$$\boxed{\epsilon_R = 4.}$$

22 W.B

$$\vec{E}_i = 20 e^{-j5z} \hat{a}_x - 25 e^{-j5z} \hat{a}_y \text{ V/m}$$

$$H = \frac{20}{120\pi} e^{-j5z} \hat{a}_y + \frac{25}{120\pi} e^{-j5z} (-\hat{a}_x)$$

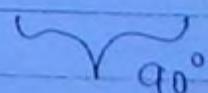
$$H = \frac{20}{120\pi} e^{-j5z} \hat{a}_y + \frac{25}{120\pi} e^{-j5z} (\hat{a}_x)$$

24-W.B

$$360^\circ \longrightarrow \lambda$$

$$2\pi \longrightarrow \lambda$$

3 mm — thickness



$$\frac{\lambda}{4} = 3\text{mm}$$

$$d = 12\text{mm}$$

(138)

$$\frac{\lambda}{4} = \frac{2\pi}{2}$$

$$\Delta f = V_p = \frac{3 \times 10^8}{\sqrt{\epsilon_R}}$$

$$\sqrt{\epsilon_R} = \frac{3 \times 10^8}{(2 \times 10^{-3} \times 10 \times 10^9)}$$

$$\epsilon_R = 6.25$$

$$E = A \cos(\omega t - \frac{\omega z}{c}) a_y$$

$$H = \frac{A}{\sqrt{\frac{\mu_0}{\epsilon_0}}} \cos(\omega t - \beta z) a_y a_y$$

$$c = 3 \times 10^8$$

$$H = \frac{A}{\sqrt{\frac{\mu_0}{\epsilon_0}}} \cos(\omega t - \beta z) (-a_x)$$

$$H = + \frac{A}{\sqrt{\frac{\mu_0}{\epsilon_0}}} \cos(\omega t - \beta z) (-a_x)$$

$$H = - \frac{A}{\sqrt{\frac{\mu_0}{\epsilon_0}}} \cos(\omega t - \beta z) a_x$$

$$H = -j A \sqrt{\frac{\mu_0}{\epsilon_0}} \sin(\omega t - \frac{\beta z}{c}) a_x$$

$$\gamma = \text{complex} \rightarrow \gamma = \alpha + j\beta.$$

$$\gamma = j\beta.$$

$$\nabla^2 E - \gamma^2 E = 0. \quad (139)$$

$\{\alpha = 0$ for free space?

Imaginary $\gamma = j\beta$ free space

$$\begin{aligned} \nabla^2 E + \beta^2 E &= 0 \\ \nabla^2 E + k^2 E &= 0 \end{aligned} \quad \beta = k$$

$$E(z) = E_0 e^{j\beta z} \quad (c)$$

$$E(x, z, t)_y = 25 \sin(\omega t - 3x + 4z) ay$$

Propagation $\rightarrow (3a_x - 4a_z)$

$$H(x, z, t) = \frac{25}{120\pi} \sin(\omega t - 3x + 4z) \left(\frac{4a_x + 3a_z}{5} \right)$$

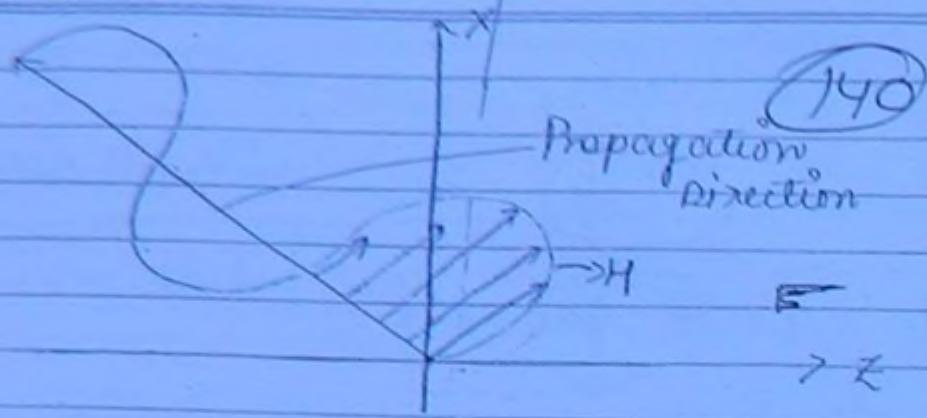
$$ay x, (?) = 3a_x - 4a_z$$

$$\sqrt{9+16} = \sqrt{25} = 5$$

$$H \rightarrow 3a_z + 4a_x$$

direction should always be a unit factor

$$H(x, z, t) = \frac{5}{120\pi} \sin(\omega t - 3x + 4z) (4a_x + 3a_z)$$



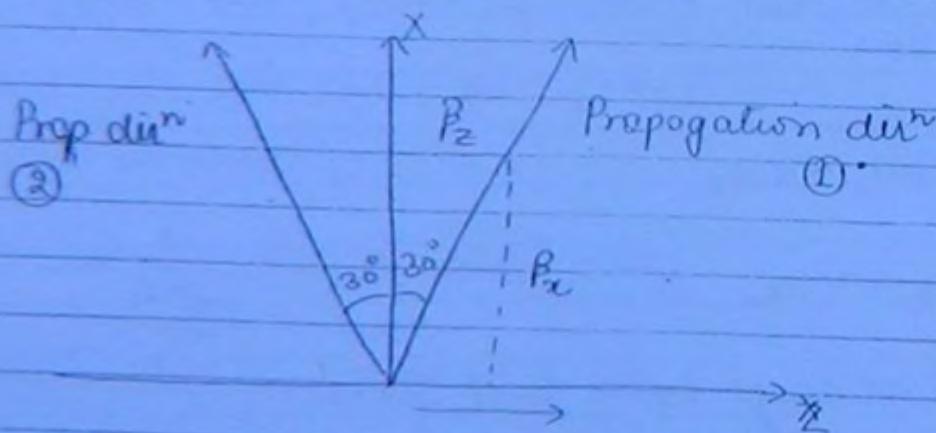
H.L.P.E.L.H

$$E(x, z, t)_y$$

- $H(x, z, t)_{(x, z)}$ is notation used to represent this wave and satisfies $E \perp H \perp P.E.$

Extention of
the question

$$\beta = \sqrt{\beta_x^2 + \beta_z^2} = 5$$



$$\left. \begin{array}{l} E(x, z, t) \\ H(x, z, t) \end{array} \right\} \tan 30^\circ = \frac{\beta_z}{\beta_x} = \frac{1}{\sqrt{3}}$$

$$\beta_x = \sqrt{3} \beta_z$$

$$e^{j(\omega t - \beta_x x - \beta_z z)}$$

$$\beta_{xz}$$

$$e^{j(\omega t - \sqrt{3}\beta_x x - \beta_z z)}$$

Conclusion from Problems:

Material has properties

σ, ϵ, μ

$$\sigma = \text{S/m}$$

$\epsilon, \mu = \text{Reactive}$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{(j\omega \mu)(\sigma+j\omega \epsilon)}$$

$$\eta = \sqrt{\frac{j\omega \mu}{\sigma+j\omega \epsilon}}$$

$$\gamma \cdot \eta = j\omega \mu$$

$$u = |\gamma \cdot \eta|$$

$$\gamma/\eta = \sigma + j\omega \epsilon$$

$$\sigma = \text{Real} [\gamma/\eta]$$

$$\epsilon = \text{Imag} [\gamma/\eta]$$

$$2. \quad \eta = \frac{E_x}{H_y} = \sqrt{\frac{j\omega \mu}{\sigma+j\omega \epsilon}} = |\eta|/|\theta| = \text{complex}$$

E_x & H_y are always orthogonal in space but have phase shift in time in general materials (when η is complex).

$$3. \gamma = \alpha + j\beta = \sqrt{j\omega u(\sigma + j\omega e)}$$

142

Square on both the sides
Imaginary Imag.

$$\underbrace{\alpha^2 - \beta^2}_{\text{Real}} + j2\alpha\beta = j\omega u\sigma - \underbrace{\omega^2 ue}_{\text{Real}}$$

Real

$$\alpha = w \sqrt{\frac{ue}{2} \left(\sqrt{1 + \left(\frac{\sigma}{we} \right)^2} - 1 \right)}$$

$$\beta = w \sqrt{\frac{ue}{2} \left(\sqrt{1 + \left(\frac{\sigma}{we} \right)^2} + 1 \right)}$$

For tangent and dissipation factor $\left[\frac{\sigma}{we} \right]$

In the value of γ is completely dependent on the term σ If it is quite large then α is very

large and hence attenuation is very high

good conductor $\left\{ \frac{\sigma}{we} \gg 1 \right. \Rightarrow \alpha \uparrow \beta \uparrow \Rightarrow \text{Propagation is difficult}$
 $\left. \text{Attenuation is high} \right\}$

The term is quite large for a good conductor because

$$\sigma \gg we$$

Hence electromagnetic (EM) waves cannot travel inside good conductor

e.g. $\frac{\sigma}{we} \gg 1$ for earth only when w upto MHz

$$\boxed{w \leq 10^6}$$

W

$$\sigma = 10^6$$

$$\omega = 10^6$$

$$\epsilon = 10^{-12}$$

$$\frac{\sigma}{\omega\epsilon} = \frac{10^6}{10^6 \times 10^{-12}} \gg 1$$

143

Whether an object is a good conductor or not depends on σ as well as frequency of the wave.

e.g. $\frac{\sigma}{\omega\epsilon} \gg 1$ for earth only when ω (freq.) upto MHz only.

Earth is a good conductor for low frequency audio and radio (AF/RF) freq.

e.g. $\frac{\sigma}{\omega\epsilon} \gg 1$ for Human Body upto High frequency. only.

$\frac{\sigma}{\omega\epsilon}$ is called as loss tangent and dissipation factor

$\frac{\sigma}{\omega\epsilon}$ is unitless because it is $\left| \frac{J_c}{J_d} \right|$

$$\left| \frac{J_c}{J_d} \right| = \frac{\sigma E}{\epsilon \frac{dE}{dt}} = \frac{\sigma E}{\epsilon \cdot j\omega E} = \left| \frac{\sigma}{j\omega\epsilon} \right| = \frac{\sigma}{\omega\epsilon}$$

$J_c \gg J_d$ very good conductor

case (iii) EM wave propagation in good conductors

$$\gamma = \sqrt{j\omega\epsilon(\sigma + j\omega\epsilon)}$$

$$\sigma \gg \omega\epsilon$$

$$= \sqrt{j\omega\epsilon\sigma}$$

$$= \sqrt{\omega u \sigma} [90^\circ] = \sqrt{\omega u \sigma} [45^\circ]$$

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$$\cos 45^\circ + j \sin 45^\circ = \frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}}$$

$$V = \sqrt{\frac{\omega u \sigma}{2}} + j \sqrt{\frac{\omega u \sigma}{2}} = X + j \beta$$

$$\alpha = \beta = \sqrt{\frac{\omega u \sigma}{2}}$$

$$\eta = \sqrt{\frac{j \omega u}{\sigma + j \omega \epsilon}}$$

$$\eta = \sqrt{\frac{j \omega u}{\sigma}}$$

$$h = \sqrt{\frac{\omega u}{\sigma}} [90^\circ] \Rightarrow \sqrt{\frac{\omega u}{\sigma}} [45^\circ]$$

$$\eta = \sqrt{\frac{\omega u}{2\sigma}} + j \sqrt{\frac{\omega u}{2\sigma}}$$

$$R = X = \sqrt{\frac{\omega u}{2\sigma}}$$

Ques - 1 If the resistance is equal to the reactance the rate of Amplitude loss is equal to the rate of phase change

$$\begin{cases} \chi = \alpha + j \beta \\ \eta = R + j X \end{cases}$$

$$\eta's \text{ phase} = 45^\circ$$

E_x / H_y are out of phase by 45° in conductor

Depth of Penetration or Skin depth:

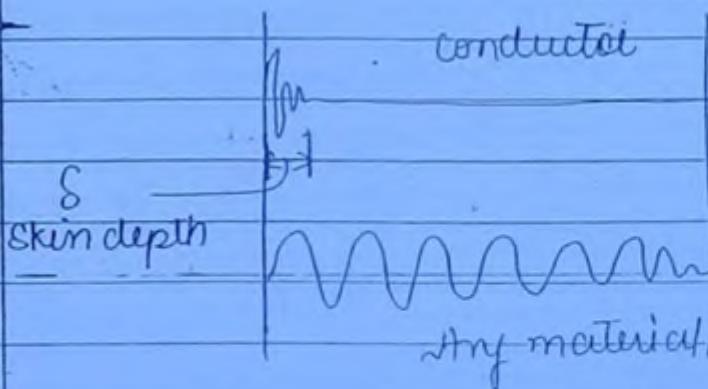
(145)

An exponentially decaying wave never reaches exact zero value but is approximately small after travelling $\frac{1}{\alpha}$ distance. The $\frac{1}{\alpha}$ distance is called as skin depth. Hence skin depth is defined as.

A distance travelled by the wave where the amplitude becomes $\frac{1}{e}$ times i.e. 37% of initial value.

$$\text{skin depth } \delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega \mu \sigma}}$$

100
 \downarrow
 $\frac{1}{e}$
37



$$\delta = \sqrt{\frac{2\sigma}{\omega \mu \sigma^2}} = \frac{1}{\sigma} \sqrt{\frac{2\sigma}{\omega \mu}} = \frac{1}{\sigma R} = f$$

$R = R_\delta = \text{Skin Resistance}$

$v_p = \text{phase velocity}$
free space

$$\frac{v}{\omega t} = 0$$

$\frac{3 \times 10^8}{\sqrt{\epsilon_r}}$ → Dielectric

$10^5 - 10^6$ → Material

few m/s → conductor (weak)

$$V_p = \frac{\omega}{\beta} = \omega \sqrt{\frac{\mu_e}{2} \left(1 + \left(\frac{\sigma}{\omega e} \right)^2 + 1 \right)} \quad (146)$$

$$V_p = \frac{1}{\sqrt{\frac{\mu_e}{2} \left(1 + \left(\frac{\sigma}{\omega e} \right)^2 + 1 \right)}}.$$

Repeat the same question for the same data if the freq. is increased by 4 times.

$$\delta = 4 \text{ mm}$$

frequency = 800 kHz,
stairneans freq. = 4 times

$$V_p = \omega \delta$$

$$V_p' = \omega' \delta'$$

$$= 4\omega \frac{\delta}{2}$$

$$V_p' = 2V_p = 10 \text{ m/s}$$

$$\omega = 2\pi f$$

$$N' = 4N$$

$$\delta' \propto \frac{1}{\sqrt{f}} = \frac{1}{\sqrt{4}} = \frac{1}{2} \delta$$

$$\delta' = \frac{1}{2} \delta$$

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$

$$\delta' = \frac{\delta}{2}$$

$$1.73 = \sqrt{3} = \frac{\sigma}{\omega e}$$

$$\frac{\sigma}{\omega e} \gg 1$$

E/H - phase - ??

$$\eta' \text{ phase} = \frac{j\omega u}{\sigma + j\omega e} = \sqrt{\frac{\omega u / 90^\circ}{K \tan^{-1} \left(\frac{\omega e}{\sigma} \right)}}$$

$$= \sqrt{K' \left[90^\circ - \tan^{-1} \left(\frac{\omega e}{\sigma} \right) \right]}$$

$$= 90^\circ - \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = \frac{90^\circ - 30^\circ}{2} = 30^\circ$$

Phase diff b/w E and H

147

0° — free space

$$\frac{\sigma}{\omega \epsilon} = 0$$

0° — Dielectrics.



22½ - 30° — Materials

45° — conductors

Note: The maximum phase difference b/w E and H is 45° which is in conductors.

$$29 \text{ vB} \quad I_c = I_d \quad \Gamma_c = \Gamma_d$$

$$\frac{I_c}{I_d} = \frac{\sigma}{\omega \epsilon} = 1$$

$$\frac{\sigma}{2\pi f \epsilon} = 1 \Rightarrow f = \frac{\sigma}{2\pi \epsilon} \Rightarrow \frac{10^2}{2\pi \epsilon \epsilon_0} = \frac{10^2}{2\pi \times 1 \times 4 \times 10^{-9}} \\ \Rightarrow 4.5 \times 10^7 = 45 \text{ MHz} \quad \underline{\text{any}}$$

$$36 \pi \times 10^9$$

$$30 \text{ vB} \quad \delta = \sqrt{\frac{2}{\omega \mu_0}} \Rightarrow \sqrt{\frac{2}{2\pi f \mu_0}}$$

$$\delta_1 = \sqrt{\frac{1}{f_1}}$$

$$\delta_2 = \sqrt{\frac{1}{f_2}}$$

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{f_2}{f_1}} \quad \text{or} \quad \frac{\delta_2}{\delta_1} = \sqrt{\frac{f_1}{f_2}} \Rightarrow \delta_2 = 25 \sqrt{\frac{10^6}{4 \times 10^6}} = \frac{25}{2} \\ = 12.5 \text{ cm}$$

$$I_c = \frac{f_c}{I_d} = \omega$$

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$$I_d = \frac{\omega E}{\sigma \times I_c} = \frac{2\pi \times 50 \times 1}{36\pi \times 10^9}$$

58

$$I_d = 4.8 \times 10^{-11} A$$

Amp

Power, Power Density, Poynting vector
 $E/H \rightarrow$ Energy formats
 \rightarrow Time
 \rightarrow Power EH wave

$$E \times H = \frac{\text{volt}}{\text{m}} \times \frac{\text{amp}}{\text{m}} = \frac{\text{watts}}{\text{m}^2}$$

$$\nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t}$$

$$(\nabla \times H) \cdot E = \sigma E^2 + \epsilon \frac{\partial E}{\partial t} \cdot E$$

$$H \cdot (\nabla \times E) - \nabla \cdot (E \times H) = \sigma E^2 + \epsilon \frac{\partial E}{\partial t} \cdot E$$

$$-\nabla \cdot (E \times H) - \sigma E^2 = \epsilon \frac{\partial E}{\partial t} \cdot E + \frac{1}{\mu} \frac{\partial H}{\partial t} \cdot H$$

$$-\nabla \cdot (E \times H) - \sigma E^2 = \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right)$$

$$\left\{ \therefore \chi d\chi = d\left(\frac{x^2}{a}\right) \right\}$$

$$\int -\nabla \cdot (E \times H) dv - \int \sigma E^2 dv = \int \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right) dv$$

$$\oint (E \times H) \cdot ds + \int \sigma E^2 dv = - \underbrace{\text{Energy}}_{\substack{\text{given} \\ \text{to system}}} \underbrace{\text{unit time}}$$

$$\frac{320}{\sqrt{2}} \text{ Vrms}$$

$$320 \sin(100\pi t) \rightarrow \text{Instantaneous}$$

230 V, 50 Hz

Date _____
Page _____

$$\oint (E \times H) \cdot d\ell + \int \sigma E^2 dv = - \text{Power crossing the closed surface}$$

(149)

The right hand side is a power term that is crossing the surface enclosing the EM wave source. Hence

$E \times H$ = power crossing per unit area.

= power density = strength of the power at any instance in the EM wave.

It is called the Poynting vector of the EM wave which indicates the power

$$E(z, t)_x \times H(z, t)_y = P(z, t)_z$$

Power is carried by the EM wave in the propagation direction due to both the energy formats. The EM wave always have a finite power with it.

$$P(z, t)_z = (E_0 e^{-\alpha z}) e^{j(\omega t - \beta z)} (H_0 e^{-\alpha z}) e^{j(\omega t - \beta z)}$$

= Instantaneous Poynting Vector

= Power density at an instance of time at an instance of (point) of space

Instantaneous never useful we always want an avg.

Time Averaged Poynting Vector

$$P(z)_{\text{avg}} = \frac{(E_0 e^{-\alpha z})}{\sqrt{2}}$$

The average of any harmonic is never its radional transidental b/c which always gives a zero value. It is called as R.M.S which is amplitude by $\sqrt{2}$ for any harmonic

Similarly for the harmonic power density the time average operation gives. $P(z)_{avg}$

$$P(z)_{avg} = E(\text{RMS}) \times H(\text{RMS}) \quad (156)$$

$$P(z)_{avg} = \left(\frac{E_0 e^{-\alpha z}}{\sqrt{2}} \right) \cdot \left(\frac{H_0 e^{-\alpha z}}{\sqrt{2}} \right) = \frac{1}{2} E_0 H_0 e^{-2\alpha z}$$

$$V_{avg} = \frac{1}{T} \int_0^T v(t) dt$$

$$V_{avg} = \sqrt{\frac{1}{T} \int_0^T V^2(t) dt} = \frac{V_{max}}{\sqrt{2}}$$

Average power of the wave exponentially decays at a rate

$$\begin{aligned} P(z)_{avg} &= \frac{1}{2} \eta E_0^2 e^{-2\alpha z} \\ &= \frac{1}{2} \eta H_0^2 e^{-2\alpha z} \end{aligned} \quad \left. \begin{array}{l} \frac{1}{2} (ExH^*) \\ \end{array} \right\}$$

As the power decays in the wave the medium acquire this power in the form of ohmic power. Hence

$$\sigma^2 E = J \cdot E$$

is the ohmic power dissipated to the medium per unit volume

The time average pointing vector can also be represented as

$$P(z)_{avg} = \frac{1}{2} ExH^*$$

where a conjugate operation means the phase and

the harmonic are cancelled and only amplitudes are used in power calculation

$\{ \therefore V^2(t) \text{ means } \rightarrow \text{conjugate} \}$

(157)

In free space when $\alpha = 0$

$$P_{\text{avg}} = \frac{1}{2} \frac{E_0^2}{\eta} = \frac{1}{2} \eta H_0^2 = \frac{1}{2} E_0 H_0$$

vB- Total Power = $\left(\frac{\text{Initial Power} - \text{final Power}}{\text{Density}} \right) \times \text{Area}$

$$= \left(\frac{1}{2} \times \frac{200 \times 200}{80} - \frac{1}{2} \times \left(\frac{200 \times e^{-0.4 \times 0.6}}{\eta} \right)^2 \right) \times \text{Area}.$$

$$= \left(\frac{1}{2} \times \frac{(200)^2}{80} - \frac{1}{2} \left(\frac{200 \times e^{-0.4 \times 0.6}}{\eta} \right)^2 \right) \times 4 \times 10^4$$

$$= 0.0381$$

formula = $P(z)_{\text{avg}} = \frac{1}{2} \frac{E_0^2}{\eta} e^{-2\alpha z}$

for initialaly $z = 0$ and for final $z = 60 \text{ cm}$

if $\sigma = 0.8$ and η = unknown

$$\text{Total Power} = \int_{\text{ext}} (\sigma E^2) dV \quad (162)$$

$$= \int \sigma (E_0 e^{-kz})^2 dA dz$$

$$= \sigma \cdot E_0^2 A \int_{z=0}^{0.6} e^{-2kz} dz$$

$$= 0.8 \times (200)^2 \times (4 \times 10^{-4}) \left[e^{-2 \times 0.4 \times 0.6} - 1 \right]$$

$$= -4.8795$$

$$(i) H = 50 \sin(\omega t - \beta z) a_x + 150 \sin(\omega t - \beta z) a_y$$

$$= 50 \sin(\omega t - \beta z) (a_x + 3a_y)$$

$$= 50\sqrt{10} \sin(\omega t - \beta z) \left(\frac{a_x + 3a_y}{\sqrt{10}} \right)$$

$$P_{avg} = \frac{1}{2} \eta H_0^2$$

$$\eta = 120\pi$$

$$P_{avg} = \frac{1}{2} (50\sqrt{10})^2 \times 120\pi$$

$$= \frac{1}{2} (2500 \times 10) \times 120\pi$$

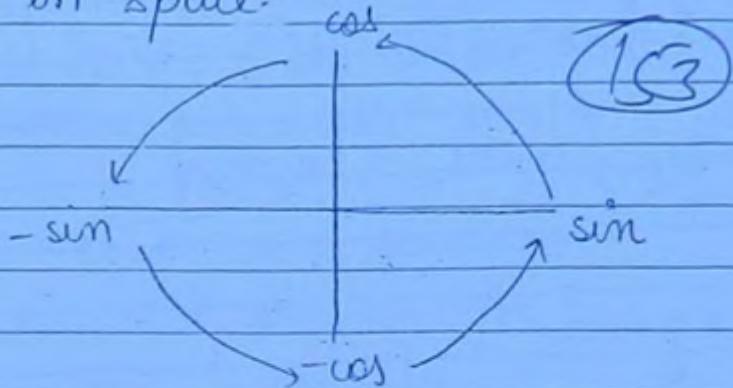
$$= 4.71 \times 10^6$$

$$H = 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_y$$

$$= 50 \sin(\omega t - \beta z) a_x [1 + 3j]$$

$$= 50(1 + 3j) \sin(\omega t - \beta z) a_x$$

Orthogonality in time has the same effect as the orthogonality in space.



$$H_0 = 50\sqrt{10}$$

$$P_{avg} = \frac{1}{2} \times 120\pi \times (50\sqrt{10})^2 = 4.71 \times 10^6$$

$$ii) H = 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_y$$

$$H = 50 \sin(\omega t - \beta z) (a_x + j3a_y)$$

$$\begin{aligned} E \times a_z &= a_z \\ \times a_y &= a_z \end{aligned}$$

$$P_{avg} = \frac{1}{2} (E \times H^*)$$

$$E(z,t) = (50 \times \eta) \sin(\omega t - \beta z) (-a_y)$$

$$\left\{ \because E \times a_x = a_z \right\}$$

$$+ (150 \times \eta) \cos(\omega t - \beta z) (a_z)$$

$$\left\{ \because E \times a_y = a_x \right\}$$

$$E(z,t) = (50 \times \eta) \sin(\omega t - \beta z) (3j a_x - a_y)$$

$$H(z,t) = 50 \sin(\omega t - \beta z) (a_x + j3a_y)$$

$$P_{avg} = \frac{1}{2} (E_x H^*)$$

(754)

$$P_{avg} = \frac{1}{2} [50\eta (3j\alpha_z - ay) \times 50 (\alpha_z - 3jay)]$$

$$P_{avg} = \frac{1}{2} [50 \times 50 \eta [0 + \alpha_z + 9\alpha_z + 0]]$$

$$P_{avg} = \frac{1}{2} \times 50 \times 50 \times \eta \times 10$$

$$\vec{E} = (\alpha_z + jay) e^{jkz - j\omega t}$$

$$\vec{H} = \left(\frac{k}{\omega u}\right) (ay + j\alpha_z) e^{jkz - j\omega t}$$

$$E = (\alpha_z + jay) \quad H = \left(\frac{k}{\omega u}\right) (ay + j\alpha_z)$$

$$\frac{1}{2} (E_x H^*) = \frac{k}{2\omega u} (\alpha_z + jay) \times (ay - j\alpha_z)$$

$$= \frac{k}{2\omega u} [\alpha_z + 0 + 0 - \alpha_z] = 0$$

= null vector.

$$\text{Average power crossing} = \text{Density} \times \text{Area}$$

$$= \frac{1}{2} (E_0 H_0) \times \text{Area}$$

$$= \frac{1}{2} \left(\frac{50 \times 5 \pi}{12 \times 10000} \right) \times \left(2 (\sqrt{2})^2 \right)^2$$

$$= 250 \text{ watt}$$

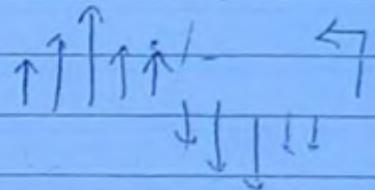
Wave Polarization

(155)

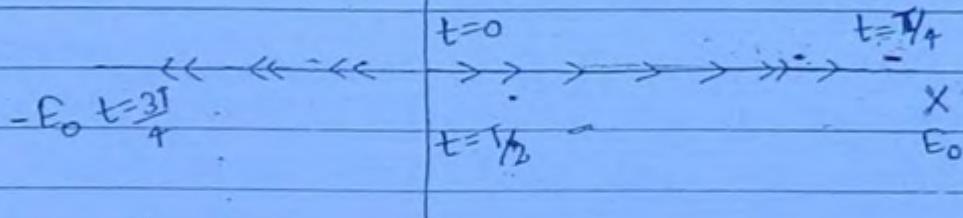
It defines the orientation of the E field in the plane wave. It is the study of the possible relative orientation of the planar components of the E -field.

case(i) Wave travel in $+z$ direction

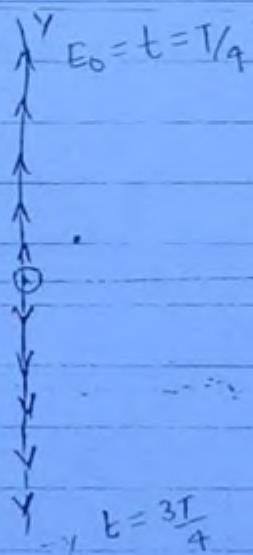
$$E(z,t)_x = E_0 \sin(\omega t - \beta z) a_x$$



y



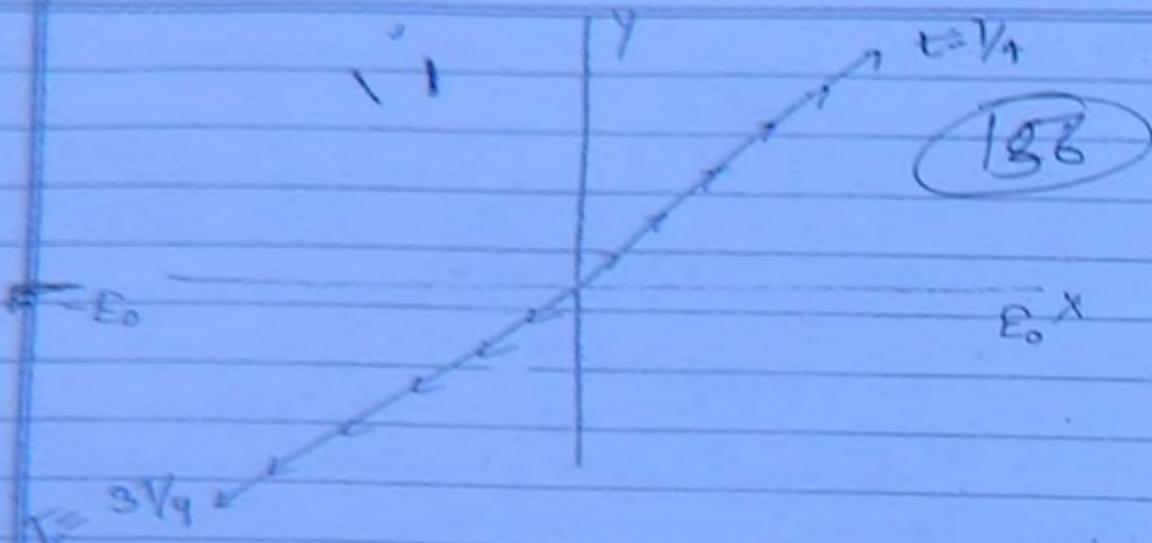
Linear Horizontal Polarization



Linear Vertical Polarization

case(ii)

$$E(z,t) = E_x \sin(\omega t - \beta z) a_x + E_y \sin(\omega t - \beta z) a_y$$



Summary for linear Polarization

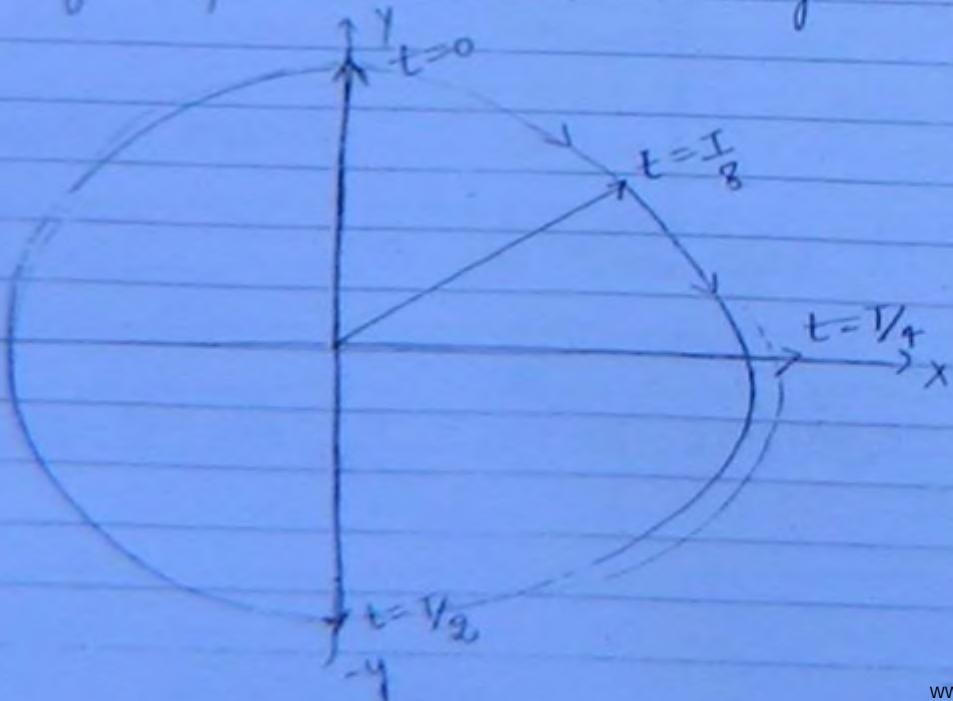
If the wave has single E field component or two planar component both in phase the wave is said to be linearly polarized.

(iv) The wave travel in +z direction

$$E(z, t) = E_0 \sin(\omega t - \beta z) a_x + E_0 \cos(\omega t - \beta z) a_y$$

latus

Linear Polarization can be defined - by tracing E field on the (z=0) surface for various advancing time.



$$t=0 \rightarrow E = E_y, E_x = 0$$

$$t=\frac{T}{8} \rightarrow E_x = \frac{E_0}{\sqrt{2}}, E_y = \frac{E_0}{\sqrt{2}}$$

$$t=\frac{T}{4} \rightarrow E = E_x, E_y = 0$$

$$t=\frac{T}{2} \rightarrow E = E_y, E_x = 0$$

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**Summary.**

If the two planar component are out of phase by 90° and have equal amplitude the wave is circularly polarized.

Sense of Rotation - Left or Right

If the left hand thumb direct towards propagation direction and closed fingers along advancing time the wave is left circularly polarized.

