

DIGITALS

NOTES

GATE 2009

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Digits

SUN.

17/08/08

Number Systems:

	<u>Base/Radix</u>	<u>Numbers</u>
1. Decimal	10	0, 1, ..., 9
2. Binary	2	0, 1
3. Octal	8	0, 1, ..., 7
4. Hexadecimal	16	0, 1, ..., 9, A, B, C, D, E, F.

Each Hexa digit \rightarrow 4 bits,

$$3F_{16} \rightarrow 0011\ 1111_2$$

Each octal digit \rightarrow 3 bits

$$316_8 \rightarrow 011\ 001\ 110_2$$

Q. $110010_2 = x_{16}$

Q. $11011.01_2 = x_{16}$

$$\begin{array}{r} \overset{\leftarrow}{0011} \overset{\leftarrow}{0010} \\ \hline 3 \quad 2 \end{array} = 32_{16} \quad \begin{array}{r} \overset{\leftarrow}{0001} \overset{\leftarrow}{1011} \overset{\rightarrow}{0100} \\ \hline 1 \quad B \quad 4 \end{array} = 4B1_{16}$$

Q. $6728_{10} = x_2$

$$6728_{10} \rightarrow 6728_{16} \rightarrow x_2$$

$$\Rightarrow \begin{array}{r} 6728 \\ \hline 16 \quad | \quad 420 - 8 \\ \hline 16 \quad | \quad 26 - 4 \\ \hline 1 \quad - 10(A) \uparrow \end{array} \quad \begin{array}{r} 1A48 \\ \hline 16 \end{array} = \underline{\underline{0001\ 1010\ 0100\ 1000}}_2$$

Q. Determine the possible bases of the following relations.

(1). $\sqrt{41} = \frac{5}{7}$ max. digit is 5 \downarrow

so min. value of base is 6. so base ≥ 6

Let base = b.

$$\sqrt{4 \times b^1 + 1 \times b^0}_{10} = 5 \times b^0_{10}$$

$$\Rightarrow \sqrt{4b+1} = 5$$

$$\Rightarrow 4b+1 = 25$$

$$\Rightarrow b = 6.$$

Q. $\frac{302}{20} = 12.1$

Let base = b .

Base ≥ 4 b'coz max digit is 3.

$$\Rightarrow \frac{3b^2+2}{2b} = b+2+\frac{1}{b}$$

$$\Rightarrow \frac{3b^2+2}{2b} = \frac{b^2+2b+1}{b}$$

$$\Rightarrow b = 4.$$

Q. $\frac{44}{4} = 11$

Let base = b . Observed base ≥ 5 , b'coz maximum value of digit = 4.

$$\frac{4b+4}{4} = b+1 \Rightarrow b+1 = b+1$$

The above relation is valid in all the no. system with base ≥ 5 .

Q. In a positional weight system x & y are two successive digits and $xy = 25_{10}$ & $yx = 31_{10}$. Determine the values of base x & y .

Here $b = ?$, $x = ?$ & $y = ?$

and $y = x+1$.

$$(x)(x+1) = 25_{10} \quad ((x+1)b+x) = 31_{10} \quad (1)$$

$$\Rightarrow [x(b+1) + (x+1)]_{10} = x(b+1) + b = 31 \rightarrow (2)$$

$$\Rightarrow x(b+1) + 1 = 25 \rightarrow (1)$$

$$(1) - (2) \Rightarrow b = 7. \text{ Then from } (1) \Rightarrow x = 3, y = 4.$$

Complementary Number Representation :-

base = 2

\Rightarrow (2-1)'s complement

\Rightarrow 1's complement

Decimal system ($\lambda = 10$)

$$\text{9's complement of } 168_{10} \Rightarrow \begin{array}{r} 999 \\ 168 \\ (-) \hline 831_{10} \end{array}$$

10's complement of $168_{10} \Rightarrow$ 9's comp + 1

$$\Rightarrow \begin{array}{r} 999 \\ 168 \\ (-) \hline 831+1 = 832_{10} \end{array}$$

Q. 862_{10}

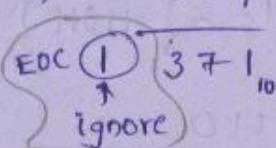
$$\begin{array}{r} 491_{10} \\ (-) \hline ? \end{array} = 862_{10} + (-491_{10})$$

(i). 862

$$\begin{array}{r} 862 \\ +(\text{9's of } 491) \\ \hline (+) \quad 508 \\ \hline 370 \end{array}$$

(iii). 862

$$\begin{array}{r} 862 \\ +(\text{10's of } 491) \\ \hline 509 \end{array}$$

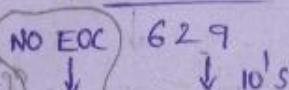


Q. 491_{10}

$$\begin{array}{r} 862_{10} \\ - \hline ? \\ -371_{10} \end{array}$$

$491 + (-862)$

$$\begin{array}{r} 491 \\ +(-862) \\ \hline (+) \quad 138 \end{array} \leftarrow \text{10's}$$



Digital System ($\lambda = 2$)

1's complement of 1011 \rightarrow 0100,

2's complement of 1011 \rightarrow 1's of 1011 + 1

$$\Rightarrow 0100 + 1 = 0101$$

$$\text{Q. } x = \overbrace{1000111}^{\leftarrow} \underline{000}$$

$$x\text{'s complement of } x = \underline{0111001}000$$

$$\text{Q. } x = 1011$$

$$2\text{'s of } x = 0101$$

$$\text{Q. } 11010_2 \quad 11010$$

$$- 01110_2 = +(-01110)$$

$$\begin{array}{r} 11010 \\ + (1\text{'s of } 01110) \\ \hline 11010 \\ + 10001 \end{array}$$

$$\begin{array}{r} 11010 \\ + (2\text{'s of } 01110) \\ \hline 11010 \end{array}$$

$$\begin{array}{r} \text{EOC} \text{ } \textcircled{1} \\ 01011 \\ \downarrow +1 \\ \hline 01100 \end{array}$$

$$\begin{array}{r} 11010 \\ + 10010 \\ \hline \end{array}$$

EOC ignore $\textcircled{1}$

$$\text{Q. } 01110_2 \quad 01110$$

$$- 11010_2 = + (2\text{'s of } 11010)$$

$$\begin{array}{r} 01110 \\ + 00110 \\ \hline \end{array}$$

$$\begin{array}{r} \xrightarrow{\text{NO EOC}} \square 10100 \\ \downarrow 2\text{'s} \\ 01100 \end{array}$$

$$\begin{array}{l} 2^4 = 16 \\ 16 - 2 = 14 \end{array}$$

i's comp

$$16 - 1 = 15$$

$$+0 = 0000$$

$$+0 = 0000$$

$$-0 = \text{i's comp of } +0$$

$$-0 = \text{2's comp. of } +0$$

$$= \text{i's of } 0000$$

$$= 0000$$

$$= 1111$$

\leftarrow (Disadv. of i's complement.)

* Range of numbers represented using 'n' bits

To represent 16 numbers
1's comp. form $\rightarrow + (2^{n-1} - 1)$ to $- (2^{n-1} - 1)$

$$\text{Let } n=4 \Rightarrow +7 \text{ to } -7 \rightarrow (14)$$

2's comp. form $\Rightarrow + (2^{n-1} - 1)$ to -2^{n-1}

$$\text{Let } n=4 \Rightarrow +7 \text{ to } -8 \rightarrow (15)$$

Q. How many bits are required to represent -64_{10} in a). 1's comp. form b). 2's form

1's form $\Rightarrow + (2^{n-1} - 1)$ to $- (2^{n-1} - 1)$

$$\text{Let } n=7 \Rightarrow +63 \text{ to } -63$$

$$\checkmark n=8 \Rightarrow +127 \text{ to } -127$$

2's form $\Rightarrow + (2^{n-1} - 1)$ to -2^{n-1}

$$\checkmark \text{Let } n=7 \Rightarrow +63 \text{ to } -64$$

Q. 10's comp for $(-731)_{11}$

$$\begin{array}{r} A A A \\ 7 3 1 \\ (-) \hline 3 7 9 \end{array}$$

Q. 9's comp of $(-731)_{10}$

$$\begin{array}{r} 999 \\ (-) 731 \\ \hline 268 \end{array}$$

Binary Numbers :

(a). Unsigned Numbers \rightarrow

n bits

magnitude

(b). Signed Numbers

↓ represented by

(i). sign magnitude

(ii). 1's comp form

(iii). 2's comp form

HSB

↓ sign bit

magnitude

0 \rightarrow +ve

1 \rightarrow -ve

These three representations are same for unsigned (+ve) numbers.

$$(i). \text{ sign magnitude} \Rightarrow +3 = \begin{array}{c} 0 \\ \downarrow \\ 111 \end{array}$$

$$-3 = \begin{array}{c} 1 \\ \downarrow \\ 111 \end{array}$$

$$(ii). 1's \text{ comp. form} \Rightarrow +3 = 011$$

$$-3 = \begin{array}{c} 1's \text{ comp of } +3 \\ \hline = 100 \end{array}$$

$$(iii). 2's \text{ comp. form} \Rightarrow +3 = 011$$

$$-3 = \begin{array}{c} 2's \text{ comp of } +3 \\ \hline = 101 \end{array}$$

Q. Decimal equivalent of 2's number $\begin{array}{c} 101 \\ \downarrow 2's \end{array}$ is -?

$$\begin{array}{r} \cancel{1} \\ - \underline{011} \\ = -3_{10} \end{array}$$

Q. Decimal equivalent of sign mag. no. 111 is -?

$$-3_{10}$$

Q. Represent $+53_{10}$ & -53_{10} in all the 3 forms of signed no. representation.

$$53_{10} \rightarrow \begin{array}{r} 53 \\ 2 \overline{)26} -1 \\ 2 \overline{)13} -0 \\ 2 \overline{)6} -1 \\ 2 \overline{)3} -0 \\ 1 -1 \uparrow \end{array} = 110101_2$$

$$+53 = 0110101$$

sign mag. form	1's form	2's form
$0\ 110101$	0110101	0110101

$+53_{10}$	1110101	$-53 = 1's \text{ of } +53$	$-53 = 2's \text{ of } +53$
		$= 1001010$	1001011

Q. What are the decimal equivalents of the following signed no.s in all the 3 forms.

	Sign mag. form	1's form	2's form
01101	$+13_{10}$	$+13_{10}$	$+13_{10}$
101010	$\begin{array}{r} 101010 \\ -10_{10} \end{array}$	$\begin{array}{r} 101010 \\ \downarrow 1's \\ -010101 \end{array}$ $= -21_{10}$	$\begin{array}{r} 101010 \\ \downarrow 2's \\ -010110 \end{array}$ $= -22_{10}$
111111	$\begin{array}{r} 111111 \\ -31_{10} \end{array}$	$\begin{array}{r} 111111 \\ \downarrow 1's \\ -0 \end{array}$	$\begin{array}{r} 111111 \\ \downarrow 2's \\ -1 \end{array}$

Q. Decimal equivalent of 2's no. 1000 is - ?

$$\begin{array}{r} 1000 \\ \downarrow 2's \\ -1000 \\ = -8_{10} \end{array}$$

Q. Decimal equivalent of 2's no. 10000 is - ?

$$\begin{array}{r} 10000 \\ \downarrow 2's \\ -10000 \\ = -16_{10} \end{array}$$

Q. What is the equivalent 2's comp representation of a 2's comp. no. 1101 if - ?

- (a). 001101 (b). 011101 (c). 101101 (d). 111101

$$+6 = 0110$$

$$-6 = 2's \text{ of } +6 = 2's \text{ of } 0110$$

$$= 1010$$

$$= 2's \text{ of } 00110 = 11010$$

$$= 2's \text{ of } 000110 = 111010$$

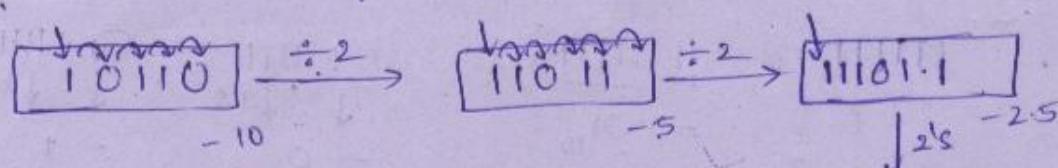
Q. A Register contains a 2's comp. no. 10110. What is the content of the register if it is divided by 2.

decimal equi. of $10110 = -01010$

$$= \frac{-10_{10}}{2} = -5$$

$-5 = 2^1$'s of +5

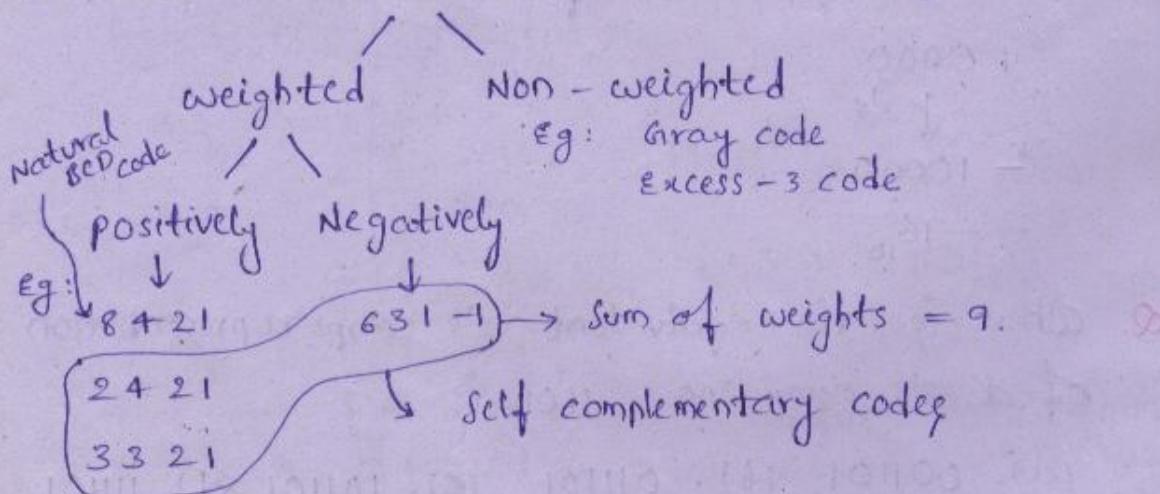
(or) $= 2^1$'s of $00101 = 11011$



Binary codes :-

a). Alpha numeric [ASCII code $000 = -2.5_{10}$
 [7 bits, $2^7 = 128$ Alphanumeric]
 EBCDIC
 { 8 bits, $2^8 = 256$ Alphanumeric]

b). Numeric $\xrightarrow{(a)} BCD$ \rightarrow each decimal digit \rightarrow 4 bits



Excess-3 : self complementary code, sequential code.

8421 : sequential code.

Dec. digit	Natural BCD	EXCESS-3	2421	631-1	Gray
0	0000	0011	0000	0000	0000
1	0001	0100	0001	0010	0001
2	0010	0101	0010	0101	0011
3	0011	0110	0011	0100	0010
4	0100	0111	0100	0110	0100
5	0101	1000	1011	1001	0111
6	0110	1001	1100	1011	0101
7	0111	1010	1101	1010	0100
8	1000	1011	1110	1101	1100
9	1001	1100	1111	1111	1101

743_{10} in (i). BCD \rightarrow 0111 0100 0011 BCD.

$$(2). \quad 3321 \rightarrow 1101\ 0101\ 0011_{3321}$$

$$\begin{array}{r} 3 \\ \downarrow \\ 1000 \\ 0100 \\ \hline \checkmark 0011 \\ (3321) \end{array} \left\{ \begin{array}{l} \text{Self} \\ \text{complementary} \end{array} \right\}$$

$$(3). \text{ Binary } \rightarrow 2^n \geq 743, n = 10.$$

$$\begin{array}{r} 16 \Big| 743 \\ 16 \Big| 46 -7 \\ 16 \Big| 2 \end{array} \quad 2E7_{16} = 0010\ 1110\ 0111_2$$

Gray code: (reflective code, unit distance code)

1-bit	2-bit	3-bit	
$0+0=0$	00	000	
$0+1=1$	01	001	
$1+0=1$		01	
$1+1=0$ Modulo-2 Addition (Exclusive OR)	0110	1110100	\rightarrow differ by 1-bit
	1000	0100110	
		11	
		01001	
Binary:	10110	100	permits : 2^n

A diagram showing the conversion of a 4-bit Gray code to a 4-bit binary code. The Gray code input is 1110. The binary output is 0110. A vertical arrow points down from the Gray code to the binary code. Below the Gray code, there are four circles with plus signs (+) inside, connected by a horizontal line. Below the binary code, there are four circles with minus signs (-) inside, also connected by a horizontal line.

BCD Addition :-

$$\begin{array}{r}
 6_{10} = 0110 \text{ BCD} \\
 + 2_{10} = 0010 \text{ BCD} \\
 \hline
 1000 \text{ BCD} \\
 \downarrow \\
 8_{10}
 \end{array}
 \quad
 \begin{array}{r}
 8_{10} = 1000 \text{ BCD} \\
 + 6_{10} = 0110 \text{ BCD} \\
 \hline
 1110 \rightarrow \text{not a valid BCD.} \\
 + 0110 \\
 \hline
 0001 \quad 0100 \text{ BCD} \\
 \hline
 14_{10}
 \end{array}$$

$$\begin{array}{r}
 + 9_{10} = 1001 \text{ BCD} \\
 8_{10} = 1000 \text{ BCD} \\
 \hline
 10001 \\
 + 0110 \\
 \hline
 10111 \text{ BCD} \\
 \hline
 17_{10}
 \end{array}$$

Decimal Binary/Hexa

0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111
10	10000
11	10001
12	10010
13	10011
14	10100
15	10101
16	10110

$8+2=10$

$(\text{Diff}=6)$

Q. In the following BCD additions how many BCD corrections are required.

$$\begin{array}{r}
 49_{10} = 0100 \text{ } \cancel{1} 001 \\
 + 57_{10} = 0101 0111 \\
 \hline
 1010 0000 \\
 + 0110 0110 \\
 \hline
 0000 0110 \\
 \hline
 106_{10}
 \end{array}$$

Ans: 2 timesq.

$$\begin{array}{r}
 176_{10} \\
 + 824_{10} \\
 \hline
 1001 1001 1010 \\
 \underline{0110} \\
 \hline
 1001 1010 0000
 \end{array}$$

Ans: 3 timesq

$$\begin{array}{r}
 176 \\
 824 \\
 \hline
 1000
 \end{array}$$

$$\begin{array}{r}
 0001 0111 0110 \\
 1000 0010 0100 \\
 \hline
 1001 1001 1010 \\
 \underline{0110} \\
 \hline
 1001 1010 0000
 \end{array}$$

$$\begin{array}{r}
 0110 \\
 \hline
 1010 0000 0000
 \end{array}$$

$$\begin{array}{r}
 0110 \\
 \hline
 0000 0000 0000
 \end{array}$$

* SUNDAY, 31. Aug. 2008 *

Boolean Algebra:

AND Law

$$A \cdot 0 = 0$$

$$A \cdot 1 = A$$

Identity element
A $\cdot A = A$

$$A \cdot \bar{A} = 0$$

OR Law

$$A + 0 = A$$

$$A + 1 = 1$$

Identity element.

$$A + A = A$$

$$A + \bar{A} = 1$$

(1). Commutative Law:

$$A + B = B + A$$

$$A \cdot B = B \cdot A$$

* AND, OR operations

are commutative &

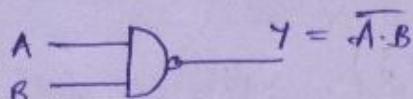
Associative

(2). Associative Law:

$$(A + B) + C = A + (B + C)$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

C find the commutative & associative operations of NAND.



$$(a). A \cdot \bar{A} \cdot B = \bar{B} \cdot A$$

$$(b). (\bar{A} \cdot \bar{B}) \text{NAND } C = \bar{A} \cdot \bar{B} \cdot C$$

$$A \text{ NAND } (B \text{ NAND } C) = A \text{ NAND } (\bar{B} \cdot \bar{C})$$

$$\rightarrow \bar{\bar{A} \cdot \bar{B} \cdot C} \neq \bar{A} \cdot \bar{\bar{B} \cdot C}$$

$$= \bar{A} \cdot \bar{B} \cdot C$$

* NAND operation is commutative but not associative.

(3). Distribution law:

$$A \cdot (B + C) = AB + AC$$

$$A + (BC) = (A + B)(A + C)$$

$$(i). A + \bar{A}B = (A + \bar{A})(A + B)$$

$$= (A + B).$$

$$(ii). \quad \bar{A} + AB = (\bar{A} + A)(\bar{A} + B) \\ = (\bar{A} + B)$$

(4). Consensus Law:

$$AB + \bar{A}C + BC = AB + \bar{A}C.$$

$$\text{eg: } xy + \bar{y}z + \bar{x}yz = xy + \bar{y}z$$

$$\begin{aligned} \text{proof: } & AB + \bar{A}C + BC(A + \bar{A}) \\ &= AB + \bar{A}C + ABC + \bar{A}BC \\ &= AB(1 + C) + \bar{A}C(1 + B) \\ &= AB + \bar{A}C. \end{aligned}$$

$$(A+B) \cdot (\bar{A}+C) \cdot (B+C) = (A+B) \cdot (\bar{A}+C)$$

(5). Transposition law:

$$AB + \bar{A}C = (A+C)(\bar{A}+B)$$

$$\text{eg: } xy + \bar{y}z = (x + \bar{y})(y + z)$$

$$\begin{aligned} \text{RHS: } (x + \bar{y})(y + z) &= xy + \bar{y}z + xz \\ &= xy + \bar{y}z. \end{aligned}$$

$$(A+B)(\bar{A}+C) = AC + \bar{A}B$$

(6). De Morgan's Law:

$$\overline{A + B + C + \dots} = \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \dots$$

$$\overline{\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \dots} = \bar{A} + \bar{B} + \bar{C} + \dots$$

Additional Laws:

$$(1). \quad x \cdot f(x, \bar{x}, \omega, y, \dots, j)$$

$$= x \cdot f(1, 0, \omega, y, \dots, j)$$

$$x + f(x, \bar{x}, w, y, \dots) \\ = x + \underline{f(0, 1, w, y, \dots)}$$

(7). Duality:

All the Boolean expressions resulting from interchanging of operators and identity elements are valid.

Eg: $A \cdot 1 = A$

$$\Rightarrow A + 0 = A$$

Adv: To findout complement of a function f .

Step 1: find dual of f ie f_D .

Step 2: Compliment of all var.f $\rightarrow \bar{F}$.

Eg: $A + B + C D \quad \bar{F} = \overline{A + B + C D}$

$$f_D = A \cdot B \cdot (C + D) \quad = \bar{A} \cdot \bar{B} \cdot (\bar{C} + \bar{D})$$

$$\bar{F} = \bar{A} \cdot \bar{B} (\bar{C} + \bar{D}) \quad = \bar{A} \cdot \bar{B} \cdot \bar{C} + \bar{A} \cdot \bar{B} \cdot \bar{D}$$

$$= \bar{A} \cdot \bar{B} \cdot \bar{C} + \bar{A} \cdot \bar{B} \cdot \bar{D}.$$

Q. Simplify following Boolean functions.

(1). $f = AB + \bar{A}C + \bar{C}D + \bar{B}\overset{\text{H}}{C}$

$$= AB + C(\bar{A} + \bar{B}) + \bar{C}D$$

$$= \underbrace{AB}_{X} + \underbrace{\bar{A}\bar{B}C}_{\bar{X}} + \bar{C}D$$

$$= AB + (C + \bar{C}D)$$

$$= AB + C + D.$$

✓ (2). $f = ABC\bar{C} + A\bar{B}C + \bar{A}BC + ABC$

$$= ABC\bar{C} + A\bar{B}C + \bar{A}BC + ABC + ABC + ABC$$

$$= AB(\bar{C} + C) + BC(\bar{A} + A) + AC(B + B)$$

$$= AB + BC + AC.$$

$$\begin{aligned}
 (3). \quad f &= \bar{x}\bar{y}z + \bar{x}y\bar{z} + \underline{\bar{x}yz} + xy\bar{z} \\
 &= \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + \bar{x}yz + \bar{x}yz + xyz \\
 &= \bar{x}z(\bar{y}+y) + \bar{x}y(\bar{z}+z) + yz(\bar{x}+x) \\
 &= \bar{x}z + \bar{x}y + yz.
 \end{aligned}$$

Q. How many two input NAND's are required to implement the following

$$\begin{aligned}
 (i). \quad f(A, B, C) &= A + AB + ABC \\
 &= A + AB(1+C) \\
 &= A + AB = A.
 \end{aligned}$$

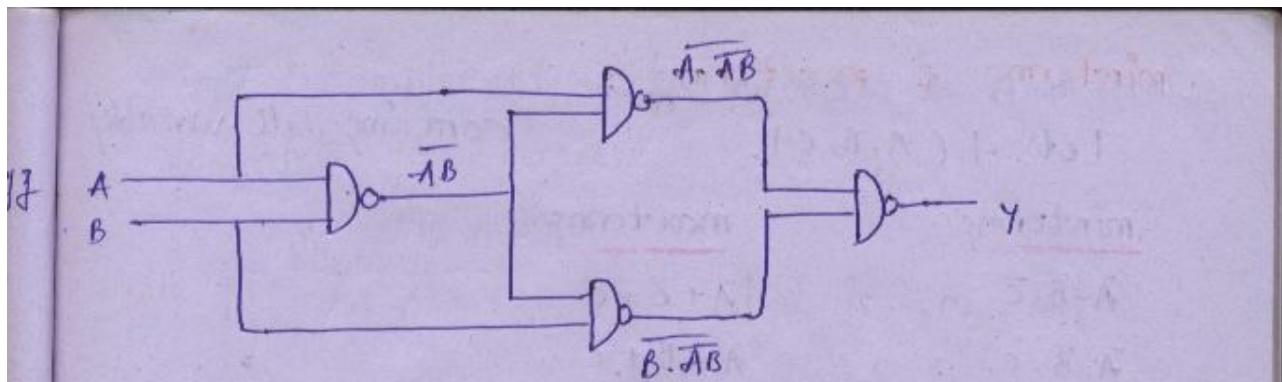
Ans: zero NAND gates.

$$\begin{aligned}
 (ii). \quad f &= ABC. \\
 \begin{array}{c} A \\ B \\ \hline \end{array} &\xrightarrow{\text{D}} \overline{AB} \\
 A &\xrightarrow{\text{D}} \overline{A} \\
 B &\xrightarrow{\text{D}} \overline{B} \\
 \begin{array}{c} \overline{A} \\ \overline{B} \\ \hline \end{array} &\xrightarrow{\text{D}} \overline{AB} \\
 C &\xrightarrow{\text{D}} ABC
 \end{aligned}$$

Each AND is replaced by two NAND's. So the total no. of NAND gates = 4.

Q. Complement EX-OR using min. no. of NAND gates.

$$\begin{aligned}
 \begin{array}{c} A \\ B \\ \hline \end{array} &\xrightarrow{\text{D}} \overline{A \oplus B} \\
 \underline{A \oplus B} &= \underline{\overline{AB} + A\bar{B}} \\
 &= \overline{AB} + A\bar{B} + A\bar{A} + B\bar{B} \\
 &= (\overline{A} + \bar{B})A + (\overline{A} + \bar{B})B \\
 &= A\bar{A}B + B\bar{A}B \\
 Y = \overline{Y} &= \frac{\overline{(A\bar{A}B + B\bar{A}B)}}{(A\bar{A}B)(B\bar{A}B)} \\
 &= \frac{\overline{(A\bar{A}B)}}{(A\bar{A}B)} \cdot \frac{\overline{(B\bar{A}B)}}{(B\bar{A}B)} \Rightarrow 5 \text{ NAND's.}
 \end{aligned}$$



Here NAND's are replaced by NOR's
then we get Ex-NOR gate.

$$\begin{aligned}
 & \overline{\bar{A} + \bar{A} \cdot \bar{B}} + \overline{\bar{B} + \bar{A} \cdot \bar{B}} \\
 = & (\bar{A} + \bar{A} \cdot \bar{B}) (\bar{B} + \bar{A} \cdot \bar{B}) \\
 = & (\bar{A} + \bar{A} \cdot \bar{B}) (\bar{B} + \bar{A} \cdot \bar{B}) \\
 = & (\bar{A} + \bar{B}) (\bar{B} + \bar{A}) \\
 = & \underline{\bar{A}\bar{B} + \bar{A}\bar{B}} = \underline{A \oplus B = } \rightarrow
 \end{aligned}$$

Operator precedence:

- (1). parenthesis ()
- (2). NOT \rightarrow
- (3). AND \cdot
- (4). OR $+$

Literal = variable (or) complement of a var.

Implement x-NOR using min. no. of NOR's.

minterms & maxterms :

Let $f(A, B, C)$.

containing all variables

minterms

maxterms

$$\bar{A} \cdot \bar{B} \cdot \bar{C}$$

$$\bar{A} + \bar{B} + \bar{C}$$

$$\bar{A} \cdot \bar{B} \cdot C$$

$$\bar{A} + \bar{B} + C$$

$$8 \quad \bar{A} \cdot B \cdot \bar{C}$$

$$\bar{A} + B + \bar{C}$$

:

:

$$AB \cdot \bar{C}$$

$$A + B + \bar{C}$$

$$ABC$$

$$A + B + C$$

* for 'n' var. function $\rightarrow 2^n$ minterms

2^n maxterms

* Sum of all minterms = 1. $\sum_{i=0}^{2^n-1} m_i = 1$

* Product of all maxterms = 0. $\prod_{i=0}^{2^n-1} M_i = 0$

* Product of any two minterms = 0.

$$m_i \cdot m_j = 0, \text{ if } i \neq j$$

$$= m_i, \text{ if } i=j$$

* Sum of any two maxterms = 1.

$$M_i + M_j = 1, \text{ if } i \neq j$$

$$= M_i, \text{ if } i=j$$

Let $f(x, y)$

$\begin{cases} 1 = \text{var} \\ 0 = \bar{\text{var}} \end{cases}$

$\begin{cases} 1 = \bar{\text{var}} \\ 0 = \text{var} \end{cases}$

$x \quad y$ minterm

maxterm

$$0 \quad 0 \quad \bar{x} \cdot \bar{y} \quad m_0 \quad x + y \quad M_0$$

$$0 \quad 1 \quad \bar{x} \cdot y \quad m_1 \quad x + \bar{y} \quad M_1$$

$$1 \quad 0 \quad x \cdot \bar{y} \quad m_2 \quad \bar{x} + y \quad M_2$$

$$1 \quad 1 \quad xy \quad m_3 \quad \bar{x} + \bar{y} \quad M_3$$

\Rightarrow complement of minterm = maxterm
and vice-versa.

$$M_j = \overline{m_j}$$

Q. If $f(A, B, C, D, E)$. what is $m_{23} = ?$

$$m_{19} = ? \quad M_{28} = ? , \quad M_{23} = ?$$

$$23 \rightarrow 10\ 111$$

$$19 \rightarrow 1\ 0011$$

$$m_{23} = A \cdot \overline{B} \cdot C \cdot D \cdot E$$

$$m_{19} \rightarrow A \cdot \overline{B} \cdot \overline{C} \cdot D \cdot E$$

$$28 \rightarrow 111\ 00$$

$$23 \rightarrow 10111$$

$$M_{28} \rightarrow \overline{A} + \overline{B} + \overline{C} + D + E \quad M_{23} = \overline{A} + B + \overline{C} + \overline{D} + \overline{E}$$

$$M_{23} = \overline{m_{23}} = \overline{A \cdot \overline{B} \cdot C \cdot D \cdot E}$$

$$= \overline{A} + B + \overline{C} + \overline{D} + \overline{E}$$

Q. $A \oplus A \oplus A \dots \oplus A = ?$

$A \oplus A \oplus A \oplus A$, if even no. of A's.

$$= 0 \oplus 0 = 0$$

$A \oplus A \oplus A$, if odd no. of A's.

$$= 0 \oplus A = A \quad * 30/01/11 TN2 *$$

* $A \oplus A \oplus A \dots \oplus A = 0$, if no. of terms = even
 $= A$, if " = odd

* $\overline{A} \oplus \overline{A} \oplus \overline{A} \oplus \dots \oplus \overline{A} = 0$, if no. of terms = Even
 $= \overline{A}$, " = odd

Q. How many Boolean fun's are possible, using 'n'-var's

Using n-var's $\rightarrow 2^n$ minterms

x minterms can be arranged in 2^x ways.

ie 2^2 boolean functions are possible.

for 2 var. $\rightarrow 2^2 = 16$ functions.

