Design of Path Following Controller for Mobile Robot

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Introduction

Technical Approach

Unicycle Model and Open-Loop Dynamics

Technical Approach

$$\dot{x} = v\cos(\theta)\ ,$$
$$\dot{y} = v\sin(\theta)\ ,$$
$$\dot{\theta} = \omega\ ,$$

$$v = v_f$$
$$\dot{\rho} = -v\cos(\frac{\pi}{2} - \phi) = -v\sin(\phi)\ ,$$
$$\phi = \theta_o - \theta\ ,$$
$$\dot{\phi} = \dot{\theta}_o - \dot{\theta} = \dot{\theta}_o - \omega = -vu\ ,$$
$$\omega = \dot{\theta}_o + vu\ ,$$

Path Constraints - Setup

Technical Approach

$$y = f(x) \ ,$$

$$f'(x) = \frac{d}{dx} f(x) \ ,$$

$$f''(x) = \frac{d^2}{dx^2} f(x) \ ,$$

$$\overrightarrow{p}(x) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ f(x) \end{bmatrix} \ ,$$

$$q_R(t) = \begin{bmatrix} x_R \\ y_R \end{bmatrix} \ ,$$

Path Constraints - Distance Minimization

Technical Approach

$$d(x) = \|q_R(t) - \overrightarrow{p}(x)\| = \sqrt{(x_R - x)^2 + (y_R - f(x))^2}$$

$$x_{d_{\text{extrema}}}(t) = \{x | \frac{d}{dx} d(x)^2 = 0, x \in \mathbb{R}\} \ ,$$

$$\rho(t) = \min(\{d(x_{d_{\text{extrema}}}(t))\}) \ ,$$

$$x_{d_{\min}}(t) = x | \rho = d(x) \ ,$$

Path Constraints - Path Direction

Technical Approach

$$\vec{v}(x) = \frac{d}{dx}\vec{p}(x) = \begin{bmatrix} 1 \\ f'(x) \end{bmatrix} ,$$

$$\hat{T}(x) = \frac{\vec{v}(x)}{\|\vec{v}(x)\|} = \begin{bmatrix} \frac{1}{\sqrt{1+f'(x)^2}} \\ \frac{f'(x)}{\sqrt{1+f'(x)^2}} \end{bmatrix} ,$$

$$\theta_o(t) = \arctan\left(\frac{T_y(x_{d_{\min}}(t))}{T_x(x_{d_{\min}}(t))}\right) = \arctan(\dot{f}(x_{d_{\min}}(t))) ,$$

$$\dot{\theta}_o(t) = \frac{d}{dt}\theta_o(t) = \frac{\ddot{f}(x_{d_{\min}}(t))}{1+\dot{f}(x_{d_{\min}}(t))^2} ,$$

Closed-Loop Dynamics

Technical Approach

$$u = -k_\rho(\rho - \rho_o) - k_\phi(\theta - \theta_o) \ ,$$

$$v = v_o \ ,$$

$$\dot{\rho} = -v_o \sin(\phi) \ ,$$

$$\dot{\phi} = -v_o(-k_\rho(\rho - \rho_o) - k_\phi(\theta - \theta_o)) \ ,$$

$$\omega = \frac{\ddot{f}(x_{d_{\min}})}{1 + \dot{f}(x_{d_{\min}})^2} + v_o(-k_\rho(\rho - \rho_o) - k_\phi(\theta - \theta_o)) \ ,$$

Stability

Technical Approach

$$\lim_{x \to 0} \cos(x) \approx 1 \ ,$$

$$\lim_{x \to 0} \sin(x) \approx x \ ,$$

$$\dot{\rho} \approx -v_o \phi \ ,$$

$$\dot{\phi} = -v_o(-k_\rho(\rho - \rho_o) - k_\phi(-\phi)) \ ,$$

$$\begin{bmatrix} \dot{\rho} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & -v_o \\ v_o k_\rho & -v_o k_\phi \end{bmatrix} \begin{bmatrix} \rho - \rho_o \\ \phi \end{bmatrix} \ ,$$

$$cp_A = \det(\lambda I_2 - \begin{bmatrix} 0 & -v_o \\ v_o k_\rho & -v_o k_\phi \end{bmatrix}) = \lambda^2 + k_\phi v_o \lambda + k_\rho v_o^2 \ ,$$

$$\text{eig}(A) = \{\text{roots } cp_A\} = v_o \frac{-k_\phi \pm \sqrt{k_\phi^2 - 4k_\rho}}{2} \ ,$$

