

1 Monty Hall problem

Let's Make a Deal (game show)

Problem named after host, Monty Hall.

3 identical doors (A, B, C). 2 of them have a goat behind, and one has an all-inclusive trip around the world.

Steps:

1. Player picks 1 door
2. Monty Hall opens one other door (he knows what the doors have behind, so he always opens a goat door)
3. Player can keep their previous choice or switch to the other closed door.

WLOG, let's pretend the player picks door A. The probability of getting the good prize is $1/3$. This probability will remain as long as the prize location has not been revealed. So, initially, there is a $2/3$ chance the player will get a bad prize.

After a goat door is opened, the $2/3$ chance is "concentrated" in the unopened door. If the player does not change doors, they will have a $1/3$ chance of winning. If they change, the chance will go up to $2/3$.

An extension of this problem involves a variation with one million dud doors, and only one correct door. If the host shows the player all duds but one, the probability $\frac{999999}{1000000}$ is "concentrated" like in the original problem - i.e., the player should switch to "the best of *all the other doors*."