ON WAVEFORM CODING USING WAVELETS

*Prashant P. Gandhi, Sathyanarayan S. Rao and Ravikanth S. Pappu**

Department of Electrical and Computer Engineering
Villanova University, Villanova, PA 19085

Abstract

We consider a novel baseband waveform coding technique based on *wavelets*. Wavelets are recognized for their temporal and spectral localization and for their orthogonality across scale and location. We exploit these fundamental properties of wavelets and propose a wavelet-based modulator-demodulator structure to improve communication efficiency. Numerical results for bandwidth occupancy and bandwidth efficiency are given, and a detailed comparison between different families of wavelets is presented.

1 Introduction

Wavelets have received considerable attention recently in fields such as signal processing, mathematics, physics, and seismology. This popularity of wavelets is primarily due to the interesting structure they provide based on scale (or dilation) and location (or translation). In the signal processing community itself, potential use of wavelets is currently being investigated in speech analysis and compression, image processing, video compression, radar signal processing, feature extraction and many others. In this paper, we describe a promising application of wavelets in digital communications. In particular, we propose a novel baseband coding of waveforms using wavelets.

In digital communications, a primary goal is to increase bandwidth efficiency, i.e. bit rate per unit of bandwidth (in bits/sec/Hz), without increasing the per bit signal-to-noise ratio (SNR) or the bit error probability (BER). This bandwidth efficiency is upper bounded by the Shannon limit, and the goal is to reach as close to this limit as possible. Many different modulation and coding techniques have been investigated for this purpose (see [1]). These include binary and M-ary modulation schemes, waveform shaping and channel coding. For instance, using binary phase shift keying (BPSK) with full-width rectangular pulse and null-to-null definition for bandwidth, bandwidth efficiency of 1 bit/sec/Hz is achieved. For the same SNR and BER, bandwidth efficiency is doubled to 2 bits/sec/Hz using quadrature PSK (QPSK). Additional improvement over QPSK is attained by shaping rectangular pulses, leading to bandwidth efficient offset QPSK (OQPSK) and minimum shift keying (MSK) modulation schemes.

*R. Pappu is now with MIT Media Lab., Cambridge, MA

In this paper, we propose a waveform coding technique based on wavelets to improve bandwidth efficiency of a communication system. A wavelet is a signal or a waveform having desirable characteristics, such as localization in time and frequency, and orthogonality across scale and translation [2][3]. Because of these appealing properties, wavelets appear to be promising waveforms in communications. Specifically, in this paper, we use wavelets and the related scaling functions for continuous-time analog representation of data bits. We note that bit-by-bit coding using discrete shift-orthogonal wavelet sequences has been recently considered [4].

The paper is organized as follows. Basic concepts of wavelets and scaling functions and some of their useful properties are described in Section 2. Waveform coding using wavelets and scaling functions is described in Section 3 where block diagrams of transmitter and receivers are given and bit error rate and bandwidth efficiency are computed for several types of wavelets We end the paper with some concluding remarks in Section 4.

2 Basic Concepts of Wavelets

Briefly, let $\psi(t)$ be a (mother) wavelet. A dilated and translated wavelet $\psi_{j,k}(t)$ of $\psi(t)$ is given by

$$\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}(t - 2^j k)), \tag{1}$$

where 2^j and $2^j k$ indicate the amount of dilation and translation for some integers j and k. It can be shown that the inner product between $\psi_{j,k}(t)$ and $\psi_{m,n}(t)$ for some integers j, k, m, n is given by

$$\begin{aligned} < \psi_{j,k}(t), \psi_{m,n}(t) > &= \int_{-\infty}^{\infty} \psi_{j,k}(t)\psi_{m,n}(t)dt \\ &= \delta_{j-m}\delta_{k-n}, \end{aligned} \tag{2}$$

which implies that dilated and translated wavelets $\psi_{j,k}(t)$ for different (j, k) values are orthogonal to each other.

Wavelets are unit-energy bandpass functions. There also exists corresponding unit-energy lowpass functions,

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called *scaling functions* $\phi_{j,k}(t)$, which are generated from a mother function $\phi(t)$. Scaling functions are orthogonal across location but *not* across scale; that is, for each integer j and some integers k and n, we have

$$< \phi_{j,k}(t), \phi_{j,n}(t) >= \delta_{k-n}. \quad (3)$$

Further, for any $j \leq m$,

$$< \psi_{j,k}(t), \phi_{m,n}(t) >= \delta_{j-m}\delta_{k-n} \quad (4)$$

which implies that wavelets at certain dilations and arbitrary translations are orthogonal to dilated and translated scaling functions. This important feature of wavelets and scaling functions is exploited in Section 3 for baseband coding application. The above properties of wavelets and scaling functions can be derived using the theory of multiresolution signal analysis [2].

