The Economics of DeFi Lending: A Model of Smart Contract Parameter Choice

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Abstract

We propose a simple model to analyze the optimal design of a decentralized finance (DeFi) lending protocol. We focus on three key smart contract parameters: the loan-to-value ratio, liquidation threshold, and liquidation bonus rate. Our model reveals fundamental trade-offs between borrower leverage and lender security, with important implications for protocol governance and design. We show that the optimal liquidation threshold should be set to its maximum value, benefiting both borrowers and liquidity providers. However, this theoretical prediction stands in contrast to observed protocol implementations, where thresholds are typically set lower. The model also demonstrates how governance structure affects parameter choices: borrower-dominated governance leads to higher loan-to-value ratios, while liquidity provider-dominated governance leads to lower ratios. Additionally, we analyze the optimal liquidation bonus rate, revealing a crucial trade-off between ensuring timely liquidations and minimizing borrower costs. Higher bonuses increase liquidation probability, protecting liquidity providers, but raise costs for borrowers who face liquidation. The optimal bonus depends on governance preferences and the efficiency of liquidation mechanisms. These findings have significant implications for protocol design, particularly in light of the efficiency of modern liquidation mechanisms enabled by flash loans and automated bots. Our analysis provides a framework for understanding how protocol parameters are determined through the interaction of governance structure, market competition, and stakeholder interests.

1 Introduction

Decentralized finance (DeFi) lending protocols have emerged as a transformative innovation in financial markets, with over $50 billion in total value locked as of early 2024. These

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protocols enable users to borrow cryptocurrency against collateral without traditional intermediaries, using smart contracts to automate lending operations. A critical yet understudied aspect of these protocols is the choice of risk parameters that govern lending activities. Three key parameters—the loan-to-value (LTV) ratio, liquidation threshold, and liquidation bonus rate—fundamentally shape borrower behavior, lender security, and overall protocol stability. These parameters are not hardcoded but can be modified through protocol governance, typically implemented through a Decentralized Autonomous Organization (DAO). This creates an intriguing economic problem: how should these parameters be optimally set to balance the competing interests of different stakeholders?

The importance of this question is highlighted by recent empirical evidence. Wang (2024) demonstrates that relaxations in collateral requirements significantly affect borrower behavior, with highly leveraged positions increasing their exposure further. Mueller (2022) shows how blockchain transaction fees can temporarily hinder liquidations, leading to "zombie debt" positions that remain undercollateralized. These findings underscore the real economic consequences of parameter choices and the complex trade-offs they entail.

This paper develops a theoretical framework to analyze optimal parameter choice in DeFi lending protocols. We consider a model with three key participants: borrowers who deposit collateral to take loans, liquidity providers (LPs) who supply loanable funds, and the protocol governance (DAO) that sets parameters to maximize a weighted average of stakeholder welfare. Our framework explicitly incorporates the uncertainty in crypto-asset prices and the efficiency of liquidation mechanisms, allowing us to study how these factors influence optimal parameter choices.

Our analysis yields several important results. First, we show that the optimal liquidation threshold should be set to its maximum value, benefiting both borrowers through lower expected costs and LPs through increased collateral provision. This theoretical prediction stands in contrast to observed protocol implementations, where thresholds are typically set lower, suggesting potential inefficiencies in current practice. Second, we demonstrate that the optimal loan-to-value ratio creates a fundamental trade-off between borrower leverage and LP security, with the optimal choice depending critically on governance structure. Borrower-dominated governance leads to higher LTV ratios, while LP-dominated governance favors lower ratios, creating natural tensions in protocol governance.

Third, we analyze the optimal liquidation bonus rate under uncertainty, revealing a crucial trade-off between ensuring timely liquidations and minimizing borrower costs. Higher bonuses increase liquidation probability, protecting LPs, but raise costs for liquidated borrowers. The optimal bonus depends on governance preferences and liquidation mechanism efficiency. This finding has particular relevance given the emergence of flash loans and automated liquidation bots, which have fundamentally altered liquidation dynamics in DeFi markets.

Our paper contributes to several strands of literature. We build on Irresberger et al. (2024)'s framework for understanding DeFi lending protocols, particularly their emphasis on how governance calibrates risk parameters. Our theoretical predictions complement empirical work by Wang (2024) on borrower responses to parameter changes and Mueller (2022) on liquidation frictions. While a growing literature focuses on optimal interest rate models (See, for instance Rivera et al. (2023) on equilibrium dynamics and Bastankhah et al. (2024), Baude et al. (2025), and Bertucci et al. (2025) on adaptive mechanisms), we place the DAO's decision-making process regarding core risk parameters at the center of analysis, offering a novel perspective on the political economy of protocol design.

Our analysis also connects to the literature on systemic risk in DeFi markets. Lehar and Parlour (2022) document how liquidations can trigger negative feedback loops through price impacts, while Cohen et al. (2023) identify a "paradox of adversarial liquidation" where incentives chosen for protocol safety can encourage price manipulation. Warmuz et al. (2022) describe "toxic liquidation spirals" and bad debt incurred by protocols, while Chiu et al. (2022) demonstrate how rigid smart contract terms can introduce price-liquidity feedback loops that make the system vulnerable to sentiment-driven cycles. Our framework provides theoretical foundations for understanding how parameter choices influence these systemic risks and how governance structure affects the trade-offs involved.

The rest of the paper is organized as follows. Section 2 presents the model setup. Section 3.1 analyzes borrower incentives and optimal collateral choices. Section 3.2 examines liquidity provider payoffs. Section 4 derives optimal parameter choices. Section 5 extends the analysis to uncertain liquidation. Section 6 discusses implications and empirical predictions. Section 7 concludes.

2 Model Setup

Consider a two-period model ($t = 0, 1$) featuring three types of agents: borrowers, liquidity providers, and a decentralized autonomous organization (DAO) that sets the protocol parameters. A lending smart contract facilitates overcollateralized borrowing by borrowers and interest earnings by liquidity providers. The smart contract operates on two crypto-assets: the loanable currency and the collateral asset. The exogenous exchange rate, defined as the price of the loanable currency in terms of the collateral asset, is denoted by p and represents the sole source of uncertainty in the model.

The model is designed to capture key features of decentralized finance (DeFi) lending protocols while maintaining analytical tractability. For simplicity, we initially treat the interest rate (denoted i) and the liquidation bonus rate (i.e., the additional compensation awarded to liquidators, denoted b) as exogenously given. These parameters analyzed in greater detail in subsequent extensions of the model.

2.1 Agents and Timeline

2.1.1 DAO

At $t = 0$, the DAO chooses three key parameters:

- The loan-to-value ratio (λ), which determines the maximum amount that can be borrowed against collateral
- The liquidation threshold (δ), which determines when a position becomes eligible for liquidation
- The liquidation bonus rate (b), which determines the compensation rate awarded to liquidators

These parameters must satisfy $0 < \lambda < \delta < 1$ to ensure economic viability of the protocol.

2.1.2 Borrowers

Borrowers are risk-neutral agents who:

- At $t = 0$, want to borrow a fixed amount L and must decide how much collateral C to deposit (subject to the borrowing constraint $L \leq \lambda C$)
- Face an opportunity cost rate r_o for the capital used as collateral
- Pay the smart contract's interest rate i on the loan if no liquidation occurs
- Face additional costs if liquidation occurs, including a bonus rate b paid to liquidators

2.1.3 Liquidity Providers

LPs are risk-neutral agents who:

- At $t = 0$, provide funds to the lending pool
- Earn interest rate i on loans that are repaid
- Face potential losses if liquidation value is insufficient

2.2 Price Uncertainty and Returns

The exchange rate p_t follows a stochastic process with known distribution $F(p)$ and density $f(p)$. We assume:

- $p_t > 0$ for all t
- The distribution $F(p)$ is common knowledge

2.3 Liquidation Mechanism

At $t = 1$, a loan position faces liquidation if:

$$\frac{p_1 L(1+i)(1+b)}{C} > \delta \tag{1}$$

This occurs when the debt value relative to collateral exceeds the liquidation threshold.[1] When liquidation occurs:

- The loan is automatically repaid by the smart contract through a liquidation mechanism, which deducts the corresponding amount from the borrower's collateral.