For system poles to be on left-half of complex plane:

$$k_\phi > 0 \ ,$$

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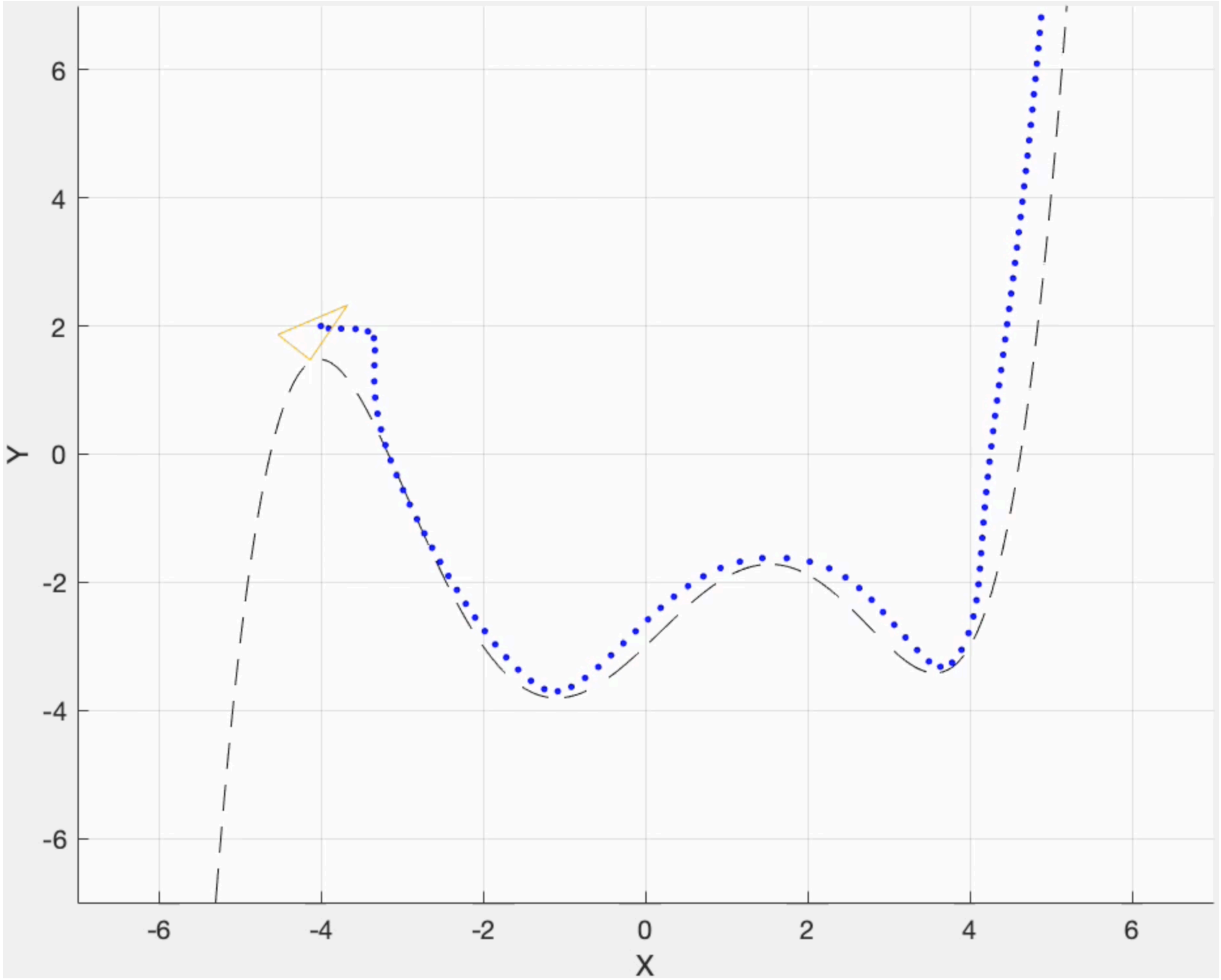
Further, for system poles to be real:

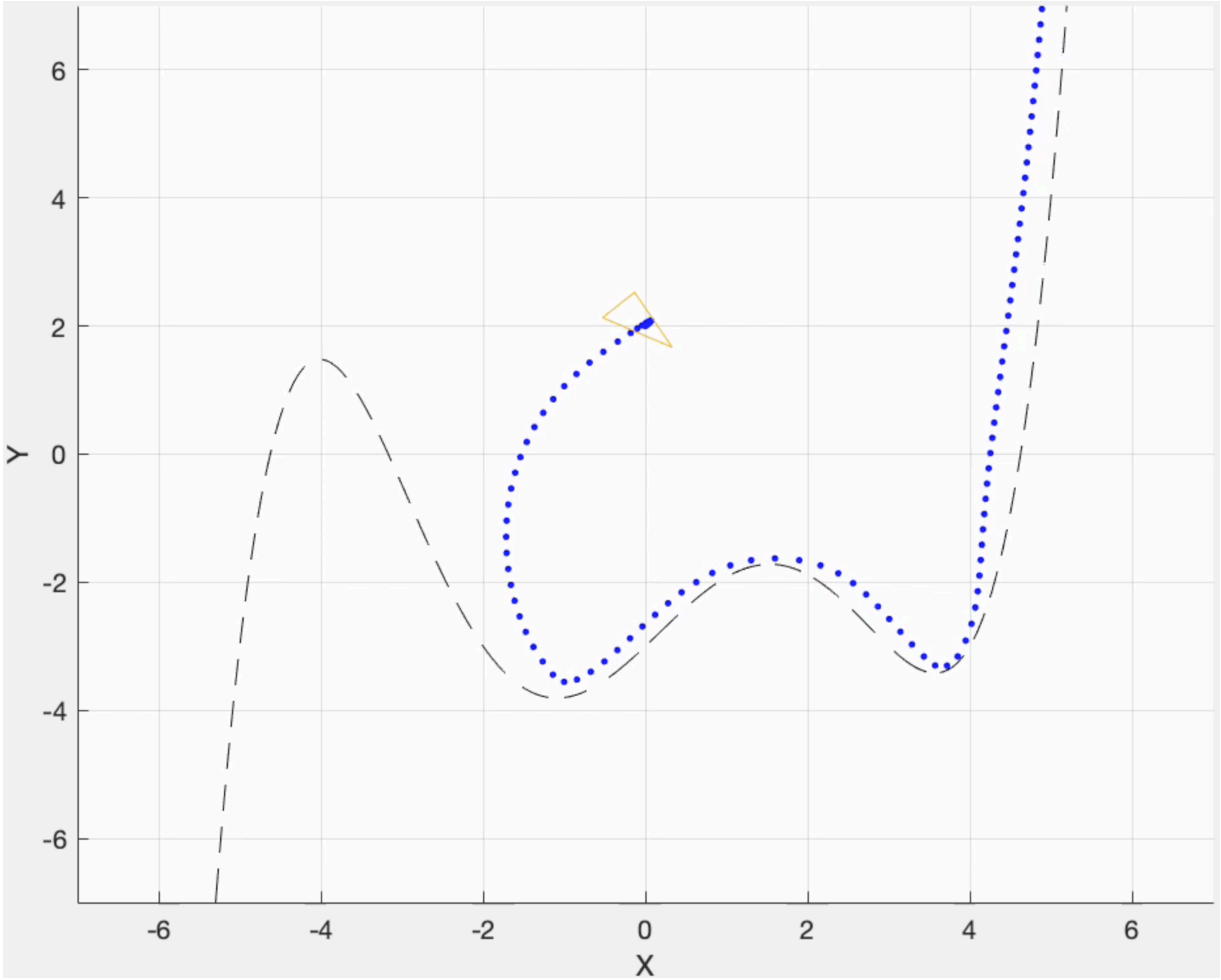
$$k_\phi^2 > 4k_\rho \ ,$$