There exists many families of wavelets and scaling functions. For example, the well-known Haar wavelet is given by $\psi(t) = 1/2$ for $0 \leq t < 1/2$ and $= -1/2$ for $1/2 \leq t < 1$ and has corresponding scaling function $\phi(t) = 1$ for $0 \leq t < 1$. Orthogonality properties (2)-(4) can be easily verified in this case. Note that both Haar wavelet and corresponding scaling function are commonly used to represent digital information in communication systems where Haar scaling function is generally referred to as full-width rectangular pulse and Haar wavelet as biphase pulse.

Due to the discontinuity, however, the spectrum of the Haar wavelet does not decay rapidly. Wavelets that are smoother than the Haar wavelet do exist, and they offer better temporal as well as spectral localization. We consider two well-known wavelet families in this paper; they are commonly referred to as Daubechies family of wavelets and Lemarie-Battle wavelets.

Daubechies Wavelets [3] The Haar wavelet is an extreme example of Daubechies family of wavelets. A Daubechies mother wavelet D_M of order M, $M \geq 2$ and even, has following properties:

- it has finite temporal support in $[0, M-1]$,
- it is generated using an M-tap FIR filter, and
- its spectrum has a null at 2 Hz.

The wavelet D_2 is the Haar function and is the only discontinuous wavelet of this family. As the order M increases, the D_M wavelet becomes increasingly smoother and longer.

Lemarie-Battle Wavelet (see [2]) This wavelet is defined using cubic spline functions and is quite appealing in communications type application because it has rapidly decaying spectrum. Although theoretically an infinite-tap filter is required to generate it, a finite-tap filter of dominant coefficients is sufficient. We show in the next section that Lemarie-Battle wavelet significantly improves bandwidth efficiency when compared to the Daubechies family of wavelets.

3 Waveform Coding Using Wavelets

In the traditional BPSK modulator (see Figure 1), polar signaling is used to encode each bit prior to modulation. That is, a binary "1" is represented by a pulse $p(t)$ of duration T_b, and a binary "0" is represented by $-p(t)$. This polar signaling scheme is efficient in that, for given bit energy, it provides maximum separation between the two bit signals and thus yields the lowest probability of bit error among other binary modulation schemes (on-off, orthogonal, etc.).

Let $\{A_j\}$ be a sequence of statistically independent symbols, each assuming the value $\sqrt{E_b}$ for bit "1" or $-\sqrt{E_b}$ for bit "0", where E_b is the bit energy. A polar (baseband) signal based on pulse $p(t)$ may then be represented as

$$x_p(t) = \sum_{j=-\infty}^{\infty} A_j p(t - jT_b) \quad (5)$$

where T_b is the bit duration ($R_b = 1/T_b$ is the bit rate) and $p(t)$ is a unit-energy continuous-time pulse of duration T_b (e.g. the full-width rectangular pulse). The optimum receiver in this case is a matched filter with impulse response function $p(T_b - t)$, followed by sampler and a hard-limiter. In the case of binary phase shift keying (BPSK) modulation, baseband signal (5) is used to modulate the carrier $\cos(\omega_c t)$, where ω_c is the carrier frequency.

With $p(t)$ being the full-width rectangular pulse, we may say that information bits in (5) are coded using the D_2 scaling function. This D_2 scaling function however does not have appealing spectral characteristics; so it is natural to consider other scaling functions that do have desirable characteristics. Replacing $p(t)$ in (5) with a suitable scaling function, we get

$$x_\phi(t) = \sum_{j=-\infty}^{\infty} \frac{A_j}{\sqrt{T_b}} \phi(\frac{t - jT_b}{T_b}). \quad (6)$$

Note that each scaling function in (6) is dilated by T_b. For orthogonality property (3) to hold, clearly T_b must be a power of 2 (i.e. $T_b = 2^j$ for some j). Further, mother scaling functions (except D_2) that we consider here have temporal support larger than unity. Thus each dilated scaling function in (6) has temporal duration $T > T_b$. A D_M scaling function, for example, is of duration $T = (M-1)T_b$. Block diagrams of scaling function-based baseband modulator and demodulator are given in Figure 2. Figure 3 shows a typical polar signal representing bit sequence $\cdots$ 1 0 0 1 $\cdots$. The same bit sequence, coded using D_4 scaling functions, is shown in Figure 4(a).

