The Economics of DeFi Lending: A Model of Smart Contract Parameter Choice

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Abstract

This paper develops a theoretical framework to analyze the economics of decentralized finance (DeFi) lending protocols. We focus on how key smart contract parameters - the loan-to-value ratio and liquidation threshold - affect the behavior and payoffs of borrowers and liquidity providers. We show that these parameters create fundamental trade-offs between borrower leverage and lender security. Borrowers benefit from higher liquidation thresholds while liquidity providers prefer lower thresholds, creating conflicting interests in protocol governance. The model provides insights into how protocol parameters should be set based on the relative influence of different stakeholders in decentralized autonomous organization (DAO) governance.

1 Introduction

Decentralized lending platforms have emerged as one of the most significant applications in decentralized finance (DeFi). These protocols allow users to borrow cryptocurrency against collateral without intermediaries, using smart contracts to automate lending operations. A critical aspect of these protocols is the choice of risk parameters that govern lending activities, particularly the loan-to-value (LTV) ratio and liquidation threshold. These parameters are typically not hardcoded but can be modified through protocol governance, usually implemented through a Decentralized Autonomous Organization (DAO). This creates an interesting economic problem: how should these parameters be set to balance the interests of different stakeholders? This paper develops a theoretical framework to analyze this question.

Our model considers three key participants: borrowers, liquidity providers (LPs), and the protocol governance (DAO). Borrowers deposit collateral to take loans, which they can invest in private projects. LPs provide the loanable funds in exchange for interest. The DAO, representing the community of stakeholders, sets the protocol parameters to maximize a weighted average of borrower and LP welfare.

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2 Related Literature

This paper contributes to the emerging literature on decentralized finance (DeFi) lending protocols and their governance mechanisms. Our work relates to several strands of literature.

First, we build on studies examining the fundamental characteristics of DeFi lending protocols. Irresberger et al. (2024) discuss how lending protocols attract liquidity by encouraging borrowers to enter debt positions and repay loans with interest, which is then distributed to lenders and protocol reserves. A key aspect of their analysis is how protocol governors calibrate liquidation thresholds and penalties to balance liquidation risk mitigation against borrowers' ability to take leveraged positions.

Second, our work relates to research on how parameter changes affect borrower and lender behavior. Wang (2024) explore the effects of margin requirement relaxations on borrower behavior using data from Aave and Compound. They find that looser requirements increase borrowing, particularly among highly leveraged positions, and lead to higher utilization ratios and interest rates. Complementing this analysis, Mueller (2022) examines the effects of liquidation incentives on liquidation behavior, documenting that insufficient incentives can result in positions remaining unliquidated. Their work also analyzes how blockchain transaction fees affect leveraged traders' behavior.

Third, we contribute to the literature on systemic risk and liquidation mechanisms in DeFi markets. Lehar and Parlour (2022) examine the price impact of collateral liquidations in decentralized lending platforms, focusing on Compound and Aave. They find that liquidations, especially those involving flash loans, can create both temporary and permanent price impacts on decentralized exchanges, potentially triggering negative feedback loops and increasing systemic fragility.

Our theoretical framework complements these empirical studies by providing a formal model on the factors driving how protocol parameters protocol optimization, taking into account the competing interests of different stakeholders. The model generates testable predictions about the relationship between governance structure and parameter choices, helping explain several empirical findings in the literature while providing new hypotheses for future research.

3 Model Setup

Consider a two-period model ($t = 0, 1$) with three types of agents: borrowers, liquidity providers, and a DAO that sets protocol parameters. The model captures the key features of DeFi lending protocols while remaining tractable.

3.1 Agents and Timeline

3.1.1 DAO

At $t = 0$, the DAO chooses two key parameters:

- The loan-to-value ratio (λ), which determines the maximum amount that can be borrowed against collateral

- The liquidation threshold (δ), which determines when a position becomes eligible for liquidation

These parameters must satisfy $0 < \lambda < \delta < 1$ to ensure economic viability of the protocol.

