

Continuously observing the spectrum of a dynamically decoupled spin-1 quantum gas

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Quantum states and spectra can be made sensitive to a particular measurand whilst simultaneously impervious to parasitic fluctuations of an environment. Here we use an atom-light interface with minimal backaction to probe the spectrum of a radiofrequency-dressed spin-1 quantum gas continuously and in-situ. The dressing amplitude sets the radiofrequency band in which oscillating magnetic fields manifest a linear measurand, and we probe the energy spectrum while the system evolves unitarily. By varying a symmetry-breaking parameter of the Hamiltonian, we find a regime in which two of the dressed states are maximally insensitive (up to fourth-order) in magnetic field fluctuations that are slow compared to the dressed-state splittings. Moreover, we demonstrate the predictive power of our continuous probe to optimize the dynamical decoupling and tune the measurement band. This robust system shares the useful hallmarks of quantum metrology platforms; the states are thus termed “synthetic clock” states in a complementary result by Lundblad *et al.* (arXiv:1706.xxxx) and are candidates for band-tunable magnetometry and emulation of quantum magnetism in solid-state systems.

From Hahn echoes to contemporary dynamical decoupling, abrupt, discrete rotations have been used to protect spin superpositions from inhomogeneities and parasitic fluctuations, prolonging quantum coherence and circumventing deleterious energy shifts [nmr,qi,ions]. A complementary strategy is to replace the pulse train with an uninterrupted coupling of ‘bare’ spin states, thus admitting new ‘dressed’ spin eigenstates, with a modified quantization direction, spectrum, and coupling, which too are protected from unwanted artifacts of their environment [1]. This *continuous* dynamical decoupling (CoDD) has proven useful across multiple platforms including nitrogen-vacancy centers [2–5] and superconducting qubits, and forms the basis for creating protected qubit [6] and decoherence-free [7] subspaces. Weak continuous measurement is a powerful instrument for appraising and refining dynamical decoupling in real time; to probe stochastic evolution, improve metrological bandwidth, or realize quantum feedback schemes [8]. Here we use minimally perturbative [dispersive] optical readout of a ‘bare’ spin component \hat{F}_x to measure the spectrum of a continuously decoupled spin-1 quantum gas in real-time. The rich time-frequency domain data reveal not only multiple dressed-state splittings and their relative immunity to noise, but also dressed state coherences and coupling strengths. This potent ability to estimate the eigenspectrum of a multi-level dressed system reveals feature absent in spin-1/2 or composite spin-1/2 qubit systems; principally, we identify a regime in which a subspace of the dressed system is maximally decoupled from noise $\propto \hat{F}_z$. This protected subspace is spanned by two of the dressed states, termed ‘synthetic clock states’ in a co-submission by Lundblad *et al.* (arXiv:1706.xxxx). The low-frequency magnetic stability and high-bandwidth detection of these states is immediately applicable to band-tunable (ac) magnetometry [2, 3, 9] and experiments preparing delicate spin-entangled many-body states [10];

whereas the unconventional cyclic coupling of all $2F + 1$ dressed states could be applied to emulation of frustrated quantum spin chains [11].

Atomic Zeeman states $|m_z = -1, 0, 1\rangle$ in a magnetic field $B_z \mathbf{e}_z$ can be decoupled from fluctuations in B_z by applying a radiofrequency (rf) field perpendicular to the quantization direction z , of amplitude $B_{\text{rf}} \mathbf{e}_x$, and oscillating near the splitting of the ‘bare’ Zeeman states $\omega_L \equiv (E_{m_z=+1} - E_{m_z=-1})/2\hbar$, the Larmor frequency. At low magnetic fields, the degeneracy of the composite spin-1/2 systems [12] renders the spin-1 behavior identical to CoDD in spin-1/2 qubits. The spin is quantized along x in a frame rotating with the radiofrequency ω_{rf} , i.e. the eigenstates of $\mathcal{H}_{\text{rwa}} = \Delta \hat{F}_z + \Omega \hat{F}_x$ are $|m_x = -1, 0, 1\rangle$, with energies $m_x \hbar \sqrt{\Omega^2 + \Delta^2}$, where $\Delta = \omega_{\text{rf}} - \omega_L$ is the detuning and $\Omega = \gamma B_{\text{rf}}/2$ is the Rabi frequency. The rf dressing reduces the leading-order linear sensitivity of the ‘bare’ state energies ($\omega_L \approx \gamma B_z$ where γ is the gyromagnetic ratio) to magnetic field variations δB_z to quadratic sensitivity, via the detuning Δ . The spin character and symmetries are otherwise unchanged; magnetic fields along y or z oscillating near the Rabi frequency drive transitions $|m_x = -1\rangle \leftrightarrow |m_x = 0\rangle$ and $|m_x = 0\rangle \leftrightarrow |m_x = +1\rangle$, the basis for ac magnetometry [2] or concatenated CoDD to mitigate fluctuations in Ω on the spectrum and coherences [4].

However, any \hat{F}_z^2 interaction – arising from nonlinear Zeeman [13], microwave ac-Stark [14], or tensor light [15] shifts – raises the degeneracy of the $|m_z = -1\rangle \leftrightarrow |m_z = 0\rangle$ and $|m_z = 0\rangle \leftrightarrow |m_z = +1\rangle$ transitions, and breaks the symmetry of $\mathcal{H}_{\text{rwa}} = \Delta \hat{F}_z + \Omega \hat{F}_x + q \hat{F}_z^2$, where $q \equiv (E_{m_z=+1} + E_{m_z=-1} - 2E_{m_z=0})_{\Omega=0}/2\hbar$. This yields dressed eigenstates $\{|1\rangle, |2\rangle, |3\rangle\}$ that are no longer SO(3) rotations of the bare Zeeman states $|m_z\rangle$, but have nematic order, and an eigenspectrum $\omega_i(\Delta) = E_{|i\rangle}/\hbar$ shown in Fig. 1(left) for $q \in [0, \Omega]$. Moreover, the coupling of these dressed states is markedly different: the



FIG. 1. (Color online) Energy spectrum and splittings of a radiofrequency coupled spin-1 for various $q \in [0, \Omega]$. The transparency of each curve is proportional to the distance of the quadratic shift q from $q_{\text{magic}} \approx 0.348\Omega$. (Left) Energies ω_n of dressed states $|n\rangle = |1\rangle$ (red) $|2\rangle$ (blue), and $|3\rangle$ (green) normalized to the rf-coupling strength (Rabi frequency) Ω as a function of detuning $\Delta = \omega_{\text{rf}} - \omega_L$. Dashed lines indicate the energies of uncoupled states ($\Omega = 0$) in a frame rotating at ω_{rf} . (Right) Splittings ω_{ij} of dressed states $|i\rangle$ and $|j\rangle$ as a function of detuning. When $q = q_{\text{magic}}$ (bold curves), energies ω_1 and ω_2 share the same curvature, and their difference ω_{12} (right, purple) is minimally sensitive to detuning and thus magnetic field variations.