[1] We include the bonus in the liquidation rule for analytical simplicity but could be removed without loss of generality. For instance, we note that in protocols such as AAVE the liquidation rule does not include the bonus. See AAVE (2020).

- Under this mechanism, third-party liquidators repay the outstanding loan and, in return, receive a liquidation bonus at rate b applied to the liquidated amount. We assume the bonus provides sufficient incentive to ensure prompt liquidation. If the value of the collateral is insufficient to cover the debt, the entire collateral is liquidated.

- LPs recover the lesser of the outstanding debt or the collateral value, with each amount adjusted to account for the liquidation bonus.

In order for the liquidation mechanism to work efficiently, the DAO must set the liquidation bonus b to a value that is sufficiently high to incentivize liquidation. As a simplyfing assumption, we assume that κ represents the cost a liquidator might face for liquidating a unit of loan value (e.g., \$1 of debt), with κ following a known stochastic process with distribution $G(\kappa)$.

2.4 Payoff Structure

The payoff structure depends on three key events that can be expressed in terms of the exchange rate price p_1:

1. Whether liquidation occurs ($p_1 > \frac{\delta C}{(1+i)(1+b)L}$)
2. Whether the liquidation occurs after debt exceeds the collateral value ($p_1 > \frac{C}{(1+i)(1+b)L}$)
3. Whether the position is liquidated before debt exceeds the collateral value ($\frac{\delta C}{(1+i)(1+b)L} < p_1 \leq \frac{C}{(1+i)(1+b)L}$)

These events determine the borrower's costs and the payoffs to both borrowers and LPs.

3 Analysis

3.1 The Borrower's Cost Minimization Problem

The borrower's objective is to choose the optimal amount of collateral to deposit, C, so as to minimize the expected borrowing cost, subject to the protocol's loan-to-value constraint.

The borrowing costs include the opportunity cost of providing collateral ($r_o C$), plus the following contingent costs:

- $p_1 L i$: The interest cost if no liquidation occurs

- $p_1 L(i + b + ib)$: The cost if liquidation occurs while collateral is sufficient to cover the debt. Notice in this case the borrower keeps the loan ($p_1 L$) but loses the value of debt plus the liquidation costs ($p_1 L(1 + i)(1 + b)$) from the collateral deposited in the protocol

- $C - p_1 L$: The cost if liquidation occurs when collateral is insufficient to cover the debt. Notice in this case the borrower loses the entire collateral deposited in the protocol.

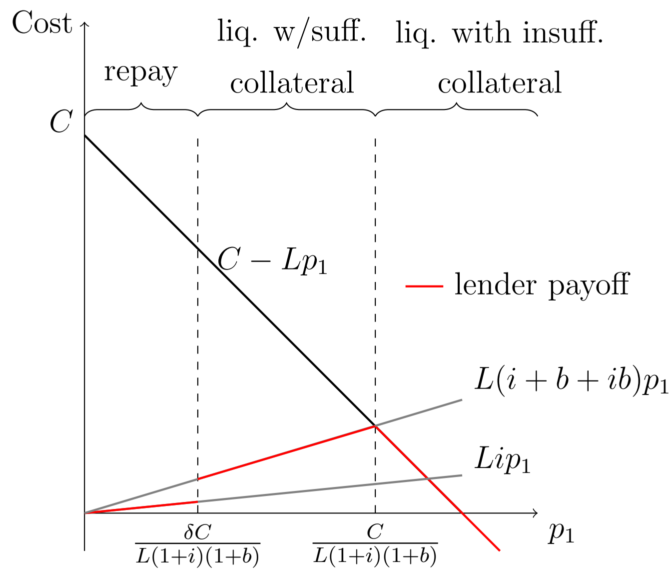

Figure 1: Borrower's cost structure and payoff regions. The graph shows the borrower's cost (in red) across different price regions: repayment (no liquidation), liquidation with sufficient collateral, and liquidation with insufficient collateral. The vertical dashed lines indicate the price thresholds that separate these regions.

The contingent costs are shown in the Figure 1.

Notice that at low values of p_1, the costs increase linearly with p_1. There is also a jump at the liquidation threshold, given by the added costs of the liquidation. Finally, the borrower's costs decrease when the debt exceeds the collateral value. This could eventually turn to a benefit if p_1 is sufficiently high.

The cost minimization problem can be formulated as:

$$\begin{aligned}\min_{C}\ E[\mathsf{BC}] =& r_o C + P\left(p_1 \le \frac{\delta C}{(1+i)(1+b)L}\right) p_1 L i \\ &+ P\left(\frac{\delta C}{(1+i)(1+b)L} < p_1 \le \frac{C}{(1+i)(1+b)L}\right) p_1 L(i+b+ib) \\ &+ P\left(p_1 > \frac{C}{(1+b)(1+i)L}\right)(C - p_1 L) \\ \text{s.t. } & L \le \lambda C\end{aligned} \tag{2}$$

Proposition 1. *For some parameter and distribution assumptions, the borrower's problem has an interior solution, meaning that the borrower will provide more collateral than what is required by the protocol's loan-to-value ratio ($C^* > \frac{L}{\lambda}$). If the marginal costs are sufficiently high, the borrower will choose to provide the minimum collateral required by the protocol's loan-to-value ratio (i.e., $C^* = \frac{L}{\lambda}$).*

Sketch. The proof follows from analyzing the Kuhn-Tucker conditions of the borrower's cost minimization problem. When the constraint is non-binding ($L < \lambda C$), setting $\mu = 0$ in the first-order condition reveals that the optimal collateral amount balances several effects:

- The marginal cost for providing collateral.
- The lower interest cost due to the increase in the probability of non-liquidation.
- The increased costs of liquidation in the event that the loan is liquidated before the debt exceeds the value of the collateral.
- The higher loss in the event that collateral is completely liquidated.

For some parameter configurations, these effects can exactly offset each other at a point where $L < \lambda C$, leading to an interior solution. See section A.1 in Appendix A for the complete proof. □

3.1.1 Effects of Protocol Parameters on the Borrower's Incentives

We now analyze how changes in the protocol parameters affect the borrower's decisions. We start by analyzing the effect of changing the liquidation threshold δ.

Proposition 2. *The borrower's expected cost decreases with the liquidation threshold δ. Specifically, $\frac{\partial}{\partial \delta} E[\mathsf{BC}] < 0$.*

This result is intuitive: a higher liquidation threshold reduces the probability of liquidation, thereby reducing the expected costs associated with liquidation events. The proof follows from analyzing the derivative of the cost function with respect to δ, which shows that all terms in the derivative are negative. See Section A.2 in Appendix A for the complete derivation.

Proposition 3. *The optimal collateral amount increases with the liquidation threshold. Specifically, $\frac{dC}{d\delta} > 0$.*

This result shows that borrowers optimally respond to higher liquidation thresholds by increasing their collateral. While higher thresholds reduce liquidation risk, they also allow borrowers to take on more risk. The optimal response is to increase collateral to maintain the same level of risk protection for the protocol. The proof follows from analyzing the implicit derivative of the first-order condition with respect to δ. See Section A.3 in Appendix A for the complete derivation.

We now analyze the effect of changing the loan-to-value ratio λ.

Proposition 4. *The effect of the loan-to-value ratio λ on the borrower's cost depends on whether the constraint $L \leq \lambda C$ is binding in the borrower's cost minimization problem:*

1. *If the constraint is not binding (interior solution), then $\frac{\partial E[\mathsf{BC}]}{\partial \lambda} = 0$*
2. *If the constraint is binding, then $\frac{\partial E[\mathsf{BC}]}{\partial \lambda} < 0$, as increasing λ allows the borrower to provide less collateral while maintaining the same loan amount*

Sketch. The proof follows from analyzing the borrower's cost minimization problem with the constraint $L \leq \lambda C$. When the constraint is not binding, changes in λ have no effect on the optimal solution. When the constraint is binding, the first-order condition implies that $\frac{\partial E[\mathsf{BC}]}{\partial C} > 0$ at the optimum. This means that increasing λ relaxes the constraint, allowing the borrower to reduce their collateral while maintaining the same loan amount, thereby reducing their expected cost. See Section A.4 in Appendix A for the complete proof. □

The first implication of this result is that increases in λ may not always imply changes in borrowers behavior. If the borrower is already overcollateralizing its position, there will be no change to the expected cost. However, if this is not the case, increasing λ will reduce the borrower's expected cost by allowing them to provide less collateral while maintaining the same loan amount.