$f(x, y)$.

x	y	f_1	f_2	f_3	\dots	f_{16}
m_0	0 0	0	0	0		1
m_1	0 1	0	0	0		1
m_2	1 0	0	0	1		1
m_3	1 1	0	1	0		1
<hr/>						
\emptyset AND (Inhibition) $\bar{x}\bar{y} = \bar{x}y$						1

Algebraic forms of Boolean functions:

①. Standard form $\begin{cases} \text{stand. SOP form} \\ \text{stand. POS form} \end{cases}$

②. Canonical form $\begin{cases} \text{cano. SOP form (or) sum of minterms} \\ \text{cano. POS form (or) product of maxterms} \end{cases}$

$$f_1(A, B, C) = (A+B+\bar{C})(\bar{A}+\bar{B}+\bar{C}) \rightarrow \text{cano. POS}$$

$$f_2(A, B, C) = A\bar{B}\bar{C} + A\bar{B}C + A\bar{B}C \rightarrow \text{stand. SOP form}$$

* SAT. 11/10/08 *

Q. Convert the following Boolean eq. into canonical SOP form.

$$1). f(A, B, C) = \bar{A} + A\bar{B}C + B\bar{C} \rightarrow \text{std. SOP}$$

$$\begin{aligned} &\rightarrow \bar{A}(B+\bar{B})(C+\bar{C}) + A\bar{B}C + B\bar{C}(A+\bar{A}) \\ &= \cancel{\bar{A}BC} + \cancel{\bar{A}B\bar{C}} + \cancel{\bar{A}\bar{B}C} + \cancel{A\bar{B}\bar{C}} + A\bar{B}C + \cancel{A\bar{B}\bar{C}} \\ &\quad + \underline{\cancel{ABC}} \\ &= m_3 + m_2 + m_1 + m_0 + m_5 + m_6 \\ &= \sum m(0, 1, 2, 3, 5, 6). \end{aligned}$$

(OR)	$\begin{array}{c} A \quad B \quad C \\ \hline \overline{A} \quad 0 \quad - \end{array}$	$\begin{array}{c} A \quad B \quad C \\ \hline \overline{\emptyset} \quad 1 \quad 0 \end{array}$	$\begin{array}{c} A \quad B \quad C \\ \hline \overline{B} \quad \overline{C} \end{array}$	$\begin{array}{c} 1 \quad 0 \quad 1 \\ A \quad \overline{B} \quad C \end{array} \rightarrow m_5$
	\downarrow			
	$\cancel{000} \rightarrow m_0$		$010 \rightarrow m_2$	
	$\cancel{001} \rightarrow m_1$		$110 \rightarrow m_6$	
	$\cancel{010} \rightarrow m_4$			
	$\cancel{011} \rightarrow m_3$			

$$f = \sum m(0, 1, 2, 3, 5, 6) \rightarrow \text{cano. SOP}$$

$$f = \pi M(4, 7) \rightarrow \text{cano. POS.}$$

Q. Convert the following Boolean eq. into cano. POS form.

$$f(A, B, C) = \overline{A} \cdot (\overline{B} + \overline{C}) \cdot (\overline{A} + \overline{B} + \overline{C}) \rightarrow \begin{matrix} M_1 \\ \text{std. pos form} \end{matrix}$$

$$\begin{aligned} f &= (\overline{A} + B\overline{B} + C\overline{C})(\overline{B} + \overline{C} + A\overline{A})(A + B + \overline{C}) \\ &= (\overline{A} + B\overline{B} + C)(\overline{A} + B\overline{B} + \overline{C})(\overline{B} + \overline{C} + A)(\overline{B} + \overline{C} + \overline{A}) \end{aligned}$$

$$\begin{aligned} &(A + B + \overline{C}) \\ &= (\overline{A} + \overline{B} + C)(\overline{A} + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C}) \\ &\quad (\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C})(A + B + \overline{C}), \\ &= M_4 \cdot M_6 \cdot M_5 \cdot M_7 \cdot M_3 \cdot M_7 \cdot M_1, \end{aligned}$$

$$= \pi M(1, 3, 4, 5, 6, 7) \rightarrow \text{cano. POS.}$$

$$= \sum m(0, 2) \rightarrow \text{cano. SOP.}$$

[OR]

$\begin{array}{c} A \quad B \quad C \\ \hline 1 \quad \overline{1} \quad - \end{array}$	$\begin{array}{c} A \quad B \quad C \\ \hline - \quad 1 \quad 1 \end{array}$
\downarrow	\downarrow
$1 \quad \underline{0} \quad \underline{0} \rightarrow M_4$	$\underline{0} \quad 1 \quad 1 \rightarrow M_3$
$1 \quad \underline{0} \quad \underline{1} \rightarrow M_5$	$1 \quad 1 \quad 1 \rightarrow M_7$
$1 \quad \underline{1} \quad \underline{0} \rightarrow M_6$	
$1 \quad \underline{1} \quad \underline{1} \rightarrow M_7$	

Q. Convert the following into cano. pos form.

$$f(x, y, z) = \bar{x}\bar{y} + \bar{x}z \rightarrow \text{std. SOP}$$

$$\rightarrow f = (\bar{x} + z)(\bar{x} + y)$$

$x \ y \ z$	$\bar{x} \ y \ z$	std pos	cano. SOP
$0 \ 0 \ 0$	$1 \ 0 \ 0$	std pos	cano. pos
$0 \ 0 \ 1$	$1 \ 0 \ 1$		
$0 \ 1 \ 0$	$1 \ 1 \ 0$		
$0 \ 1 \ 1$	$1 \ 1 \ 1$		

$$m_0 \ 0 \ 0 \ 0 \quad 1 \ 0 \ 0 \quad M_4$$

$$m_2 \ 0 \ 1 \ 0 \quad 1 \ 0 \ 1 \quad M_5$$

$$f = \pi M(0, 2, 4, 5) \rightarrow \text{cano. POS}$$

K-maps :-

2-variable k-map

		0	1
		0	0
		1	2
A	B	0	1

neighbours

$$m_0 \rightarrow m_1, m_2$$

$$m_2 \rightarrow m_0, m_3$$

		3-var. k-map			
		Gray code			
		00	01	11	10
A	B	0	1	3	2
	C	0	1	3	2
	D	4	5	7	6
		12	13	15	14
		8	9	11	10

To make
neighbours
differ by
only one
bit

neighbours

$$m_0 \rightarrow m_1, m_4, m_2, m_3$$

$$m_9 \rightarrow m_8, m_{11}, m_{13}, m_1$$

4 var. k-map

		CD				
		00	01	11	10	
		00	0	1	3	2
		01	4	5	7	6
		11	12	13	15	14
		10	8	9	11	10

neighbours:

$$m_0 \rightarrow m_1, m_4, m_2, m_3$$

$$m_9 \rightarrow m_8, m_{11}, m_{13}, m_1$$

[Ans]

group of 8 \rightarrow octet

group of 4 \rightarrow quad

group of 2 \rightarrow pair

\rightarrow single minterm

Qn 3-var. k-map: Quads: 0145, 1357,

3276, 0246, 0132, 4576 : Total = 6.

a. Simplify $f(A, B, C) = \sum m(0, 2, 3, 4, 5, 6)$

[That is from cano sop into std sop].

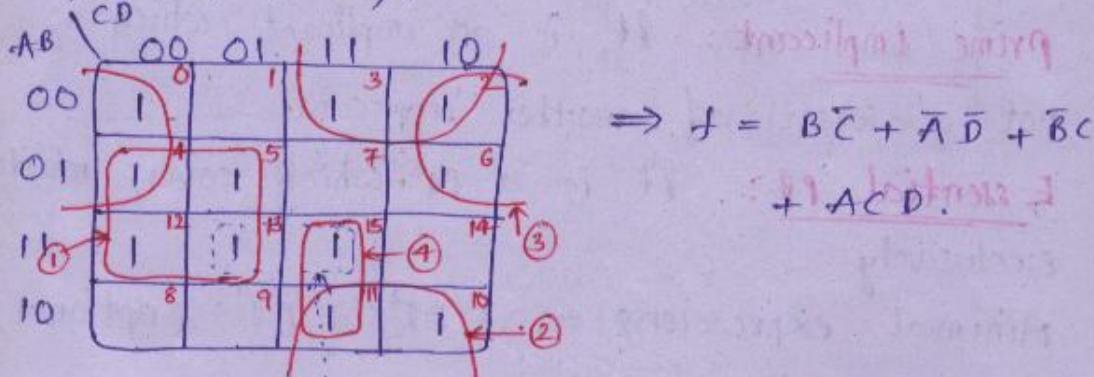
		BC	00	01	11	10
		A	0	1	1	1
		B	0	0	1	1
0			1			
1			1	1		

$$= \overline{A}B + A\overline{B} + \overline{C}$$

Qn 4-var. k-map:

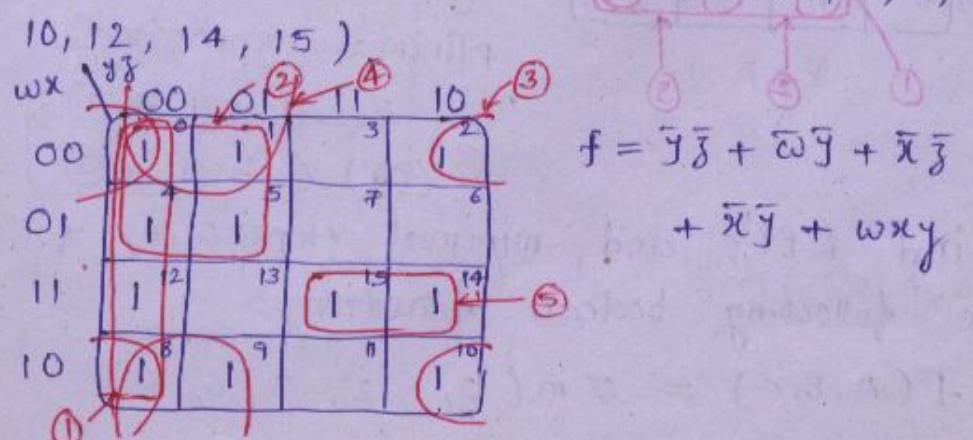
Total Octets = 8 ; columns Rowg
 $12, 23, 34, 12, 23, 34$
 41 41

a. Simplify $f(A, B, C, D) = \sum m(0, 2, 3, 5, 6, 10,$
 $4, 11, 12, 13, 15)$.



Simplified k-map eq. is a minimal eq.
 but not unique.

a. Simplify $f(w, x, y, z) = \sum m(0, 1, 2, 4, 5, 8, 9,$
 $10, 12, 14, 15)$



Q. $f(A, B, C) = \pi m(0, 1, 2, 4, 5, 6) \rightarrow$ cano. pos

		BC		00		01		11		10	
		A	B	0	1	0	1	1	0	1	0
		0	0	0	0	0	0	1	1	1	0
		1	0	0	0	0	0	1	1	1	0

{ convert it into
std pos form }

$$f = B \cdot (A + C)$$

Q. $f(w, x, y, z) = \pi m(0, 1, 2, 4, 5, 9, 11, 13, 14, 15)$

		yz		00		01		11		10	
		wx	z	0	1	0	1	1	0	1	0
		00	0	0	0	0	0	1	1	1	0
		01	0	0	0	0	0	1	1	1	0
		11	1	0	0	0	0	1	1	1	0
		10	0	0	0	0	0	1	1	1	0

$$f = (w + y)(\bar{w} + \bar{z})(\bar{w} + \bar{x} + \bar{y})$$

$$(\bar{x} + w + \bar{z})$$

Implicant: It indicates the set of all adjacent minterms.

Prime Implicant: It is an implicant which is not a subset of another implicant.

Essential PI: It is a PI which covers minterms exclusively.

Minimal expression = EPI's + PI's (optional)

eg: $f(A, B, C) = \sum m(1, 2, 5, 6, 7)$

		BC		00		01		11		10	
		A	B	0	1	0	1	1	0	1	0
		0	0	0	1	1	0	1	1	1	0
		1	0	0	1	1	0	1	1	1	0

All are PIs.

EPI's = ①, ④

Minimal expression

$$= ① + ④ + ②$$

$$(or) ① + ④ + ③$$

Q. find EPI's and minimal expressions for the following boolean functions.

$$f(A, B, C) = \sum m(0, 1, 2, 5, 6, 7)$$

A	BC	00	01	11	10	
0	1	1	1	1	1	1
1	1	1	1	1	1	1
		2	3	4	5	6

$$EPF = 0 \text{ (Nil)}$$

Minimal expression

$$= 1 + 3 + 5$$

$$(\text{or}) 2 + 4 + 6$$

Dont care conditions :-

for non-occurring imp's the o/p can be assumed as 0 or 1. and this is called as Dont care condition.

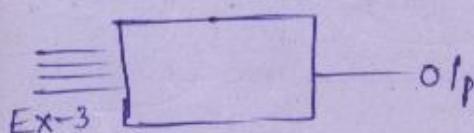
Eg : $\begin{array}{c} BCD \\ \hline \text{ilp} \end{array}$ \rightarrow o/p

valid BCP ilp's

0 - 0000	10 - 1010 \rightarrow x
1 - 0001	11 - 1011 \rightarrow x
:	12 - 1100 \rightarrow x
9 - 1001	13 - 1101 \rightarrow x
	14 - 1110 \rightarrow x
	15 - 1111 \rightarrow x

Non-occurring
ilp's. o/p

} dont care's



Dont care's : $0000 \rightarrow x$
 $0001 \rightarrow x$
 $0010 \rightarrow x$

Q. $f(A, B, C, D) = \sum m(0, 1, 3, 6, 10, 13, 15) + d(2, 5, 8, 11)$

AB \ CD

A\B	00	01	11	10	
00	1	1	1	x	1
01	x			1	4
11	1	1			
10	x	x		1	2

$$f = \bar{A}\bar{B} + \bar{B}C + ABD$$

$$+ \bar{A}C\bar{D}$$

: signl lost out

Q. $f_1 = \sum m(0, 2, 4, 7); f_2 = \sum m(1, 2, 4, 6)$
 $f = f_1 \cdot f_2 \Rightarrow f = ?$

$$f = \sum m(2, 4)$$

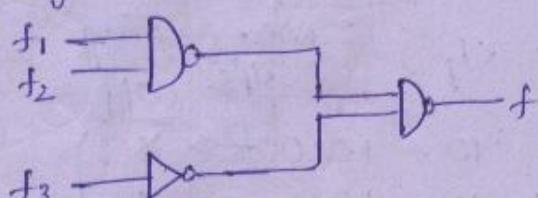
Similarly $f_3 = f_1 - f_2 \Rightarrow f_3 = ?$

$$f_3 = \sum m(0, 7).$$

$$f_4 = f_2 - f_1 \Rightarrow f_4 = ?$$

$$f_4 = \sum m(1, 6).$$

Q. Determine the function f_3 in the following logic ckt.



$$\text{where } f = \sum m(0, 1, 3, 5)$$

$$f_1 = \sum m(2, 3, 6, 7)$$

$$f_2 = \sum m(0, 1, 5).$$

$$f = \overline{\overline{f_1} \overline{f_2} \cdot \overline{f_3}}$$

$$= f_1 f_2 + f_3$$

$$\Rightarrow f_3 = f - f_1 \cdot f_2$$

$$\text{But } f_1 \cdot f_2 = \emptyset$$

$$\Rightarrow f_3 = f = \sum m(0, 1, 3, 5).$$

Q. $f = f_1 \cdot f_2$ where $f_1 = \sum m(0, 1, 5) + d(2, 3, 7)$

$$f_2 = \sum m(1, 2, 4, 5) + d(0, 7).$$

$$f = f_1 \cdot f_2 = \sum m(1, 5) + d(0, 2, 7).$$

minterm
in one fun. ↓ don't care
in another fun.
1. d = d

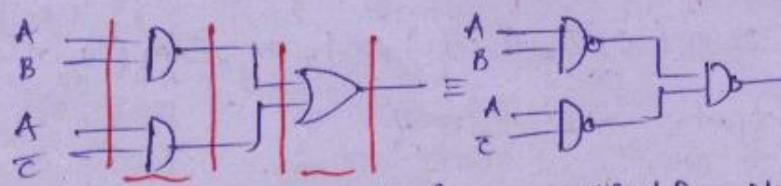
$$0. d = 0$$

$$1 + d = 1$$

$$0 + d = d$$

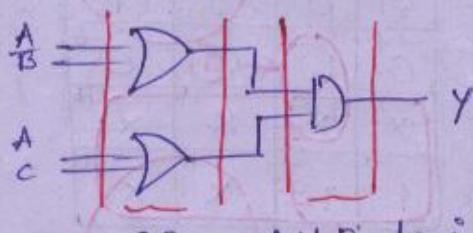
Two level logic :-

$$\text{SOP form} \rightarrow Y = AB + A\bar{C}$$



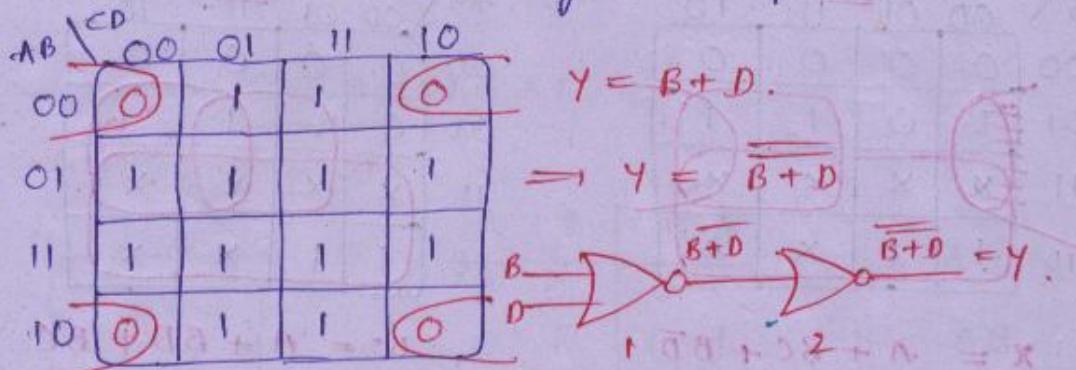
AND - OR logic \equiv NAND - NAND

$$\text{POS form : } \rightarrow y = (A + \bar{B})(A + C)$$

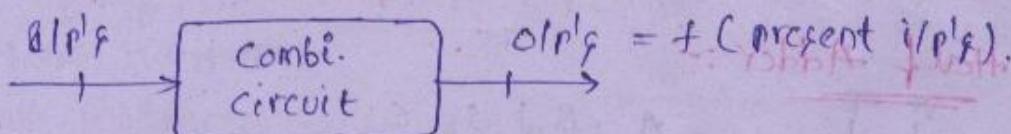


OR - AND logic \equiv NOR - NOR.

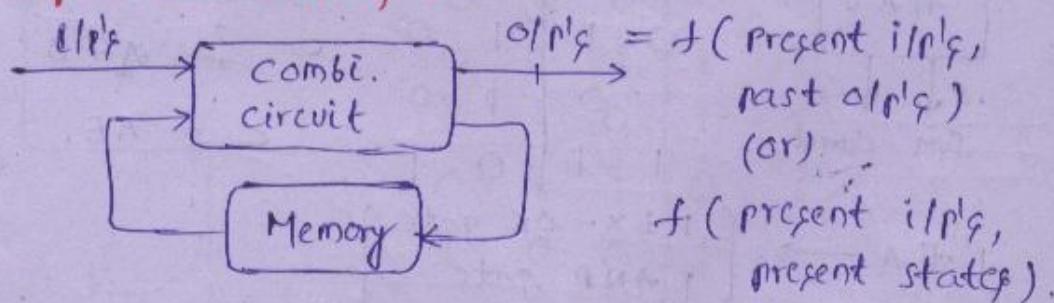
- Q. How many two i/p NOR gates are required to implement the following k-map.



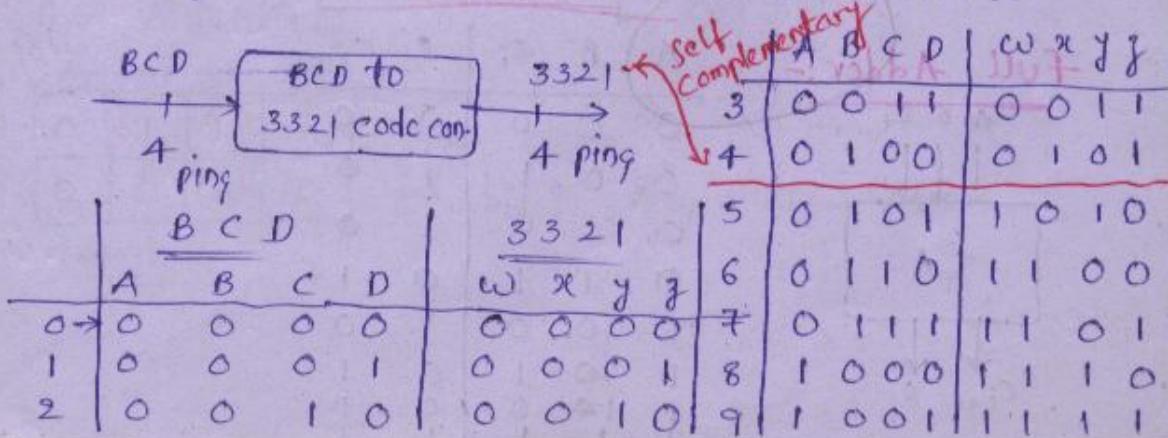
combinational circuits :- idios. combinational



Sequential circuits :-



- Q. Design a BCD to 3321 code converter.



AB \ CD	00	01	<u>Z</u>	10
00	0	1	1	0
01	1	0	1	0
11	x	x	x	x
10	0	1	x	x

$$Z = \overline{BD} + CD + B\overline{C}$$

AB \ CD	00	01	<u>X</u>	10
00	0	0	0	0
01	1	0	1	1
11	x	x	x	x
10	1	1	x	x

$$x = A + BC + BD$$

AB \ CD	00	01	<u>y</u>	10
00	0	0	1	0
01	0	1	0	0
11	x	x	x	x
10	1	1	x	x

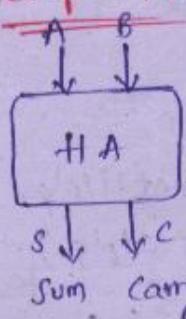
$$y = A + \overline{BC} + B\overline{C}$$

AB \ CD	00	01	<u>w</u>	10
00	0	0	0	0
01	0	1	1	1
11	x	x	x	x
10	1	1	x	x

$$w = A + BD + BC$$

Arithmetic combi. circuit :- (don't care)

Half Adder :-



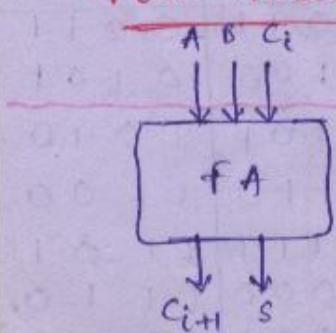
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$\begin{aligned} S &= A'B + AB' \\ &\Rightarrow A \oplus B \\ C &= AB \end{aligned}$$

$$1 \text{ HA} \rightarrow \left\{ \begin{array}{l} 1 \text{ EX-OR gate} \\ 1 \text{ AND gate} \end{array} \right\}$$

* SUNDAY, 12/10/08 *

full Adder :-



A	B	ci	S	ci+1
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	1	1
1	1	1	0	1

	BC_i	for. S	00	01	11	10
A	0	0	0	1	0	0
0	0	1	0	1	0	0
1	1	0	0	1	1	0

diagonal Adjacency

$$S = \overline{B} \left(\frac{A \oplus C_i}{x} \right) + B \left(\frac{A \otimes \overline{C}_i}{x} \right)$$

$$= B \oplus x = B \oplus A \oplus C_i$$

$$\Rightarrow S = \underline{A \oplus B \oplus C_i}$$

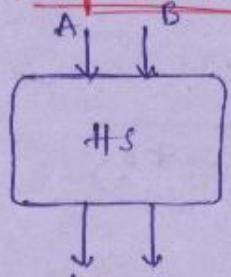
$$C_{i+1} = \overline{A} \underline{B} \underline{C_i}$$

$$+ \underline{A} \overline{B} \underline{C_i} + \underline{A} \underline{B} \overline{C_i} + \underline{A} \underline{B} \underline{C_i}$$

$$= \underline{\underline{A} \underline{B}} + \underline{B} \underline{C_i} + \underline{C_i} \underline{A}$$

$$(or) \quad \underline{C_i} (\underline{A} \oplus \underline{B}) + \underline{A} \underline{B}.$$

Half subtractor:-



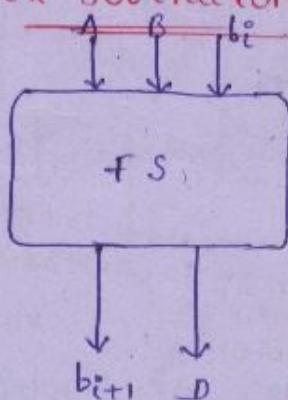
(borrow) (Difference)

A	B	D	b
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$D = A \oplus B$$

$$b = \overline{A} \overline{B}$$

full subtractor:-



A	B	bi	D	bi+1
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0

	B_{bi}	00	01	11	10
A	0	0	1	1	1
0	0	0	1	1	0
1	0	0	1	1	0

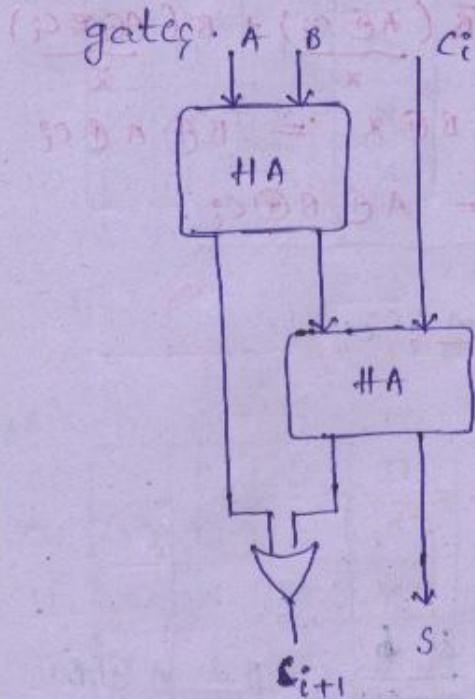
$$\Rightarrow b_{i+1} = \overline{A} b_i + B b_i + \overline{A} B$$

for b_{i+1} ,

$$> (\overline{A} + A) + \overline{B} B = 0 \quad : 102$$

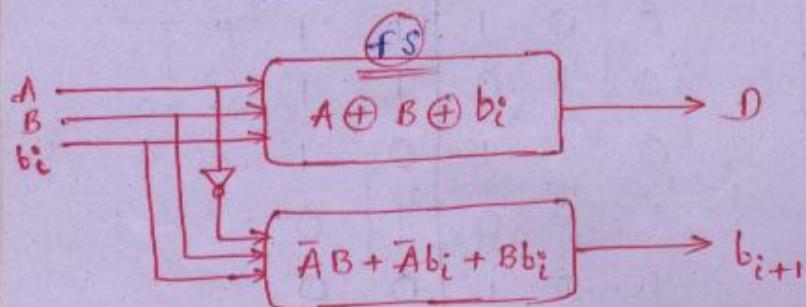
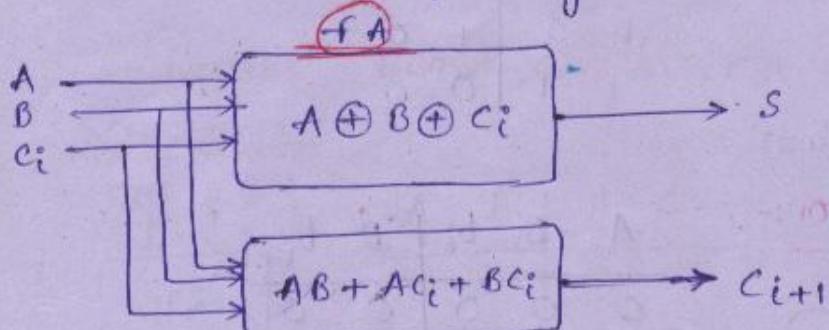
$$\therefore B b_i = B \cdot \overline{B} + B B =$$

Q. Implement a FA by using HA's and logic gates.



FA requires, one OR gate and two HA's.

Q. Convert the following FA into a FS.



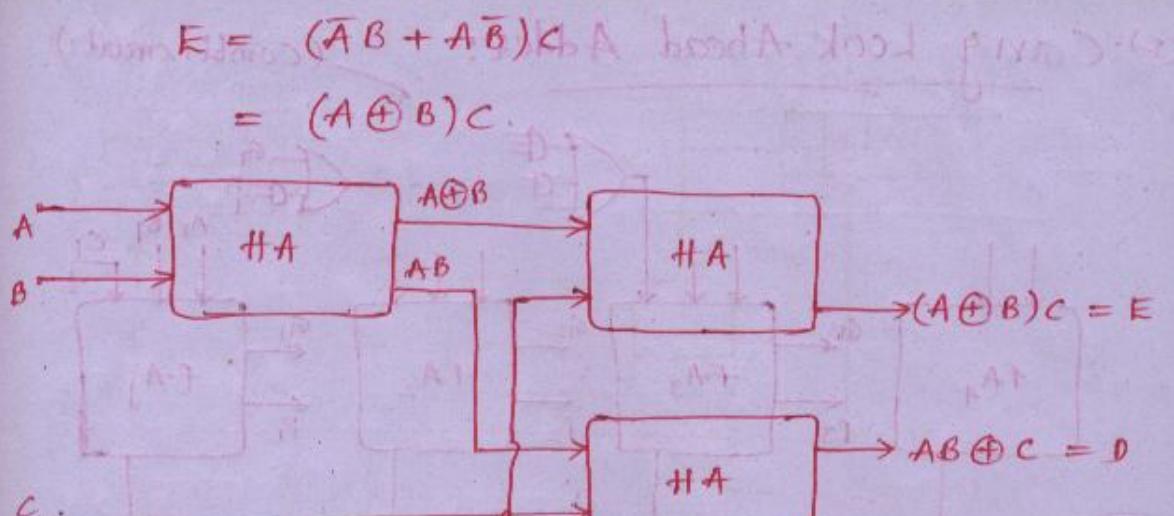
Q. Implement the following boolean exp's using only HA's.

$$D = AB\bar{C} + \bar{A}C + \bar{B}C$$

$$E = \bar{A}BC + A\bar{B}C$$

$$Sol: D = AB\bar{C} + (\bar{A} + \bar{B})C$$

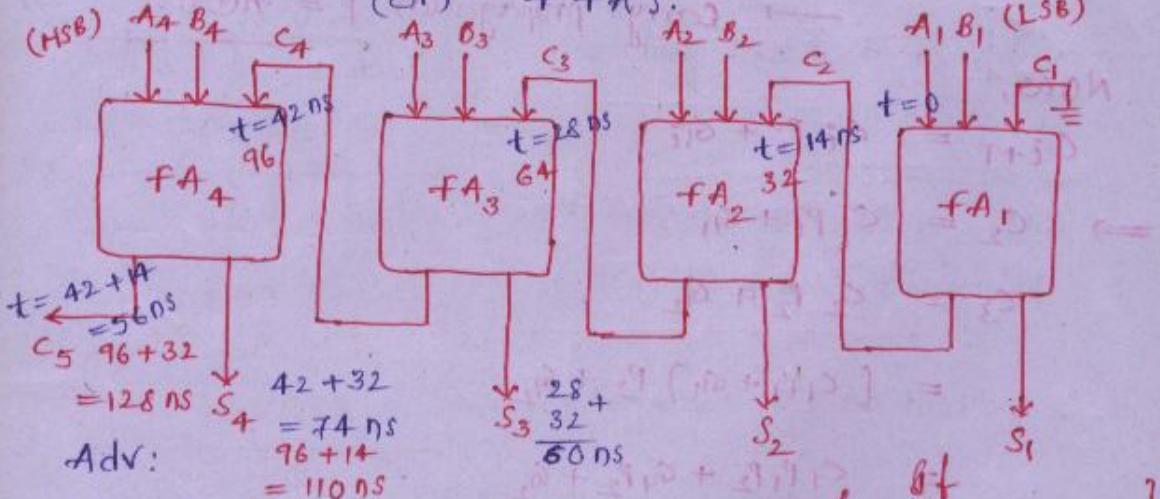
$$= AB\bar{C} + \bar{A}\bar{B} \cdot C = AB \oplus C$$



(i) 4-bit parallel binary Adder :-

$$\begin{array}{r}
 A \rightarrow A_4 \ A_3 \ A_2 \ A_1 \\
 B \rightarrow B_4 \ B_3 \ B_2 \ B_1 \\
 \hline
 \text{Required : } 3 \text{ fA}'s + 1 \text{ HA}
 \end{array}$$

(combi. circuit)



Simple to construct

Drawback:

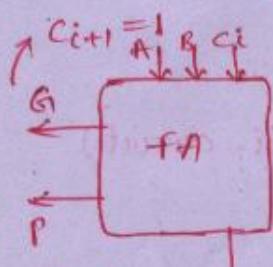
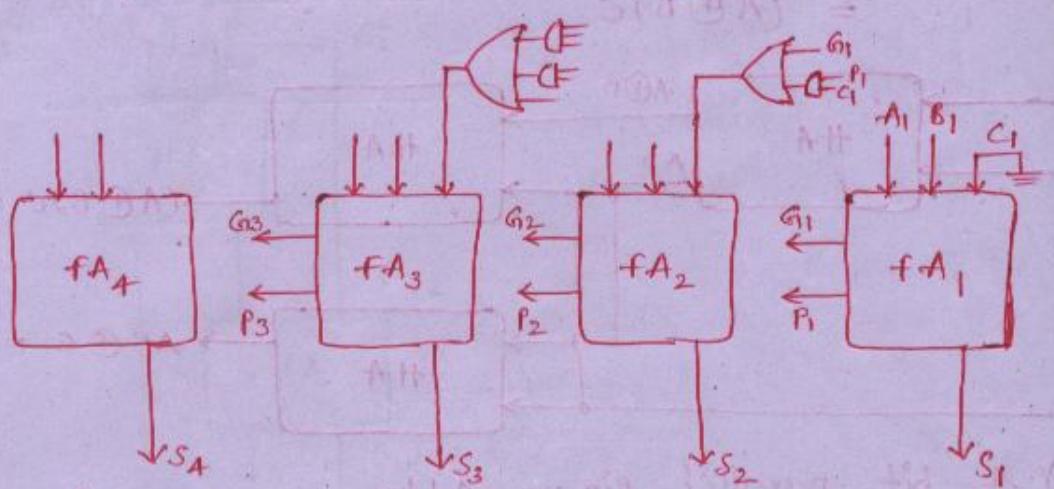
Speed of operation is less if the size of the adder increases.