spin operators $\hat{F}_{x,y,z}$ in the dressed basis span more of SU(3) for $q \neq 0$, e.g. $\langle i|\hat{F}_x|j\rangle$ has a non-zero projection onto the fourth Gell-Mann matrix $Q_{x^2-y^2}$ signifying the direct coupling of dressed states $|1\rangle$ and $|3\rangle$. The dressed-state transitions are thus cyclic and non-degenerate (Fig. 1(right)), characterized by a dressed Larmor frequency

$$\begin{aligned} \omega_D &\equiv (\omega_3 - \omega_2)_{\Delta=0}/2 = (\omega_{12} + \omega_{23})_{\Delta=0}/2 \\ &= \sqrt{\Omega^2 + q_D^2}, \end{aligned} \quad (1)$$

and dressed quadratic shift

$$\begin{aligned} q_D &\equiv (\omega_3 + \omega_1 - 2\omega_2)_{\Delta=0}/2 \\ &= (\omega_{23} - \omega_{12})_{\Delta=0}/2s \\ &= -q/2. \end{aligned} \quad (2)$$

The curvature of the dressed state energies $\omega_i(\Delta)$ depends on the ratio $q_R = q/\Omega$, and is equal for states $|1\rangle$ and $|2\rangle$ when

$$q_R = q_{R,\text{magic}} = \sqrt{(3\sqrt{2} - 4)}/2 \approx 0.348, \quad (3)$$

resulting in vanishing quadratic dependence of their transition frequency $\omega_{12} = \omega_2 - \omega_1$ on Δ [16]. The leading-order sensitivity of these states to field variations δB_z at $q_{R,\text{magic}}$ is quartic [17], amounting to their optimal CoDD and we join in Lundblad et al. in terming them synthetic clock states in this configuration.

Literature review notes

- Minimally sensitive states in other systems, e.g. $|F = 1, m = -1\rangle \leftrightarrow |F = 2, m = +1\rangle$ at $B = 3.23 \text{ G}$ [18], clocks.
- ✓ Wide utility of these states for clocks, magnetometers (including microwave, e.g. Treutlein [9]), quantum information, and quantum emulation, e.g. elusive many-body spin-singlet state which requires unforgiving field stability [10].
- ✓ In electronically/magnetically sensitive spin experiments, magnetic field noise manifest as small perturbations $\Delta \hat{F}_z$ which shift the energy eigenvalues.
- Continuous measurement of an $F = 1$ system using the Faraday effect necessitates the state be magnetically sensitive $\langle \hat{F}_x \rangle \neq 0$ but dark states [incorrect use of dark states?], protected from magnetic field fluctuations only exist in the nullspace of \hat{F}_z i.e. $\hat{F}_z|\psi\rangle = \hat{0}$ [6]. Therefore, no state exists which is both insensitive to magnetic fields fluctuations and capable of generating a continuous Faraday signal; the two are not mutually exclusive.
- Whereas the aim of a protected qubit subspace is to create *states* that are insensitive to fields, the impetus of metrology is to create *superpositions* and *transitions* insensitive to fields.
- ✓ Continuous dynamical decoupling (CoDD) [1, 7] occurs when the spin undergoes rotations in a plane perpendicular induce perturbation, i.e. \hat{F}_x or \hat{F}_y most readily achieved through Rabi coupling where $\Omega \gg \delta B$.
- ✓ Decoupling can be used to increase the coherence time of single qubits [5] which can be extended even further by doubly dressing the system and decoupling from noise induced by the first coupling field [4] (concatenated continuous decoupling, or CCD).
- It has also been shown theoretically that CoDD is superior than pulsed sequence dynamical decoupling for single solid-state qubits for magnetometry [2].
- ✓ Motivate continuous measurement, especially in context of measurement bandwidth; it doesn't make sense to measure something in kHz–MHz band using a shot-based (0.1 Hz or less) readout. Why? Can't react, can't feedback, can't always assume periodicity/repeatability.
- Whilst the paper is not focused on introducing spectrograms, we can say they provide a new mechanism for appraising spectrum of a quantum system in real-time.

- From a magnetometry perspective, breaking rotational symmetry is bad because you want there to be no anisotropy to the sensitivity. How does this relate to this work?
- ✓ (How) The \hat{F}_z^2 interaction has been controlled using static magnetic fields, microwave ac Stark shifts [14] and tensor-light shifts of off-resonant electric dipole transitions [15], respectively. In collective psuedo-spins, it can arise from nonlinear collisional interactions [19].
- (What) This nonlinear term has been used to traverse the magnetic phase space of a spinor quantum gas, drive quantum quenches of same, initiate spin dynamics in optical lattices [14], enact the canonical spin-squeezing of Kitagawa and Ueda [20]: one-axis twisting (shear of coherent spin state uncertainty region) that has squeezed atomic spins in cavities [21], Bose-Einstein condensates [19], and superconducting qubits.

Continuous measurement using the linear Faraday effect:

- Paramagnetic Faraday effect has been used to continuously measure spin-mixing dynamics of a polar spinor condensate [22].
- Quantum state tomography (QST) [15, 22] using continuous weak measurement and dynamical control. Ref. [15, 2004] foreshadows “real-time estimation of the spin density matrix” via weak measurement.
- Real-time nonperturbing polarization probe (tensor-based at $\Delta \sim \Delta_{HF}$, not Faraday) in Ref. [23] measured hyperfine Rabi oscillations of the collective clock-transition pseudospin. The birefringence is modulated near baseband, i.e. polarization rotation oscillates at the Rabi frequency. This paper highlights the fact that “atoms in $m_F = 0$ clock states cannot contribute to Faraday rotation, and therefore in the limit $\Delta \gg \Delta_{HF}$ the probe polarization does not couple to the clock pseudospin”. In contrast, the synthetic clock states presented here can yield Faraday rotation, including in the $\Delta \gg \Delta_{HF}$ limit. Moreover, the probe-induced differential light shift changed the resonance of the clock-transition (detected as a modified total Rabi frequency). Also from this paper:

...if a measurement can resolve the quantum fluctuations associated with a collective observable, then backaction will be induced on the collective state and the observable can be squeezed.

- Faraday QND measurement of collective spin projection of an atomic beam as a way of preparing squeezed spin states [24]. This is an odd regime where the background field is modulated faster than the peak Larmor frequency, but it is a rare example of the polarimetry signal being fed to a spectrum analyzer/lock-in detector.

