MAE 598: Design Optimization Project-2

Design Optimization using DOE and Response Surface in ANSYS

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Introduction

This project is performed to find the optimum solution and primarily reduce the volume of the required material used as well as for emergency braking conditions while maintaining every other aspect of the design under a certain limit. This analysis is performed on a brake disc model. The project is separated into 5 parts: Design problem statement, Mathematical problem formulation, Brake design model analysis, Design optimization, and Results and Conclusion.

Design Problem Statement

The primary objective is to minimize the brake disc volume for emergency braking conditions.

The secondary objectives are as follows:

- Minimize the maximum von-mises stress in the brake disc (*structural analysis*).
- Maximize the first natural frequency of the brake disc (*modal analysis*).
- Minimize the maximum temperature in the brake disc (*thermal analysis*).

Mathematical Problem Formulation

The primary objective is to minimize the disc volume.

Constraints: Here, the maximum von-mises stress and the maximum temperature is to be upper bounded. The first natural frequency of the brake disc should be higher than that of the engine fire frequency.

Variables: The variables that are changed to find the optimum solution are the brake disc's inner diameter, outer diameter, and thickness.

Brake design model analysis

Here, there are three analyses are performed: 1. Static structural analysis, 2. Modal analysis, 3. Transient thermal analysis. The project schematics of the work are shown below. The initial design of the brake disc has design variables thickness (P1), outer diameter (P2), and inner diameter (P3) are 25 mm, 125 mm, and 75 mm respectively.

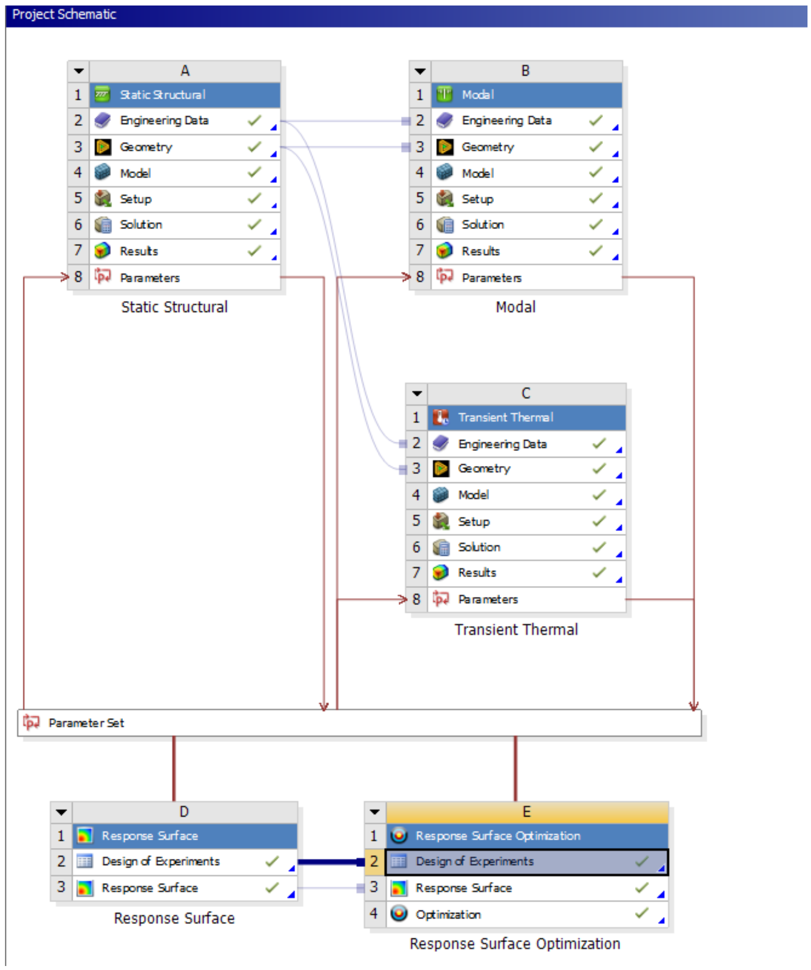

Figure 1: Ansys Optimization Flowchart

Static Structural Analysis

Static structural analysis is implemented to find the maximum von-mises stress that is produced in the model. Gray cast iron and structural steel are used as materials for the brake disc and brake pads respectively. Tetrahedrons are used as a meshing method for the brake disc as well as the brake pads. The inner face of the brake pads which are in contact with the brake disc has smaller elements so that it can generate more accurate results. There are some boundary conditions are applied to the model to get accurate results. The rotational velocity of 250 rad/s is given for Y-axis. The brake pads are restricted to moving in the X and Z-axis. They can only move along Y-axis. The pressure of 10.495 MPa is applied to the outer side of the brake pads to press the brake pads against the brake disc. A revolute joint is created between the brake disc and the ground so that the brake can rotate with respect to the ground. The contact between the brake pads and the brake disc is created to simulate the connection between the two. After solving the simulation, it has been found that the maximum stress of 13.539 MPa is produced in the contact region as shown in the figure below.