Proposition 5. *The borrower's expected cost increases with the liquidation bonus rate b. Specifically, $\frac{\partial E[\mathsf{BC}]}{\partial b} > 0$.*

This result is intuitive: a higher liquidation bonus rate both increases the cost of liquidation and the probability of liquidation, thereby increasing the expected cost of the borrower. The proof follows from analyzing the derivative of the cost function with respect to b. See Section A.5 in Appendix A for the complete derivation.

3.2 Liquidity Provider Payoffs

The liquidity provider (LP) deposits loanable currency with the expectation of receiving an interest rate i. The payoff for the LP is guaranteed as long as the collateral is sufficient to cover for the lender's debt and liquidation costs. If this does not happen, the LP will receive only what can be recovered from the collateral after liquidation costs.

Therefore, the contingent payoffs for the LP, in collateral currency units, are the following:

- Lip_1: Obtains interest i in the scenarios where there is repayment (no liquidation) or there is a liquidation before debt exceeds the collateral value (plus liquidation costs) ($p_1 < \frac{C}{L(1+i)(1+b)}$)
- $\frac{C}{(1+b)}$ when the collateral cannot cover the the lender's debt.

Therefore the expected return for a lender that deposits L is

$$E[\mathsf{I}] = p_1 Li \cdot P\left(p_1 \leq \frac{C}{(1+i)(1+b)L}\right) + \frac{C}{(1+b)} \cdot P\left(p_1 > \frac{C}{(1+i)(1+b)L}\right) \tag{3}$$

The expected return for the LP is shown in the Figure 2.

3.3 Effects of Protocol Parameters in the Liquidity Provider Payoff

Proposition 6. *A higher collateral provision increases the expected return for the LP.*

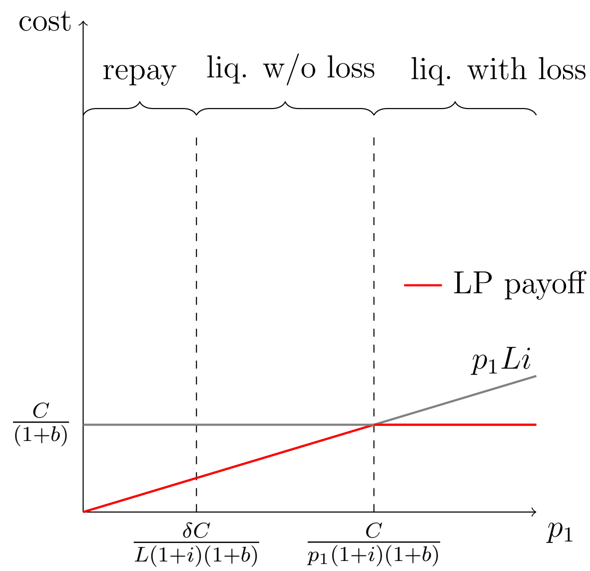

Figure 2: Liquidity provider's payoff. The graph shows the liquidity provider's payoff in collateral currency units across different price regions: repayment (no liquidation), liquidation before debt exceeds the collateral value, and liquidation after debt exceeds the collateral value. The vertical dashed lines indicate the price thresholds that separate these regions.

A higher collateral provision implies higher interest payments in the case there is repayment (or early liquidation) and a higher recovery amount in case of liquidation (debt exceeding the collateral value case).

The proof follows from analyzing the derivative of the expected return function with respect to collateral. See Section A.6 in Appendix A for the complete derivation.

Corollary 6.1. *A higher liquidation threshold increases the expected return for the LP.*

This follows from Proposition 3 where we showed that increases in the liquidation threshold increase the collateral provision.

Proposition 7. *If the borrower its optimally allocating collateral above of what is required by the protocol (interior solution) there will be no marginal effect on the LP expected return from changing the loan-to-value parameter (λ). When the collateralization requirement is binding, increasing λ implies a reduction the LP expected return ($\frac{\partial E[\mathsf{l}]}{\partial \lambda} < 0$).*

Increasing λ implies less collateral required per value loan, meaning a reduction in the required optimal collateralization of the borrower (Proposition 4). Since we have shown that collateralization increases the expected return of the LP (Proposition 6), therefore increasing λ means a reduction in the LP expected return.

Proposition 8. *The expected return for the LP decreases with the liquidation bonus rate b. Specifically, $\frac{\partial E[\mathsf{l}]}{\partial b} < 0$.*

This result can be observed in Figure 2. A higher liquidation bonus rate both decreases the recovery payoff in the event the collateral is insufficient to cover the debt, and increases the probability of such outcome. Meanwhile, the payoff does not change when the liquidation happens while the collateral is sufficient.

4 Optimal Parameter Choice

4.1 The DAO's Problem

The DAO's objective function reflects the interests of the protocol community, which comprises both the borrower and the LP. The governance chooses parameters λ, δ and b to maximize the weighted surplus:

$$W(\lambda, \delta, b) = \alpha(-E[\mathsf{BC}])(\lambda, \delta, b) + (1-\alpha)E[\mathsf{I}](\lambda, \delta, b) \tag{4}$$

where $\alpha \in [0, 1]$ represents the relative weight given to borrower interests in the governance process. This weight could reflect either the explicit voting power distribution in the DAO or the implicit influence of different stakeholder groups. Note that we maximize the negative of the borrower's cost (-E[BC]) since we have been working with the borrower's cost minimization problem throughout the paper.

4.2 Participation Constraints

The optimization is subject to participation constraints that ensure both borrowers and LPs find it optimal to use the protocol:

$$E[\mathsf{BC}](\lambda, \delta) \leq \mathsf{BC}^0 \tag{PC-B}$$
$$E[\mathsf{I}](\lambda, \delta) \geq \mathsf{I}^0 \tag{PC-I}$$

where BC^0 and I^0 represent the outside options (costs and expected returns respectively) for the borrower and the LP respectively. These outside options might include alternative DeFi lending protocols, centralized exchanges, etc.

In addition, the DAO must ensure that the liquidation threshold is sufficiently high to incentivize liquidation. This is captured by the following constraint:[2]

$$b \geq \bar{\kappa} \tag{PC-L}$$

where $\bar{\kappa} = \sup\{\kappa : G(\kappa) < 1\}$ denotes the essential supremum (maximum value in the support) of κ under the distribution G.

4.3 Optimal Parameter Choice

We now analyze the optimal parameter choice for the DAO.

Proposition 9. *The optimal liquidation threshold δ^* is equal to 1, the maximum possible value.*

Sketch. The proof follows from analyzing the DAO's optimization problem with respect to δ. From Proposition 2, we know that $\frac{\partial E[\mathsf{BC}]}{\partial \delta} < 0$, meaning the borrower's cost decreases with

[2] We allow for the possibility of setting lower values in Section 5.

δ. From Corollary 6.1, we know that $\frac{\partial E[\mathsf{I}]}{\partial \delta} > 0$, meaning the LP's expected return increases with δ.

Since both the borrower's cost decreases and the LP's return increases with δ, the participation constraints will never bind for $\delta < 1$. This means the objective function is strictly increasing in δ for all $\delta < 1$, and therefore the optimal value is $\delta^* = 1$. See Section A.7 in Appendix A for the complete proof. □

Proposition 10. *The optimal loan-to-value ratio λ^* depends on the relative weights of borrower and LP interests in the DAO's objective function:*

1. *If the borrower's weight α is sufficiently high, λ^* will be set to its maximum value (1) to minimize borrower costs*

2. *If the LP's weight $(1 - \alpha)$ is sufficiently high, λ^* will be set to its minimum value to maximize LP returns*

3. *For intermediate values of α, λ^* will be determined by the binding participation constraint*

Sketch. The proof follows from analyzing the DAO's optimization problem with respect to λ. From Proposition 4, we know that $\frac{\partial E[\mathsf{BC}]}{\partial \lambda} < 0$ when the constraint is binding, and from Proposition 7, we know that $\frac{\partial E[\mathsf{I}]}{\partial \lambda} < 0$ when the constraint is binding.