Point? clockwise - time advancement

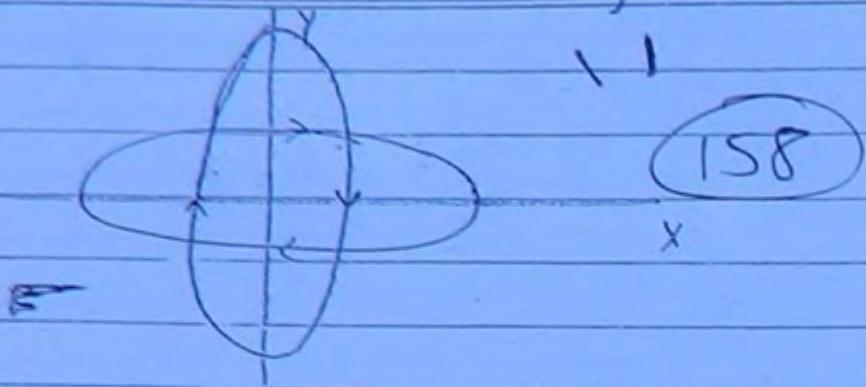
out of the paper - propagation

- LEFT -

Ex(+) The wave travel in +z direction

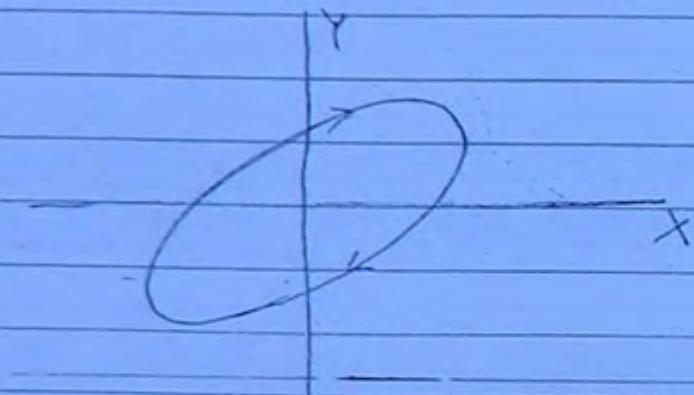
$$E(x,t) = E_{x0} \sin(\omega t - \beta x) a_x + E_{y0} \cos(\omega t - \beta x) a_y$$

$$\text{locus} = \left(\frac{x}{E_{x0}}\right)^2 + \left(\frac{y}{E_{y0}}\right)^2 = 1$$



(vi) the wave travel in $\pm z$ direction

$$\mathbf{E}(z,t) = E_x \sin(\omega t - \beta z) \mathbf{a}_x + E_y \sin(\omega t - \beta z + \theta^\circ) \mathbf{a}_y$$



Summary (of elliptical Polarization)

If the planar component has any phase and unequal amplitude the wave is elliptically polarized

Every ellipse has Axial Ratio [AR]

$$\text{Axial Ratio [AR]} = \frac{\text{major axis}}{\text{minor axis}}$$

$$\text{(Range)} \text{ AR} = (1, \infty)$$

circle linear

42 n.B (i) Amplitudes are unequal and both are in phase
same plane and two planar \rightarrow linear

(159)

$$(ii) 25 \sin(\omega t + 4x)(a_y + ja_z)$$

$$25 \sin(\omega t + 4x)a_y + 25 \cos(\omega t + 4x)a_z \quad (\text{amplitude same})$$

circular-

$$(iii) 25 \sin(\omega t + 4x + 60^\circ)(a_y + ja_z)$$

Polarization is always relative phase difference b/w the planar components.
(circular)

$$(iv) 25 \sin(\omega t + 4x)(a_y + (1+j)a_z)$$

$$25 \sin(\omega t + 4x)a_y + 25 \sin(\omega t + 4x)a_z \\ + 25 \sin(\omega t + 4x)j a_z$$

$$\Rightarrow 25 \sin(\omega t + 4x)a_y + 25 \cos(\omega t + 4x)a_z + 25 \sin(\omega t + 4x)a_z$$

$$j = e^{j90^\circ}$$

$$(1+j) = \sqrt{2} e^{j45^\circ}$$

$$25 \sin(\omega t + 4x)(a_y + \sqrt{2} e^{j45^\circ} a_z) \quad \text{Elliptical}$$

$$(v) 25 \sin(\omega t + 4x)(a_y + 2e^{j60^\circ} a_z) \quad \text{Elliptical}$$

$$(vi) 25 \sin(\omega t + 4x)((1-j)a_y + (1+j)a_z)$$

$$25 \sin(\omega t + 4x) \left(\sqrt{2} e^{-j45^\circ} a_y + \sqrt{2} e^{j45^\circ} a_z \right)$$

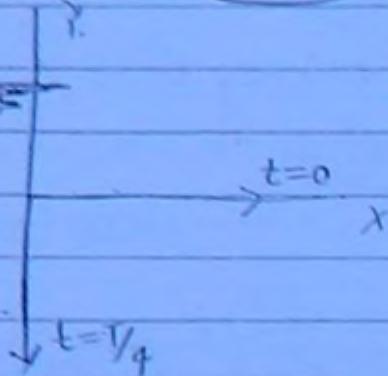
circular

$$44 \quad \bar{E}(t) = [E_x \cos \omega t \alpha_x - E_y \sin \omega t \alpha_y] e^{-jkz}$$

at $t=0$: $E_x, E_y = 0$

at $t=\frac{T}{4}$: $E_y, E_z = 0$

(160)



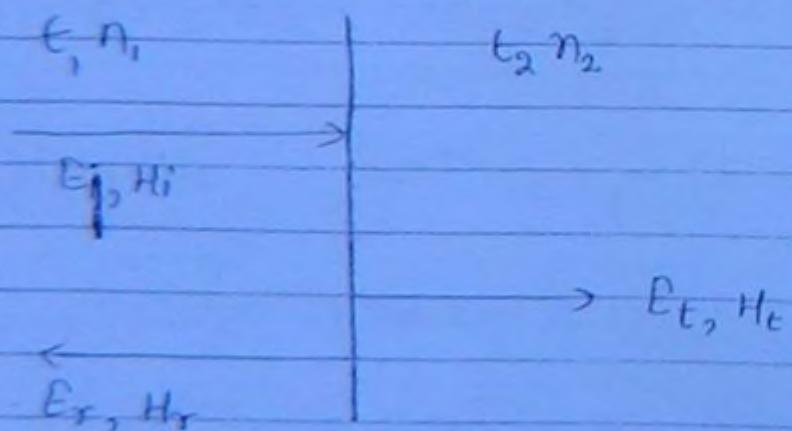
Elliptically polarized \rightarrow left

Table:

E_x	E_y	P_z	Polarization
\sin	\cos	$+z$	LCP
\sin	\cos	$-z$	RCP
\cos	\sin	$+z$	RCP
\sin	$-\cos$	$+z$	RCP

Reflection / Transmission of Plane waves

Normal Incidence \rightarrow Dielectric-Dielectric



Put $\tau = (1 + \Gamma)$

$$1 - \Gamma = \frac{n_1}{n_2} (1 + \Gamma)$$

(61)

$$\Rightarrow \Gamma = \frac{n_2 - n_1}{n_2 + n_1}$$

$$n_1 = \sqrt{\frac{\mu_0}{\epsilon_1}} \quad n_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}$$

for dielectric-dielectric

$$\left[\Gamma = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \right]$$

extension 1. $1 + \Gamma_H = \tau_H$

2. $\Gamma_H = -\Gamma_E = \frac{n_1 - n_2}{n_1 + n_2}$

3. $1 + \Gamma_p = \tau_p$

$$\Gamma_p = \Gamma_E \Gamma_H = -\Gamma_E^2 - \Gamma_H^2$$

Summary:

If $\epsilon_1 > \epsilon_2 \quad \Gamma_E = +ve$

$$1 + \Gamma_E = \tau_E = \frac{E_t}{E_i} > 1$$

$$\Gamma_H = -ve$$

$$1 + \Gamma_H = \tau_H \quad H_t < 1$$

$|\tau_p| < 1$ Always

$$G = E_0 \quad E_R = 40$$

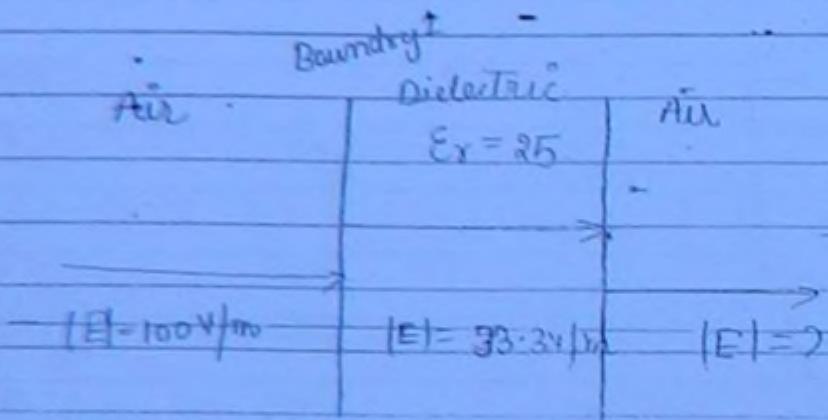
(162)

$$\Gamma_E = \frac{1-\sqrt{4}}{1+\sqrt{4}} = \frac{-1}{3} \rightarrow T_E = 1 + \left(\frac{-1}{3}\right) = \frac{2}{3}$$

$$\Gamma_H = \frac{+1}{3} \rightarrow \Gamma_H = 1 + \frac{1}{3} = \frac{4}{3}$$

$$\Gamma_P = \frac{-1}{3}$$

$$T_P = 1 - \frac{1}{3} = \frac{8}{9} \rightarrow T_P = T_E \cdot T_H = \frac{8}{9}$$



Boundary 1

$$\Gamma_E = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} = \frac{1-5}{1+5} \Rightarrow -\frac{2}{3}$$

$$T_E = 1 + \Gamma_E = 1 - \frac{2}{3} = \frac{1}{3}$$

$$T_E = \frac{1}{3} = \frac{E_t}{E_i}$$

$$E_t = \frac{1}{3} \times 100 = 33.33 \text{ V/m}$$

Boundary 2

$$\Gamma_E = \frac{5-1}{5+1} = \frac{4}{6} = \frac{2}{3}$$

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$$E_t > E_i \text{ or } E$$

for any interface E_{incident} and $E_{\text{transmitted}}$ follows.
the relationship which depends on ϵ_1 and ϵ_2 .
such that.

$$E_t > E_i \text{ if } E_t < E_i$$

if depends on $\epsilon_1 + \epsilon_2$.

If the transmitted E field is greater than E_{incident}
it has to be compensated $H_t < H_{\text{incident}}$ so that
the transmitted power will always less than incident
power.

$$H_t < H_i \text{ if } E_t > E_i$$

$$H_t > H_i \text{ if } E_t < E_i$$

$$P_t < P_i \text{ always}$$

$$E_i = \eta_1 H_i$$

$$a_x \times a_y = a_z$$

$$E_t = \eta_2 H_t$$

$$a_x \times -a_y = -a_z$$

$$E_r = -\eta_1 H_r$$

$$(-a_x) \times a_y = -a_z$$

when the propagation direction changes only E or H
will negate its sign i.e either E or H undergoes a
phase reversal but not go both.

Applying Boundary Condition

$$E_{t1} = E_{t2} \quad (\text{when propagation is normal})$$

(the both fields are tangential)

$$E_i + E_r = E_t \checkmark$$

$$H_{t1} = H_{t2}$$

$$H_i + H_s = H_t$$

(164)

divide the eqⁿ $E_i + E_r = E_t$ by E_i

$$\frac{E_i + E_r}{E_i} = \frac{E_t}{E_i}$$

F

the fraction of the field component reflecting back into the first medium is measured by the term $\frac{E_r}{E_i} = \text{Reflection coefficient}$ and similarly

$\frac{E_t}{E_i} = \text{(r)}$

the transmitted power $\frac{E_t}{E_i} = \text{transmission coefficient}(\tau)$

$$1 + \frac{E_r}{E_i} = \frac{E_t}{E_i}$$

$$\frac{E_r}{E_i} = r, \quad \frac{E_t}{E_i} = \tau$$

$$[1 + r = \tau]$$

Again using the same analyzee $H_i + H_s = H_t$

$$\frac{E_i}{n_1} - \frac{E_r}{n_1} = \frac{E_t}{n_2}$$

$$E_i - E_r = \frac{n_1}{n_2} E_t$$

divide the above eq by E_i

$$1 - \frac{E_r}{E_i} = \frac{n_1}{n_2} \frac{E_t}{E_i}$$

$$1 - r = \frac{n_1}{n_2} \tau$$

$$T_E = \frac{1 + F_E}{3} \Rightarrow 1 + 2 = \frac{5}{3}$$

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$$T_E = \frac{5}{3} = \frac{E_t}{E_i}$$

$$E_t = \frac{5}{3} \times 33.3 = 55.5 \text{ V/m}$$

~~46WB~~ $E_i = E_0 \cos(\omega t - \beta z) ay.$

$$\frac{\omega}{\beta} = \frac{3 \times 10^9 \pi}{10 \pi} = 3 \times 10^8$$

It means $E_R = 1$

$$F'_E = \frac{1 - 2}{1 + 2} = \frac{-1}{3}$$

$$T_E' = \frac{1 + F'_E}{3} = \frac{1 + 1}{3} = \frac{2}{3}$$

(ω is a source property)
It doesn't change
from medium to
medium

$$E_t = \frac{2}{3} E_0 \cos(\omega t - 2\beta z) ay. \quad \underline{\text{Any}}$$

$$\beta =$$

Note Among the properties of a wave $\alpha, \beta, \gamma, \eta, \lambda, v_p$
all these properties are material dependent.

They change from medium to medium.

E_t should be adjusted accordingly.

ω - source property and never changes in any medium

For E_r calculations all parameters are same except propagation direction

$$E_i \rightarrow H_r = ?$$

Air - dielectric

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$$\frac{E_i}{\epsilon_r} \xrightarrow{\Gamma_E} H_i \xrightarrow{\Gamma_H} H_r$$

ϵ_r

$$\downarrow \eta_r$$

H_r

F

$$\frac{E_t}{\epsilon_r} = -2 \Rightarrow \frac{E_t}{\epsilon_i \times \epsilon_r} \xrightarrow{\Gamma} \frac{\tau}{\Gamma} \Rightarrow \frac{1 + \Gamma}{\Gamma} = -2$$

$$\Gamma = \frac{-1}{3}$$

$$\Gamma_E = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} = \frac{1 - \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}}{1 + \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}} = \frac{1}{3} \Rightarrow \frac{4}{3} = \frac{2}{\sqrt{\epsilon_1}}$$

$$= 4 \quad \underline{\text{Ans}}$$

$$\frac{\sigma}{\omega \epsilon} \gg 1 \Rightarrow \frac{5}{8\pi \times 25 \text{ kHz} \times 80 \times 1 / 36\pi \times 10^9} =$$

$$\frac{\sigma}{\omega \epsilon} \gg 1 \text{ so it is a conductor} \quad \chi = \sqrt{\frac{\omega \mu_r}{2}} \quad (\text{for good conductor})$$

$$\chi = \sqrt{\frac{8\pi \times 25 \times 10^3 \times 4\pi \times 10^{-7} \times 5}{2}}$$

$$\chi = 0.702$$

$$10 = 100 e^{-\chi z}$$

$$\ln\left(\frac{10}{100}\right) = -0.702$$

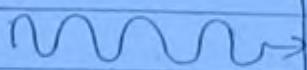
$$\frac{-0.702}{-0.70} = 2$$

$$z = 3.28 \text{ m}$$

Dielectric - Conductor Interface.

Dielectric conductor

$$\eta_1 \quad | \quad \eta_2$$



in 1% travel.
99%
reflected

$$\Gamma_E^I = +1$$

$$\Gamma_E^R = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \approx -1$$

$$\eta_1 = 377/\epsilon_{fr}$$

$$\eta_2 = \sqrt{\mu_0 \epsilon_0} \approx 0$$

$$E_i - E_r$$

$$\Gamma_E = -1 = \frac{E_r}{E_i}$$

Note :- ①

• when Γ_E^I is very close to -1 Γ_E^R is very close to +1.

• Γ_E^R being -ve means the reflected electric field is changing its sign and hence cancels with the incident electric field, and hence at the Boundary electric field is zero and magnetic field is maximum

Note ② when the wave is normally incident the fields are obviously tangential and hence E field should be zero along the conductor surface as $E_{tan} = 0$

(c) Electric field is minimum and magnetic field is maximum.

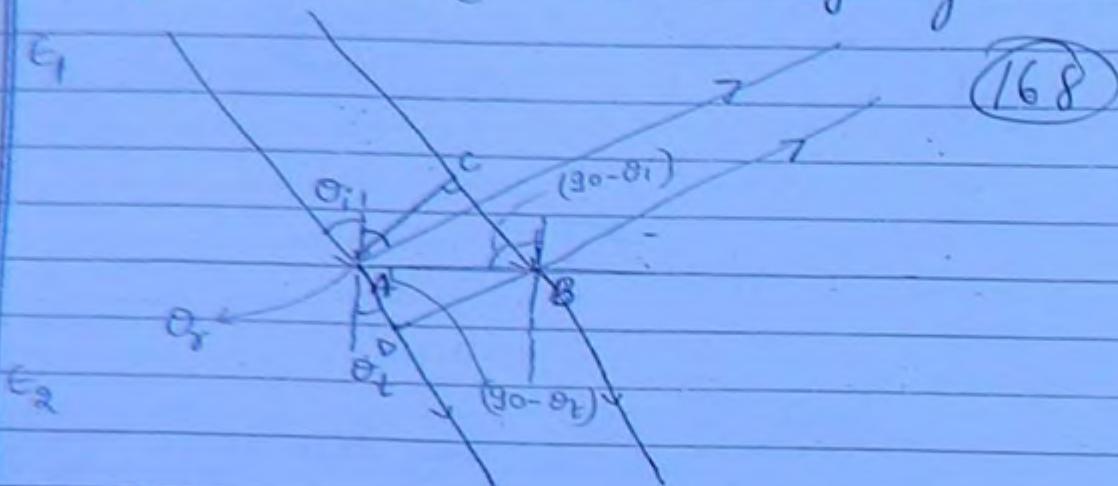
~~48W3~~ $\eta_1 = \eta_1 = 377$

Magnetic $\eta_2 = \sqrt{\frac{\mu}{\epsilon}} = 1$ or $\mu^* = \epsilon^*$

$$\Gamma = \frac{1 - 377}{1 + 377} \approx -1$$

(Real parts are also equal if complex is equal.)

(B) Oblique Incidence (Inclined at any angle)



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Snell's law.

- Relate the θ_i , θ_r , θ_t angles

Proof:

A/c \rightarrow Equidistance from the source i.e. A/c are in-phase points

CB = distance travelled by the wave in medium 1,

B/D are in-phase points

AD = distance travelled by the wave in medium 2.

The ratio of the distance in different media should be equal to the ratio of the velocities in those medium

$$\frac{CB}{AD} = \frac{v_1}{v_2}$$

$$\frac{AB \cos(90 - \theta_i)}{AB \cos(90 - \theta_t)} = \frac{v_1}{v_2} = \frac{1}{\sqrt{\mu_0 \epsilon_1}} \cdot \frac{1}{\sqrt{\mu_0 \epsilon_2}} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\frac{\sin \theta_i}{\sin \theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

Refractive index of medium 2
w.r.t to medium 1

Law(2) However angle of incidence is equal to angle of reflection so

$$\theta_i = \theta_r$$

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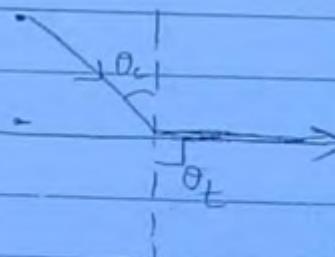
Critical Angle and Total internal Reflection (TIR)

If $\theta_t = 90^\circ$; the wave in the II medium is along the surface not into the medium

$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{\sin \theta_i}{\sin 90^\circ} = \frac{\sin \theta_i}{1} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\sin \theta_c = \sin \theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$\theta_i = \theta_c$ = critical angle

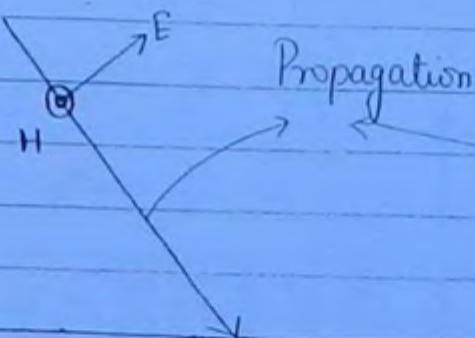


If $\theta_i > \theta_c$

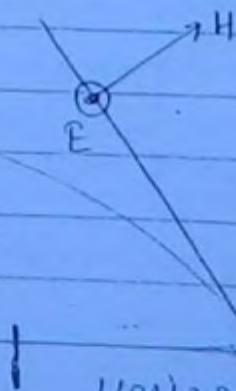
Total internal reflection takes place and zero transmission

S and P-polarized waves:

Medium 1



vertical
polarization
P-polarization



Horizontal Polarization

S-polarization

Medium 2

In oblique incidence electric field orientation is crucial in boundary condition and hence subsequent Γ and τ calculation

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S-Polarized : Boundary condition

$$E_{t1} = E_{t2}$$

$$E_i + E_r = E_t$$

$$1 + \Gamma_s = \tau_s$$

P-polarized : Boundary condition

$$E_{t1} = E_{t2}$$

$$E_i \cos \theta_i + E_r \cos \theta_i = E_t \cos \theta_t$$

Apply magnetic boundary condition in the same way,
we can derive for Γ and τ

$$1 + \Gamma_p = \tau_p \frac{\cos \theta_t}{\cos \theta_i}$$

$$\Gamma_s^t = \frac{\eta_2 \sec \theta_t - \eta_1 \sec \theta_i}{\eta_2 \sec \theta_t + \eta_1 \sec \theta_i}$$

$$1 + \Gamma_s = \tau_s$$

$$\Gamma_p^t = \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i}$$

$$1 + \Gamma_p = \tau_p \frac{\cos \theta_t}{\cos \theta_i}$$

Substituting the parameters of dielectric-dielectric interface
and converting the incident angle into transmitted angle

$$\Gamma_s^t = \frac{\sqrt{\epsilon_1} \cos \theta_i - \sqrt{\epsilon_2} \cos \theta_t}{\sqrt{\epsilon_1} \cos \theta_i + \sqrt{\epsilon_2} \cos \theta_t}$$

(71)

$$= \cos \theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1}} \sqrt{1 - \frac{\epsilon_1 \sin^2 \theta_i}{\epsilon_2}}$$

$$\cos \theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1}} \sqrt{1 - \frac{\epsilon_1 \sin^2 \theta_i}{\epsilon_2}}$$

$$\Gamma_s^t = \cos \theta_i - \sqrt{\frac{\epsilon_2 - \sin^2 \theta_i}{\epsilon_1}}$$

$$\cos \theta_i + \sqrt{\frac{\epsilon_2 - \sin^2 \theta_i}{\epsilon_1}}$$

$$\Gamma_p^t = \frac{\epsilon_2 \cos \theta_i - \sqrt{\epsilon_2 - \sin^2 \theta_i}}{\epsilon_1}$$

$$\frac{\epsilon_2 \cos \theta_i + \sqrt{\epsilon_2 - \sin^2 \theta_i}}{\epsilon_1}$$

Let $\Gamma_s^t = 0$ i.e zero reflection for s polarized wave

$$0 = \cos \theta_i - \sqrt{\frac{\epsilon_2 - \sin^2 \theta_i}{\epsilon_1}}$$

$$\cos \theta_i + \sqrt{\frac{\epsilon_2 - \sin^2 \theta_i}{\epsilon_1}}$$

$$\cos^2 \theta_i = \frac{\epsilon_2 - \sin^2 \theta_i}{\epsilon_1}$$

$$\frac{\epsilon_2}{\epsilon_1} = 1$$

$$[\epsilon_1 = \epsilon_2]$$

It is impossible for any incident angle to have zero reflections for s-polarized wave

(172)

Let $r_p = 0$

$$0 = \frac{\epsilon_2 \cos \theta_i}{\epsilon_1} - \sqrt{\frac{\epsilon_2 - \sin^2 \theta_i}{\epsilon_1}}$$

$$\left(\frac{\epsilon_2}{\epsilon_1}\right)^2 \cos^2 \theta_i = \frac{\epsilon_2 - \sin^2 \theta_i}{\epsilon_1}$$

$$\left(\frac{\epsilon_2}{\epsilon_1}\right)^2 (1 - \sin^2 \theta_i) = \frac{\epsilon_2 - \sin^2 \theta_i}{\epsilon_1}$$

$$\sin^2 \theta_i \left(1 - \left(\frac{\epsilon_2}{\epsilon_1}\right)^2\right) = \frac{\epsilon_2}{\epsilon_1} - \left(\frac{\epsilon_2}{\epsilon_1}\right)^2$$

$$\sin^2 \theta_i = \frac{\epsilon_2}{\epsilon_1} \left(1 - \frac{\epsilon_2}{\epsilon_1}\right)$$

$$\left(\frac{1 + \frac{\epsilon_2}{\epsilon_1}}{\epsilon_1}\right) \left(1 - \frac{\epsilon_2}{\epsilon_1}\right)$$

$$\boxed{\sin \theta_i = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1 + \epsilon_2}}}$$

$$\boxed{\tan \theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

Zero reflection & complete po transmission is possible for the p-polarized wave at a specific angle called as Brewster's angle.

Brewster's Angle

$$\tan \theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

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Total Internal Reflection
critical angle

$$\sin \theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$\epsilon_1 > \epsilon_2$ is the condition

Zero Reflection
Brewster angle

$$1. \tan \theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

No such restriction. Any two media can have.

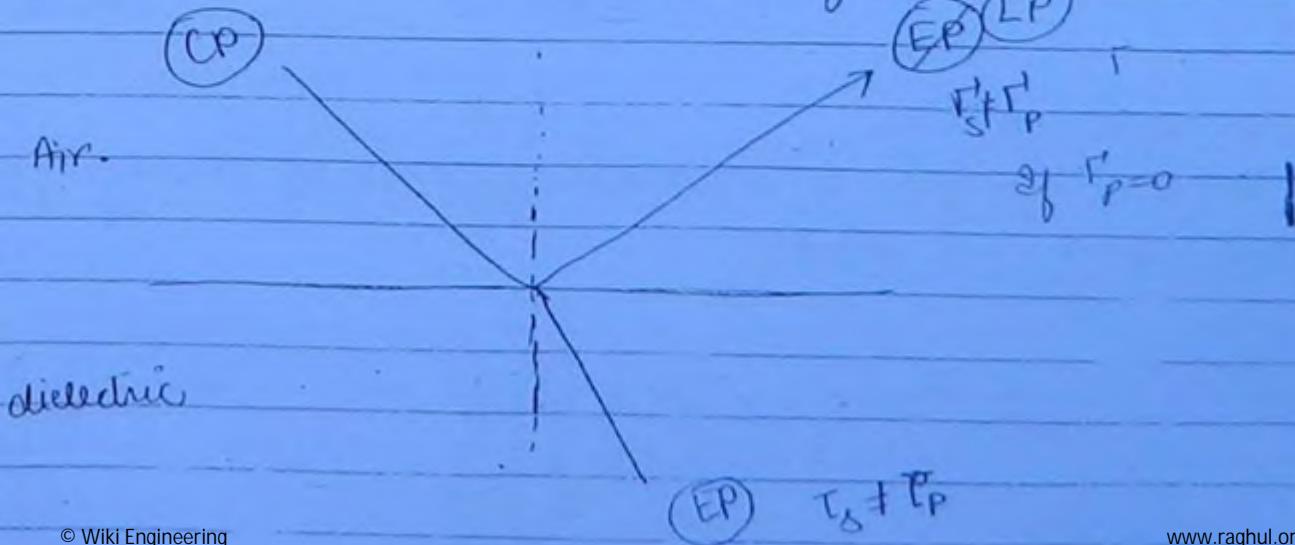
All incidence angle is greater than critical angle have the same phenomena.

All $\theta_i > \theta_c$ have the total internal reflection

either s or p polarized both can have critical angle

2. At exactly one single angle $\theta_i = \theta_B$ - zero reflection -

3. only p polarized wave can have Brewster's angle.

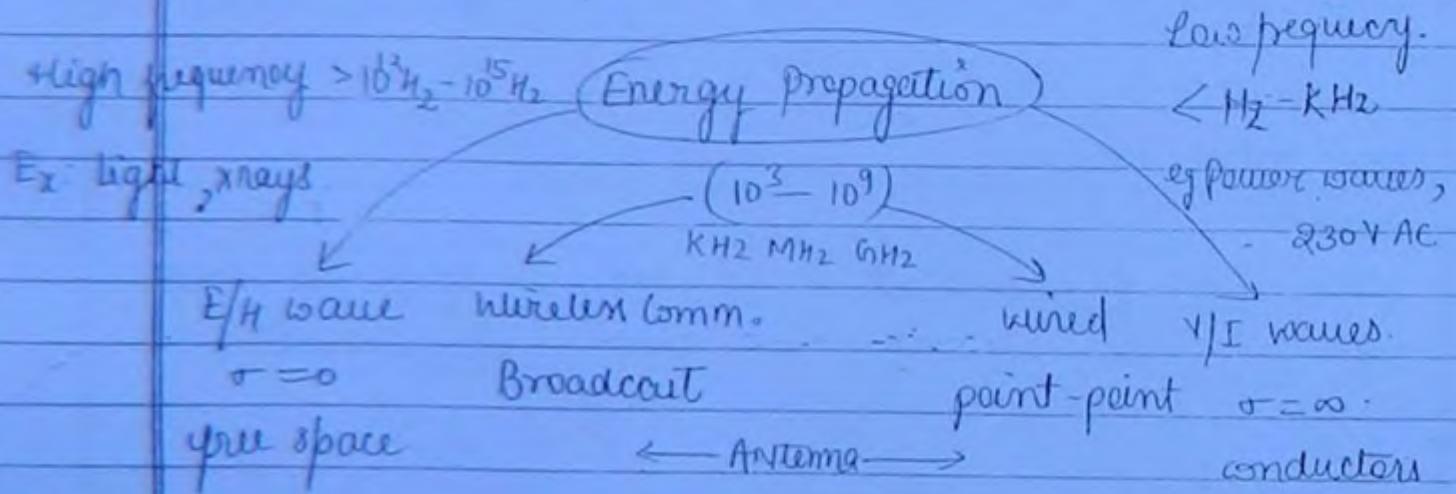


Note: When the incident wave is circularly polarized, with equal S.P., the transmitted and reflected wave must be elliptically polarized, with unequal S and P polarized components as $E_s \neq E_p$.
 The transmitted is also elliptically polarized as $T_s \neq T_p$.

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If $\frac{E_s}{E_p} = 0$ the only chance for the reflected wave to be linearly polarized is when $\frac{T_s}{T_p} = 0$ so that the reflected wave has only S component. However the transmitted wave is always elliptically polarized.

Hence at Brewster angle of incidence the reflected is linearly polarized and transmitted is elliptically polarized.



Transmission line:

A transmission line is a conducting mean of energy propagation using V-I waves.

EM wave
Medium
 σ , ϵ , μ

Harmonic
source
 w

$E_s \left\{ \begin{matrix} E_x \\ H_y \end{matrix} \right\} e^{j\omega t}$ → Propagation
Space
Harmonic

$$V = \sqrt{j\omega \mu} (\sigma + j\epsilon\omega)$$
$$Z_0 = \frac{\sigma + j\epsilon\omega}{j\omega \mu}$$

Medium

$$1 + \Gamma = \tau$$

$$\Gamma = \frac{\sigma - \mu}{\sigma + \mu}$$

$$1 + \Gamma = \tau$$

Load

Z_L

classmate

Date _____

Page _____

(175)

$$V = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$Z_R = \frac{(R + j\omega L)}{(G + j\omega C)}$$

V_x
 I_x

Propagation
length
Harmonic

V_s
 I_s

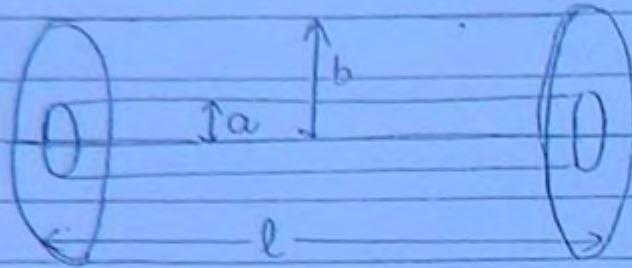
Harmonic
source
 ω

VI wave
Transmission
time
 R, G, C, L

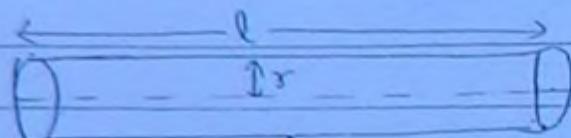
Time
 T

Geometries of a Transmission line.

(176)



coaxial
cable



$D \gg r$

Parallel
line



Primary Constants of a line (R, L, G, C)

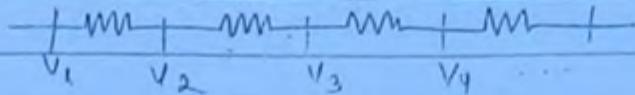
Resistance (R)

- It is the resistance of the conducting material along the length of the line

$$R = \frac{\rho l}{A} \quad \text{distributed}$$

$$R = \frac{R}{l} = \frac{\Omega \text{hm}}{\text{metre}}$$

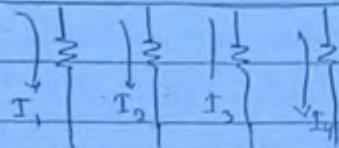
- The primary constt is the resistance per unit length i.e the instantaneous value as the study is made on a per unit length basis. extending it for any length.
- Resistance causes voltage decay all along the line as it appears in wires.



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conductance (G_l)

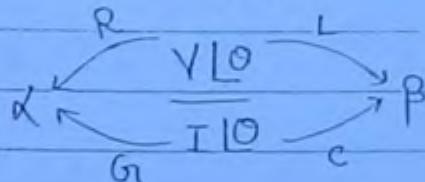
It is the conductance of the dielectric (insulating material) b/w the lines



- Conductance b/w the line leads to current leakage and it is distributed everywhere b/w the lines
- Hence the primary constt is G_l per unit length

$$G_l = \frac{G_l}{l} = \frac{\text{mho}}{\text{meter}} \quad \text{--- distributed}$$

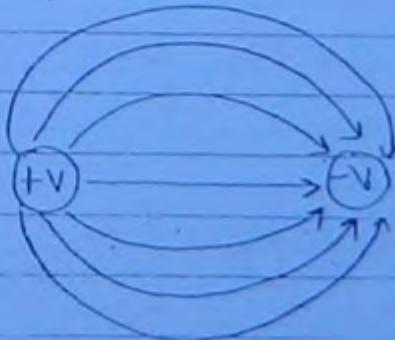
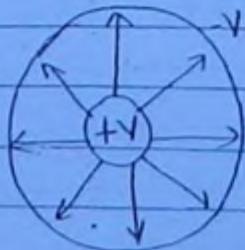
- It is the shunt property b/w the lines



Both R and G_l cause attenuation on the line.

capacitance (C)

The voltages on the line have charge accumulation and hence have E-field surrounding the line and have capacitance



When we give voltage or the line then electric field
 " " current then get magnetic field.

Date _____
 Page _____

$$C = \frac{2\pi G_0 l}{\ln(b/a)}$$

coaxial cable 178

$$C = \frac{\pi \epsilon_0 l}{\ln(D/r)}$$

parallel wire

- The capacitance is proportional to the length and hence it is said to be distributed.