Other exotica:

- Spin-orbit coupled spin-1 Bose gases exhibit e.g. tricriticality in (Ω, q) phase diagrams [25], where the optimal decoupling $q = q_{R, \text{magic}} \Omega$ is a line traversing multiple phases, in the vicinity of a tricritical point of polar-striped and plane-wave phases. Although here the wavevector of the coupling is zero, the essential results may carry over, and the Faraday probe – used to detect magnetization – could constitute one of the Raman beams used to generate the spin-orbit coupling.
- Lipkin-Meshkov-Glick Hamiltonian [26] (see Refs. [16] of [27]).

BACKGROUND + RANDOM

Structure of background theory:

- ✓ Introduce Hamiltonian for CoDD (clock states have been introduced so splitting will come naturally).
- ✓ Show eigenenergies of for varying q and separations.
- ✓ Point out in figure that curvature of ω_{12} starts positive but becomes negative. By mean value theorem, it must become vanishing.
- ✓ Use second order perturbation theory to derive q_{magic} . Only in supp. info. if forced to do so.

Hamiltonians (quasi-static field along z , coupling field along x :

$$\begin{aligned}\hat{H}_{\text{lab}} &= -\omega_L \hat{F}_z + q \hat{F}_z^2 + 2\Omega \cos(\omega_{\text{rf}} t) \hat{F}_x \\ \Rightarrow \hat{H}_{\text{rwa}} &= \Delta \hat{F}_z + q \hat{F}_z^2 + \Omega \hat{F}_x, \text{ where} \\ \omega_L(B) &\equiv (E_{m=-1} - E_{m=+1})/2\hbar, \text{ and} \\ q(B) &\equiv (E_{m=+1} + E_{m=-1} - 2E_{m=0})/2\hbar\end{aligned}$$

are the Larmor frequency and quadratic shift, respectively, which can be gleaned from the Breit-Rabi equation. The rf Rabi frequency $\Omega = \gamma B_{\text{rf}}/2$ and detuning $\Delta(B) = \omega_{\text{rf}} - \omega_L(B)$.

The control field $\mathbf{B}(t \geq 0) = -B_{\text{rf}} \cos(\omega_{\text{rf}} t) \mathbf{e}_x + B_z(t) \mathbf{e}_z$, where $B_z(t) = B_0 + \alpha t + B_{\text{ac}}(t)$ is the sum of a linear ramp and parasitic power-line magnetic noise.

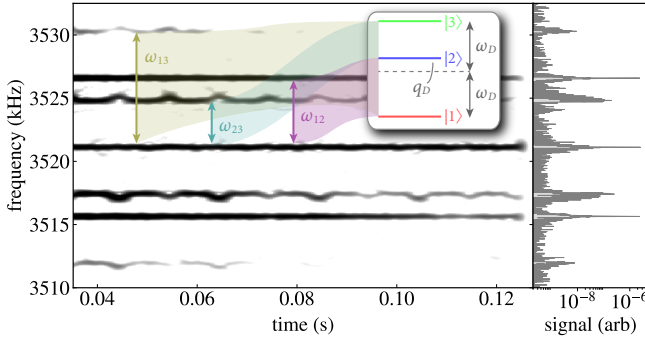


FIG. 2. Continuous measurement of the dressed energy spectra for $q_R = 0.402(3)$, $f_{\text{rf}} = 3.521$ MHz and $B_0 = 5.013$ G (left) and the log of the fitted moving average signal strength in normalized units (right). The shape of the energy spectra in the Fourier domain are Gaussian, but magnetic field fluctuations skew the time average shape. Skewed Gaussian curves are fitted to each peak resulting in a significant skew parameter of $\gamma = 2.4(1)$ for the ω_{23} splitting width $\sigma_{23} = 243(2)$ Hz. The fit of the transition most decoupled from magnetic field fluctuations ω_{12} has a vanishing skew and is well approximated by a Gaussian of width $\sigma_{12} = 109(1)$ Hz.

- At low magnetic field strengths, $\omega_L \propto B_z$ and $q \propto B^2$, and for our parameters we are justified in taking $\omega_L = \gamma B$ and $q = q_Z B_z^2$, where $\gamma = 2\pi \times 702.379$ kHz/G is the gyromagnetic ratio for ^{87}Rb $F = 1$ and $q_Z = 2\pi \times 71.89$ Hz/G² the quadratic Zeeman coefficient.
- For most of the analysis presented here (with ω_L and q defined as above) these proportionalities need not be met, or the results, e.g. value of q_{magic} require a small correction.
- For $q = 0$, \hat{H}_{lab} and $\hat{H}_{\text{rwa}} \propto \mathbf{B} \cdot \hat{\mathbf{F}}$ and are thus a generator of rotations, but $q\hat{F}_z^2 \neq 0$ breaks the $SU(2)$ symmetry and $\langle \hat{F}_z^2 \rangle$ is not preserved.
- This broken symmetry lifts the degeneracy of the dressed state splittings making them distinguishable in our spectrogram measurements.
- Moreover, \hat{H}_{rwa} meets the requirements of reconstructing pure $SU(2F+1)$ states from a weak measurement of $\langle \hat{F}_x \rangle$ [28]; thus one can use the measurement record for quantum state estimation, but the spectrogram is similarly rich in Hamiltonian and density matrix information.
- We vary the magnetic field to affect a change in the detuning of $\Delta \in [0, 2\Omega]$, the domain of Fig. 1(B).
- Variations in $B_z \mapsto B_0 + \delta B_z$ of order $B_{\text{rf}} = 2\Omega/\gamma$ affect the detuning linearly, *viz.* $\Delta = -\gamma \delta B_z$ for $\omega_L(B_0) = \omega_{\text{rf}}$ (i.e. rf is resonant when $B_z = B_0$), and do not affect q appreciably for sufficiently small field strengths.