Results

Controller Success

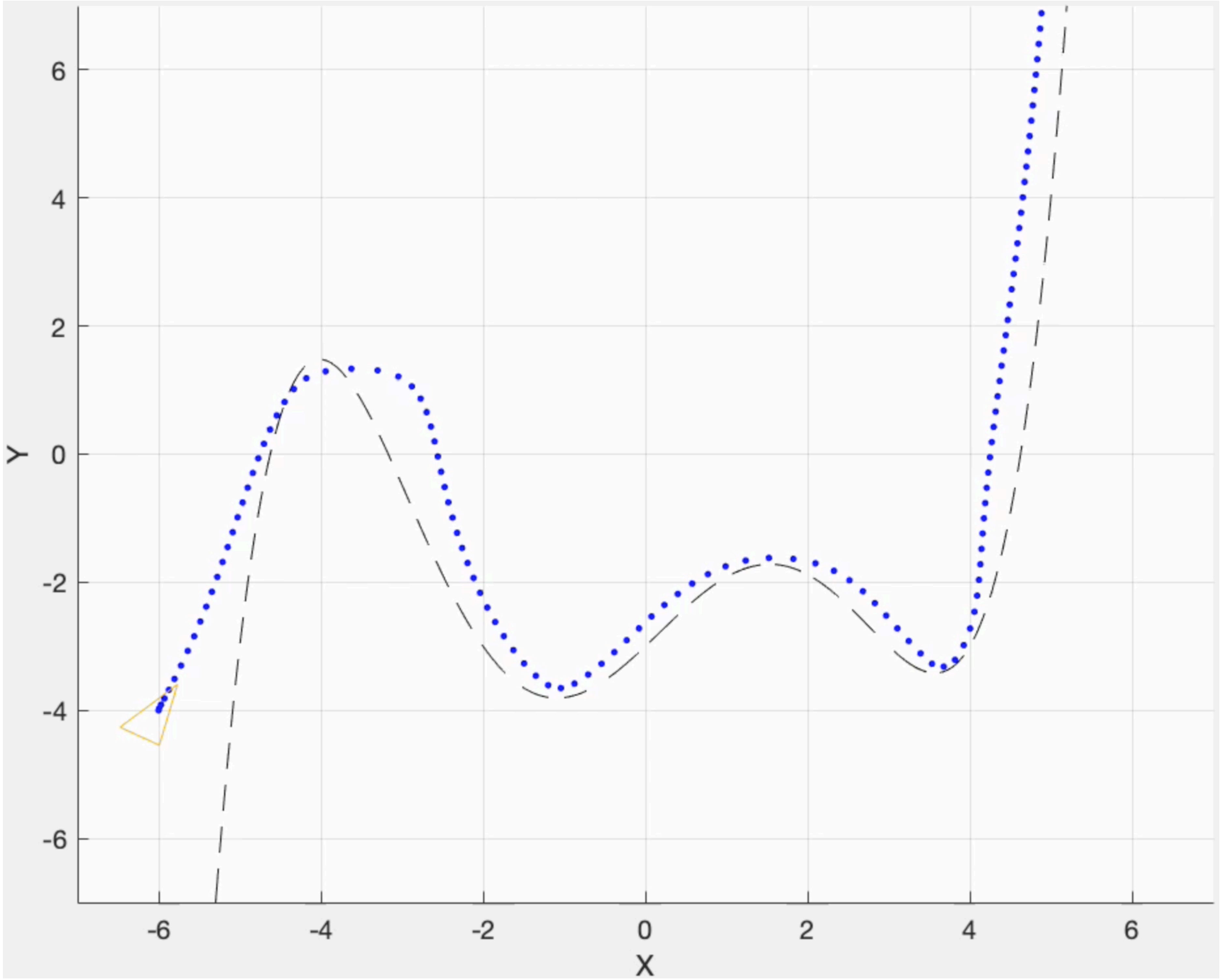
