PAULOS, RAE S.   COM232

PERSON	HEIGHT (CM)	WEIGHT (KG)	WOMEN	MAN
A	158	52	0.98	0.02
B	162	56	0.95	0.05
C	166	60	0.99	0.00
D	175	70	0.10	0.90
E	180	78	0.05	0.95
F	185	84	0.02	0.98

1. COMPUTE THE RESPONSIBILITY

$$(C - \mu_{women})^T \Sigma^{-1} (C - \mu_{women}) = \left(\frac{X_H - \mu_H}{}\right)^2 + \left(\frac{X_W - \mu_W}{}\right)^2$$

$$= \frac{(166 - 160)^2}{9} + \frac{(60 - 55)^2}{16}$$

$$= 4 + 1.56$$

$$= 5.56$$

$$\left| \Sigma_{women} \right| = \begin{bmatrix} 9 & 0 \\ 0 & 16 \end{bmatrix} = 144$$

$$144^{1/2} = \sqrt{144}$$
$$= 12$$

$$2(3.14)^{2/2}(12) = 75.36$$

$$\Sigma_{women}^{-1} = \frac{1}{144} \begin{bmatrix} 9 & 0 \\ 0 & 16 \end{bmatrix} = \begin{bmatrix} 0.0625 & 0 \\ 0 & 0.1111 \end{bmatrix}$$

$$N(C | \mu_{women}, \Sigma_{women}) = \frac{1}{75.36} \exp\left(-\frac{1}{2} \times 5.56\right)$$

$$= \frac{1}{75.36} \exp(-2.78)$$

$$= 0.000823$$

$$(c - \mu_{MEN})^T \Sigma^{-1} (c - \mu_{MEN}) = \left(\frac{x_M - \mu_M}{}\right)^2 + \left(\frac{x_W - \mu_W}{}\right)^2$$

$$= \frac{(166 - 180)^2}{16} + \frac{(60 - 78)^2}{25}$$

$$= 12.25 + 12.96$$

$$= 25.21$$

$$|\Sigma_{MEN}| = \begin{bmatrix} 16 & 0 \\ 0 & 25 \end{bmatrix} = 400$$

$$400^{1/2} = \sqrt{400} = 20 \qquad 2 \cdot (3.14)^{2/2} \, 20 = 125.6$$

$$N(c | \mu_{MEN}, \Sigma_{MEN}) = \frac{1}{125.6} \exp\left(-\frac{1}{2} \times 25.21\right)$$

$$= \frac{1}{125.6} \exp(-12.605)$$

$$= 0.0000002671$$

$$R_{WOMEN} = \frac{(0.5)(0.000823)}{(0.5)(0.000823) + 0.5(0.0000002671)} = 0.99$$

$$R_{MEN} = \frac{(0.5)(0.0000002671)}{(0.5)(0.000823) + 0.5(0.0000002671)} = 0.000033$$

2. DATA POINTS FOR EACH CLUSTER

WOMEN = 3

MEN = 3

3. UPDATED MIXING COEFFICIENT

$$\pi_{WOMEN} = \frac{3}{6} = \frac{1}{2} = 0.5$$

$$\pi_{MEN} = \frac{3}{6} = \frac{1}{2} = 0.5$$

PAOLOS , RAE S.   COM232

4. COMPUTE THE MEAN

$HEIGHT_{WOMEN} = (158 + 162 + 166) / 3$

$\qquad = 162$

$WEIGHT_{WOMEN} = (52 + 56 + 60) / 3$

$\qquad = 56$

$WOMEN = \begin{bmatrix} 162 \\ 56 \end{bmatrix}$

$HEIGHT_{MEN} = (175 + 180 + 185) / 3$

$\qquad = 180$

$WEIGHT_{MEN} = (72 + 78 + 84) / 3$

$\qquad = 78$

$MEN = \begin{bmatrix} 180 \\ 78 \end{bmatrix}$

5. UPDATED COVARIANCE

$A = \left(\begin{bmatrix} 158 \\ 52 \end{bmatrix} - \begin{bmatrix} 162 \\ 56 \end{bmatrix} \right) = \begin{bmatrix} -4 \\ -4 \end{bmatrix} \qquad A_W^T = \begin{bmatrix} -4 & -4 \end{bmatrix}$

$B = \left(\begin{bmatrix} 162 \\ 56 \end{bmatrix} - \begin{bmatrix} 162 \\ 56 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad B_W^T = \begin{bmatrix} 0 & 0 \end{bmatrix}$

$C = \left(\begin{bmatrix} 166 \\ 60 \end{bmatrix} - \begin{bmatrix} 162 \\ 56 \end{bmatrix} \right) = \begin{bmatrix} 4 \\ 4 \end{bmatrix} \qquad C_W^T = \begin{bmatrix} 4 & 4 \end{bmatrix}$

$D = \left(\begin{bmatrix} 175 \\ 72 \end{bmatrix} - \begin{bmatrix} 180 \\ 78 \end{bmatrix} \right) = \begin{bmatrix} -5 \\ -6 \end{bmatrix} \qquad D_M^T = \begin{bmatrix} -5 & -6 \end{bmatrix}$

$E = \left(\begin{bmatrix} 180 \\ 78 \end{bmatrix} - \begin{bmatrix} 180 \\ 78 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad E_M^T = \begin{bmatrix} 0 & 0 \end{bmatrix}$

$F = \left(\begin{bmatrix} 185 \\ 84 \end{bmatrix} - \begin{bmatrix} 180 \\ 78 \end{bmatrix} \right) = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \qquad F_M^T = \begin{bmatrix} 5 & 6 \end{bmatrix}$

$$A = \begin{bmatrix} -4 \\ -4 \end{bmatrix} \begin{bmatrix} -4 & -4 \end{bmatrix} = \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\frac{1}{3}(A + B + C)$$

$$C = \begin{bmatrix} 4 \\ 4 \end{bmatrix} \begin{bmatrix} 4 & 4 \end{bmatrix} = \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 10.67 & 10.67 \\ 10.67 & 10.67 \end{bmatrix}$$

$$D = \begin{bmatrix} -5 \\ -6 \end{bmatrix} \begin{bmatrix} -5 & -6 \end{bmatrix} = \begin{bmatrix} 25 & 30 \\ 30 & 36 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\frac{1}{3}(D + E + F)$$

$$F = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \begin{bmatrix} 5 & 6 \end{bmatrix} = \begin{bmatrix} 25 & 30 \\ 30 & 36 \end{bmatrix}$$

$$= \begin{bmatrix} 16.67 & 20 \\ 20 & 24 \end{bmatrix}$$