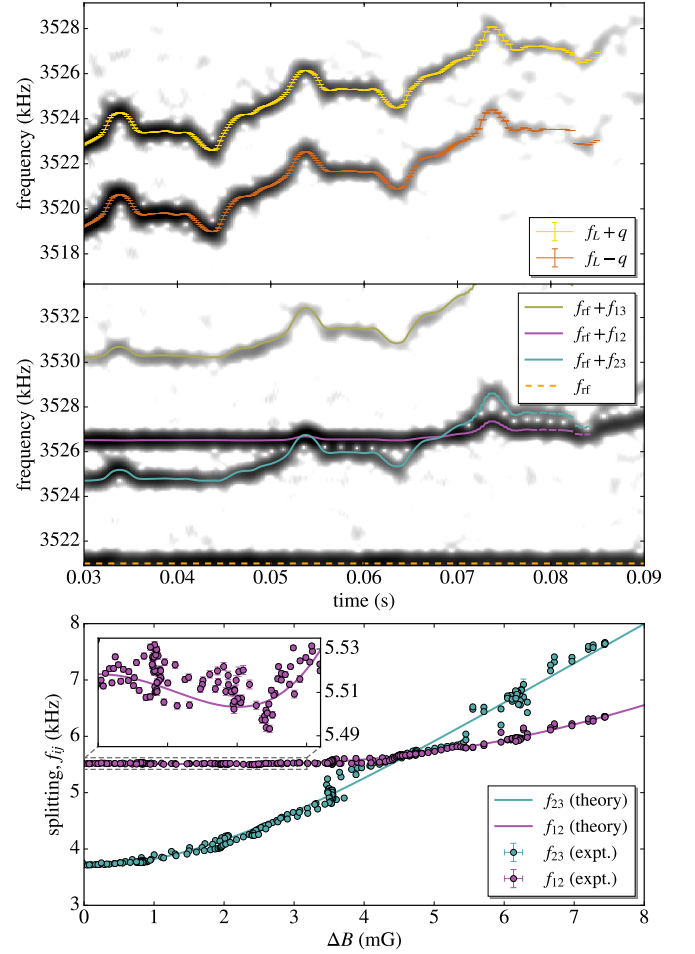


FIG. 3. (Color online) Acquisition and analysis pipeline of the continuous spectrum observation for $q_R = 0.402(3)$ ($f_{\text{rf}} = 3.521$ MHz, $B_0 = 5.013$ G). The underlay of (A) and (B) are spectrograms of raw acquisitions from the polarimeter measuring Faraday rotation of the probe beam. (A) Time-resolved magnetometry is used to calibrate the instantaneous magnetic field $B_z(t) = B_0 + \delta B_z(t)$ over the interrogation interval, in which the field [detuning] varies over a range $\sim B_{\text{rf}}$ [2Ω] and the spinor gas is left to Larmor precess in the absence of a dressing field ($\Omega = 0$). We fit sinc peaks in the frequency domain for each spectrogram window (time domain) to determine the instantaneous Larmor frequency $f_L(t)$ and quadratic shift $q(t)$. (B) The field is swept over the same range but the radiofrequency dressing is applied ($\Omega > 0$). Three sidebands appear above (shown) and below the carrier at f_{rf} (dashed, orange), revealing the dressed state splittings ω_{ij} . (C) A parametric plot of $f_{12}(t)$ and $f_{23}(t)$ versus $\delta B(t)$ by combining analysis of (A) and (B). Solid curves in (B) and (C) are theoretical splittings from an eigenspectrum calculation, provided only f_{rf} , $B_z(t)$, and Ω , i.e. no free parameters. The synthetic clock transition f_{12} varies by 39 Hz for $0 \leq \delta B_z \leq B_{\text{rf}}/4 = 3.2$ mG, equivalent to $0 \leq |\Delta/\Omega| \leq 0.5$ (C, inset).

- Indeed, our data corroborate this since we directly measure $q(B_z(t))$ during the field sweep and find that $\sigma(q)/(2\pi) = 11.7 \text{ Hz}$ on average (alternatively, inferring q from ω_L via the Breit-Rabi equation gives $\sigma(q)/(2\pi) = 1.2 \text{ Hz}$).
- Thus the horizontal axis Δ/Ω in Fig. 1 is a proxy for δB_z , and ω_{12} at $q = q_{\text{magic}}$ has leading-order quartic sensitivity to δB_z .
- At high fields ($B \approx 30 \text{ G}$) this approximation is no longer valid; $q \approx q_{\text{magic}}$ varies appreciably across $\delta B_z \in [0, B_{\text{rf}}]$ and ω_{12} has weak linear dependence on δB_z [29].
- *On varying Ω or q to change $q_R = q/\Omega$:* For a given static magnetic field, q_R can be modified via the Rabi frequency. However, this is not what is represented in Fig. 1, as the normalization of the horizontal and vertical axes would vary for each q_R . Importantly, the insensitivity of ω_{12} to detuning only gets better for increasing Ω in absolute terms; if the rf amplitude is unlimited, use it. However, doing so also modifies the absolute dressed state splittings on resonance, and thus the bandwidth of the dressed spin-1 as an ac magnetometer. The take home message is then: use as high an rf amplitude as you can afford (or want to tune the ac-band to), and then modify q_R via q to realize the synthetic clock states.
- *Transitions between dressed states:* $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ driven by fields oscillating along y or z near frequencies $\omega_D \mp q_D$, respectively. Alternatively, $|1\rangle \leftrightarrow |3\rangle$ driven by fields oscillating along x near frequency $2\omega_D$. This is very different to the fully polarized bare states $|m = \pm 1\rangle$, which are coupled by a single-photon transition as this would conserve neither photon number nor angular momentum. There is no such restriction on the dressed states however as they are neither eigenstates of \hat{F}_z nor photon number.

Lab frame eigenstates:

- ✓ Mean splitting ω_L ; quadratic shift q .
- ✓ Pairwise coupling between $|m = -1\rangle \leftrightarrow |m = 0\rangle$ and $|m = 0\rangle \leftrightarrow |m = +1\rangle$ via \hat{F}_x and/or \hat{F}_y , i.e. affected by fields transverse to the static field oscillating near ω_L .
- The $F = 1$ spin operators in the undressed basis do not span all of $\text{SU}(3)$, i.e. $\sigma_x = (\lambda_1 + \lambda_6)/\sqrt{2}$, $\sigma_y = (\lambda_2 + \lambda_7)/\sqrt{2}$, and $\sigma_z = (\lambda_3 + \sqrt{3}\lambda_8)/2$ (λ_4 and λ_5 which include off diagonal terms in rows/columns 1 and 3 are absent) where $\lambda_i, i = 1, \dots, 8$ are the Gell-Mann matrices.

Dressed states on resonance:

- ✓ Dressed Larmor frequency:

$$\begin{aligned}\omega_D &\equiv (\omega_3 - \omega_2)_{\Delta=0}/2 = (\omega_{12} + \omega_{23})_{\Delta=0}/2 \\ &= \sqrt{\Omega^2 + q_D^2}.\end{aligned}$$

- ✓ Dressed quadratic shift:

$$\begin{aligned}q_D &\equiv (\omega_3 + \omega_1 - 2\omega_2)_{\Delta=0}/2 \\ &= (\omega_{23} - \omega_{12})_{\Delta=0}/2 \\ &= -q/2.\end{aligned}$$

