

Problem Session 2 Problems

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This is a collection of problems that are starting points for discussion in a problem session.

Question 1. Write down an example of a first-order separable differential equation as well as an example of a first-order exact differential equation. Can you get a two-in-one?

Question 2. Write down a differential equation that is satisfied by $y(x) = \pm\sqrt{C^2 - x^2}$ for any $C \in \mathbb{R}$ and either choice of sign.

Hint: There are many ways to do this, but the one that is in my opinion simplest is to think about what the relationship between x and $y(x)$ is.

Question 3. Sometimes, first-order equations that only depend on y come up (e.g. $\frac{dy}{dx} = y$). Do these equations fit into one of the types of differential equation that we've discussed already?

Question 4. Let's think more about integrating factors, and why they are what they are. Consider the equation

$$\frac{dy}{dx} + p(x)y = q(x). \tag{1}$$

Assume that we want $\mu(x)$ to be our integrating factor.

- (a) Compute $\frac{d}{dx}(y\mu)$.
- (b) Multiply the differential equation 1 by μ and equate the left hand side to what you got above.
- (c) You should now be able to reduce this to a differential equation for μ ; do this and solve the equation.

Question 5. Solve the equation $x\frac{dy}{dx} = ay$ where $a \in \mathbb{R}$ is an arbitrary constant.