$fA \rightarrow \left\{ \begin{array}{l} 32 \text{ ns} \rightarrow \text{sum} \\ 14 \text{ ns} \rightarrow \text{carry} \end{array} \right\}$ Then the total time required for the operation

ANS: $42 + 32$ for S_4 ; for $C_5 \Rightarrow 42 + 14 = 56$
 $= 74 \text{ ns.}$

To complete addition
Total time = 74 ns.

(2) Carry Look Ahead Adder: - (combi. circuit)



$$C_{i+1} = c_i (A \oplus B) + AB.$$

(1). $C_{i+1} = 1$ if $AB = 1$

→ Carry Generation $G = AB$.

(2). when $C_{i+1} = C_i$ then $A \oplus B = 1$

→ Carry propagation $P = A \oplus B$.

Now,

$$C_{i+1} = C_i P_i + G_i$$

$$\Rightarrow C_2 = C_1 P_1 + G_1$$

$$C_3 = C_2 P_2 + G_2$$

$$= [C_1 P_1 + G_1] P_2 + G_2$$

$$= C_1 P_1 P_2 + G_1 P_2 + G_2$$

$$\therefore C_4 = C_3 P_3 + G_3$$

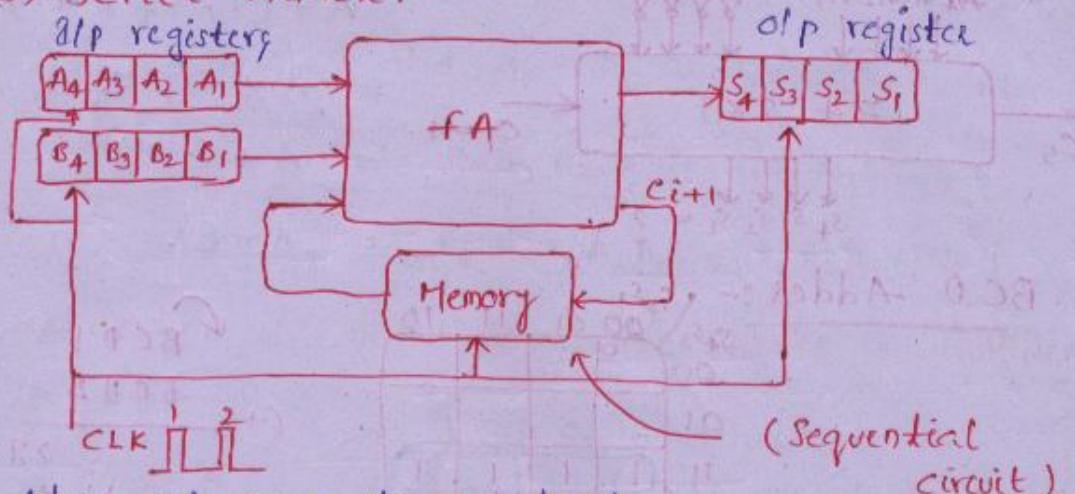
$$= C_1 P_1 P_2 P_3 + G_1 P_2 P_3 + G_2 P_3 + G_3$$

Adv: speed is more

Dis Adv: More hardware complexity

Advantages still not better

(3). Serial Adder :-



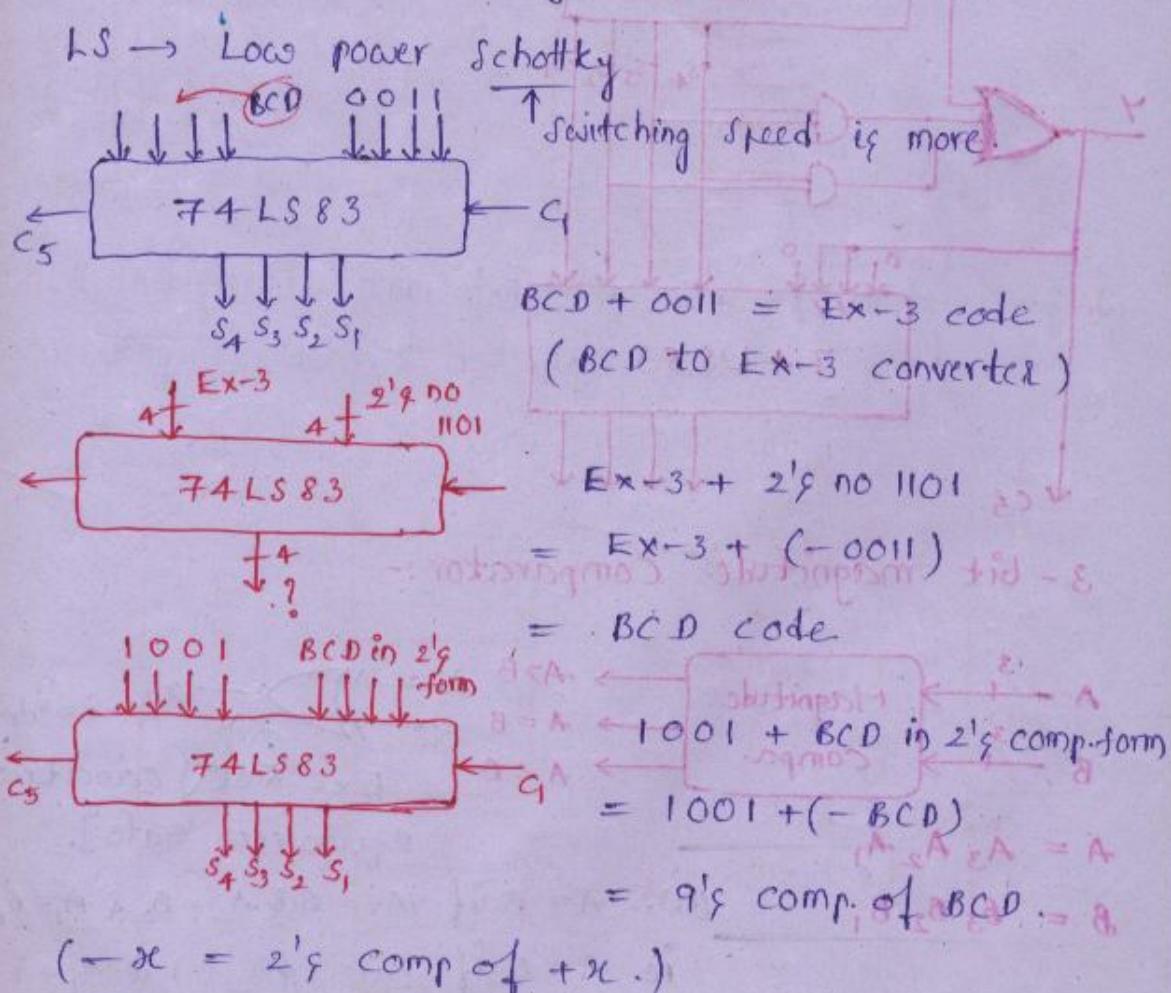
Adv: (1) easy to construct

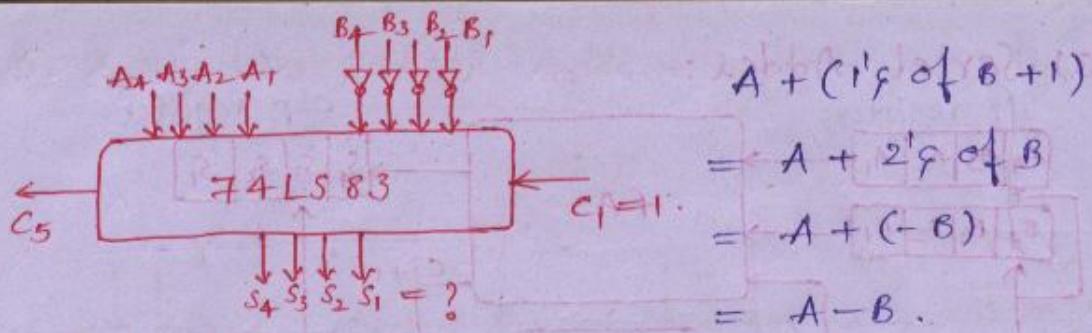
(2) only one FA is used.

Dis Adv:

speed of operation is less.

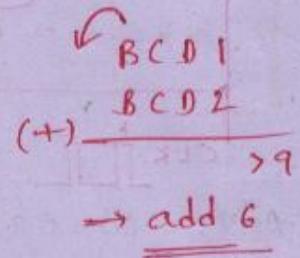
4 Bit parallel Binary Adder (74 LS 83)





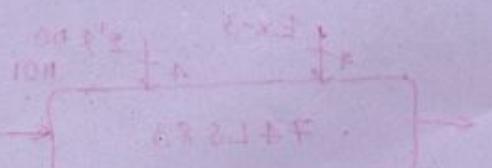
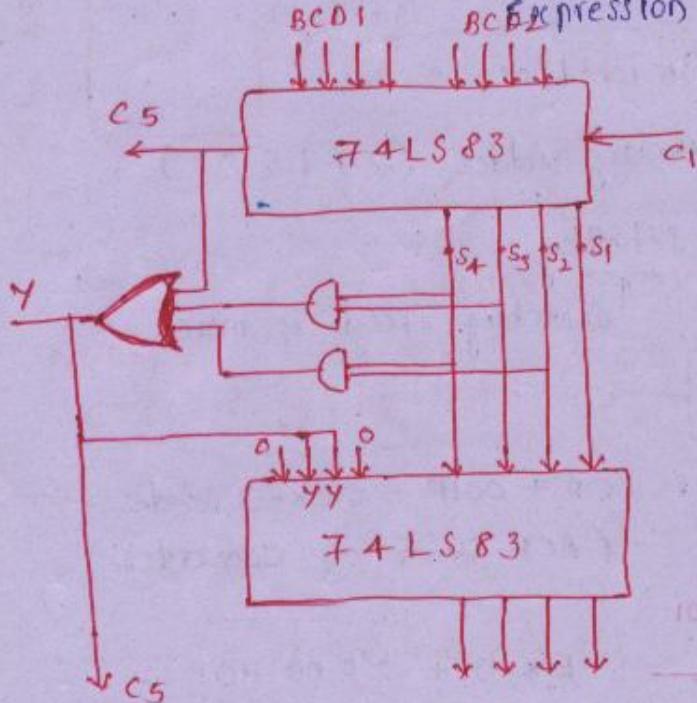
BCD Adder :-

	$S_4 S_3$	00 01	11	10
$S_4 S_3$	00	0		
00	01			
01		1 1	1 1	
11		1 1	1 1	1 1
10		1 1	1 1	1 1

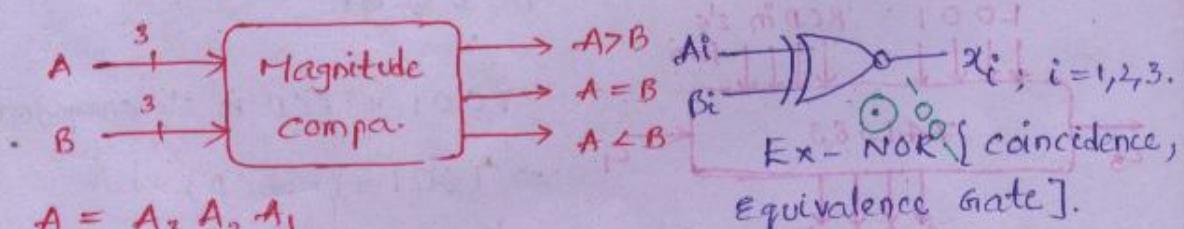


Invalid BCD = BCD Expression

$$\begin{aligned}
 Y &= S_4 S_3 + S_4 S_2 \\
 &\quad + C_5
 \end{aligned}$$



✓ 3-bit magnitude comparator :-



$$A = A_3 A_2 A_1$$

$$B = B_3 B_2 B_1$$

(a). $A = B$ if $A_3 = B_3 \& A_2 = B_2 \& A_1 = B_1$

i.e. $A = B$ if $x_3 = 1 \& x_2 = 1 \& x_1 = 1$

i.e. $A = B$ if $x_3 \& x_2 \& x_1 = 1$

Ex- NOR { coincidence, equivalence gate].

(b). $A > B$ if $A_3 > B_3$ (or) $A_3 = B_3$ and $A_2 > B_2$

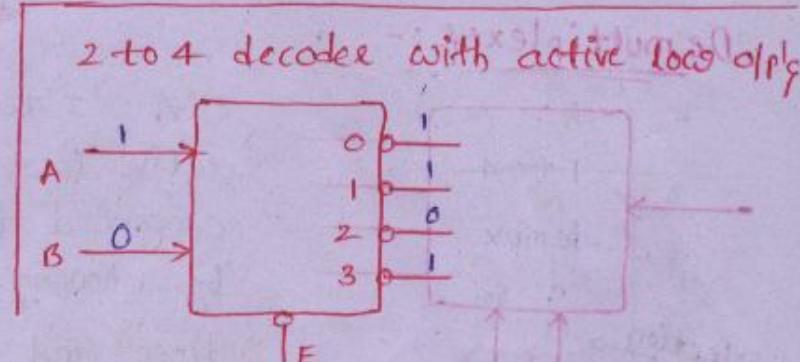
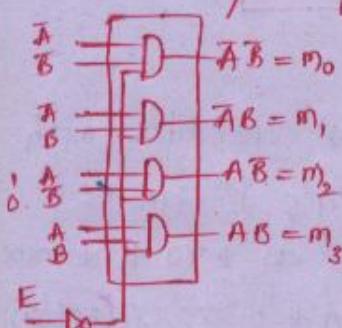
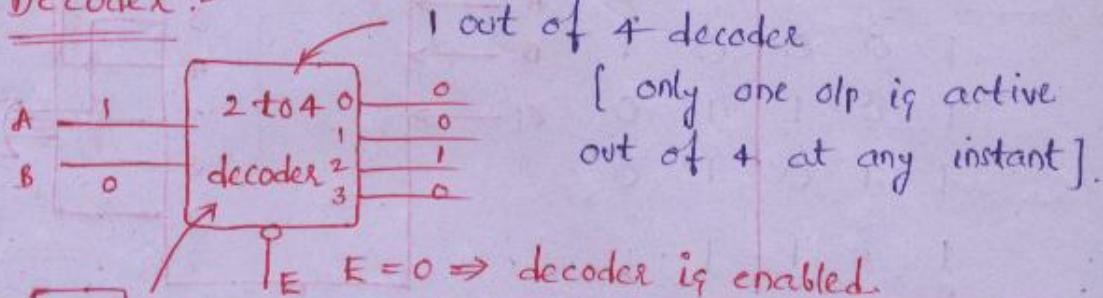
(or) $A_3 = B_3$ and $A_2 = B_2$ and $A_1 > B_1$

$A > B$ if $A_3 \bar{B}_3 + x_3 A_2 \bar{B}_2 + x_3 x_2 A_1 \bar{B}_1 = 1$.

(c). $A < B$ if $\bar{A}_3 B_3 + x_3 \bar{A}_2 B_2 + x_3 x_2 \bar{A}_1 B_1 = 1$.

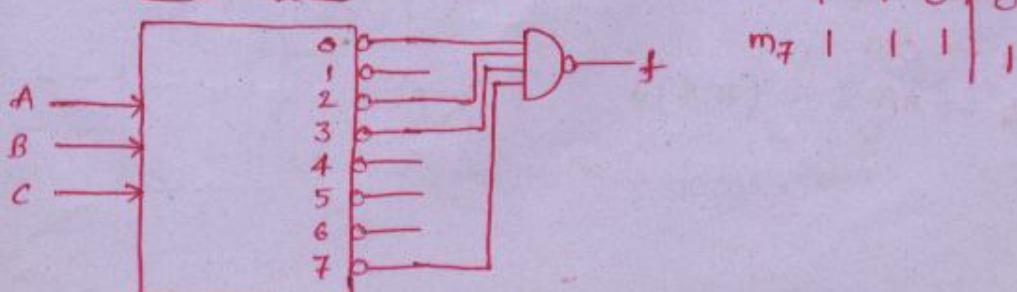
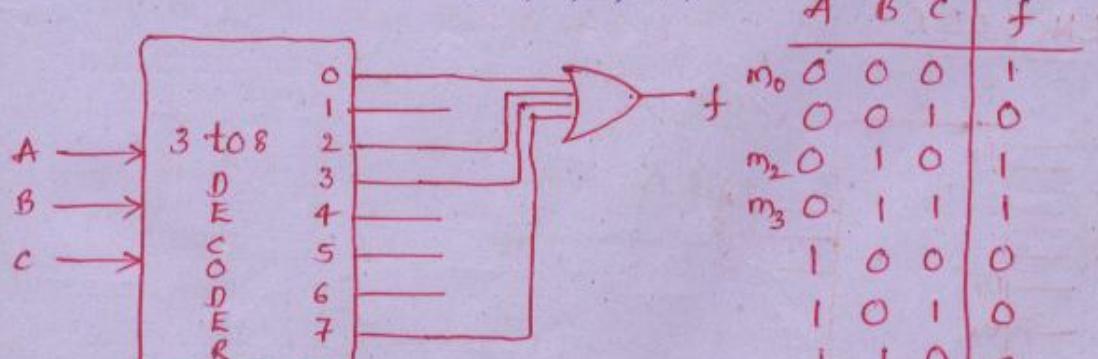
(1). decoder (2). demultiplexer (3). Encoder (4). Multiplexer

Decoder :-



d. Implement the following sum of minterm eq by using a decoder and logic gates.

$$f(A, B, C) = \sum m(0, 2, 3, 7)$$



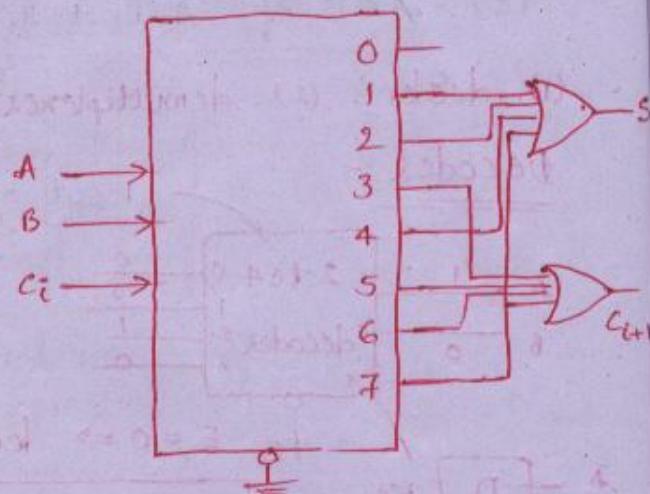
A	B	C	f
0	0	0	1
0	0	1	0
m ₂ 0	1	0	1
m ₃ 0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
m ₇ 1	1	1	1

Q. Implement a ffa by using decoder and logic gates.

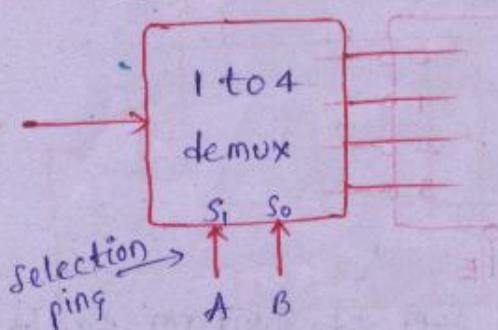
A	B	C_i	C_{i+1}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$S = \sum m(1, 2, 4, 7)$$

$$C_{i+1} = \sum m(3, 5, 6, 7)$$



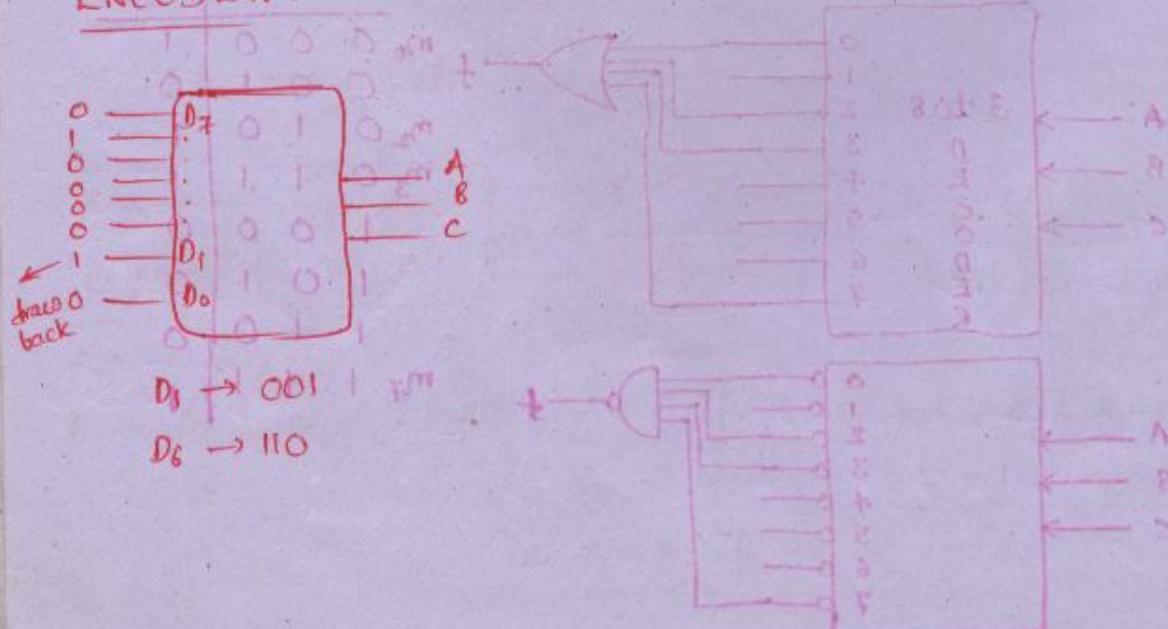
Demultiplexer:-



A 2 to 4 decoder [with active low output] can be converted to a 1 to 4 demux by choosing A & B as selection lines and the enable pin as the serial ip.

* 29/11/08 *

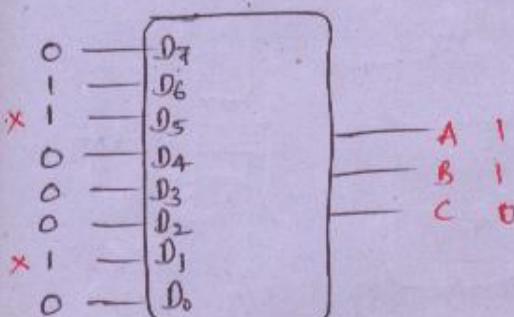
ENCODER:



PRIORITY ENCODER: (74 LS 148)

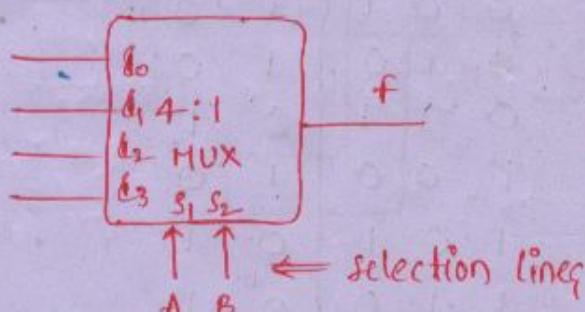
D_7 - highest priority

D_0 - lowest priority



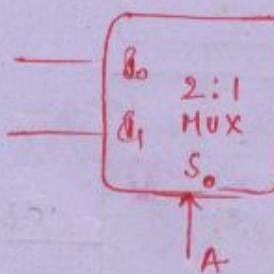
x - ignored

MULTIPLEXER:



for 4:1 MUX,

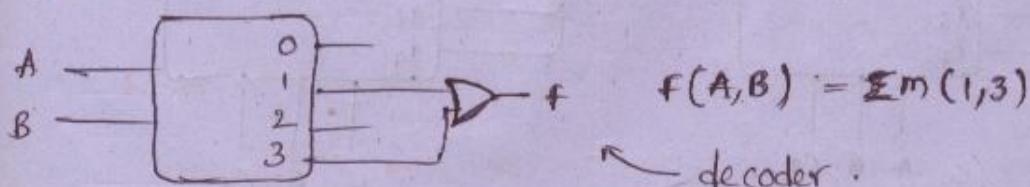
$$\begin{aligned} f &= \bar{A}\bar{B}d_0 + \bar{A}Bd_1 + A\bar{B}d_2 + ABd_3 \\ &= m_0d_0 + m_1d_1 + m_2d_2 + m_3d_3. \end{aligned}$$



for 2:1 MUX,

$$f = \bar{A}d_0 + Ad_1$$

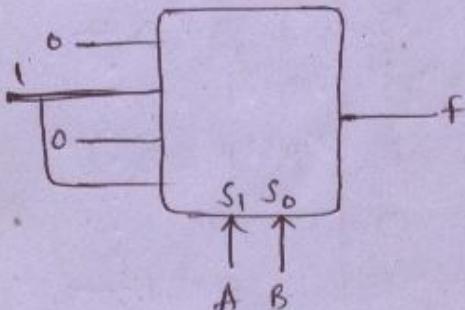
Q.



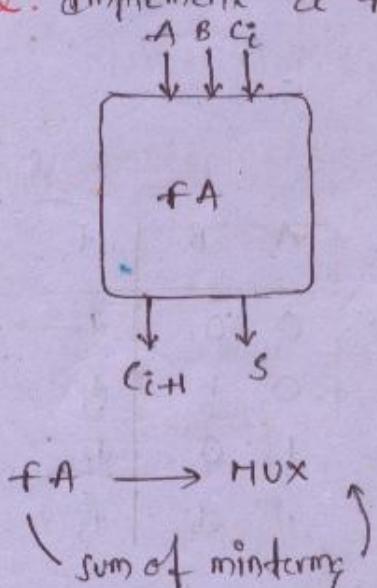
Q. Implement the following sum of minterms exp. by using multiplexer.

\downarrow
(sum of minterms)

$$f(A, B) = \sum m(1, 3).$$



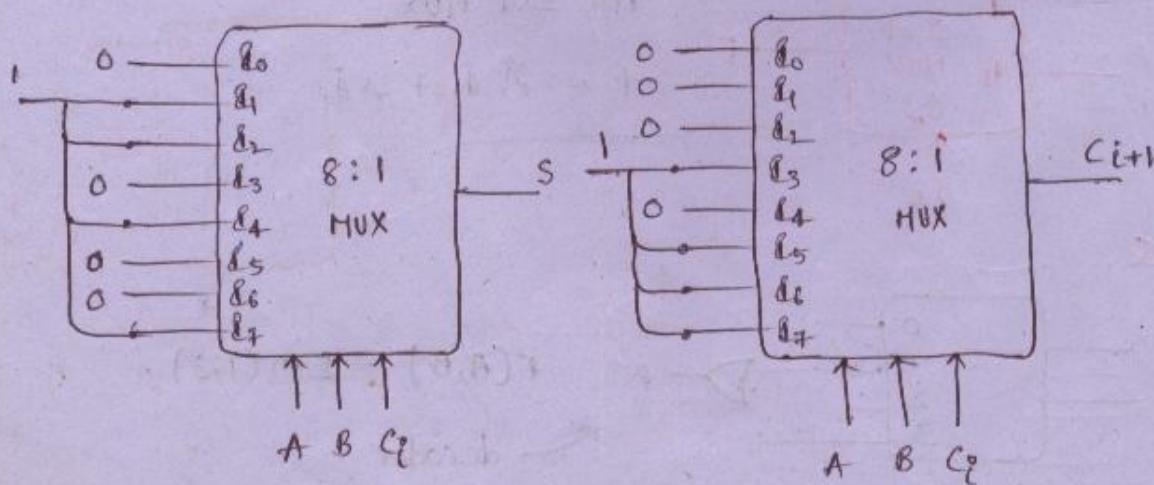
Q. Implement a FA by using multiplexers:

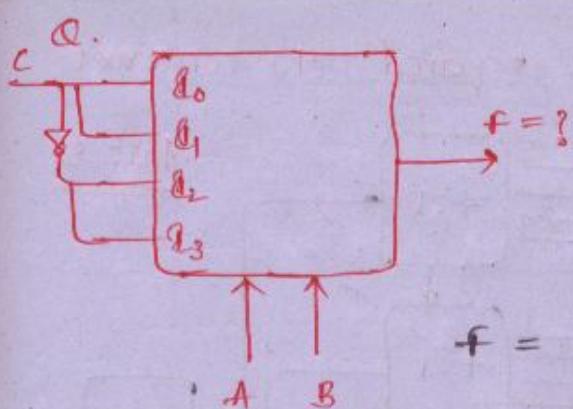


A	B	C _i	S	C _{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = \sum m(1, 2, 4, 7)$$

$$C_{i+1} = \sum m(3, 5, 6, 7).$$





Given that

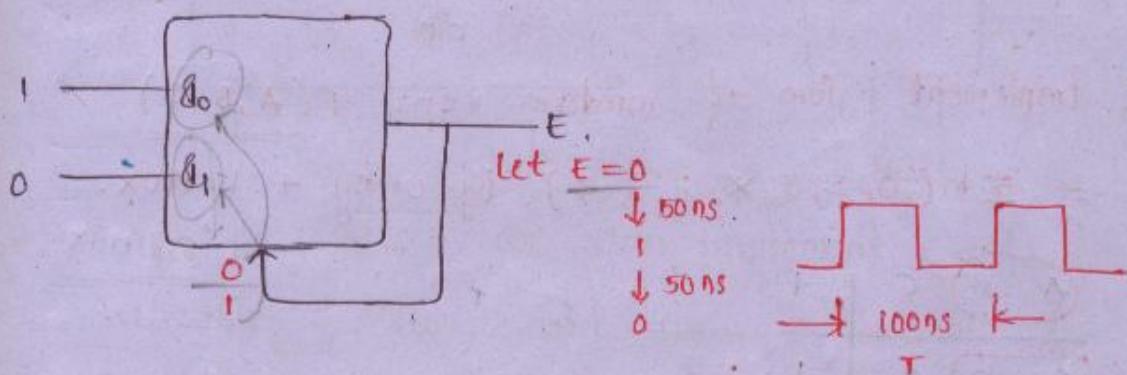
$$d_0 = d_1 = c$$

$$d_2 = d_3 = \bar{c}$$

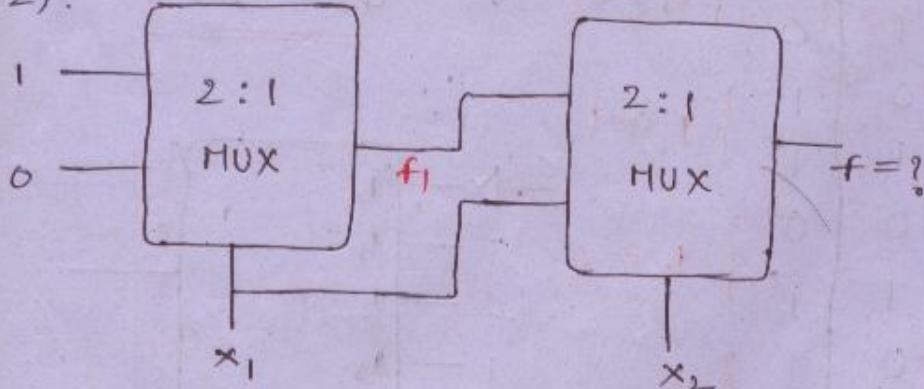
$$\begin{aligned} f &= A\bar{B}c + \bar{A}Bc + A\bar{B}\bar{c} + A\bar{B}\bar{c} \\ &= \bar{A}c(\bar{B}+B) + A\bar{c}(\bar{B}+B) \\ &= A \oplus c. \end{aligned}$$

Q. Determine the op's of the following MUX's?

1). switching speed is 50 ns.