The DAO's first-order condition with respect to λ is:

$$\frac{\partial W}{\partial \lambda} = -\alpha \frac{\partial E[\mathsf{BC}]}{\partial \lambda} + (1 - \alpha) \frac{\partial E[\mathsf{I}]}{\partial \lambda} \tag{5}$$

Since both derivatives are negative when the constraint is binding, the sign of $\frac{\partial W}{\partial \lambda}$ depends on the relative weights α and $(1 - \alpha)$. This creates a trade-off between borrower costs and LP returns, leading to the three cases described in the proposition. See Section A.8 in Appendix A for the complete proof. □

Proposition 11. *The optimal liquidation bonus b^* is equal to $\bar{\kappa}$, the minimum bonus value that guarantees liquidation in the highest liquidation cost scenario.*

Proof. This result follows from analyzing the DAO's optimization problem with respect to b. From Proposition 5, we know that $\frac{\partial E[\mathsf{BC}]}{\partial b} > 0$, meaning the borrower's cost increases with b. From Proposition 8, we know that $\frac{\partial E[\mathsf{I}]}{\partial b} < 0$, meaning the LP's expected return decreases with b. This implies that the optimal bonus is the minimum bonus that satisfies the liquidation constraint (PC-L). □

4.4 Comparative Statics and Predictions

This framework generates several testable predictions about protocol parameter choices:

1. **Governance Composition:** Higher values of α (greater borrower influence) lead to:
   - Higher loan-to-value ratios (λ)

2. **Competition Effects:** Increased competition (higher BC^0 or I^0) leads to:
   - Parameter choices that favor the group with more competitive outside options
   - Possible specialization across protocols in serving different user types

These predictions suggest that observed parameter choices should systematically vary with:

- The composition of protocol governance
- The competitive environment
- The relative bargaining power of borrowers versus LPs

The model thus provides a framework for understanding how protocol parameters are determined through the interaction of governance structure, market competition, and stakeholder interests.

5 Setting Incentives for Liquidators

In this section, we extend our model to analyze the choice of the liquidation bonus, b. We relax the assumption that liquidations are always successful if the bonus is sufficiently high. Instead, we introduce a more realistic scenario where the probability of a successful liquidation, denoted by $q(b)$, is an increasing function of the bonus rate ($q'(b) > 0$). This modification acknowledges that a higher bonus provides a stronger incentive for liquidators to act, but does not guarantee execution, for instance due to network congestion further increasing liquidation costs or other frictions.

This change introduces a crucial trade-off for the DAO. A higher bonus b increases the likelihood of successful liquidation, which protects liquidity providers from losses. However, it also raises the expected cost for borrowers who are liquidated. Conversely, a lower bonus reduces costs for liquidated borrowers but increases the risk for LPs that undercollateralized positions may not be liquidated in a timely manner. The DAO's task is to set an optimal bonus that balances these competing interests.

5.1 Borrower's Cost with Uncertain Liquidation

With uncertain liquidation, the borrower's costs are modified. When a position becomes eligible for liquidation (i.e., $p_1 > \frac{\delta C}{L(1+i)(1+b)}$), it is successfully liquidated with probability $q(b)$. If liquidation fails, with probability $1-q(b)$, the borrower may be able to simply repay the loan and avoid liquidation penalties. This is advantageous for the borrower. However, if the collateral value falls below the debt value ($p_1 > \frac{C}{L(1+i)}$), the borrower will default regardless of whether the liquidation is triggered, losing the entire collateral. The borrower's costs under these scenarios are summarized in Table 1.

Proposition 12. *Under uncertain liquidation, the expected cost for the borrower increases with the liquidation bonus b. Specifically, $\frac{\partial E[\mathsf{BC}]}{\partial b} > 0$.*

Table 1: Borrower's Cost with Uncertain Liquidation

Scenario	**Price Range**	**Cost (Liquidation Succeeds)**	**Cost (Liquidation Fails)**
Sufficient collateral for liquidation costs	$\frac{\delta C}{L(1+i)(1+b)} < p_1 \leq \frac{C}{L(1+i)(1+b)}$	$p_1 L(i+b+ib)$	$p_1 Li$
Insufficient collateral for bonus, sufficient for loan	$\frac{C}{L(1+i)(1+b)} < p_1 \leq \frac{C}{L(1+i)}$	$C - p_1 L$	$p_1 Li$
Borrower defaults	$p_1 > \frac{C}{L(1+i)}$	$C - p_1 L$	$C - p_1 L$

Sketch. The effect of increasing b can be deduced by comparing with the baseline model in Section 3.1. Since increasing b increases the likelihood of liquidation, and this implies higher costs in some scenarios (See 1), it follows that increasing b increases the expected costs for borrowers. The baseline model probability effects are maintained. For a complete proof, see Section A.9 in Appendix A. □

5.2 Liquidity Provider's Payoff with Uncertain Liquidation

The LP's payoffs are also affected by liquidation uncertainty. If liquidation is not triggered, or if it is triggered but the collateral is sufficient to cover debt and bonus, the LP's payoff is unaffected. This happens because, independently of the liquidation, the position is repaid. However, when the collateral value is insufficient to cover the full debt plus bonus, the LP's payoff depends on whether liquidation succeeds. If it fails, the borrower might still repay the loan, which is beneficial for the LP compared to a partial recovery from a costly liquidation. If the borrower defaults, a failed liquidation implies the LP will only recover the collateral value at a later, potentially less favorable, time, which we model as a discounted recovery. The LP's payoffs are summarized in Table 2.

Table 2: Liquidity Provider's Payoff with Uncertain Liquidation

Scenario	**Price Range**	**Payoff (Liquidation Succeeds)**	**Payoff (Liquidation Fails)**
Sufficient collateral for liquidation	$p_1 \leq \frac{C}{L(1+i)(1+b)}$	$p_1 L(1+i)$	$p_1 L(1+i)$
Insufficient collateral for bonus	$\frac{C}{L(1+i)(1+b)} < p_1 \leq \frac{C}{L(1+i)}$	$C/(1+b)$	$p_1 L(1+i)$
Borrower defaults	$p_1 > \frac{C}{L(1+i)}$	$C/(1+b)$	$d \cdot C/(1+b)$

In the default scenario, $d \in (0,1)$ represents a discount factor on the recovered collateral value in case of a failed liquidation, reflecting the costs and risks of delayed recovery. The effect of the bonus b on the LP's expected payoff is now ambiguous. A higher b increases the probability of a successful (but costly) liquidation while reducing the probability of a failed liquidation, which could be more or less favorable depending on the price level.

5.3 Optimal Liquidation Bonus Choice

The DAO must choose the bonus rate b to maximize its objective function, considering the trade-off between borrower costs and LP returns under uncertainty. The optimal bonus b^* will balance the marginal benefit of increasing liquidation probability for LPs against the marginal cost imposed on borrowers. This choice will depend on the governance weight α, the specific form of the liquidation probability function $q(b)$, and the distribution of the asset price. A protocol with a higher weight on LP interests (lower α) would be expected to set a higher bonus to ensure greater security, at the expense of higher costs for borrowers.

6 Discussion

Our theoretical framework generates several important implications for DeFi lending protocol design and governance. The model's predictions regarding optimal liquidation thresholds and loan-to-value ratios provide insights into the trade-offs faced by protocol designers and governance participants.

6.1 Theoretical Predictions versus Protocol Implementations

A key finding of our model is that the optimal liquidation threshold should be set to its maximum value of 1. This prediction stands in contrast to observed protocol implementations, where liquidation thresholds are typically set below 1, and often below the liquidation bonus-adjusted value of 1/(1+b). This discrepancy warrants careful consideration.

Protocols commonly set different thresholds for different collateral types, with the stated aim of preventing scenarios where price increases above the threshold eliminate repayment incentives. However, our model shows that such concerns may be misplaced, as the threshold does not alter the underlying probability distribution. Partial liquidations, when they occur, impose costs on borrowers while benefiting liquidators.