Results

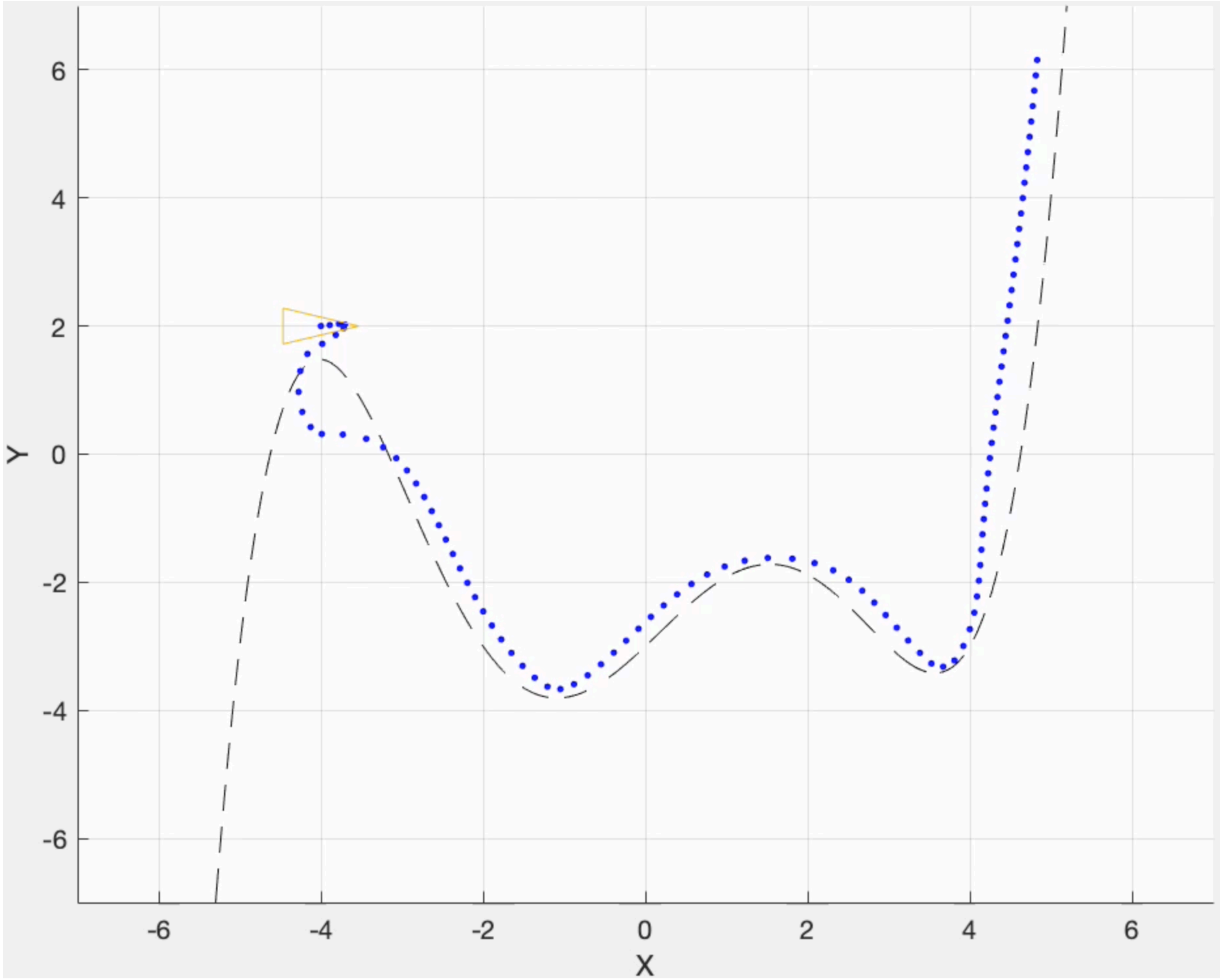




Controller Semi-Failure