2).



$$f_1 = \bar{A}d_0 + Ad_1$$

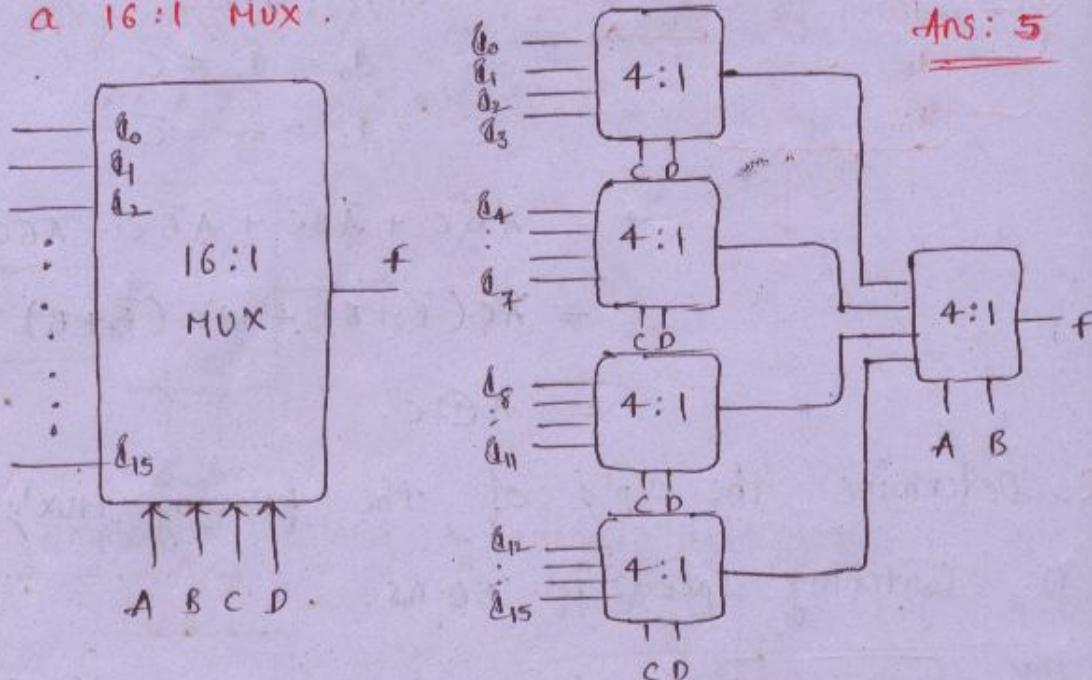
$$f_1 = \bar{x}_1 \cdot 1 + x_1 \cdot 0 = \bar{x}_1$$

$$f = \bar{A}d_0 + Ad_1$$

$$= \bar{x}_2 \cdot \bar{x}_1 + x_2 \cdot x_1$$

$$= x_2 \oplus x_1$$

Q. How many 4:1 mux's are required to construct a 16:1 MUX.

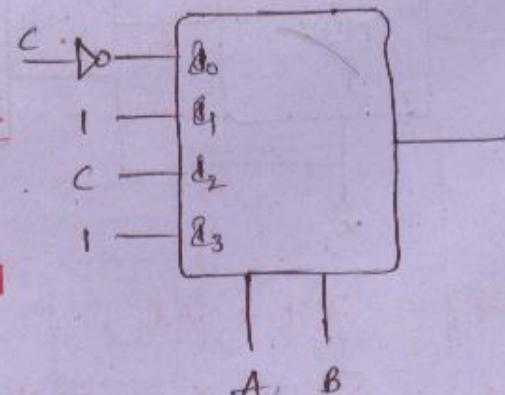


Q. Implement sum of minterm exp. $f(A, B, C)$

$= \sum m(0, 2, 3, 4, 5, 6, 7)$. by using 4:1 MUX.

A	B	C	f
0	0	0	1 } $\{ d_0 = \bar{C}$
0	0	1	0 }
0	1	0	1 } $\{ d_1 = 1$
0	1	1	1 }
1	0	0	0 } $\{ d_2 = C$
1	0	1	1 }
1	1	0	1 } $\{ d_3 = 1$
1	1	1	1 }

↓ 8:1 MUX



[OR]		\oplus			
		00	01	10	11
		d_0	d_1	d_2	d_3
0	\bar{C}	000	010	100	110
1	C	0	2	4	6
		001	1	3	5
				7	

<u>AB</u>	d_0	d_1	d_2	d_3
\bar{C}	0	2	4	6
C	1	3	5	7

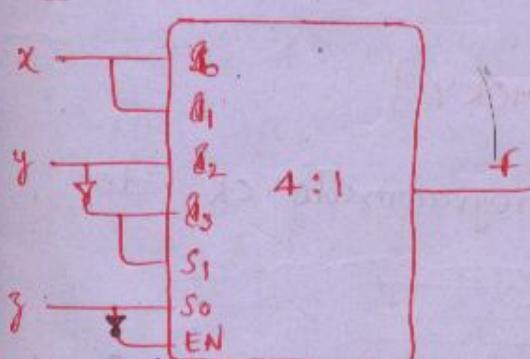
$\bar{C} \quad 1 \quad C \quad 1$

Implement above problem by choosing B & C as selection lines.

<u>BC</u>	d_0	d_1	d_2	d_3
0 \bar{A}	0	1	4	6
1 A	4	5	6	7

$\bar{A} \quad A \quad 1 \quad 1$

* Using 4:1 MUX, we can implement all 2 variable functions and some 3 variable functions. $f(A, B)$ \rightarrow Requires some logic gates like NOT GATE.



If $Z=0$, MUX is enabled and with $Z=1$, MUX is disabled.

$$S_1 = \bar{Y}$$

$$S_0 = Z$$

x	y	\bar{z}	s_1	s_0	f
0	0	0	1	0	$\bar{s}_2 = y = 0$
0	1	0	0	0	$\bar{s}_0 = x = 0$
1	0	0	1	0	$\bar{s}_2 = y = 0$
1	1	0	0	0	$\bar{s}_0 = x = 1$
$1 \rightarrow \text{disabled}$			$f = xy\bar{z}$		

ANOTHER WAY :

$$f = \bar{A}\bar{B}\bar{s}_0 + \bar{A}B\bar{s}_1 + A\bar{B}\bar{s}_2 + AB\bar{s}_3$$

$$\text{where } s_1 \ A = \bar{y} \quad \bar{s}_0 = \bar{s}_1 = x$$

$$s_0 \ B = \bar{z} \quad \bar{s}_2 = y; \bar{s}_3 = \bar{y}$$

$$\Rightarrow f = \bar{y}\bar{z}x + \bar{y}\bar{z}x + \bar{y}\bar{z}y + \bar{y}\bar{z}\bar{y}$$

$$= xy\bar{z} + \cancel{xy\bar{z}} + 0 + \cancel{y\bar{z}}$$

$$x=1 \quad x=1 \quad y=0$$

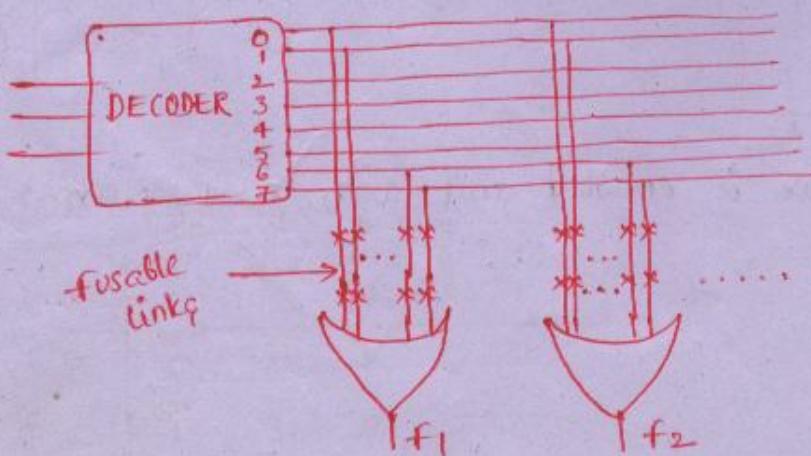
$$y=1 \quad y=1 \quad z=1$$

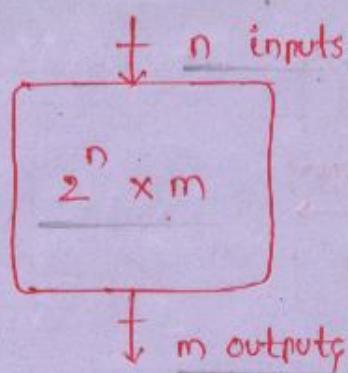
$$z=0 \quad z=1$$

$$\Rightarrow f = xy\bar{z}$$

ROM [READ ONLY] MEMORY]

ROM \Rightarrow DECODER + programmable OR gates





Size of the ROM indicates the no. of fuses at the beginning.

PLA : programmable AND gates & programmable OR gates.

PAL : Programmable AND gates & fixed OR gates.

Decoder }
MUX }
ROM } ← sum of minterms ie $\sum m(\dots)$.
(canonical SOP form).

PLA ← std. SOP form. is sufficient.

Determine the size of the ROM for the following

$$(i) . f_1(x, y, z) = \sum m(0, 1, 3).$$

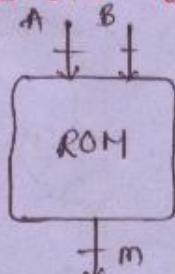
$$f_2(x, y, z) = x\bar{y} + \bar{x}\bar{y}\bar{z} + \bar{y}z$$

$$f_3(x, y, z) = \bar{x}yz.$$

$$n = 3 \text{ & } m = 3.$$

$$\text{ROM size} = 2^3 \times 3 = 24.$$

(iii). 3 bit binary Multiplier

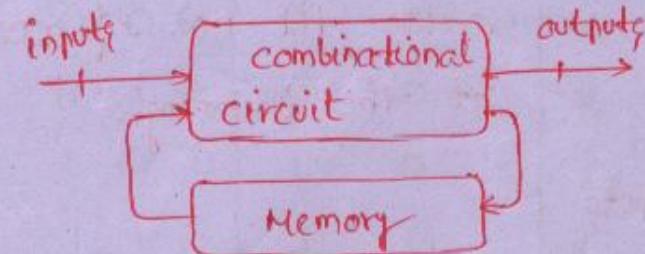


$$\begin{array}{r}
 111 \times 111 \\
 \hline
 110 \times 110 = 49_{10} \\
 \Rightarrow 2^m > 49 \\
 \Rightarrow m = 6
 \end{array}$$

$$\therefore \text{Size} = 2^6 \times 6$$

=

SEQUENTIAL CIRCUITS:



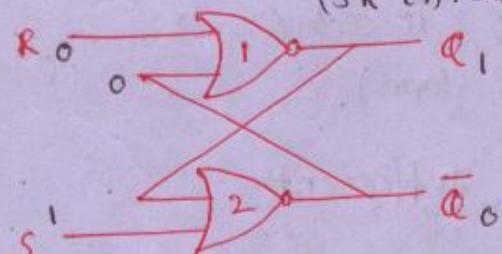
Output = $f(\text{present inputs, past outputs})$

or

$+ (\text{" , present state})$.

1 Bit Memory Element:

(SR LATCH)



$\begin{matrix} \leftarrow \text{SET} \\ S \\ \leftarrow \text{RESET} \\ R \end{matrix}$

$\begin{matrix} 0 & 0 & \text{NO change in output} \end{matrix}$

$\begin{matrix} 0 & 1 & 0 \end{matrix}$

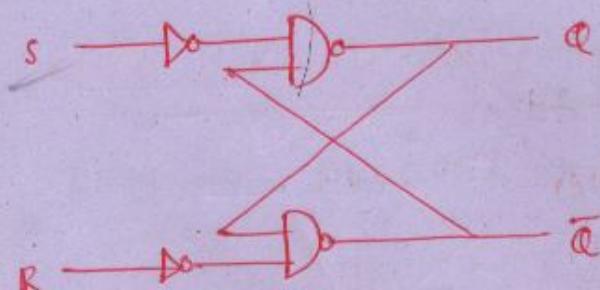
$\begin{matrix} 1 & 0 & 1 \end{matrix}$

$\begin{matrix} 1 & 1 & \text{Impractical state} \end{matrix}$

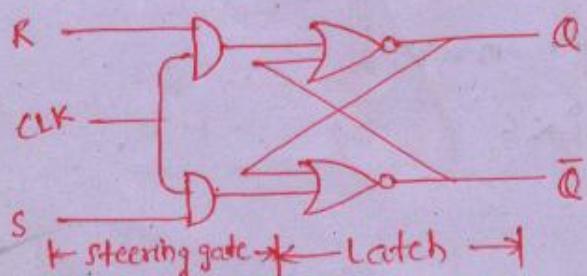
$\begin{matrix} S & R & Q \\ \hline 1 & 0 & 1 \end{matrix}$

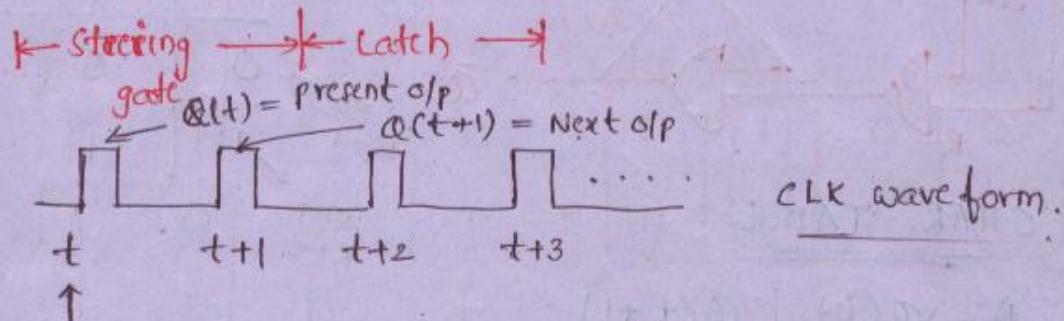
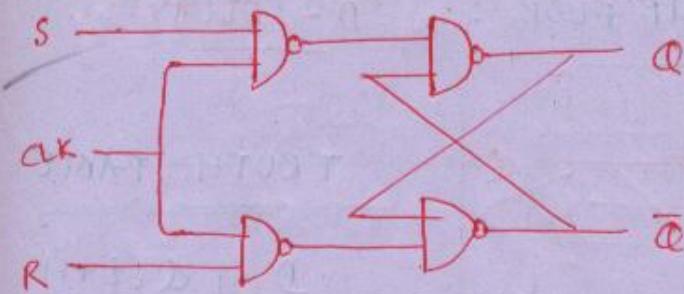
$\begin{matrix} 0 & 0 & 1 \end{matrix}$

\rightarrow Even if inputs are removed, the output will be 1 i.e. it stored the output. \rightarrow memory unit



CLOCKED S-R FLIP FLOP:





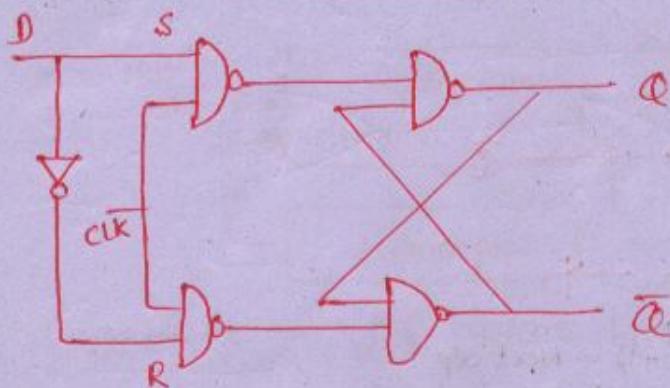
TRUTH TABLE :

S	R	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	(Ambiguous state)

CHAR. TABLE :

S	R	$Q(t)$	$Q(t+1)$
0	0	0	0
	0	1	1
0	1	0	0
	1	1	0
1	0	0	1
	0	1	1
1	1	0	x
	1	1	x

$Q(t+1) = S + \bar{R}Q$.

CLOCKED D - FLIP FLOP :TRUTH TABLE

D	$Q(t+1)$
0	0
1	1
s=0 R=1	1
s=1 R=0	0

CHAR. TABLE :

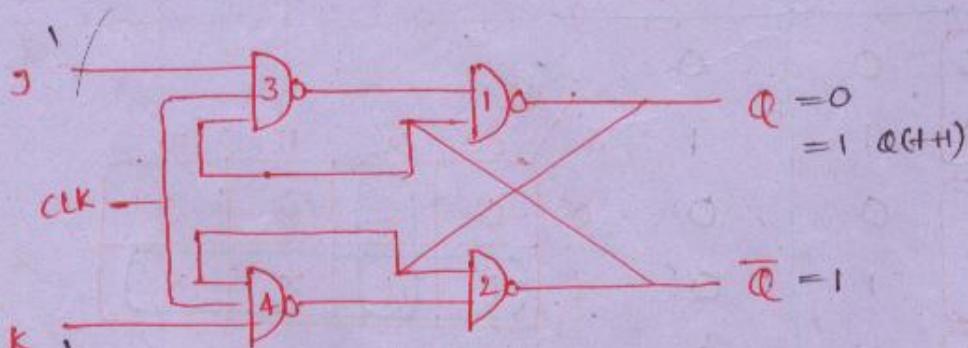
D	$Q(t)$	$Q(t+1)$
0	0	0
0	1	0
1	0	1
1	1	1

$$\begin{aligned} Q(t+1) &= D\bar{Q} + \bar{D}Q \\ &= D. \end{aligned}$$

CLOCKED JK FLIP FLOP :

$$S = J\bar{Q}$$

$$R = KQ$$

TRUTH TABLE :

J	K	$Q(t+1)$	$Q(t)$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

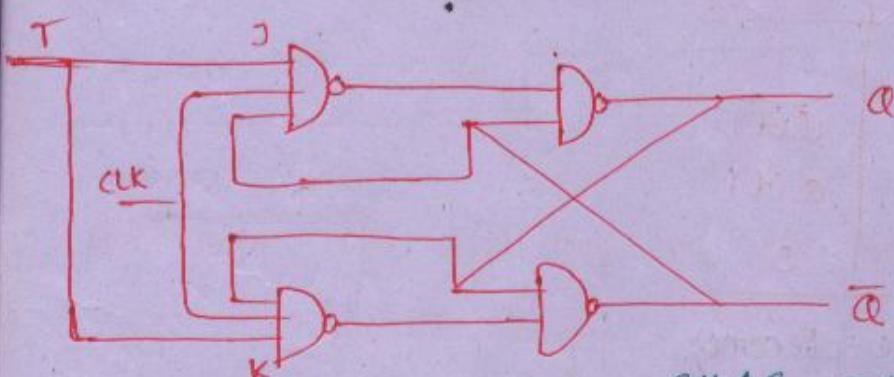
CHAR. TABLE:

J	K	$Q(t)$	$Q(t+1)$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$\frac{Q(t+1) = J\bar{Q} + \bar{K}Q}{}$

CLOCKED T- FLIP FLOP:

T - TOGGLE

TRUTH TABLE :-

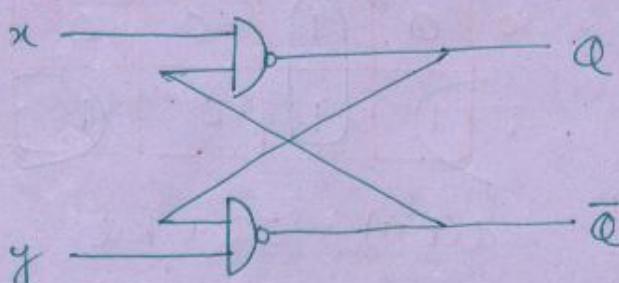
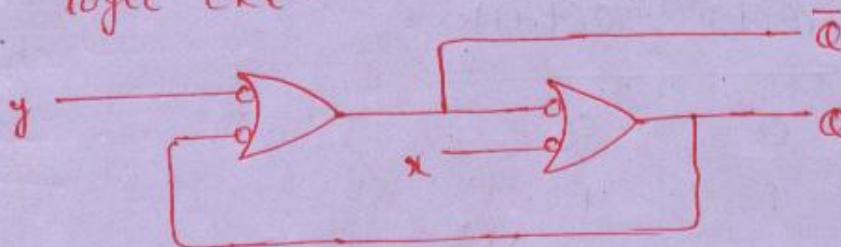
T	$Q(t+1)$
$J=K=0$	$Q(t)$
$J=K=1$	$\bar{Q}(t)$

CHAR. TABLE :-

T	$Q(t)$	$Q(t+1)$
0	0	0
0	1	1
1	0	1
1	1	0

$$\therefore Q(t+1) = T \oplus Q$$

Q. Determine the fun. table of the following logic ckt.



x	y	Q
0	0	$Q=1, \bar{Q}=1$
0	1	1
1	0	0
1	1	No change

a obtain char. eq. of x-y flip flop whose truth table as shown below -

x	y	$Q(t+1)$
0	0	1
0	1	$\bar{Q}(t)$
1	0	$Q(t)$
1	1	0

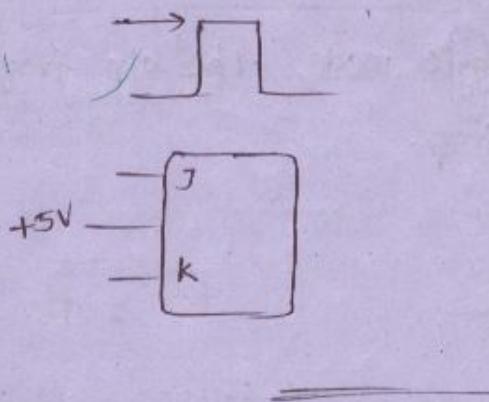
char. table becomes :-

x	y	$Q(t)$	$Q(t+1)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

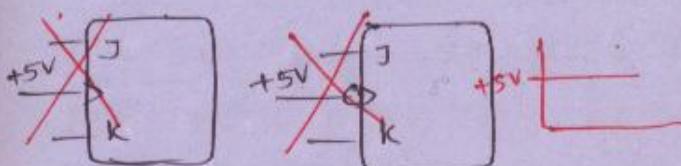
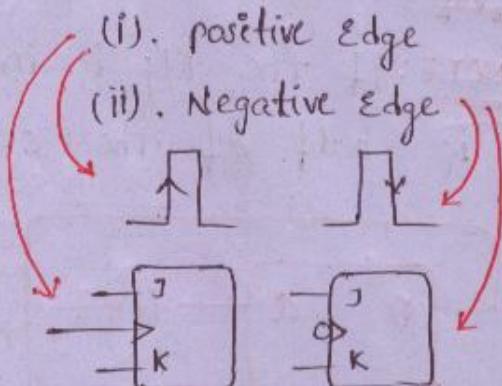
$Q(t+1) = \bar{x}\bar{Q} + \bar{y}Q$

TYPES OF TRIGGERING:

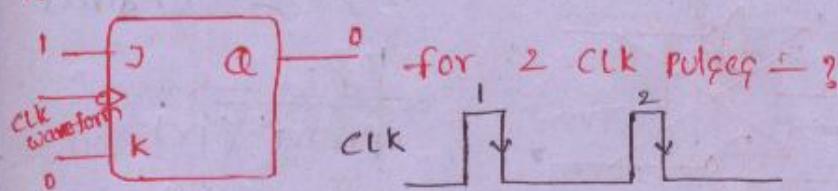
(1). LEVEL TRIGGER



(2). EDGE TRIGGERED



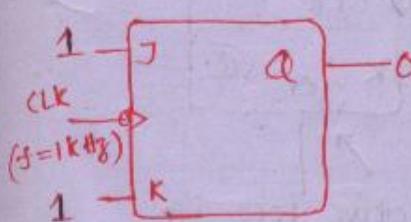
Q.



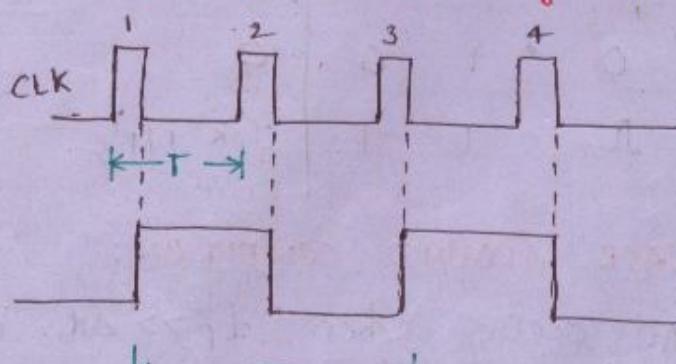
$$Q(t) = 0$$

$$Q(t+1) = 1$$

Q. Determine the off freq. of the following f_o - ?



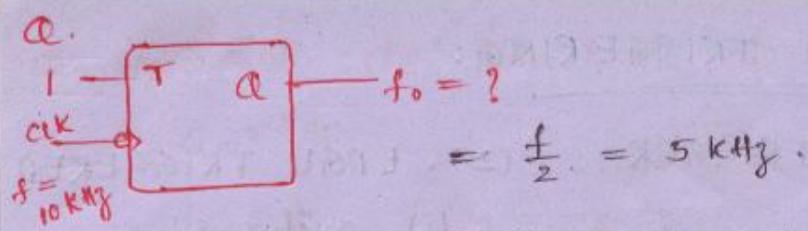
$$\begin{matrix} J & K \\ \hline 1 & 1 \end{matrix} \rightarrow Q(t+1) = \overline{Q(t)}$$



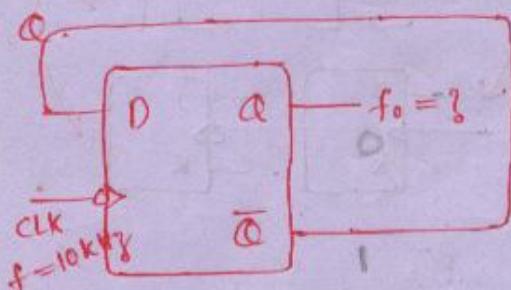
$$T_0 = 2T$$

$$f_o = \frac{1}{T_0}$$

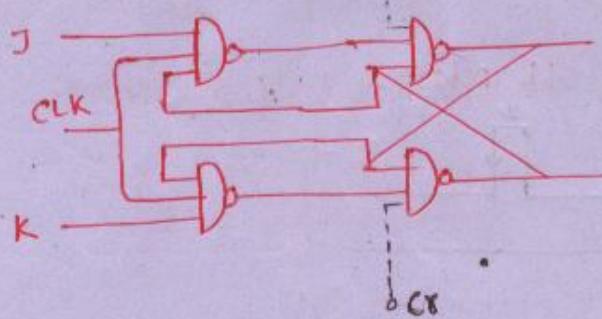
$$f_o = \frac{f}{2} = \frac{1\text{ kHz}}{2} = 500\text{ Hz}$$



NOTE: If the flf is in toggle mode, the o/p freq. is half of the clk freq.



* SUM. OF (12108) *



CLK	Pr	Cr	Q
0	0	1	1
0	1	0	0
1	1	1	J, K i/p's

RACE AROUND CONDITION:

RAC occurs when $t_p \gg \Delta t$ and $J = K = 1$.

$t_p \rightarrow$ applied clk pulse width

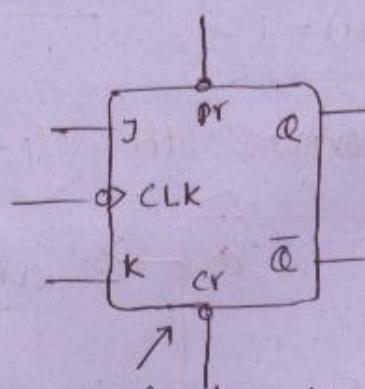
$\Delta t \rightarrow$ propagation delay of flf.

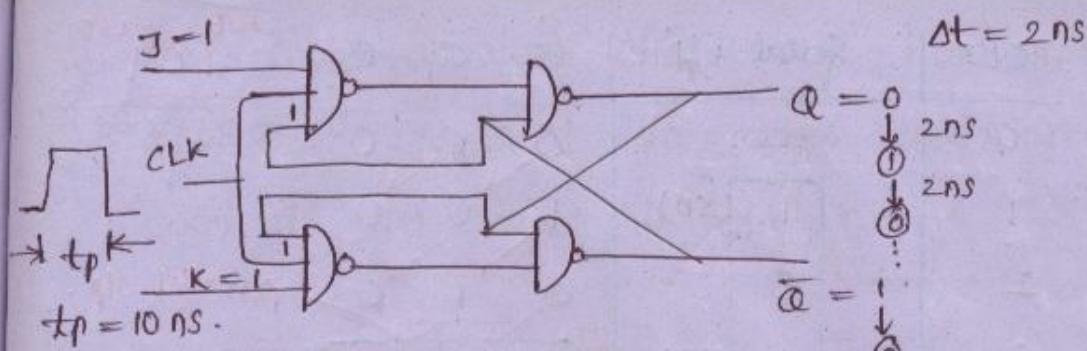
Asynchronous / direct

Blng:

preset (Pr)

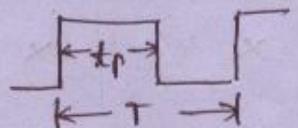
clear (Cr)





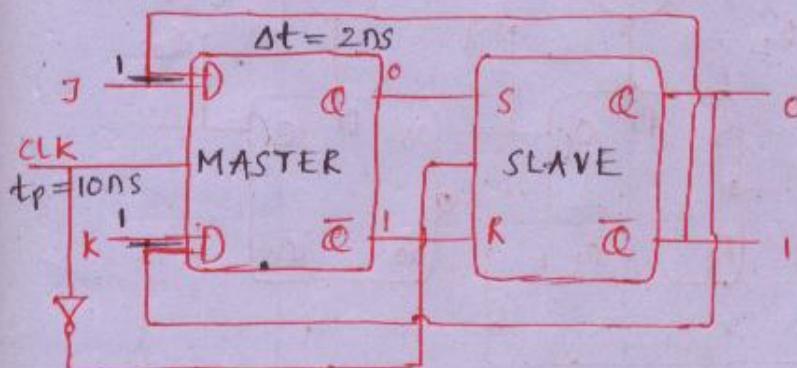
TO AVOID RAC:

$$t_p \leq \Delta t < T$$

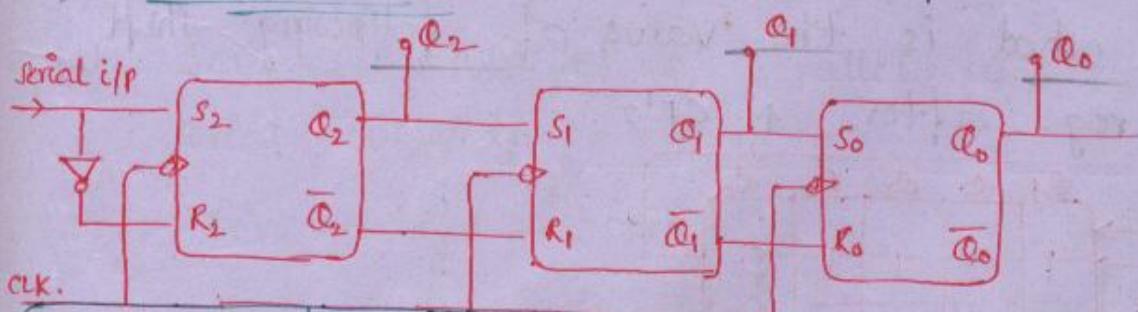


RAC occurs only in level triggered f/f but not in edge triggered f/f.