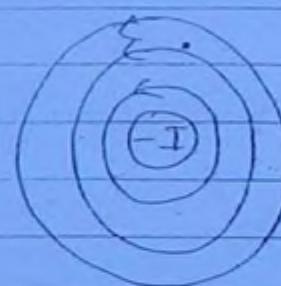
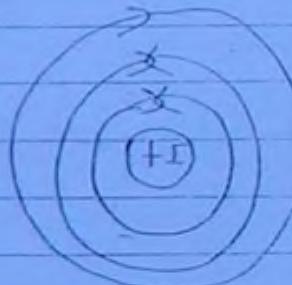
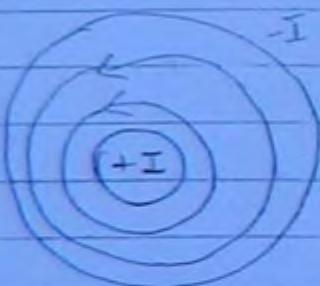
The primary constt c is $\frac{C}{l}$

$$\frac{C}{l} = \frac{\text{Total}}{\text{length}} \quad - (\text{distributed}) -$$

- The capacitance appears in shunt b/w the lines

Inductance (L)

The currents on the line have a charge flow and hence a H field surrounding the line and hence inductance



8 shape

$$L = \frac{\mu_0 \cdot l}{2\pi} \ln(b/a)$$

coaxial cable

$$L = \frac{\mu_0 \cdot l}{\pi} \ln(D/r)$$

parallel wire

Inductance is proportional to the length and is distributed
on the line hence

the primary current therefore is $I = \frac{L}{l}$

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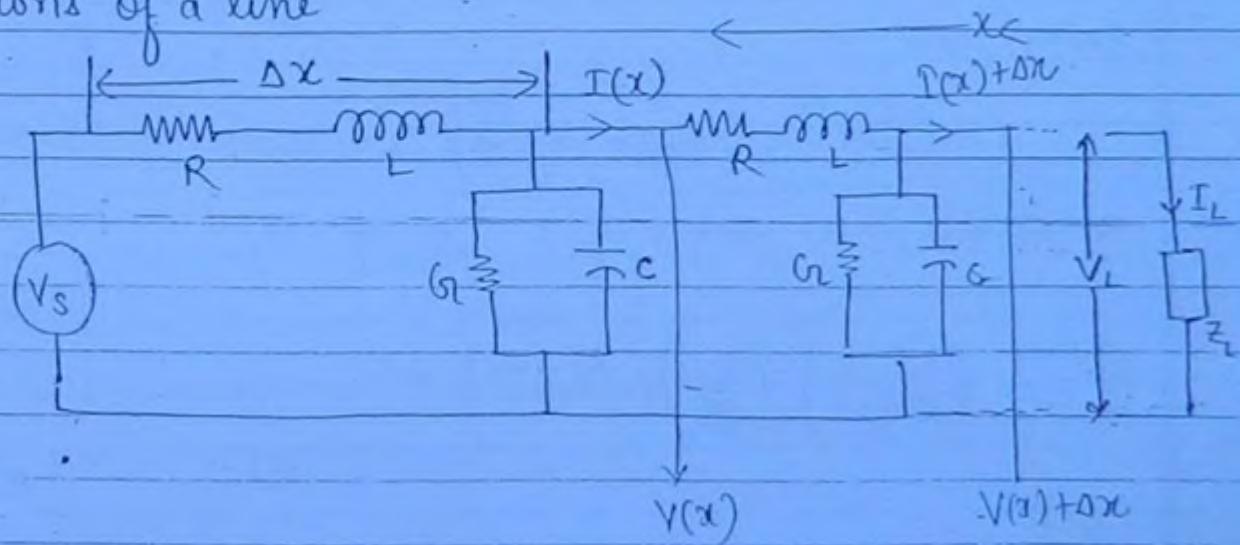
$$L = \frac{L}{l} = \frac{\text{Henry}}{\text{meter}}$$

Inductance comes in series due to the flowing current.

$$I_C = \mu_0 e_0$$

Distributed inductance and distributed conductance product
is $\mu_0 e_0$ for any transmission line.

VI Equations of a line



x — always starts from the load and increases towards the source.

i.e. at $x=0$ $V=V_s$

$$I = I_L$$

at $x=l$, $V=V_L$
 $I = I_L$

Monday

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Impedance

Z of the line of Δx length = $(R+j\omega L) \Delta x$ } in series.

y " " " = $(G+j\omega C) \Delta x$ } Admittance
in shunt

$$\Delta V = I(z) (R+j\omega L) \cdot \Delta x$$

$$\frac{dV}{dz} = I \cdot (R+j\omega L) \quad (1)$$

$$\Delta I = V(z) (G+j\omega C) \Delta x \quad I = \frac{V}{R} = V \times \text{Admittance}$$

$$\Delta V = I(z) (R+j\omega L) \cdot \Delta x \quad (2)$$

from (1)

$$I = \frac{1}{(R+j\omega L)} \left(\frac{dV}{dz} \right)$$

Substitute this value of I in eq (2)

$$\Delta V = \left(\frac{1}{R+j\omega L} \right) \left(\frac{dV}{dz} \right) (R+j\omega L) \cdot \Delta x$$

$$\frac{d}{dx} \left(\left(\frac{1}{R+j\omega L} \right) \frac{dV}{dx} \right) = V \cdot (G+j\omega C)$$

$$\frac{d^2V}{dx^2} = (R+j\omega L)(G+j\omega C) V \quad (3)$$

$$\frac{d^2I}{dx^2} = (R+j\omega L)(G+j\omega C) I \quad (4)$$

In transmission line:

Since the V - I Eqⁿ are harmonic functions of length of the line; since energy propagates along the length of a line

$$\sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{(Par m)(m)} \left(\frac{\text{ohm}}{m} \right) \times \left(\frac{mho}{m} \right)$$

$$\frac{d^2V}{dx^2} - \gamma^2 V = 0 \quad \dots \quad (3)$$

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$$\frac{d^2V}{dx^2} - \gamma^2 I = 0 \quad \dots \quad (4)$$

Solutions of (3) & (4)

$$V(x) = (C_1 e^{-\gamma x} + C_2 e^{\gamma x})$$

$$I(x) = (C_3 e^{-\gamma x} + C_4 e^{\gamma x})$$

Using Boundary conditions called initial conditions

$$x=0, \quad V=V_L, \quad I=I_L$$

$$V_L = C_1 + C_2 \quad \dots \quad (7)$$

$$I_L = C_3 + C_4 \quad \dots \quad (8)$$

using eq (1)

$$\frac{dV}{dx} = I(R+j\omega L)$$

$$-\gamma C_1 e^{-\gamma x} + \gamma C_2 e^{\gamma x} = I(R+j\omega L)$$

at $x=0$

$$-\gamma C_1 + \gamma C_2 = I_L(R+j\omega L)$$

$$(C_2 - C_1) = \frac{I_L(R+j\omega L)}{\gamma}$$

$$C_2 - C_1 = \frac{I_L(R+j\omega L)}{\sqrt{(R+j\omega L)(G+j\omega C)}} \quad (8)$$

$$C_2 - C_1 = I_L \cdot \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad (8)$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$C_2 - C_1 = I_L \times Z_0 \quad (9)$$

Again using $\frac{dI}{dx} = v(x) (G+j\omega C)$

$$\frac{dI}{dx} = v(x) (G+j\omega C)$$

$$-r c_3 e^{-rx} + r c_4 e^{rx} = v(G+j\omega C)$$

at $x=0$

$$r(c_4 - c_3) = v(G+j\omega C)$$

$$c_4 - c_3 = \frac{v(G+j\omega C)}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

$$\frac{v_L}{Z_0} = c_4 - c_3 \quad (10)$$

On solving (7) (8) (9) (10) we get the values of
 c_1, c_2, c_3, c_4

$$C_1 = \frac{V_L - I_L Z_0}{2}$$

$$C_2 = \frac{V_L + I_L Z_0}{2}$$

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$$C_3 = \frac{I_L - V_L / Z_0}{2}$$

$$C_4 = \frac{I_L + V_L / Z_0}{2}$$

$$V(x) = C_1 e^{-fx} + C_2 e^{fx}$$

Put C_1 & C_2 in the above eqn.

$$V(x) = \frac{V_L - I_L Z_0}{2} e^{-fx} + \frac{V_L + I_L Z_0}{2} e^{fx}$$

$$= \frac{V_L}{2} \left[\left(1 - \frac{Z_0}{Z_L} \right) e^{-fx} + \left(1 + \frac{Z_0}{Z_L} \right) e^{fx} \right]$$

$$= \frac{V_L}{2Z_L} \left[(Z_L - Z_0) e^{-fx} + (Z_L + Z_0) e^{fx} \right]$$

$$V(x) = \frac{V_L (Z_L + Z_0)}{2Z_L} \left[e^{fx} + \frac{(Z_L - Z_0) e^{-fx}}{(Z_L + Z_0)} \right]$$

$$I(x) = (C_3 e^{-fx} + C_4 e^{fx})$$

$$= \frac{I_L - V_L}{2Z_0} e^{-fx} + \frac{I_L + V_L}{2Z_0} e^{fx}$$

$$= \frac{I_L}{2} \left[\left(1 - \frac{Z_L}{Z_0} \right) e^{-fx} + \left(1 + \frac{Z_L}{Z_0} \right) e^{fx} \right]$$

$$I(x) = \frac{I_L (Z_L + Z_0)}{2Z_0} \left[e^{fx} + \left(\frac{Z_0 - Z_L}{Z_0 + Z_L} \right) e^{-fx} \right]$$

forward wave
source-load
direction

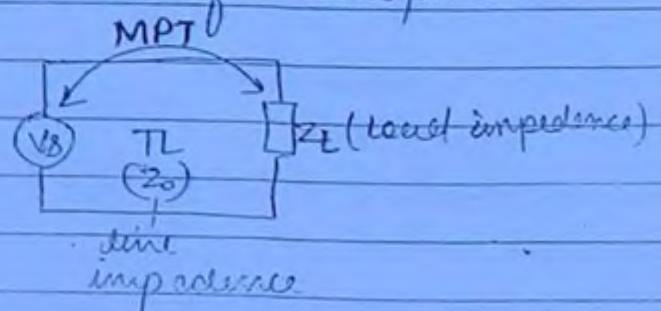
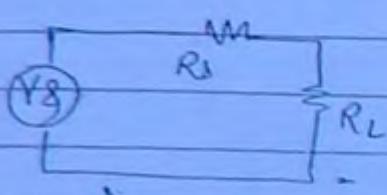
backward source
direction
D.L. & T.M.

1. Every transmission line has two waves on it
the forward wave and the reflected wave.

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2. The reflected wave becomes zero only under one condition i.e. $Z_L = Z_0$. In any other condition the reflected wave is due to unabsorbed power in the load.

3. If $Z_L = Z_0$ the complete source power is transmitted to the load. Maximum power transfer takes place



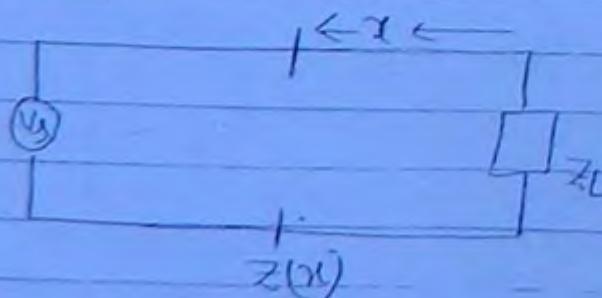
MPT = Maximum Power Transformation

Characteristic Impedance:

Z_0 is a unique impedance of the line which depends on its primary constt and it is the impedance with which the load should be terminated for maximum power transfer and hence called as characteristic impedance.

Impedance on the line and Input impedance

Every transmission line has impedance anywhere on the line which is due to the load at one end and primary constt R, L, G, C



$$Z(x) = \frac{V(x)}{I(x)}$$

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$$Z_{in} = \text{Input impedance} = \left. \frac{V(x)}{I(x)} \right|_{x=0}$$

$$\begin{aligned} V(x) &= \frac{V_L(z_L + z_0)}{2z_L} \left[e^{rx} + \left(\frac{z_L - z_0}{z_L + z_0} \right) e^{-rx} \right] \\ &= \frac{V_L}{z_L} \left[\left(\frac{z_L + z_0}{2} \right) e^{rx} + \left(\frac{z_L - z_0}{2} \right) e^{-rx} \right] \\ &= \frac{V_L}{z_L} \left[z_L \left(\frac{e^{rx} + e^{-rx}}{2} \right) + z_0 \left(\frac{e^{rx} - e^{-rx}}{2} \right) \right] \\ &= \frac{V_L \times z_L}{z_L} \left(\frac{e^{rx} - e^{-rx}}{2} \right) + \frac{V_L \times z_0}{z_L} \left(\frac{e^{rx} - e^{-rx}}{2} \right) \end{aligned}$$

$$= V_L \cosh(rx) + I_L z_0 \sinh(rx)$$

Similarly

$$I(x) = I_L \cosh(rx) + \frac{V_L}{z_0} \sinh(rx)$$

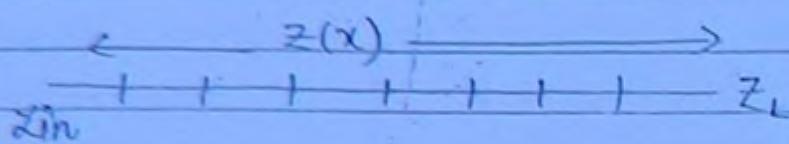
$$\boxed{\frac{Z(x)}{I(x)} = \frac{V_L \cosh(rx) + I_L z_0 \sinh(rx)}{I_L \cosh(rx) + \frac{V_L}{z_0} \sinh(rx)}}$$

$$Z(x) = \frac{I_L \left(\frac{V_L}{I_L} \cosh(rx) + z_0 \sinh(rx) \right)}{I_L \left(\cosh(rx) + \frac{z_L}{z_0} \sinh(rx) \right)}$$

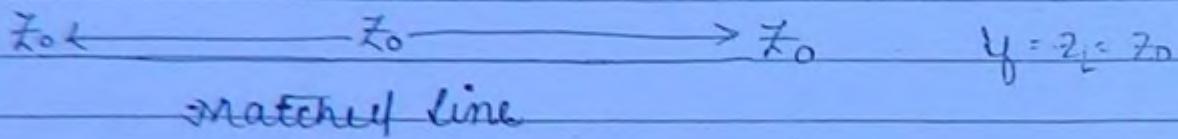
$$z(x) = z_0 \begin{bmatrix} z_L \cosh(\gamma x) + z_0 \sinh(\gamma x) \\ z_0 \cosh(\gamma x) + z_L \sinh(\gamma x) \end{bmatrix}$$

mismatched line 

Line 1



Line 2



If a line is terminated with its characteristic impedance the impedance anywhere on the line and at the S₁₁ port is also the same such a line is called a matched line

Short circuit and open circuit line

If $z_L = 0 \Rightarrow$ short circuit line

then $Z_{in} = Z_{sc} = z_0 \tanh(\gamma x)$

If $z_L = \infty \Rightarrow$ open circuit line

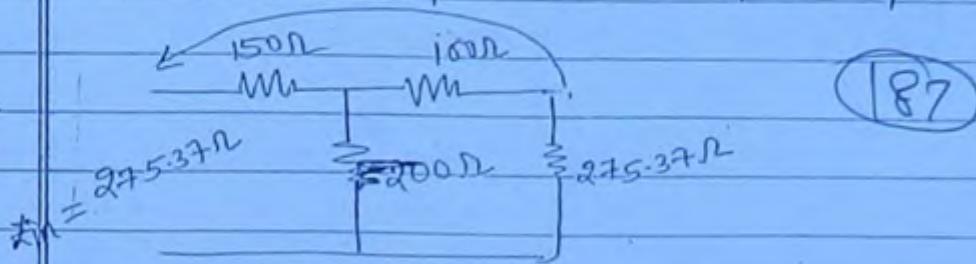
then $Z_{in} = Z_{oc} = z_0 \coth(\gamma x)$

where $x =$ the complete length - length of line

$$Z_0 = \sqrt{Z_{sc} \cdot Z_{oc}}$$

Note characteristic Impedance is always the geometric mean of short ckt and open circuit input impedance.

eg.

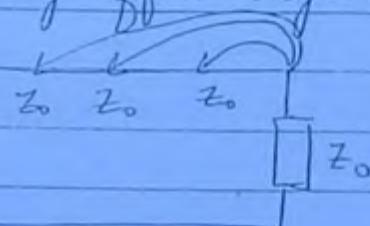


$$Z_{sc} = 150 + (100 \parallel 200) = 216.66\Omega$$

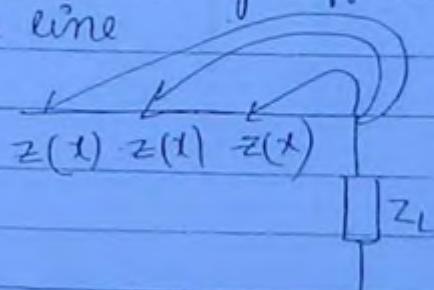
$$Z_{oc} = 3.50\Omega$$

$$Z_0 = \sqrt{350 \times 216.66} = 275.37\Omega$$

For any discrete N/w (3-4 elements N/w) if it is terminated with Z_0 its O/p impedance is also the same i.e. the N/w behaves like a mirror and the O/p is reflected in the I/p. similarly for a continuous RLC G. N/w on a distributed transmission line if the termination is Z_L the loading effect anywhere is Z_0 as shown.



Hence the Impedance is same everywhere instead if the load is Z_L the loading effect is different at different points on the line

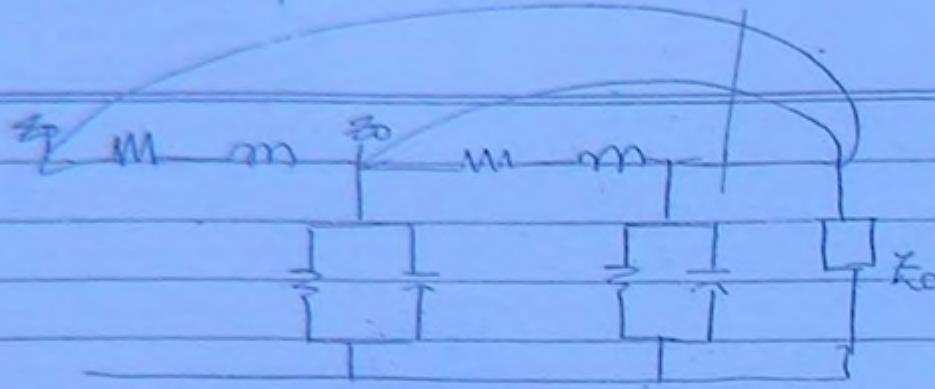


NTL (Nodal and transmission line)

classmate

Date _____

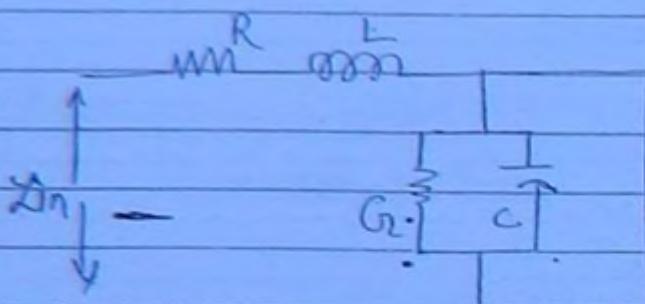
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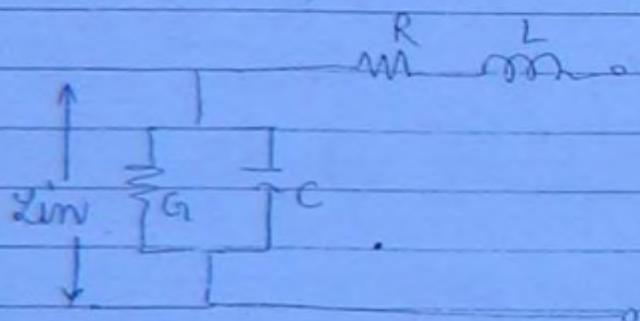
(88)

F

$$Z_0 = \sqrt{Z_{sc} \cdot Z_{oc}}$$



$$Z_{oc} = (R + j\omega L)$$



$$Z_{oc} = \frac{1}{G_1 + j\omega C}$$

$$Z_0 = \sqrt{Z_{sc} \cdot Z_{oc}} = \sqrt{\frac{R + j\omega L}{G_1 + j\omega C}}$$

Lossless Line & Distortionless Line:

γ = propagation constt.

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$$= \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

If $\alpha = 0$ — No attenuation on the line
No loss on the line
Lossless line

series - No resistance

shunt - No leakage

(i) $R = G_L = 0$

$$\gamma = j\omega \sqrt{LC} = 0 + j\beta$$

$\boxed{\beta = \omega \sqrt{LC}}$

(ii) $\omega L \gg R$, $\omega C \gg G_L$

• High freq. lines

• Inductive / capacitive lines are also lossless

For Z_0 $Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$

If $R = G_L = 0$

$\boxed{Z_0 = \sqrt{\frac{L}{C}}} = \text{real};$

$Z_0 = \sqrt{\frac{\mu_0 \ln(b/a)}{2\lambda} \frac{\ln(b/a)}{2\pi \epsilon_0}}$ — for coaxial cable

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \frac{\ln(b/a)}{2\pi}$$

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$$Z_0 = \frac{120\pi}{2\pi} \ln(b/a) \Rightarrow 60 \ln(b/a) \quad \left\{ \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \right\}$$

$$Z_0 = 60 \ln(b/a) \quad \text{--- for air-filled coaxial line cable}$$

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln(b/a) \quad \text{--- for dielectric filled coaxial cable}$$

for parallel line

$$Z_0 = 120 \ln(D/r) \quad \text{--- for air filled parallel line}$$

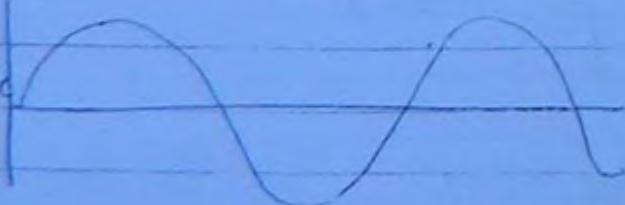
$$Z_0 = \frac{120}{\sqrt{\epsilon_r}} \ln(D/r) \quad \text{--- for dielectric filled parallel line}$$

Phase velocity $v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega \sqrt{LC}} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \epsilon}}$

$$v_p = \frac{1}{\sqrt{\mu_0 \epsilon}}$$

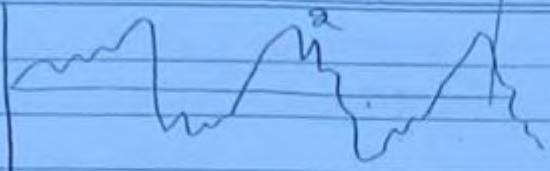
Distortionless line

Distortionless
Simple Harmonic



$$\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j} \quad \cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\cosh \theta = \frac{e^{\theta} + e^{-\theta}}{2} \quad \sinh \theta = \frac{e^{\theta} - e^{-\theta}}{2}$$



Distorted or multi-Harmonic

classmate

Date _____
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If $\beta \propto w$



β is a linear function of $w \Rightarrow$ the line is distortionless

Phase-shift varies with space & time should be linear

Rise time of the wave = fall time of the wave.

Series arm time constt = shunt arm time constt

$$\frac{L}{R} = C \Rightarrow L G_1 = R C$$

WorkBook - chapter 3. (Transmission line)

$$Z_{in} = Z_0 \left[\frac{Z_L \cosh(j\beta l) + Z_0 \sinh(j\beta l)}{Z_0 \cosh(j\beta l) + Z_L \sinh(j\beta l)} \right]$$

$$\cosh(j\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} = \cos(j\theta)$$

$$Z_{in} = Z_0 \left[\frac{Z_L \cos(\beta l) + j Z_0 \sin(\beta l)}{Z_0 \cos(\beta l) + Z_L \sin(\beta l)} \right]$$

$$\sinh(j\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2} = j \sin \theta$$

e

$$\text{(i)} \quad \lambda_8 = l$$

$$\beta l = \frac{2\pi \times d}{\lambda} \Rightarrow \frac{\pi}{4}$$

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$$Z_{in} = Z_0 \left[\frac{Z_L \cos \frac{\pi}{4} + j Z_0 \sin \frac{\pi}{4}}{Z_0 \cos \pi/4 + j Z_L \sin \pi/4} \right]$$

$$Z_{in} = Z_0 \left[\frac{Z_L + j Z_0}{Z_0 + j Z_L} \right]$$

$$\text{(ii)} \quad \lambda_4 = l$$

$$\beta l = \frac{2\pi \times d}{\lambda} = \frac{\pi}{2}$$

$$Z_{in} = Z_0 \left[\frac{Z_L \cos \pi/2 + j Z_0 \sin \pi/2}{Z_0 \cos \pi/2 + Z_L \sin \pi/2} \right]$$

$$Z_{in} = Z_0 \left[\frac{j Z_0}{Z_L} \right] = \frac{Z_0^2}{Z_L}$$

$$\text{(iii)} \quad \lambda_2 = l$$

$$\beta l = \frac{2\pi \times d}{\lambda} = \pi$$

$$Z_{in} = Z_0 \left[\frac{Z_L \cos \pi + j Z_0 \sin \pi}{Z_0 \cos \pi + Z_L \sin \pi} \right]$$

$$= Z_0 \left[\frac{-Z_L}{-Z_0} \right]$$

$$Z_{in} = Z_L$$

$$\text{(iv)} \quad \lambda = l$$

$$\beta l = \frac{2\pi \times d}{\lambda} = 2\pi$$

$$Z_{in} = Z_0 \left[\frac{Z_L \cos(\alpha x) + j Z_0 \sin(\alpha x)}{Z_0 \cos(\alpha x) + Z_L \sin(\alpha x)} \right]$$

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$$Z_{in} = Z_L$$

Conclusion: The input impedance Z_{in} and the behaviour of the line always depends on in term αl hence length to wavelength relationship is crucial for any line.
Hence αl is called as Electrical length of the line
units = Radians or degrees.

~~2 w~~ $Z_0 = 50\Omega$: $Z_0 \neq Z_L$ (Not a matched line)
 $Z_L = j50\Omega$

50Ω = Inductive load. \rightarrow Line is not matched
 $(j50\Omega)$

$$Z_{in} = \frac{50}{50 + j50} \left(\frac{j50 + j50}{50 - j50} \right) \Rightarrow \frac{50 \times j50}{50 + j50} \left(\frac{2}{50 - j50} \right) = \frac{50 \times j50 \times 2}{50 + j50} \times \frac{1}{50 - j50} = \frac{50 \times j50 \times 2}{50(1+j)}$$

$$Z_{in} = \text{--} \quad \text{O.C.} \quad \boxed{\rightarrow}$$

$$Z_{in} = \frac{50}{j50} = \frac{50}{j50} - \frac{50}{j}$$

$$\frac{1}{j} = \frac{j^4}{j} = j^3 = (-j)$$

$$Z_{in} = -j50 \quad (\text{capacitive})$$

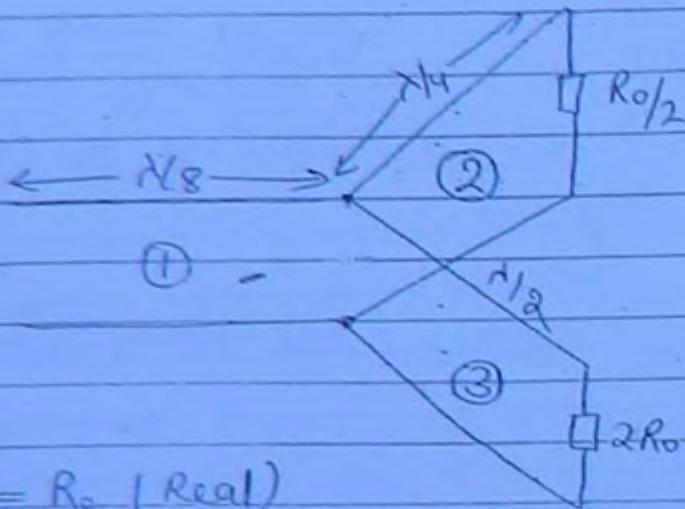
(4) $Z_{in} = Z_L$

$$Z_{in} = j50 \quad (\text{inductive})$$

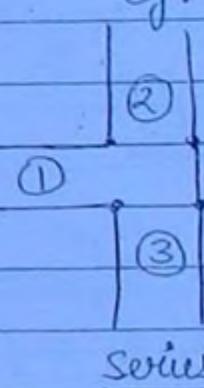
$$Z_{in} = \frac{Z_0^2}{Z_L}$$

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- A $\lambda/4$ line at one end offers an exact opposite impedance to the impedance at the other end. Hence it is called as impedance inverter.
- Similarly a $\lambda/2$ line this is simple impedance reflector.



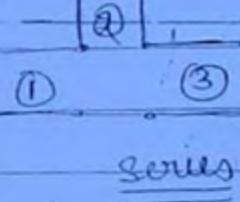
eg.



series

$$Z_0 = R_o \text{ (Real)}$$

means transmission line



series

$$Z_{L1} = (Z_{in2} \parallel Z_{in3})$$

Load of first line is input impedance for (2) and (3) that are in shunt

$$Z_{in2} = \frac{Z_0^2}{Z_L} \quad (\text{for } \lambda/4 \text{ line})$$

$$= \frac{R_o^2}{R_o/2} - 2R_o$$

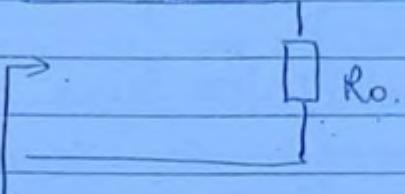
$$Z_{in3} = Z_L \quad (\text{for } \lambda/2 \text{ line})$$

$$= 2R_o$$

$$Z_L = (2R_0 \parallel 2R_0) = R_0$$

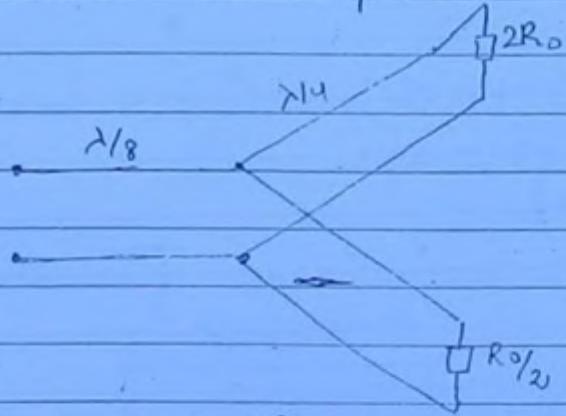
$\leftarrow \lambda_{18} \rightarrow$

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$$Z_{in} = R_0$$

Repeat the same question with the load interchange.



$$Z_0 = R_0 \text{ (Real)}$$

$$Z_{in} = (Z_{in2} \parallel Z_{in3})$$

$$Z_{in2} = \frac{Z_0^2}{Z_L} \quad (\text{for } \lambda/4 \text{ line})$$

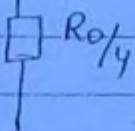
$$= \frac{R_0^2}{2R_0} = \frac{R_0}{2}$$

$$Z_{in} = (R_0/2 \parallel R_0/2)$$

$$= R_0/4$$

$$Z_{in3} = Z_L = \frac{R_0}{2}$$

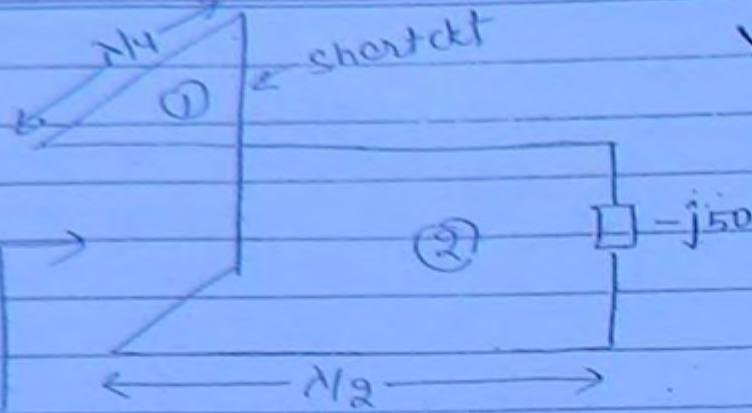
$\leftarrow \lambda_{18} \rightarrow$



$$Z_{in} = R_0 \left(Z_L + jZ_0 \right)$$

$$\left[Z_0 + jZ_L \right]$$

$$Z_{in1} = R_0 \left(\frac{R_0 + jR_0}{j} \right) \Rightarrow R_0 \left(\frac{1 + j4}{2 + j} \right)$$



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$$\gamma_{in} = ?$$

$$\gamma_{in} = \gamma_{in1} + \gamma_{in2}$$

$$\gamma_{in1} = \frac{Z_L}{Z_0^2} \quad (\text{for } l = \lambda l_1) \\ = -j50$$

$$\gamma_{in2} = \frac{Z_L}{Z_0^2} = \frac{1}{-j50} = \frac{j}{50}$$

$$\gamma_{in} = 0 + j = j(0.02)$$

$$\text{other method} \quad \gamma_{in} = \frac{1}{Z_{in}}$$

$$Z_{in1} = \infty = 0/C$$

$$Z_{in2} = Z_L = -j50$$

$$Z_{in} = (\infty || -j50) \\ = -j50$$

$$\gamma_{in} = \frac{1}{Z_{in}} = \frac{1}{-j50} = \frac{j}{50} = j(0.02)$$

1	$R \parallel 0$	$= 0$
2	$R \parallel \infty$	$= R$

9

$$f = 5 \times 10^6 \text{ Hz}$$

(197)

$$\lambda = \frac{v_p}{f} = \frac{2 \times 10^8}{5 \times 10^6} = 2000 \div 5 = 400.$$

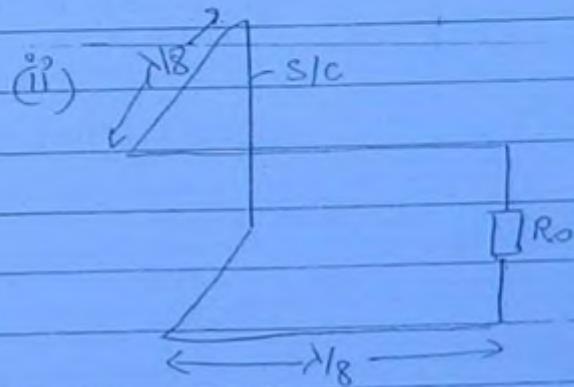
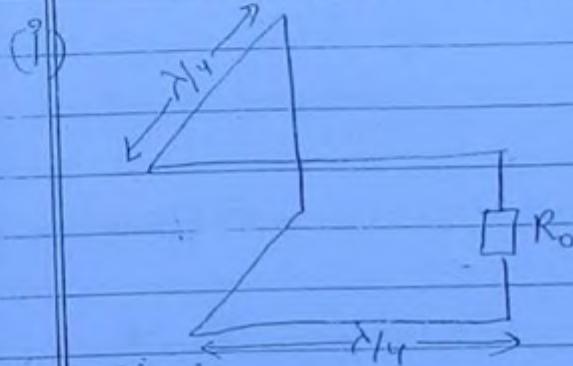
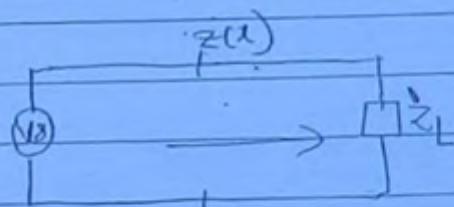
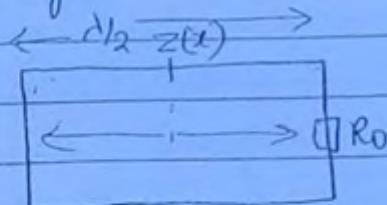
$$\beta l = \frac{2\pi \times 10}{\lambda} \Rightarrow \frac{2\pi \times 10}{400} \Rightarrow \frac{\pi}{2}$$

$$l = \frac{\lambda}{4}$$

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{(30 + j40)^2}{(30 - j40)}$$

