Bézier Curves

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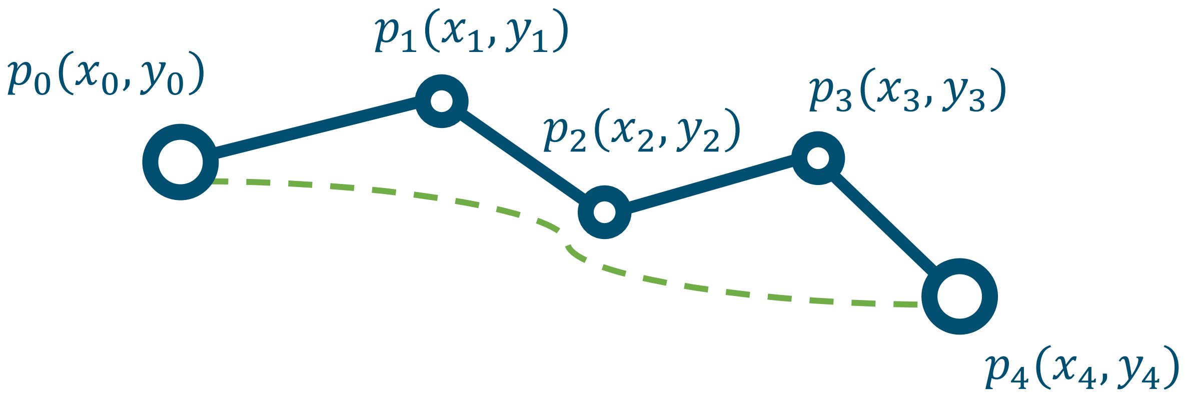
The Bézier curve is a parametric curve used in computer graphics and related fields. The curve associated with the Bernstein polynomial is named after Pierre Bézier, who used it in the 1960s to design curves for Renault car bodies.

Other uses include computer font design, animation, and motion planning. Bézier curves can be combined in the form of a Bézier-Spline or generalized to higher dimensions in the form of a Bézier surface.

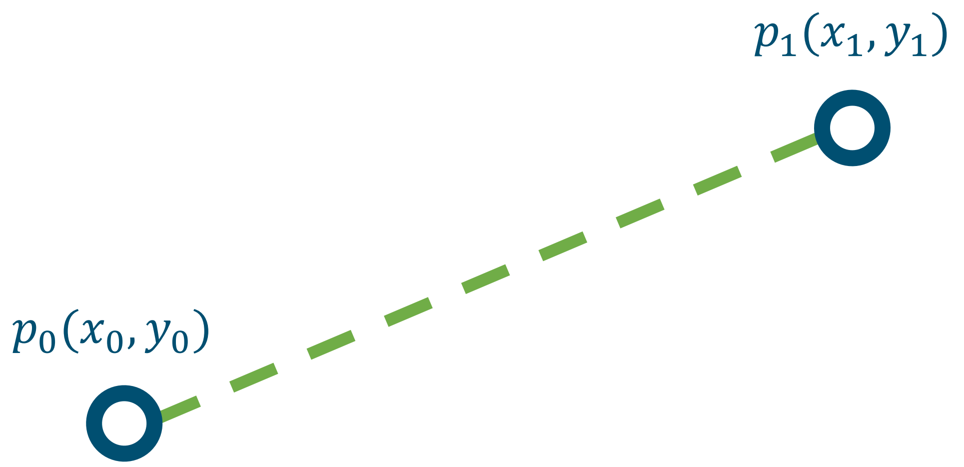
If we want to save a curve or surface, it is impractical to save each point. The solution is to somehow represent entire curves using a small amount of data.



Linear Bézier Curve

Definition.
Given two control points p_0 and p_1 we define the Linear Bézier Curve to be the curve parametrized by:

