1 4.1

If $a, b \in \mathbb{Z}$ and $m \in \mathbb{Z}^+$, then a is congruent to b modulo m, if m divides $a - b$.

We write $a \equiv b \pmod{m}$.

Examples:

$7 \equiv 13 (\mod 6)$, $7 - 13 = -6$, $6 | -6$? (yes, multiplier -1)

$3 \not\equiv 10 (\mod 2)$, $10 - 3 = 7$, $2 \not| 7$?, $3 \mod 2 \neq 10 \mod 2$

Let $a, b \in \mathbb{Z}, m \in \mathbb{Z}^+$. Then, $a \equiv b (\mod m) \leftrightarrow \exists k \in \mathbb{Z}$ s.t. $a = b + km$.

If $a \equiv b (\mod m)$ and $c \equiv d (\mod m)$, then $a + c \equiv b + d (\mod m)$?

By definition of congruency: $a = b + km, k \in \mathbb{Z}$

$c = d + k^{'}m, k \in \mathbb{Z}$

Add LHSs, RHSs: $a + c = b + km + d + k^{'}m$

Regroup, refactor: $a + c = b + d + (k + k^{'})m$

By definition of congruency modulo m, $a + c \equiv b + d (\mod m)$.

Conclusion: yes, the statement above is true.

$a \equiv b (\mod m) \wedge c \equiv d (\mod m) \rightarrow a + c \equiv b + d (\mod m)$ as shown, with $a, b, c, d \in \mathbb{Z}$ and $m \in \mathbb{Z}^+$.

2 4.2: Binary Numbers

$123_{10} = 1 \cdot 10^2 + 2 \cdot 10^1 + 3 \cdot 10^0$

$10110_2 = 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 22_{10}$