3.1.2 Borrowers

Borrowers are risk-neutral agents who:

- At $t = 0$, deposit collateral C and borrow amount L_0 in a different cryptocurrency
- Invest borrowed funds in a private project with stochastic return r
- At $t = 1$, receive project returns and decide whether to repay the loan

The borrowing constraint at $t = 0$ is:

$$p_0 L_0 \leq \lambda C \tag{1}$$

where p_t is the exchange rate between the loan and collateral currencies at time t.

3.1.3 Liquidity Providers

LPs are risk-neutral agents who:

- At $t = 0$, provide funds to the lending pool
- Earn interest rate i on loans that are repaid
- Face potential losses if borrowers default or if liquidation value is insufficient

3.2 Price Uncertainty and Returns

The exchange rate p_t follows a stochastic process with known distribution $F(p)$ and density $f(p)$. The private project returns r are distributed according to $G(r)$ with density $g(r)$. We assume:

- $p_t > 0$ for all t
- r and p_t are independently distributed
- The distributions $F(p)$ and $G(r)$ are common knowledge

3.3 Liquidation Mechanism

At $t = 1$, a loan position faces liquidation if:

$$\frac{p_1 L_0(1+i)}{C} > \delta \tag{2}$$

This occurs when the debt value relative to collateral exceeds the liquidation threshold. Using the borrowing constraint, we can express the liquidation condition in terms of price movements:

$$p_1 > \frac{\delta}{\lambda}\frac{p_0}{1+i} \tag{3}$$

When liquidation occurs:

- The borrower loses their collateral
- LPs receive the minimum of the debt value and collateral value

3.4 Payoff Structure

The payoff structure depends on three key events:

1. Whether liquidation occurs ($p_1 > \frac{\delta}{\lambda}\frac{p_0}{1+i}$)
2. Whether the project return exceeds the interest rate ($r > i$)
3. Whether the collateral value exceeds the debt value ($C > p_1 L_0(1+i)$)

These events determine whether borrowers choose to repay, default strategically, or face liquidation, and consequently affect the payoffs to both borrowers and LPs.

4 Analysis

4.1 Borrower Payoffs

A risk-neutral borrower with collateral C has the following expected payoff:

$$\begin{aligned} E[\mathrm{B}] = & \underbrace{\frac{p_1}{p_0}\lambda C(r-i)P(r \geq i)P(p_1 \leq \frac{\delta}{\lambda}\frac{p_0}{(1+i)})}_{\text{Repay}} \\ & + \underbrace{\big(\frac{p_1}{p_0}\lambda C(1+r) - C\big)P(r < i)P(p_1 \leq \frac{\delta}{\lambda}\frac{p_0}{(1+i)})}_{\text{Unable to repay}} \\ & + \underbrace{\big(\frac{p_1}{p_0}\lambda C(1+r) - C\big)P(p_1 > \frac{\delta}{\lambda}\frac{p_0}{(1+i)})}_{\text{Liquidated}} \end{aligned} \tag{4}$$

The first term represents the borrower's payoff when they successfully repay the loan. In such a case they keep the private project's excess return over the interest rate. The second term captures the case where the borrower is unable to repay due to insufficient project returns. In such a case they lose the collateral. The third term represents the payoff when the position is liquidated due to adverse price movements. In such a case they also lose the collateral.

4.2 Liquidity Provider Payoffs

The expected payoff for a liquidity provider is:

$$
\begin{aligned}
E[\mathrm{I}] = &\underbrace{\big(0 \cdot P(r < i) + p_1 L_0 (1+i) P(r \geq i)\big) P(p_1 \leq \frac{\delta}{\lambda} \frac{p_0}{(1+i)})}_{\text{No liquidation}} \\
&+ \underbrace{p_1 L_0 (1+i) P\left(\frac{\delta}{\lambda} \frac{p_0}{(1+i)} \leq p_1 \leq \frac{1}{\lambda} \frac{p_0}{(1+i)}\right)}_{\text{Liquidated without loss}} \\
&+ \underbrace{C P\left(p_1 > \frac{1}{\lambda} \frac{p_0}{(1+i)}\right)}_{\text{Liquidated with loss}}
\end{aligned}
\tag{5}
$$

The first term represents the payoff when there is no liquidation. In such a case the LP receives the expected return that involves the repayment of the loan. For simplicity, we assume that in the case there is no repayment (and no liquidation) the LP receives zero. This might represent an iliquidity scenario where the LP is not able to withdraw their funds. The second term represents the payoff when the position is liquidated without loss. In such a case the LP receives the interest rate on the loan and the collateral. The third term represents the payoff when the position is liquidated with a loss for the LP. In such a case the LP receives the collateral, which is worth less than the debt.