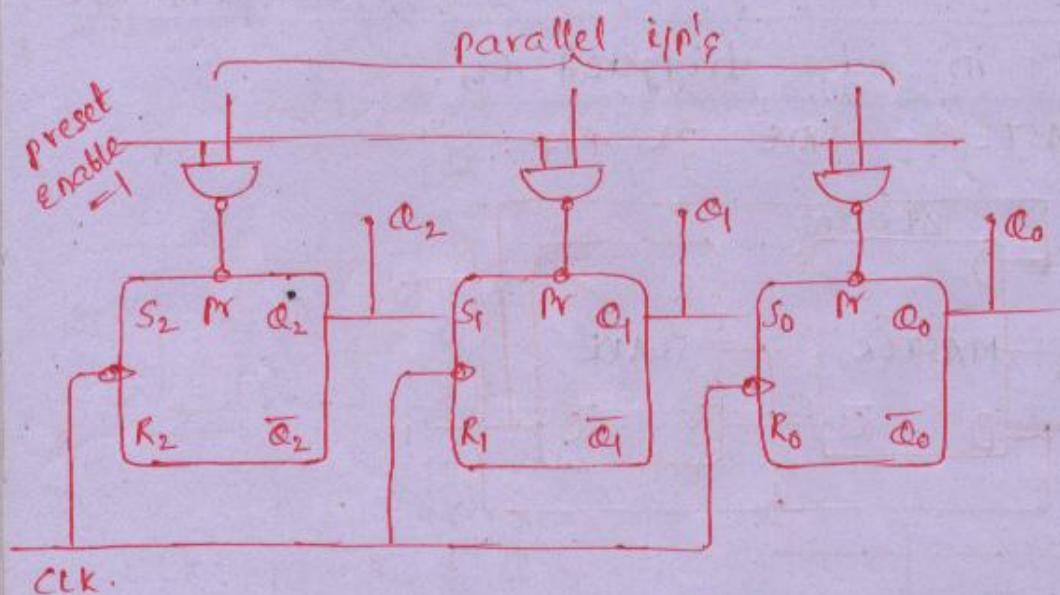
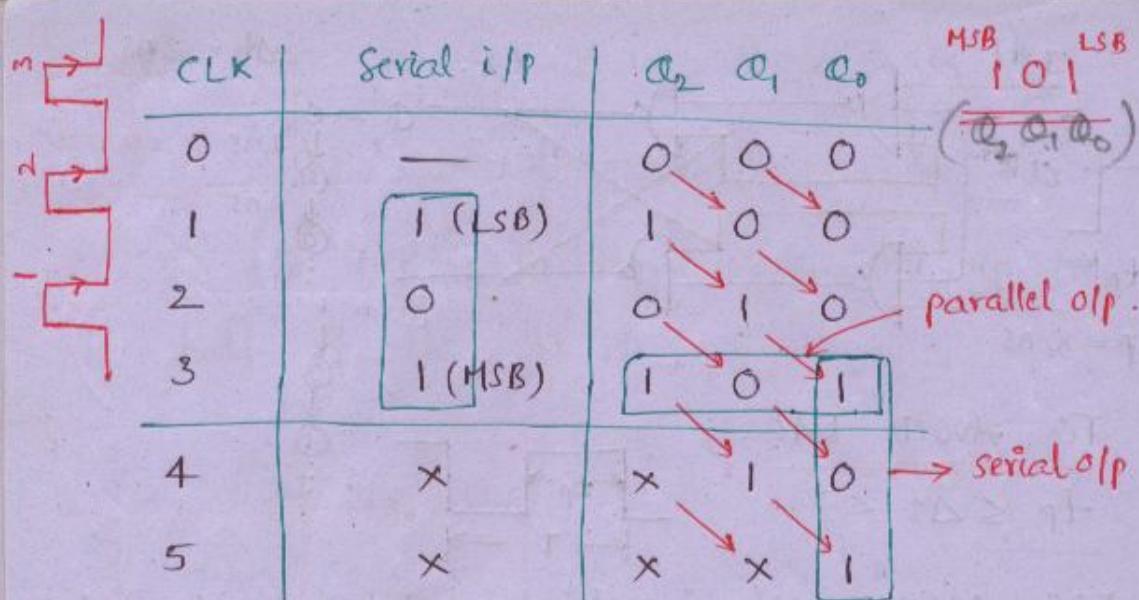
MASTER - SLAVE JK f/f:



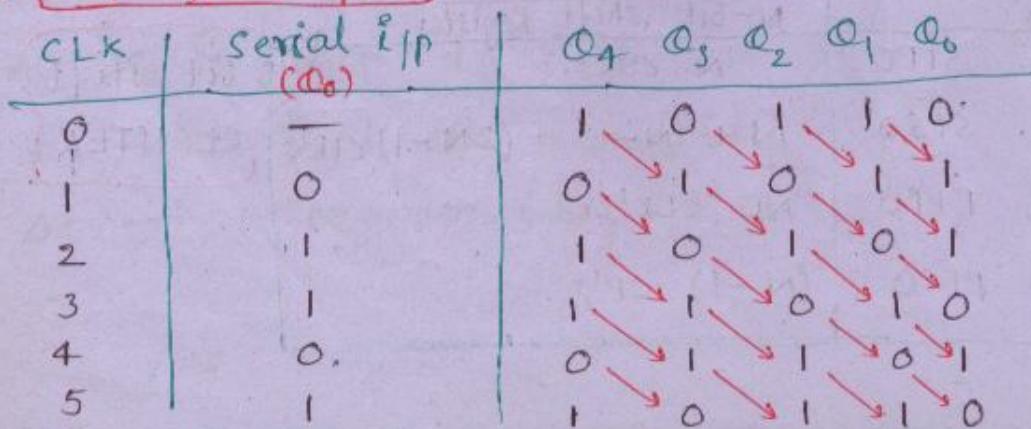
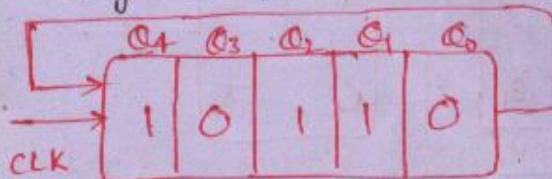
SHIFT REGISTER \rightarrow D - f/f's.



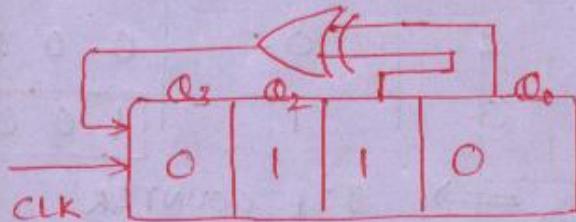
N-bit shift Register	
(1). SIPO	'N' CLK's.
(2). SISO	$N + (N-1) = (2N-1)$ CLK's.
(3). PIPO	NO CLK's.
(4). PISO	$(N-1)$ CP's.



Q. what is the value of following shift reg. after 4 CP's.



Q. In the following shift reg. how many CP's are required to make shift reg. content to have all one's.



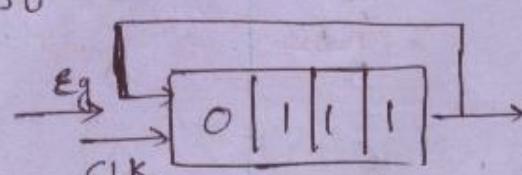
CLK	Serial I/P ($Q_3 + Q_0$)	$Q_3 \quad Q_2 \quad Q_1 \quad Q_0$
0	-	0 1 1 0
1	1	1 0 1 1
2	0	0 1 0 1
3	1	1 0 1 0
4	1	1 1 0 1
5	1	1 1 1 0
6	1	1 1 1 1

APPLICATIONS OF SHIFT REG'S:

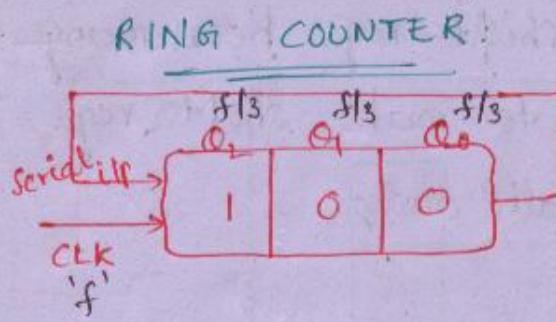
(1). Serial to parallel & parallel to serial conversion

(2). Time delays - SISO

(3). Sequence Generator



(4). Counter
RING
JOHNSON.

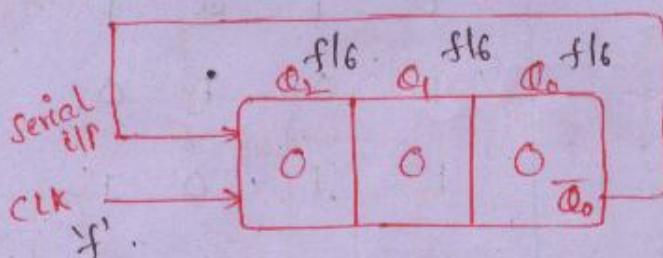


CLK	Serial i/p (Q0)	Q2 Q1 Q0
0	-	1 0 0
1	0	0 1 0
2	0	0 0 1
3	1	1 0 0

N-bit Ring Counter: \Rightarrow 3:1 COUNTER

- Counting capacity = $N:1$
- Output frequency = f/N .

JOHNSON COUNTER: [TWISTED RING COUNTER]



CLK	Serial i/p (Q0)	Q2 Q1 Q0
0	-	0 0 0
1	1	1 0 0
2	1	1 1 0
3	1	1 1 1
4	0	0 1 1
5	0	0 0 1
6	0	0 0 0

\Rightarrow 6:1 COUNTER.

N-bit Johnson Counter:

- Counting capacity = $2N:1$
- Output frequency = $f/2N$.

Q. what is the o/p freq. of a 3bit Johnson counter if its clk freq is 18 kHz. The initial content of the reg. is 101.

clk	\bar{Q}_0	serial o/p		
		Q_2	Q_1	Q_0
0	—	1	0	1
1	0	0	1	0
2	1	1	0	1

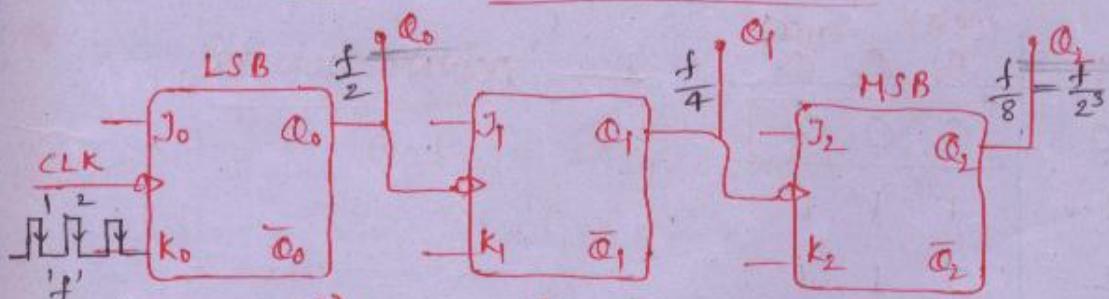
\Rightarrow 2:1 COUNTER.

$$2N = 6$$

COUNTERS:

- (1) Asynchronous / Ripple. $\rightarrow T \cdot f_{IF}$.
- (2) Synchronous / parallel.

3-bit Asynchronous / Ripple counter:



clk	(LSB)		(MSB)		UP COUNTER
	Q_0	Q_1	Q_2	\bar{Q}_2	
0	0	0	0	1	000
1	1	0	0	1	100 → 10ns
2	0	1	0	1	010 → 20ns
3	1	1	0	1	110 → 30ns
4	0	0	1	0	001 → 8:1 COUNTER
5	1	0	1	0	
6	0	1	1	0	
7	1	1	1	0	
8	0	0	0	1	

{ CLK PULSE is given to LSB f/F }

→ N-bit Asynchronous counter:

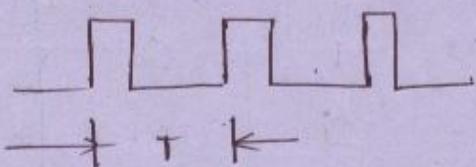
→ $2^N : 1$ counter

→ final output freq = $f/2^N$.

Let $t_{pd/ff} = 10 \text{ ns}$.

Then Max. conversion time = 30 ns .

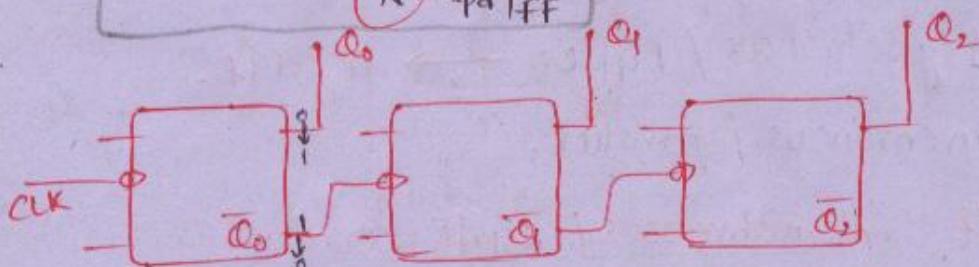
$\Rightarrow T \geq 30 \text{ ns}$.



$$f = \frac{1}{T} \leq \frac{1}{30 \text{ ns}}$$

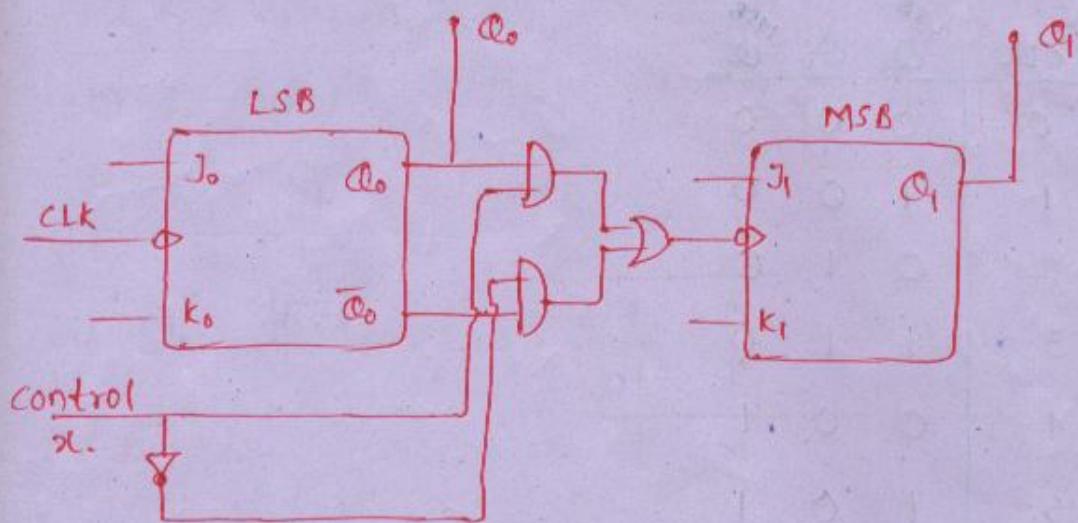
→ $f_{\max} = \frac{1}{30 \text{ ns}}$.

$f_{\max} = \frac{1}{N \cdot t_{pd/ff}}$ no. of flip-flops.



CLK	(LSB)			(MSB)			↓	DOWN COUNTER.
	Q_0	Q_1	Q_2	\bar{Q}_0	\bar{Q}_1	\bar{Q}_2		
0	0	0	0	1	1	1		
1	1	1	1	0	0	0		
2	0	1	1	1	0	1		
3	1	0	1	0	1	0		
4	0	0	1	1	1	0		
5	1	1	0	0	0	1		
6	0	1	0	1	0	0		
7	1	0	0	0	1	1		
8	0	0	0	1	1	1		

2-Bit Asynchronous up/down counter:



$X = 1 \rightarrow Q_0 \rightarrow CLK \rightarrow$ up counter. (00, 01, 10, 11, 00..)

$X = 0 \rightarrow \bar{Q}_0 \rightarrow CLK \rightarrow$ down counter. (00, 11, 10, 01, 00..)

MODULUS OF A COUNTER:

→ It is the no. of cp's required to bring the counter to the initial state.

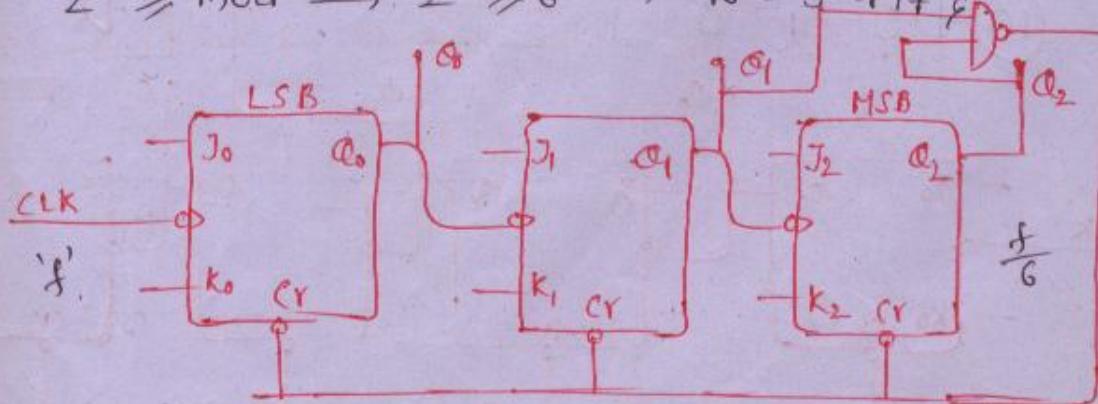
→ A Mod-N counter counts from 0 to $(N-1)$.

and clk freq. = $\frac{f}{N}$

Q. Construct Mod-6 Asy. counter.

Mod-6 Asy. COUNTER:

$$2^N \geqslant \text{mod} \Rightarrow 2^N \geqslant 6 \Rightarrow N = 3$$

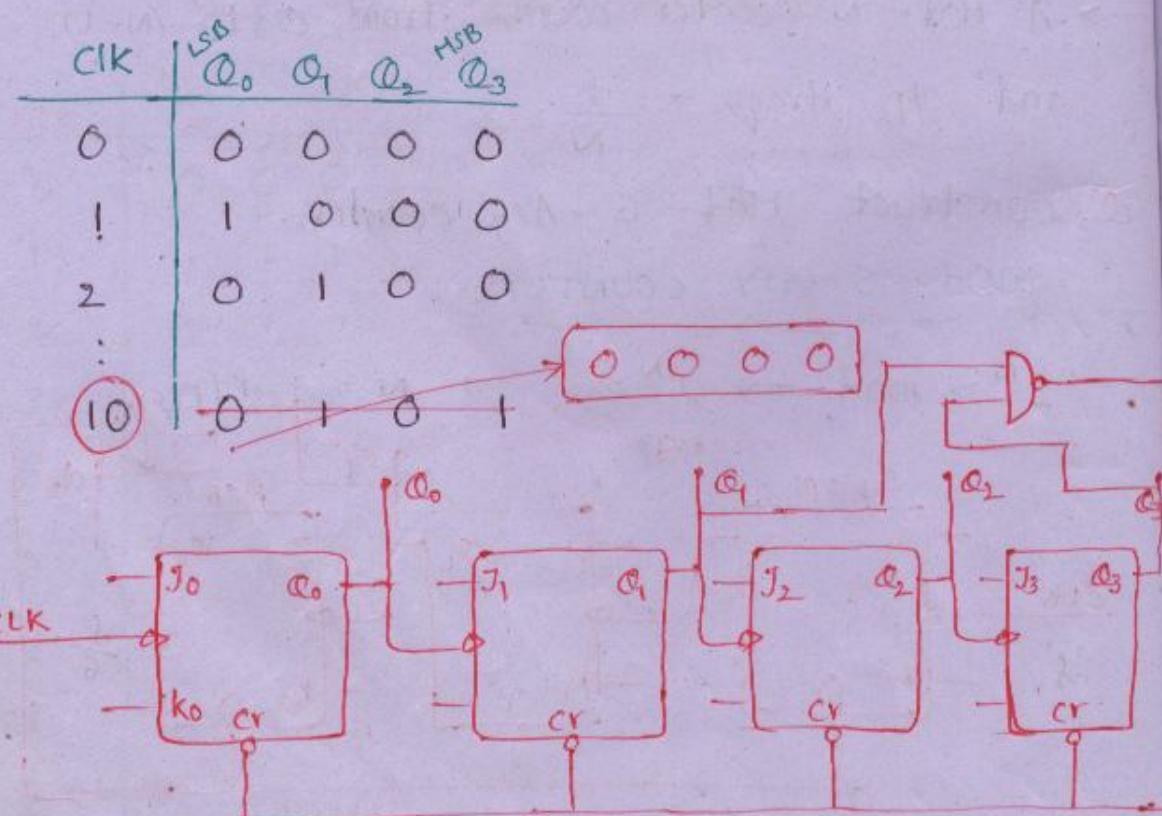


UP COUNTER

<u>CLK</u>	<u>LSB</u> <u>Q_0</u>	<u>Q_1</u>	<u>MSB</u> <u>Q_2</u>
0	0	0	0
1	1	0	0
2	0	1	0
3	1	1	0
4	0	0	1
5	1	0	1
6	0	1	1
7			

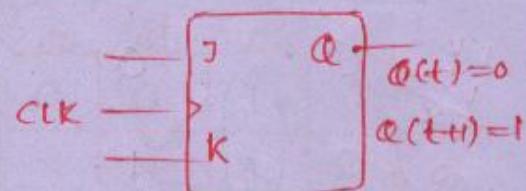
Q. Construct a Asy. decade counter ?
Mod - 10

$$2^N \geq 10 \rightarrow N = 4 + \text{fif's.}$$



Excitation Table:

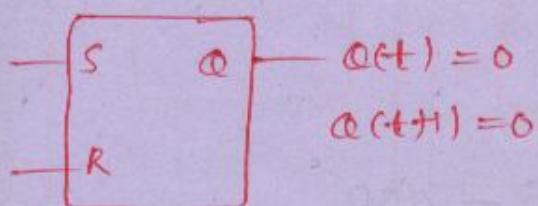
J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0



1	0	1
1	1	$\overline{Q(t)}$

J	K
1	0
1	1

SR flip-flop:



S	R
0	1
0	0

$Q(t)$	$Q(t+1)$	J	K	S	R	T	D
0	0	0	x	0	x	0	0
1	1	1	x	1	0	1	1
0	0	x	1	0	1	1	0
1	0	x	0	x	0	0	1

c. Obtain excitation table of XY flipflop:-

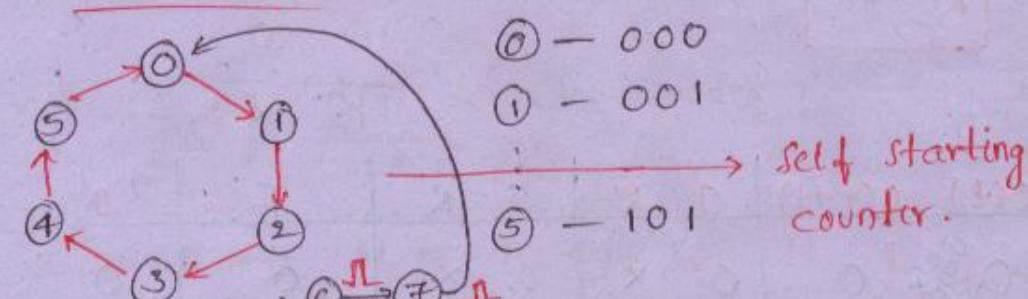
X	Y	$Q(t+1)$
0	0	1
0	1	$\overline{Q(t)}$
1	0	$Q(t)$
1	1	0

$Q(t)$	$Q(t+1)$	x	y
10	0	1	x
00	1	0	x
01	0	x	1
11	1	x	0

Q. Design a mod-6 syn. counter using JK flip-flops.

$$\text{mod-6} \Rightarrow 0 \text{ to } 5.$$

state diagram



Present state $Q_2\ Q_1\ Q_0$	Next state $Q_2\ Q_1\ Q_0$	FF inputs		
		$J_2\ K_2$	$J_1\ K_1$	$J_0\ K_0$
0 0 0	0 0 1	0 x	0 x	1 x
0 0 1	0 1 0	0 x	1 x	x 1
0 1 0	0 1 1	0 x	x 0	1 x
0 1 1	1 0 0	1 x	x 1	x 1
1 0 0	1 0 1	x 0	0 x	1 x
1 0 1	0 0 0	x 1	0 x	x 1

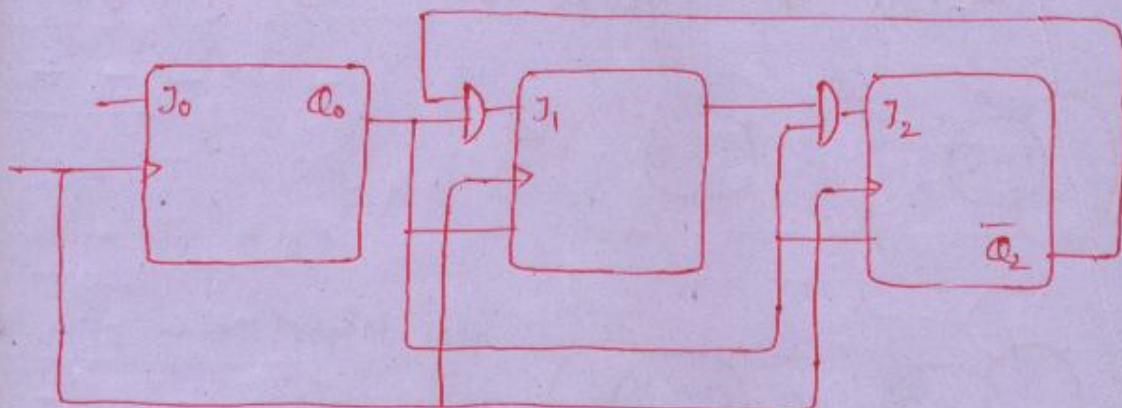
				$J_0 = k_0 = 1.$
				J_1
				$Q_2 \quad Q_1 \quad Q_0$
0	0	1	X	X
1	0	0	X	X

$$J_1 = \bar{Q}_2 Q_0$$

				J_2
				$Q_2 \quad Q_1 \quad Q_0$
				$00 \quad 01 \quad 101 \quad 10$
0	0	0	1	0
1	X	X	X	X

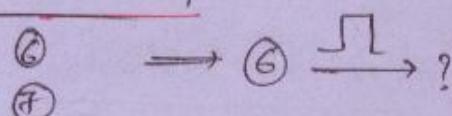
$$J_2 = Q_1 Q_0$$

and $k_1 = Q_0$, $k_2 = Q_0$.



$$f_{\max} = \frac{1}{t_{pd/ff}}$$

unspecified states



present state			flip flops			Next state		
Q_2	Q_1	Q_0	$J_2 \ k_2$	$J_1 \ k_1$	$J_0 \ k_0$	Q_2	Q_1	Q_0
1	1	0	00	00	11	1	1	1
1	1	1	11	01	11	0	0	0

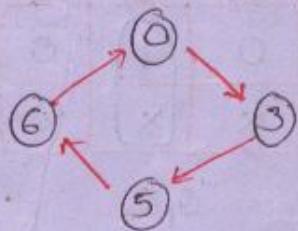
c. Design a syn. counter using T flip-flops.

which counts 10 0, 3, 5, 6, 0, ...

Is it a self-starting counter?

state diagram

→ Not a self starting counter.



(1).

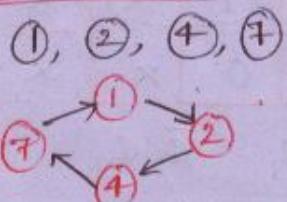
P.S.

 $Q_2\ Q_1\ Q_0$

N.S.

 $Q_2\ Q_1\ Q_0$

"LOCK OUT"
Unused states



f/f i/p's

 $T_2\ T_1\ T_0$

(2).

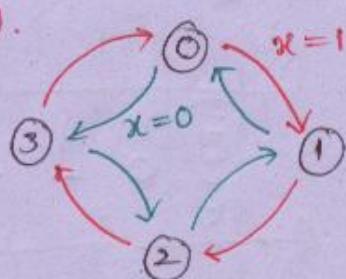
P.S

f/f i/p's

N.S.

a. Draw the state diagram of following digital circuit (ix) 2 bit syn. up/down counter).

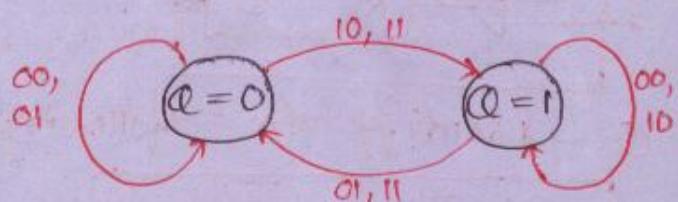
(ii).



(iii). JK - f/f

present
input {
j
k
} → branches of
each state.
p.s { Q → states

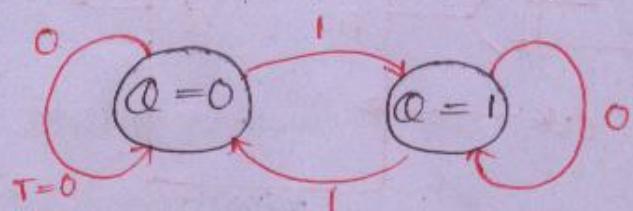
(iv). T - f/f



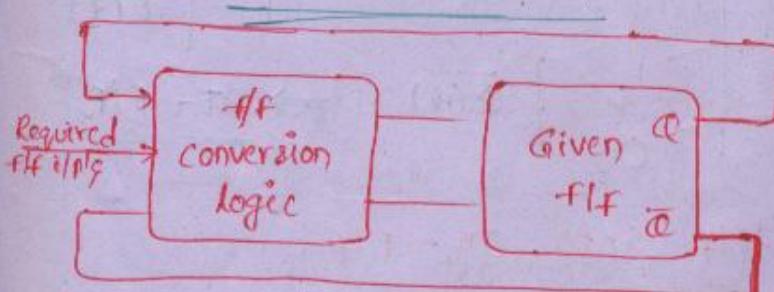
T → Present input

Q → P.S.

T	$Q(t+1)$
0	$Q(t)$
1	$\bar{Q}(t)$



CONVERSION OF f/f's:



a. Convert SR-f/f into T-f/f.

SR-f/f
exc. table

T-f/f
char. table

T	$Q(t)$	$Q(t+1)$	S	R
0	0	0	0	x
0	1	1	x	0
1	0	1	1	0
1	1	0	0	1

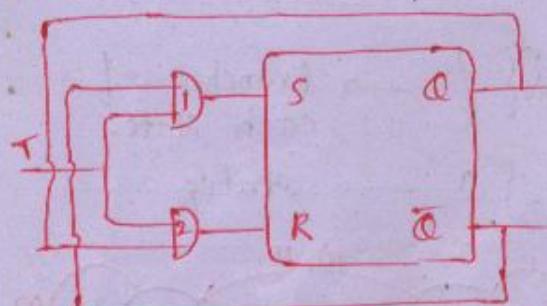
T	Q	\bar{Q}
0	0	X
1	1	0

$$S = T\bar{Q}$$

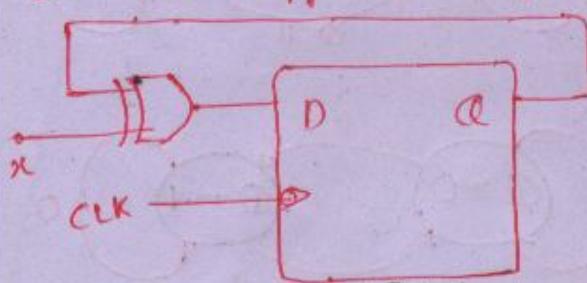
T	Q	\bar{Q}
0	X	0
1	0	1

$$R = TQ$$

$T - \text{ff}$



a) Identify the following ff.



x	$Q(t)$	D	$Q(t+1)$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

x	$Q(t)$	D	$Q(t+1)$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

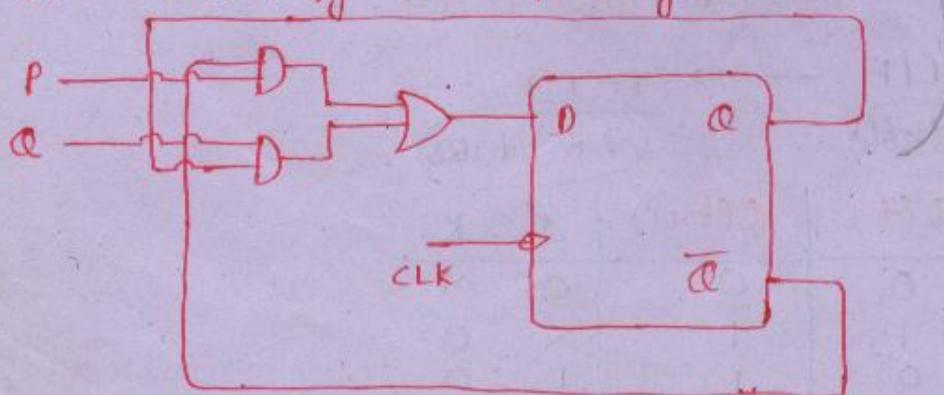
$x \quad Q(t+1)$

$0 \quad Q(t)$
1 $\bar{Q}(t)$

$\Rightarrow T - \text{ff}$

a) Convert D-ff into JK-ff.

a2) Identify the following ff.