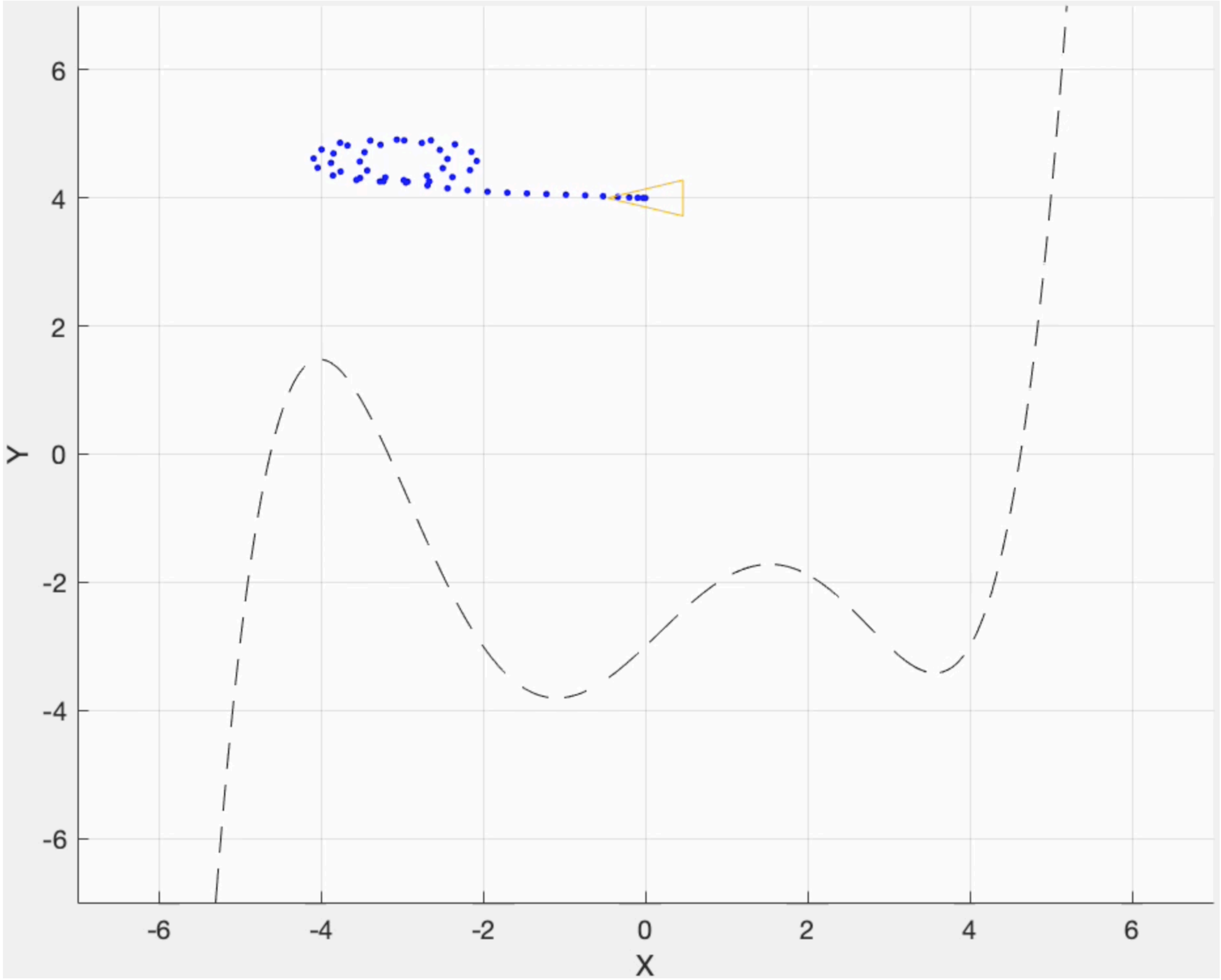
Results