- Thus $\Omega = \sqrt{\omega_{12}\omega_{23}}_{\Delta=0}$ and $q_D = (\omega_{23} - \omega_{12})_{\Delta=0}/2$, both of which can be attained from the dressed sideband splittings on resonance. Such high-bandwidth measurement of Ω (magnetic field oscillating along x with amplitude B_{rf} and frequency ω_L) allows (in principle) closed-loop control of Ω using the atoms.
- ✓ For $q = 0$ (low-field limit), the dressed states at $\Delta = 0$ are eigenstates of \hat{F}_x , viz. $|m_x = -1, 0, +1\rangle \equiv |1\rangle, |2\rangle$, and $|3\rangle$ respectively. (i) the spectrum has vanishing linear sensitivity to magnetic fields, with the leading quadratic sensitivity (as in spin-1/2) of the degenerate $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ transitions, and (ii) fields along y or z oscillating near the Rabi frequency Ω drive transitions between different $|m_x\rangle$ states. [Cite other dressed-ception papers on both of these.]
- ✓ The $F = 1$ spin operators in the resonantly dressed basis span more of $\text{SU}(3)$, as $[\hat{F}_x]_D = (q_D/\omega_D)Q_{x^2-y^2} - (\Omega/\omega_D)\sigma_z = (q_D/\omega_D)\lambda_4 - \Omega/\omega_D(\lambda_3 + \sqrt{3}\lambda_8)/2$, $[\hat{F}_y]_D$ is a sum of λ_1, λ_2 , and λ_7 , and $[\hat{F}_z]_D$ is a sum of λ_1 and λ_6 . The presence of $Q_{x^2-y^2} = \lambda_4$ for $q \neq 0$ signifies the coupling of dressed states $|1\rangle$ and $|3\rangle$, i.e. a non-zero $\langle 1|\hat{F}_x|3\rangle$. [The spin operators have no projection onto λ_5 , even for an rf field oscillating along y .]
- To drive the $|1\rangle \leftrightarrow |2\rangle$ transition exclusively (remain in the $\{|1\rangle, |2\rangle\}$ subspace) with an oscillating field along y or z , the concatenated coupling strength must be $\ll q_D$, the separation of the two lowest dressed state transitions. This sets the *dynamic range* of ac magnetometry using the synthetic clock states alone.
- ✓ Curvature of the dressed-state energies can be evaluated using perturbation theory;

$$\frac{\partial^2 \omega_n}{\partial \Delta^2} = \sum_{k \neq n} \frac{|\langle k|\hat{F}_z|n\rangle|^2}{\omega_n - \omega_k}.$$

- ✓ Thus the curvature of the dressed-state splittings can be found. In particular, the dimensionless curvature of ω_{12} is (presuming $|\partial q/\partial \Delta| \ll 1$)

$$\frac{\partial^2(\omega_{12}/\Omega)}{\partial(\Delta/\Omega)^2} = \Omega \frac{\partial^2 \omega_{12}}{\partial \Delta^2} = -\frac{3q_R \sqrt{4 + q_R^2} - q_R^2 - 2}{2\sqrt{4 + q_R^2}}.$$

This vanishes when $q = q_{R,\text{magic}}$, given by

$$q_{R,\text{magic}} = \sqrt{(3\sqrt{2} - 4)/2} \approx 0.348.$$

For $q_R = 0$, we recover the spin-1/2 result, $\Omega \partial^2 \omega_{12}/\partial \Delta^2 = 1$.

- ✓ Similarly, perturbation theory can be used to show that the third-order derivatives of ω_n with respect to detuning all vanish when $|\partial q/\partial \Delta| \approx |\gamma^{-1} \partial q/\partial B_z| \ll 1$, and thus the leading sensitivity to detuning (and thus B_z) is fourth-order.
- ✓ The above can be quantified by noting that $\gamma^{-1} \partial q/\partial B_z = 2B_z q_Z/\gamma \lesssim 10^{-3}$ for $B_z \lesssim 5$ G.
- This validates the choice of our phenomenological even-polynomial model for fitting to $(\delta B_z(t), \omega_{12}(t))$ data extracted from Faraday spectrograms.
- Near $q = q_{\text{magic}}$, the ratio of the Rabi frequency to the Larmor frequency is approximately:

$$\begin{aligned} \frac{\Omega}{\omega_L} &= \frac{B_{\text{rf}}}{B_0} \\ &\approx \frac{q_Z B_0}{\sqrt{2} \gamma q_{R,\text{magic}}} \\ &= 2.1 \times 10^{-4} B_0, \end{aligned}$$

with B_0 is in Gauss. Thus for $B_0 \lesssim 5$ G, $\Omega/\omega_L \lesssim 10^{-3}$ and the rotating-wave approximation is justified.

- For $0 \leq \delta B_z \leq B_{\text{rf}}/4 = 3.2$ mG ($0 \leq |\Delta/\Omega| \leq 0.5$) we observe a variation in the splitting f_{12} of 39 Hz for the data in Fig. 3, compared to the theoretical estimate of 26 Hz. These correspond to a normalized variation in ω_{12}/Ω of 8.6×10^{-3} and 5.8×10^{-3} , respectively. By comparison, the normalized variation at $q_R = 0$ is $(\sqrt{5} - 2)/2 \approx 0.118$; 14 [20] times higher than the observed [predicted] variation. Alternatively, the normalized variation of the $|m = \pm 1\rangle \leftrightarrow |0\rangle$ transitions at $q = 0$ is 0.5; 58 [86] times higher than the observed [predicted] variation in the synthetic clock transition frequency. [Both of these comparisons depend a lot on the range of δB ! They are far more impressive (theoretically) for smaller ranges.]

TABLE I. Upper sidebands of the carrier (at ω_{rf}) of the Faraday rotation signal $\propto \langle \hat{F}_x \rangle$ of a state $|\psi(t=0)\rangle = |m=1\rangle$ driven on resonance ($\Delta = 0$) in the laboratory frame. Frequency and phase are reported relative to the carrier, along with the transition that each sideband corresponds to. For each upper sideband, there is a lower sideband of the same amplitude, relative frequency and opposite $[\pi]$ relative phase.

frequency	amplitude	transition
0	$\frac{q_D \Omega}{2\omega_D^2}$	—
$\omega_D - q_D$	$\frac{\Omega}{4\omega_D}$	$ 1\rangle \leftrightarrow 2\rangle$
$\omega_D + q_D$	$\frac{\Omega}{4\omega_D}$	$ 2\rangle \leftrightarrow 3\rangle$
$2\omega_D$	$\frac{q_D \Omega}{4\omega_D^2}$	$ 1\rangle \leftrightarrow 3\rangle$

Analytic Faraday signal on resonance: The Faraday signal we detect is proportional to $\langle \hat{F}_x \rangle$ in the laboratory frame, which is given by $\langle \psi(t) | \hat{S}^\dagger \hat{F}_x \hat{S} | \psi(t) \rangle$ where $|\psi(t)\rangle = \exp(-i\hat{H}_{\text{rwa}}t/\hbar) |\psi(t=0)\rangle$, and $\hat{S} = \exp(-i\omega_{\text{rf}}\hat{F}_z t)$. For an initially polarized $|\psi(t=0)\rangle = |m=1\rangle$ state driven on resonance, we get

$$\begin{aligned} \langle \hat{F}_x \rangle_{\text{lab}} &= -\frac{\Omega}{\omega_D} \cos(q_D t) \sin(\omega_D t) \sin(\omega_{\text{rf}} t) - \\ &\quad \frac{q_D \Omega}{\omega_D^2} \sin^2(\omega_D t) \cos(\omega_{\text{rf}} t). \end{aligned}$$

The first term has equal-amplitude sidebands at $\pm(\omega_D \pm q_D)$ and the second term has smaller amplitude sidebands at $\pm 2\omega_D$, as summarized in Table I. The ratio of the ω_{13} sideband amplitude to the ω_{12} and ω_{23} sideband amplitudes is $q_R/\sqrt{4 + q_R^2} \approx 0.197$ for $q_R = 0.402$ in Fig. 3. For the data in Fig. 2 we measure the ratio of sideband amplitudes to be 0.699 and 0.210 of the ω_{23} and ω_{13} peaks relative to the ω_{12} peak respectively.