Q. In which of the following counters lockout doesn't occur.

(1). Mod - 13 counter (2). Mod - 30 counter

(3). Mod - 32 " (4). Mod - 36 "

$$2^4 - 13 = 3 \text{ unused states}$$

$$2^5 - 32 = 0 \text{ unused states}$$

$$2^5 - 30 = 2 \text{ unused states}$$

$$2^6 - 36 = 28 \text{ unused states}$$

MULTI VIBRATORS USING LOGIC GATES:

1. ASTABLE MV:

→ 2 quasi stable states

→ Square wave Generator.

2. BISTABLE MV:

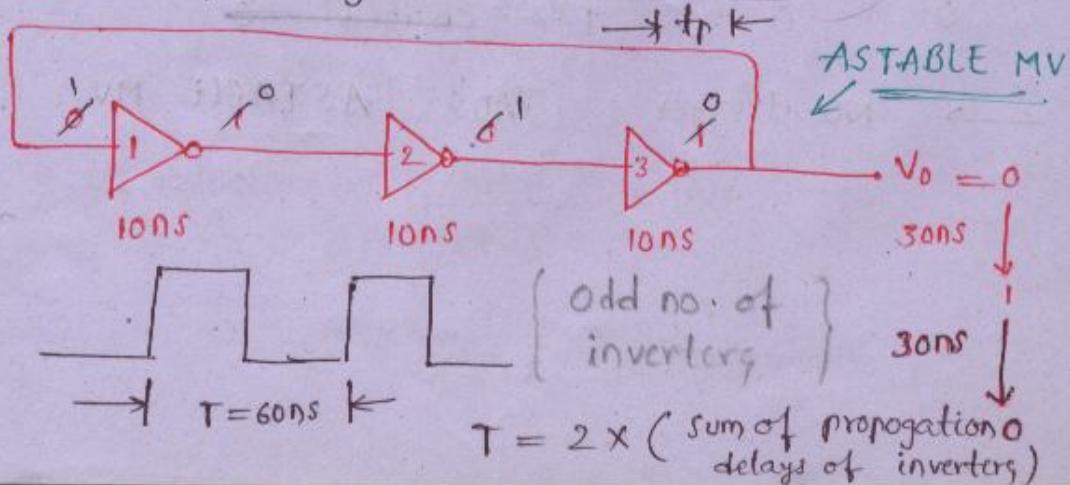
→ 2 stable states

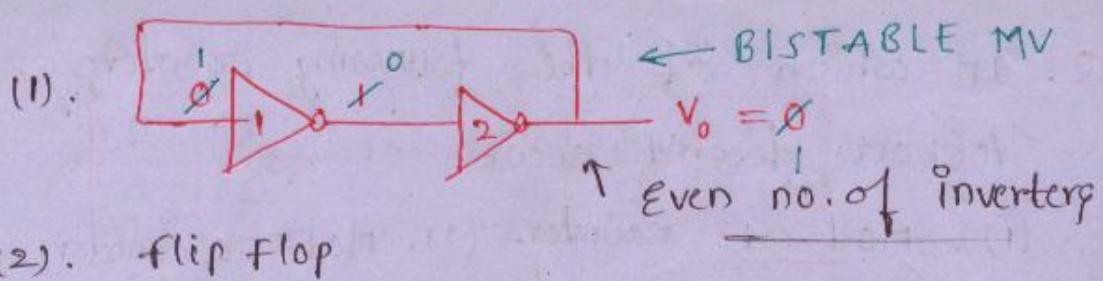
→ 1-bit memory element

3. MONOSTABLE MV: [One shot]

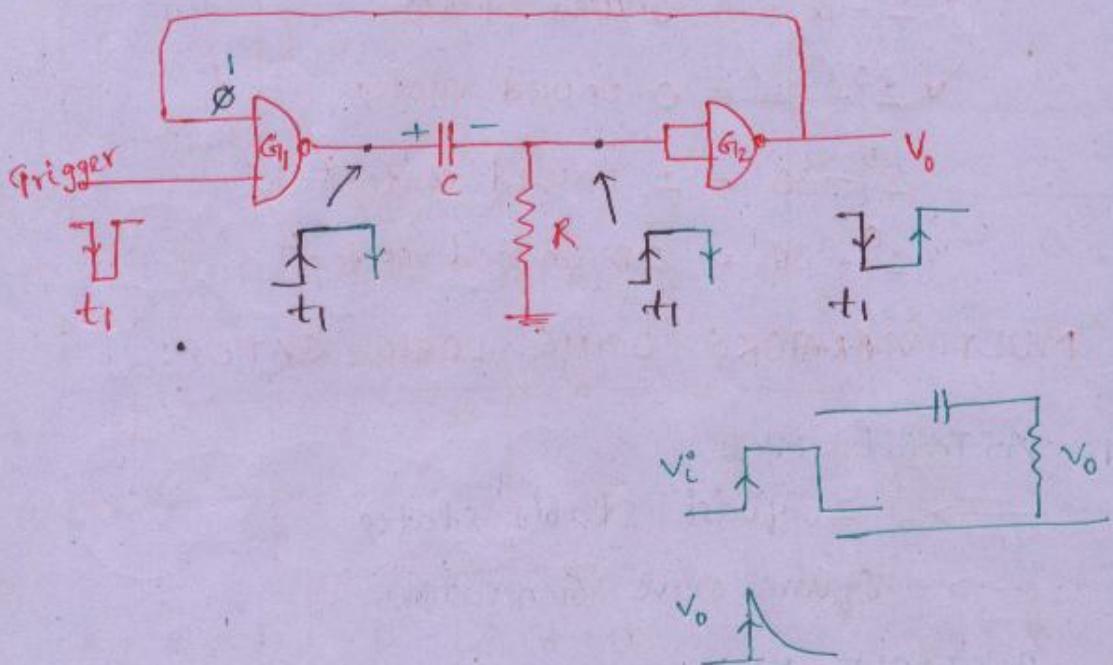
→ 1 quasi & 1 stable

→ pulse generator

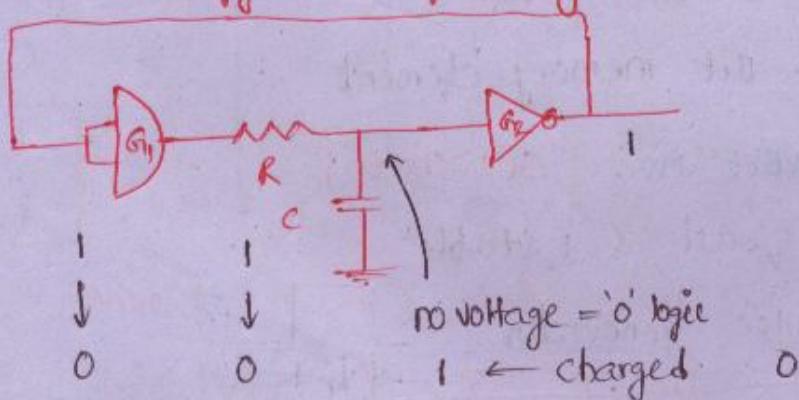




MONO STABLE :



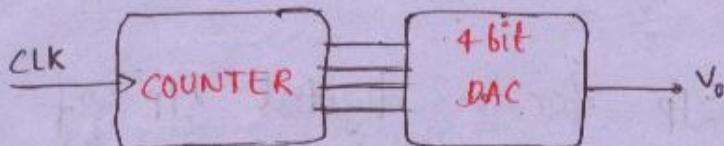
c. Identify the following MV's.



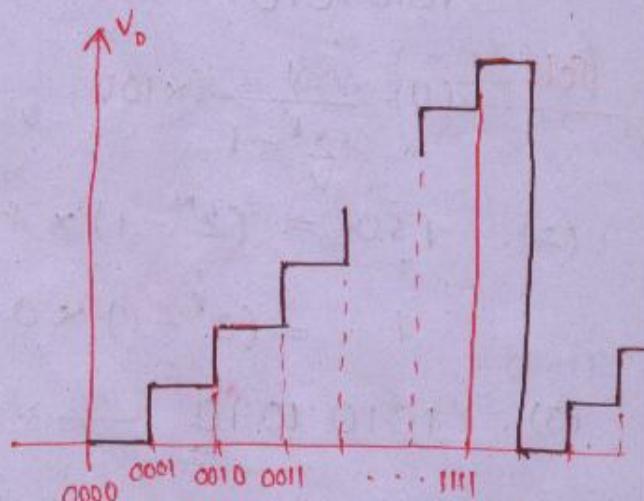
→ NO trigger ; Ans: ASTABLE MV.

DATA CONVERTERS :

- (1). DAC [Digital to Analog]
- (2). ADC [Analog to Digital].



CLK	count	V_o
0	0000	0V
1	0001	1V
2	0010	2V
:	:	:
15	1111	15V
16	0000	0V



no. of steps = 15

fs0 (full scale o/p) = 15V

Resolution = step size (V)

It is the smallest possible change at the o/p of DAC for any change in i/p.

'N' bit DAC $\rightarrow (2^N - 1)$

= no. of steps \times step size $\leftarrow f_{s0}$

$= (2^N - 1) \times$ step size.

$\rightarrow \text{Resolution} = \frac{\text{Step size}}{f_{s0}} \times 100$

$$= \frac{1}{2^N - 1} \times 100$$

Q The o/p of a 8-bit DAC is 0.15V when the i/p is 00000001.

Determine (1). v. resolution

(2). FSO

(3). DAC o/p for a digital i/p of 10101010.

Sol: (1). $\frac{1}{2^8 - 1} \times 100$

(2). $\therefore \text{FSO} = (2^N - 1) \times \text{step size}$
 $= (2^8 - 1) \times 0.15 \text{ V}$

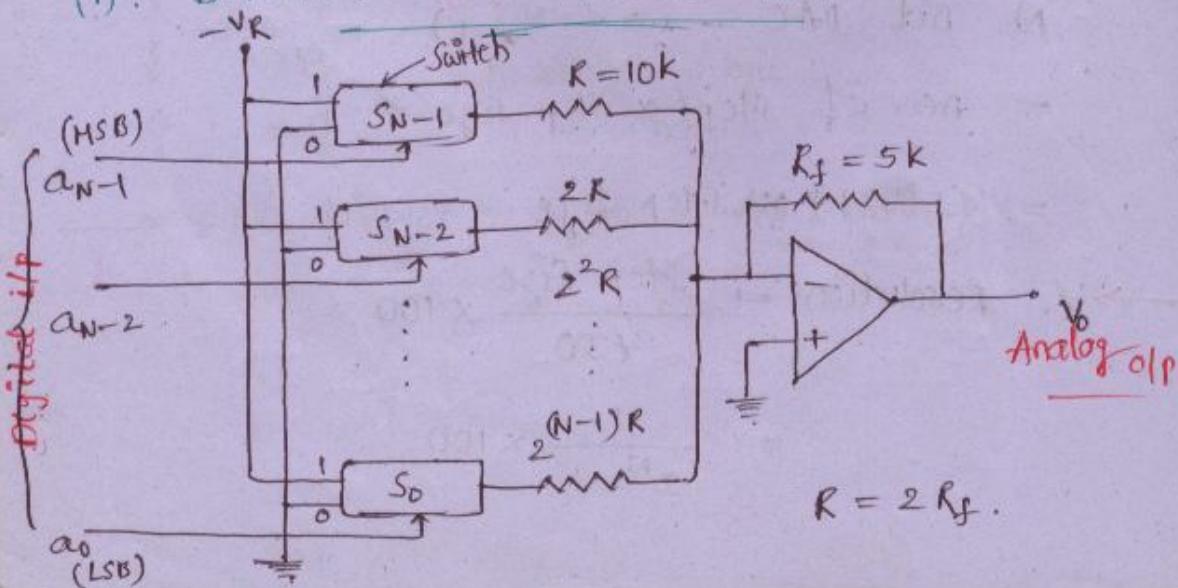
(3). $1010\ 1010_2 \rightarrow 170_{10}$

$\therefore \text{o/p} = 170 \times 0.15 \text{ V.}$

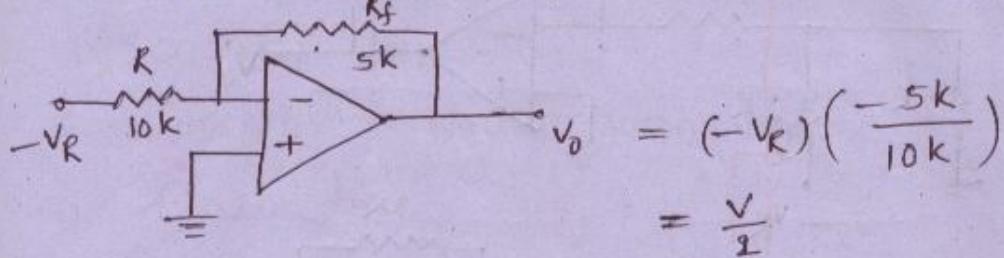
Resolution \leftarrow voltage (should be less)

8 bit DAC	0.1V
16 "	0.5V
32 "	1V

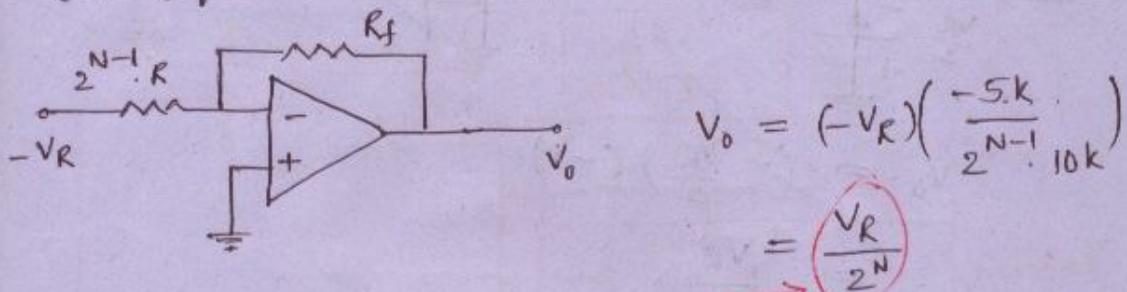
(1). BINARY WEIGHTED DAC:



(1). If $a_{N-1} = 1; a_{N-2} = \dots = a_1 = a_0 = 0$



(2). If $a_0 = 1, a_{N-1} = \dots = a_1 = 0$.



Resolution

$$V_0 = (a_{N-1} \cdot 2^{-1} + a_{N-2} \cdot 2^{-2} + \dots + a_1 \cdot 2^{-(N-1)} + a_0 \cdot 2^{-N}) VR$$

Eg: for a 3 bit DAC.

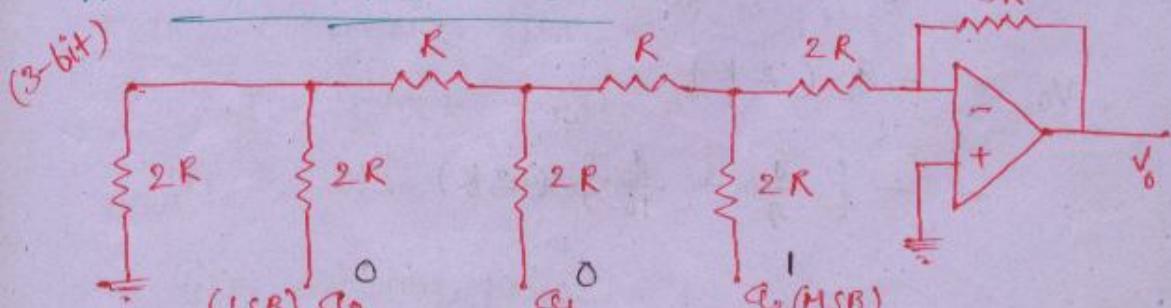
$$V_0 = (a_2 \cdot 2^{-1} + a_1 \cdot 2^{-2} + a_0 \cdot 2^{-3}) VR$$

$$\text{Resolution} = \frac{VR}{2^3}$$

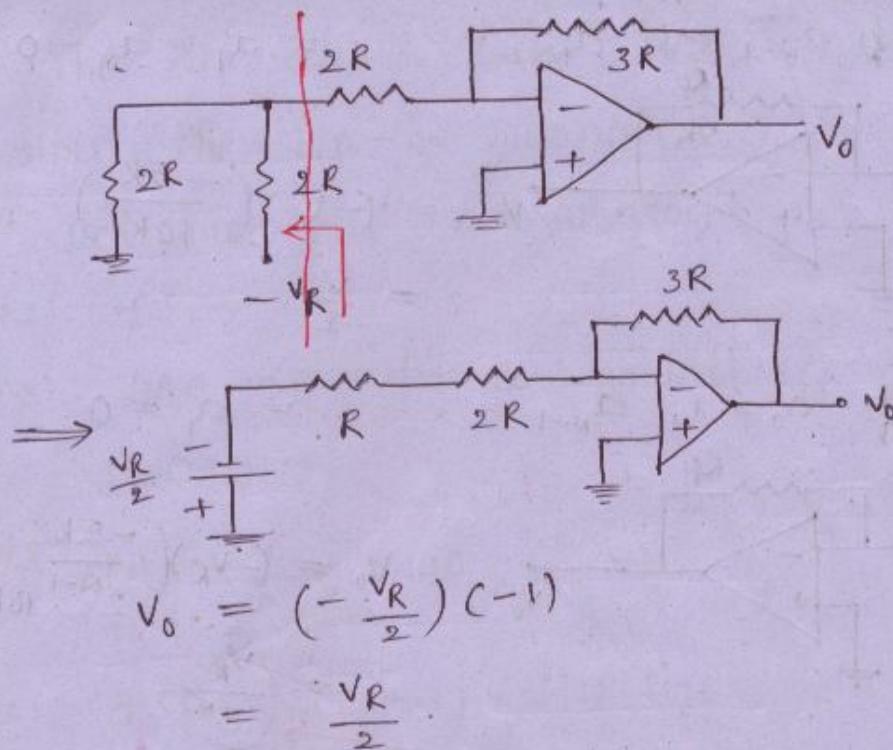
Draw back \rightarrow for 32 bit DAC $\rightarrow 2^{31} \times R$

if required .. and so.

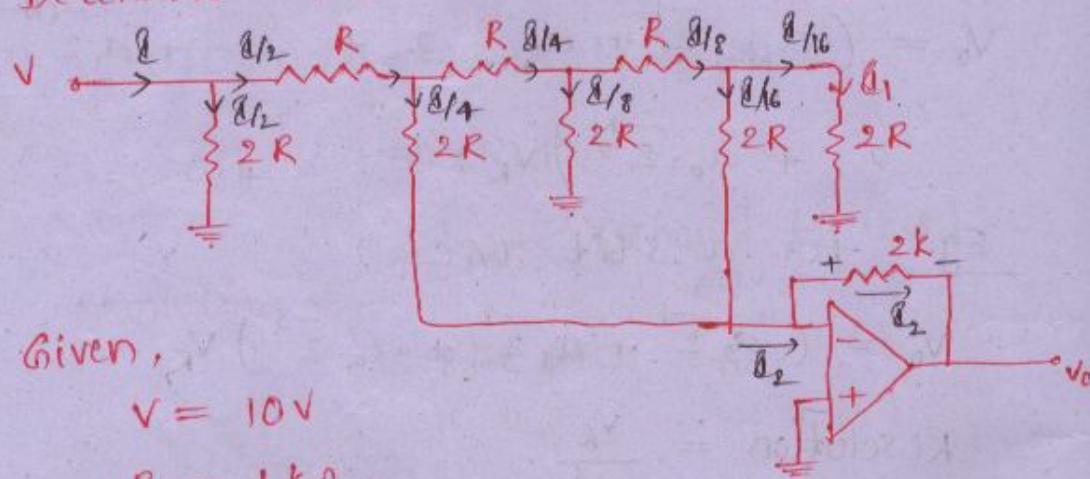
R - 2R LADDER DAC:



'1' \rightarrow -VR volt
'0' \rightarrow 0 volt



Q: Determine δ_1 & v_o in the following circuit.



Given,

$$V = 10V$$

$$R = 1k\Omega$$

$$\delta = \frac{V}{R} = \frac{10}{1k} = 10mA$$

$$\delta_1 = \frac{\delta}{16} = \frac{10mA}{16}$$

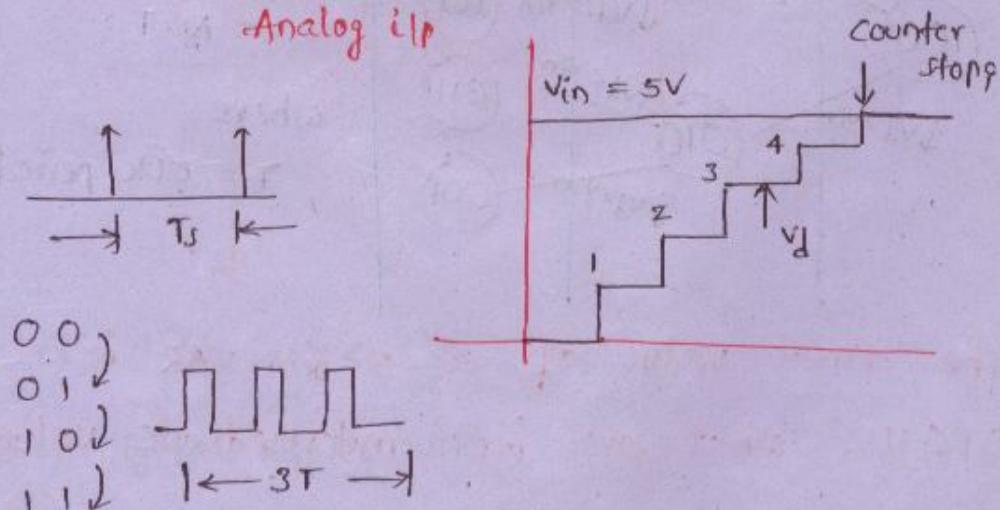
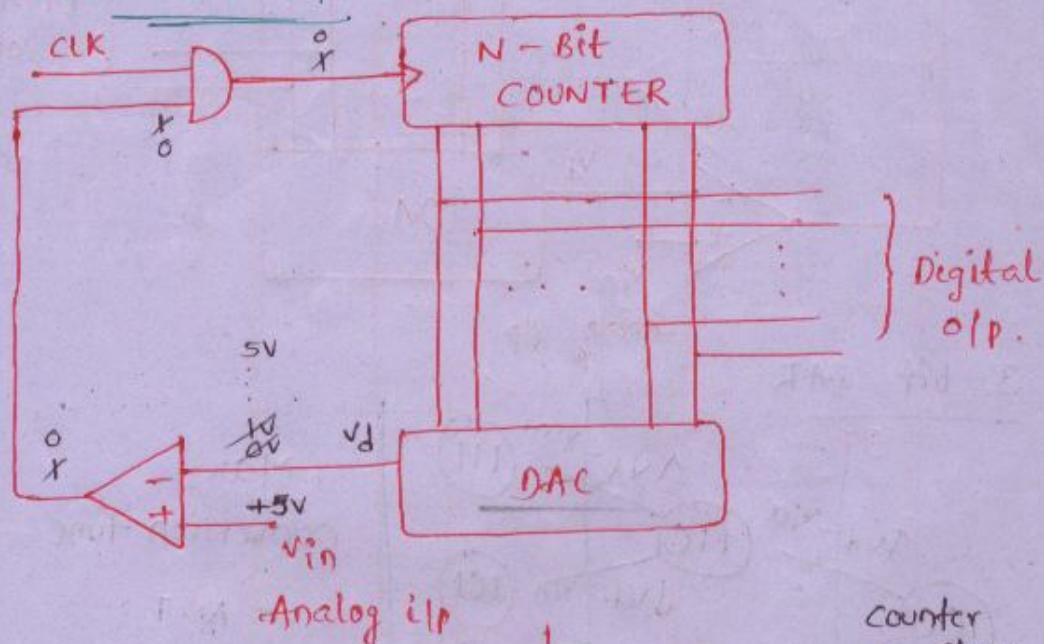
$$v_o = -\delta_2 (2k)$$

$$= - \left[\frac{\delta}{4} + \frac{\delta}{16} \right] (2k)$$

ADC's:

1. counter type
2. successive approximation type.
3. flash type
4. dual slope.

COUNTER TYPE:-



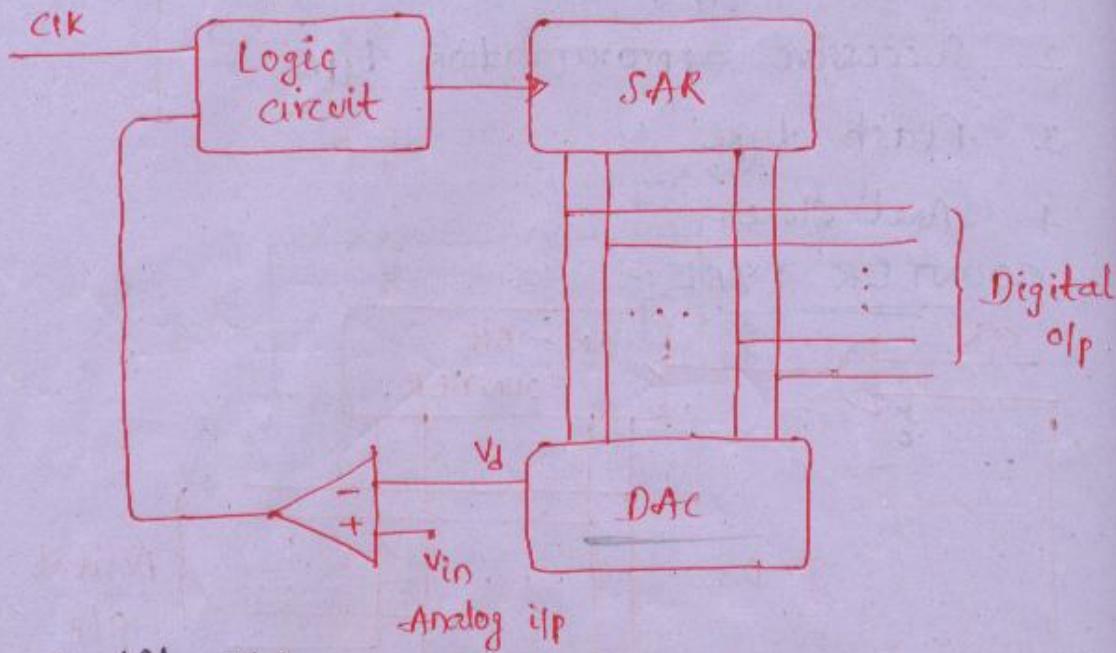
$$\text{Max. conversion time} = (2^N - 1) \cdot T ;$$

T - clock period

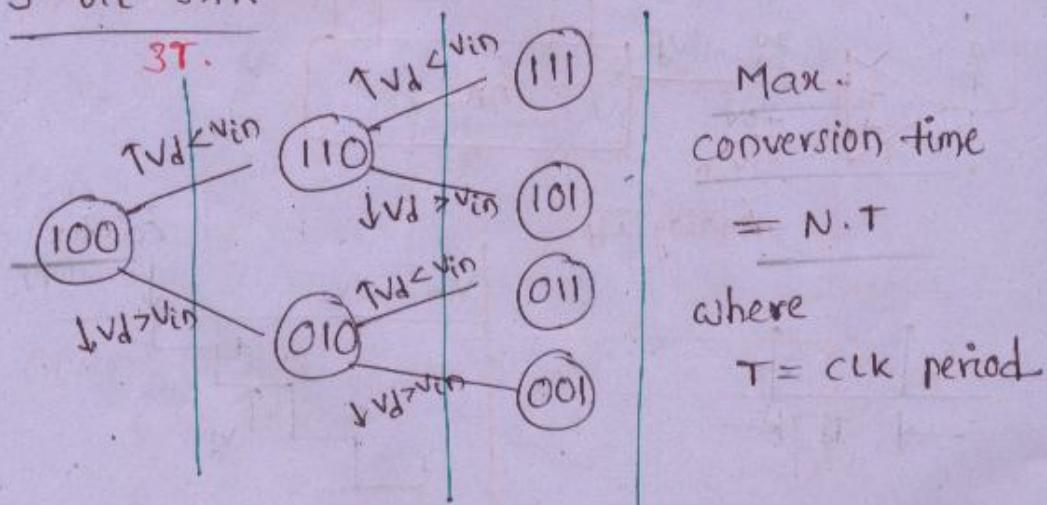
$$T_s \geq \text{Max. conversion time}$$

$\underline{T_s}$ = Sampling period

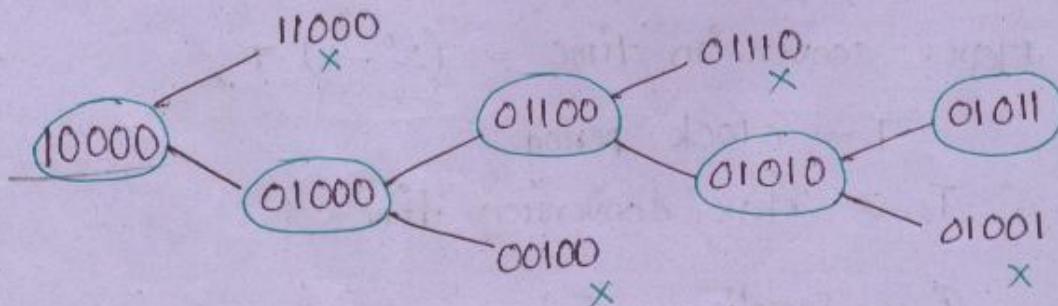
SUCCESSIVE APPROXIMATION ADC :