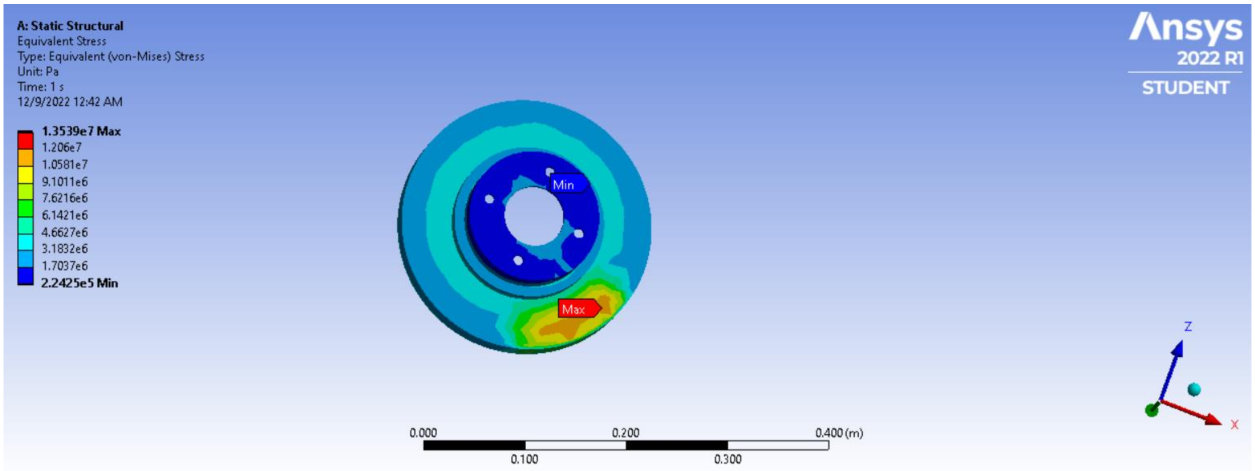

Figure 2: Static Structural Analysis

Modal Analysis

Modal analysis is performed using the same geometry as well as the same meshing method of tetrahedrons. It is implemented to find the first natural frequency of the brake disc. As it is shown in the figure the first natural frequency of the brake disc is found as 1612.6 Hz.

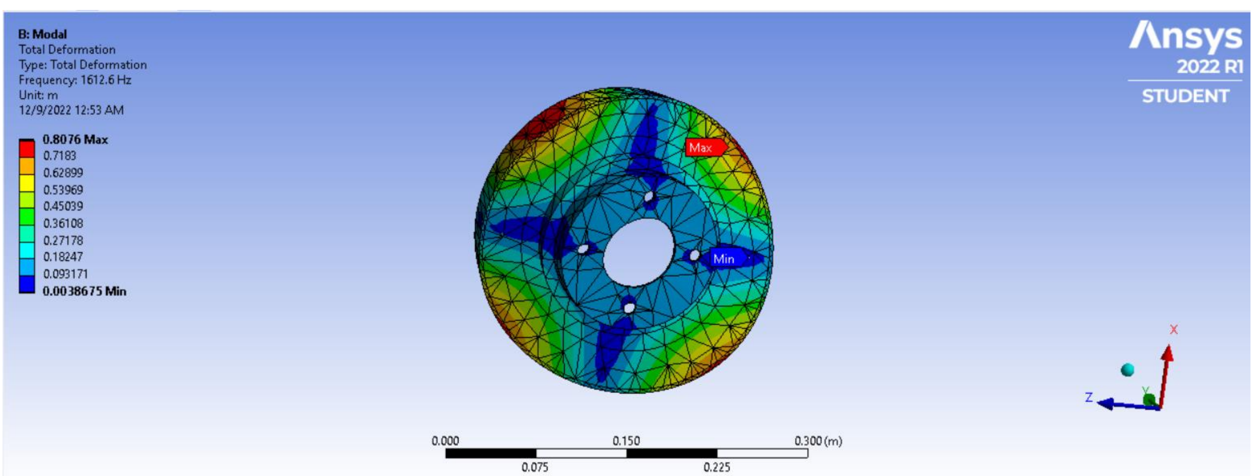

Figure 3: Modal Analysis

Transient Thermal Analysis

Transient thermal analysis is implemented to observe the maximum temperature generated in the model. The initial temperature of the brake disc is 35 ℃. Convection is applied to all the faces of the brake disc with a film coefficient of 5 W/m^2. Heat flux of 1.5395 MW/m^2 is applied to the surface of the brake disc where the brake pads are in contact while the disc is rotating. After solving the problem, we found that the maximum temperature is 339.97 ℃.

