

LEON MARCO A. DEVELA
COM232ADVANCED MACHINE LEARNING
EXERCISE 5

Dataset:

PERSON	HEIGHT (cm)	WEIGHT (kg)	r_{women}	r_{men}
A	158	52	0.98	0.02
B	162	56	0.95	0.05
C	166	60	0.99	0.00
D	175	72	0.10	0.90
E	180	78	0.05	0.95
F	185	84	0.02	0.98

PARAMETER	WOMEN	MEN
mean (μ)	$\begin{bmatrix} 160 \\ 55 \end{bmatrix}$	$\begin{bmatrix} 180 \\ 78 \end{bmatrix}$
mixing coefficient (π)	0.5	0.5
covariance (Σ)	$\begin{bmatrix} 9 & 0 \\ 0 & 16 \end{bmatrix}$	$\begin{bmatrix} 16 & 0 \\ 0 & 25 \end{bmatrix}$

(B) Compute the responsibility for sample C

$$|\Sigma_{\text{women}}| = \begin{vmatrix} 9 & 0 \\ 0 & 16 \end{vmatrix} = 144$$

$$144^{1/2} = \sqrt{144} = 12$$

$$2(3.14)^{2/2} * 12 = [75.36]$$

$$\Sigma_{\text{women}}^{-1} = \frac{1}{144} \begin{bmatrix} 9 & 0 \\ 0 & 16 \end{bmatrix} = \begin{bmatrix} 0.0625 & 0 \\ 0 & 0.0625 \end{bmatrix}$$

$$(C - \mu_{\text{women}})^T \Sigma_{\text{women}}^{-1} (C - \mu_{\text{women}}) = \frac{(166 - 160)^2}{9} + \frac{(60 - 55)^2}{16} = 5.56$$

$$N(C | \mu_{\text{women}}, \Sigma_{\text{women}}) = \frac{1}{75.36} \exp\left(-\frac{1}{2} \times 5.56\right)$$

$$= \frac{1}{75.36} \exp(-2.78)$$

$$= \frac{1}{75.36} (0.062) = \boxed{0.000823}$$

$$|\Sigma_{\text{men}}| = \begin{bmatrix} 16 & 0 \\ 0 & 25 \end{bmatrix} = 400$$

$$400^{1/2} = \sqrt{400} = 20$$

$$2(3.14)^{2/2} * 20 = 125.6$$

$$\Sigma_{\text{men}}^{-1} = \frac{1}{400} \begin{bmatrix} 16 & 0 \\ 0 & 25 \end{bmatrix} = \begin{bmatrix} 0.04 & 0 \\ 0 & 0.0625 \end{bmatrix}$$

$$(C - \mu_{\text{men}})^T \Sigma_{\text{men}}^{-1} (C - \mu_{\text{men}}) = \frac{(166 - 180)^2}{16} + \frac{(60 - 78)^2}{25} = 25.21$$

$$N(C | \mu_{\text{men}}, \Sigma_{\text{men}}) = \frac{1}{125.6} \exp\left(-\frac{1}{2} \times 25.21\right)$$

$$= \frac{1}{125.6} \exp(-12.605)$$

$$= \frac{1}{125.6} (0.0000003355) = \boxed{0.0000002671}$$

$$r_{\text{women}} = \frac{0.5 \times 0.000823}{0.5 \times 0.000823 + 0.5 \times 0.0000002671} = \boxed{0.999}$$

$$r_{\text{men}} = \frac{0.5 \times 0.000823}{0.5 \times 0.000823 + 0.5 \times 0.0000002671} = \boxed{0.000325}$$

② Compute the data points for each cluster

$$\text{Women} = 1+1+1+0+0+0 = \boxed{3} \quad \text{Men} = 0+0+0+1+1+1 = \boxed{3}$$

③ Compute the updated mixing coefficient for each cluster

$$\pi_{\text{women}} = \frac{3}{6} = \frac{1}{2} = \boxed{0.5} \quad \text{coefficient for men} = \frac{3}{6} = \frac{1}{2} = \boxed{0.5}$$