6/7/4

Tuesday



(i) $\lambda/4$ at centre $= \infty \parallel R_0 = R_0$

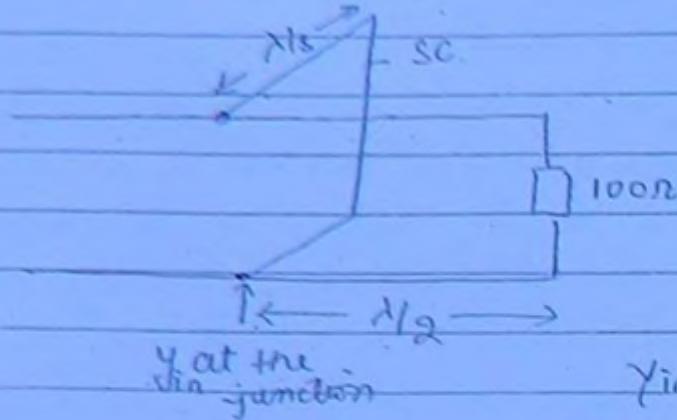
$$\frac{Z_0^2}{Z_L} = \frac{Z_0^2}{R_0} = \infty \quad (\text{for } R_0)$$

$$\frac{Z_0^2}{Z_L} = \frac{R_0^2}{R_0} = R_0 \quad (\text{for } R_0)$$

(ii) at $l = \frac{\lambda^2}{4}$ means $\lambda/8$

$$\lambda/8 \text{ at centre} = jR_0 \parallel R_0 = \frac{jR_0 \times R_0}{jR_0 + R_0} = jR_0$$

$$Z = Z_0 \left[\frac{Z_L + jZ_0}{Z_0 + Z_L} \right] = R_0 \left[\frac{R_0 + jR_0}{R_0 + R_0} \right] = jR_0$$



$$Y_{in1} = \frac{Z_0(Z_L + jZ_0)}{(Z_0 + jZ_L)} = \frac{Z_0(0 + jZ_0)}{(Z_0 + j0)} = jZ_0 = j50.$$

$$Y_{in2} = \frac{1}{Z_{in1}} = \frac{1}{j50} = \frac{-j}{50} \Rightarrow -j(0.02) \quad \frac{1}{100} = 0.01$$

Iterative : for $\lambda/8$ line

method.

$$\begin{aligned} h_{sc} &= \frac{Z_0 \tanh(j\beta l)}{jZ_0 \tanh(\beta l)} & l = \lambda/8 \\ &= \frac{jZ_0 \tanh(\beta l)}{\beta l} & \beta l = \gamma_1 \\ &= jZ_0 \end{aligned}$$

$$Y_{in2} = \frac{1}{Z_L} = \frac{1}{100}$$

8- $l = 10m$

$$V_p = 2 \times 10^8 \text{ m/s}$$

$$f = 10 \text{ MHz}$$

$$\lambda = \frac{2 \times 10^8}{10 \times 10^6} = \frac{V_p}{f} = 20 \text{ m}$$

$$\beta l = \frac{\alpha \lambda \times 10}{\lambda} \Rightarrow \frac{\alpha \lambda \times 10}{20} = \pi \quad (\lambda/2 \text{ line})$$

$$Z_{in} = Z_L$$

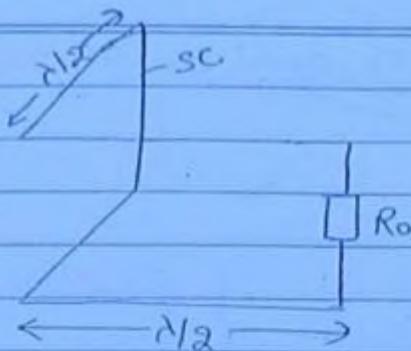
$$Z_{in} = (30 - j40)$$

$$Z = R_0 \left[\frac{R_0 + jR_0}{R_0 - jR_0} \right] \Rightarrow R_0 \left(\frac{j}{2} \right) = R_0$$

classmate

Date _____
Page _____

(ii)



$$\cancel{\text{At the centre}} = 0 || R_0 = 0 \quad (199)$$

$$Z = Z_L = 0 \quad (\text{for } \lambda/2)$$

$$Z = Z_L = R_0$$

10

$$l = \lambda/4$$

$$\beta l = \frac{2\pi}{\lambda} \times \left(\frac{\lambda}{4}\right)$$

$$\beta l = \frac{\lambda}{4} \times \frac{\lambda}{2\lambda} = \frac{\lambda}{8}$$

$$d = \lambda/8$$

It is short circuited at one end:

$$Z_{in} = j60\Omega$$

$$Z_0 = ?$$

$$Z_{sc} = jZ_0 \tan \beta l$$

$$j60 = jZ_0 \tan \frac{\lambda}{4}$$

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0}{Z_0 + jZ_L} \right)$$

$$j60 = Z_0 \left(\frac{0 + jZ_0}{Z_0 + j0} \right)$$

$$j60 = jZ_0$$

$$Z_0 = 60$$

$$jZ_0 = j60\Omega \Rightarrow Z_0 = 60\Omega$$

11

$$\begin{aligned} d &= \lambda/8 & f & \quad f \propto \frac{1}{\lambda} \\ f' &= 2f & & \\ \lambda' &= \lambda/2 & = d' & = 2d' \end{aligned}$$

$$d = \frac{2d'}{8} = \frac{d'}{4}$$

when $l = \lambda/4$ — short ckt at one end.

$$Z_{in} = \infty \quad \text{as} \quad Z_{in} = \frac{Z_0^2}{Z_L} - \frac{Z_0^2}{0} = \infty$$

12.

$$f = 12 \text{ MHz}$$

200

$$Z_{in} = j60$$

$$j60 = j\omega L$$

$$L = \frac{60}{2 \times \pi \times 12 \times 10^6}$$

$$L = \frac{30}{\pi \times 12 \times 10^6}$$

$$L = \frac{9.5}{\pi} \mu\text{H}$$

Tabular Column:

short circuit

$$1. Z_{sc} = jZ_0 \tan \beta l$$

$$0 < \beta l < \frac{\lambda}{2}$$

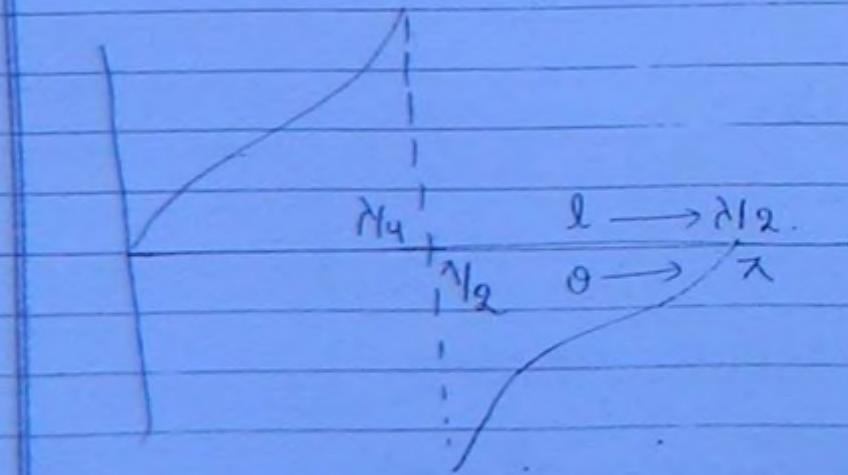
$$0 < l < \frac{\lambda}{4}$$

Z_{in} = Purely Inductive

$$\frac{\lambda}{4} < l < \frac{\lambda}{2}$$

$$\frac{\lambda}{2} < \beta l < \pi$$

Z_{in} = Purely capacitive



2. open circuit line

(20)

$$Z_{OC} = Z_0 \coth(j\beta l) = -jZ_0 / \tanh \beta l$$

$$\left. \begin{array}{l} 0 < \beta l < \pi/2 \\ 0 < l < \lambda/4 \end{array} \right\} Z_{in} = \text{Purely capacitive}$$

$$\left. \begin{array}{l} \lambda/4 < l < \lambda/2 \\ \pi/2 < \beta l < \pi \end{array} \right\} Z_{in} = \text{Purely Inductive}$$

$$\begin{aligned} & 13 \text{ V}^B \\ & Z_0 = ? \\ & C = \text{capacitance} \\ & V = c = 3 \times 10^8 \text{ m/sec.} \end{aligned}$$

ϵ_R

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{L \times C}{C \times C}} = \frac{1}{C} \sqrt{LC} = \frac{1}{C \times V} = \frac{1}{\epsilon_R \times c}$$

$$\text{as } V_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \epsilon_R}}$$

$$Z_0 = \frac{1}{3 \times 10^8 \times C} = \frac{\sqrt{\epsilon_R}}{3 \times 10^8 \times C}$$

Note: Every lossless line is by default distortionless but the vice versa is not true because distortionless is not lossless and but lossless is always distortionless.

$$(i) L C \eta = R C$$

$$R = C \eta = 0$$

$$(ii) \quad \checkmark \quad \beta = \omega \sqrt{LC}$$

$$\beta \propto \omega$$

Note:

$$(1) Z_0 = \text{real} = \sqrt{\frac{L}{C}}$$

$$\Rightarrow \alpha = 0$$

$$\beta = \omega \sqrt{LC}$$

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Lossless Line

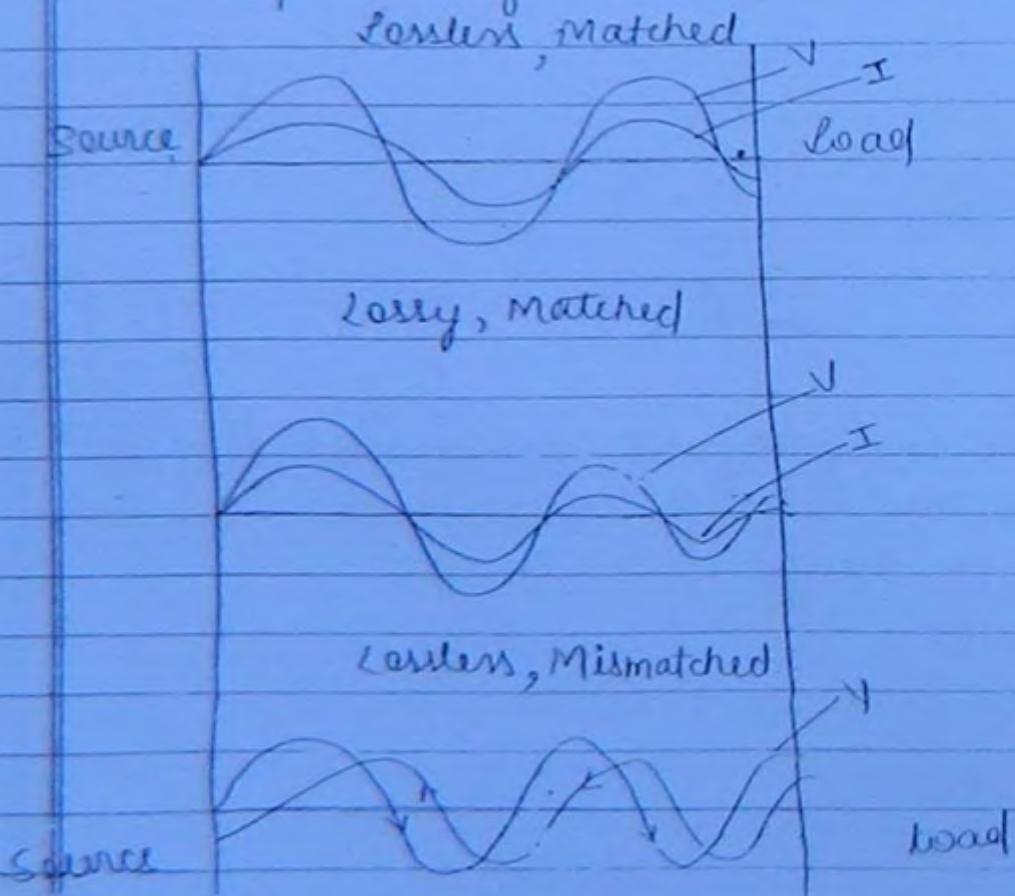
(2) If $\omega = 0$ for DC line (It is not necessary, we can take $Z_0 = \text{real}$)

$$Y = \sqrt{RG} = \alpha + j\beta$$

$$Z_0 = \sqrt{\frac{R}{G}}$$

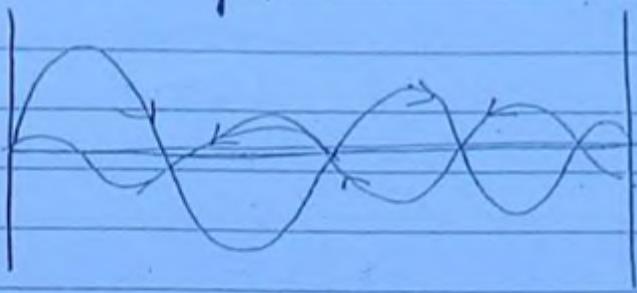
$$= \text{real}$$

- Note
- Loss is a line property. It is R, L, G, C dependent.
 - Attenuation is a load property.
 - Match is a load property. It is Z_L dependent.
 - Power reflections at the load.



Lossy, mismatched

203



Reflection coefficient, Standing wave, SWR.

$$V(x) = V_L \frac{(Z_L + Z_0)}{2Z_L} \left[e^{j\beta x} + \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-j\beta x} \right]$$

consider lossless line

$$= V_0 e^{j\beta x} + V_0 \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-j\beta x}$$

↓
forward wave ↓
reflected wave

$$\begin{aligned} V_r &= \text{Reflection} \\ V_f &= \text{coefficient} \end{aligned} = \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-j2\beta x}$$

for the voltage
anywhere on the line = $\Gamma(x)$

at the load $x=0$

$$\Gamma \text{ at the load} = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| 10$$

$$\text{start of the line} = \left| \frac{V_r}{V_f} \right| 10$$

$|\Gamma| = \left| \frac{V_r}{V_f} \right| = \frac{\text{fraction of the reflected voltage with respect to forward voltage}}{\text{forward voltage}}$

Γ can be a complex no when Z_L is any complex impedance. Hence Γ is

$$|\Gamma| \leq 1$$

Hence $\Gamma = \frac{|V_r|}{|V_f|} \leq 1$

204

Hence $|\Gamma|$ stands for the fraction of the reflected voltage w.r.t to the forward voltage.

It is the ratio of the amplitude of reflected to forward voltages.

$$|\Gamma| = 0 \leq |\Gamma| \leq 1$$

$$|\Gamma| = 0 \text{ when } Z_L = Z_0 \text{ (matched case)}$$

$$|\Gamma| = 1 \text{ complete mismatch}$$

No power absorption at the load
(all reflect back)

$\theta \rightarrow$ the phase difference b/w V_r and V_f

$\Gamma \rightarrow$ It is a measure of mismatch b/w the expected impedance Z_0 and the actual impedance Z_L at the load.

$F(z) \rightarrow$ It is a measure of mismatch b/w the expected impedance Z_0 and the actual impedance $z(x)$ anywhere on the line

$$\Gamma(x) = \frac{z(x) - Z_0}{z(x) + Z_0}$$

Note 1 $\Gamma_x = \text{Reflection coefficient for currents}$
 $= -\Gamma_y$

$$\boxed{\Gamma_x = -\Gamma_y}$$

Ques

$$I(x) = I_0 e^{j\beta x} + I_0 \left(\frac{z_0 - z_L}{z_0 + z_L} \right) \bar{e}^{j\beta x}$$

$$2. I(t) = I_0 e^{j\beta t} - I_0 \left(\frac{z_1 - z_0}{z_L + z_0} \right) \bar{e}^{j\beta t}$$

If forward voltage - are in phase, ref voltage - are out of phase by
forward current ref current phase by 180°

Four cases of complete mismatch on the line ($\Gamma = 1$)

case(i) $z_L = jR_0 = \text{pure Inductive load}$

$z_0 = R_0 = \text{lossless line}$

$$\Gamma = \frac{z_1 - z_0}{z_L + z_0} = \frac{jR_0 - R_0}{jR_0 + R_0} = \frac{j-1}{j+1} = \frac{\sqrt{2} |135^\circ|}{\sqrt{2} |45^\circ|} = j$$

Note Inductance cannot consume any real power hence the complete voltage reflects back with a phase shift of 90° such that if $V_f = \sin$ then $V_r = \cos$

case(ii) $z_L = -jR_0 = \text{pure capacitive load}$

$z_0 = R_0 = \text{lossless line}$

$$\Gamma = \frac{z_L - z_0}{z_L + z_0} = \frac{-jR_0 - R_0}{-jR_0 + R_0} = \frac{-(j+1)}{-j+1} = \frac{j+1}{j-1} = -1$$

case(iii) $\tilde{Z}_L = 0 \times 1$ (short circuit line)

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$\tilde{Z}_0 = R_0$ (loss less line)

$$H = \frac{\tilde{Z}_L - R_0}{\tilde{Z}_L + R_0} = \frac{0 - R_0}{0 + R_0} = -1 = -118^\circ$$

$|H| = 1$ (no power consumed)

Note: A short circuit does not consume any real power such that if the forward voltage is \sin the reflected voltage is $-\sin$ so the net voltage is 0.

$$V_f = \sin$$

$$V_r = -\sin$$

$$\text{Net } V = 0$$

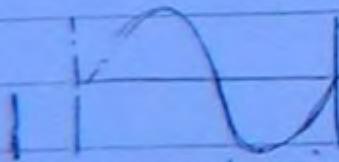
voltage does not exist in short ckt.

case(iv) $\tilde{Z}_L = \infty$ (open circuit line)

$\tilde{Z}_0 = R_0$ (lossless line)

$$H = \frac{\tilde{Z}_L - R_0}{\tilde{Z}_L + R_0} = \frac{1 - \frac{R_0}{Z_L}}{1 + \frac{R_0}{Z_L}} = 1 \Rightarrow 0^\circ$$

$$V_f = \sin, V_r = \sin \quad |H| = 1$$



current does not exist in open ckt only voltage exist

Standing Waves & SWR

207

$$V(x) = V_0 e^{j\beta x} + V_0 |r| e^{j\theta} e^{-j\beta x}$$

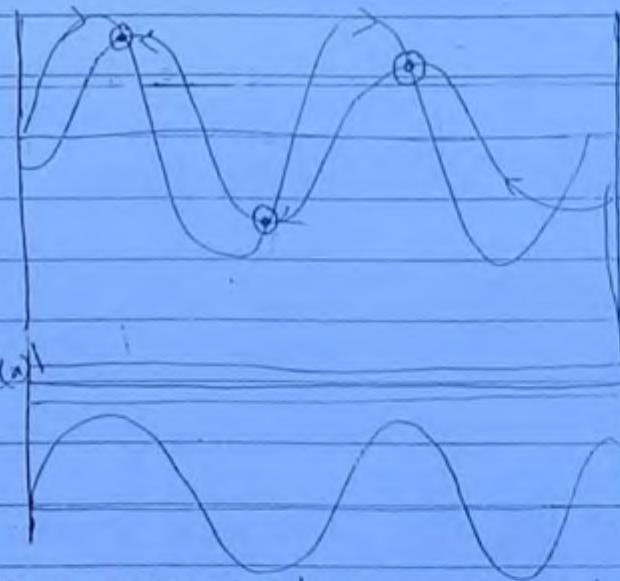
$$= V_0 e^{j\beta x} + V_0 |r| e^{-j(\beta x - \theta)}$$

$$= A \sin \psi_1 + B \sin \psi_2$$

- If there were two waveforms they have unequal amplitudes and different phases depending on reflection coefficient
- If the magnitude plot $|V(x)|$ is plotted for only the forward wave existing matched line. It is a constant valued straight line

$A \sin \psi_1 \propto V \propto \theta \rightarrow$ Harmonic plot

Instead $A \propto V \propto \theta \rightarrow$ Amplitude plot



for
single
harmonic

If two wffs travelling in opposite directions and a phase diff. b/w them interfere among themselves. There is a chance of periodic addition and cancellation of amplitude. This never $V(x)$ occur for wave in same dir. The two wffs are the maxima when they come inphase and a minima when they go out of phase.

$$|V(x)| = A + B = V_0 + V_0 |r| = |V(x)|_{\max} = V_{\max}$$

$$|V(x)| = A - B =$$

$$\beta x - (-\beta x + \theta) = 2n\pi \quad n=0, 1, 2$$

$$\beta x_{\max} = 2n\pi + \theta$$

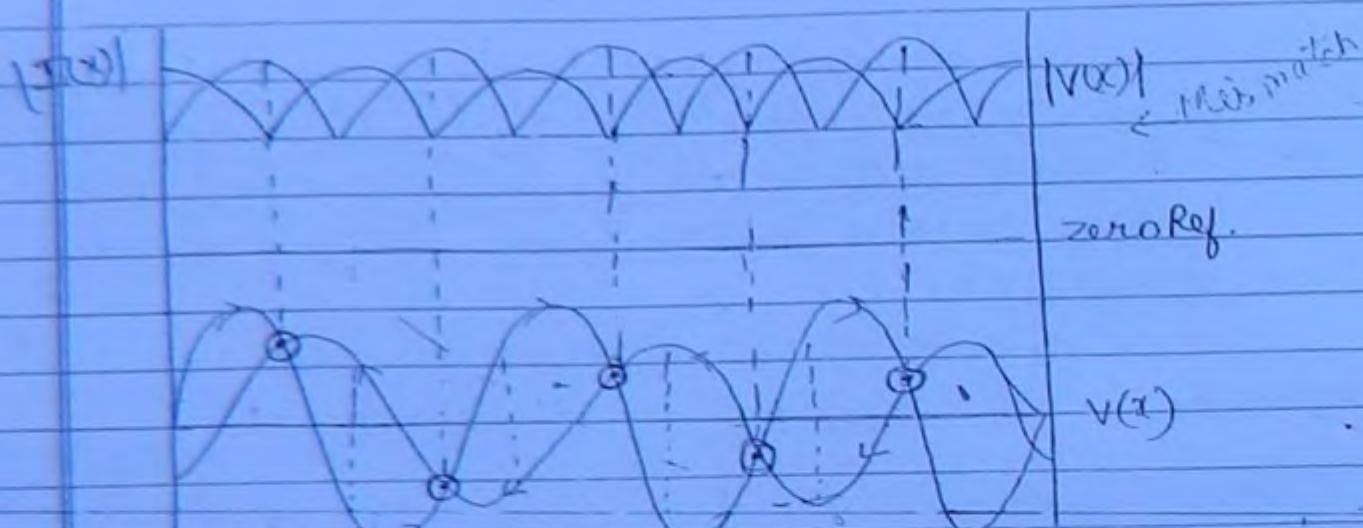
Position of voltage maxima

$$V(x) = (A - B) = V_0 - V_0 |\Gamma| \quad \lambda = |V(x)|_{\min} = V_{\min}$$

$$\beta x - (-\beta x + \theta) = (2n+1)\lambda$$

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$$2\beta x_{\min} = (2n+1)\lambda + \theta$$

Position of voltage
minima

The resultant amplitude plot of two wfs travelling in opposite direction is called as Standing wave formation.

Interference b/w n-sources always leads to points of permanent maxima's or minima's anywhere in space.

i.e. On a standing wave the gap b/w two successive maxima is $\frac{\lambda}{2}$.

$\beta x - 2\pi \rightarrow$ Harmonic

On the Harmonic of $\frac{\lambda}{2}$ period βx repeats such that x repeats for $\frac{1}{2}\lambda$.

On the standing wave of $\frac{\lambda}{2}$ period x_{\max} repeats hence x_{\max} repeats for $\frac{1}{2}\lambda$.

$$2\beta x_{\text{max}} = 2\lambda \quad \text{Standing wave.}$$

$$2x \frac{2\lambda}{x_{\text{max}}} = 2\lambda$$

$$\frac{2\lambda}{x_{\text{max}}} = 2\lambda$$

$\left(\frac{\lambda}{2}\right)$

209

Note 2. Current also forms an identical standing wave but the voltage maxima coincide with current minimal.

$$I(x) = I_0 e^{j\beta x} - I_0 |\Gamma| e^{j\beta x} e^{j\theta}$$

Note 3 At the points of voltage maxima and current minima, the impedance should have been maximum i.e $Z_{\text{max}} = V_{\text{max}} / I_{\text{min}}$

$$Z_{\text{min}} = \frac{V_{\text{min}}}{I_{\text{max}}}$$

SWR \rightarrow Standing Wave Ratio

It can be defined for both CNWR or VSWR.

$$VSWR = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{V_0 + V_0 |\Gamma|}{V_0 + V_0 |\Gamma|} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = SINR$$

$$|\Gamma| = \frac{SWR - 1}{SWR + 1}$$

SWR = Measure of mismatch on the line

$$\text{Range} = [1, \infty]$$

$SWR = 1 \Rightarrow \text{Matched line}$

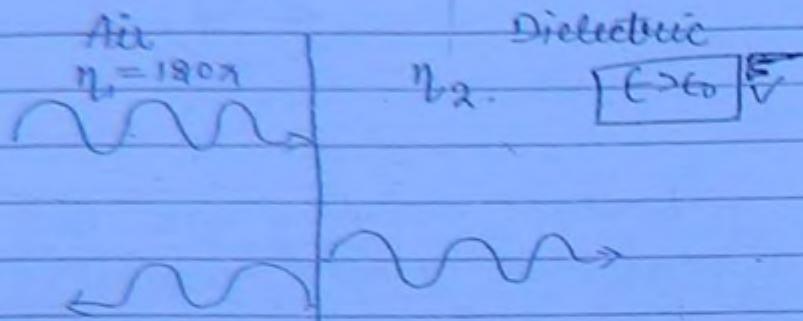
$$\Rightarrow V_{\text{max}} = V_{\text{min}}$$

\Rightarrow No standing wave.

$\text{SWR} = \infty \Rightarrow \text{completely mismatch}$
 $\Rightarrow |T'| = 1$
 $\Rightarrow V_{\min} \text{ or } P_{\min} \text{ is zero.}$

(210)

Q5 W.B.



- given $E\text{SWR} = 5$

$$|T'| = \frac{5-1}{5+1} = \frac{2}{3}$$

$$T' = \frac{n_2 - 120\pi}{n_2 + 120\pi}$$

$$T' = |T'|/10 = \frac{n_2 - n_1}{n_2 + n_1}$$

$\frac{120\pi}{\sqrt{\epsilon_r}}$ Real

Real

$$\theta = 0^\circ \text{ or } 180^\circ$$

SWR always gives $|T'|$ which is $T' = |T'|/10$

In this problem T' is real as n_1, n_2 are real
and hence T' is $\frac{2}{3}$

$$\theta = 0^\circ \text{ or } 180^\circ$$

$$n_2 < n_1 = -\text{ve}$$

$$n_2 = \frac{n_1}{\sqrt{\epsilon_r}}$$

at $\epsilon > \epsilon_0$ so Γ is -ve.

$$-\frac{2}{3} = \frac{\eta_2 - 120\pi}{\eta_2 + 120\pi}$$

(21)

$$\eta_2 = 84\pi \Omega$$

$$\eta = \sqrt{j\omega u}$$

$$\text{if } \sigma = 0 \quad \eta = \sqrt{\frac{u}{\epsilon}} = \eta_{\max}$$

$$\text{So } \eta \leq 120\pi$$

|v(z)|

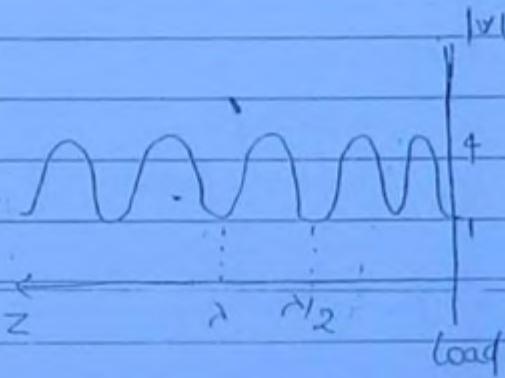
$$26 \text{ wB } Z_0 = 50 \Omega \text{ real}$$

$$|\Gamma| = \frac{Z_L - Z_0}{Z_L + Z_0}$$

real.
 $\frac{1}{R_{\text{eff}}(\text{resistive})}$

$$|\Gamma| = \frac{4-1}{4+1} = \frac{3}{5}$$

$$\frac{+3}{5} \quad \theta = 0^\circ \text{ or } 180^\circ$$



voltage is minimum at load $Z_L < Z_0$

$$2\beta x_{\min} = (n+1)\pi + \theta$$

$$\theta = 0 + \lambda + \theta$$

$$\theta = -\lambda = 180^\circ \quad \Gamma's \text{ phase} = 180^\circ$$

$$\frac{-3}{5} = \frac{Z_L - 50}{Z_L + 50} \Rightarrow -3Z_L - 150 = 5Z_L - 250$$

$$100 = 8Z_L$$

$$Z_L = \frac{100}{8} - \frac{50}{4} = 25 - 12.5 = 12.5 \Omega$$

Note 1

$$\Gamma = \frac{z_L - z_0}{z_L + z_0}$$

212

$$2\pi f = Rn\lambda + \theta$$

If $z_L = \text{resistive}$ } Real.
 $z_0 = \text{lossless line}$ }
 $\Gamma = \text{Real}$

F

If $z_L > z_0$ then $\epsilon \Gamma = \text{positive}$
 voltage maxima occur at the load.

$$x_{\max} = 0 \text{ if } n=0$$

e.g.: S/C line $z_L = \infty$. $|\Gamma| = 1$

If $z_L < z_0$ & $\Gamma = \text{negative}$
 voltage minima occur at the load.
 $x_{\min} = 0 \text{ if } n=0$

e.g. S/C line $z_L = 0$ $|\Gamma| = -1$

Note 2. If z_L, z_0 are real

$$\text{SWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + \left| \frac{z_L - z_0}{z_L + z_0} \right|}{1 - \left| \frac{z_L - z_0}{z_L + z_0} \right|} = \frac{1 + \frac{z_L - z_0}{z_L + z_0}}{1 - \frac{z_L - z_0}{z_L + z_0}}$$

If $z_L > z_0$

$$\boxed{\text{SWR} = \left| \frac{z_L}{z_0} \right|}$$

$$\boxed{\text{SWR} = \left| \frac{z_0}{z_L} \right|}$$

Wednesday
 VSWR = 5
 $\Gamma = 1/10$

(213)

$$|\Gamma| = \frac{5-1}{5+1} = \frac{2}{3}$$

$$17\text{ cm} - 27\text{ cm}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\frac{\lambda}{2} = 10 \Rightarrow \lambda = 20$$

1^{st} min	2^{nd} min	3^{rd} min
7	17	27
$n=0$	$n=1$	$n=2$

$$2\beta x_{\min} = (2n+1)\frac{\lambda}{2} + \theta$$

$$2 \times \frac{2\pi}{\lambda} (4) = \pi + \theta$$

$$\theta = \frac{2 \times 7\pi - \pi}{10}$$

$$\theta = \frac{2 \times 7\pi - \pi}{5}$$

$$= \frac{14\pi - 5\pi}{5} =$$

$$\theta = \frac{8\pi}{20}$$

$$VSWR = 5$$

$$Z_0 = 50\Omega$$

$$|\Gamma| = \frac{5-1}{5+1} = \frac{2}{3}$$

χ_q — 1st voltage maximum

$$\Rightarrow |\Gamma| = \frac{Z_L - Z_0}{Z_L + Z_0}$$

(214)

γ_4 load.

If maxima is at λ_1 , the minima is at the load.
means $Z_L < Z_0$ means $|E| = -ve$

$$\frac{-2}{3} = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow \frac{Z_L - 50}{Z_L + 50}$$

$$-2Z_L - 100 = 3Z_L - 3 \times 150 \\ 50 = 5Z_L$$

$$Z_L = 50 \Omega$$

$$VSWR = \frac{Z_0}{Z_L}$$

short

Ans - B

$$Z_0 = 30\Omega$$

$$Z_0 = 30\sqrt{2}\Omega$$

$\lambda_1 = 1.4\lambda$

$$(2) \rightarrow Z_{in} = \frac{Z_0(Z_L + jZ_0)}{(Z_0 + jZ_L)} = \frac{30(0 + j30)}{(30 + j0)}$$

(for S.C)
 $\lambda/8$

$$Z_{in} = j30$$

$$(3) \rightarrow Z_{in} = \frac{Z_0^2}{Z_L} = \frac{(30\sqrt{2})^2}{30} = \frac{900 \times 2}{30} = 30 \times 2 = 60\Omega$$

(2) and (3) are in series.

$$(j30 + 60)$$

215

W.B
51

$$\text{phase} = -150^\circ$$

$n=0$

$$\beta = \frac{2\pi}{150} = \frac{\pi}{75}$$

$$2\beta x_{\max} = (2n\pi) + \theta$$

$$2 \times \frac{2\pi}{150} \times x_{\max} = (2 \times 0 \times \pi) + \theta = -\frac{5\pi}{6}$$

$n=1$

$$2\beta x_{\max} = (2\pi) + \theta$$

$$2 \times \frac{2\pi}{150} \times x_{\max} = 2\pi - \frac{5\pi}{6} = \frac{7\pi}{6}$$

$$x_{\max} = \frac{7 \times 25}{4} = 43.75 \text{ m}$$

$$1^{\text{st}} \text{ max} \quad 43.75 \text{ m}$$

$$2^{\text{n}} \text{ max} \quad 43.75 + 75 = 118.75 \text{ m}$$

$$3^{\text{rd}} \text{ max} \quad 118.75 + 75 = 193.75 \text{ m}$$

$$4^{\text{th}} \text{ max} \quad 193.75 + 75 = 268.75 \text{ m}$$

$$5^{\text{th}} \text{ max} \quad 268.75 + 75 = 343.75 \text{ m}$$

$$6^{\text{th}} \text{ max} \quad 343.75 + 75 = 418.75 \text{ m}$$

$$7^{\text{th}} \text{ max} \quad 418.75 + 75 = 493.75 \text{ m}$$

F at input

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$$Z_{in} = Z_{in1} \parallel Z_{in2}$$

=

$$Z_{in1} = \frac{Z_0^2}{Z_L} = \frac{(50)^2}{100} = \frac{2500}{100} = 25$$

$$Z_{in2} = \frac{Z_0^2}{Z_L} = \frac{(50)^2}{200} = \frac{2500}{200} = \frac{25}{2}$$

$$Z_{in} = \frac{25 \parallel 25}{2} \Rightarrow \frac{1}{25} + \frac{1}{25} = \frac{3}{25} = \frac{3}{3} = \frac{25}{25} = \frac{1}{3}$$

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \Rightarrow \Gamma = \frac{\frac{25}{3} - 50}{\frac{25}{3} + 50} \neq \frac{25 - 150}{25 + 150} = \frac{-135}{175}$$

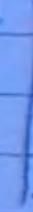
$$\Gamma = \frac{-135}{175}$$

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{(50)^2}{25} \times 3 = \frac{2500 \times 3}{25} = 300$$

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{300 - 50}{300 + 50} = \frac{250}{350} = \frac{5}{7}$$

Source

20V



$$\begin{aligned} & \text{6000} \\ & \downarrow V_s \quad V_s \\ & 30+10 = 40V \\ & 10 \parallel 20 \end{aligned}$$

400 μs

A+B = then

 $\Gamma = 1/V_s$

$$\Gamma = \frac{V_s}{V_F} = \frac{+10}{30} = \frac{1}{3}$$

$$\frac{1}{3} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\frac{1}{3} = \frac{Z_L - 50}{Z_L + 50} \Rightarrow Z_L + 50 = 3Z_L - 150$$

$$\Rightarrow 200 = 2Z_L \quad Z_L = 100$$

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Steady state current = $I = \frac{30}{100} = \frac{3}{10} = 0.3$ Amp

29 W B Two sources are there

$$20. W^B \quad Z_0 = 50\Omega \quad Z_L = 40 + j30\Omega \quad \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{90 + j30 - 50}{40 + j30 + 50}$$

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = \frac{4}{2} = 2 \quad F = \frac{j30 + 10}{j30 + 90}$$

$$= \frac{3+1}{3} \times \frac{3}{3-1} = \frac{4}{2} = 2 \quad \Gamma = \sqrt{900 + 100} = \sqrt{1000}$$

$$\sqrt{900 + 8100} \quad \sqrt{9000}$$

$$\Gamma = \sqrt{\lambda g} = \frac{1}{3}$$

31. W B $Z_0 = 50\Omega \quad \Gamma = 0.268$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow 0.268 = \frac{Z_L - 50}{Z_L + 50}$$

$$\Rightarrow Z_L = 86$$

$$\text{transmitted power} = (1 + \Gamma) (1 - \Gamma) \cdot \frac{V_a^2}{Z_L}$$

$$\frac{\text{Transmitted power}}{\text{power}} = (1+E) V_L \cdot (1-\Gamma) \frac{V_L}{Z_L}$$

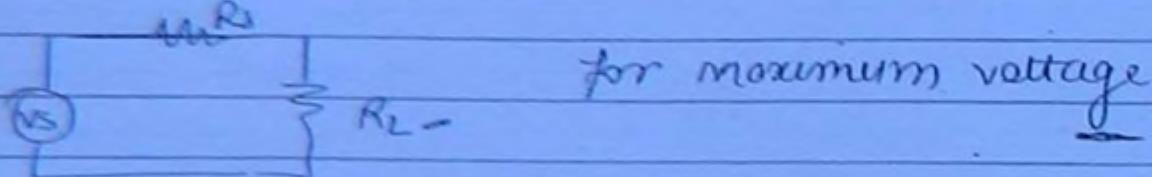
$$= (8+0.268)$$

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$$= 2.428$$

$$P_{\max} = V_{\max} \times I_{\min}$$

$$= I_{\max} \times V_{\min}$$



N.B.

$$\text{VSWR} = 3 = \frac{V_r}{V_s}$$

$$|\Gamma_E| = \frac{\text{VSWR}-1}{\text{VSWR}+1} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

$$(\Gamma_p) = -\Gamma_E^2$$

$$= \frac{1}{4} = 25\%$$

N.B.

$$\Gamma = 0.6 \angle 60^\circ$$

$$|\Gamma| = 0.6$$

$$\lambda = \lambda/8$$

$$\Gamma(x) = Z_L z_0 e^{-j2\beta x}$$

$$\Gamma(x) = \Gamma_L e^{-j2x \frac{2\lambda}{\lambda/8}}$$

$$\Gamma(x) = 0.6 e^{j60^\circ} e^{-j30^\circ}$$

$$= 0.6 e^{j(-30^\circ)}$$

$$\Gamma(x) = 0.6 | -30^\circ \rangle$$

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35 W.B. $V_f = 25 \sin(\beta x - 45^\circ)$

$$\Gamma = 0.6 | -30^\circ \rangle$$

$$\Gamma = \frac{V_f}{V_f} \Rightarrow \frac{25 \sin(\beta x - 45^\circ)}{25} = 0.6 | -30^\circ \rangle$$

$$V_f \Rightarrow \frac{25 \sin(\beta x - 45^\circ)}{0.6 | -30^\circ \rangle}$$

$$0.6 = \frac{V_f}{V_f} \Rightarrow V_f = 25 \times 0.6 = 15$$

$$V_f = 15 \sin(\beta x - 105^\circ)$$

$$\Gamma_{\text{phase}} = V_{r\text{phase}} - V_{f\text{phase}}$$

$$V_{r\text{phase}} = F_{\text{phase}} + V_{f\text{phase}}$$

$$= -30^\circ - 45^\circ$$

$$= -75^\circ$$

Q6 $2\beta x_{\min} = (2n+1)\pi + \theta$

for $n=0$

$$2x \frac{\lambda}{\lambda} x_{\min} = (\pi) - 30^\circ$$

$$2x \frac{\lambda}{\lambda} x_{\min} = \pi - \frac{\pi}{6}$$

$$x_{\min} = \frac{5\lambda}{6} \times \frac{\lambda}{4\lambda}$$

$$x_{\min} = \frac{5\lambda}{24}$$

Impedance Matching Techniques

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Loads are typically high impedances of the order of 100Ω to a few $k\Omega$ whereas line impedance are few Ω to 100Ω . Hence intermediate devices are placed b/w the load and the line this is called as impedance matching devices.