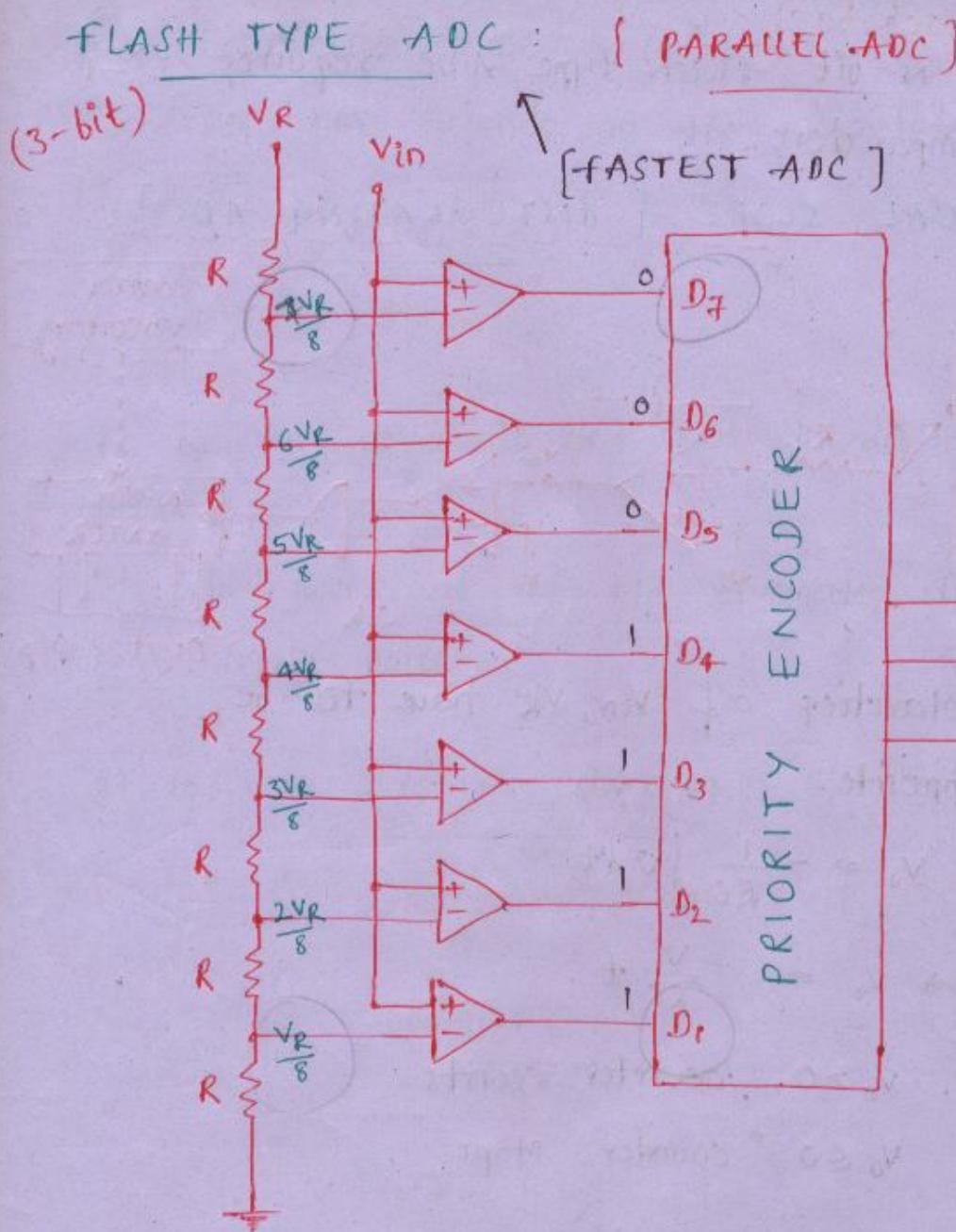


3-bit SAR



e. The final value of a 5-bit SAR is 01011. what are its intermediate values?





$$\text{let } \frac{4VR}{8} < V_{in} < \frac{5VR}{8}$$

\Rightarrow Digital output = 100

$$\text{let } VR = 8$$

$$4V < V_{in} < 5V$$

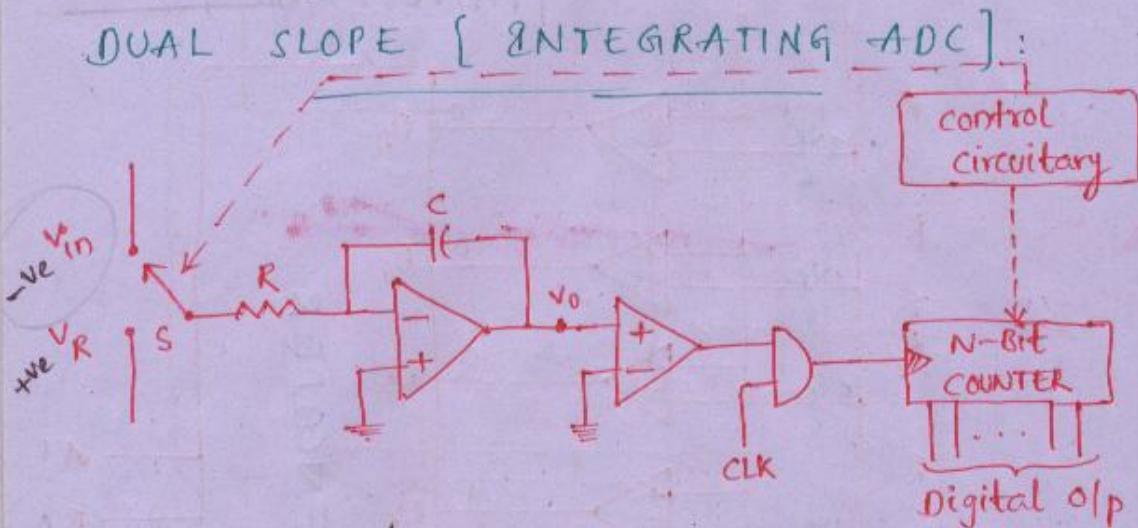
$\rightarrow 100$

$$\text{if } \frac{1.VR}{8} < V_{in} < \frac{2VR}{8}$$

$\rightarrow 001.$

Draw back :-

A N -bit flash type ADC requires $2^N - 1$ comparators.



polarities of V_{in} , V_R have to be opposite.

$$V_o = -\frac{1}{RC} \int v dt$$

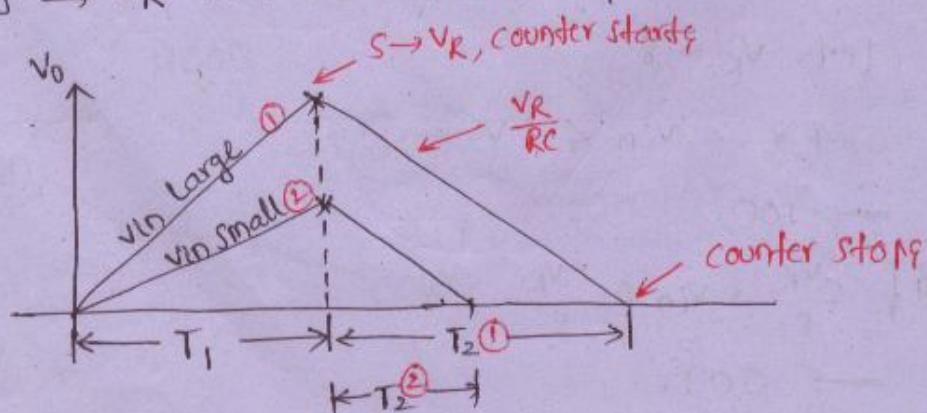
$$\Rightarrow V_o = -\frac{V}{RC} t$$

- (1). $V_o > 0$, counter counts
 $V_o \leq 0$, counter stops.

(2). Control circuitry :

$S \rightarrow V_{in}$ for fixed time ' T_1 '

$S \rightarrow V_R$ and counter starts.



→ Max. conversion time = $(2^N - 1) T$.

Conversion time depends on the magnitude of i/p.

$$T_2 = \frac{|V_{in}| T}{|V_R|}$$

Advantages :

1. It is very accurate and used in digital voltmeters.
2. The integrator at the i/p eliminates the power supply noise.

Draw back :

It is very slow in conversion.

Q. 8-bit ADC, i/p voltage range is -10 to $+10$

Resolution = ?

$$\text{Resolution} = \frac{+10 - (-10)}{2^8}$$

$$= \frac{20}{256}$$

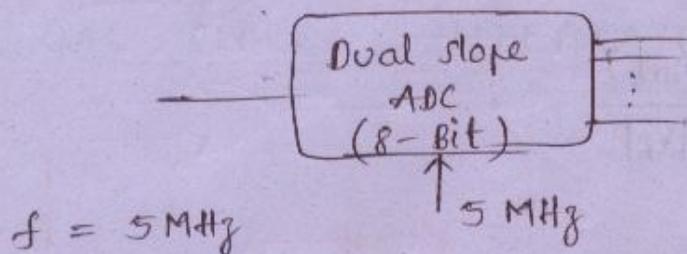
Q. To convert $V_{in} = 5V$ into digital, a SAR ADC takes 10s and dual slope takes 10ns.

Then for $V_{in} = 2.5V$, what is time required.?

$$V_{in} = 2.5V$$

SAR ADC → 10s
Dual ADC → 5s.

c. what is sampling rate of 8 bit dual slope if its CLK freq. is 5 MHz.



$$f = 5 \text{ MHz}$$

$$T = \frac{1}{f} = 0.2 \mu\text{sec}$$

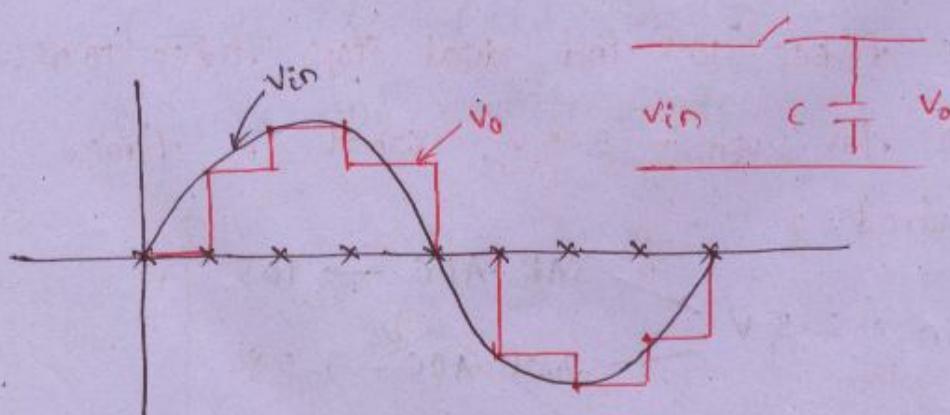
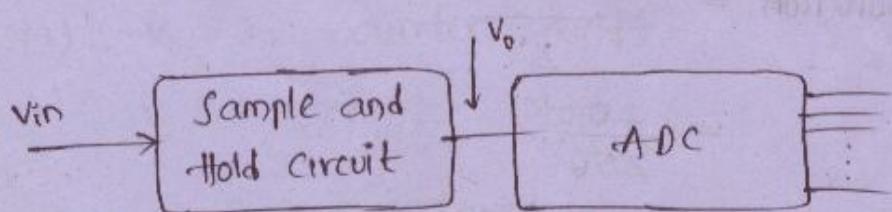
$T_s \geq \text{max conversion time}$

$$\text{ie } T_s \geq (2^8 - 1) T$$

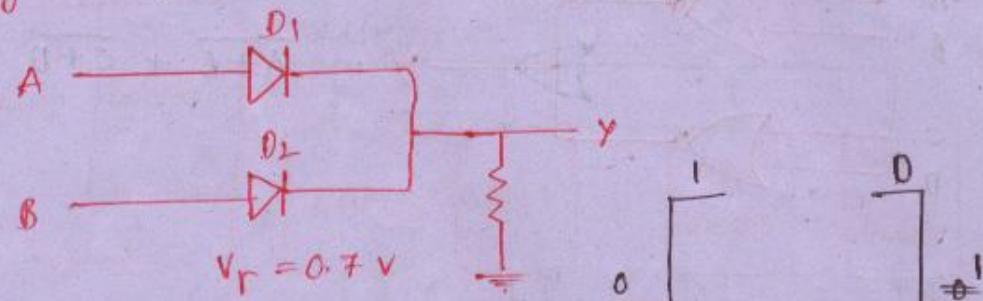
$$T_s \geq (255 * 0.2 \mu\text{sec})$$

$$T_s = 51 \mu\text{sec}$$

$$\begin{aligned} \text{Sampling rate } f_s &= \frac{1}{T_s} \\ &= \frac{1}{51 \mu\text{sec}} \text{ samples/sec.} \end{aligned}$$



Q. Identify the following logic gate in -ve logic - ?



A	B	Y
0	0	0
0	+5	$4.3V \leq 5V$
+5	0	$4.3V \leq 5V$
+5	+5	$4.3V \leq 5V$

+ve logic -ve logic

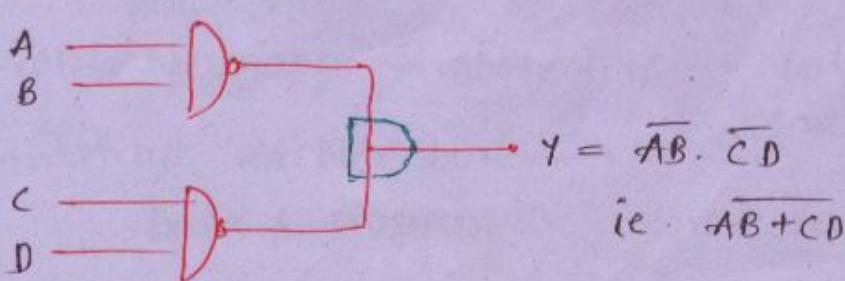
A	B	Y
1	1	1
1	0	0
0	1	0
0	0	0

AND
gate

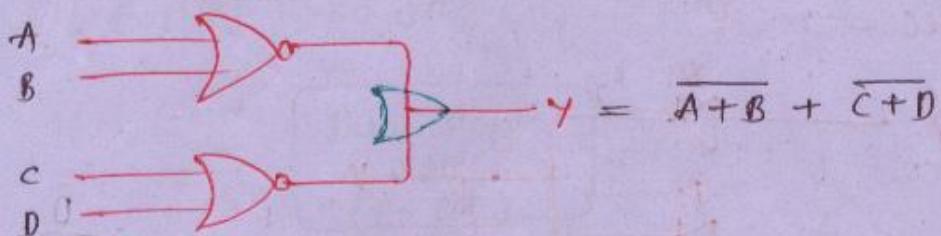
* The OR gate in +ve logic is equal to AND gate in -ve logic.

+ve logic	-ve logic
NAND	NOR
NOR	NAND
Ex-OR	Ex-NOR
Ex-NOR	Ex-OR

WIRED- AND LOGIC :-



WIRED-OR LOGIC:



$$Y = \overline{A+B} + \overline{C+D}$$

	0	1	0	1	0	1	0	1
A	0	1	0	1	0	1	0	1
B	0	0	1	1	0	0	1	1
C	1	1	0	0	1	1	0	0
D	0	0	1	1	0	0	1	1

Output Y is 1 if any one signal is 1 in the AND gate.

It is 0 if all signals are 0.

So, it is called wired OR.

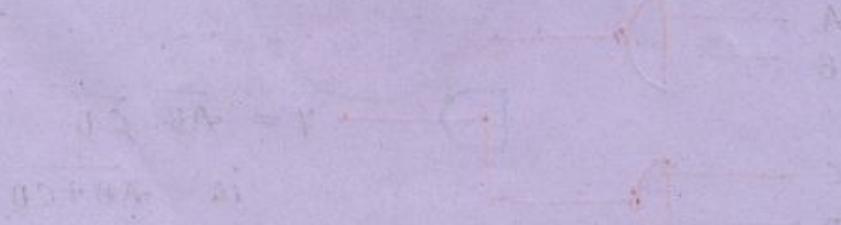
Wired OR = $\overline{A+B} + \overline{C+D}$

Wired OR = $\overline{A+B+C+D}$

Wired OR = $\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}$

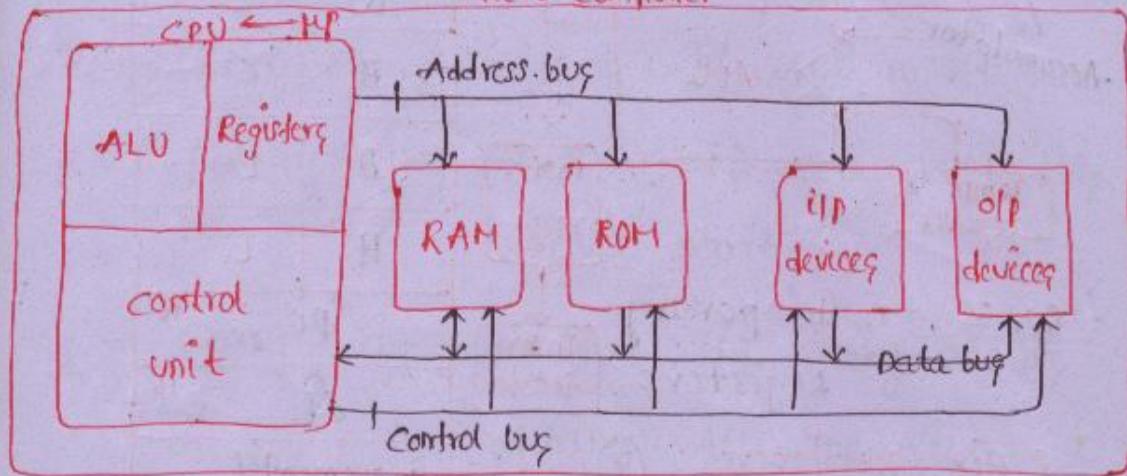
Wired OR = $\overline{A} + \overline{B} + \overline{C} + \overline{D}$

Wired OR = $\overline{A} \cdot \overline{B} \cdot \overline{C} \cdot \overline{D} + \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D + \overline{A} \cdot \overline{B} \cdot C \cdot \overline{D} + \overline{A} \cdot B \cdot \overline{C} \cdot \overline{D}$



MICRO PROCESSORS

Micro computer.



8085 MP :-

(1). 16 Adr. lines → A_0 to A_{15}

$$\begin{aligned}
 \text{Memory capacity} &= 2^{16} \\
 &= 2^6 \cdot 2^{10} \\
 &= 64 \cdot 1 \text{ KB} \\
 &= 64 \text{ kB}.
 \end{aligned}$$

$A_8 - A_{15}$

$A_0 - A_7$

(2). 8 Data lines → D_0 to D_7 .

(3). freq of MP = ~~3.68~~ 3.072 MHz .
(f).

(4). Clock freq 'f',

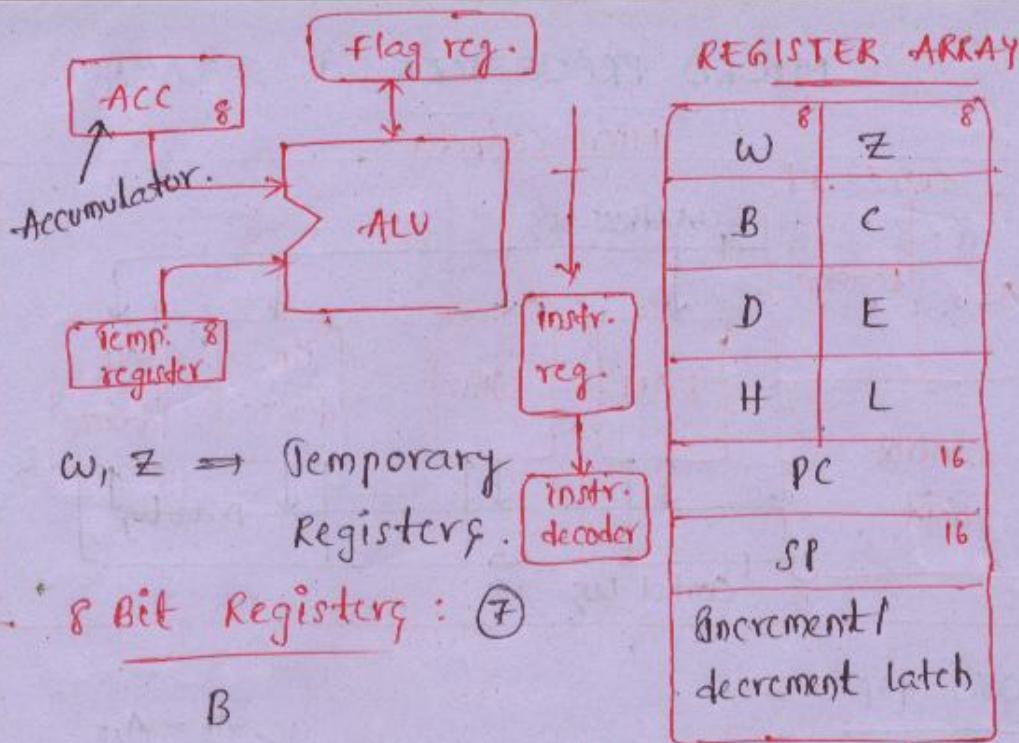
$$\text{clock period } T = \frac{1}{f} = 320 \text{ ns.}$$

'NMOS' Tech :

Von Neumann Architecture → Data & program stored in the same

Harvard Architecture →

Data & program are stored separately



8 Bit Registers : 7

B
C
D
E
H
L
ACC

16 Bit Registers : 3

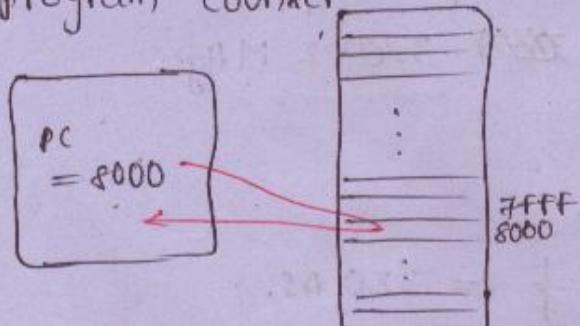
PSW_{16} = program status word
 $= ACC_8 + \text{flag reg}_8$

BC
DE

HL → Memory pointer

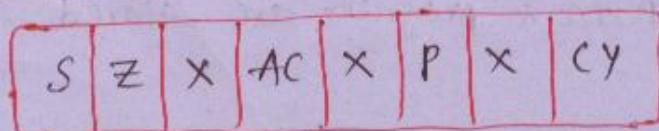
PC :

program counter



It indicates memory location from where MP has to fetch its next instr.

FLAG REGISTER:



S - Sign flag $\Rightarrow S=1$, if MSB of ALU result = 1.

Z - Zero flag $\Rightarrow Z=1$, if ALU result = 0.

P - Parity flag $\Rightarrow P=1$, if ALU result has even parity.

Cy - Carry flag $\Rightarrow Cy=1$, if carry occurs during ALU operations.

AC - Auxiliary carry flag $\Rightarrow AC=1$, if carry occurs from D_3 to D_4 bit.

\hookrightarrow Can't accessed by the programmer.

\rightarrow Used in BCD arithmetic operations.

$$\begin{array}{r}
 & \overbrace{1} & 1 & 0 & | & \overbrace{1} & 1 & 0 & 1 \\
 & | & 1 & 1 & | & | & 1 & 0 & 0 \\
 \hline
 & 1 & 1 & 1 & 1 & | & 1 & 0 & 0 & 1 \\
 \hline
 & 1 & 1 & 1 & 0 & 1 & | & 1 & 0 & 0 & 1 \\
 \hline
 \end{array}$$

$$S=1, P=0, Z=0, Cy=1, AC=1.$$

Over flow flag \rightarrow Signed Addition

$$+ 011 (+3) (\text{or}) \quad 110 (-2) \quad \begin{array}{r} 101 \\ \downarrow 2^3 \end{array}$$

$$+ 010 (+2) \quad 101 (-3) \quad \begin{array}{r} -011 -3 \\ \hline \end{array}$$

$$\hline 101 \quad \hline 011 \leftarrow +ve \text{ number}$$

\uparrow -ve number.

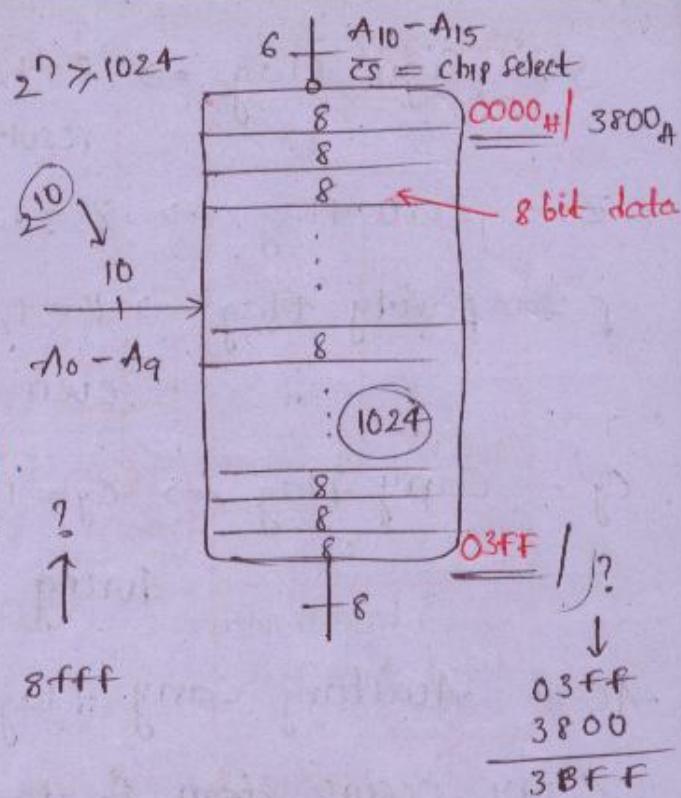
\rightarrow So over flow flag will set in this case.

MEMORY AC's:

$$\begin{aligned} \text{1 KB Memory} \\ = 1024 \times 8 \end{aligned}$$

$$\begin{aligned} 03FF &\leftarrow 16 \text{ bit Address} \\ = 0000 \quad 0011 \quad 1111 \quad 1111 \end{aligned}$$

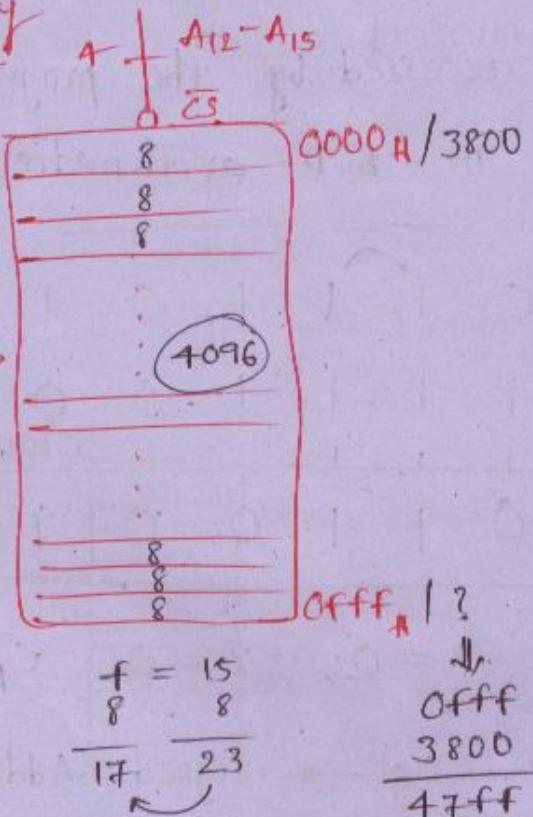
$$\begin{array}{r} 8FFF \\ -03FF \\ \hline 8C00 \end{array}$$



4 KB Memory

$$\begin{aligned} 4 \text{ KB} \\ = 2^2 \cdot 2^{10} \\ = 2^{12} \\ = 4 \times 1024 \times 8 \\ = 4096 \times 8 \end{aligned}$$

$$\begin{array}{l} (2^7 \cdot 4096) \\ n=12 \end{array} \xrightarrow{12} \quad \begin{array}{l} A_0-A_{11} \end{array}$$



e. for a 32 KB memory the ending location address is "AFFF". what is its starting address. ?

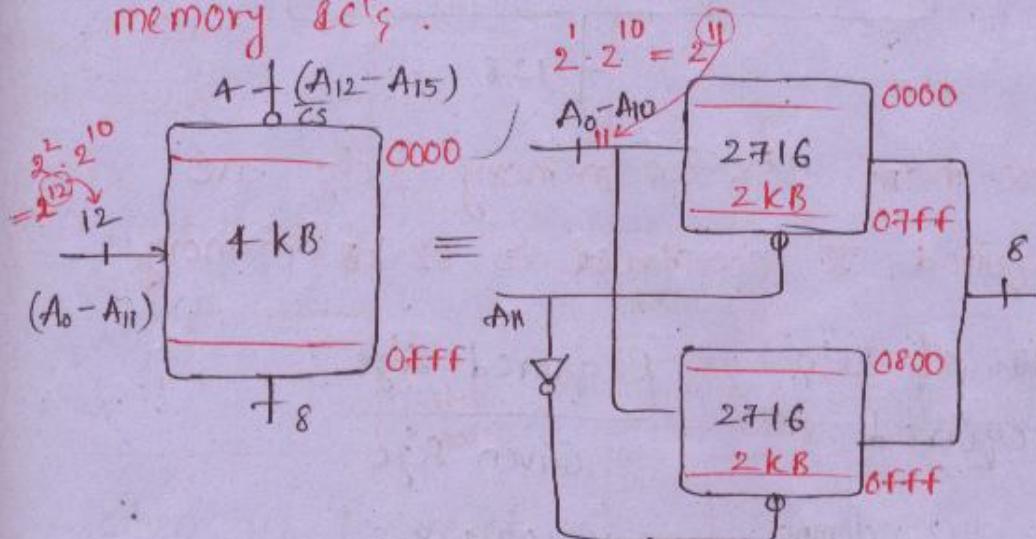
ANS: 3000H

EPROM

<u>2716</u>	= 2 kB	\leftarrow	<u>RAM</u>
<u>2732</u>	= 4 kB	\leftarrow	6132
<u>2764</u>	= 8 kB	\leftarrow	6164
<u>27128</u>	= 16 kB	\leftarrow	61128

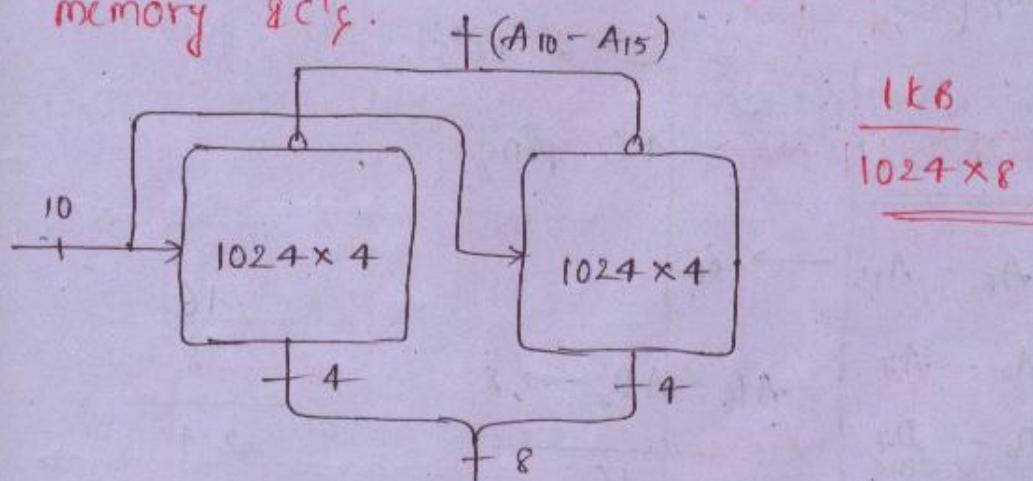
Q. Construct a 4 kB memory using 2716

memory 8C's.



Q. Construct a 1 kB memory using 1024x4

memory 8C's.