Also an implicit assumption that should be clarified in our model is related to the abscence of lender default. In our model there is no lender default while there are incentives to repayment (i.e. $P_1 < \frac{C}{L(1+i)}$). The existence of services that allow the borrower to automatically repay the loan before positions are liquidated justifies this assumption. This is native to the DeFi ecosystem and overcollateralized lending.

6.2 Governance and Parameter Choice

Our model highlights the fundamental trade-offs in setting protocol parameters, particularly the loan-to-value ratio. Higher LTV ratios benefit borrowers by allowing more leverage, while lower LTV ratios benefit liquidity providers by ensuring greater security. The optimal LTV ratio depends on the relative influence of different stakeholders in protocol governance, creating a natural tension in DAO governance between borrower and LP interests.

The model's predictions regarding governance structure have important implications for protocol design. Borrower-dominated governance (high α) leads to higher LTV ratios, while LP-dominated governance (low α) leads to lower LTV ratios. This suggests that protocol governance should carefully consider stakeholder representation and the potential impact on parameter choices.

6.3 Competition and Protocol Design

The model also provides insights into how competition affects protocol parameter choices. Increased competition (higher outside options) will lead to parameter choices favoring the group with better alternatives. This may result in protocols specializing in serving different user types based on competitive pressures. Parameter choices should reflect the relative bargaining power of borrowers versus LPs, which can be influenced by the competitive landscape.

6.4 Liquidation Incentives

The efficiency of liquidation mechanisms in DeFi protocols is another important consideration. The widespread availability of flash loans and automated liquidation bots ensures that any liquidation opportunity is likely to be exploited with minimal delay, but, as we have discussed, this can be guaranteed as far as enough incentives are provided to liquidators. DAOs need to set incentives in advance, and as we have shown this leads to setting a bonus that is sufficiently high to guarantee liquidation in stress scenarios. This point has been mentioned by Cohen et al. (2023), who add that high bonuses may also incentivize malicious price manipulation in less costly liquidation scenarios. In our model, we show that the lower bonuses will benefit borrowers, but at the cost of higher liquidation costs for LPs.

7 Conclusion

This paper provides a theoretical framework for understanding parameter choice in DeFi lending protocols. Our results highlight the fundamental trade-offs between borrower leverage and LP security, and how these trade-offs are resolved through DAO governance. The model generates testable predictions about how protocol parameters should vary with stakeholder composition and market conditions.

The analysis reveals several important implications for protocol design and governance. First, the optimal liquidation threshold should be set to its maximum value, though this stands in contrast to current protocol implementations. Second, the loan-to-value ratio creates a natural tension between borrower and LP interests, with the optimal choice depending on governance structure. Third, competition between protocols can lead to specialization and parameter choices that reflect the relative bargaining power of different stakeholders.

7.1 Next Steps and Research Directions

There are several avenues for future research related with the analysis presented in this paper:

1. **Empirical Testing:** The model's predictions can be tested using on-chain data from major DeFi lending protocols. This includes analyzing the relationship between governance structure and parameter choices, the effects of parameter changes, studying the impact of competition on parameter choices, and examining actual collateralization behavior. We are currently working on this.

2. **Model Extensions:** The framework could be extended by endogenizing interest rates, and the impact of multiple collateral types. This would provide a more comprehensive understanding of protocol design choices.

These research directions would help bridge the gap between theoretical predictions and observed protocol implementations, providing valuable insights for protocol designers and governance participants.

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A Detailed Proofs

A.1 Proof of Proposition 1: Optimal Overcollateralization by Borrowers

Here we show that for some parameters and density function configurations, the borrower's problem has an interior solution, meaning that the borrower will borrow less than the maximum allowed by the protocol's loan-to-value ratio. To solve the borrower's cost minimization problem, we use the Kuhn-Tucker conditions. We first define the Lagrangian:

$$\begin{aligned}\mathcal{L}(C,\mu) =& r_o C + P\left(p_1 \leq \frac{\delta C}{(1+i)(1+b)L}\right) p_1 L i \\ &+ P\left(\frac{\delta C}{(1+i)(1+b)L} < p_1 \leq \frac{C}{(1+i)(1+b)L}\right) p_1 L(i+b+ib) \\ &+ P\left(p_1 > \frac{C}{(1+b)(1+i)L}\right)(C - p_1 L) + \mu(L - \lambda C)\end{aligned} \tag{6}$$

The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial C} = 0 \tag{7}$$

$$\mu(L - \lambda C) = 0 \tag{8}$$

$$\mu \geq 0 \tag{9}$$

$$L \leq \lambda C \tag{10}$$

According to the complementary slackness condition, if $\mu > 0$, then $L = \lambda C$ (the constraint is binding), and if $\mu = 0$, the optimal C is determined solely by the first-order condition. Taking the partial derivative with respect to C and applying the product and chain rules:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial C} = r_o &+ \frac{\partial}{\partial C}\left[P\left(p_1 \leq \frac{\delta C}{(1+i)(1+b)L}\right) p_1 L i\right] \\ &+ \frac{\partial}{\partial C}\left[P\left(\frac{\delta C}{(1+i)(1+b)L} < p_1 \leq \frac{C}{(1+i)(1+b)L}\right) p_1 L(i+b+ib)\right] \\ &+ \frac{\partial}{\partial C}\left[P\left(p_1 > \frac{C}{(1+b)(1+i)L}\right)(C - p_1 L)\right] - \mu\lambda\end{aligned} \tag{11}$$

Using the product rule and chain rule for each term:

$$\frac{\partial}{\partial C}\left[P\left(p_1 \leq \frac{\delta C}{(1+i)(1+b)L}\right)p_1 Li\right] = p_1 Li \cdot f\left(\frac{\delta C}{(1+i)(1+b)L}\right)\cdot\frac{\delta}{(1+i)(1+b)L} \tag{12}$$

$$\begin{aligned}\frac{\partial}{\partial C}&\left[P\left(\frac{\delta C}{(1+i)(1+b)L} < p_1 \leq \frac{C}{(1+i)(1+b)L}\right)p_1 L(i+b+ib)\right]\\ &= p_1 L(i+b+ib)\cdot\left[f\left(\frac{C}{(1+i)(1+b)L}\right)\cdot\frac{1}{(1+i)(1+b)L}\right.\\ &\qquad\left.-f\left(\frac{\delta C}{(1+i)(1+b)L}\right)\cdot\frac{\delta}{(1+i)(1+b)L}\right]\end{aligned} \tag{13}$$

$$\begin{aligned}&\frac{\partial}{\partial C}\left[P\left(p_1 > \frac{C}{(1+b)(1+i)L}\right)(C-p_1L)\right]\\ &= (C-p_1L)\cdot\left[-f\left(\frac{C}{(1+b)(1+i)L}\right)\cdot\frac{1}{(1+b)(1+i)L}\right]\\ &\qquad +P\left(p_1 > \frac{C}{(1+b)(1+i)L}\right)\end{aligned} \tag{14}$$

Setting $\frac{\partial \mathcal{L}}{\partial C} = 0$:

$$\begin{aligned}&r_o + p_1 Li \cdot f\left(\frac{\delta C}{(1+i)(1+b)L}\right)\cdot\frac{\delta}{(1+i)(1+b)L}\\ &+ p_1 L(i+b+ib)\cdot\left[f\left(\frac{C}{(1+i)(1+b)L}\right)\cdot\frac{1}{(1+i)(1+b)L}\right.\\ &\left.-f\left(\frac{\delta C}{(1+i)(1+b)L}\right)\cdot\frac{\delta}{(1+i)(1+b)L}\right]\\ &+(C-p_1L)\cdot\left[-f\left(\frac{C}{(1+b)(1+i)L}\right)\cdot\frac{1}{(1+b)(1+i)L}\right]\\ &+P\left(p_1 > \frac{C}{(1+b)(1+i)L}\right) - \mu\lambda = 0\end{aligned} \tag{15}$$

Grouping the terms:

$$\begin{aligned}&\underbrace{r_o}_{\geq 0} + \underbrace{[p_1Li - p_1L(i+b+ib)]}_{<0}\cdot f\left(\frac{\delta C}{(1+i)L}\right)\cdot\frac{\delta}{(1+i)L}\\ &+\underbrace{[p_1L(i+b+ib)-(C-p_1L)]}_{>0}\cdot f\left(\frac{C}{(1+b)(1+i)L}\right)\cdot\frac{1}{(1+b)(1+i)L}\\ &+\underbrace{P\left(p_1 > \frac{C}{(1+b)(1+i)L}\right)}_{>0} = 0\end{aligned}$$

There are several effects at play:

- The first term is positive — a constant marginal cost for providing collateral.
- The second term negative, and represents the lower interest cost due to the increase in the probability of non-liquidation.
- Due to the increment in collateral, there is also an increase probability that the loan will be liquidated before the debt exceeds the value of the collateral. The third term is positive reflecting the increased costs of liquidation in such a scenario.
- Lastly the forth term reflects the higher loss in the event that collateral is completely liquidated.