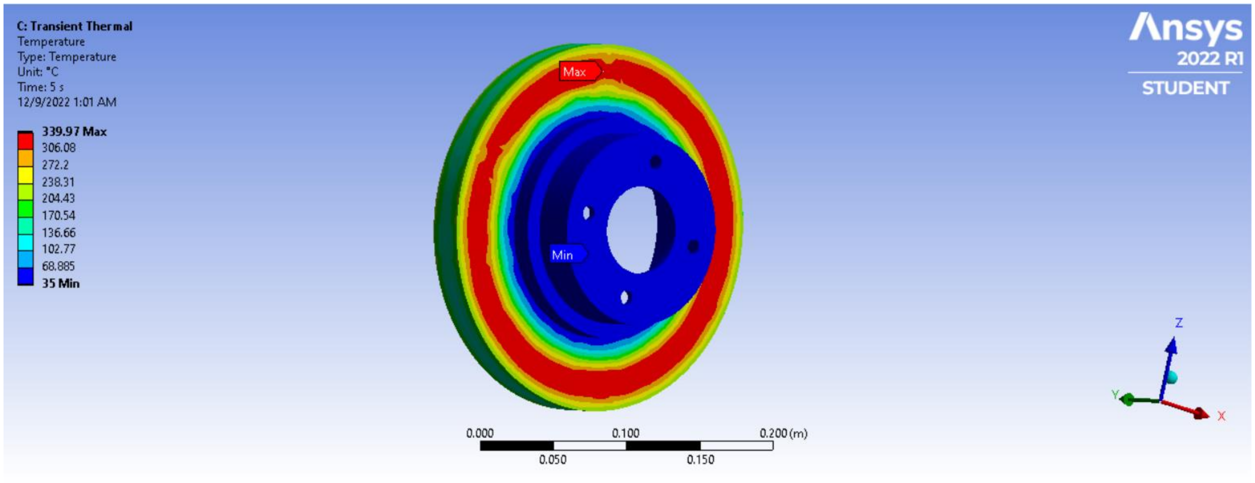

Figure 4: Thermal Analysis

Design Optimization

The values of the output parameters are known from the static structural analysis, modal analysis, and thermal analysis performed on the initial values of the input parameters (P1, P2, and P3 as discussed earlier). Here, DOE and ANSYS built-in optimization method is implemented to find the optimum solutions when the objective functions are maximum stress, solid volume, maximum temperature, and first natural frequency of the brake disc.

Design of Experiments

Latin hypercube sampling (LHS) is used to generate the input data points from the given region by the user. The range for the brake disc thickness, outer diameter, and inner diameters are taken as 9-26 mm, 127-150 mm, and 68-89 mm respectively. Here, a total of 50 DOE input parameters are generated in the bounds mentioned above. All the output parameters are found by solving the analyses according to the input parameters generated. Bounds for each input parameter are also shown in the table below. The results for all the DOE points are also shown in the figures shown below.

Design Variables	Lower Bound (mm)	Upper Bound (mm)
Thickness (P1)	9	26
Inner Diameter (P2)	127	150
Outer Diameter (P3)	68	89