Impedance Matching techniques.

Active
(Bias)

CB amp
CC amp.

Passive

shunt

Stub Matching

series

$\lambda/4$ quarter wave
transformer

$\lambda/4$ quarter wave transformer

A quarter wave transformer is a $\lambda/4$ length line placed in series b/w the line and the load.

The load of the Z_0 line = Z_m of the Z'_0 line of $\lambda/4$ length.

$$= \frac{Z_0}{Z_L}$$

$$= Z_0$$

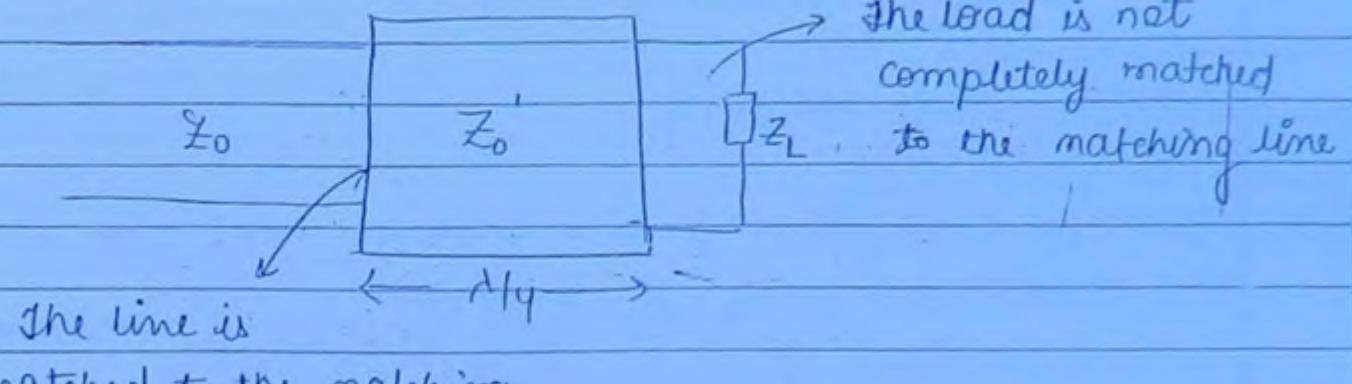
Hence Z'_0 should be such that it is the geometric mean of line and load impedances. so that the impedance converter offers the exact low impedance to the line due to high impedance load.

Hence the line is perfectly matching i.e. no matching device

However the Z_0' matching device this is not matched to.

Z_L load

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e.g.: 50 Ω - line
100 Ω - load.

$$\Gamma = \frac{100-50}{100+50} = \frac{1}{3}$$

with matching device

$$Z_0' = \sqrt{50 \times 100} = 71$$

$$\Gamma = \frac{100-71}{100+71} = \frac{1}{3}$$

The mismatch b/w the matching device and the load is certainly less than the mismatch without the device hence an improvement

Disadvantage.

1. As the transformer unit is Δ_y it cannot be used over a wide range of frequencies
2. The technique is useful only for resistive loads or lossless line.

so $Z_0' = \text{real and positive}$

(222) 1

$$Z_0' = \sqrt{Z_0 Z_L}$$

Shunt - stub Matching

Step 1. Identify a length on the line from the load side where the impedance is $Z_0 + jx = z(x)$

$$y(x) \Big|_{x=ls} = Y_0 \pm jB$$

This x value is called as ls or position of the stub

Mathematically

$$\text{Real } \left[\frac{Z_L \cos \beta x + j Z_0 \sin \beta x}{Z_0 \cos \beta x + j Z_L \sin \beta x} \right] = 1$$

$\Rightarrow x = ls$

Rationalize the eqn.

$$\frac{Z_L \cos \beta x + j Z_0 \sin \beta x}{Z_0 \cos \beta x + j Z_L \sin \beta x} \times \frac{Z_0 \cos \beta x - j Z_L \sin \beta x}{Z_0 \cos \beta x - j Z_L \sin \beta x}$$

$$\Rightarrow Z_L^2 \cos^2 \beta x$$

$$Z = U = \frac{1}{2\lambda} \tan^{-1} \sqrt{\frac{Z_L}{Z_0}}$$

Step 2 At this position an equal and opposite reactance is placed such that $y(l_s) = Y_0 + jB$

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as $-jB$ added to it

$$y(l_s) = Y_0 \pm jB - jB \text{ such that}$$

$$y(l_s) = Y_0$$

so that $-jB$ is a stub Reactance.

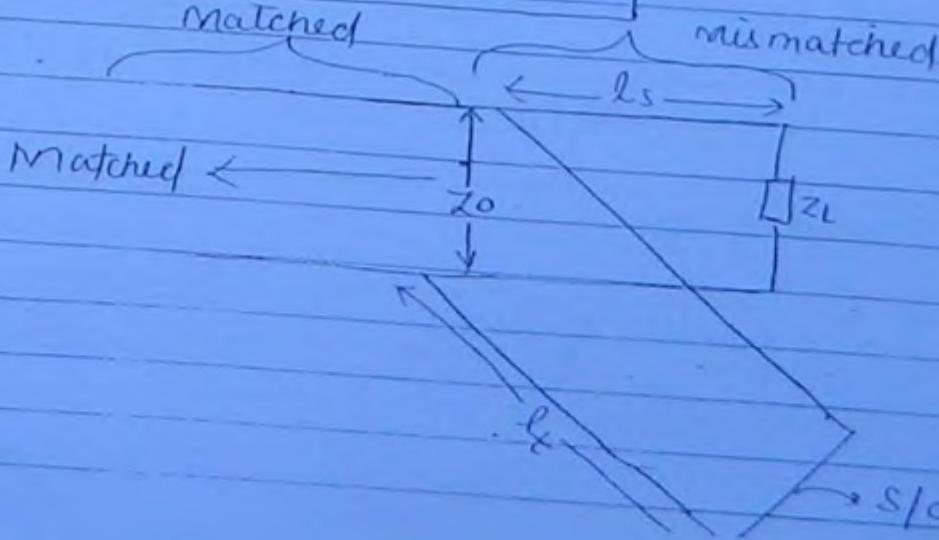
Step 3 This reactance is provided by a short circuited line of length. because every short circuit line has.

$$Z_{sc} = jZ_0 \tan \beta l$$

Hence a stub is short circuit line of length which is precalculated such that

$$Z_0 \tan(\beta l_t) = -\text{Imag} \left[Z_0 \begin{bmatrix} Z_L \cos \beta l_t + jZ_0 \sin \beta l_t \\ Z_0 \cos \beta l_t + jZ_L \sin \beta l_t \end{bmatrix} \right]$$

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \sqrt{\frac{Z_L}{Z_0}}$$



Note

The stub should always be placed at close as to the load as possible

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Disadvantage:

l_s and l_{t2} are λ dependent hence the stub has to be moved each time of frequency changes.

Note

The stub length can be adjusted by moving a short to and fro on the stub or hence short circuited stub are preferred over open circuited stub.

- The disadvantage is partly removed using a double stub matching which is l_s , and l_{t2} both fixed.
- l_s and l_{t2} both are variable hence it can be used for a wide range of frequencies

Smith chart - circle diagram:

- It is a rectangular graph.
- Polar plot
- Calculate Γ_s , VSWR but known $(\frac{Z_L}{Z_0})$ i.e. Z_L & Z_0 should be known.

 Γ_s vs Γ_i $|\Gamma|$ vs θ

$$\frac{Z_L}{Z_0} = \text{Normalized load Impedance} = R + jX$$
 Z_0

- Dividing by Z_0 - Normalization
- It is a graph Γ_s and Γ_i axis

Graph consist of → 2 families of circles

$$R + jX$$

- Const R circle
- Const X circle

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$$\Gamma_s + j\Gamma_i = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} = \frac{R + jX - 1}{R + jX + 1}$$

$$\Gamma_s + j\Gamma_i = \frac{(R-1) + jX}{(R+1) + jX}$$

Const R circle Eqⁿ:

$$\left(\frac{\Gamma_s - R}{R+1}\right)^2 + \left(\frac{\Gamma_i}{R+1}\right)^2 = \left(\frac{1 - \frac{R}{R+1}}{R+1}\right)^2$$

Const X circle Eqⁿ

$$\left(\frac{\Gamma_s - 1}{R+1}\right)^2 + \left(\frac{\Gamma_i - \frac{X}{R+1}}{R+1}\right)^2 = \left(\frac{1 - \frac{X}{R+1}}{R+1}\right)^2$$

Properties of const R circle:

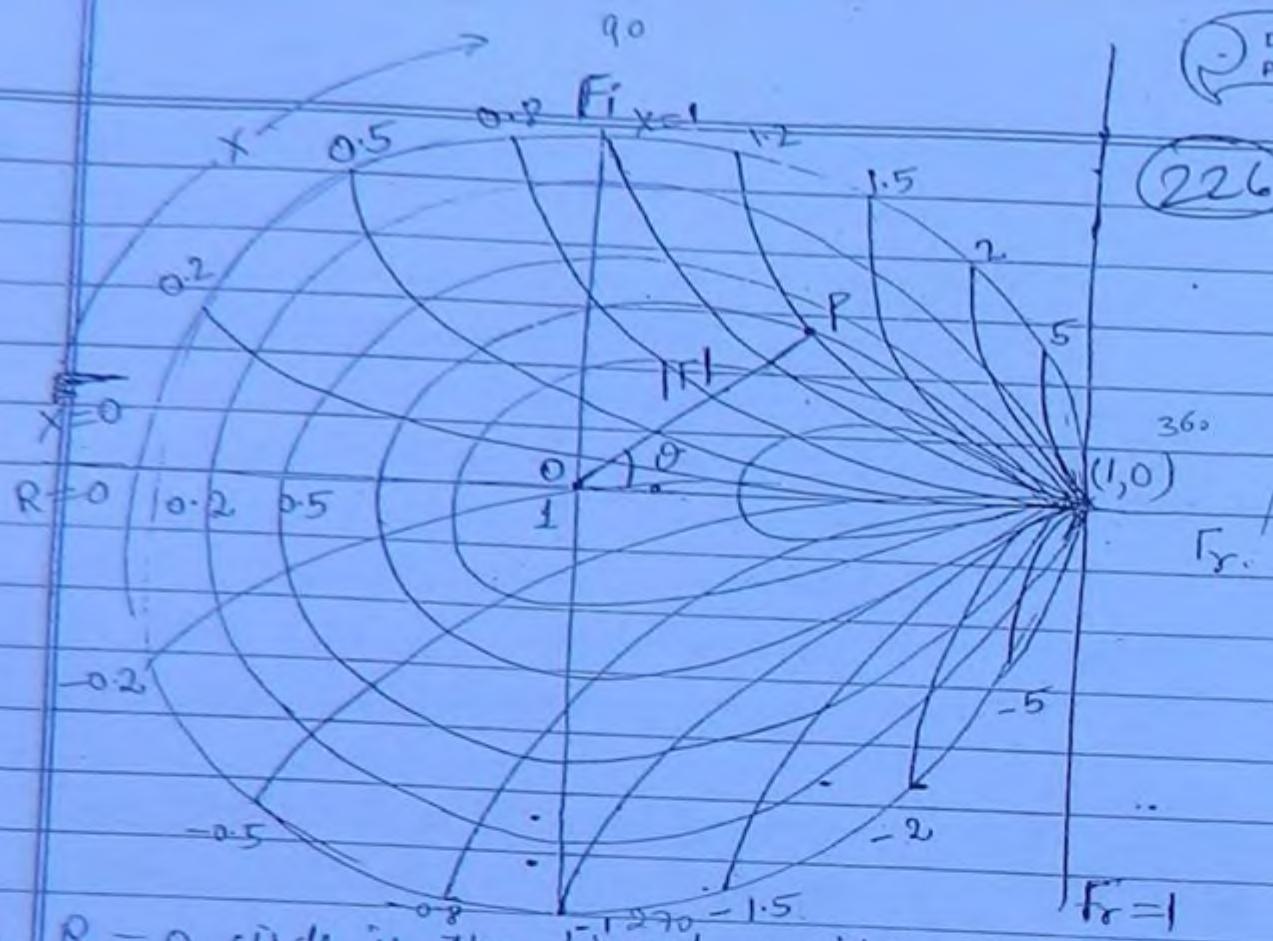
i) Centres $(\frac{R}{R+1}, 0)$

ii) Radius $\left(\frac{1}{R+1}\right)$

iii) passes through $(1, 0)$

iv) Range R $[0, \infty]$

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$R=0$ circle is the biggest possible circle and it is the boundary of the Smith chart. The centre is at all the origin and radius = 1.

$R=\infty$ circle passes through the origin and centre is $(\frac{1}{2})$ and radius is also $(\frac{1}{2})$

As R increases the circle.

$R=\infty$ is the point $(1,0)$

They all have common tangent. $r_{real} = 1$ cm
they all concurrent circles with their centers on the r_{real} axis

All the circles with R 0 to 1 lie in four quadrants
the circles with $R > 1$ lie only in the 1st and 4th quadrant

Properties of constt x circles:

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- (i) centre $(1, \frac{1}{x})$
- (ii) Radius $\left(\frac{1}{x}\right)$
- (iii) will pass through $(1, 0)$
- (iv) Range of $x [-\infty, \infty]$

Note: All the x circles have common tangent the real axis. They are all concave to the right.

2. Circles with $x > 0$ to 1 lie in the first and second quadrant. Circles with $x > 1$ lie in the 3rd and fourth only in the first quadrant

If we traverse on the $R=0$ circle clockwise the x increases inductively.

Similarly if we traverse on the $x=0$ circle horizontally R increases.

A common tangent of R -circles contains the centres of x circles and vice versa. Hence R and x form a orthogonal family of circles.

$$z_L = jR_0, z_0 = R_0$$

$$\frac{z_L}{z_0} = \frac{jR_0}{R_0} = j \Rightarrow 0+j = R+jx$$

$$j = j \quad R=0$$

$$x=1$$

$$z_L = -jR_0, z_0 = R_0$$

$$\frac{z_L}{z_0} = \frac{-jR_0}{R_0} = 0-j = R+jx$$

$$R=0$$

$$x=-1$$

$$j = -j$$

3. $Z_L = 0 \quad Z_0 = R_0$

$$\frac{Z_L}{Z_0} = \frac{0}{R_0} = 0+0 = R+jX$$

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$$R=0, X=0 \quad F = -1$$

F

4. $Z_L = \infty \quad Z_0 = R_0$

$$\frac{Z_L}{Z_0} = \frac{\infty}{R_0} = \infty = R+jX$$

$$R=\infty, X=\infty \quad F=-1$$

5. $Z_L = R_0 \rightarrow Z_0 = R_0$

$$\frac{Z_L}{Z_0} = \frac{R_0}{R_0} = 1+0j = R+0jX$$

$$\therefore R=1, X=0, \quad F=0$$

Procedure for Calculation of I'

Identify the intersection of the $R + X$ circle.
Called pt p and join'd the origin O.

- OP length = $|r'|$

- Phase of $\{r'\}$ is the inclination angle of the segment op with positive r' real axis is a phase of I'

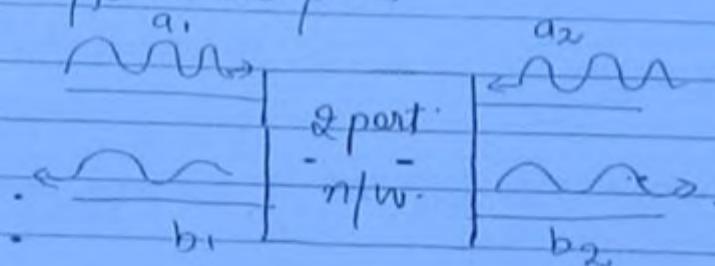
Thursday

S - parameters - Scattering matrix of a TL

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Regular two port n/w models relate the voltage and currents across the two ports but at high frequencies these models fail to explain because we do not have V and I at high frequencies hence S parameters are used.

Static matrix relates the incident wfs at the ports to the outward wfs at the ports



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$b_1 = s_{11}a_1 + s_{12}a_2$$

$$b_2 = s_{21}a_1 + s_{22}a_2$$

$$s_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} = \text{Reflection coeff. at port 1}$$

$$s_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0} = \text{Reflection coeff. at port 2.}$$

$$s_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} \quad \begin{array}{l} \text{Transmission coefficient} \\ \text{from port 2 to port 1} \end{array}$$

$$s_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \quad \begin{array}{l} \text{Transmission coefficient} \\ \text{from port 1 to port 2.} \end{array}$$

Note 1

$$S_{11} = S_{22}$$

Symmetric Network

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2. If $S_{11} = S_{22} = 0$

Symmetric and matched

3. $S_{12} = S_{21}$

Reciprocal N/W

e.g. (Linear N/W, Power supply)

If the n/w is linear elements based. It is always reciprocal.

4. $|S_{12}| = |S_{21}| = 1$

Lossless N/W.

$$\begin{aligned} Z_0 &= 100\Omega \quad | \quad \frac{500 - 100}{500 + 100} = \frac{400}{600} = \frac{2}{3} \\ Z_L &= 500\Omega \end{aligned}$$

~~W.B.~~ $Z_L = 225 \text{ ohms}$ $Z_0 = 256 \text{ ohm}$

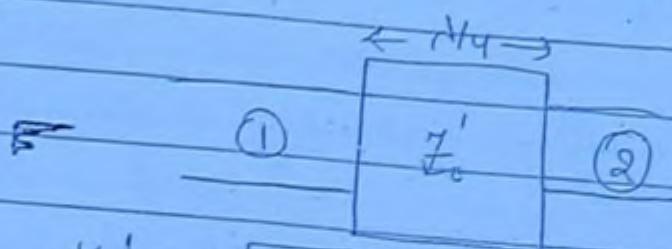
$$Z'_0 = \sqrt{225 \times 256} \quad \text{as } Z'_0 = \sqrt{Z_0 Z_L}$$

43^{W.B.}
=

$$50\Omega \rightarrow Z_0 \quad L_1$$

$$72\Omega \rightarrow Z_L \quad L_2$$

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$$Z_0' = \sqrt{Z_0 Z_L} = \sqrt{50 \times 72} = 60$$

$$60 \ln\left(\frac{b}{a}\right) = 60$$

$$\ln\left(\frac{b}{a}\right) = 1$$

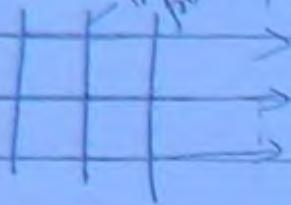
$$\left(\frac{b}{a}\right) = e^{(1)}$$

$$b = 27$$

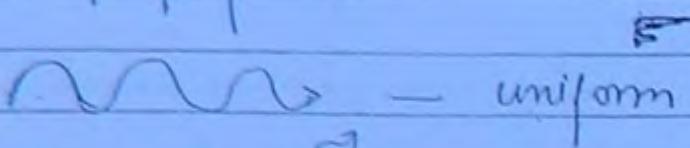
52(n.B) 1. the impedance on any line of length $\lambda/2$ values of hence a circle diagram repeats for 360° and hence one revolution around the smith chart $\lambda/2$ distance

planar
wavefront
Waveguide

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uniform, plane wave

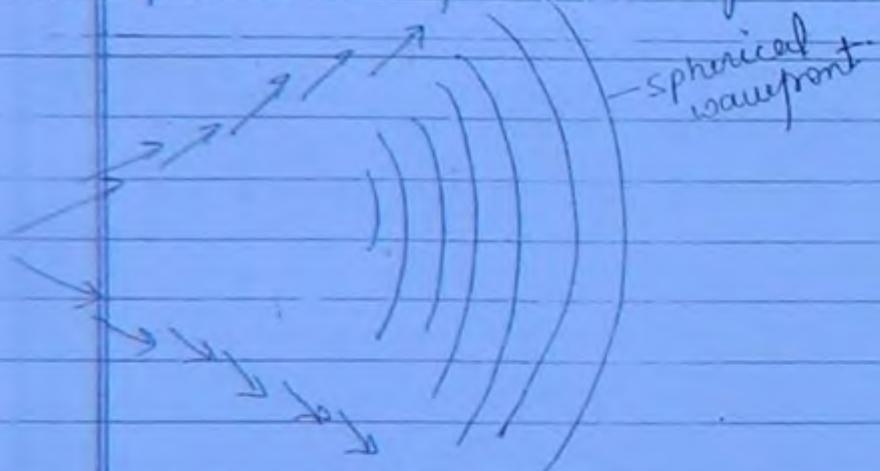


→ uniform

→ Expanding wave (Dispersive wave)

uniform plane plane wave travel in straight line and have a planar wavefront hence the strength everywhere can be assumed to be same.

However most EM waves come from various sources which have dispersive nature and hence they form spherical wavefront as they travel forward as shown.



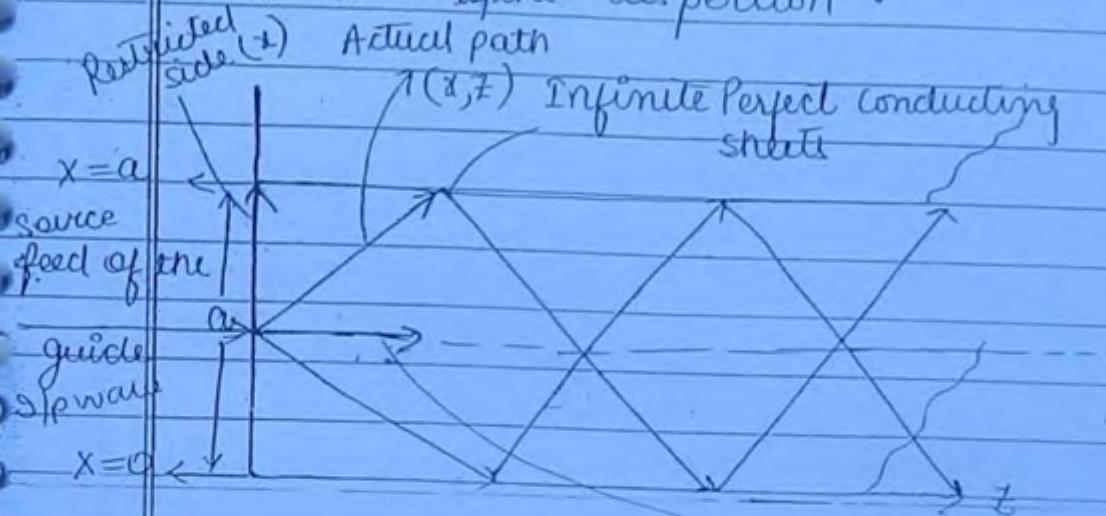
This is the cause of diffraction and diffraction properties of EM waves. This is called as Huygen's Principle

Hence waveguides are used confined electromagnetic wave within specific boundary. We can use parallel plane waveguide and rectangular waveguide with parallel plane waveguide one dimensional confinement. Rectangular waveguide for two dimensional.

- Parallel plane w/or - $E(x, y, t) / H(x, z, t)$
 Rectangular w/or $E(x, y, z, t) / H(x, y, z, t)$ (233)

Parallel Plane Waveguides

- The wave is assumed to travel in z but also disperse in x only.
- Two infinitely large conducting sheets confine the wave and avoid dispersion



- Every conductor should have $E_{tan} = 0$

$$\boxed{E_{tan} = 0}$$

final path
guide axis (z)

i.e. $E(z)$ at $x=0$

$E(z)$ at $x=a$

should not have component parallel to the conductor

$$\boxed{E(z)_{tan} = 0}$$

at $x=0 \& a$

Inverse Eqs in waveguide

$$\nabla^2 F = \gamma^2 F$$

$$\nabla^2 H = \gamma^2 H$$

Solving Helmholtz's Eqns for $x \& z$ dimensions

Assumption:

(234)

$\gamma = \text{freespace propagation constt b/w the guides}$

$$\gamma = j\omega \sqrt{\mu_0 \epsilon_0}$$

$E(z)/H(z) \rightarrow \text{Propagation in the guide axis}$

No Boundaries

No Restriction

Any natural Harmonic of space-z

$-\bar{r}_z$

e

$\gamma = \gamma_z = \bar{r} = \text{propagation constt along the guide axis}$

$$E(z) = e^{\bar{r}z}$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu_0 \epsilon_0 E$$

$$\frac{\partial^2 E}{\partial x^2} + \bar{r}^2 E = -\omega^2 \mu_0 \epsilon_0 E$$

$$\frac{\partial^2 E}{\partial x^2} = -(\omega^2 \mu_0 \epsilon_0 + \bar{r}^2) E$$

Hence the wave is following the harmonic solution in x -dimension also such that

$$\bar{r}^2 + \omega^2 \mu_0 \epsilon_0 = \gamma_z^2$$

Propagation constt

in restricted dimension (x -dim.)

$$\frac{\partial^2 E}{\partial x^2} + \gamma_z^2 E = 0$$

$(D^2 + m^2 = 0)$ (Trigonometric Harmonic)
 $(D^2 - m^2 = 0)$ Natural Harmonic

Solution of the eqn.

$$E(x) = C_1 \sin(r_x \cdot x) + C_2 \cos(r_x \cdot x)$$

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Applying boundary conditions.
at $x=0$ $E(x)_{\text{tan}} = 0$

$$E(x) = 0 + C_2$$

$$\Rightarrow C_2 = 0$$

Conclusion - In the restricted dimension the wave solution is always a trigonometric harmonic such that in tangential component of a wave Electric field should be \sin harmonic only.

at $x=a$

$$E(x) = C_1 \sin(r_x \cdot a) = 0$$

$$r_x \cdot a = m\pi$$

$$m = 0, 1, 2, 3, \dots$$

$$r_x = \frac{m\pi}{a}$$

Hence
$$\bar{r} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \epsilon_0 \mu_0}$$

Concept 1:

cut-off frequency of a guide

$$\bar{r} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \epsilon_0 \mu_0}$$

$$\left(\frac{m\pi}{a}\right)^2 > \omega^2 \mu_0 \epsilon_0$$

$$\bar{r} = \text{real} = \alpha + j\beta$$

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No propagation - No wave.

The \bar{r} is either real or imaginary but never complex
Hence

Hence the wave is not propagating but is exponentially
removed by the guide walls

$$\omega^2 \mu_0 \epsilon_0 > \left(\frac{m\pi}{a}\right)^2$$

$$\bar{r} = 0 + j\beta$$

No attenuation and propagation const along the guide
axis exist

Hence every waveguide has the minimum cut off
frequency below which there cannot be propagation

Hence

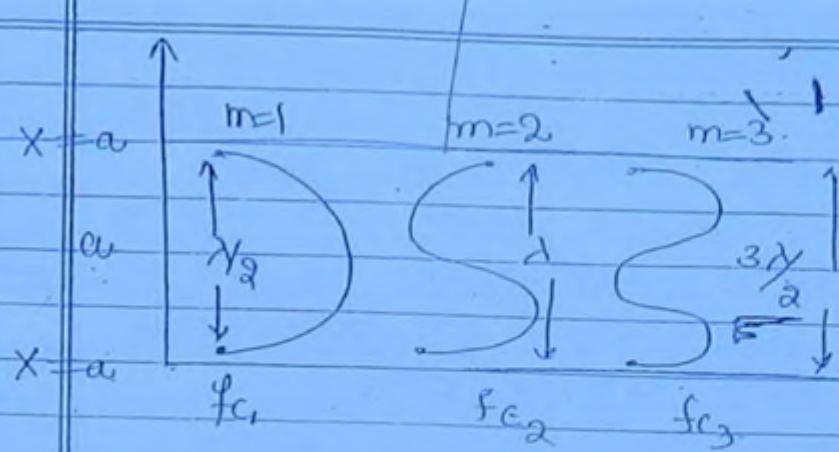
$$\frac{m\pi}{a} = \sqrt{\omega^2 \mu_0 \epsilon_0}$$

$$\frac{m\pi}{a \sqrt{\mu_0 \epsilon_0}} = \omega = \frac{mc}{a}$$

$$f_c = \frac{mc}{2a}$$

$$\lambda_c = \frac{c}{f}$$

$$\lambda_c = \frac{2a}{m}$$



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At exact cut off frequency there is no propagation along the guide axis but the wave resonates b/w the guide walls.

concept 2. Wave angle or tilt angle.

$$\bar{\gamma} = j\bar{\beta} = j\sqrt{\omega^2 u_0 \epsilon_0 - \left(\frac{m\pi}{a}\right)^2}$$

\bar{V}_p = phase velocity along the guide axis

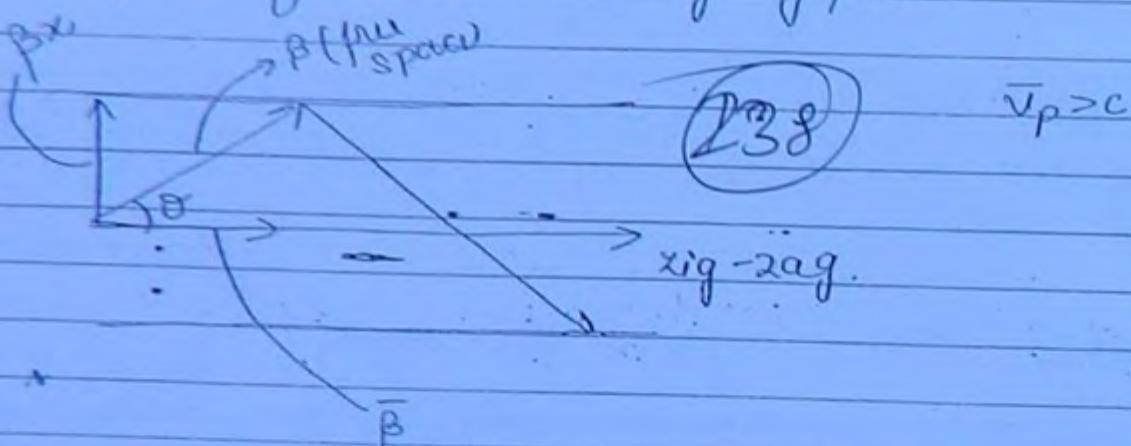
$$= \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\omega^2 u_0 \epsilon_0 - \left(\frac{m\pi}{a}\right)^2}}$$

$$\begin{aligned} \bar{V}_p &= \frac{\omega}{\sqrt{u_0 \epsilon_0 - \left(\frac{m\pi}{a\omega}\right)^2}} \\ &= \frac{1}{\sqrt{u_0 \epsilon_0} \left(\sqrt{1 - \left(\frac{m\pi c}{a\omega}\right)^2} \right)} \end{aligned}$$

$$\boxed{\bar{V}_p = \frac{c}{\sqrt{1 - \left(\frac{w_c}{w}\right)^2}}}$$

Date _____
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~~velocity~~
the phase along the guide axis is apparently greater than free-space velocity because when the wave goes in a zig-zag path with inclination the guide axis the wavelength along the guide axis appears longer than the free space wavelength. i.e. the phase shift rate and its dynamics are altered due to multiple reflections and zig-zag path



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$$\bar{v}_p > c$$

$$\bar{\lambda} = \lambda \cos \theta$$

$$\frac{\partial \pi}{\lambda} = \frac{\partial \pi}{\bar{\lambda}} \cos \theta$$

$$d = \frac{d}{\cos \theta}$$

$$\bar{v}_p = \frac{c}{\cos \theta}$$

$$\therefore \bar{\lambda} = \bar{c}/f$$

$$\bar{c} = \bar{\lambda} f$$

$$c = \frac{d}{\cos \theta} f = \frac{c}{\cos \theta}$$

Hence by comparison.

$\sin \theta = \omega_c / \omega$

where $\theta = \text{tilt angl: with } \text{guide axis}$

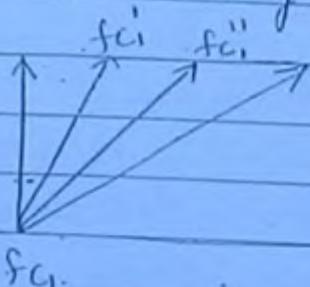
No two frequencies will ever have the same tilt angle and hence no two frequencies will never ever have the same velocity.

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At $\omega = \omega_c$, $\theta = 90^\circ$

No propagation, only Resonance.