Given the presence of both marginal costs and benefits in the optimization problem, there exist parameter configurations where the optimal solution C^* satisfies the first-order condition without binding the constraint $L \leq \lambda C$. This interior solution emerges when the marginal benefit of additional collateral (in terms of reduced liquidation risk) exactly offsets its marginal cost (in terms of opportunity cost and potential losses).

A.2 Proof of Proposition 2: Effect of δ on Borrower Cost

The liquidation threshold δ is a parameter chosen by the protocol DAO. Taking the derivative of the cost function with respect to δ reveals how changes in this parameter affect the overall cost:

$$\begin{aligned}
\frac{\partial}{\partial \delta} E[\mathsf{BC}] =& p_1 L i \cdot f\left(\frac{\delta C}{(1+i)(1+b)L}\right) \cdot \frac{C}{(1+i)(1+b)L} \\
&- p_1 L(i+b+ib) \cdot f\left(\frac{\delta C}{(1+i)(1+b)L}\right) \cdot \frac{C}{(1+i)(1+b)L} \\
=& \frac{C}{(1+i)(1+b)L} \cdot f\left(\frac{\delta C}{(1+i)(1+b)L}\right) \cdot [p_1 L i - p_1 L(i+b+ib)] \\
=& \frac{C}{(1+i)(1+b)L} \cdot f\left(\frac{\delta C}{(1+i)(1+b)L}\right) \cdot p_1 L(-b-ib) \\
=& -\frac{C p_1 L(b+ib)}{(1+i)(1+b)L} \cdot f\left(\frac{\delta C}{(1+i)(1+b)L}\right)
\end{aligned} \tag{16}$$

The derivative is negative because:

- All terms in the numerator and denominator are positive
- The negative sign comes from the term $(-b - ib)$
- The probability density function $f(\cdot)$ is always non-negative

This confirms that increasing δ decreases the cost function, which means that a higher liquidation threshold leads to lower expected costs for the lender.

A.3 Proof of Proposition 3: Effect of δ on Collateral Amount

We now show that the optimal response from borrowers is to overcollateralize more when the liquidation threshold is higher.

We use implicit differentiation since we have an equation that implicitly defines C as a function of δ.

Let's start with the first-order condition from the interior solution (where $\mu = 0$):

$$\begin{aligned} &r_o + [p_1Li - p_1L(i+b+ib)] \cdot f\left(\frac{\delta C}{(1+i)L}\right) \cdot \frac{\delta}{(1+i)L} \\ &+ [p_1L(i+b+ib) - (C - p_1L)] \cdot f\left(\frac{C}{(1+b)(1+i)L}\right) \cdot \frac{1}{(1+b)(1+i)L} \\ &+ P\left(p_1 > \frac{C}{(1+b)(1+i)L}\right) = 0 \end{aligned}$$

Let's call this equation $F(C, \delta) = 0$. To find how C changes with δ, we'll use implicit differentiation:

$$\frac{dC}{d\delta} = -\frac{\partial F/\partial \delta}{\partial F/\partial C}$$

First, let's compute $\frac{\partial F}{\partial \delta}$:

$$\frac{\partial F}{\partial \delta} = [p_1Li - p_1L(i+b+ib)] \cdot \left[f'\left(\frac{\delta C}{(1+i)L}\right) \cdot \frac{C}{(1+i)L} \cdot \frac{\delta}{(1+i)L} + f\left(\frac{\delta C}{(1+i)L}\right) \cdot \frac{1}{(1+i)L}\right]$$

In terms of the sign of the denominator ($\frac{dF}{dC}$), this effect is positive since this is a cost minimization problem, and at the minimum point, the second derivative with respect to C must be positive.

Therefore, $\frac{dC}{d\delta} = -(-)/(+) = (+)$, which means that as the liquidation threshold δ increases, the optimal collateral amount C also increases.

This makes economic sense because:

1. A higher liquidation threshold means the position is less likely to be liquidated 2. This reduces the expected cost of liquidation 3. However, this also means the borrower can take on more risk 4. To maintain the same level of risk protection for the protocol, the optimal response is to increase the collateral amount C.

A.4 Proof of Proposition 4: Effect of λ on Borrower Cost

The borrower's cost minimization problem is:

$$\begin{aligned}\min_{C}\ E[\mathsf{BC}] =& r_o C + P\left(p_1 \leq \frac{\delta C}{(1+i)(1+b)L}\right) p_1 L i \\ &+ P\left(\frac{\delta C}{(1+i)(1+b)L} < p_1 \leq \frac{C}{(1+i)(1+b)L}\right) p_1 L(i+b+ib) \\ &+ P\left(p_1 > \frac{C}{(1+b)(1+i)L}\right)(C - p_1 L) \\ \text{s.t. } & L \leq \lambda C\end{aligned} \tag{17}$$

The Lagrangian for this problem is:

$$\mathcal{L}(C,\mu) = E[\mathsf{BC}] + \mu(L - \lambda C) \tag{18}$$

The first-order condition with respect to C is:

$$\frac{\partial \mathcal{L}}{\partial C} = \frac{\partial E[\mathsf{BC}]}{\partial C} - \mu\lambda = 0 \tag{19}$$

The complementary slackness condition is:

$$\mu(L - \lambda C) = 0 \tag{20}$$

There are two cases to consider:

1. If the constraint is not binding ($L < \lambda C$), then $\mu = 0$ and the optimal C is determined solely by $\frac{\partial E[\mathsf{BC}]}{\partial C} = 0$. In this case, changes in λ have no effect on the optimal C or the expected cost, so $\frac{\partial E[\mathsf{BC}]}{\partial \lambda} = 0$.

2. If the constraint is binding ($L = \lambda C$), then $\frac{\partial E[\mathsf{BC}]}{\partial C} > 0$ at the optimal solution. This is because if $\frac{\partial E[\mathsf{BC}]}{\partial C} < 0$, the borrower could reduce their expected cost by increasing C, which would violate the constraint being binding. Therefore, when λ increases, it relaxes the constraint, allowing the borrower to reduce C while maintaining the same loan amount, thereby reducing their expected cost. This implies $\frac{dC}{d\lambda} < 0$ and consequently $\frac{\partial E[\mathsf{BC}]}{\partial \lambda} < 0$.