4.3 Effects of Protocol Parameters

We now analyze how changes in the protocol parameters affect borrower and LP payoffs.

Proposition 1. *For any borrower position where the collateral value maintains a positive recovery incentive, the borrower's expected return exhibits monotonic growth with respect to the liquidation threshold parameter δ.*

Sketch. Computing the first-order condition with respect to δ yields:

$$
\frac{\partial E[\mathrm{B}]}{\partial \delta} = P(r \geq i) C \left[1 - \frac{p_1}{p_0} \lambda (1+i)\right] \cdot f\left(\frac{\delta}{\lambda} \frac{p_0}{(1+i)}\right) \frac{p_0}{\lambda(1+i)} > 0 \tag{6}
$$

This derivative maintains strict positivity under the assumption that collateral value dominates the debt obligation (i.e., $C > p_1 L_0 (1+i)$), which ensures rational participation in the protocol. For complete derivation, see Appendix A. □

This result aligns with economic intuition: elevated liquidation thresholds expand the feasible region for position maintenance, thereby enhancing the expected value of the borrower's position.

Proposition 2. *The expected return for liquidity providers decreases with the liquidation threshold δ.*

Sketch. The derivative of the LP's expected return with respect to δ is:

$$\frac{\partial E[\mathrm{I}]}{\partial \delta} = \underbrace{(P(r \geq i) - 1)}_{<0} \underbrace{\frac{\lambda p_1 C(1+i)}{p_0} \cdot f\left(\frac{\delta}{\lambda} \frac{p_0}{(1+i)}\right) \cdot \frac{p_0}{\lambda(1+i)}}_{>0} < 0 \tag{7}$$

See Appendix A for the complete proof. □

The economic intuition reflects the fundamental trade-off in the protocol: a higher liquidation threshold reduces the frequency of liquidations, which increases the probability that the LP faces a scenario where the borrower defaults without prior liquidation. In such cases, the LP receives nothing, whereas with a lower δ, the position would have been liquidated earlier, allowing the LP to recover at least part of their investment through the collateral.

Proposition 3. *For any borrower position where the collateral value maintains a positive recovery incentive, the derivative of $E[B]$ with respect to λ involves a trade-off between increased leverage benefits and higher liquidation risks.*

Sketch. The effect can be decomposed into direct and indirect effects:

- Direct effect (always positive): increased leverage potential
- Indirect effect (always negative): increased liquidation risk

The total effect is negative when:

$$P(r \geq i)\, C \left[1 - \frac{p_1}{p_0}\lambda(1+i)\right] \left| \frac{\partial}{\partial \lambda} P\left(p_1 \leq \frac{\delta}{\lambda} \frac{p_0}{1+i}\right) \right| > \frac{\partial B_{\text{direct}}}{\partial \lambda}$$

See Appendix A for the complete derivation. □

Increasing the loan per collateral ratio increases the leverage potential of the borrower, which increases their expected payoff. However, it also increases the probability of liquidation, which reduces the expected payoff of the borrower. The total effect depends on which effect dominates.

Proposition 4. *The effect of increasing the loan-to-value ratio λ on LP returns is characterized by a trade-off between:*

- *Positive direct effects from increased debt value and larger liquidation proceeds*
- *Indirect effects from changes in liquidation probabilities and recovery rates*

The total effect tends to be positive when leverage benefits dominate liquidation risks.

