# Homework 2

MATH 218: Differential Equations for Engineers

Due: October $9^{th}$ at 11:59 PM EST

This homework is primarialy based on material from unit 3.

**Question 1** (6 points)**.**

(a) Assume that you have a function $y_1(x)$ that is a solution to $y'' + f(x)y' + g(x)y = h_1(x)$ and a function $y_2(x)$ that is a solution to $y'' + f(x)y' + g(x)y = h_2(x)$. Show that $y_1 + y_2$ is a solution to $y'' + f(x)y' + g(x)y = h_1(x) + h_2(x)$.

(b) Explain how this proves the theorem for non-homogeneous equations.

**Question 2** (3 points)**.** Explain why the method of undetermined coefficients will always work to find a particular solution for an equation of the form $y'' + ay' + b = p(t)$ where $p(t)$ is a polynomial.

**Question 3** (5 points)**.** For which choices of $a, b \in \mathbb{R}$ does one have that $\lim_{t\to\infty} y(t) = 0$ for *all* solutions to $y'' + ay' + by = 0$?

**Question 4** (10 points)**.** In principle, everything discussed in this unit applies to any order of differential equation. Interestingly, this gives an alternative method of solving some first-order linear equations: instead of finding an integrating factor, one can find a particular solution by undetermined coefficients and then find all solutions by using the theorem for non-homogeneous equations. Use this technique to find all solutions to $y' + 2y = 2t^2$. I suggest you think about whether this was easier or harder than the integrating factor method, and just so you know you can use either method if it comes up in the future.

**Question 5** (8 points)**.** Continuing on with the theme of the previous question, come up with initial guesses for undetermined coefficients for the following equations. N.B.: you don't need to solve them. For example on part c) you cound submit something like $y(t) = At^2 + B\ln(t)$ (although that isn't close to correct).

(a) $y' - y = \sin(t)$

(b) $y'' - y' - 2y = t^2$

(c) $y'' - y = e^{2t}$

(d) $y^{(3)} - y' + y = 2t^2 + t$

(e) $y'' + 4y = \cos(2t)$

(f) $y'' - y' - 2y = e^{2t}$

(g) $y^{(4)} + 2y'' + y = \sin(t) + \cos(t)$

(h) $y^{(3)} - 3y'' + 3y' - y = e^t$

**Question 6** (5 points)**.** Assume you have a differential equation of the form $a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = 0$. Find all solutions to this equation.

*N.B.*: Your answer should be of the form "Here are $n$ solutions $y_1(x), \ldots, y_n(x)$ and every solution is of the form $C_1 y_1(x) + \cdots + C_n y_n(x)$."

**Question 7** (10 points)**.** Differential equations like the one $y'' + 4y' + 5y = \alpha \cos(\omega t)$ will come up in the next unit. Find all solutions to this eqaution. Additionally, explain why all solutions to this equation oscillate sinusoidally as $t \to \infty$. Finally, compute the frequency and amplitude of the oscillations.