Table of Outline A2: Design Points of Design of Experiments

	A	B	C	D	E	F	G	H
1	Name	P1 - rotor_thickness (mm)	P2 - rotor_OD (mm)	P3 - rotor_ID (mm)	P4 - Equivalent Stress Maximum (Pa)	P5 - Total Deformation Reported Frequency (Hz)	P6 - Temperature Maximum (C)	P7 - Solid Volume (m^3)
2	1	15.29	128.61	88.37	1.1318E+07	1206.8	408.29	0.00067969
3	2	18.35	141.95	76.61	1.1137E+07	1215.3	376.71	0.0010401
4	3	9.51	146.55	69.89	1.0226E+07	859.34	565.36	0.00068974
5	4	16.65	140.57	87.95	2.0906E+07	1117.9	390.44	0.00088764
6	5	10.19	132.29	71.57	9.9554E+06	1172.6	533.54	0.00059587
7	6	24.81	134.13	80.81	1.2061E+07	1393.1	341.96	0.0011252
8	7	16.31	143.79	87.11	1.595E+07	1091.9	395.72	0.00092602
9	8	24.47	141.03	68.21	1.1437E+07	1330.3	343.45	0.0013599
10	9	12.23	135.51	74.51	1.0079E+07	1224.3	470.58	0.00070181
11	10	11.21	149.77	70.31	1.0097E+07	871.43	501.62	0.00081127
12	11	13.59	127.23	71.15	1.1705E+07	1398.1	435.13	0.00067318
13	12	10.87	138.27	77.45	1.0141E+07	1151.3	512.16	0.00066787
14	13	14.95	137.35	73.67	1.1454E+07	1249.5	414.41	0.00083783
15	14	19.37	148.39	75.77	1.1337E+07	1124.8	369	0.0012045
16	15	13.25	131.37	88.79	1.8717E+07	1152.1	441.94	0.00065213
17	16	22.43	145.17	85.43	1.2168E+07	1148.4	350.72	0.0012198
18	17	21.75	138.73	71.99	1.1378E+07	1327.6	353	0.001162
19	18	18.01	148.85	84.59	1.1416E+07	1051.3	378.65	0.0010946
20	19	10.53	136.43	79.97	1.0104E+07	1204.9	523.69	0.00063302
21	20	9.85	149.31	81.65	1.0087E+07	938.64	550	0.00071851
22	21	14.61	141.49	78.29	1.0307E+07	1164.4	417.44	0.00086035
23	22	22.77	130.91	85.01	1.2458E+07	1322.4	349.38	0.00095639
24	23	25.15	147.01	82.91	1.1928E+07	1191.8	340.43	0.001404
25	24	11.55	129.99	76.19	9.8878E+06	1357.8	484.29	0.0006179
26	25	20.39	139.19	70.73	1.1351E+07	1292.5	360.99	0.0011174

Figure 5: DOE Points

Table of Outline A2: Design Points of Design of Experiments

	A	B	C	D	E	F	G	H
1	Name	P1 - rotor_thickness (mm)	P2 - rotor_OD (mm)	P3 - rotor_ID (mm)	P4 - Equivalent Stress Maximum (Pa)	P5 - Total Deformation Reported Frequency (Hz)	P6 - Temperature Maximum (C)	P7 - Solid Volume (m^3)
27	26	17.67	135.97	79.55	1.1085E+07	1299.4	382.33	0.00090233
28	27	21.41	135.05	83.33	1.1495E+07	1304.1	355.5	0.0010008
29	28	19.03	130.45	74.09	1.104E+07	1454.3	368.46	0.00089733
30	29	25.49	134.59	69.47	1.1436E+07	1499.1	340.53	0.0012568
31	30	13.93	144.71	79.13	1.0012E+07	1095.3	434.29	0.00086822
32	31	20.73	146.09	73.25	1.1623E+07	1167.2	359.56	0.0012458
33	32	22.09	144.25	69.05	1.1625E+07	1243.4	352.2	0.0013044
34	33	15.63	136.89	81.23	1.1352E+07	1251.5	405.95	0.00082955
35	34	23.11	143.33	84.17	1.1928E+07	1189.7	347.8	0.0012214
36	35	14.27	145.63	80.39	1.0122E+07	1083.2	427.37	0.00089141
37	36	12.57	129.07	72.41	1.0319E+07	1379.9	458.96	0.00065324
38	37	24.13	139.65	78.71	1.178E+07	1326.1	343.58	0.0012331
39	38	23.79	132.75	83.75	1.2004E+07	1340.1	347.64	0.0010356
40	39	17.33	127.69	77.87	1.0917E+07	1443.3	384.18	0.00077888
41	40	25.83	147.93	82.07	1.223E+07	1195.5	338.82	0.0014657
42	41	16.99	133.21	75.35	1.1284E+07	1349.8	388.12	0.00085658
43	42	15.97	147.47	74.93	1.0093E+07	1063.8	400.7	0.0010204
44	43	19.71	129.53	72.83	1.107E+07	1491.5	363.22	0.00091431
45	44	11.89	133.67	85.85	1.0162E+07	1205.6	477.65	0.00064273
46	45	21.07	131.83	87.53	1.4354E+07	1241.9	356.29	0.00090028
47	46	20.05	142.41	86.69	1.4833E+07	1143.8	362.9	0.0010579
48	47	23.45	128.15	77.03	1.1626E+07	1516.9	346.12	0.00099107
49	48	18.69	137.81	68.63	1.1179E+07	1274.2	373.17	0.0010285
50	49	12.91	142.87	86.27	1.1598E+07	1088.7	451.87	0.00077822
51	50	9.17	140.11	82.49	1.0424E+07	1115.2	577.96	0.00060754

Figure 6: DOE Points (Continued)

Response Surface

When DOE is completed, the response surface is created using the method of Standard Response Surface- Full 2nd Order Polynomial by taking the refinement type as manual and generating 3 verification points to verify the results. The goodness of fit is provided below. The response surface table is also given below.