Sketch. The derivative of LP returns with respect to λ can be decomposed into:

- Direct effects (positive):
  - Increased debt value in repayment scenarios
  - Larger liquidation proceeds when positions are liquidated
- Indirect effects (mixed):
  - Changes in liquidation probabilities
  - Altered recovery rates in default scenarios

The total effect tends to be positive when:

- The increased payoffs from higher leverage are substantial
- The price distribution is relatively flat (reducing indirect effects)
- The initial value of λ is low (maximizing marginal benefits)

See Appendix A for complete derivation. □

5 Optimal Parameter Choice

5.1 The DAO's Problem

The DAO's objective function reflects the interests of the protocol community, which comprises both borrowers and LPs. The governance chooses parameters λ and δ to maximize the weighted surplus:

$$W(\lambda, \delta) = \alpha E[B](\lambda, \delta) + (1 - \alpha) E[I](\lambda, \delta) \tag{8}$$

where $\alpha \in [0, 1]$ represents the relative weight given to borrower interests in the governance process. This weight could reflect either the explicit voting power distribution in the DAO or the implicit influence of different stakeholder groups.

5.2 Participation Constraints

The optimization is subject to participation constraints that ensure both borrowers and LPs find it optimal to use the protocol:

$$E[B](\lambda, \delta) \geq U_B^0 \tag{PC-B}$$
$$E[I](\lambda, \delta) \geq U_I^0 \tag{PC-I}$$

where U_B^0 and U_I^0 represent the outside options for borrowers and LPs respectively. These outside options might include:

- Alternative DeFi lending protocols
- Traditional financial intermediaries
- Direct investment opportunities

5.3 Competitive Environment

In a competitive DeFi ecosystem, these participation constraints become particularly relevant as protocols compete for both borrowers and liquidity. The outside options U_B^0 and U_I^0 are endogenously determined by:

$$U_j^0 = \max_{k \neq i} \{E[j](\lambda_k, \delta_k)\} \tag{9}$$

where $j \in \{B, I\}$ and k indexes competing protocols.

5.4 Optimal Parameter Choice

The first-order conditions for the DAO's optimization problem are:

$$\frac{\partial W}{\partial \delta} = \alpha \frac{\partial E[B]}{\partial \delta} + (1-\alpha) \frac{\partial E[I]}{\partial \delta} + \mu_B \frac{\partial E[B]}{\partial \delta} + \mu_I \frac{\partial E[I]}{\partial \delta} = 0 \tag{10}$$
$$\frac{\partial W}{\partial \lambda} = \alpha \frac{\partial E[B]}{\partial \lambda} + (1-\alpha) \frac{\partial E[I]}{\partial \lambda} + \mu_B \frac{\partial E[B]}{\partial \lambda} + \mu_I \frac{\partial E[I]}{\partial \lambda} = 0 \tag{11}$$

where μ_B and μ_I are the Lagrange multipliers on the participation constraints.

From Propositions 1 and 2, we know that $\frac{\partial E[B]}{\partial \delta} > 0$ and $\frac{\partial E[I]}{\partial \delta} < 0$. Therefore, the optimal choice of δ balances these opposing effects, weighted by both governance power (α) and the tightness of participation constraints (μ_B, μ_I).

5.5 Comparative Statics and Predictions

This framework generates several testable predictions about protocol parameter choices:

1. **Governance Composition:** Higher values of α (greater borrower influence) lead to:
   - Higher liquidation thresholds (δ)
   - Higher loan-to-value ratios (λ)
2. **Competition Effects:** Increased competition (higher U_B^0 or U_I^0) leads to:
   - Parameter choices that favor the group with more competitive outside options
   - Possible specialization across protocols in serving different user types

3. **Parameter Substitution:** Protocols may trade off λ and δ to satisfy participation constraints while maintaining risk control:

$$\frac{d\lambda}{d\delta} = -\frac{\partial E[j]/\partial \delta}{\partial E[j]/\partial \lambda} \tag{12}$$

where j is the binding constraint.

These predictions suggest that observed parameter choices should systematically vary with:

- The composition of protocol governance
- The competitive environment
- The relative bargaining power of borrowers versus LPs

The model thus provides a framework for understanding how protocol parameters are determined through the interaction of governance structure, market competition, and stakeholder interests.

6 Empirical Implications

Our model generates several testable predictions:

1. DAOs with greater LP versus borrower representation will set lower liquidation thresholds
2. DAOs might substitute between liquidation thresholds and LTV ratios in attracting participants. For instance, increasing LTV ratios while increasing the liquidation threshold might cancel out the effects for borrowers.