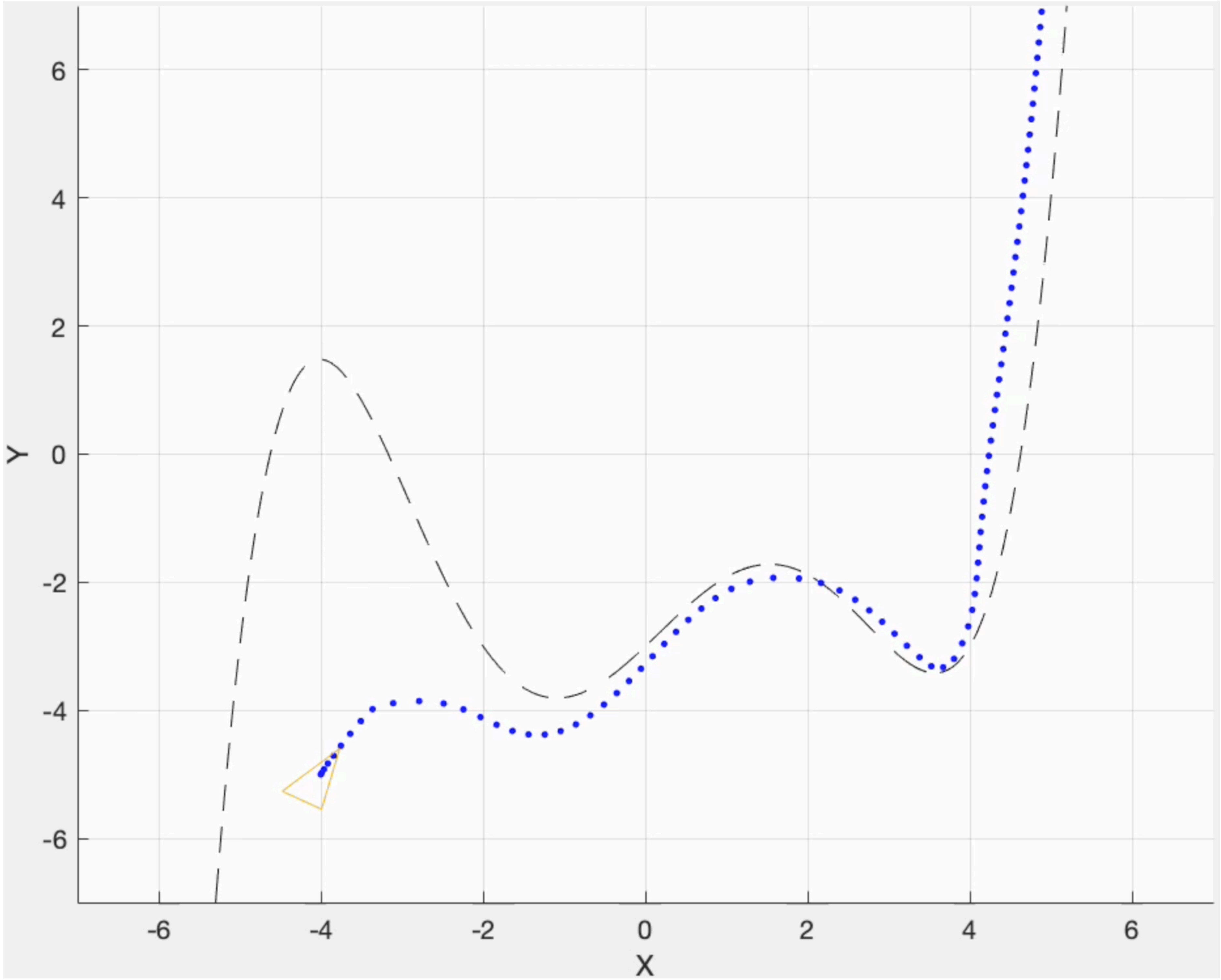




Controller Failure

Results





Questions?