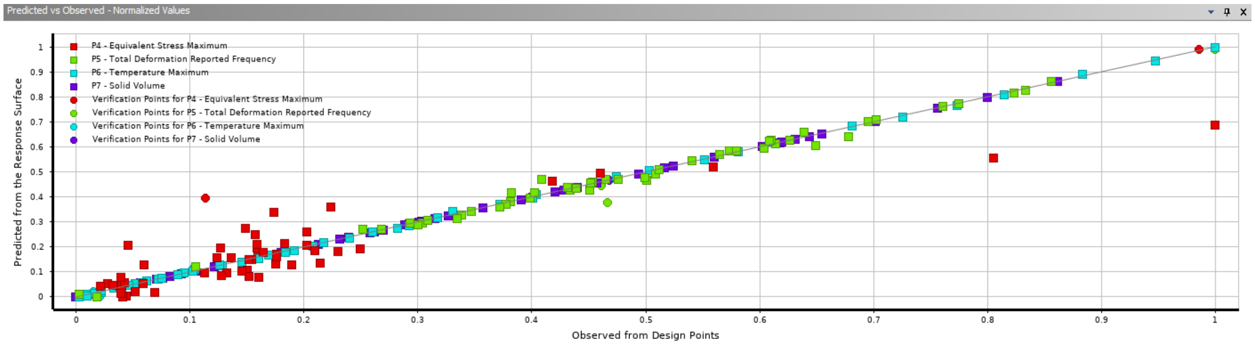

Figure 7: Goodness of fit plot

Table of Schematic D3: Response Surface

	A	B	C	D	E
1		P4 - Equivalent Stress Maximum	P5 - Total Deformation Reported Frequency	P6 - Temperature Maximum	P7 - Solid Volume
2	Coefficient of Determination (Best Value = 1)				
3	Learning Points	0.80297	0.99208	0.99971	1
4	Root Mean Square Error (Best Value = 0)				
5	Learning Points	9.115E+05	12.977	1.1184	1.0153E-07
6	Verification Points	1.8403E+06	39.073	3.0666	4.4576E-07
7	Relative Maximum Absolute Error (Best Value = 0%)				
8	Learning Points	169.44	32.066	4.7932	0.12913
9	Verification Points	134.12	43.329	7.6659	0.24652
10	Relative Average Absolute Error (Best Value = 0%)				
11	Learning Points	29.684	6.2309	1.2933	0.033362
12	Verification Points	47.928	18.408	3.3291	0.17027

Figure 8: Response Surface table

Here, Local sensitivity analysis and local sensitivity curves are also shown below.

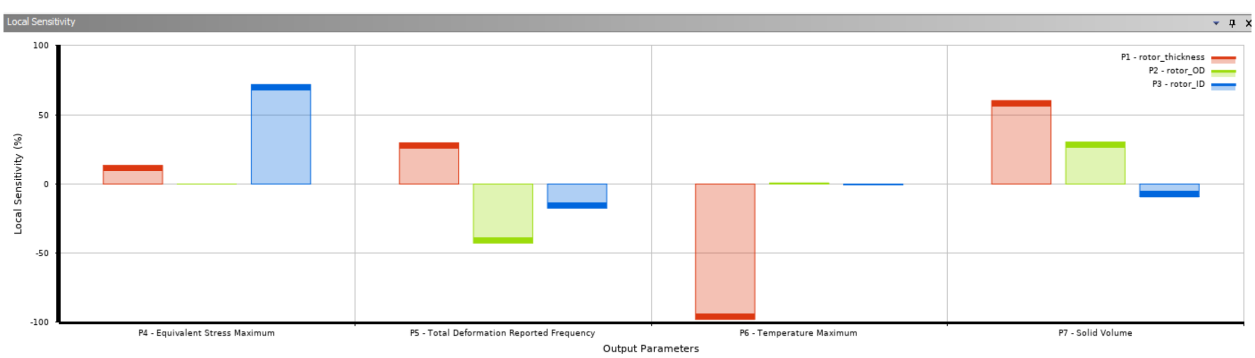

Figure 9: Local Sensitivity

Here, the given figure shows the maximum equivalent stress (Y-axis) with respect to the design variables (P1, P2, P3) (X-axis).

