Problem Session 2 Solutions

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1. An example of a separable differential equation is:

$$\frac{dy}{dt} = y^2(t^2+1)$$

An example of an exact differential equation is:

$$y + x\frac{dy}{dx} = 0$$

This is also separable, because it can be written as $\frac{dy}{y} = -\frac{dx}{x}$. The equation that we use to see that this is exact is $\Psi = xy$.

2. This is an example of moving backwards and will require us to remind ourselves of implicit differentiation. We're asked to find a DE satisfied by $y = \pm\sqrt{C^2 - x^2}$, a simple way of deriving one is as follows, square both sides:

$$y^2 = C^2 - x^2$$

Differentiate with respect to x,

$$y\frac{dy}{dx} = -2x$$

and then we can write that in standard form as:

$$y\frac{dy}{dx} + 2x = 0$$

3. This type of equation (which is called autonomous) in one variable is always separable, we see this as follows, let f be any nice function,

$$\frac{dy}{dx} = f(y)$$

$$\frac{dy}{f(y)} = dx$$

Thus it will be separable, with appropriate notice about the case where $f(y) = 0$.

4. This question is **excellent studying material** and I recommend everybody reading this go through it themselves, for the first part we get:

$$\frac{d(y\mu)}{dx} = y\frac{d\mu}{dx} + \mu\frac{dy}{dx}$$

by the product rule.
Equating this with the left-hand side of our equation multiplied by μ we get,

$$y\frac{d\mu}{dx} + \mu\frac{dy}{dx} = \mu\frac{dy}{dx} + p(x)\mu y$$

And finally this gives us the following differential equation for μ:

$$\frac{d\mu}{dx} = p(x)\mu$$

5. The problem given is a simple separation of variables problem. We get:

$$\frac{dy}{y} = a\frac{dx}{x}$$

After sufficient constant shuffling around we get:

$$\log(|y|) = a\log(|x|) + c$$

$$y = k|x|^a$$

for $k \in \mathbb{R}$. The absolute value only matters for $|a| < 1$.