7 Conclusion

This paper provides a theoretical framework for understanding parameter choice in DeFi lending protocols. Our results highlight the fundamental trade-offs between borrower leverage and LP security, and how these trade-offs are resolved through DAO governance. The model generates testable predictions about how protocol parameters should vary with stakeholder composition and market conditions.

References

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A Detailed Proofs

A.1 Proof of Proposition 1: Effect of δ on Borrower Returns

We can rewrite the expected payoff as:

$$
\begin{aligned}
E[\mathrm{B}] =& \frac{p_1}{p_0}\lambda C(r-i)P(r \geq i)P(p_1 \leq \frac{\delta}{\lambda}\frac{p_0}{(1+i)}) \\
&+ \big(\frac{p_1}{p_0}\lambda C(1+r) - C\big)P(r<i)P(p_1 \leq \frac{\delta}{\lambda}\frac{p_0}{(1+i)}) \\
&+ \big(\frac{p_1}{p_0}\lambda C(1+r) - C\big)\left(1 - P(p_1 \leq \frac{\delta}{\lambda}\frac{p_0}{(1+i)})\right)
\end{aligned}
$$

The derivative of the borrower's expected return with respect to δ is:

$$
\begin{aligned}
\frac{\partial E[\mathrm{B}]}{\partial \delta} =& \Big[\frac{p_1}{p_0}\lambda C(r-i)P(r \geq i) + \big(\frac{p_1}{p_0}\lambda C(1+r) - C\big)P(r<i) \\
&- \big(\frac{p_1}{p_0}\lambda C(1+r) - C\big)\Big] \underbrace{f\left(\frac{\delta}{\lambda}\frac{p_0}{(1+i)}\right)\frac{p_0}{\lambda(1+i)}}_{>0}
\end{aligned}
$$

The multiple on the right is strictly positive as it is the product of a probability density function and positive parameters. The expression in brackets can be rewritten as:

$$
P(r \geq i)\big[-\frac{p_1}{p_0}\lambda C(1+i) + C\big]
$$

Since $P(r \geq i) > 0$, the sign depends on $C - \frac{p_1}{p_0}\lambda C(1+i)$. This is positive by assumption, as the collateral value (C) must exceed the debt value ($\frac{p_1}{p_0}\lambda C(1+i)$) for the position to remain active. Therefore, $\frac{\partial E[\mathrm{B}]}{\partial \delta} > 0$.

A.2 Proof of Proposition 2: Effect of δ on LP Returns

The derivative of the LP's expected return with respect to δ is:

$$
\begin{aligned}
\frac{\partial E[\mathrm{I}]}{\partial \delta} =& \left(\frac{p_1}{p_0}\lambda C(1+i)P(r \geq i)\right) f\left(\frac{\delta}{\lambda}\frac{p_0}{(1+i)}\right)\frac{p_0}{\lambda(1+i)} \\
&- \frac{\lambda p_1 C(1+i)}{p_0} \cdot f\left(\frac{\delta}{\lambda}\frac{p_0}{(1+i)}\right) \cdot \frac{p_0}{\lambda(1+i)}
\end{aligned}
$$

This can be simplified to:

$$\frac{\partial E[\mathrm{I}]}{\partial \delta} = \Big(P(r \geq i) - 1\Big) \underbrace{\frac{\lambda p_1 C(1+i)}{p_0} \cdot f\left(\frac{\delta}{\lambda} \frac{p_0}{(1+i)}\right) \cdot \frac{p_0}{\lambda(1+i)}}_{>0} < 0$$

The second term is strictly positive as it is the product of positive parameters and a probability density function. The first term $(P(r \geq i) - 1)$ is strictly negative since $P(r \geq i)$ is a probability and thus less than 1. Therefore, $\frac{\partial E[\mathrm{I}]}{\partial \delta} < 0$.

A.3 Proof of Proposition 3: Effect of λ on Borrower Returns

The effect can be decomposed into direct and indirect effects ($\frac{\partial T}{\partial \lambda} = \frac{\partial T_{\text{direct}}}{\partial \lambda} + \frac{\partial T_{\text{indirect}}}{\partial \lambda}$).