Faraday signal for arbitrary states: Evaluating $\langle \hat{F}_x \rangle_{\text{lab}}$ in the dressed basis, one can show analytically that the sideband frequencies are the dressed state splittings ω_{ij} with amplitudes proportional to the relevant dressed-state coherence $\text{Re}\{\rho_{ij}\}$ where $\rho_{ij} = \langle i|\psi\rangle\langle\psi|j\rangle$. The carrier amplitude is $(\rho_{33} - \rho_{11})\Omega/\omega_D$. This explains the relative longevity of the sidebands; the transition most sensitive to detuning is ω_{13} , and thus the coherence ρ_{13} decays the fastest in the presence of detuning noise with a measured $1e$ signal decay time of $\tau_s = 44(1)$ ms from Fig. 2. Furthermore, the decay time of the ω_{12} and ω_{23} sidebands are 74(1) ms and 71(2) ms respectively. Contrasting these results to an undressed measurement decay time $\tau_s = 21(1)$, it is clear that rf dressing shields the quantum state from magnetically induced decoherence. The constant of proportionality is the coupling strength of the dressed states.

Adiabatic considerations: The rf-dressing is applied non-adiabatically, and as a result we project $|m=-1\rangle$ onto the dressed basis at the initial detuning $\Delta(t=0)$

transition	sideband frequency	sideband amplitude
—	ω_{rf}	$(\langle 3 \hat{F}_x 3\rangle - \langle 1 \hat{F}_x 1\rangle)(\rho_{33} - \rho_{11})$
$ 1\rangle \leftrightarrow 2\rangle$	$\omega_{\text{rf}} + \omega_{12}$	$-2i\langle 1 \hat{F}_y 2\rangle\text{Re}\{\rho_{12}\} = -2\langle 2 \hat{F}_z 3\rangle\text{Re}\{\rho_{12}\}$
$ 2\rangle \leftrightarrow 3\rangle$	$\omega_{\text{rf}} + \omega_{23}$	$2i\langle 2 \hat{F}_y 3\rangle\text{Re}\{\rho_{23}\} = 2\langle 1 \hat{F}_z 2\rangle\text{Re}\{\rho_{23}\}$
$ 1\rangle \leftrightarrow 3\rangle$	$\omega_{\text{rf}} + \omega_{13}$	$2\langle 1 \hat{F}_x 3\rangle\text{Re}\{\rho_{13}\}$

into a state

$$\begin{pmatrix} \langle 1|\psi(t=0)\rangle \\ \langle 2|\psi(t=0)\rangle \\ \langle 3|\psi(t=0)\rangle \end{pmatrix} \approx \begin{pmatrix} \frac{1}{\sqrt{4+q_R(q_R+\sqrt{q_R^2+4})}} \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+q_R(q_R-\sqrt{q_R^2+4})}} \end{pmatrix}$$

for $\Delta(t=0) \ll \Omega$. In the adiabatic limit, the dressed state populations remain constant and the above superposition evolves under phase acquisition $e^{-i\omega_i t}$ by each dressed state $|i\rangle$ [30]. We quantify the stasis of the dressed superposition using the generalized adiabatic parameter $\Gamma \equiv |\mathbf{\Omega}(t)/\dot{\theta}(t)|$, where $\mathbf{\Omega} = \mathbf{B}_{\text{eff}}/\gamma \equiv \Omega \mathbf{e}_x + \Delta \mathbf{e}_z$ and $\tan \theta \equiv \Omega/\Delta$. For constant coupling amplitude $\dot{\Omega} = 0$, $\dot{\theta}(t) = -\Omega\dot{\Delta}(t)/|\mathbf{\Omega}|^2$. For the magnetic field sweeps used here, $\Gamma > 200$ and $\sqrt{\langle \Gamma^2 \rangle}_t \gtrsim 600$ where $\langle \cdot \rangle_t$ denotes the time-average over the duration of the sweep. Nevertheless, the continuous spectra when plotted parametrically (Fig. 3(bottom)) exhibit some evidence of non-adiabatic following. This is corroborated by numerically integrating the Schrödinger equation of the known control Hamiltonian; the dressed state populations vary slowly (compared to ω_D^{-1}) but by a few percent near minima of $\Gamma(t)$, i.e. when the sweep is least adiabatic.

APPARATUS

Our spinor quantum gas apparatus [31] and Faraday atom-light interface are described in greater detail elsewhere [32]. We prepare an ultracold gas ($\sim 1 \mu\text{K}$) of approximately 10^6 ^{87}Rb atoms in a crossed-beam optical dipole trap ($\lambda = 1064 \text{ nm}$). The three Zeeman states $|m = -1, 0, +1\rangle$ of the lowest hyperfine ($F = 1$) ground state are coupled using a radiofrequency field with $\Omega/(2\pi) \leq 100 \text{ kHz}$, generated by a single-turn coil placed immediately atop the glass vacuum cell, fed by an amplified radiofrequency source generated using direct-digital synthesis. A component of the spin (e.g. $\langle \hat{F}_x \rangle$) transverse to the static magnetic field direction (along z) rotates the polarization of an off-resonant probe beam via the paramagnetic Faraday effect. By tuning the probe to a magic-zero wavelength at $\lambda = 790.0 \text{ nm}$, and ensuring it is linearly polarized, the probe exerts no scalar or vector light shift on the atoms. The former would enact a dipole force on the cloud, perturbing its total density, whereas