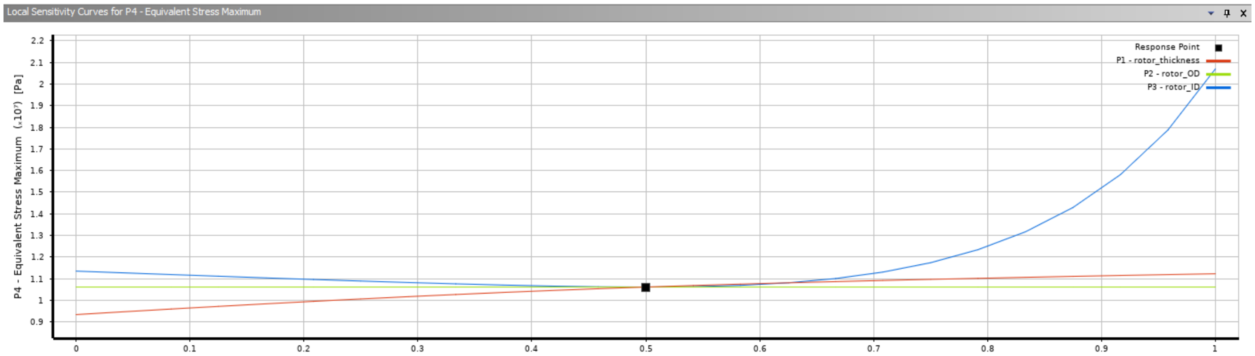

Figure 10: Local Sensitivity Curve: Stress

Here, the given figure shows the solid volume (Y-axis) with respect to the design variables (P1, P2, P3) (X-axis).

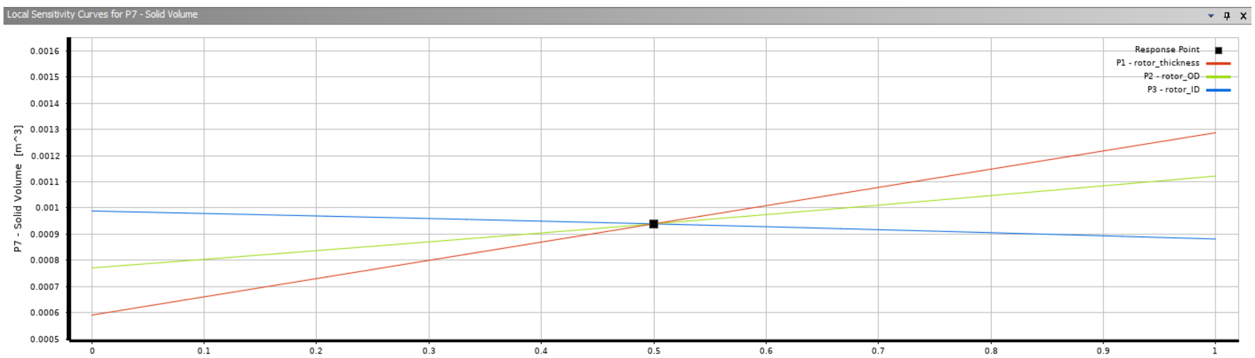

Figure 11: Local Sensitivity Curve: Volume

Here, the given figure shows the first natural frequency (Y-axis) with respect to the design variables (P1, P2, P3) (X-axis).

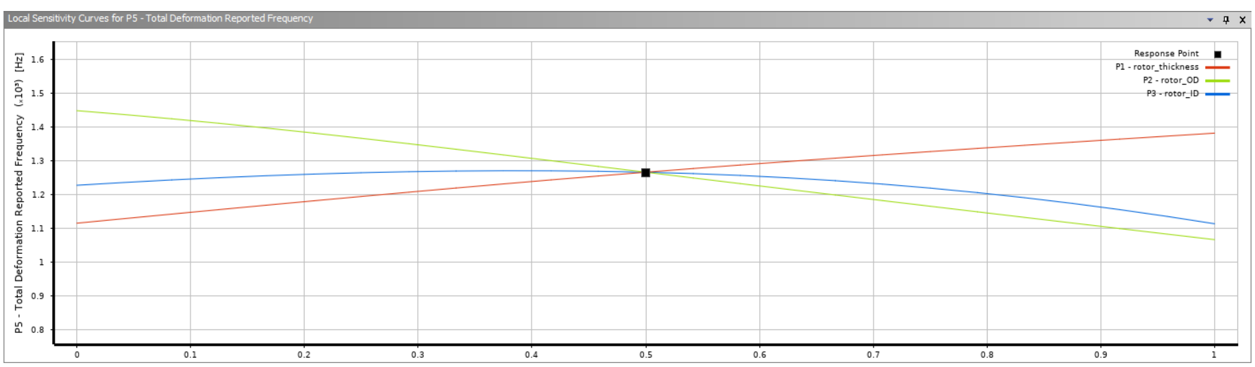

Figure 12: Local Sensitivity Curve: Frequency

Here, the given figure shows the maximum temperature(Y-axis) with respect to the design variables (P1, P2, P3) (X-axis).