The direct effect is always positive:

$$\begin{aligned}\frac{\partial T_{\text{direct}}}{\partial \lambda} = & \underbrace{\frac{p_1}{p_0} C(r-i) P(r \geq i) P\Big(p_1 \leq \frac{\delta}{\lambda} \frac{p_0}{1+i}\Big)}_{>0} \\ & + \underbrace{\frac{p_1}{p_0} C(1+r) P(r < i) P\Big(p_1 \leq \frac{\delta}{\lambda} \frac{p_0}{1+i}\Big)}_{>0} \\ & + \underbrace{\frac{p_1}{p_0} C(1+r) P\Big(p_1 > \frac{\delta}{\lambda} \frac{p_0}{1+i}\Big)}_{>0}\end{aligned}$$

The indirect effect is always negative:

$$\frac{\partial T_{\text{indirect}}}{\partial \lambda} = \underbrace{P(r \geq i)}_{\geq 0} \, C \underbrace{\left[1 - \frac{p_1}{p_0} \lambda(1+i)\right]}_{\geq 0} \underbrace{\frac{\partial}{\partial \lambda} P\left(p_1 \leq \frac{\delta}{\lambda} \frac{p_0}{1+i}\right)}_{<0} \leq 0$$

The total effect is negative when:

$$P(r \geq i) \, C \left[1 - \frac{p_1}{p_0} \lambda(1+i)\right] \left| \frac{\partial}{\partial \lambda} P\left(p_1 \leq \frac{\delta}{\lambda} \frac{p_0}{1+i}\right) \right| > \frac{\partial T_{\text{direct}}}{\partial \lambda}$$

A.4 Proof of Proposition 4: Effect of λ on LP Returns

Taking the derivative of $E[\mathrm{I}]$ with respect to λ separately for each term, we can decompose the effect into three components:

1. For the first term:

$$\begin{aligned}& \frac{\partial}{\partial \lambda}\left[\left(\frac{p_1}{p_0} \lambda C(1+i) P(r \geq i)\right) P\left(p_1 \leq \frac{\delta}{\lambda} \frac{p_0}{(1+i)}\right)\right] = \\ & \frac{p_1}{p_0} C(1+i) P(r \geq i) P\left(p_1 \leq \frac{\delta}{\lambda} \frac{p_0}{(1+i)}\right) + \\ & \left(\frac{p_1}{p_0} \lambda C(1+i) P(r \geq i)\right) f\left(\frac{\delta}{\lambda} \frac{p_0}{(1+i)}\right)\left(-\frac{\delta}{\lambda^2} \frac{p_0}{(1+i)}\right)\end{aligned}$$

This first term has a positive and a negative effect. The positive effect comes from the larger payoff in case of recovery. The negative effect is driven by the lower chances of recovery, reflecting the increased riskiness.

2. For the second term:

$$\frac{\partial}{\partial \lambda}\left[\frac{\lambda p_1 C(1+i)}{p_0} P\left(\frac{\delta}{\lambda}\frac{p_0}{(1+i)} \leq p_1 \leq \frac{1}{\lambda}\frac{p_0}{(1+i)}\right)\right] =$$
$$\frac{p_1 C(1+i)}{p_0} P\left(\frac{\delta}{\lambda}\frac{p_0}{(1+i)} \leq p_1 \leq \frac{1}{\lambda}\frac{p_0}{(1+i)}\right) +$$
$$\frac{\lambda p_1 C(1+i)}{p_0}\left[f\left(\frac{\delta}{\lambda}\frac{p_0}{(1+i)}\right)\left(\frac{\delta}{\lambda^2}\frac{p_0}{(1+i)}\right) - f\left(\frac{1}{\lambda}\frac{p_0}{(1+i)}\right)\left(\frac{1}{\lambda^2}\frac{p_0}{(1+i)}\right)\right]$$

The first term is positive, showing the effect of larger payoff in case the loan is liquidated. The second term depends on the probability function. If the probability function is uniform, this term is negative. The overall sign will tend to be positive because the payoff effect will dominate.