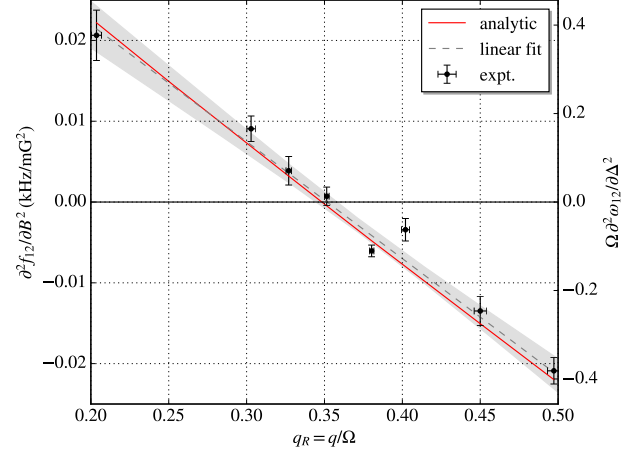


FIG. 4. Curvature of the synthetic clock transition for various quadratic shifts $q_R \in [0.2, 0.5]$. The measured curvature (black points) was determined from polynomial fitting to $(\delta B, f_{12})$ data shown in Fig. 3(B). Vertical and horizontal error bars correspond to the standard error of the regression and uncertainty in q_R (via $u(q)$ and $u(\Omega)$ at each field B_0), respectively. A linear fit (black, dashed) with 1-sigma confidence band (gray, shaded) are shown, whose intercept can be used to impute $q_{R,\text{magic}}(\text{expt.}) = 0.350(6)$. The analytic expression for the curvature (red) is consistent with the data-driven analysis of the curvature, cf. $q_{R,\text{magic}}(\text{theory}) = 0.348$. The left [right] vertical axis shows the curvature $\partial^2 f_{12}/\partial B^2$ [$\Omega \partial^2 \omega_{12}/\partial \Delta^2$] in absolute units of kHz/G^2 [dimensionless units, with the splitting and detuning normalized to the Rabi frequency]. cf. The normalized curvature is unity when $q_R = 0$.

the latter would be manifest as a fictitious magnetic field and gradient, dephasing the collective spin [33]. Here we used a wavelength of $\lambda = 781.15 \text{ nm}$ to increase the SNR with respect to the rf pickup at f_{rf} ; the measured $1/e$ decay time of calibration peaks $\tau_s = 23.8(2) \text{ ms}$ is consistent with scattering in [32] for a 10.6 mW probe with $1/e^2$ diameter of $150 \mu\text{m}$ (peak intensity $I_0 = 120.1(4) \text{ W}/\text{cm}^2$).

We detect the Faraday rotation of the probe light using a shot-noise limited balanced polarimeter, with bandwidth up to 8 MHz , and record the signal using an AlazarTech ATS9462 digitizer (16-bit, 180 MS/s) [34]. Upon applying the radiofrequency (rf) dressing field, $|\langle \hat{F}_x \rangle| > 0$ and the signature of the coupled spin-1 system is a Faraday signal frequency modulated (FM) about a

carrier at the Larmor frequency. The frequency difference of each FM sideband from the carrier is a calibration-free measure of each dressed state splitting ω_{ij} .

As we seek to appraise the robustness of the rf-dressed states to varying magnetic fields, we apply a time dependent $\delta B_z(t)$ and observe the dynamical change in the frequency composition of the Faraday signal using the short-time Fourier transform, or spectrogram. The time-dependent magnetic field shift $\delta B_z(t)$ is the sum of an applied linear ramp and the parasitic background fluctuations at the power-line frequency of 50 Hz and its odd harmonics, and typically ranges from 0 ($\Delta = 0$, resonance) to B_{rf} ($\Delta = 2\Omega$), cf. Fig. 1. For each realization (or ‘shot’) of the experiment, we directly calibrate this time-dependent field using ac magnetometry; an rf $\pi/2$ -pulse (rather than continuous coupling) initiates Larmor precession of the collective spin the x - y plane, and the Faraday signal is composed of two tones, at $\omega_{\pm} = \omega_L \pm q$. For $q\tau_f \geq 2\pi$, where τ_f is the length of the overlapping spectrogram windows, the two tones are spectrally resolved and their mean and difference yields the instantaneous $\omega_L(t)$ and $q(t)$, the former of which is used to find $\delta B(t)$ by inverting the Breit-Rabi equation [13]. The experiment is synchronized to the AC power line; the harmonic composition of which varies little between contiguous shots (20 s apart), and thus the measured $\delta B_z(t)$ and $q(t)$ from the calibration [fiducial?] shot serve as a good proxy for the values experienced by the atoms in the subsequent rf-dressed shot.

We measured the dressed spectrum for a range of $q_R \in [0.2, 0.5]$ by varying the resonant magnetic field B_0 [applied rf frequency f_{rf}] from 3.549 G [2.493 MHz] to 5.568 G [3.911 MHz], with a fixed Rabi frequency of $\Omega/(2\pi) = 4.505(3)$ kHz ($B_{\text{rf}} = 12.83(1)$ mG). For each resonant field B_0 , we ensured the Rabi frequency was fixed by measuring the voltage drop across the coil at f_{rf} with a lock-in amplifier which – in concert with an impedance analyzer – could be used to ensure the rf current in the coil and thus B_{rf} and Ω were constant. The Rabi frequency was ultimately measured using the atoms, by analyzing a subset of the dressed energy spectrum during the magnetic field sweep when $|\Delta|/(2\pi) \leq 100$ Hz. The measured Rabi frequencies had a standard deviation $\sigma(\Omega) = 9.4$ Hz, validating the above method.

Despite long coherence times, the low duty cycle ($D < 0.01$) and large dead time ($T_{\text{shot}} \gtrsim 10$ s) of cold quantum gas experiments make challenging achieving metrological sensitivities per unit bandwidth that are competitive with other platforms. Here $D = 0.005$ and $T_{\text{shot}} = 20$ s, yet we make many more spin projection measurements ($[N_m = \text{blah}]$ at a shot-noise limited SNR of 10–100 [32]) than traditional cold atom experiments ($N_m = 1$ to several, e.g. absorption or dispersive imaging). This intra-shot revelation of the time and frequency domain renders the measurement of these spectra orders of magnitude more efficient. For example, the single spectrum shown

in Fig. 3 would take $\sim (10 \text{ shots per } \delta B_z \text{ per } \omega_{ij}) \times (100 \text{ different } \delta B_z \text{ values}) \times (3 \text{ different transition frequencies } \omega_{ij}) = 3000 \text{ shots}$ (2.5 times fewer δB_z values than shown here), or $\sim 6 \times 10^4 \text{ s} = 1000 \text{ minutes}$ of data acquisition. We acquire this spectrum in a single shot [two shots accounting for the field calibration, but that would require $\sim 2 \times$ more traditional shots], i.e. 20 s. The data used to generate Fig. 4 was acquired in only 6 minutes.

THE DATA - THINK OF SEXIER TITLE

Our technique directly measures all the quantities in the dressed energy eigen spectrum using sinc fitting to the spectrogram data in a two shot experiment. The first observation calibrates essential quantities of the magnetic field ($q_z, \omega_L, \frac{\partial B}{\partial t}$) Fig. 3A characterising the horizontal axis of Fig. 1B. In the second shot, we repeat the experiment but dress the atoms with RF Fig. 3B from which we measure the energy splitting.

