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CUNY SPS Data Analytics Summer Bridge
Math Assignment #1

① $f(x) = \underbrace{x}_{\text{price per doll}} \overbrace{(80-2x)}^{\text{\# of dolls sold}} = -2x^2 + 80x$

② $f(x) = 8x^3 + 7x^2 - 5$

$f(3) = 8(3)^3 + 7(3)^2 - 5 = 8(27) + 7(9) - 5 = 216 + 63 - 5 = 274$

$f(-2) = 8(-2)^3 + 7(-2)^2 - 5 = 8(-8) + 7(4) - 5 = -64 + 28 - 5 = -41$

$f(x+c) = 8(x+c)^3 + 7(x+c) - 5 = 8(x^3 + x^2c + 3xc^2 + c^3) + 7(x+c) - 5$

$= 8x^3 + 8x^2c + 24xc^2 + c^3 + 7x + 7c - 5$

③ Step 1: $\lim_{x \to 1^-} f(x) = 2$

Step 2: $\lim_{x \to 1^+} f(x) = -5$

Step 3: $\lim_{x \to 1} f(x) =$ does not exist

④ $f(x) = -2x^3$

$f'(x) = -2x^2 \cdot 3 = -6x^2$

⑤ $f(x) = \frac{-8}{x^2} = -8x^{-2}$

$f'(x) = -8x^{-3} \cdot -2 = 16x^{-3} = \frac{16}{x^3}$

⑥ $g(x) = 5\sqrt[3]{x} = 5x^{\frac{1}{3}}$

$g'(x) = \frac{1}{3} \cdot 5x^{-\frac{2}{3}} = \frac{5}{3}x^{-\frac{2}{3}} = \frac{5}{3x^{\frac{2}{3}}} = \frac{5}{3\sqrt[3]{x^2}}$

(7) $y = -2x^{\frac{9}{8}}$

$y' = \frac{9}{8} \cdot -2x^{\frac{1}{8}} = -\frac{9}{4}x^{\frac{1}{8}} = -\frac{9}{4}\sqrt[8]{x}$

(8) $f(0) = 40$

$f(4) = 35$

$\frac{\Delta f}{\Delta x} = \frac{35-40}{4-0} = \frac{-5}{4}$

(9) $C(x) = 630 + 2.4x$

$AC(x) = \frac{630+2.4x}{x}$

(10) $f(x) = (-2x^2+1)(-5x+9)$

$f'(x) = (-4x)(-5x+9) + (-5)(-2x^2+1)$

$= 20x^2 - 36x + 10x^2 - 5$

$= 30x^2 - 36x - 5$

(11) $f(x) = \frac{5x^{\frac{1}{2}}+7}{-x^3+1}$

$f'(x) = \frac{(\frac{5}{2}x)(-x^3+1) - (-3x^2)(5x^{\frac{1}{2}}+7)}{(-x^3+1)^2}$

$= \frac{-\frac{5}{2}x^4 + \frac{5}{2}x - (-15x^{\frac{3}{2}} - 21x^2)}{x^6 - 2x^3 + 1} = \frac{-\frac{5}{2}x^4 + 21x^2 + 15x^{\frac{3}{2}} + \frac{5}{2}x}{x^6 - 2x^3 + 1}$

(12) $f(x) = (3x^{-3} - 8x + 6)^{\frac{4}{3}}$

$f'(x) = \frac{4}{3}(3x^{-3} - 8x + 6)(-3 \cdot 3x^{-4} - 8)$

$= (4x^{-3} - \frac{32}{3}x + 8)(-9x^{-4} - 8)$

$= -36x^{-7} + 96x^{-3} - 72x^{-4} - 32x^{-3} + \frac{256}{3}x - 64$

$= -36x^{-7} - 72x^{-4} + 64x^{-3} + \frac{256}{3}x - 64$

(13) $f(t) = \frac{550t^2}{\sqrt{t^2+15}} = (550t^2)(t^2+15)^{\frac{1}{2}}$

$f'(t) = 1100t(t^2+15)^{\frac{1}{2}} + \frac{1}{2}(t^2+15)^{-\frac{1}{2}}(2t)(550t^2)$

$f'(3) = 1100(3)(3^2+15)^{\frac{1}{2}} + \frac{1}{2}(3^2+15)^{-\frac{1}{2}}(2\cdot 3)(550(3)^2)$

$= 3300(24)^{\frac{1}{2}} + \cancel{\frac{1}{2}}(18)^{-\frac{1}{2}}\cancel{(6)}\,3\,(4950)$

≈ 17333 ft

(14) $N(t) = 1000(6+.1t)^{\frac{1}{2}}, \quad 0 \le t \le 14$

step 1: $N(3) = 1000(6+.1(3))^{\frac{1}{2}}$

≈ 2510

step 2: $N'(x) = 1000 \cdot \frac{1}{2}(6+.1t)^{-\frac{1}{2}}(.1)$

$= \frac{50}{(6+.1t)^{\frac{1}{2}}}$

$N'(5) = \frac{50}{(6+.1(5))^{\frac{1}{2}}} \approx 20$

At week 5 in the season, they gain approximately 20 new attendees each week.