3. For the third term:

$$\frac{\partial}{\partial \lambda}\left[CP\left(p_1 > \frac{1}{\lambda}\frac{p_0}{(1+i)}\right)\right] =$$
$$\frac{\partial}{\partial \lambda}\left[C\left(1 - F\left(\frac{1}{\lambda}\frac{p_0}{(1+i)}\right)\right)\right] =$$
$$Cf\left(\frac{1}{\lambda}\frac{p_0}{(1+i)}\right)\left(\frac{1}{\lambda^2}\frac{p_0}{(1+i)}\right)$$

This term is positive, indicating an increase in the probability of receiving the collateral in the event of a liquidation with a loss.

The complete derivative is therefore:

$$\begin{aligned}\frac{\partial E[\mathrm{I}]}{\partial \lambda} =& \frac{p_1}{p_0} C(1+i) P(r \geq i) P\left(p_1 \leq \frac{\delta}{\lambda}\frac{p_0}{(1+i)}\right) + \\ & \left(\frac{p_1}{p_0}\lambda C(1+i)P(r \geq i)\right) f\left(\frac{\delta}{\lambda}\frac{p_0}{(1+i)}\right)\left(-\frac{\delta}{\lambda^2}\frac{p_0}{(1+i)}\right) + \\ & \frac{p_1 C(1+i)}{p_0} P\left(\frac{\delta}{\lambda}\frac{p_0}{(1+i)} \leq p_1 \leq \frac{1}{\lambda}\frac{p_0}{(1+i)}\right) + \\ & \frac{\lambda p_1 C(1+i)}{p_0}\left[f\left(\frac{\delta}{\lambda}\frac{p_0}{(1+i)}\right)\left(\frac{\delta}{\lambda^2}\frac{p_0}{(1+i)}\right) - f\left(\frac{1}{\lambda}\frac{p_0}{(1+i)}\right)\left(\frac{1}{\lambda^2}\frac{p_0}{(1+i)}\right)\right] + \\ & Cf\left(\frac{1}{\lambda}\frac{p_0}{(1+i)}\right)\left(\frac{1}{\lambda^2}\frac{p_0}{(1+i)}\right)\end{aligned}$$

The effects can be organized into direct and indirect effects:

1. Direct effects (positive):

$$\frac{\partial I_{\text{direct}}}{\partial \lambda} = \underbrace{\frac{p_1}{p_0} C(1+i) P(r \geq i) P\left(p_1 \leq \frac{\delta}{\lambda} \frac{p_0}{(1+i)}\right)}_{\geq 0} + \underbrace{\frac{p_1 C(1+i)}{p_0} P\left(\frac{\delta}{\lambda} \frac{p_0}{(1+i)} \leq p_1 \leq \frac{1}{\lambda} \frac{p_0}{(1+i)}\right)}_{\geq 0}$$

2. Indirect effects (mixed sign):

$$\frac{\partial I_{\text{indirect}}}{\partial \lambda} = \underbrace{\left(\frac{p_1}{p_0} \lambda C(1+i) P(r \geq i)\right) f\left(\frac{\delta}{\lambda} \frac{p_0}{(1+i)}\right)\left(-\frac{\delta}{\lambda^2} \frac{p_0}{(1+i)}\right)}_{\leq 0} + \underbrace{\frac{\lambda p_1 C(1+i)}{p_0}\left[f\left(\frac{\delta}{\lambda} \frac{p_0}{(1+i)}\right)\left(\frac{\delta}{\lambda^2} \frac{p_0}{(1+i)}\right) - f\left(\frac{1}{\lambda} \frac{p_0}{(1+i)}\right)\left(\frac{1}{\lambda^2} \frac{p_0}{(1+i)}\right)\right]}_{\gtreqless 0} + \underbrace{C f\left(\frac{1}{\lambda} \frac{p_0}{(1+i)}\right)\left(\frac{1}{\lambda^2} \frac{p_0}{(1+i)}\right)}_{\geq 0}$$

We conclude that the overall effect is undetermined, but will tend to be positive when:

- The increased payoffs from higher leverage are large
- The probability function f is relatively flat (making indirect effects smaller)
- The initial value of λ is small (making the marginal benefit of increased leverage higher)