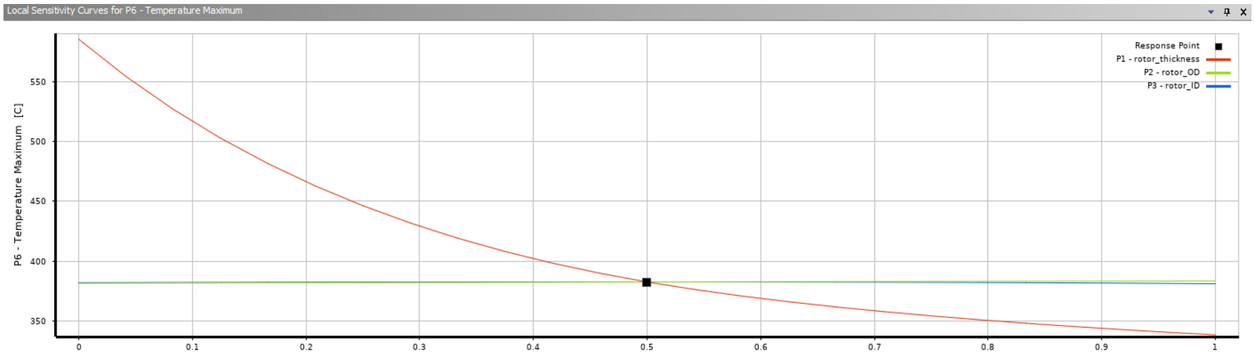

Figure 13: Local Sensitivity Curve: Temperature

Optimization

Here, the Manual optimization method of Multiobjective Genetic Algorithm (MOGA) is chosen for the optimization of multiple objectives. Here, the candidate points are also verified. We have provided objectives and constraints as shown below in the figure. Here, the maximum stress is upper bounded by 14 MPa. The maximum temperature is upper bounded by 400 °C. The solid volume of the brake disc is also upper bounded by $0.00099667\ m^3$. Here, these three parameters are minimized. The frequency is lower bounded by -1200 Hz to get accurate results. The frequency is to be maximized. The figure below shows the objectives and constraints and the method of optimization in detail.

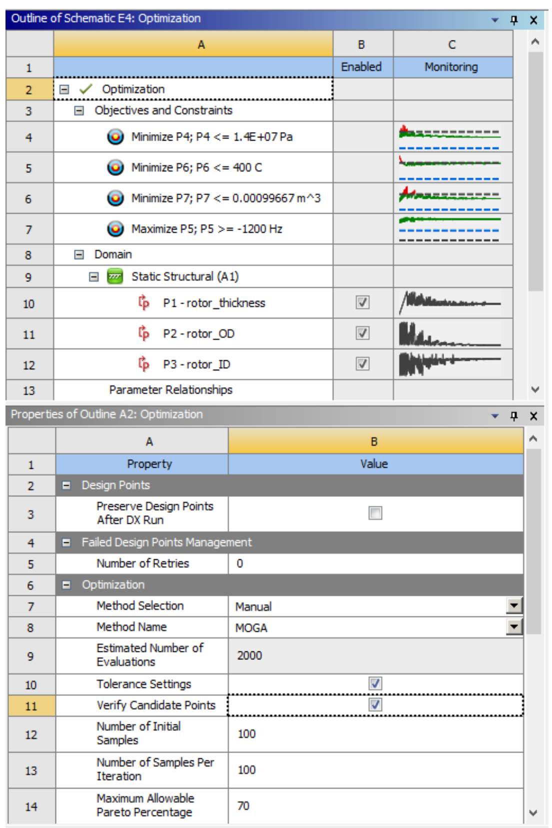

Figure 14: Optimization Properties

After the solution of optimization, the highlighted candidate point is chosen as the final optimized result because that particular point has the least volume compared to the other two candidate points. The optimization results are shown in the table below:

