FiveThirtyEight's September 11, 2020 Riddler

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Today's riddler is about the Tour de FiveThirtyEight:

Question 1. At the moment, you are racing against three other riders up a mountain. The first rider over the top gets 5 points, the second rider gets 3, the third rider gets 2, and the fourth rider gets 1.

All four of you are of equal ability - that is, under normal circumstances, you all have an equal chance of reaching the summit first. But there's a catch - two of your competitors are on the same team. Teammates are able to work together, drafting and setting a tempo up the mountain. Whichever teammate happens to be slower on the climb will get a boost from their faster teammate, and the two of them will both reach the summit at the faster teammate's time.

As a lone rider, the odds may be stacked against you. In your quest for the polka dot jersey, how many points can you expect to win on this mountain, on average?

Let's call the riders A, B, C1, and C2, with A being you. There are 24 possible finishes for the four racers, but since distinguishing between C1 and C2 is irrelevant, there are only 12 relevant orders. I will list them below together with the number of points you get:

- ABCC-5
- ACBC-5
- ACCB-5
- BACC 3
- CABC-2
- CACB 2
- BCAC 1
- CBAC-1
- CCAB-2

- BCCA 1
- CBCA 1
- CCBA 1

One gets a total score of 29, so the average result is $\frac{29}{12}$.