(15) $3x^3 + 4y^3 = 77$

step 1: $\frac{d}{dx}(3x^3 + 4y^3) = \frac{d}{dx}(77)$

$\frac{d}{dx}(3x^3) + \frac{d}{dx}(4y^3) = 0$

$3\cdot 3x^2 + 3\cdot 4y^2\frac{dy}{dx} = 0$

$9x^2 + 12y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{9x^2}{12y^2} = -\frac{3x^2}{4y^2}$

step 2: Slope at $(3,-1) = -\frac{3(3)^2}{4(-1)^2}$

$= -\frac{27}{4}$

(16) $f(x) = \frac{x+3}{x-8}$

$f'(x) = \frac{(x-8)-(x+3)}{(x-8)^2} = \frac{-11}{(x-8)^2}$

Since $f'(x)$ is always negative, the slope is always decreasing. That is $f'(x)$ is negative for x over $(-\infty, \infty)$

(17) $F(t) = 14 + \frac{367t^2}{t^2+100}$

Step 1: $F'(t) = \frac{2\cdot 367t(t^2+100) - 2t(367t^2)}{(t^2+100)^2}$

$= \frac{2t(367(t^2+100) - 367t^2)}{(t^2+100)^2}$

$= \frac{734t(t^2+100-t^2)}{(t^2+100)^2}$

$= \frac{73400t}{(t^2+100)^2}$

Since $F'(x)$ is always positive, it $F(x)$ is always increasing over $[0, \infty)$

Step 2: $\lim_{t \to \infty} F(t) \approx 14 + 367$
≈ 381

(18) Max packages delivered = .41 million

Day delivered most packages = 23

See attached code

(19) $f(x) = 7x^2 + 28x - 35$

$f'(x) = 14x + 28 \Rightarrow 0 = 14x + 28$
$x = -2$

$f''(x) = 14$

Minimum at $(-2, -63)$

(20) $f(x) = -6x^3 + 27x^2 + 180x$

$f'(x) = -18x^2 + 54x + 180 \Rightarrow$ Roots at -2 & 5

Critical points: $(-2, -204)$, $(5, 825)$

$f''(x) = -36x + 54$

$f''(-2) = 126 \Rightarrow$ Min

$f''(5) = -126 \Rightarrow$ Max

See attached code

(21)

(22) $V = x^2 y \Rightarrow 18432 = x^2 y$

$y = \frac{18432}{x^2}$

$C = 5x^2 + 3(4xy + x^2)$

$= 5x^2 + 12xy + 3x^2$

$= 8x^2 + 12x\left(\frac{18432}{x^2}\right)$

Min at $x = 24 \Rightarrow$ See attached code

$V = x^2 y$

$18432 = 24^2 y$

$y = 32$

Final dimension: 24 ft x 24 ft x 32 ft

(23) $A = xy \Rightarrow 1056 = xy$

$x = \frac{1056}{y}$

$C = 14.4(2x+2y) + 2(12)(x)$

$= 28.8x + 28.8y + 24x$

$= 52.8x + 28.8y$

$= 28.8y + 52.8\left(\frac{1056}{y}\right)$

$= 28.8y + \frac{55756.8}{y}$

$C' = 28.8 - \frac{55756.8}{y^2}$

$0 = 28.8 - \frac{55756.8}{y^2}$

$y^2 = \frac{55756.8}{28.8}$

$y = 44 \text{ ft}$

$A = xy$

$1056 = x(44)$

$x = 24 \text{ ft}$

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(24) * See included code *

$y = a(1-r)^x$

$\ln\left(\frac{y}{a}\right) = x\ln(1-r)$

$1 - e^{\ln\left(\frac{y}{a}\right)/x} = r$

$r \approx .08132692$

$y_{9\,years} = 31226.53$

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(25)