Table of Schematic E4: Optimization

	A	B	C	D	E	F	G
1	Optimization Study						
2	Minimize P4; P4 <= 1.4E+07 Pa	Goal, Minimize P4 (Default importance); Strict Constraint, P4 values less than or equals to 1.4E+07 Pa (Default importance)					
3	Maximize P5; P5 >= -1200 Hz	Goal, Maximize P5 (Default importance); Strict Constraint, P5 values greater than or equals to -1200 Hz (Default importance)					
4	Minimize P6; P6 <= 400 C	Goal, Minimize P6 (Default importance); Strict Constraint, P6 values less than or equals to 400 C (Default importance)					
5	Minimize P7; P7 <= 0.00099667 m^3	Goal, Minimize P7 (Default importance); Strict Constraint, P7 values less than or equals to 0.00099667 m^3 (Default importance)					
6	Optimization Method						
7	MOGA	The MOGA method (Multi-Objective Genetic Algorithm) is a variant of the popular NSGA-II (Non-dominated Sorted Genetic Algorithm-II) based on controlled elitism concepts. It supports multiple objectives and constraints and aims at finding the global optimum.					
8	Configuration	Generate 100 samples initially, 100 samples per iteration and find 3 candidates in a maximum of 20 iterations.					
9	Status	Converged after 1121 evaluations.					
10	Candidate Points						
11		Candidate Point 1	Candidate Point 1 (verified)	Candidate Point 2	Candidate Point 2 (verified)	Candidate Point 3	Candidate Point 3 (verified)
12	P1 - rotor_thickness (mm)	16.685		16.634		16.78	
13	P2 - rotor_OD (mm)	127.05		127.08		127.05	
14	P3 - rotor_ID (mm)	83.463		83.463		83.463	
15	P4 - Equivalent Stress Maximum (Pa)	9.3499E+06	1.1847E+07	9.3589E+06	1.1752E+07	9.3597E+06	1.1825E+07
16	P5 - Total Deformation Reported Frequency (Hz)	1355	1324.4	1354.4	1323.9	1355.5	1327.8
17	P6 - Temperature Maximum (C)	389.57	391.69	390.14	391.25	388.53	389.61
18	P7 - Solid Volume (m^3)	0.00072225	0.00072256	0.00072123	0.00072152	0.00072499	0.0007253

Figure 15: Optimization Results

The convergence criteria plot is given for reference as follows:

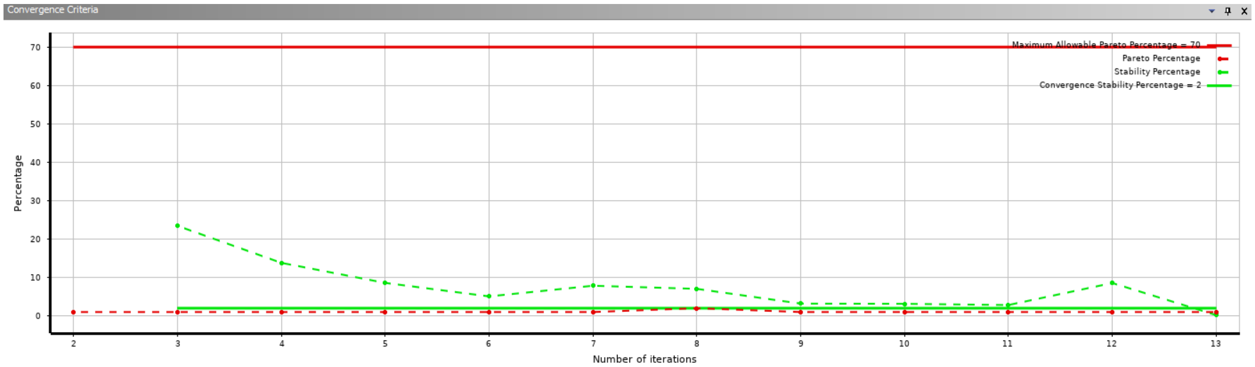

Figure 16: Convergence Criteria plot

Results and Conclusion

Please consider the table below:

Parameter	Initial Values	Optimal values from ANSYS	% increment/% decrement
Thickness (P1)	25 mm	16.634 mm	
Outer diameter (P2)	125 mm	127.08 mm	
Inner diameter (P3)	75 mm	83.463 mm	
Max. Stress	13.539 MPa	11.752 MPa	-13.2%
Natural Frequency	1612.6 Hz	1323.9 Hz	-17.9%
Max. Temperature	339.97 ℃	391.25 ℃	+15.08%
Solid Volume	**0.00099667** $\boldsymbol{m^3}$	**0.00072152** $\boldsymbol{m^3}$	**-27.6%**

As observed from the above table, The improvement in the reduction in the solid volume can be seen, which is **27.6%** less than the volume that was given as the initial design. So, the design has improved by that margin. For, the optimal design, the brake disc thickness, outer diameter, and inner diameter are as 16.634 mm, 127.08 mm, and 83.463 mm. Not only, the solid volume of the design has been successfully reduced, but also the maximum stress is reduced by **13.2%**, Here, all the values found, satisfy the constraints that are provided. The maximum temperature has risen by **15.08%**, which is logical since the thickness of the disc has been reduced. However, the maximum temperature still satisfies the given constraint of 400 ℃. Also, the first natural frequency of the disc is 1323,9 Hz in the improved design which is less than 1612.6 Hz by **17.9%**. Though, it satisfies the constraint of engine fire frequency of 1200 Hz. So, Overall, the design is improved by **27.6%** for the primary objective with every other secondary objective under the desired limitations.