Problem Session 1 Problems

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This is a collection of problems that are starting points for discussion in a problem session.

Question 1. Write down an example of the following:

(a) A linear, first order differential equation

(b) A nonlinear, first order differential equation

(c) A linear, higher order differential equation

(d) A nonlinear, higher order differential equation

Question 2. The equation $\frac{dy}{dx} = y^4$ has a solution of the form $y(x) = ax^b$ for some real numbers a and b. Find a and b.

Is the same true of $\frac{dy}{dx} = y^3$?

Question 3. The equation $\frac{dy}{dx} = y$ has all solutions of the from $y(x) = Ce^x$ for some constant C. Can you find all solutions to $\frac{dy}{dx} = 3y$? $\frac{dy}{dx} = ky$ for any constant k?

Question 4. Find some solutions to $\frac{d^2y}{dx^2} + y = 0$. If I tell you that I'm thinking of a solution to this equation that has $y(0) = 1$, can you tell me the solution I'm thinking of? If so, what is it? If not, what else would you need to know?

Question 5. Consider the equation $\frac{dy}{dx} = \cos(y)$. If $y(0) = 1$, what is the smallest upper bound you can give for $y(t)$?

Question 6. Consider the differential equation $\frac{dy}{dx} = f(x)$ for some continuous function $f : \mathbb{R} \to \mathbb{R}$. What is the solution to this equation?