Since information bits arrive at rate $R_b = 1/T_b$, it is easy to see that scaling functions corresponding to neighboring bits *overlap*. This is evident from Figure 4(a). The resulting polar signal is depicted in Figure 4(b). Decoding of bits is possible because, according to (3), any pair of scaling functions separated by integer multiple of

T_b are orthogonal. Specifically, at the receiver (see Figure 2), impulse response of the matched filter is given by $\phi(\frac{JT_b - t}{T_b})/\sqrt{T_b}$ where $T = JT_b$ is the duration of $\phi(t/T_b)$ for some integer $J > 1$. Assuming noiseless transmission medium, the matched filter output $y(t)$, sampled at $t = nT_b$, is

$$\begin{aligned} y(t = nT_b) &= x_\phi(t) * T_b^{-1/2}\phi(\frac{JT_b - t}{T_b})|_{t=nT_b} \\ &= \sum_{j=-\infty}^{\infty} \frac{A_j}{T_b} \int_{(n-J)T_b}^{nT_b} \phi(\frac{\tau - jT_b}{T_b})\phi(\frac{\tau - (n-J)T_b}{T_b})d\tau \\ &= A_{n-J} \end{aligned} \quad (7)$$

Clearly, from (7), information bits are recovered at the receiver at rate R_b after an initial delay of JT_b.

Bit error probability (BER), in the presence of additive white Gaussian noise, can also be computed in a similar manner. It is not surprising to see that, for equal bit energies, BER for (6) is *identical* to the one for (5). Hence, comparison between the two methods can only be based on their bandwidth utilizations.

The key features of waveform coding using scaling functions are

- neighboring waveforms overlap;
- decoder outputs are delayed by JT_b;
- output bit rate is identical to the input rate;
- T_b must be a power of 2; and
- BER is equal to the BER of BPSK scheme.

The above scheme can be generalized to include both scaling functions and wavelets as shown in Figure 5. Here the composite baseband signal is a sum of two polar signals - one coded using scaling functions and the other using wavelets:

$$x_{\phi\psi}(t) = \sum_{j=-\infty}^{\infty} \frac{A_j}{\sqrt{T_b}}\phi(\frac{t - jT_b}{T_b}) + \sum_{j=-\infty}^{\infty} \frac{B_j}{\sqrt{T_b}}\psi(\frac{t - jT_b}{T_b}). \quad (8)$$

Here zero-mean random variables $\{A_j\}$ and $\{B_j\}$ are assumed to be individually and jointly independent. A corresponding passband signal is obtained by modulating a single carrier $\cos(\omega_c t)$ using (8). Decoding is possible, as shown in Figure 5, due to the self- and cross-orthogonality properties of $\phi(t/\ T_b)$ and $\psi(t/\ T_b)$ functions. The bit error rate again remains equal to the BPSK scheme as we are dealing with two independent polar signals. Further, the data rate is doubled and so is the overall bandwidth, as seen in Figure 6(a).

Bandwidth Efficiency To compare different wavelet-based coding techniques, we employ the bandwidth efficiency (BE) measure defined by

$$BE = \frac{\text{Total Bit Rate}}{\text{Bandwidth}} \quad (\text{bits/sec/Hz}) \quad (9)$$

where we assume 99% power bandwidth. Power spectral densities (psd) of the composite scheme (8) for several Daubechies waveforms as well as Lemarie-Battle waveforms are shown in Figure 6(b). Clearly, psd of composite coding scheme based on Lemarie-Battle waveforms has better decay characteristics than those based on Daubechies waveforms. Numerical comparisions are given in the table given below. Here the 99% bandwidth (BW) is normalized with respect to the bit period T_b.

Wavelet	*BW*	*BE*
D_2 (Haar)	3.5	0.57
D_4	3.1	0.65
D_6	2.8	0.71
D_8	1.4	1.43
D_{10}	1.35	1.48
Lemarie-Battle	1.15	1.74

Table 1: Comparison of different coding techniques

As seen from the table, BE improves with higher orders of Daubechies waveforms; Lemarie-Battle waveforms give the best performance.

4 Concluding Remarks

We have proposed an interesting method for coding of waveforms using wavelets. Its operation is possible due to the scale and location orthogonality of wavelets and scaling functions. Further work in extending the proposed technique to M-ary QAM is underway.

References

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[4] M. Tzannes and M. Tzannes, "Bit-by-bit channel coding using wavelets," In *Proc. of GLOBECOM*, 1992.

