1 Practice exam 4

1. Basis step:

$f(0) = 6$

$f(1) = 10$

Recursive step: $f(n+2) = f(n+1) + f(n)$

2. Shortest members: $\{1, 111, 11111, 1111111, ...\}$

3. Given $f(0) = 3,\ f(n) = 2 \cdot f(n-1) + 6$

$f(1) = 12;\ f(2) = 30$

4. $\Sigma = \{0, 1\}$

Base case: $\lambda \in P,\ 0 \in P, 1 \in P$

Recursive step: if $(x \in P$ and $w \in \Sigma) \rightarrow wxw \in P$

5. Base case: $1 \in S$

Recursive step: $x \in S \rightarrow x/3 \in S$

6. flipbits(λ) = λ

flipbits(0) = 1; flipbits(1) = 0

Recursive step: $\Sigma = \{0, 1\}$. Σ^* is the Kleene closure (all possible strings that can be formed from the alphabet Σ)

(b $\in \Sigma$ and $x \in \Sigma^*$)

flipbits(bx) = flipbits(b) + flipbits(x)

2 6.5 Practice problems

1. TATTLETALES

Permutations: $\frac{11!}{4!2!2!2!}$

2. 200 students, 4 houses. Using *Stars and bars*, we have $C(200 + (4-1), (4-1)) = C(203, 200) = C(203, 3)$ possible ways to assign students to houses.

3. (a) Equation $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 100$. Using *Stars and bars*, we have $C(100 + (6-1), (6-1)) = C(105, 5)$ possible ways to solve it.

(b) If we reqire $a_1, a_2 \geq 2$, then we have $C(96 + (6-1), (6-1)) = C(101, 5) = C(101, 96)$ ways.

(c) Equation $a_1+a_2+a_3+a_4+a_5+a_6 \leq 100$, where a_n are in $\mathbb{Z}^{\geq 0}$. We have $C(100+(7-1),(7-1)) = C(106,6)$ ways. There is a "hidden" category created.

3 In-lecture worksheet

7. Laundry: 5 T-shirts, 6 shorts, 3 dress pants, 4 shirts.

$5!6! + 3!4!$

$5 \cdot 6 + 3 \cdot 4 = 42$

8. String

CABDEFGH

Treat CAB as one letter and permute. This results in 6!.

9. Bit string

Length 8, twice as many 0s than 1s. The answer is 0.