$\omega > \omega_c$ any finite tilt angle.



concept 3. Group Velocity (v_g) along the guide axis

$$\text{Dispersion Relationship} = \beta = \sqrt{\omega^2 u_{\infty} - \left(\frac{m\pi}{a}\right)^2}$$

When β is proportional to ω .

like in uniform plane waves, lensless transmission lines
velocity is correctly $\frac{\omega}{\beta}$.

$$\beta \propto \omega \text{ (dispersion law)} \quad \left. \begin{array}{l} \text{one ray} \\ \hline \end{array} \right.$$

$$v_p = \frac{\omega}{\beta}$$

But when β is the non linear function of ω like in waveguides velocity is correctly $\frac{d\omega}{d\beta}$ but not $\frac{\omega}{\beta}$.
This is called as dispersion condition $d\omega/d\beta$.

$$\beta = \sqrt{\omega^2 u_{\infty} - \left(\frac{m\pi}{a}\right)^2} \quad \left. \begin{array}{l} \text{multiple ray} \\ \hline \end{array} \right.$$

per unit beam

$$v_p \neq \frac{\omega}{\beta} = \frac{d\omega}{d\beta}$$

$$V_g = \frac{dw}{d\beta}$$

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This velocity is called as group velocity. This has to be used has dispersive because (group of rays or group of frequencies)

$$y = mx$$

$$\frac{y}{x} = \frac{dy}{dx} = m$$

linear
funⁿ

$$\bar{y} = mx^2$$

$$\frac{y}{x^2} = mx$$

$$\frac{dy}{dx} = 2mx$$

non linear funⁿ.

$$\text{derivation } V_g = \frac{dw}{d\beta}$$

$$\frac{d\bar{\beta}}{dw} = \frac{1}{2\sqrt{w^2 u_0 e_0 - \left(\frac{mz}{a}\right)^2}} \cdot \frac{dw}{\sqrt{u_0 e_0}}$$

$$\frac{d\bar{\beta}}{dw} = \frac{\sqrt{u_0 e_0}}{\sqrt{u_0 e_0 - \left(\frac{mz}{aw}\right)^2}}$$

$$\frac{d\bar{\beta}}{\frac{dw}{d\beta}} = \frac{u_0 e_0}{\sqrt{u_0 e_0} \left(1 - \left(\frac{mz}{aw\sqrt{u_0 e_0}}\right)^2\right)}$$

$$\frac{d\bar{\beta}}{dw} = \frac{\sqrt{u_0 e_0}}{\left[1 - \left(\frac{mz}{aw\sqrt{u_0 e_0}}\right)^2\right]}$$

$$\frac{d\beta}{dw} = \frac{c}{\left[1 - \left(\frac{m\pi c}{aw} \right)^2 \right]} \quad (24)$$

$$= \frac{1}{}$$

$$c \sqrt{1 - \left(\frac{w_c}{w} \right)^2}$$

$$\bar{v}_g = c \sqrt{1 - \left(\frac{w_c}{w} \right)^2} = c \cos \theta$$

$$V_p \cdot \bar{v}_g = c^2$$

Hence group velocity defines a physical rate at which the wave is travelling or the energy is propagating along the guide axis.

The rate of change of phase with space i.e. is longer than λ in free space

$$\lambda' > \lambda$$

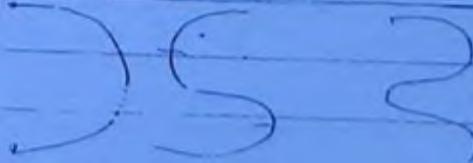
Hence $V_p > c$

but $v_g < c$ because it represents the decay in time.

Concept 4 Modes of operation

m - stands for connection mechanism for the feed...
 It is an integer standing for the no. mixed feed connection. Mathematically the no. of half cycles the wave complete b/w the guide wall. or the no. of maxima b/w the guide wall

$$-m- \quad m=1 \quad m=2 \quad m=3$$



Relation between λ , λ_c , and λ_g

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$$\bar{\lambda} = \lambda_g = \frac{\lambda}{\cos \theta} = \frac{\lambda}{\sqrt{1 - \left(\frac{\mu_r}{f}\right)^2}} = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

Squaring both sides

$$1 - \left(\frac{\lambda}{\lambda_c}\right)^2 = \left(\frac{\lambda}{\lambda_g}\right)^2$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$$

$$\lambda = \frac{c}{f} \quad (\text{free space})$$

$$\lambda_c = \frac{2\pi}{m} \quad (\text{cut off freq.})$$

Transverse electric, TM, TEM waves.

Consider a dispersive beam of EM waves which are $E(x, z, t)$ (x, y, z) $H(x, z, t)$ (x, y, z)
Let us use time harmonic Maxwell's eqn.

$$\nabla \times H = j\omega \epsilon E$$

$$\nabla \times E = -j\omega \mu H$$

a_x	a_y	a_z
$\partial / \partial x$	$\partial / \partial y$	$\partial / \partial z$
E_x	E_y	E_z

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega \mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega \mu H_y$$

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$$\frac{\partial E_y}{\partial z} - \frac{\partial E_x}{\partial y} = -j\omega \mu H_z$$

$$\nabla \times H = -j\omega \epsilon E$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega \epsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega \epsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega \epsilon E_z$$

All y derivatives are zero and all z derivatives are the same function with \bar{r} scaling as.

$$E(z) \quad \left\{ \begin{array}{l} e^{-\bar{r}z} \\ \text{any natural harmonic} \end{array} \right.$$

$$\nabla \times H = -j\omega \epsilon E$$

$$\nabla \times E = -j\omega \mu H$$

$$\bar{Y}H_y = j\omega \epsilon E_x$$

$$\bar{Y}E_y = -j\omega \mu H_x$$

$$-\bar{r}H_x - \frac{\partial H_z}{\partial x} = j\omega \epsilon E_y$$

$$-\bar{r}E_x - \frac{\partial E_z}{\partial x} = -j\omega \mu H_y$$

$$\frac{\partial H_y}{\partial x} = j\omega \epsilon E_z$$

$$\frac{\partial E_y}{\partial x} = -j\omega \mu H_z$$

$$-\bar{r}H_x - \frac{\partial H_z}{\partial x} = j\omega \epsilon \left(-\frac{j\omega \mu H_x}{\bar{r}} \right)$$

$$-\bar{r}H_x - \frac{\partial H_z}{\partial x} = \frac{\omega^2 \epsilon \mu H_x}{\bar{r}}$$

7
multiply by \bar{r} both sides

$$-\bar{r}^2 H_2 - \bar{r} \frac{\partial H_2}{\partial x} = \omega^2 \epsilon u H_x$$

$$-\bar{r} \frac{\partial H_2}{\partial x} = \omega^2 \epsilon u H_x + \bar{r}^2 H_x$$

$$-\bar{r} \frac{\partial H_2}{\partial x} = (\omega^2 \epsilon u + \bar{r}^2) H_x$$

$$-\bar{r} \frac{\partial H_2}{\partial x} = \bar{r}_x^2 H_x$$

Eq 1

$$\frac{\partial H_2}{\partial x} = -\frac{\bar{r}_x^2}{\bar{r}} H_x$$

Eq 3

$$\frac{\partial H_2}{\partial x} = \frac{\bar{r}_x^2 E_y}{j\omega u}$$

Eq 2.

$$\frac{\partial E_2}{\partial x} = -\frac{\bar{r}_x^2}{\bar{r}} F_x$$

Eq 4

$$\frac{\partial E_2}{\partial x} = -\frac{\bar{r}_x^2}{j\omega u} F_y$$

Note

The axial field components are the central components upon which the fields in the other direction also depend upon these components.

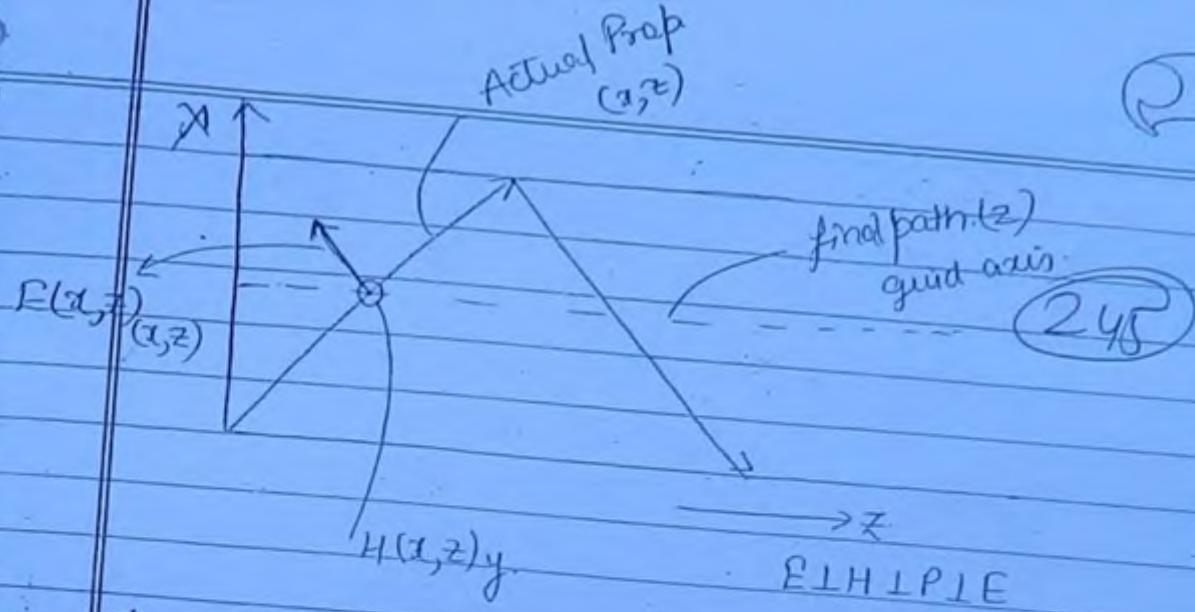
Note

If the axial component of magnetic field is zero i.e. the field mechanism is such that magnetic field is not directed along the guide axis the other dependent component is also vanish hence the wave becomes

If $H_2 = 0$ we are left with E_x, H_y, F_z

and the wave is

$$H(x, z, t)_y = E(x, z, t)_{(x, z)}$$

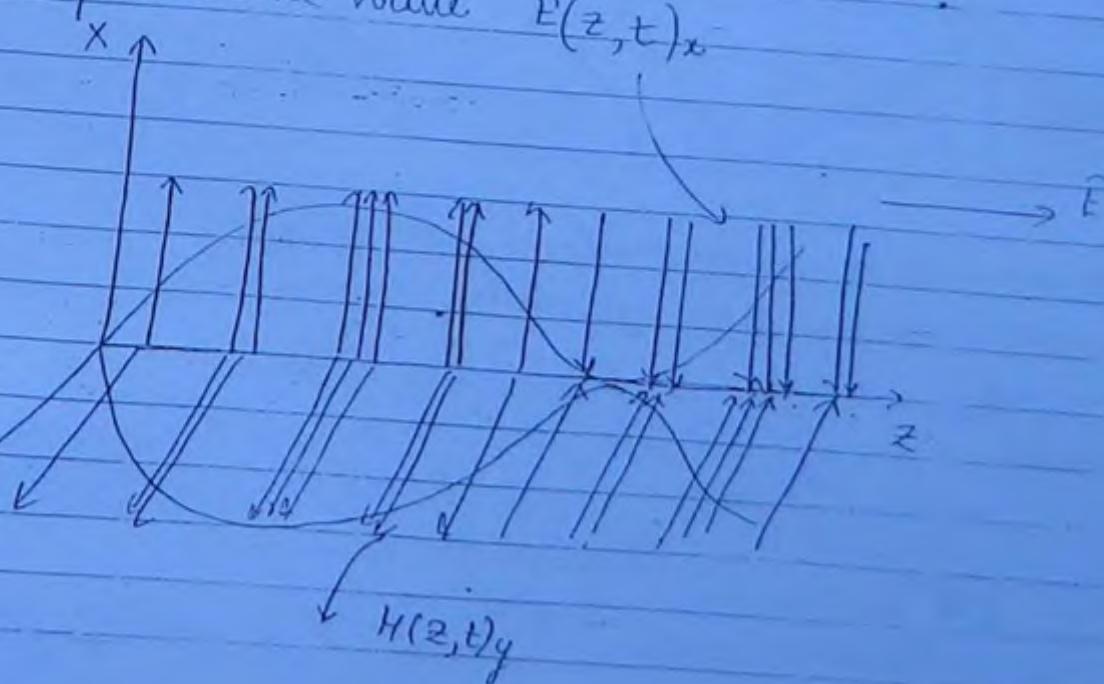


The magnetic field is not only perpendicular to the electric field and the actual propagation but it is also \perp to the guide axis. Hence this wave is called as transverse Magnetic wave. (TM wave)

Similarly if $E_z = 0$ the components we are left are H_x , E_y , H_z and the wave is

$E(x, z, t)_y$, $H(x, z, t)(x, z)$. This wave is called transverse electric wave or (TE wave)

uniform Plane wave $E(z, t)_x$



$$E_x H = P$$

$$X \times Y = Z$$

$$E_x = H_y$$

$$E_y = H_x$$

$$\frac{E_x}{H_x} = \frac{1}{2}$$

$$\frac{E_y}{H_y} = \frac{1}{2}$$

$$\frac{E_z}{H_z} = \text{classmate}$$

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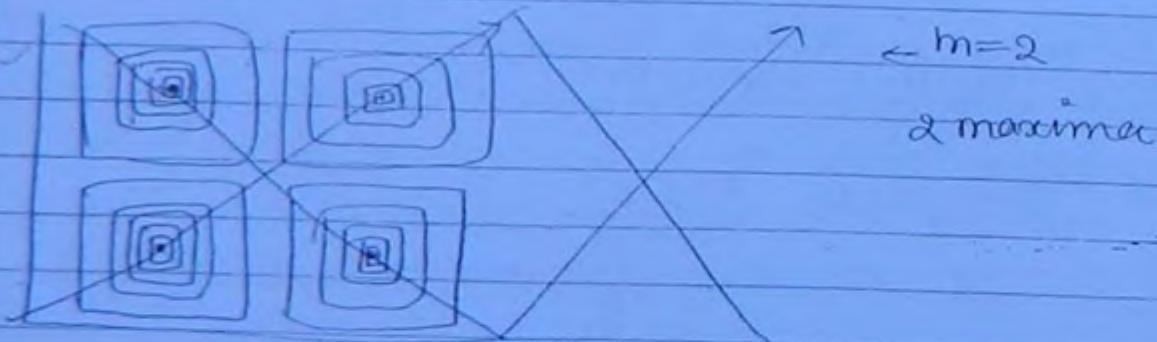
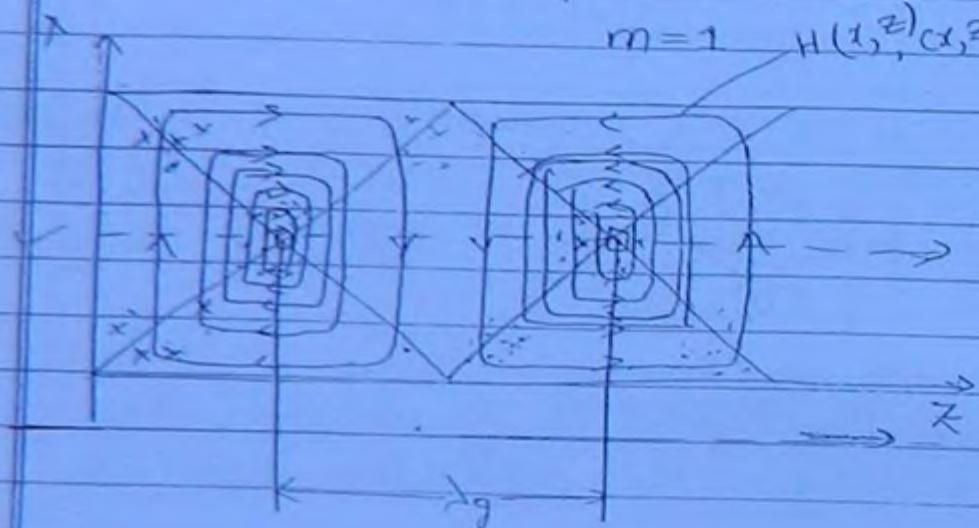
TE wave solutions in Parallel, plane waveguide

$$E(x, z, t) = E_{x0} \sin\left(\frac{m_1 x}{a}\right) e^{j k_z z} e^{j \omega t} a_y$$

$$H(x, z, t)_x = H_{x0} \sin\left(\frac{m_1 x}{a}\right) e^{-j k_z z} e^{j \omega t} a_x$$

$$H(x, z, t)_z = H_{z0} \cos\left(\frac{m_1 x}{a}\right) e^{-j k_z z} e^{j \omega t} a_z$$

$$m=1 \quad H(x, z, a_x, z)$$



If $m=0$ EM wave don't exist in TE format.
i.e $m=1$ is the least possibility for TE existence

$TE_{10}, TE_{20}, TE_{30}, \dots, TE_{m0}$ representation for
TE waves.

TE_{m0}

feed mechanism
in side

guides are not placed in?

TM wave solutions in parallel plane w/o.

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$$E(x, z, t)_x = E_{x0} \cos\left(\frac{m\pi}{a}x\right) e^{-rz} e^{j\omega t} a_x$$

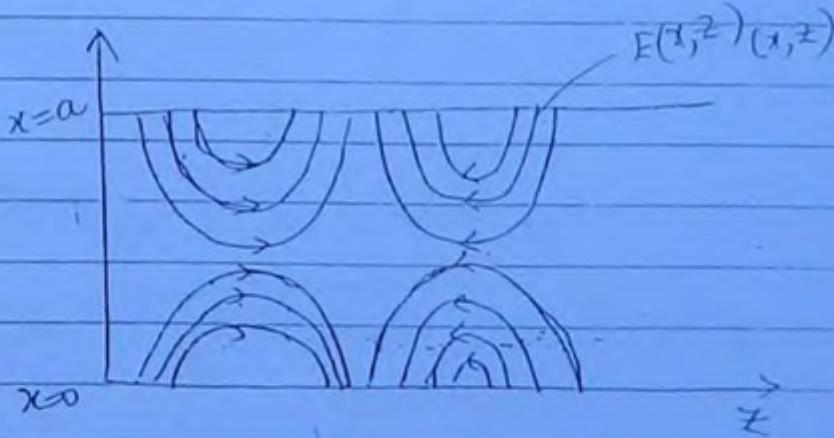
$$E(x, z, t)_z = E_{z0} \sin\left(\frac{m\pi}{a}x\right) e^{-rz} e^{j\omega t} a_z$$

$$H(x, z, t)_y = H_{y0} \cos\left(\frac{m\pi}{a}x\right) e^{-rz} e^{j\omega t} a_y$$

The $E(x)_x$ directed in x or $E(x)_z$ directed in z will follow trigonometric harmonic such that

$E(x)$ tang should be equal to sin harmonic as the guide walls are $x=0$ and $x=a$ surface.

$E(x)$ directed in x is normal component. $E(x)$ directed in y or $E(x)$ directed in z is tangential component. Hence $E(x)_z$ = directed in sin harmonic in x .



In the TM operations if $m=0$ only $E(x)$ and $H(y)$ components are left out and the wave become.

$$E(z, t)_x = E_0 e^{-rz} e^{j\omega t} a_x$$

$$H(z, t)_y = H_0 e^{-rz} e^{j\omega t} a_y$$

This wave is called as transverse Electromagnetic wave

It has neither E_z or H_z and has $m=0$ condition.

TM_{10} , TM_{20} , TM_{30} ... TM_{m0}
TM modes representation

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Properties of TEM wave:

- (i) They travel only along the guide axis i.e. they are not dispersive in x -axis.
- (ii) Their wave angle is zero always.
- (iii). $w_c = 0$ because $m=0$. No cut off freq. Any freq. can travel as TEM.

Summary:

1. $E(x,t)/H(z,t)$

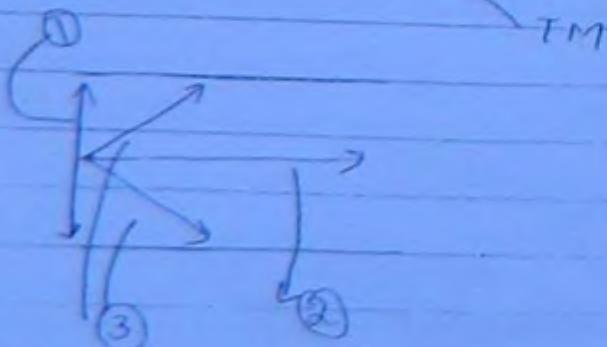
Wave at cut off frequency, $f_1, f_2 \dots f_m \quad \theta = 90^\circ$

$\bar{r} = 0$, Not propagating; it is resonating between the guide walls.

2. $E(z,t)/H(z,t) \rightarrow$ TEM wave, $\theta = 0^\circ$, $\bar{r} = j\omega\sqrt{\mu_0\epsilon_0}$

$m = 0$, $w_c = 0$; perfectly travelling along z axis,
no modes

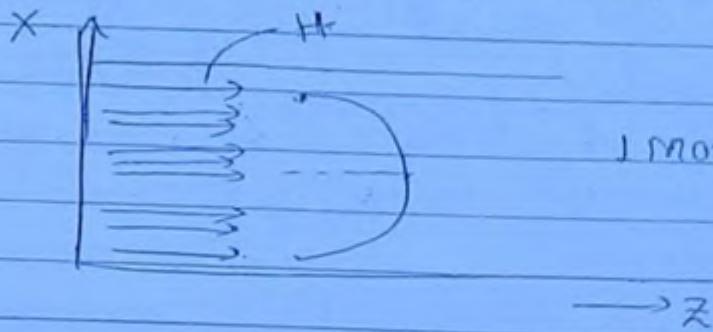
3. $E(x,z,t)/H(x,z,t)$ nodes in, w_c, \bar{r}, Y, Y_{se}



Q Identify the following field lines belonging to mode

- (i) TE₁₀ (ii) TM₁₀, (iii) TE₂₀ (iv) TM₂₀ (v) TEM
 (vi) TM or TEM (vii) TE or TM.

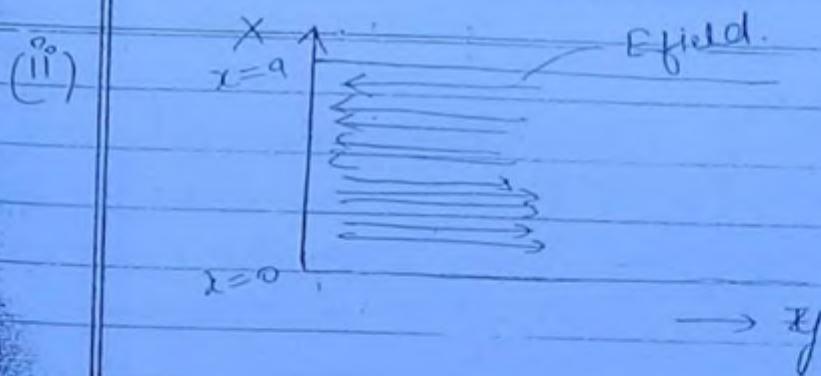
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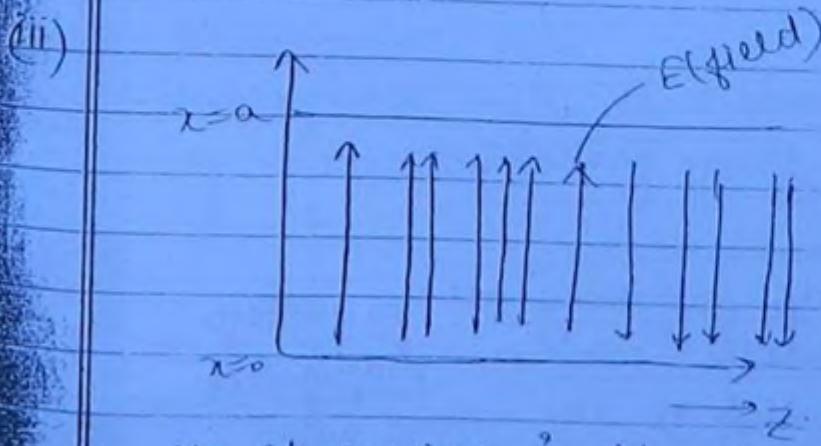
1 maximum = m=1.

$$\left. \begin{array}{l} E_y \\ H_x \\ H_z \end{array} \right\} TE \quad \left. \begin{array}{l} H_y \\ E_x \\ E_z \end{array} \right\} TM$$

H(z) \rightarrow TE₁₀



E(y) \rightarrow TE₂₀.



E(z) \rightarrow TM or TEM

No. of maxima is b/w the guidewalls bt not along the guide axis.

yield (varying) Director

250
310
210
110

field (propagation) Director

250

For a parallel plate waveguide of 1 cm separation calculate the total no. of possible modes of energy transfer = for all frequency $f \leq 40 \text{ GHz}$

$$\text{Since } f_c = \frac{mc}{2a} = \frac{m \times 3 \times 10^8}{2 \times 10^{-2}} = 40 \times 10^9$$

$$m = \frac{40 \times 10^9 \times 2 \times 10^{-2}}{3 \times 10^8}$$

$$m = \frac{80 \times 10^7}{3 \times 10^8}$$

$$m = 26.6 \times 10^1$$

$$m = 2.66 \times 10^2$$

$$1 \text{ cm} \rightarrow f_c = \frac{c}{2a} = 15 \text{ GHz}$$

15 ∞ TE₁₀ / TM₁₀

30 ∞ TE₂₀ / TM₂₀

0 ∞ TEM

5 modes ∞

Dominant mode

In a mode of a lower cut off frequency all frequency permissible in the higher modes can be propagated since dominant mode which is the mode with lowest cut off frequency can be used for all frequencies in guide support.

Dominant mode for parallel plane wave guide TEM mode

(27)

Note: Wave impedance of TE waves or TM waves is defined only with respect to the transverse component to the guide axis. The total components are not considered.

$$\eta_{TE} = \text{wave impedance of TE wave.} = \frac{E_y}{H_x} = \frac{E_T}{H_T \cos\theta} = \frac{120\pi}{\sqrt{1 - (\beta_c/f)^2}}$$

$E_T - H_T \rightarrow$ total field.

$E_x, E_y, H_x, H_y \rightarrow$ transverse to guide axis.

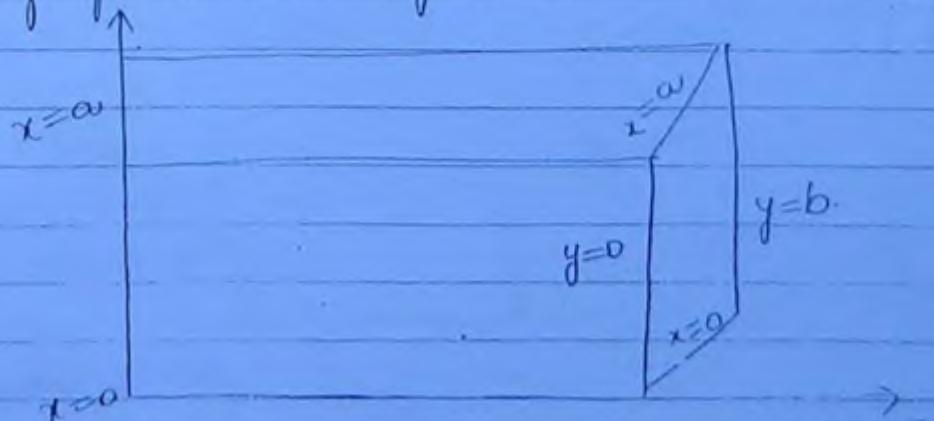
$$\eta_{TM} = \frac{E_x}{H_y} = \frac{E_T \cos\theta}{H_T} = 120\pi \cos\theta$$

$$\eta_{TEM} = 120\pi$$

Parallel Plate wave guide may not be a practical w/o. But ground wave propagation over a conducting earth and duct propagation are excellent example of parallel plate w/o.

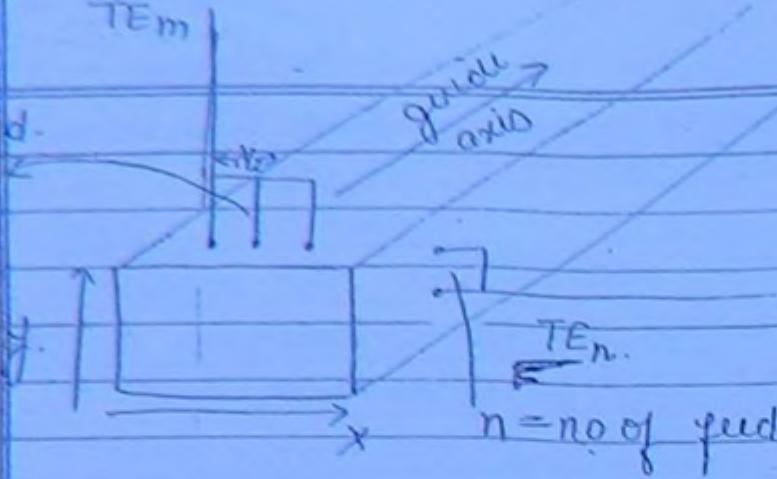
Rectangular w/o

Rectangular w/o is a closed and confined structure out of four conducting wall.



TEM

no. of feed
in points

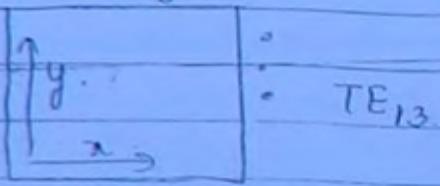


$E_2 = 0 \rightarrow$ TE feed. } Horizontal/vertical
 E_x or E_y exist } Axial - - -

$H_2 = 0 \rightarrow$ TM feed. } Axial - - -
 H_x or H_y exist } - - -

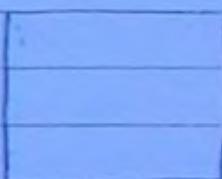
Q Identify the field connection in the following cases.

①



TE₁₃

②



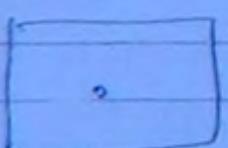
TE₂₁

③



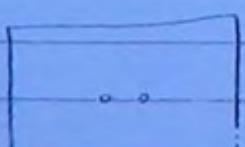
TE₀₂

④



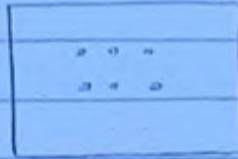
TM₁₁

⑤



TM₂₁

(6)

TM₃₂

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TE₃₂ → does not exist

TM_{mo} & TM_{on} → Evanescent modes Non-existing modes in
Rectangular W/G.

Chapter 4 waveguide (workbook)

$$f_c = \frac{mc}{2a}$$

$$\text{mode} = 1 \quad f_c = \frac{3 \times 10^8}{2 \times 2.5 \times 10^{-2}} = \frac{30 \times 10^9}{5} = 6 \times 10^9 \text{ Hz}$$

$$\sin \theta = \left(\frac{f_c}{f} \right)$$

$$\theta = \sin^{-1} \left(\frac{6 \times 10^9}{22 \times 10^9} \right)$$

$$\text{mode} = 3 \quad f_c = \frac{3 \times 3 \times 10^8}{5 \times 10^{-2}} = \frac{90 \times 10^9}{5} = 18 \times 10^9 \text{ Hz}$$

$$\sin \theta = \left(\frac{f_c}{f} \right)$$

$$\theta = \sin^{-1} \left(\frac{18}{22} \right)$$

$$2. \quad f_c = \frac{mc}{2a} \Rightarrow a = \frac{mc}{2f_c} \Rightarrow \frac{3 \times 3 \times 10^8}{2 \times 36 \times 10^9} = \frac{1}{80 \times 10^6}$$

$$= 0.0125 \text{ m} \Rightarrow 1.25 \text{ cm}$$

$$\sin \theta = \frac{f_c}{f}$$

$$\sin \theta = \frac{10}{36}$$

$$\sin \theta = \frac{10}{36}$$

F

$$3) f_c = \frac{mc}{2a} = \frac{0 \times 10^8}{2 \times 3 \times 10^{-2}} = \frac{10^{10}}{2} = 0.5 \times 10^{10} = 5 \times 10^9$$

(Normal condition) X

$$f_c = \frac{1 \times 3 \times 10^8}{2 \times 3 \times 3 \times 10^{-2}} = \frac{3 \times 10^{10}}{18} = \frac{10^{10}}{6} = 0.166 \times 10^{10} = 1.66 \times 10^9$$

$$\sin \theta = \left(\begin{array}{l} +6 \times 10^9 \\ -2 \times 10^9 \end{array} \right)$$

$$\theta = \sin^{-1}(0.8)$$

$$\sin \theta = \frac{10^{10}}{6} \times \frac{1}{2 \times 10^9}$$

$$\theta = \sin^{-1}\left(\frac{5}{6}\right) \text{ Ans}$$

Monday.

Propagation along the guide axis

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu_0 \epsilon_0}$$

$\gamma_x \quad \gamma_y \quad \gamma$ of free space

$$\text{If } \bar{\gamma} = 0 \quad \omega_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} c$$

(253)

$$f_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \frac{c}{2}$$

$$\sin \theta = \frac{\omega_c}{\omega}$$

$$\bar{v}_p = \frac{c}{\cos \theta}$$

$$\bar{v}_g = c \cos \theta$$

$$\frac{1}{d^2} = \frac{1}{d_c^2} + \frac{1}{d_g^2}$$

$$\eta_{TE} = \frac{\eta_0}{\cos \theta} \quad \eta_0 = 120\pi$$

$$\eta_{TM} = \eta_0 \cos \theta$$

TM wave solutions in Rectangular waveguide

$$(H_x, H_y, E_x, E_y, E_z) \quad H_z = 0$$

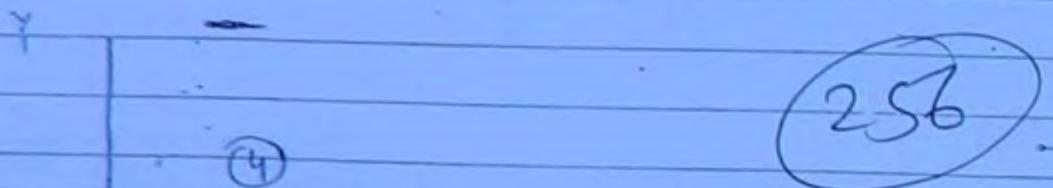
$$E_z = E(x, y, z, t)_z = E_{zo} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_z$$

$$E_x = E(x, y, z, t)_x = E_{xo} \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_x$$

$$E_y = E(x, y, z, t)_y = E_{yo} \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_y$$

$$H_z = H(x, y, z, t)_z = H_{zo} \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_z$$

$$H_y = H(x, y, z, t)_y = H_{yo} \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} e^{j\omega t} a_y$$



$E(t) / H(t) \rightarrow e^{j\omega t} \rightarrow$ source Harmonic

$E(z) / H(z) \rightarrow e^{-\gamma z} \rightarrow$ Harmonic natural

$E(x, y) / H(x, y) \rightarrow \sin \text{ or } \cos$ Harmonic trigonometric

$E(x, y)$ i.e. the wave in the restricted dimensions should be strictly a trigonometric harmonic following the rule.

$E(x \text{ or } y)_{\text{long}}$ parallel component to the guide wall should be an harmonic

$$E(x \text{ or } y)_{\text{long}} = \sin$$

If $m=0$ & $n \neq 0$

TM_{0n} mode does not exist

TM_{m0} " " "

(25)

The non-existent modes for a waveguide are called as errant modes.

If two different modes have the same cut off frequency they are said to be degenerate modes.