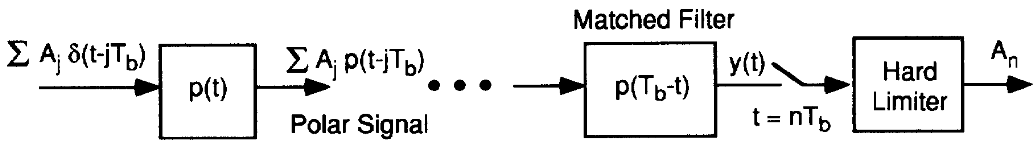

Figure 1: Block diagrams of baseband polar transmitter and receiver

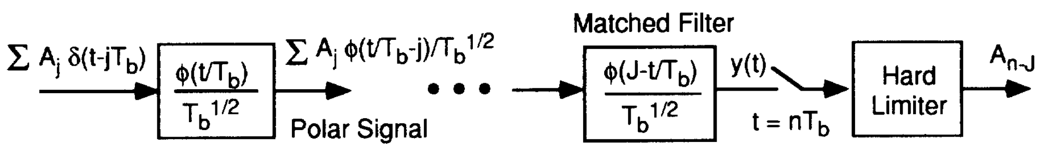

Figure 2: Block diagrams of transmitter and receiver based on scaling functions

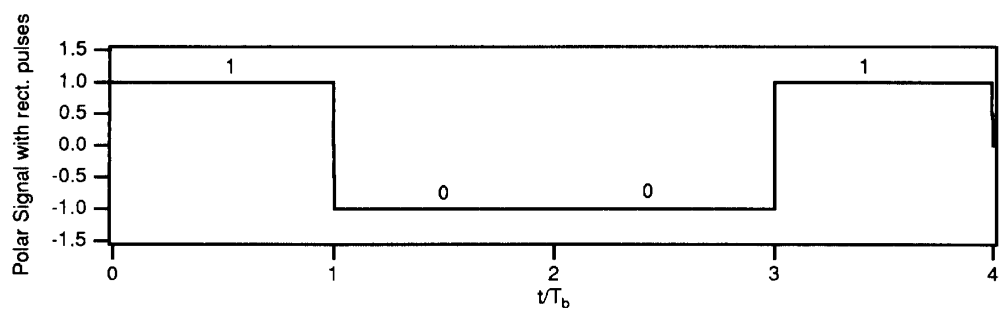

Figure 3: Polar Signal

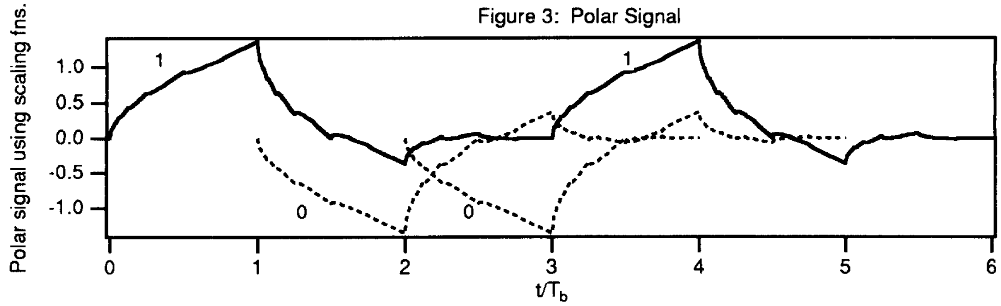

Figure 4a: D4 scaling functions corresponding to information bits

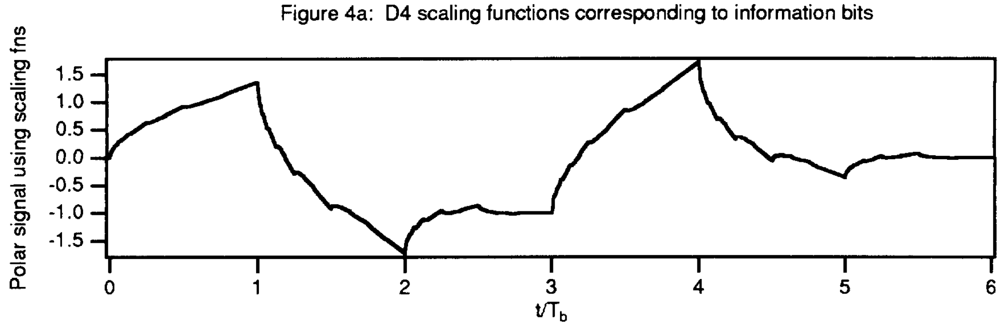