A.5 Proof of Proposition 5: Effect of b on Borrower Cost

The borrower's expected cost function is:

$$\begin{aligned}E[\mathsf{BC}] = & r_o C + P\left(p_1 \leq \frac{\delta C}{(1+i)(1+b)L}\right) p_1 L i \\ &+ P\left(\frac{\delta C}{(1+i)(1+b)L} < p_1 \leq \frac{C}{(1+i)(1+b)L}\right) p_1 L(i+b+ib) \\ &+ P\left(p_1 > \frac{C}{(1+b)(1+i)L}\right)(C - p_1 L)\end{aligned} \tag{21}$$

Taking the partial derivative with respect to b:

$$
\begin{aligned}
\frac{\partial E[\mathsf{BC}]}{\partial b} = {} & p_1 L i \cdot f\left(\frac{\delta C}{(1+i)(1+b)L}\right) \cdot \frac{-\delta C}{(1+i)(1+b)^2 L} \\
& + p_1 L(i+b+ib) \cdot \left[f\left(\frac{C}{(1+i)(1+b)L}\right) \cdot \frac{-C}{(1+i)(1+b)^2 L} \right. \\
& \left. - f\left(\frac{\delta C}{(1+i)(1+b)L}\right) \cdot \frac{-\delta C}{(1+i)(1+b)^2 L} \right] \\
& + P\left(\frac{\delta C}{(1+i)(1+b)L} < p_1 \leq \frac{C}{(1+i)(1+b)L}\right) \cdot p_1 L(1+i) \\
& + (C - p_1 L) \cdot f\left(\frac{C}{(1+b)(1+i)L}\right) \cdot \frac{C}{(1+b)^2(1+i)L}
\end{aligned} \tag{22}
$$

Rearranging and factoring:

$$
\begin{aligned}
\frac{\partial E[\mathsf{BC}]}{\partial b} = {} & \underbrace{[p_1 L(i+b+ib) - p_1 L i]}_{>0} \cdot f\left(\frac{\delta C}{(1+i)(1+b)L}\right) \cdot \frac{\delta C}{(1+i)(1+b)^2 L} \\
& + \underbrace{[(C - p_1 L) - p_1 L(i+b+ib)]}_{>0} \cdot f\left(\frac{C}{(1+i)(1+b)L}\right) \cdot \frac{C}{(1+i)(1+b)^2 L} \\
& + \underbrace{\left[F\left(\frac{C}{(1+i)(1+b)L}\right) - F\left(\frac{\delta C}{(1+i)(1+b)L}\right)\right]}_{>0} \cdot p_1 L(1+i)
\end{aligned} \tag{23}
$$

Since all probability density functions $f(\cdot) \geq 0$, and the remaining terms are positive, we have $\frac{\partial E[\mathsf{BC}]}{\partial b} > 0$.

A.6 Proof of Proposition 6: Effect of Collateral on Liquidity Provider Return

To analyze the effect of C on E[I], we take the derivative with respect to C. Let $F(p_1)$ be the cumulative distribution function of p_1, and $f(p_1)$ be the probability density function of p_1. Then:

$$
\begin{aligned}
\frac{dE[I]}{dC} = {} & p_1 L i \cdot f\left(\frac{C}{(1+i)(1+b)L}\right) \cdot \frac{1}{(1+i)(1+b)L} + \frac{1}{(1+b)} \cdot \left[1 - F\left(\frac{C}{(1+i)(1+b)L}\right)\right] \\
& - \frac{C}{(1+b)} \cdot f\left(\frac{C}{(1+i)(1+b)L}\right) \cdot \frac{1}{(1+i)(1+b)L}
\end{aligned}
$$

Rearranging terms

$$\frac{dE[I]}{dC} = \underbrace{\underbrace{\left(p_1 Li - \frac{C}{(1+b)}\right)}_{>0} \cdot f\left(\frac{C}{(1+i)(1+b)L}\right) \cdot \frac{1}{(1+i)(1+b)L}}_{<0}$$
$$+ \underbrace{\frac{1}{(1+b)} \cdot \left[1 - F\left(\frac{C}{(1+i)(1+b)L}\right)\right]}_{>0}$$

The sign of the derivative depends on two effects:

1. A net positive effect of increasing higher interest payments in the case there is repayment or early liquidation.

2. A positive effect of a higher recovery amount in case of liquidation (debt exceeding the collateral value case).

Therefore, increasing collateral values increases the expected outcome for the LP.

A.7 Proof of Proposition 9: Optimal Liquidation Threshold

From Proposition 2, we know that $\frac{\partial E[\mathsf{BC}]}{\partial \delta} < 0$, meaning the borrower's cost decreases with δ. From Corollary 6.1, we know that $\frac{\partial E[\mathsf{I}]}{\partial \delta} > 0$, meaning the LP's expected return increases with δ.

The DAO's optimization problem can be written as:

$$\max_{\lambda,\delta} W(\lambda,\delta) = \alpha(-E[\mathsf{BC}])(\lambda,\delta) + (1-\alpha)E[\mathsf{I}](\lambda,\delta) \tag{24}$$

subject to:

$$E[\mathsf{BC}](\lambda,\delta) \leq \mathsf{BC}^0 \tag{PC-B}$$
$$E[\mathsf{I}](\lambda,\delta) \geq \mathsf{I}^0 \tag{PC-I}$$
$$0 \leq \delta \leq 1 \tag{BC-δ}$$

Since both the borrower's cost decreases and the LP's return increases with δ, the participation constraints will never bind for $\delta < 1$. This is because:

- For the borrower: if $E[\mathsf{BC}](\lambda,\delta) \leq \mathsf{BC}^0$ holds for some δ, it will also hold for any higher δ
- For the LP: if $E[\mathsf{I}](\lambda,\delta) \geq \mathsf{I}^0$ holds for some δ, it will also hold for any higher δ

Therefore, the optimal δ^* must be 1, as this maximizes both the borrower's welfare (by minimizing costs) and the LP's welfare (by maximizing returns).

This can be seen from the first-order condition with respect to δ:

$$\frac{\partial W}{\partial \delta} = -\alpha\frac{\partial E[\mathsf{BC}]}{\partial \delta} + (1-\alpha)\frac{\partial E[\mathsf{I}]}{\partial \delta} \tag{25}$$

Since $\frac{\partial E[\mathsf{BC}]}{\partial \delta} < 0$ and $\frac{\partial E[\mathsf{I}]}{\partial \delta} > 0$, we have:

$$\frac{\partial W}{\partial \delta} = \alpha |\frac{\partial E[\mathsf{BC}]}{\partial \delta}| + (1-\alpha)\frac{\partial E[\mathsf{I}]}{\partial \delta} > 0 \tag{26}$$

This means the objective function is strictly increasing in δ for all $\delta < 1$, and therefore the optimal value is $\delta^* = 1$.

A.8 Proof of Proposition 10: Optimal Loan-to-Value Ratio

The DAO's optimization problem with respect to λ is:

$$\max_{\lambda} W(\lambda, \delta) = \alpha(-E[\mathsf{BC}])(\lambda, \delta) + (1-\alpha)E[\mathsf{I}](\lambda, \delta) \tag{27}$$

subject to:

$$E[\mathsf{BC}](\lambda, \delta) \leq \mathsf{BC}^0 \tag{PC-B}$$
$$E[\mathsf{I}](\lambda, \delta) \geq \mathsf{I}^0 \tag{PC-I}$$
$$0 < \lambda < \delta \tag{BC-λ}$$

The first-order condition with respect to λ is:

$$\frac{\partial W}{\partial \lambda} = -\alpha \frac{\partial E[\mathsf{BC}]}{\partial \lambda} + (1-\alpha)\frac{\partial E[\mathsf{I}]}{\partial \lambda} \tag{28}$$

From Proposition 4, we know that $\frac{\partial E[\mathsf{BC}]}{\partial \lambda} < 0$ when the constraint is binding, and from Proposition 7, we know that $\frac{\partial E[\mathsf{I}]}{\partial \lambda} < 0$ when the constraint is binding.