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- [1] F. F. Fanchini, J. E. M. Hornos, and R. d. J. Napolitano, *Physical Review A* **75**, 022329 (2007).
 - [2] M. Hirose, C. D. Aiello, and P. Cappellaro, *Physical Review A* **86**, 062320 (2012).
 - [3] M. Loretz, T. Rosskopf, and C. L. Degen, *Physical Review Letters* **110**, 017602 (2013).
 - [4] J.-M. Cai, B. Naydenov, R. Pfeiffer, L. P. McGuinness, K. D. Jahnke, F. Jelezko, M. B. Plenio, and A. Retzker, *New Journal of Physics* **14**, 113023 (2012); J. Cai, F. Jelezko, N. Katz, A. Retzker, and M. B. Plenio, *New Journal of Physics* **14**, 093030 (2012).
 - [5] D. A. Golter, T. K. Baldwin, and H. Wang, *Physical Review Letters* **113**, 237601 (2014).
 - [6] N. Aharon, M. Drewsen, and A. Retzker, *Physical Review Letters* **111**, 230507 (2013).
 - [7] P. Facchi and S. Pascazio, *Physical Review Letters* **89**, 080401 (2002); P. Facchi, D. A. Lidar, and S. Pascazio, *Physical Review A* **69**, 032314 (2004).
 - [8] R. Vijay, C. Macklin, D. H. Slichter, S. J. Weber, K. W. Murch, R. Naik, A. N. Korotkov, and I. Siddiqi, *Nature* **490**, 77 (2012).
 - [9] C. F. Ockeloen, R. Schmied, M. F. Riedel, and P. Treutlein, *Physical Review Letters* **111**, 143001 (2013); A. Horsley and P. Treutlein, *Applied Physics Letters* **108**, 211102 (2016).
 - [10] D. M. Stamper-Kurn and M. Ueda, *Reviews of Modern Physics* **85**, 1191 (2013).
 - [11] H.-J. Mikeska and A. K. Kolezhuk, in *Quantum Magnetism*, Lecture Notes in Physics No. 645, edited by U. Schollwck, J. Richter, D. J. J. Farnell, and R. F. Bishop (Springer Berlin Heidelberg, 2004) pp. 1–83, DOI: 10.1007/BFb0119591.
 - [12] E. Majorana, *Il Nuovo Cimento* (1924-1942) **9**, 43 (1932).
 - [13] N. Ramsey, *Molecular beams* (Clarendon Press, 1956).
 - [14] F. Gerbier, A. Widera, S. Fölling, O. Mandel, and I. Bloch, *Physical Review A* **73**, 041602 (2006).

- [15] G. A. Smith, S. Chaudhury, A. Silberfarb, I. H. Deutsch, and P. S. Jessen, *Physical Review Letters* **93**, 163602 (2004).
- [16] The curvature of the dressed-state energies is evaluated using perturbation theory. In particular, the dimensionless curvature of ω_{12} is $\partial^2(\omega_{12}/\Omega)/\partial(\Delta/\Omega)^2 = \Omega \partial^2\omega_{12}/\partial\Delta^2 = -(3q_R\sqrt{4+q_R^2} - q_R^2 - 2)/(2\sqrt{4+q_R^2})$. For $q_R = 0$, we recover the spin-1/2 result, $\Omega \partial^2\omega_{12}/\partial\Delta^2 = 1$.
- [17] We take $\Delta = -\gamma\delta B_z$ for $|\Delta| \leq 2\Omega$ ($|\delta B_z| \leq B_{\text{rf}}/2$) and $|\partial q/\partial\Delta| \approx |\gamma^{-1}\partial q/\partial B_z| = |2B_z q_z/\gamma| \ll 1$, valid to 10^{-3} for the field strengths $B_z \lesssim 5$ G used here, resulting in vanishing third-order derivatives of ω_i with respect to detuning. In general, the variation of q with Δ (or δB_z) can be accounted for using the Breit-Rabi equation, leading to a residual linear and cubic variation of ω_{12} with δB_z , and a small correction to $q_{R,\text{magic}}$ in Eq. (3).
- [18] M. R. Matthews, *Physical Review Letters* **81**, 243 (1998).
- [19] M. F. Riedel, P. Bhi, Y. Li, T. W. Hnsch, A. Sinatra, and P. Treutlein, *Nature* **464**, 1170 (2010); C. Gross, T. Zibold, E. Nicklas, J. Estve, and M. K. Oberthaler, *Nature* **464**, 1165 (2010).
- [20] M. Kitagawa and M. Ueda, *Physical Review A* **47**, 5138 (1993).
- [21] I. D. Leroux, M. H. Schleier-Smith, and V. Vuleti, *Physical Review Letters* **104**, 073602 (2010).
- [22] Y. Liu, S. Jung, S. E. Maxwell, L. D. Turner, E. Tiesinga, and P. D. Lett, *Physical Review Letters* **102**, 125301 (2009); G. A. Smith, A. Silberfarb, I. H. Deutsch, and P. S. Jessen, *Physical Review Letters* **97**, 180403 (2006).
- [23] S. Chaudhury, G. A. Smith, K. Schulz, and P. S. Jessen, *Physical Review Letters* **96**, 043001 (2006).
- [24] A. Kuzmich, L. Mandel, J. Janis, Y. E. Young, R. Eijnisman, and N. P. Bigelow, *Physical Review A* **60**, 2346 (1999); A. Kuzmich, L. Mandel, and N. P. Bigelow, *Physical Review Letters* **85**, 1594 (2000).
- [25] G. I. Martone, *Physical Review Letters* **117** (2016), 10.1103/PhysRevLett.117.125301.
- [26] H. J. Lipkin, N. Meshkov, and A. J. Glick, *Nuclear Physics* **62**, 188 (1965).
- [27] W. Muessel, H. Strobel, D. Linnemann, T. Zibold, B. Juli-Daz, and M. K. Oberthaler, *Phys. Rev. A* **92**, 023603 (2015).
- [28] S. T. Merkel, C. A. Riofro, S. T. Flammia, and I. H. Deutsch, *Physical Review A* **81**, 032126 (2010).
- [29] N. Lundblad and I. B. Spielman, *Physical Review Letters* **118**, xxxxxx (2017).
- [30] A. Messiah, *Quantum mechanics*, Vol. 2 (North-Holland, 1962).
- [31] A. A. Wood, L. M. Bennie, A. Duong, M. Jasperse, L. D. Turner, and R. P. Anderson, *Physical Review A* **92**, 053604 (2015).
- [32] M. Jasperse, M. J. Kewming, S. N. Fischer, P. Pakkiam, R. P. Anderson, and L. D. Turner, *arXiv:1705.10965* (2017).
- [33] A. Wood, L. Turner, and R. Anderson, *Physical Review A* **94**, 052503 (2016).
- [34] The maximum Larmor frequency and thus static magnetic field we can detect Faraday rotation at is limited by the bandwidth of the detector.