Figure 4b: Polar signal generated using D4 scaling functions

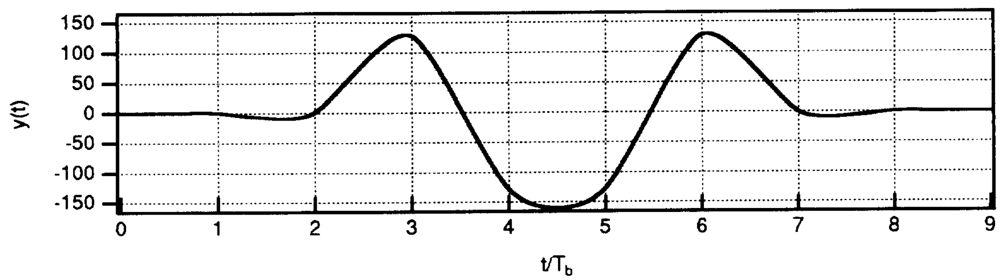

Figure 4c: Output of the matched filter

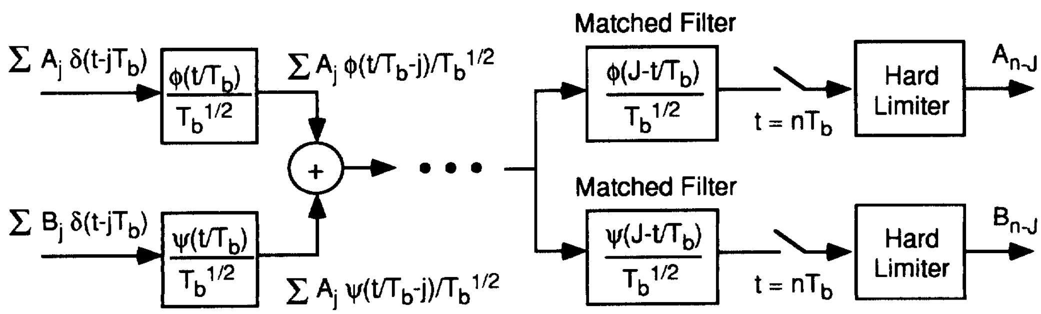

Figure 5: Block diagrams of transmitter and receiver based on scaling functions and wavelets

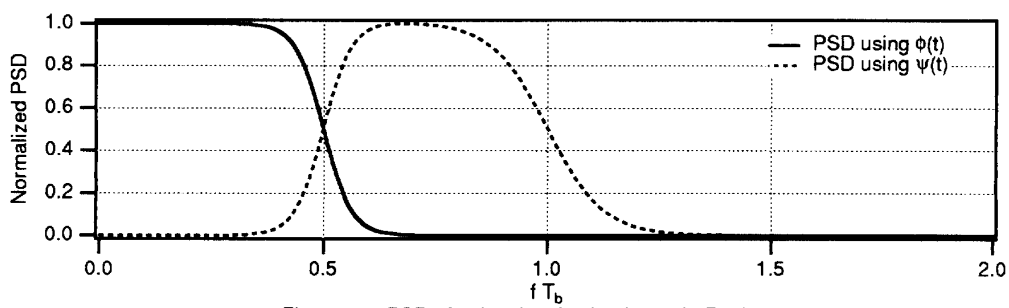

Figure 6a: PSD of polar signal using Lemarie-Battle $\phi(t)$ and $\psi(t)$

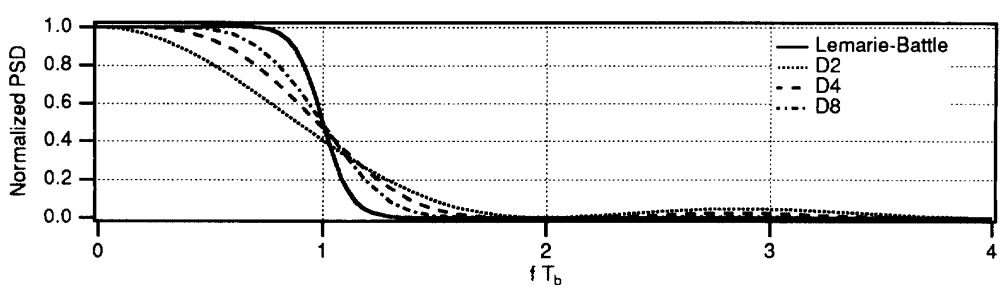

Figure 6b: Normalized PSD of composite signal