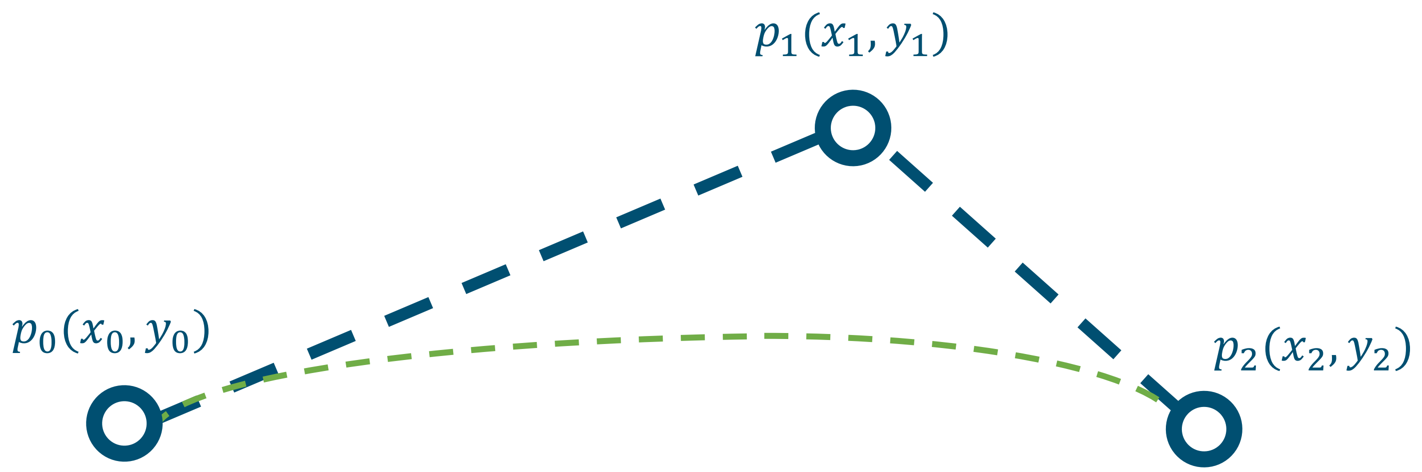
$$p(t) = (1-t)p_0 + tp_1, t \in [0,1]$$



Quadratic Bézier Curves

Definition.
Given three control points p_0, p_1 and p_2 we define the Quadratic Bézier Curve (Degree 2 Bézier Curve) to be the curve parametrized by:

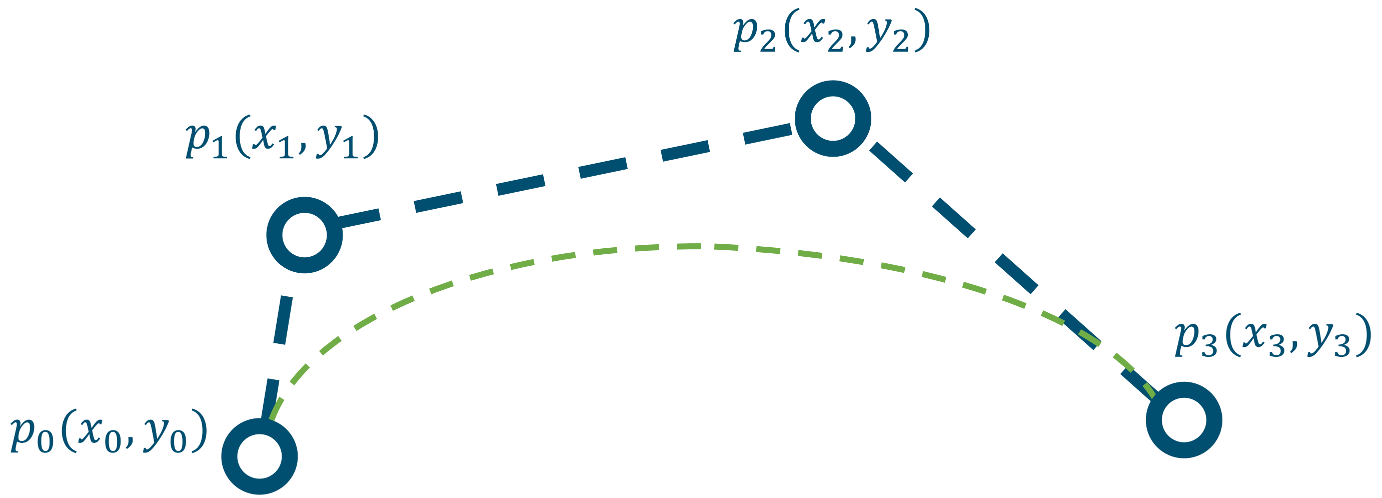
$$p(t) = (1-t)^2 p_0 + 2t(1-t)p_1 + t^2 p_2, t \in [0,1]$$



Cubic Bézier Curve

Definition.
Given four control points p_0, p_1, p_2 and p_3 we define the Cubic Bézier Curve (Degree 3 Bézier Curve) to be the curve parametrized by:

$$p(t) = (1-t)^3 p_0 + 3t(1-t)^2 p_1 + 3t^2(1-t)p_2 + t^3 p_3, t \in [0,1]$$



N – Degree Bézier Curve

Definition.
Given $n+1$ control points $p_0, p_1, \dots, p_n$ we define the degree n Bézier Curve to be the curve parametrized by:

$$p(t) = \sum_{i=0}^{n} \binom{n}{i} t^i (1-t)^{n-i} p_i \, , t \in [0,1]$$