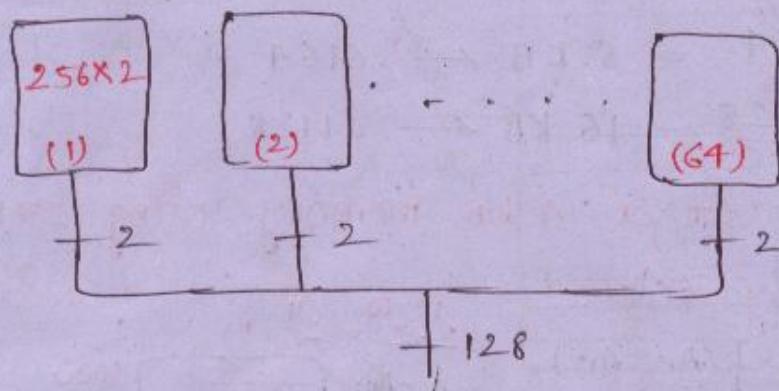


Q. The add. lines of 64 memory 8C having capacity of 256×2 are connected together. What is the size of resulting memory?

64 Memory 8C's.

$$256 \times 2$$

$$256 \times (64 \times 2) \\ = \underline{256 \times 128}$$



Q. How many 256×4 memory 8C's are required to construct a 32 kB memory.

$$\text{No. of 8C's required} = \frac{\text{Required size}}{\text{Given size}}$$

$$128 \text{ rows} \left\{ \begin{array}{c} \text{2 columns} \\ \boxed{\text{ }} \end{array} \right. = \frac{32 \times 1024 \times 8}{256 \times 4} \\ = 256 \text{ 8C's.}$$

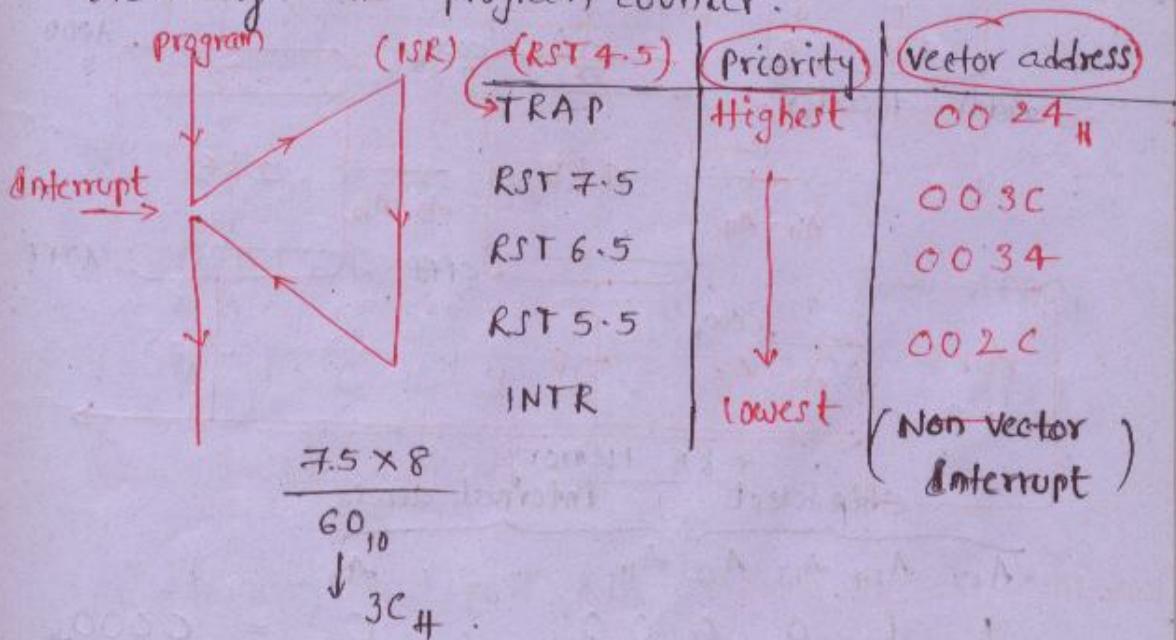
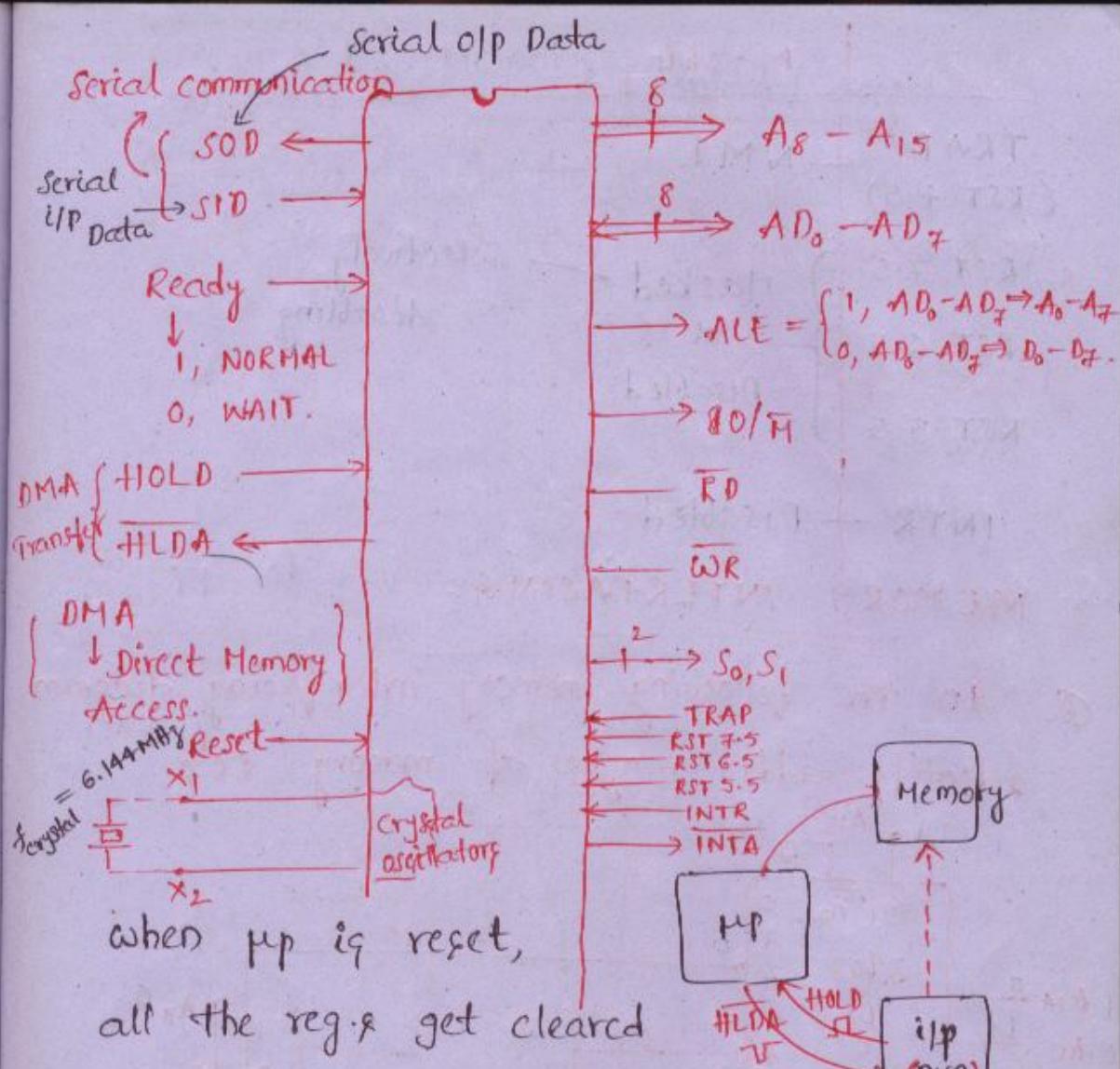
8085 CPU \rightarrow 10 pins

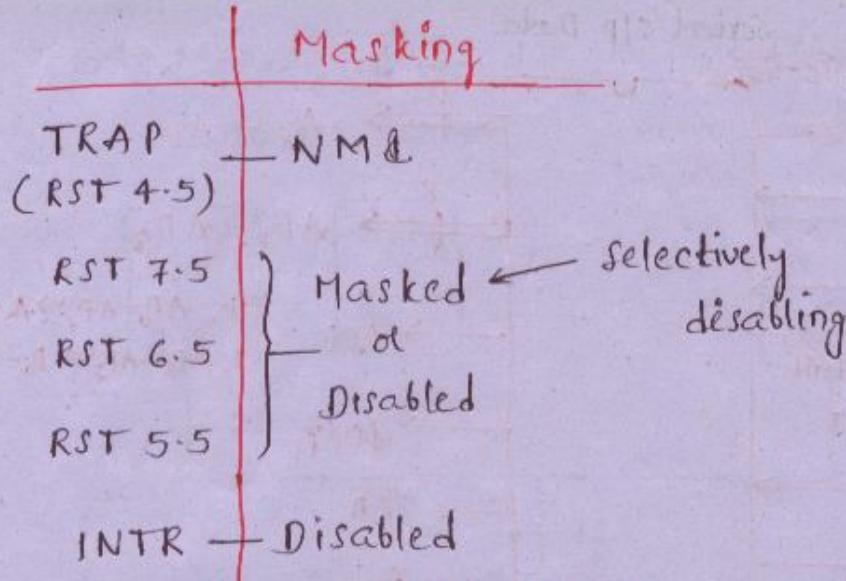
$$A_8 - A_{15} \rightarrow 8$$

$$A_0 - A_7 \\ D_0 - D_7 \} \quad AD_8 - AD_7 \rightarrow 8$$

$$\begin{array}{r} 16 \\ 8 \\ \hline 24 \end{array}$$

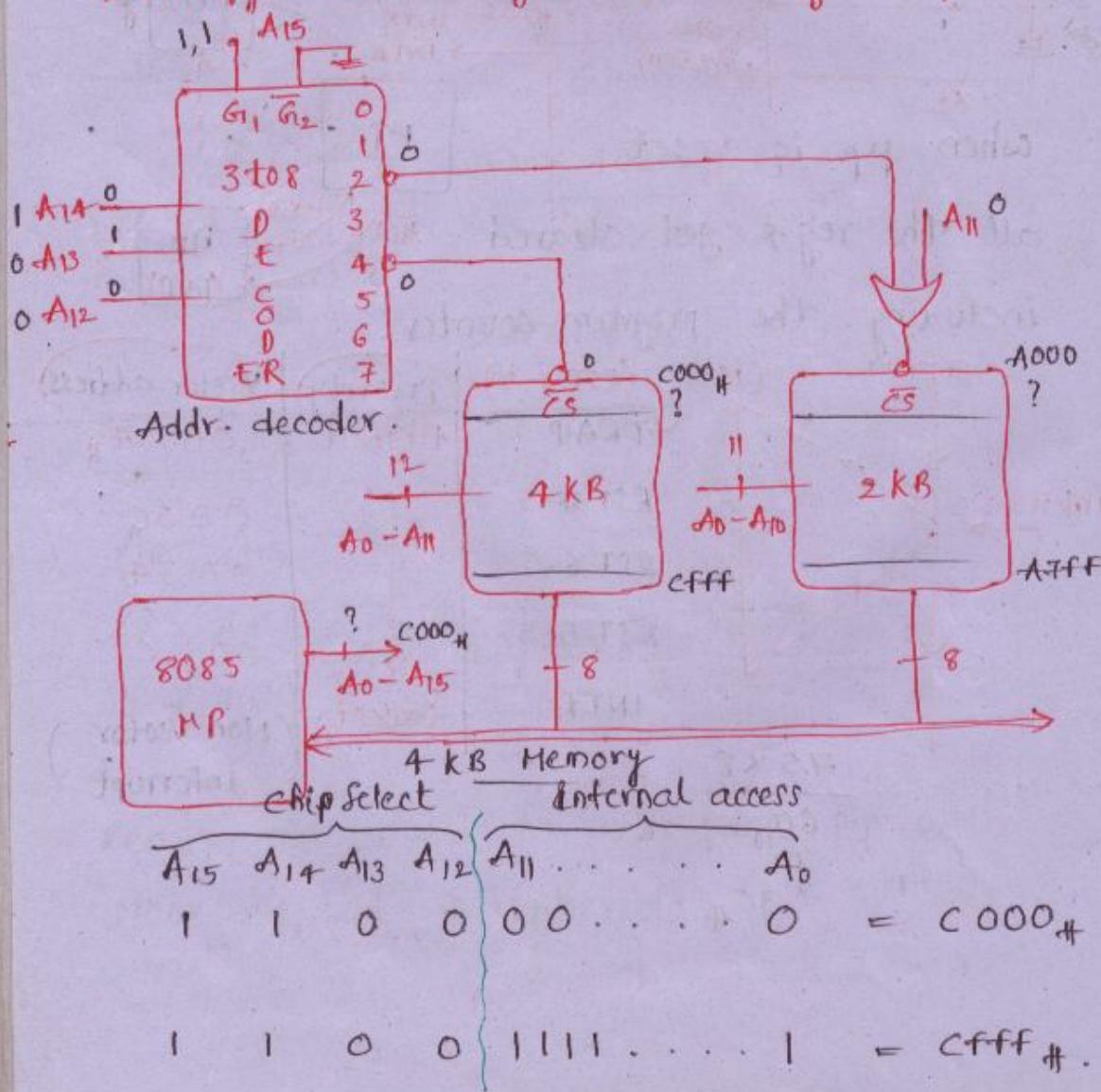
* Ready pin used to interface CPU with slow speed peripherals.

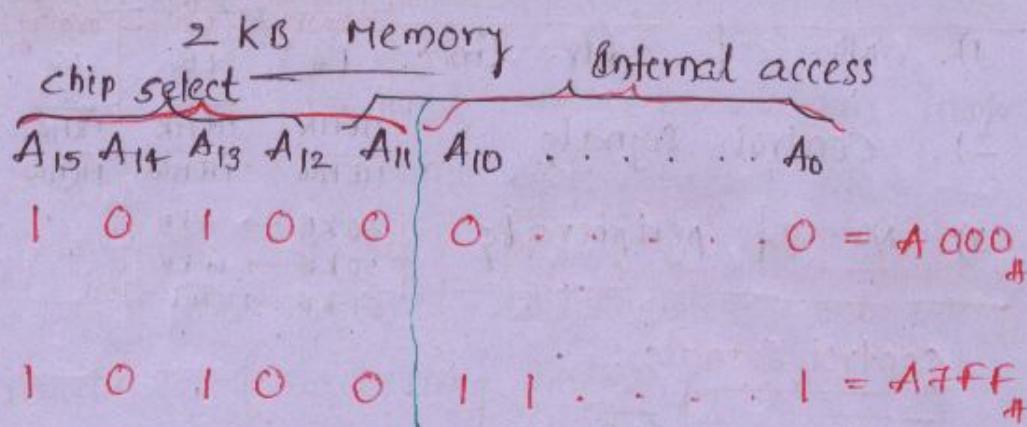




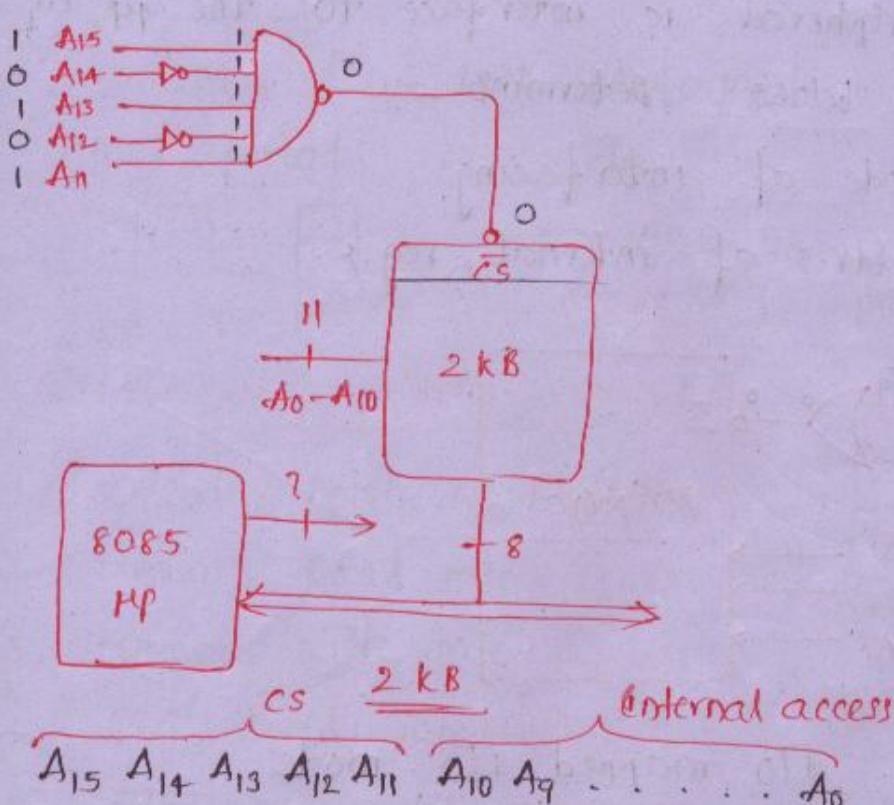
MEMORY INTERFACING:

- Q In the following memory interfacing diagram identify addr. ranges of memory & Cfg.





Q.



$1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \dots 0 = A_{800}_{\text{H}}$

$1 \ 0 \ 1 \ 0 \ 1 \ 1 \ 1 \dots 1 = A_{FFF}_{\text{H}}$

I/O INTERFACING:

1. Memory mapped I/O → I/O devices are considered as memory I/C.
2. I/O mapped I/O. → I/O & Memory are considered separately.

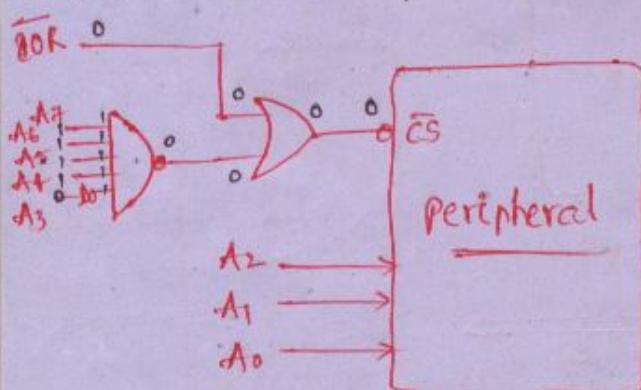
	Memory mapped		I/O mapped	
1). NO. of addr. Lines	16 MEHR MEHW	16 MEHR MEHW	16 MEHR MEHW	8 IOR IOW
2). Control signals				
3). NO. of peripherals		60 kB → 4 kB 50 kB → 14 kB 64 kB → NIL		$2^8 = 256$ 8 I/O devices

control signals:

MEHR IOR
MEHW IOW

Q. A peripheral is interface to the CPU as shown below. Determine —

- (1). Mode of interfacing
- (2). Addr.s of internal reg.s



Ans: (1). I/O mapped I/O mode.

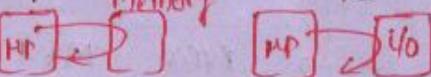
(2). peripheral has 8 reg.s, peripheral is I/O device b'coz

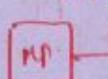
	A ₇	A ₆	A ₅	A ₄	A ₃	A ₂	A ₁	A ₀	= f0
R ₁	1	1	1	1	0	0	0	0	
R ₂									
R ₃									
R ₄									
R ₅									
R ₆									
R ₇									
R ₈	1	1	1	0	1	1	1		= f7

INSTRUCTION CYCLE :

Time required to execute an instr.

Range : 1 machine cycle to 5 m/c.

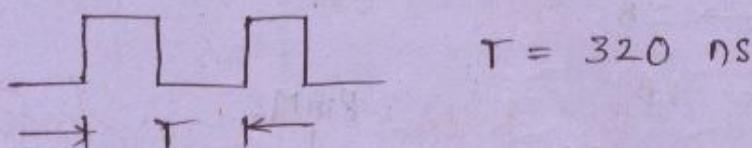
MACHINE CYCLE : 

 Time required to complete one operation of accessing memory, accessing I/O devices & sending an acknowledgement.

Range : 3 T states to 6T.

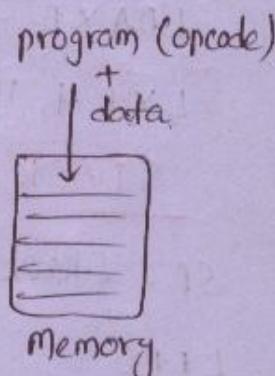
T - STATE :

It is sub task performed in one clock period.



TYPES OF MACHINE CYCLES:

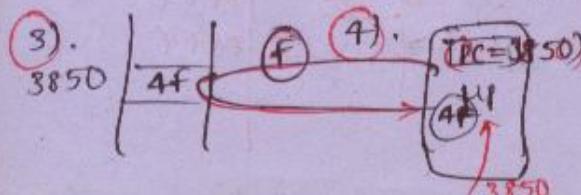
1. Opcode fetch m/c → 4T
2. Memory Read m/c → 3T
3. Memory write m/c → 3T
4. I/O Read m/c → 3T
5. I/O write m/c → 3T
6. Hold ACK m/c
7. Interrupt ACK m/c



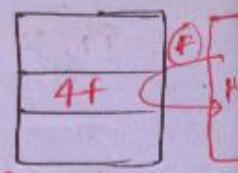
decoding ↓

Opcode fetch m/c → 4T = 3T + 1T

①. MOV C, A → ②. opcode ↑ fetching
 $= 01001111_2 = 4F_H$

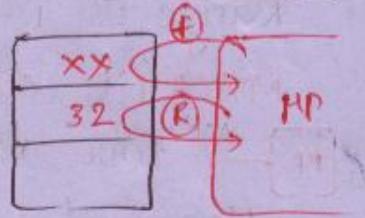


1 - Byte Instruction : $\rightarrow \text{MOV C, A}$



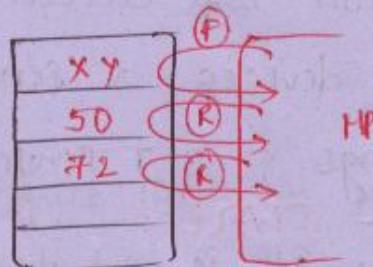
2 - Byte Instruction :

$\rightarrow \text{MOV} \quad \underline{\text{MV}} \quad \underline{\text{C, 32}}$
let xx



3 - Byte Instruction :

$\rightarrow \text{LDA} \quad \underline{\text{F250}}$
xy



XTHL \rightarrow 1B

ANI F2 \rightarrow 2B

LDA XB \rightarrow 1B

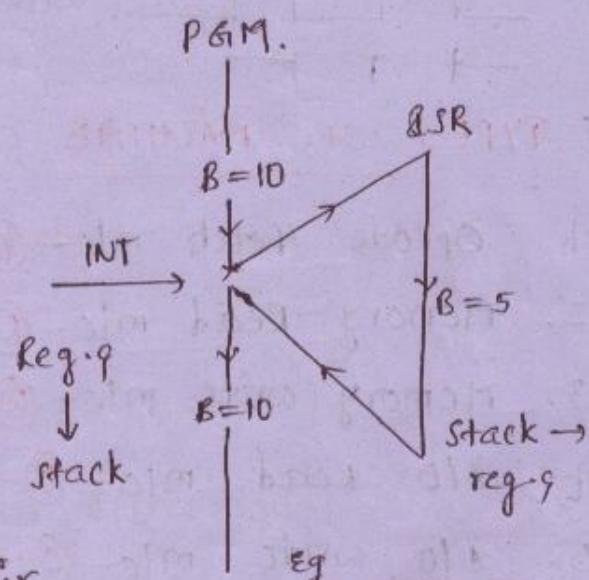
LXI H, 1122 \rightarrow 3B

STACK:

SP : Stack pointer

LIFO :

Last in first out



(1). PUSH R.P

↑ reg. pair

Eg PUSH B

decrement sp + push higher reg.

PUSH D

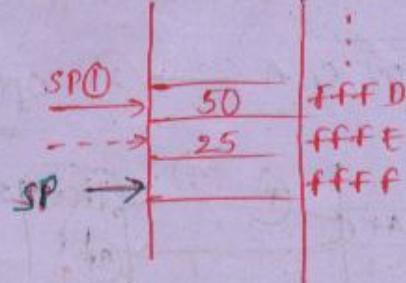
decrement sp + push lower reg.

PUSH H

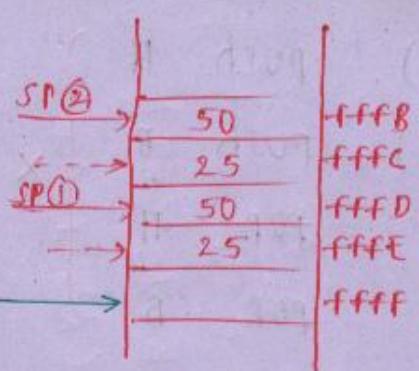
Eg: Let SP = ffff

HL = 2550

1). PUSH H $\frac{SP}{ffff}$



2). **PUSH H** 1000 $\frac{SP}{FFFFB}$

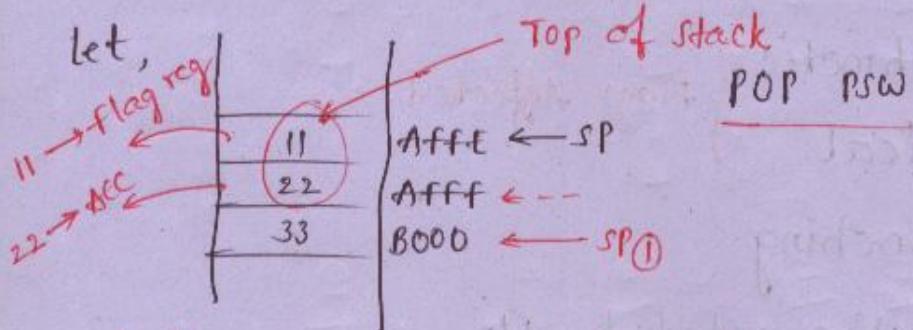


POP & P:

S1

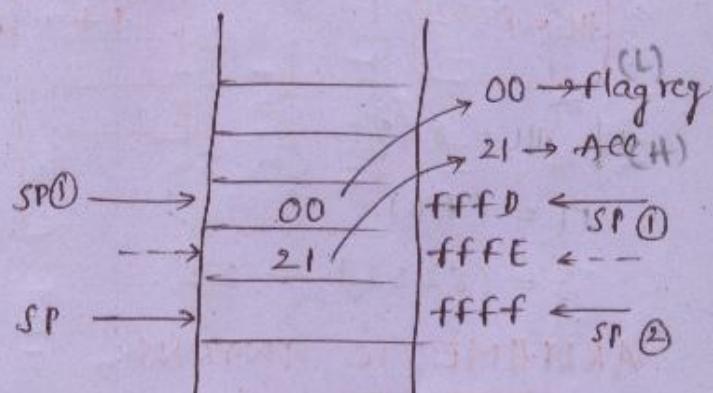
Get 1 Byte into lower reg + increment sp

Get 1 byte into higher reg + increment sp



Q what are the contents of acc & flag reg after executing following instruction.

- (i). $SP = ffff$ (ii). $push \#L$
 $L = 2100$ (iii). $pop PSW$



The above program is used to clear the flag register.

1). push H
 2) push B
 POP H
 POP B } X

2). push H
 push B
 POP B
 POP H } ✓

INSTRUCTIONS :

1. Data Transfer
2. Arithmetic } Flags Affected.
3. Logical
4. Branching
5. Machine related, 810
6. Additional

NP
 BC = 8250
 HL = 8252

$$\text{if } \text{HL} = 8252$$

$$M = (\text{HL})$$

$$= (8252) = 22.$$

Memory	
data	Addr.
11	8250 $(BC) = (8250)$
22	8251 = 11
33	8252 $(HL) = (8252)$
44	8253 = 33.
	$M = (HL) = 33.$

ARITHMETIC INTRNS:

MLC → get the instr + operation

⑧ 1 + 0
 ADD FF 2 + 1
 47 1 + 0

Instruction Operation Byteq/MC/1R Types of MC Flags affected

1). ADD R \rightarrow $R, C, Z, N, H, L, A.$ $A + R \rightarrow A$ 1/1/4 f

ADD M $A + (HL) \rightarrow A$ 1/2/7 f, R

ADD 8bit data $A + (8\text{bit data}) \rightarrow A$ 2/2/7 f, R

2). SUB R

SUB H

SUB 8bit data

3). ADC R $CY + A + R \rightarrow A$ 1/1/4

ADC M $CY + A + (HL) \rightarrow A$ 1/2/7

ADC 8bit data $CY^A + (8\text{bit data}) \rightarrow A$ 2/2/7

4). SBB R

SBB H $A - (HL) - CY \rightarrow A$ 1/2/7

SBB 8bit data

5). INR R
DCR R

1/1/4
1/1/4

INR M
DCR M

1/2/10
1/2/10

(HL)+1 → (HL)
(HL)-1 → (HL)

1/1/6
1/1/6

S = Opcode fetch mle (6+)
B = Bus idle mle (3+) flags

ALL
F
F

ALL
F, R, W
F, R, W

F, R, W
F, R, W

1/3/10

only 'C₁' flag.

ALL
but C₁ = 0.

6). DAD RP
ORI 8bit data

1/1/4
1/1/4

INR B
B = 83

1/1/4
1/1/4

8N× B
B = 8251.

1/1/6
1/1/6

ALL
F
F

ALL
F, R, W
F, R, W

F, R, W
F, R, W

1/3/10

only 'C₁' flag.

LOGICAL INSTRUCTIONS:

- 1). ORA R
ORA M
ORI 8bit data

A + R → A
A + (HL) → A
A + 8bit data → A

2). ANA R
~~OR = X~~
~~AND = X~~
~~XNOR = X~~
~~XNAND = X~~

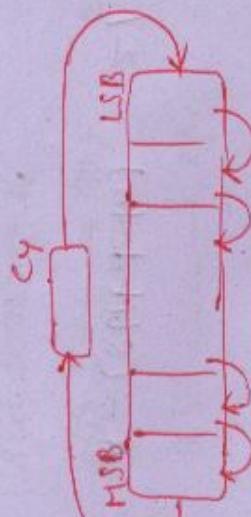
ANA H
~~OR = X~~
~~AND = X~~
~~XNOR = X~~

XRA R
~~OR = X~~
~~AND = X~~

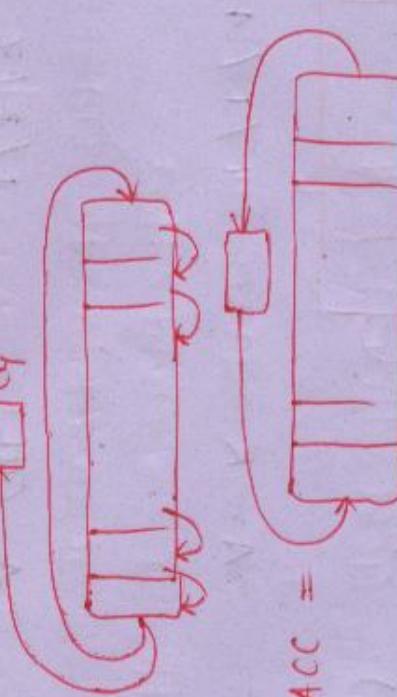
XRA H
~~OR = X~~
~~AND = X~~

A \oplus R \rightarrow A

XRL 8bit data
 4). RAL (with c_q) ACC =



RLC (without c_q)



RAR (with c_q) ACC =

Affect only
 c_q.



RRC
(without C_q)

$A_{CC} =$

$$C_q \cdot R \quad A - R \quad 1 / 1 / 4 \quad + \begin{cases} A > R \\ A < R \\ A = R \end{cases}$$

$C_q = 0, \quad Z = 0$

$C_q = 1, \quad Z = 0$

$C_q = 0, \quad Z = 1$

$S, P, AC \text{ affected}$

$$CRRH \quad A - (\#L) \quad 1 / 2 / 7$$

$$f, R \quad \begin{cases} -d0 - \\ exp \cdot R \rightarrow (\#L) \end{cases}$$

$$CR8 \quad A - (8 \text{ bit data}) \quad 2 / 2 / 7$$

$$f, R \quad \begin{cases} -d0 - \\ \end{cases}$$

$$A = f_2 \quad \boxed{\begin{array}{c} \text{AND OR} \\ \text{Masking} \end{array}}$$

$$A = \frac{1111\ 0000}{0000\ 1111} \quad \underline{02}$$

No flags
only ' C_q '

$\overline{A} \rightarrow A$
 $C_q \rightarrow C_q$
 $C_{CC} = 1$

CMA
 CRC
 STR

DATA TRANSFER INSTR.S:

- 1). $MOV R_d, R_s \quad R_s \rightarrow R_d$
 $MOV R, M \quad (HL) \rightarrow R$
 $MOV M, R \quad R \rightarrow (HL)$

$MV\# R, 8bit\text{ data} \quad 8bit\text{ data} \rightarrow R$ $fR \omega$
 $\uparrow 2+1$

$MUL M, 8bit\text{ data} \quad 8bit\text{ data} \rightarrow (HL)$ $2/3/10$
 $\uparrow fR_3+0$
- 2). $L\times 8 \quad 8P, 16bit\text{ data} \rightarrow 8P$
 $(Load \text{ immediate})$ $3/3/10$ f, R, R
- 3). $LDA \quad 16bit\text{ address} \quad (16bit\text{ addr.}) \rightarrow A$ $3/4/13$ $fRR R$
 $(Load \text{ Accumulator})$
- 4). $STA \quad 16bit\text{ address} \quad A \rightarrow (16bit\text{ addr.})$ $3/4/13$ $fRR \omega$
 $(Store \text{ Accumulator})$
- 5). $LDX \quad 8P \quad (8P) \rightarrow A$ $1/2/7$ fR
 $STAX \quad 8P \quad A \rightarrow (8P)$ $1/2/7$ $f\omega$

- 6). LHD 16 bit addr. (16 bit addr.) \rightarrow L
 (16 bit addr + 1) \rightarrow H
 \downarrow
 fRR RR
 ie (8252) \rightarrow L
 (8253) \rightarrow H
- LHD 16 bit addr. L \rightarrow (16 bit addr.)
 H \rightarrow (16 bit addr. + 1)
- SHL D 16 bit addr.
 \downarrow
 SHD 8254, (H) = 8090
- | | |
|----|------|
| 44 | 8253 |
| 10 | 8254 |
| 60 | 8255 |