④ Compute the updated means of each cluster

$$\mu_{\text{height women}} = \frac{158 + 162 + 166}{3} = \boxed{162} \quad \mu_{\text{height men}} = \frac{175 + 180 + 185}{3} = \boxed{180}$$

$$\mu_{\text{weight women}} = \frac{52 + 56 + 60}{3} = \boxed{56} \quad \mu_{\text{weight men}} = \frac{72 + 78 + 84}{3} = \boxed{78}$$

$$\mu_{\text{women}} = \begin{bmatrix} 162 \\ 56 \end{bmatrix}$$

$$\mu_{\text{men}} = \begin{bmatrix} 180 \\ 78 \end{bmatrix}$$

⑤ Compute the updated covariance of each cluster

$$A_w = \left(\begin{bmatrix} 158 \\ 52 \end{bmatrix} - \begin{bmatrix} 162 \\ 56 \end{bmatrix} \right) = \begin{bmatrix} -4 \\ -4 \end{bmatrix} \quad A_w^T = \begin{bmatrix} -4 & -4 \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \end{bmatrix} \begin{bmatrix} -4 & -4 \end{bmatrix}$$

$$B_w = \left(\begin{bmatrix} 162 \\ 56 \end{bmatrix} - \begin{bmatrix} 162 \\ 56 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad B_w^T = \begin{bmatrix} 0 & 0 \end{bmatrix} =$$

$$C_w = \left(\begin{bmatrix} 166 \\ 60 \end{bmatrix} - \begin{bmatrix} 162 \\ 56 \end{bmatrix} \right) = \begin{bmatrix} 4 \\ 4 \end{bmatrix} \quad C_w^T = \begin{bmatrix} 4 & 4 \end{bmatrix} =$$

$$D_m = \left(\begin{bmatrix} 175 \\ 72 \end{bmatrix} - \begin{bmatrix} 180 \\ 78 \end{bmatrix} \right) = \begin{bmatrix} -5 \\ -6 \end{bmatrix} \quad D_m^T = \begin{bmatrix} -5 & -6 \end{bmatrix} =$$

$$E_m = \left(\begin{bmatrix} 180 \\ 78 \end{bmatrix} - \begin{bmatrix} 180 \\ 78 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad E_m^T = \begin{bmatrix} 0 & 0 \end{bmatrix} =$$

$$F_m = \left(\begin{bmatrix} 185 \\ 84 \end{bmatrix} - \begin{bmatrix} 180 \\ 78 \end{bmatrix} \right) = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \quad F_m^T = \begin{bmatrix} 5 & 6 \end{bmatrix} =$$

$$A_w = \begin{bmatrix} -4 \\ -4 \end{bmatrix} \begin{bmatrix} -4 & -4 \end{bmatrix} = \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$$

$$B_w = [0]$$

$$C_w = \begin{bmatrix} 4 \\ 4 \end{bmatrix} \begin{bmatrix} 4 & 4 \end{bmatrix} = \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$$

$$D_m = \begin{bmatrix} -5 \\ -6 \end{bmatrix} \begin{bmatrix} -5 & -6 \end{bmatrix} = \begin{bmatrix} 25 & 30 \\ 30 & 36 \end{bmatrix}$$

$$E_m = [0]$$

$$F = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \begin{bmatrix} 5 & 6 \end{bmatrix} = \begin{bmatrix} 25 & 30 \\ 30 & 36 \end{bmatrix}$$

$$\sum_{\text{women}} = \frac{1}{3} \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix} + [0] + \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 32 & 32 \\ 32 & 32 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 10.67 & 10.67 \\ 10.67 & 10.67 \end{bmatrix}} \quad \sum_{\text{men}} = \frac{1}{3} \begin{bmatrix} 25 & 30 \\ 30 & 36 \end{bmatrix} + [0] + \begin{bmatrix} 25 & 30 \\ 30 & 36 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 50 & 60 \\ 60 & 72 \end{bmatrix}}$$

$$= \boxed{\begin{bmatrix} 16.67 & 20 \\ 20 & 24 \end{bmatrix}}$$