Therefore, the sign of $\frac{\partial W}{\partial \lambda}$ depends on the relative weights α and $(1-\alpha)$:

1. If α is sufficiently high (borrower-dominated governance), then:

$$-\alpha \frac{\partial E[\mathsf{BC}]}{\partial \lambda} > (1-\alpha)|\frac{\partial E[\mathsf{I}]}{\partial \lambda}| \tag{29}$$

This implies $\frac{\partial W}{\partial \lambda} > 0$, meaning the objective function is increasing in λ. However, the optimal λ^* may not reach its maximum value of δ if the LP's participation constraint (PC-I) becomes binding. Specifically:

- If $E[\mathsf{I}](1, \delta) \geq \mathsf{I}^0$, then λ^* could be set arbitrarily close to δ.
- If $E[\mathsf{I}](1, \delta) < \mathsf{I}^0$, then λ^* will be set to satisfy $E[\mathsf{I}](\lambda^*, \delta) = \mathsf{I}^0$

2. If $(1-\alpha)$ is sufficiently high (LP-dominated governance), then:

$$-\alpha \frac{\partial E[\mathsf{BC}]}{\partial \lambda} < (1-\alpha)|\frac{\partial E[\mathsf{I}]}{\partial \lambda}| \tag{30}$$

This implies $\frac{\partial W}{\partial \lambda} < 0$, meaning the objective function is decreasing in λ. However, the optimal λ^* may not reach its minimum value if the borrower's participation constraint (PC-B) becomes binding. Specifically:

- If $E[\mathsf{BC}](0,\delta) \leq \mathsf{BC}^0$, then λ^* could be set arbitrarily close to 0.
- If $E[\mathsf{BC}](0,\delta) > \mathsf{BC}^0$, then λ^* will be set to satisfy $E[\mathsf{BC}](\lambda^*,\delta) = \mathsf{BC}^0$

3. For intermediate values of α, there exists a value λ^* where:

$$-\alpha\frac{\partial E[\mathsf{BC}]}{\partial\lambda} = (1-\alpha)|\frac{\partial E[\mathsf{I}]}{\partial\lambda}| \tag{31}$$

Also in this case, participation constraints should be checked to determine if the optimal value of λ is at the boundary or at the interior solution. Specifically:

- If the borrower's participation constraint (PC-B) is binding, λ^* will be set to satisfy $E[\mathsf{BC}](\lambda^*,\delta) = \mathsf{BC}^0$
- If the LP's participation constraint (PC-I) is binding, λ^* will be set to satisfy $E[\mathsf{I}](\lambda^*,\delta) = \mathsf{I}^0$

This analysis shows that the optimal loan-to-value ratio depends crucially on the governance structure of the DAO, as captured by the weight α, and the participation constraints of both borrowers and LPs. The trade-off between borrower costs and LP returns leads to different optimal values of λ depending on which stakeholder group has more influence in the governance process and which participation constraints are binding. The analysis also shows that the values of the participation constraints (i.e. outside financing costs and returns) are important to determine the optimal λ. In that sense the competition between DeFi protocols is also important to determine the optimal λ.

A.9 Proof of Proposition 12: Effect of b on Borrower Cost with Uncertain Liquidation

The borrower's expected cost function under uncertain liquidation is given by:

$$\begin{aligned}
E[\mathsf{BC}]_{uncertain} =& r_o C + P\left(p_1 \leq \frac{\delta C}{L(1+i)(1+b)}\right)\cdot p_1 L i \\
&+ P\left(\frac{\delta C}{L(1+i)(1+b)} < p_1 \leq \frac{C}{L(1+i)(1+b)}\right) \\
&\cdot [q(b)\cdot p_1 L(i+b+ib) + (1-q(b))\cdot p_1 L i] \\
&+ P\left(\frac{C}{L(1+i)(1+b)} < p_1 \leq \frac{C}{L(1+i)}\right) \\
&\cdot [q(b)\cdot(C-p_1 L) + (1-q(b))\cdot p_1 L i] \\
&+ P\left(p_1 > \frac{C}{L(1+i)}\right)\cdot(C-p_1 L)
\end{aligned} \tag{32}$$

Taking the partial derivative with respect to b yields:

$$
\begin{aligned}
\frac{\partial E[\mathsf{BC}]_{uncertain}}{\partial b} =& p_1 Li \cdot f\left(\frac{\delta C}{L(1+i)(1+b)}\right) \cdot \frac{-\delta C}{L(1+i)(1+b)^2} \\
&+ \left[f\left(\frac{C}{L(1+i)(1+b)}\right) \cdot \frac{-C}{L(1+i)(1+b)^2} - f\left(\frac{\delta C}{L(1+i)(1+b)}\right) \cdot \frac{-\delta C}{L(1+i)(1+b)^2}\right] \\
&\cdot [q(b) \cdot p_1 L(i+b+ib) + (1-q(b)) \cdot p_1 Li] \\
&+ P\left(\frac{\delta C}{L(1+i)(1+b)} < p_1 \leq \frac{C}{L(1+i)(1+b)}\right) \\
&\cdot [q'(b) \cdot p_1 L(i+b+ib) - q'(b) \cdot p_1 Li + q(b) \cdot p_1 L(1+i)] \\
&+ f\left(\frac{C}{L(1+i)(1+b)}\right) \cdot \frac{C}{L(1+i)(1+b)^2} \cdot [q(b) \cdot (C - p_1 L) + (1-q(b)) \cdot p_1 Li] \\
&+ P\left(\frac{C}{L(1+i)(1+b)} < p_1 \leq \frac{C}{L(1+i)}\right) \cdot [q'(b) \cdot (C - p_1 L) - q'(b) \cdot p_1 Li]
\end{aligned} \tag{33}
$$

We proceed with a sign analysis through term grouping.

Grouping terms involving $f\left(\frac{\delta C}{L(1+i)(1+b)}\right)$ yields:

$$
\begin{aligned}
&[q(b) \cdot p_1 L(i+b+ib) + (1-q(b)) \cdot p_1 Li - p_1 Li] \cdot f\left(\frac{\delta C}{L(1+i)(1+b)}\right) \cdot \frac{\delta C}{L(1+i)(1+b)^2} \\
&= [q(b) \cdot p_1 L(i+b+ib) - q(b) \cdot p_1 Li] \cdot f\left(\frac{\delta C}{L(1+i)(1+b)}\right) \cdot \frac{\delta C}{L(1+i)(1+b)^2} \\
&= q(b) \cdot [p_1 L(i+b+ib) - p_1 Li] \cdot f\left(\frac{\delta C}{L(1+i)(1+b)}\right) \cdot \frac{\delta C}{L(1+i)(1+b)^2} > 0
\end{aligned} \tag{34}
$$

Since $p_1 L(i+b+ib) > p_1 Li$ and $q(b) \geq 0$, this term is strictly positive.

Grouping Terms Involving $f\left(\frac{C}{L(1+i)(1+b)}\right)$ yields:

$$
\begin{aligned}
&f\left(\frac{C}{L(1+i)(1+b)}\right) \cdot \frac{C}{L(1+i)(1+b)^2} \cdot \\
&[q(b) \cdot (C - p_1 L) + (1-q(b)) \cdot p_1 Li \\
&- q(b) \cdot p_1 L(i+b+ib) - (1-q(b)) \cdot p_1 Li] \\
&= f\left(\frac{C}{L(1+i)(1+b)}\right) \cdot \frac{C}{L(1+i)(1+b)^2} \cdot [q(b) \cdot (C - p_1 L) - q(b) \cdot p_1 L(i+b+ib)] \\
&= f\left(\frac{C}{L(1+i)(1+b)}\right) \cdot \frac{C}{L(1+i)(1+b)^2} \cdot q(b) \cdot [(C - p_1 L) - p_1 L(i+b+ib)] > 0
\end{aligned} \tag{35}
$$

Since $(C - p_1 L) > p_1 L(i+b+ib)$ in this price range and $q(b) \geq 0$, this term is strictly positive.

Grouping the remaining probability function terms yields:

$$\begin{aligned}
&P\left(\frac{\delta C}{L(1+i)(1+b)} < p_1 \leq \frac{C}{L(1+i)(1+b)}\right) \cdot [q'(b) \cdot p_1 L(i+b+ib) - q'(b) \cdot p_1 Li + q(b) \cdot p_1 L(1+i)] \\
&= P\left(\frac{\delta C}{L(1+i)(1+b)} < p_1 \leq \frac{C}{L(1+i)(1+b)}\right) \cdot [q'(b) \cdot p_1 L(b+ib) + q(b) \cdot p_1 L(1+i)] > 0
\end{aligned} \tag{36}$$

And:

$$P\left(\frac{C}{L(1+i)(1+b)} < p_1 \leq \frac{C}{L(1+i)}\right) \cdot [q'(b) \cdot (C - p_1 L) - q'(b) \cdot p_1 Li] > 0 \tag{37}$$

Since $q'(b) > 0$ by assumption and all terms in brackets are positive in their respective price ranges, these terms are strictly positive.

Since all grouped terms are strictly positive, we conclude that:

$$\frac{\partial E[\mathsf{BC}]_{uncertain}}{\partial b} > 0 \tag{38}$$

This establishes that under uncertain liquidation, the borrower's expected cost increases with the liquidation bonus rate b.