$TM_{11} \rightarrow$ is the least fc possible TM operation

TE wave solutions in Rectangular w/G ($E_z=0$)

(H_x, H_y, E_x, E_y, H_z)

$E(x, y, z, t)_{(x, y)}$

$H(x, y, z, t)_{(x, y, z)}$

$$H_z = H(x, y, z, t)_z = H_{z0} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{iz} e^{j\omega t} a_2$$

$$E_x = E(x, y, z, t)_x = E_{x0} \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-iz} e^{j\omega t} a_x$$

$$E_y = E(x, y, z, t)_y = E_{y0} \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-iz} e^{j\omega t} a_y$$

$$H_x = H(x, y, z, t)_x = H_{x0} \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-iz} e^{j\omega t} a_x$$

$$H_y = H(x, y, z, t)_y = H_{y0} \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-iz} e^{j\omega t} a_y$$

$$E(x \text{ or } y)_{\text{tan}} = \sin \quad \checkmark \quad H(x \text{ or } y)_{\text{tan}} = \cos$$

$$E(x \text{ or } y)_{\text{normal}} = \cos$$

$$H(x \text{ or } y)_{\text{norm}} = \sin$$

E field can never be parallel to the guide wall at the boundaries

E field always starts normally from the guide walls.

25P

If $m \neq 0 \& n=0$

$$E(x, z, t)_y = E_{y0} \sin\left(\frac{m\pi}{a}x\right) e^{\frac{-rz}{a}} e^{j\omega t} a_y$$

$$H(x, z, t)_x = H_{x0} \sin\left(\frac{m\pi}{a}x\right) e^{-\frac{rz}{a}} e^{j\omega t} a_x$$

$$H(x, z, t)_z = H_{z0} \cos\left(\frac{m\pi}{a}x\right) e^{-\frac{rz}{a}} e^{j\omega t} a_z$$

If $n \neq 0 \& m=0$

$$E(y, z, t)_x$$

$$H(y, z, t)_y$$

$$H(y, z, t)_z$$

TE_{10} or TE_{01} can exist and has the least cut off frequency

Dominant mode for TE or for the guide in values

$$TE_{10} \text{ or } TE_{01}$$

$$TE_{10}; f_c = \frac{c}{2a}$$

$$TE_{01}; f_c = \frac{c}{2b}$$

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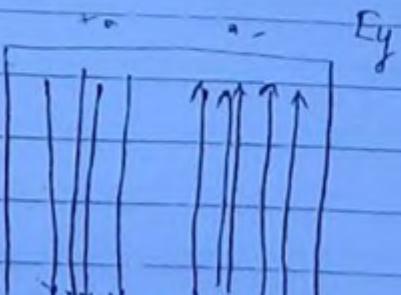
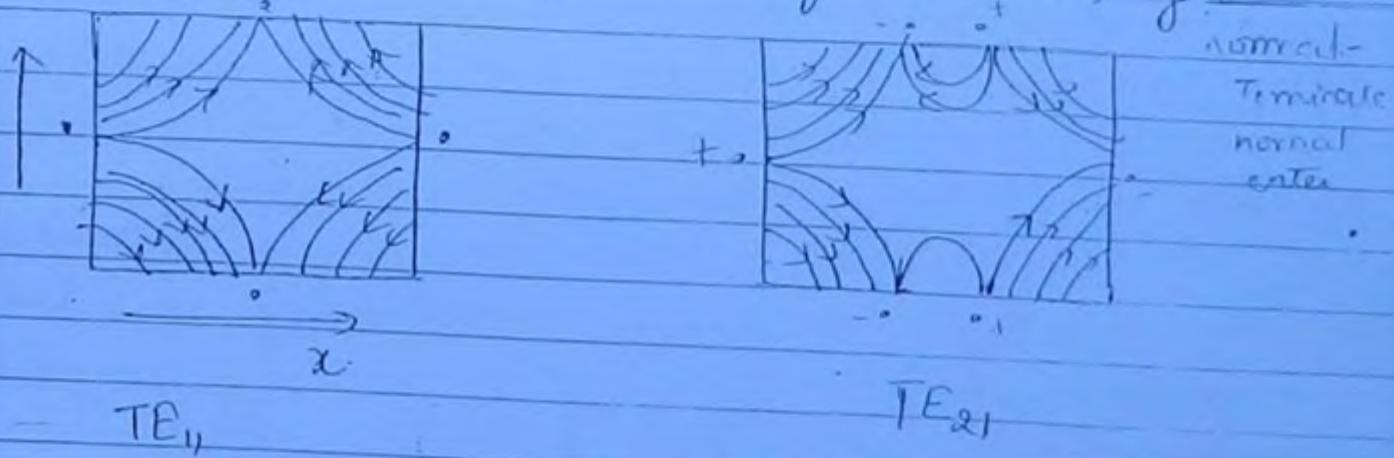
If $a \geq b$ TE_{10} dominant mode
 $a < b$ TE_{01} " "

The broad side dimensions of the guide decide the dominant mode.

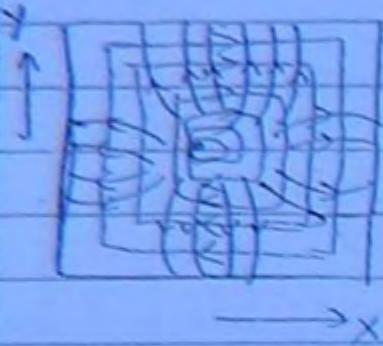
TEM waves cannot exist in rectangular waveguide

In all single conductor guides TEM wave cannot exist
 eg: Rectangular w/o ground and cylindrical w/G. $\circ \square$

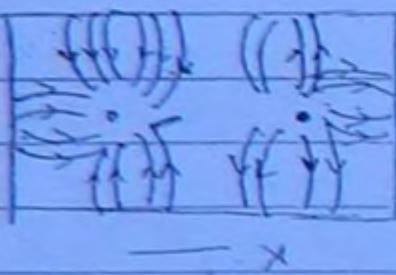
TE waves in Rectangular waveguide Ex, Ey



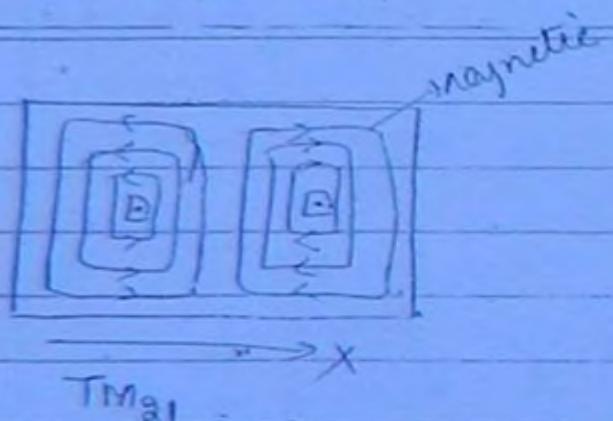
TM waves in Rectangular waveguide Ex E_x, E_y, E_z



TM₁₁

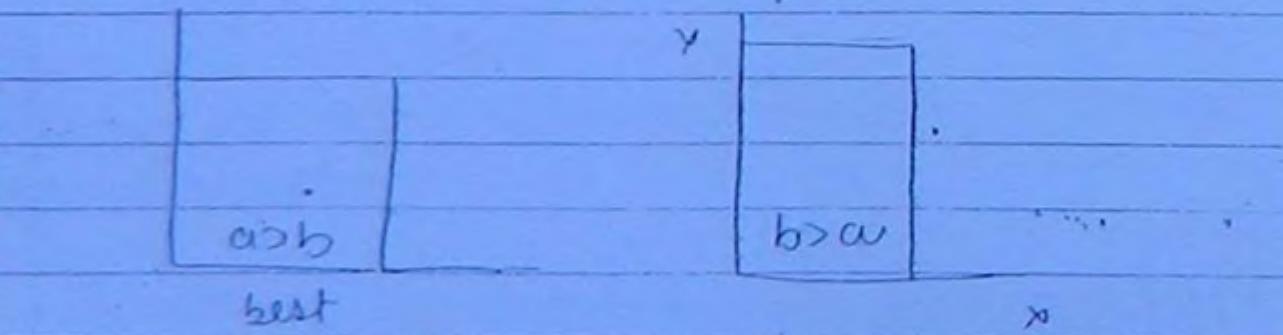


TM₁₂



TM₂₁

260



13

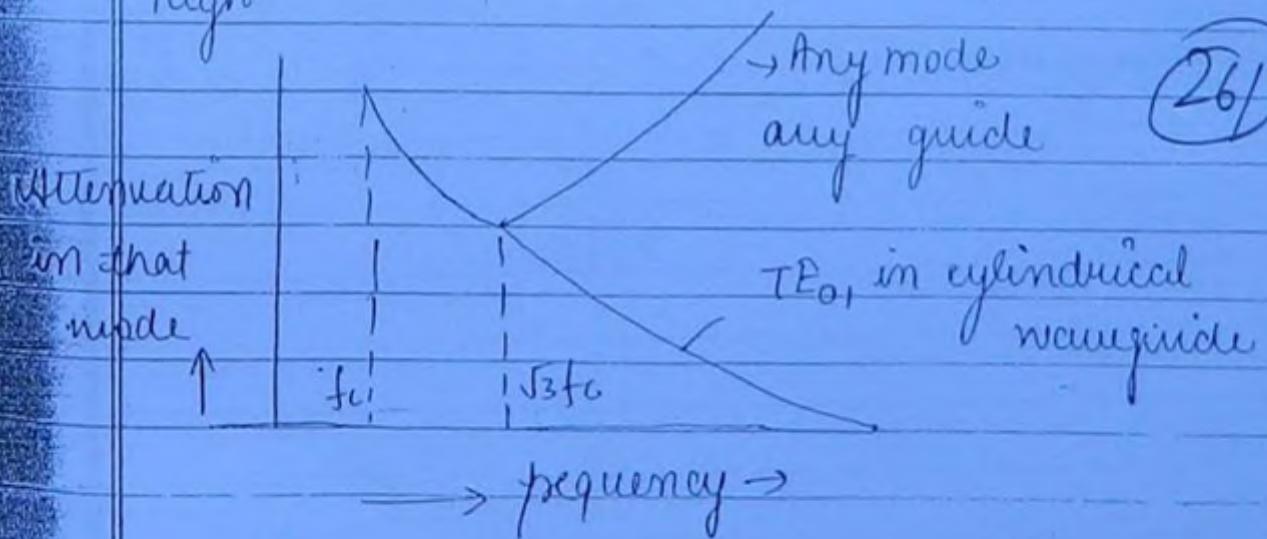
11

1

2

5

Higher frequencies are not preferred for lower modes if the frequency is quite larger than cut off-attenuation in a non conducting walls. is very high.



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5. Basic of Antennal.

1) Hertzian Dipole
Half wave dipole

5) classification of Antenna

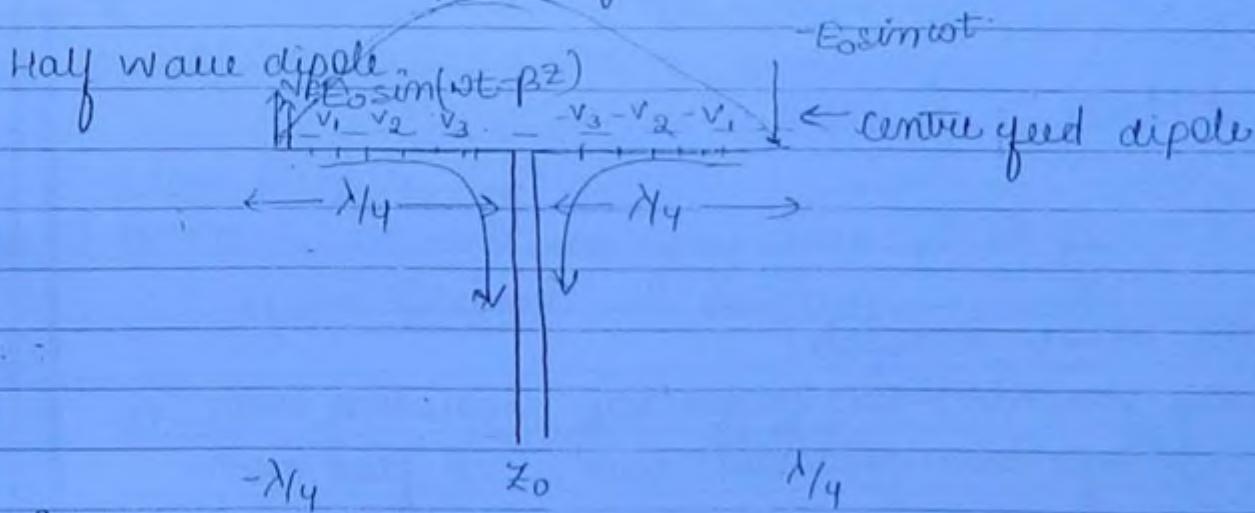
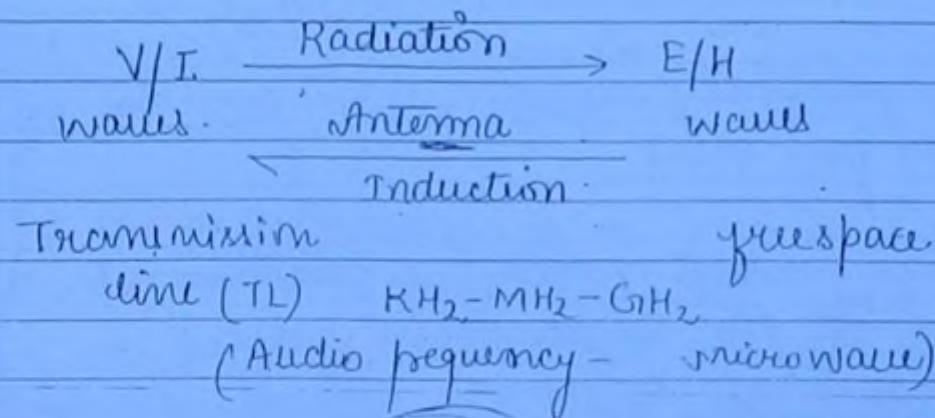
2. Basic Definitions &

Terminology

3. Antennas / Arrays

4. FRISS free space
propagation eqn

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$$f = 100 \text{ MHz}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^8}$$

$$\lambda = 3$$

$$\lambda/2 = 1.5 \text{ m}$$

- A centre feed half wave dipole is a transmission line opened out by $\lambda/4$ on either side i.e. the frequency to be received decides the length of the antenna.
- An EM wave of this frequency when travels along the length of the antenna, axis of the antenna induces voltage all along the conducting length such that at edges of the antenna equal and opposite voltages kind hence maximum potential differences at any time.

(26)

These voltages progressively decrease as we come to the centre or the feed point

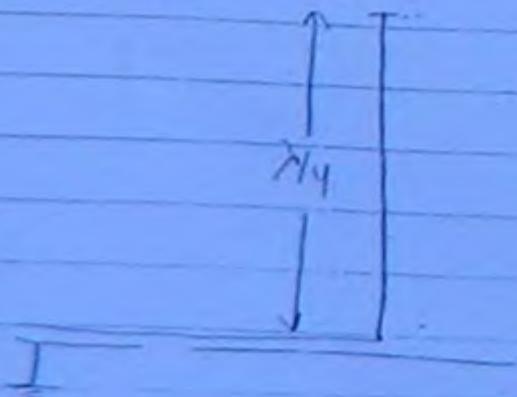
These voltages drive current which are maximum at the centre and 0 at the edges when graph is shown. This is called as asymptotic current distribution i.e.

$$|I(z)| = I_0 \cos \beta z$$

$$z = -\frac{\lambda}{4} \text{ to } \frac{\lambda}{4}$$

Quarter wave monopole:

vertically grounded-less frequency, conducting Earth at is a single wire grounded earth and feed mechanism



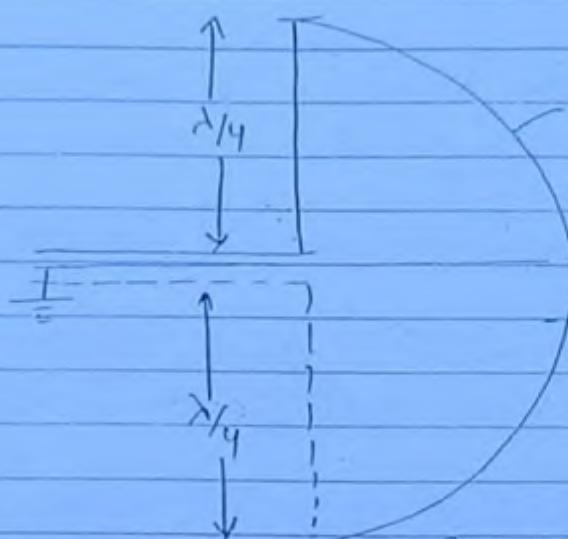
$$900 \text{ MHz} \quad 1800 \text{ MHz} \quad 0.5 \text{ M} \quad \lambda = \text{cm}$$

$$100 \text{ MHz} \quad \lambda = 3 \text{ m}$$

classmate

Date _____

Page _____



Asymptotic current ground is giving reflection
 $5 - (-5) = 10 \text{ V}$
 reflection.

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The image of the antenna in the earth makes it behave like a half wave dipole.

Note for the entire working of the half wave dipole the wave should be travelling along the axis of the antenna i.e. the antenna should be correctly aligned with the propagation of the wave. The wave polarization should be the same as antenna polarization.

Hertzian Dipole

length - current carrying wire

$I(t) = I_{\text{m}} \sin \omega t \rightarrow$ Oscillatory current element

$$\text{dl} \rightarrow 0 \Rightarrow \text{dl} \ll \lambda \quad \text{dl} < \lambda_{30}$$

It Radiates & produces EM waves

$$I \text{ dl} \rightarrow f(t) \text{ dl}$$

If an oscillatory time harmonic current $A = \omega^2 \text{ dl}$ is send in a wire it has electromagnetic waves all around. It can be proved for an Hertzian dipole as shown.

Below

$$E = \frac{1}{2} (\omega \times H)^T$$

$$\int dl \rightarrow I(t) dt$$

$$1. \bar{A} = \frac{\mu_0 I dl}{4\pi r}$$

$$2. \bar{B} = \nabla \times \bar{A}$$

$$3. \nabla \times \bar{H} = \frac{E_o E}{dt}$$

$$E = \frac{1}{c} \int (\nabla \times H) dt$$

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If the derivation is carried out in spherical coordinates
 It can be proved that radiated waves exist all around hence radiation is defined as the production of propagation of EM waves around any time harmonic current.

Tuesday.

Hertzian dipole:

$$E(z, \theta, \phi, t)_\theta = -\frac{I_m dl \sin \theta \omega \sin \omega t e^{j\beta z}}{4\pi \epsilon_0 c^2 r} \left. \begin{array}{l} \text{time harmonic} \\ \text{(I in wire)} \end{array} \right\} \text{space harmonic directed prep.}$$

$$H(z, \theta, \phi, t)_\phi = -\frac{I_m dl \sin \theta \omega \sin \omega t e^{j\beta z}}{4\pi \epsilon_0 r} \left. \begin{array}{l} \text{Radiation} \\ \text{of a Hertzian Dipole} \end{array} \right\}$$

$$E(z, t)_z = E_0 e^{j\omega t} e^{-jz} a_x$$

$$H(z, t)_y = H_0 e^{j\omega t} e^{-jz} a_y$$

uniform plane wave

TEM wave

Radiated wave travels radially outward in the direction and have a time harmonic same as the current, and an amplitude which depends on various factors

Properties of Radiation

i. The amplitudes of radiated waves decrease at $\frac{1}{r}$ as the wave disperses radially outward.

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ii. This is not due to attenuation but it is dispersion.

iii. The amplitude of the radiation is not the same in all the directions around the antenna i.e. radiation is directive or angle dependent.

iv. The amplitude of the radiation is dependent on the length to wavelength relationship of the dipole.

$\text{length} \left(\frac{\lambda}{\lambda_0} \right)$

Total Radiated Power from a Hertzian dipole.

$$W_r = \iint_{\text{any enclosing spherical surface}} \text{Power density} \times \text{Enclosing area}$$

$$= \iint \frac{1}{2} \frac{E_0^2}{\eta} \cdot dS$$

$$W_r = \int_0^\pi \int_0^{2\pi} \frac{1}{2} \left(\frac{I_m dL \sin \theta \cdot w}{4\pi E_0 c^2 \eta} \right)^2 \times \frac{1}{120\pi} r^2 \sin \theta d\theta d\phi$$

$$\theta = 0 \quad \phi = 0$$

$$W_r = I_{rms}^2 80 \pi^2 \left(\frac{dL}{\lambda} \right)^2$$

Hence an Hertzian dipole dissipates power as a resistor which dissipates heat but in an antenna the dissipated power is in the form of radiated EM waves hence

radiation Resistance $R_r = 80\pi^2 \left(\frac{dL}{\lambda} \right)^2$

for Hertzian dipole $dL \ll d$

Radiation Resistance:

Radiation resistance is a measure of radiated power for an given sp current. It should be as large as possible for practical antenna.

radiating Half wave Dipole:

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- Array or group of Hertzian dipoles where dl is from $-N_y$ to N_z
- $|I(l)| = I_m = I_0 \cos \beta l$
Asymptotic current distribution

$$E(\lambda, \theta, \phi, t)_0 = \left[\frac{60 I_m}{r} \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right] \sin \omega t e^{-j\beta r} a_\theta$$

$$H(\lambda, \theta, \phi, t)_\phi = \left[\frac{I_m}{2\pi r} \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right] \sin \omega t e^{-j\beta r} a_\phi$$

note
 $\frac{E_\theta}{H_\phi} = 120\pi$ for any EM wave

Hertzian
Dipole) $\frac{E_\theta}{H_\phi} = \frac{1}{\epsilon c} = \frac{1}{c \sqrt{\epsilon}} = \sqrt{\frac{4}{\epsilon}} = 120\pi$

Total Radiated Power from a half wave dipole

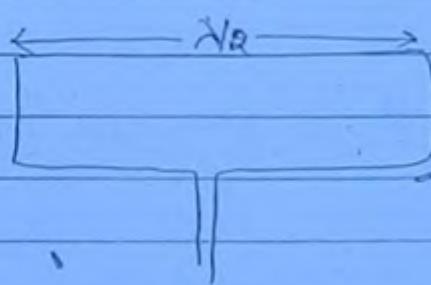
$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{2} \times \left(\frac{60 I_m}{r} \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right)^2 \times \frac{1}{120\pi} \times r^2 \sin \theta d\theta d\phi$$

$$= \frac{I_{max}^2}{\rho} (73)$$

$$W_r = \frac{I_{max}^2}{\rho} (73)$$

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Radiation Resistance of Half wave Dipole = 73Ω /
" " Quarter wave Monopole = 36.5Ω
" " Folded Dipole = $2^2 \times 73$
= 292Ω



- Basic Terms and definitions :

1. Isotropic Antenna
2. Radiation Power density
3. " " Intensity.
4. Gain
5. Effective Effective length
6. " Area
7. Radiation Pattern

Isotropic Antenna

It is also called Omnidirectional antenna and radiates power in all directions uniformly.

It's E field is independent of θ and ϕ

2. Radiation Power Density:

It is the strength of the radiated EM wave anywhere around the antenna.

Power area

$$\frac{dW_r}{ds} = \text{watts/m}^2 = \text{Poynting vector of the EM wave}$$

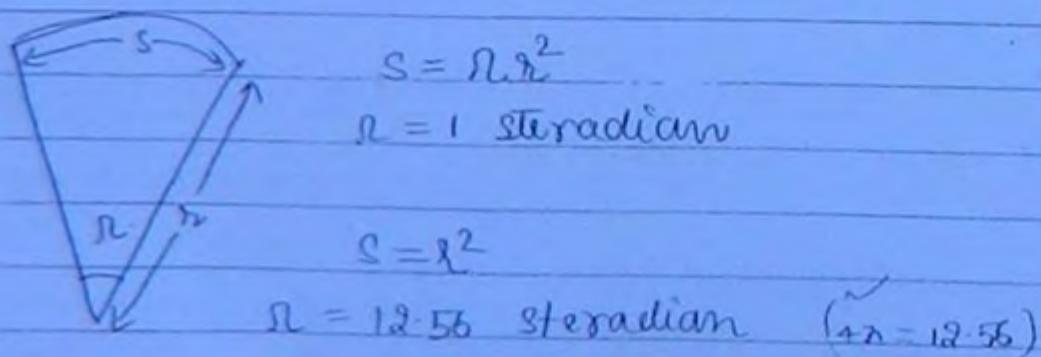
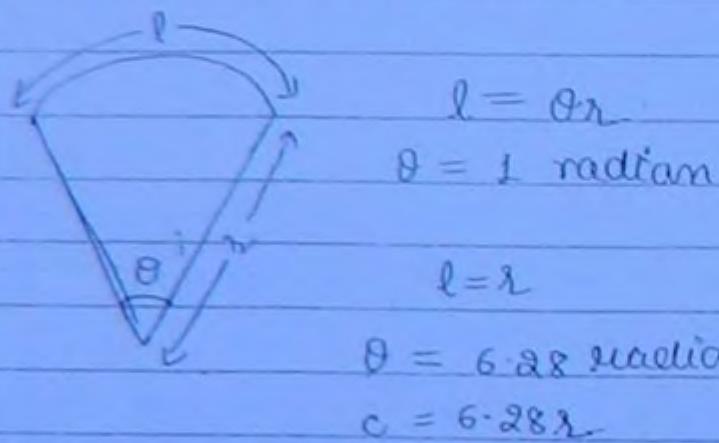
$$= \frac{1}{2} \frac{E_0^2}{\eta} (r; \theta, \phi)$$

$$= V(r, \theta, \phi)$$
2.70

3. Radiation Power Intensity

It is the strength of the radiated EM wave in any direction from the antenna.

$$= \psi(\theta, \phi) = \frac{\text{Power}}{\text{solid angle}} = \frac{\text{Power}}{\text{solid angle}} = \frac{dW_r}{d\Omega} = \frac{\text{watts}}{\text{steradian}}$$



$$TSA = 12.56 \pi^2$$

One steradian is a solid angle subtended by a cone whose base curvature is equal to square of radius of curvature
 Hence $\left| d\Omega = d\theta \sin^2 \theta = r^2 \sin \theta d\theta d\phi \right|$

$$d\Omega = \sin \theta d\theta d\phi$$

(27)

(Density)

$$\Psi(\theta, \phi) = \frac{dw_r}{dr} \Rightarrow dw_r \cdot \frac{ds}{dr} \Rightarrow V(r, \theta, \phi) \cdot r^2 \checkmark$$

$$= \frac{1}{2} \frac{E_0^2(\theta, \phi)}{r}$$

e.g: $V(r, \theta, \phi) = V_{avg} = \frac{wr}{4\pi r^2}$ = Average Rad. Power

$$\Psi(\theta, \phi) = \Psi_{avg} = \frac{wr}{4\pi} = \text{Average Power per direction}$$

4. Gain
- $\rightarrow G_D$ - Directive Gain
 - $\rightarrow G_p$ - Power Gain
 - $\rightarrow D$ - Directivity

G_D - Directive Gain

G_D = Radiation Intensity of the given antenna in a given direction

Radiation Intensity of isotropic antenna

$$G_D = \frac{\Psi(\theta, \phi)}{\Psi_{avg}} = \frac{\Psi(\theta, \phi)}{\frac{wr}{4\pi}} = \frac{4\pi \Psi(\theta, \phi)}{\iint \Psi(\theta, \phi) \cdot d\Omega}$$

$$G_D = \frac{4\pi \Psi(\theta, \phi)}{\int \Psi(\theta, \phi) d\Omega}$$

G_p - Power gain

$$G_p = \frac{4\pi \Psi(\theta, \phi)}{W_{IN}} - \frac{4\pi \Psi(\theta, \phi)}{W_r} \cdot \frac{W_r}{W_{IN}}$$

$G_p = G_D \times \text{Efficiency of Radiation}$

(272)

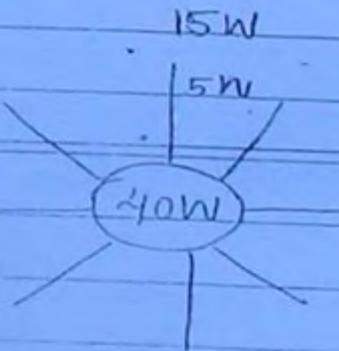
$$G_p = G_D \times \frac{W_r}{W_r + W_e}$$

$$G_p = G_D \times \left(\frac{R_r}{R_r + R_e} \right)$$

W_r = loss power

R_r = loss resistance

W_{IN} = total S/P power



$D = 3$

Directivity (D)

$D = G_D|_{max} = \text{Maximum value of directivity gain}$

Antenna chapter 5 (Watt Rank)

| $D \rightarrow G_D \rightarrow \Psi(\theta, \phi) \rightarrow V(r, \theta, \phi) \rightarrow E(\eta, \theta, \phi)$

for Hertzian dipole $|E| = \frac{E_0 \sin \theta}{d}$

$$V(r, \theta, \phi) = \frac{1}{2} \frac{E_0^2 \sin^2 \theta}{r^2 \eta}$$

$$6 \text{ W-B} \quad d = 792 \text{ m}$$

$$\frac{\lambda}{4} = 123 \text{ m}$$

$$l = 124 \text{ m}$$

It is a quarter monopole.

$$R_r = 36.5 \Omega$$

(273)

7W-B

6 dB

$$10 \log = 6 \text{ dB}$$

$$\log G_{TD} = \frac{6}{10} \text{ dB}$$

$$G_{TD} = 10^{0.6}$$

Antenna is passive element. So whenever Φ_p is given in lesser antenna the same power is transmitted.

$$10 \text{ W-B} \quad \psi(\theta, \phi) = \Psi_{avg} = \frac{4\pi}{8} = \frac{Wr}{4\pi} = \frac{100}{4\pi} = \frac{50}{2\pi} = 7.96 \text{ W}$$

$$U(r, \theta, \phi) = U_{avg} = \frac{Wr}{4\pi r^2} = \frac{100}{4\pi \times (10 \times 10^3)^2} = \frac{100}{4\pi \times 10^8} = 0.079 \times 10^{-6}$$
$$= 0.08 \mu\text{W}$$

$$12 \text{ W-B} \quad 40\pi = W_{IN}$$

$$\text{Efficiency } \eta = 90\%$$

$$36\pi = Wr$$

$$\text{Efficiency} = \frac{Wr}{W_{IN}}$$

$$\frac{90}{100} = \frac{Wr}{40\pi}$$

$$Wr = \frac{40\pi \times 90}{100}$$

$$Wr = 36\pi$$

$$\psi_{max} = 150 \text{ W/m}$$

$$D = (G_D)_{max} = 4\pi \psi(\theta, \phi)|_{max} = 4\pi \times 150 = 150$$

$$10 \log(16.67) = 11.67 \text{ dB}$$

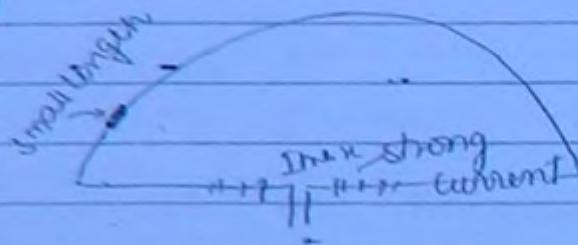
5. Effective length (l_{eff})

(274)

Half wave Dipole

$\tau(2)$ — asymptotic — w_r — l_{phy}
 $(0 \rightarrow I_{max})$

P_{avg} — uniform — w_r — l_{eff} .



If l is the length required to radiate the power w_r with uniform currents assuming the same power, is radiated with non uniform currents over the physical length

for a half wave dipole.

$$\left\{ \begin{array}{l} P_{avg} = \int_{-\frac{l}{2}}^{\frac{l}{2}} I_m \sin \omega t \\ E = \frac{2 I_{max}}{\pi} \end{array} \right.$$

where I_{max} = current at the centre

The same relationship is share for effective length and physical length.

$$\boxed{L_{eff} = \frac{2 l_{phy}}{\pi}}$$

$$G_{TD} = \alpha \omega^2$$

$$\psi(\theta, \phi) = \frac{1}{2} \frac{\epsilon_0^2 \sin^2 \theta}{\eta}$$

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$$G_{TD} = 4\pi \cdot \frac{1}{2} \frac{\epsilon_0^2 \sin^2 \theta}{\eta}$$

$$\int \int \int \frac{1}{2} \frac{\epsilon_0^2}{\eta} \sin^2 \theta \cdot \sin \theta \, d\theta \, d\phi$$

$\theta=0 \quad \phi=0$

$$= \frac{4\pi \cdot \sin^2 \theta}{2\pi \int_0^\pi \sin^3 \theta \, d\theta}$$

$$= \frac{2 \sin^2 \theta}{\left(\frac{4}{3}\right)} = \frac{3 \sin^2 \theta}{2}$$

$$G_{TD} = \frac{3}{2} \sin^2 \theta$$

$$D = \frac{3}{2} = 1.5$$

Repeat the same problem for a half wave dipole.

$$D \rightarrow G_{TD} \rightarrow \psi(\theta, \phi) \rightarrow v(r, \theta, \phi) \rightarrow E(r, \theta, \phi)$$

for half wave
dipole

$$|E| = \frac{\epsilon_0 \sin \theta \cos(k_{1/2} \cos \theta)}{\sin \theta}$$

$$v(r, \theta, \phi) = \frac{1}{2} \frac{\epsilon_0^2 \cos^2(k_{1/2} \cos \theta)}{r^2 \eta} \frac{\sin^2 \theta}{\sin \theta}$$

For a half wave dipole Directivity = "1.63"

~~2 W-1~~

21 - half wave dipoles.

(276)

$$W_r = \frac{I_{rms}^2}{2} \cdot 7.3 \quad (\text{for one half wave dipole})$$
$$= \left(\frac{I}{2 \times \sqrt{2}} \right) \cdot 7.3$$

$$W_r = \frac{\frac{I^2}{2} \times 1}{2 \sqrt{2}} \cdot 7.3 \times 4 \quad (4 \text{ dipole})$$

$$W_r = 36.5 \text{ watt.}$$

~~21. W-6~~

$$\frac{\lambda}{2} - \frac{\lambda}{4} - \frac{\lambda}{8} \text{ upto } \frac{\lambda}{10}$$

Half wave dipole

$$R_r = 7.3 \Omega - \lambda/2 \text{ dipole}$$

$$R_r = 18.25 \Omega - \lambda/8 \text{ dipole}$$

$$R_t = 1.5 \text{ (given)}$$

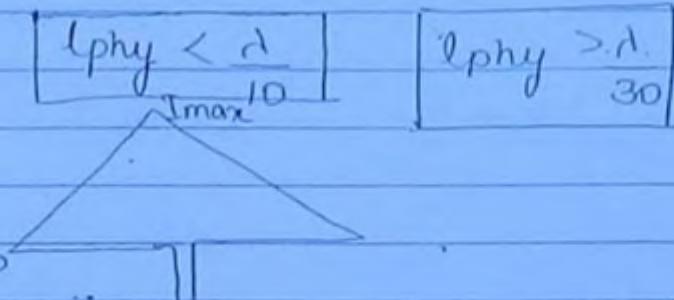
$$\text{Efficiency} = \frac{18.25}{19.25} = 89\%$$

Electrically short dipole

For electrically short dipole i.e. length is less than $\lambda/10$.

$$l_{phy} < \frac{\lambda}{10}$$

The currents are non uniform but they are linear
i.e. $I(z) \propto z$



$$I(z) = \text{linear} \quad \Rightarrow \quad w_z = \text{Phy.} \\ (I(z) \propto z)$$

$$I_{avg} = \frac{I_{max}}{2}$$

$$L_{eff} = \frac{l_{phy}}{2}$$

Hertzian Dipole length $\in [l_{phy} < \lambda/30]$

for the Hertzian dipole the currents are uniform as a length is very small and hence

$$I(z) = I_{max.} \\ = \text{uniform}$$

$$L_{eff} = l_{phy.}$$

$$\text{As Hertzian dipole} = R_r = 80\pi^2 \left(\frac{d_4}{\lambda}\right)^2$$

$$\text{for any Antenna} \quad R_r = 80\pi^2 \left(\frac{L_{eff}}{\lambda}\right)^2$$

Effective length assume uniform current everywhere
hence λ can be replaced with λ_{eff} for any antenna.