(26) See attached code

cost $\approx$ \$476.99

(27) Step 1: $p = \int (380 - 4x)\,dx = 380x - \frac{\cancel{4}^{2} x^2}{\cancel{2}} + C$

Given $p = 1700$
$x = 38$

$1700 = 380(38) - 2(38)^2 + C$

$C = 1700 - 380(38) + 2(38)^2 = -9852$

$\therefore\ p = 380x - 2x^2 - 9852$

Step 2: $p(56) = 380(56) - 2(56)^2 - 9852 = 5156$

See attached code

(28) $\int \frac{-5(\ln(y))^3}{y}\,dy = \int -5u^3\,du = -5\left(\frac{u^4}{4}\right) + C = -5\left(\frac{\ln^4 y}{4}\right) + C$

$u = \ln y$

$du = \frac{1}{y}\,dy$

(29) See code

$P(9) \approx 513$ animals

(30) See code

Area $= 2$ units^2

(31) $\frac{dy}{dx} = x^3 y$

$\int \frac{1}{y}\,dy = \int x^3\,dx$

$\ln y + C_1 = \frac{x^4}{4} + C_2$

$y = e^{\frac{x^4}{4} + C}$

(32) See code

$$\int_{-7}^{2} x\sqrt{x+7}\,dx = -\frac{144}{5}$$

(33) Step 1: $\sum_{i=0}^{k=20} 46(.22)^k = 46\left(\frac{1-.22^{20}}{1-.22}\right) \approx 58.97$ meters

See code for calculation

Step 2: $\sum_{i=0}^{k=100} 46(.22)^k = \frac{46}{1-.22} \approx 58.97$ meters

Same numbers probably due to rounding & truncation in R

(34) See code for solution

$3.890297x^5 + 1.887153\times10^9 x^4 - 2.868476\times10^{10} x^3 + 1.639562\times10^{11} x^2$

$-4.175303\times10^{11} x^1 + 3.996072\times10^{11}$

```r
#Richard Pantoliano
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# Math Assignment 1
install.packages("Deriv")
install.packages("rootSolve")
install.packages("mosaic")
install.packages("pracma")
library(pracma)
library(Deriv)
library(rootSolve)
library(mosaic)

# Problem 18
fx = function(x){(x^3)/3340000 - (7*x^2)/9475 + (42417727*x)/1265860000 + 1/33}
s = seq(1,30)
a = fx(s)
print(max(a))    # Max number of packages delivered
match (max(a),a) # Day max number delivered

# Problem 20
fx = function(x){-6*x^3 + 27*x^2 + 180*x}
fpx = Deriv(fx)
fppx = Deriv(fpx)
roots = uniroot.all(fpx, c(-100,100)) # determine x coords of critical points
print(roots)
fx(roots) # Show critical points

# problem 22
cx = function(x){8*x^2 + 221184 / x} # Cost function with one variable
dx = Deriv(cx)
roots = uniroot.all(dx, c(0,10000))
print(dx)
print(roots) # Gives critical points
ddx = Deriv(dx)
ddx(roots) # Positive; shows that it is a minimum
print(18432/roots^2) # Give missing height dimension using the volume equation

# Problem 23
cx = function(x){ 28.8* x + 52.8 * (1056 / x)}
dx = Deriv(cx)
print(dx) # Shows a non-quadratic as first derivative, so no roots to check, must solve to find critical point at dx = 0

# Problem 24
a = 67000
x = 7
x1 = 9
y = 37000
r = 1 - exp(1)^(log(y/a)/x)
print(r)
y1 = a * (1 - r) ^ x1
print(y1)


#Problem 27
antiD(380 - 4 * x ~ x) # Gives the integral
c = 1700 - 380*38 + 2*(38^2) # Calculate the constant from the integral
px = function(x){ 380 * x - 2*x^2 + c}
px(56)

#problem 29
pt = function(t){ 75 - 9 * t^.5}
integrate(pt,0,9)


#problem 30
x = function(x){6*sqrt(x) - 6*x^2} #Bounded area = integral of (higher function - lower function)
integrate(x,0,1) #region bounded between 0 and 1, where both functions intersect

#problem 32
fx = function(x){x*sqrt(x+7)}
integrate(fx,-7,2)

#problem 33
fx = function(x){46 * ((1-.22^x)/(1-.22))}
fx(20) #Finds the sum up to the 20th term of the geometric sum
46 / (1-.22) #Finds the infinite some of the geometric series

#problem 34
fx = function(x){3*exp(1)^(5*x-3)}
taylor(fx, 4, 5)
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