(a) (i) Electrically short Dipole:

$$R_r = 80\pi^2 \left(\frac{L_{phy}}{\lambda} \right)^2$$

$$R_r = 20\pi^2 \left(\frac{L_{phy}}{\lambda} \right)^2$$

(a) (ii) Electrically short Monopole:

$$R_r = 10\pi^2 \left(\frac{L_{phy}}{\lambda} \right)^2$$

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(a) (iii) Electrically short monopole: vertically grounded conducting surface.

$$R_r = 10\pi^2 \left(\frac{2h}{\lambda} \right)^2$$

$$L_{phy} = 2h$$

h = height of the monopole.

$$R_r = 40\pi^2 \left(\frac{h}{\lambda} \right)^2$$

Radiation Pattern

It is a pattern graph it is a polar plot of radiation intensity showing the regions where the radiation strength is finite.

e.g.: Hertzian Dipole has $E = \frac{E_0 \sin \theta}{r}$

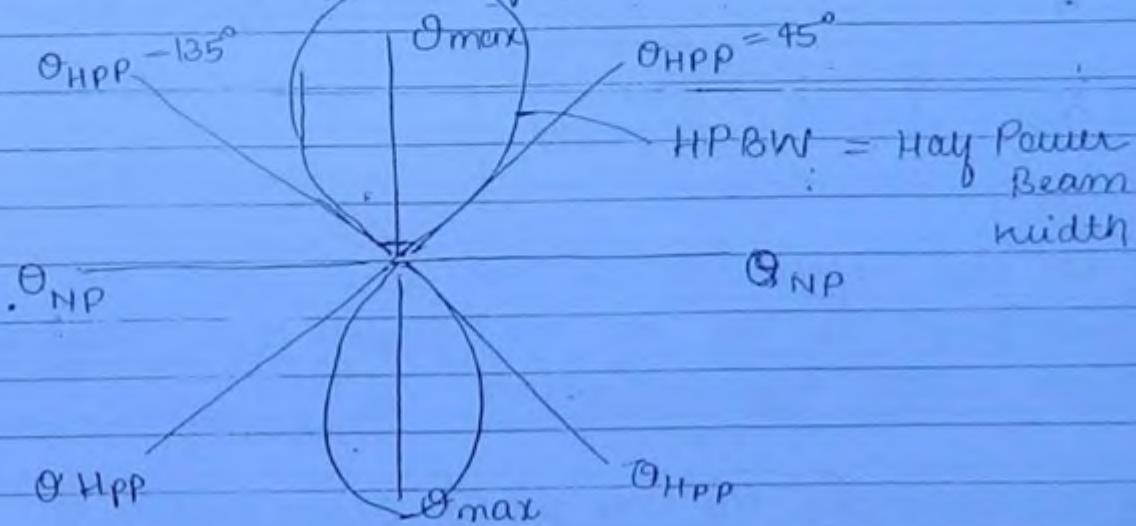
$$\psi = \psi_0 \sin^2 \theta$$

If $\theta = 0^\circ / 180^\circ \rightarrow \psi = 0$
 Q_{NP} \rightarrow null points

If $\theta = 90^\circ / 270^\circ \rightarrow \psi = \psi_{\max}$. 279
 $\rightarrow \theta = \theta_{\max}$

If $\theta = 45^\circ / 315^\circ / 135^\circ / 225^\circ \rightarrow \psi = \frac{\psi_0}{2}$

$\rightarrow \theta_{HPP} \rightarrow$ half power points.



- The Hertzian dipole has radiation intensity depending upon θ but independent of ϕ . Hence the polar plot has half power beam width defined for θ half power points enclosing the maxima.
- It is the same for all ϕ . Hence the beam is circular in the top view.

If the antenna is θ and ϕ dependent it has half power beam width in θ side and the half power beam width in ϕ side such that Beam solid angle is given by.

$$\text{HPBW} = \theta_{\text{HPBW}}$$

$$\text{HPBW} = \phi_{\text{HPBW}}$$

Beam solid angle Ω_A

$$\begin{aligned} & \theta \times \phi \\ & \text{HPBW} \quad \text{HPBW} \\ & = \text{steradians} \\ & = \text{radians}^2 \end{aligned}$$

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But if the ϕ side is a circular beam then $\theta \Omega_A$ is

$$\Omega_A = (\theta_{\text{HPBW}})^2$$

If the plot represents all non-zero values from the null point to the next null point it is called as beam width b/w full nulls.

In this example the half power Beam width is 90° and the beam width b/w full null is 180°

$D \propto 1 / \Omega_A$

$$D = \frac{4\pi}{\Omega_A}$$

If $D=1$ isotropic

Effective Area of an Antenna. (A_e) or Capture Area

$A_e = \frac{\text{Power Induced}}{\text{Poynting vector of the EM wave}} = \text{watts/watts/m}^2$



MW - 61 Hz - Dipole
 $d = \text{cm}$.

$$A_e = \frac{\lambda^2 D}{4\pi}$$

$$44 \text{ dB} = G$$

$$10 \log G = 44$$

$$\log G = 4.4$$

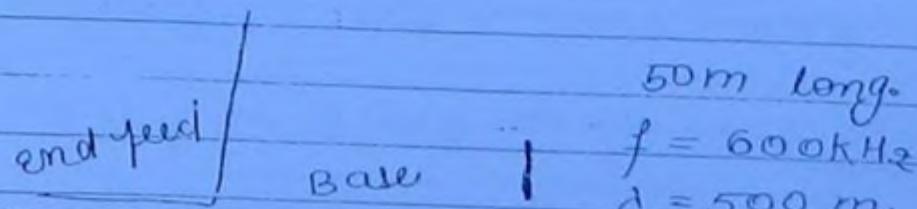
$$D = 10^{4.4}$$

$$1 \text{ Radian} = 57^\circ$$

$$3.14 \times 57^\circ = 180^\circ$$

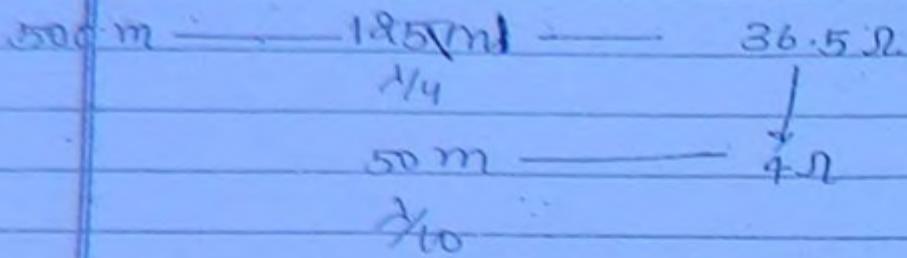
(Radian = no)

$$10^{4.4} = \frac{4\pi}{(\theta_{HPBW})^2} = \frac{4 \times 3.14 \times (57)^\circ}{(\theta_{HPBW})^2}$$



$$d = \frac{\lambda}{10}$$

$$R_f = 240\pi^2 \left(\frac{h}{\lambda}\right)^2 = 240\pi^2 \left(\frac{50}{500}\right)^2 = \frac{2\pi^2}{5} = 4\pi$$



Q3 W.B

$$1 \text{ m} = l$$

$$10 \text{ MHz} = f$$

$$\lambda = \frac{3 \times 10^8}{10 \times 10^6} = \frac{c}{f} = 30 \text{ m}$$

$$l = \frac{\lambda}{30}$$

$$R_r = 80\pi^2 \left(\frac{dl}{\lambda} \right)^2 = 80\pi^2 \left(\frac{d}{\lambda 30} \right)^2 = 0.88 \Omega$$

length is small than wavelength the radiating ability it become poor.

Q4 W.B

$$l = 0.03 d$$

$$R_r = 80\pi^2 \left(\frac{dl}{\lambda} \right)^2$$

$$= 80\pi^2 \left(\frac{0.03 d}{d} \right)^2 \Rightarrow 80\pi^2 \times (0.03)^2 \\ \Rightarrow 0.072 \pi^2 \Omega$$

Q5 W.B

$$l = 5 \text{ m}$$

$$\lambda = c = \frac{3 \times 10^8}{3 \times 10^6} = 100 \text{ nm}$$

$$R_r = 80\pi^2 \left(\frac{dl}{\lambda} \right)^2$$

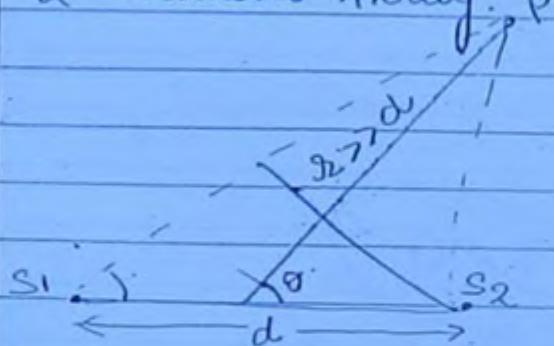
$$= 80\pi^2 \left(\frac{5}{100} \right)^2 \Rightarrow$$

$$= 0.197 \Omega \approx 2 \Omega$$

Wednesday.

Array of Isotropic sources.

2- Element Array:



To analyse the radiation at a far-zone i.e. $\sigma \gg d$.

$$E_T = E_0 + KE_0 e^{j\psi}$$

$$\text{If } |I_1| = |I_2| \text{ in } S_1 \text{ & } S_2 \\ K=1.$$

ψ = phase diff b/w the radiated waves from S_1 and S_2 .

ψ = phase difference in currents in sources :

+

path difference due to inclination

$$\alpha\pi - \delta$$

$$\gamma = d \cos \theta$$

$$\boxed{\psi = \alpha + \beta \cos \theta}$$

$$E_T = E_0 (1 + \cos \psi + j \sin \psi)$$

$$|E_T| = |E_0| \sqrt{(1 + \cos \psi)^2 + \sin^2 \psi}$$

$$= |E_0| \sqrt{1 + \cos^2 \psi + 2 \cos \psi + \sin^2 \psi}$$

$$= |E_0| \sqrt{1 + 2 \cos \psi}$$

$$= E_0 \sqrt{2 \cdot 2 \cos^2 \psi / 2}$$

$$\boxed{|E_T| = 2E_0 \cos \psi / 2}$$

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Note Two individually isotropic antennas at an array
are not isotropic due to interference of the radiation

$$E_T = 2E_0 \cos \psi/2$$

$$\psi = \alpha + \beta d \cos \theta$$

case(i) $d = \lambda/2, \alpha = 0^\circ$

$$E_T = 2E_0 \cos \psi/2$$

(264)

$$\psi = 0 + \frac{2\pi \times \lambda}{\lambda} \cos \theta$$

$$\psi = \pi \cos \theta$$

$$E_T = 2E_0 \cos \left(\frac{\pi \cos \theta}{2} \right)$$

$$\theta = 0^\circ \text{ or } 180^\circ$$

$$\begin{cases} E_T = 0 \\ \theta_{NP} \end{cases}$$

$$\theta = 90^\circ \text{ or } 270^\circ$$

$$E_T = 2E_0 \cos \left(\frac{\pi}{2} \times 0.75 \times 90^\circ \right)$$

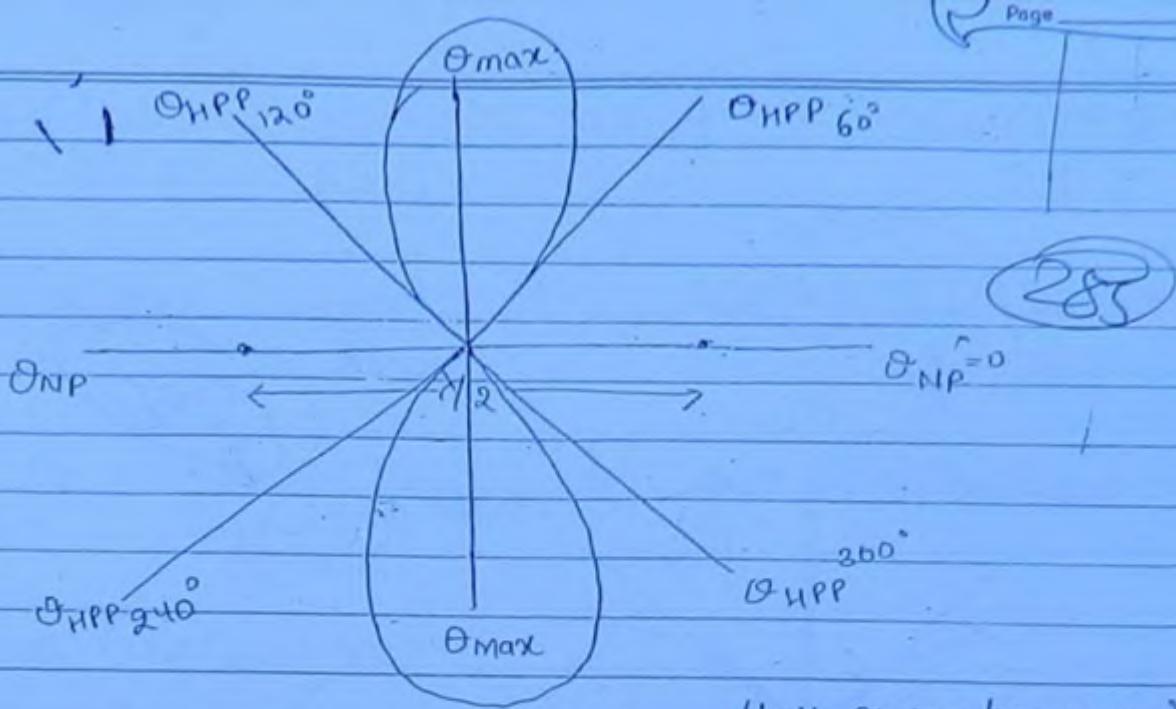
$$E_T = 2E_0$$

$$E_T = E_{max} = 2E_0$$

$$\theta_{max}$$

$$\theta = 60^\circ / 120^\circ / 240^\circ / 300^\circ$$

$$\begin{cases} E_T = \frac{E_{max}}{\sqrt{2}} \\ \theta_{HPP} \end{cases}$$



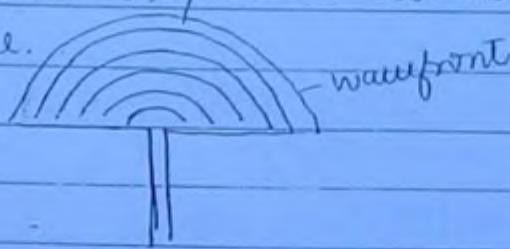
Half power beam width
= 60°

Broad side Array.

$$\text{i.e. } \theta_{\max} = 90^\circ / 270^\circ$$

Maximum Radiation normal to the axis of the array.

i) The broad side array and a half wave dipole always have identical radiation pattern as the currents are maximum at the centre.



$$d = \lambda/2 \quad \nu = \pi$$

$$E_T = 2 E_0 \cos(\chi + \pi \cos \theta)$$

$$\psi = \chi + \frac{2\pi \times d}{\lambda} \cos \theta$$

(Half power beam width is 60° for the broadside array)

$$E_T = 2 E_0 \sin(\pi/2 \cos \theta)$$

$$\text{If } \theta = 0^\circ / 180^\circ$$

$$E_T = E_{\max}$$

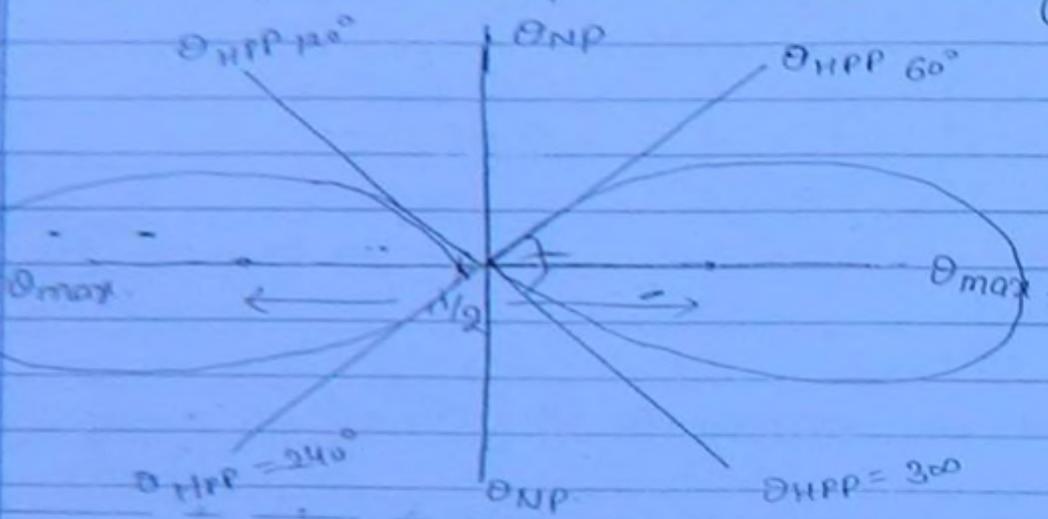
$$\theta_{\max}$$

$$\begin{aligned} \cos \psi/2 &= \cos \left(\frac{\chi + \pi \cos \theta}{2} \right) \\ &= \cos \left[\frac{\chi}{2} + \frac{\pi \cos \theta}{2} \right] \\ &= \sin \left[\frac{\chi}{2} \cos \theta \right] \end{aligned}$$

$$2) \theta = 90^\circ / 270^\circ \quad E_T = 0$$

$$3) \theta = 60^\circ / 120^\circ / 240^\circ / 300^\circ \quad E_T = \frac{E_{max}}{\sqrt{2}}$$

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End fire Array.

$$\text{i.e. } \theta_{max} = 0^\circ / 180^\circ$$

maximum Radiation along axis of the array.

Half power Beam width = 120°

$$d = \lambda, \alpha = 0^\circ$$

$$E_T = 2E_0 \cos \psi I_2$$

$$\psi = 0 + \frac{2\pi \times d \cos \theta}{\lambda}$$

$$\psi = 2 \times \cos \theta$$

$$E_T = 2E_0 \cos \left(\frac{2\pi \cos \theta}{\lambda} \right)$$

$$E_T = 2E_0 \cos (\lambda \cos \theta)$$

$$\theta = 0^\circ / 180^\circ$$

$$E = E_{max}$$

$$\theta = 90^\circ / 270^\circ$$

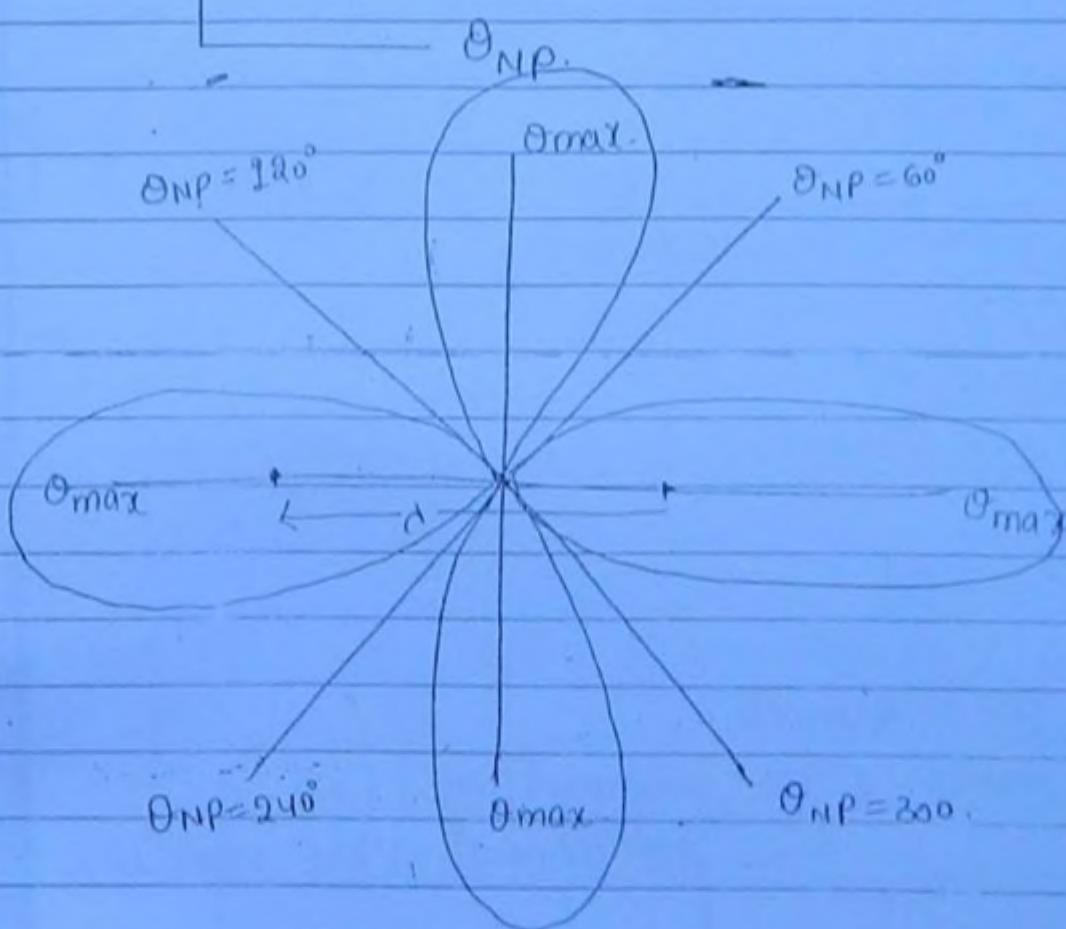
$$E = E_{max}$$

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$$E_T = 2E_B = E_{max} \quad \text{for } \theta = 0^\circ / 90^\circ / 180^\circ / 270^\circ$$

θ_{max}

$$\theta = 60^\circ / 120^\circ / 240^\circ / 300^\circ \quad E_T = 0.$$



Scanning Array.

It is designed to have maxima in specific direction by changing a in the source.

Generalization or Desinged principle

$$\psi \rightarrow 0 \Rightarrow E_{max} = \text{Max Radiation}$$

$$\chi + \beta d \cos \theta_{max} = 0$$

$$\boxed{\cos \theta_{max} = -\frac{\chi}{\beta d}}$$

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If $\theta_{max} = 90^\circ / 270^\circ$
 \Rightarrow Broad side array
 $\Rightarrow \alpha = 0$

In phase currents Broadside array.

If $\theta_{max} = 0^\circ / 180^\circ$
 \Rightarrow End fire Array.
 $\Rightarrow \alpha = \pm \beta d$

Extension for N elements

N -elements - linear, uniform array

All elements on the same line

Equal spacing (d)

Equal phase diff. progressively
 (α) between the elements.

$$E_T = E_0 + E_0 e^{j\psi} + E_0 e^{j2\psi} + E_0 e^{j3\psi} + \dots + E_0 e^{j(N-1)\psi}$$

$$E_T = E_0 [1 + e^{j\psi} + e^{j2\psi} + E_0 e^{j3\psi} \dots + e^{j(N-1)\psi}]$$

Geometric Progression

first element = $l = a$

common factor $e^{j\psi} = r$

(2pg)

$$E_T = E_0 \left(\frac{1 - e^{jN\psi}}{1 - e^{j\psi}} \right) = E_0 \left(\frac{1 - \cos N\psi - j \sin N\psi}{1 - \cos \psi - j \sin \psi} \right)$$

$$E_T = E_0 \frac{\sin(N\psi)}{\sin(\psi/2)}$$

$$\sin(\psi/2)$$

If $\psi \rightarrow 0$

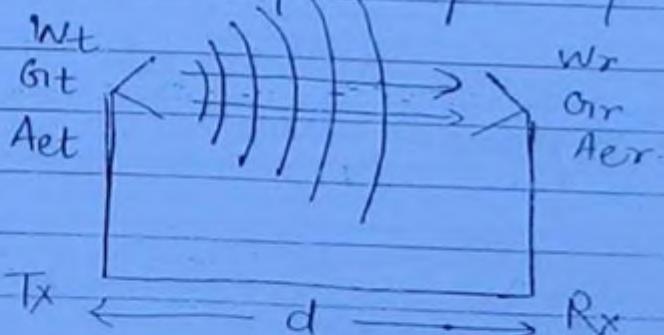
$$E_T = E_0 \left(\frac{N\psi}{2} \right) \left(\frac{2}{\psi} \right) = NE_0$$

$$E_T = NE_0$$

If $N = 2$.

$$E_T = 2E_0 \cos \psi/2$$

FRIIS - free - space propagation eqn.



$$\text{Power density at the receiver} = \frac{W_t G_t}{4\pi d^2} A_{et}$$

$$\text{Power Received} = \frac{W_t G_t}{4\pi d^2} A_{er}$$

$$E_{\text{rr}} = \frac{E_0}{r}$$

Page

$$A_{\text{eff}} = \frac{d^2}{4\pi} G_{\text{rr}}$$

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(Power Received) $M_r = \frac{W_t G_t \cdot G_{\text{rr}}}{\left(\frac{4\pi d}{c}\right)^2}$

FRIIS - free space propagation Eqⁿ.

$$\left(\frac{4\pi d}{c}\right)^2 = \begin{array}{l} \text{loss due to spatial attenuation} \\ \text{- due to dispersion} \end{array}$$

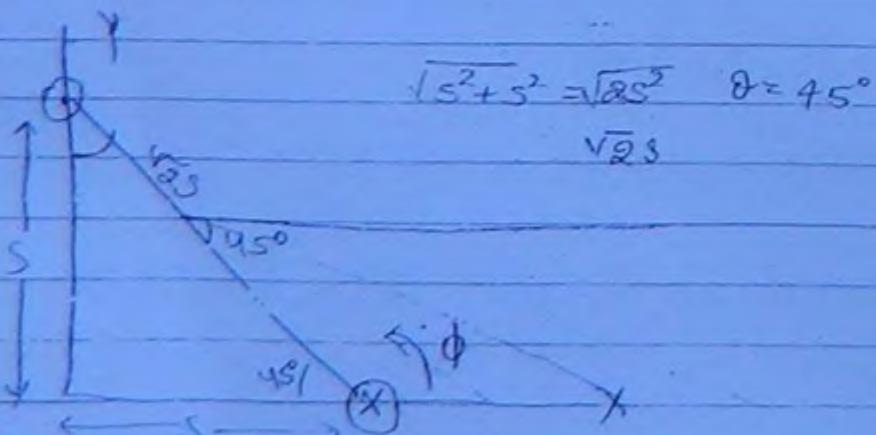
Power density at the receiver = $\frac{W_t G_t}{4\pi d^2} = \frac{1}{2} \frac{E_0^2}{\eta} = \frac{E_{\text{rms}}^2}{\eta} = \frac{E_{\text{rr}}^2}{120\eta}$

$$E_{\text{rms}} = \frac{30 W_t G_t}{d}$$

(RMS E field at the receiver) $E_{\text{rr}} = \sqrt{\frac{30 W_t G_t}{d}}$

$$[W_r (\text{dBW}) = W_t (\text{dBW}) + G_t (\text{dB}) + G_{\text{rr}} (\text{dB}) - L_s (\text{dB})]$$

~~16 W~~ $\theta = \pi/2$ plane }
 $Z = \sin \theta = 0$ } x-y plane.



$$\lambda = \lambda$$

$$d = \sqrt{2} \lambda$$

$$\theta = 45^\circ$$

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$$E_T = 2E_0 \cos \left(\frac{\alpha + \beta d \cos \theta}{\lambda} \right)$$

$$E_T = 2E_0 \cos \left(\frac{\pi}{2} + \frac{2\pi \cdot \sqrt{2} s \cos 45^\circ}{\lambda} \right)$$

$$\frac{E_T}{E_0} = 2 \cos \left(\frac{\pi}{2} + \frac{\alpha \lambda - \sqrt{2} s \times 1}{\lambda} \right)$$

$$\frac{E_T}{E_0} = 2 \cos \left(\frac{\pi}{2} + \frac{\alpha \lambda s}{\lambda} \right)$$

$$\frac{E_T}{E_0} = 2 \sin \left(\frac{\pi s}{\alpha} \right)$$

+3dB ↑ (2) (double)

-3dB ↓ (2) (i.e half).

5km → d.

$$10 \log (\text{value}) = 3 \text{ dB}$$

$$\log (\text{value}) = 0.3$$
$$\text{value} = 10^{0.3} = 2.$$

$$E \propto \frac{1}{d}$$

$$\frac{E_1}{E_2} = \frac{d_2}{d_1}$$

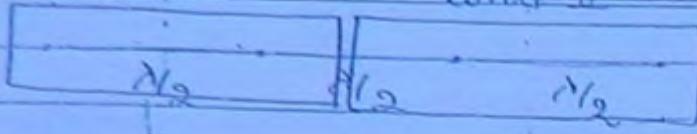
$$E_2 = \frac{E_1}{\sqrt{2}}$$

$$d_2 = \sqrt{2} d_1 = \sqrt{2} \times 5 = 7 \text{ km}$$

$$7 \text{ km} - 5 \text{ km} = 2 \text{ km}$$

14 W-B

UNIT I UNIT II



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Multiplication of patterns (symmetric arrays)

Two symmetric units (same)

unit : 2 element array.

$$d = d_{I_2}$$

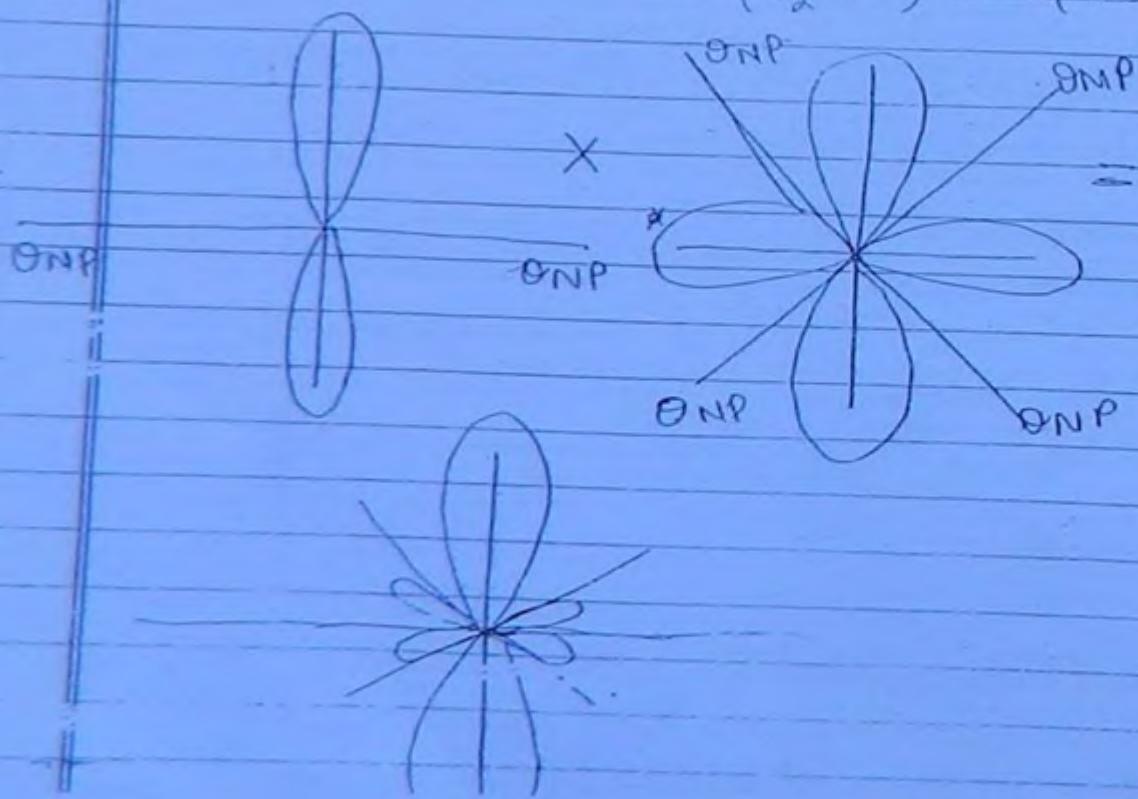
$$E_T = 2 E_0 \cos(\pi/2 \cos \theta)$$

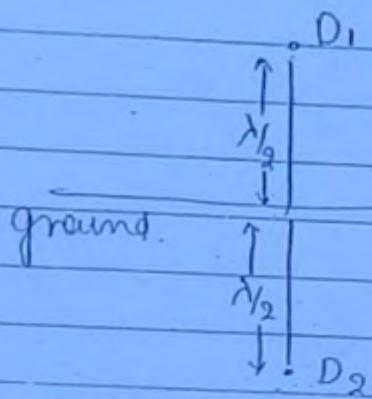
The Two units forming a group.

Group \rightarrow combination of unitsGroup \rightarrow 2 units $\Rightarrow d = d$; $\theta = 0$

$$E_T = 2 E_0 \cos(\pi \cos \theta)$$

Resultant Pattern = unit Pattern \times group pattern
 $= \cos(\pi/2 \cos \theta) \times \cos(\pi \cos \theta)$





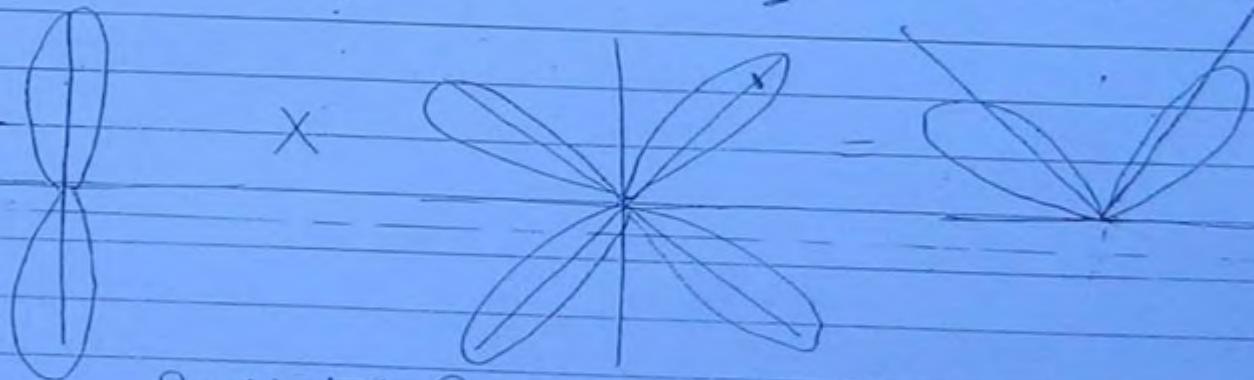
(293)

unit dipole $\lambda/2$

$$E_T = 2E_0 \cos(\pi/2 \cos\theta)$$

group. - 2 dipoles - d - $\lambda = \pi$

$$E_T = 2E_0 \sin(\pi \cos\theta)$$

at 60° we get major lobe.

Radiation Pattern above the ground.

N.B

$$\cos\theta_{\max} = -\frac{\alpha}{\beta d}$$

$$\cos 60^\circ = -\frac{\alpha}{\beta d}$$

$$\frac{2\pi \times \pi \lambda}{\beta}$$

$$\frac{1}{\alpha} = -\frac{\alpha}{\pi \lambda}$$

$$-\alpha = \frac{\pi \lambda}{2}$$

$$-\alpha = \frac{\pi}{2} \text{ radians}$$