1 2.2: Sets

Use set builder notation and logical equivalencies to show De Morgan's law for sets holds.

$A \bar{\cap} B = \bar{A} \cup \bar{B}$
$A \bar{\cap} B = \{x \mid x \notin A \cap B\}$ (definition of complement)
$= \{x \mid \neg(x \in A \cap B)\}$ (definition of $\notin$)
$= \{x \mid \neg(x \in A \wedge x \in B)\}$ (definition of $\cap$)
$= \{x \mid \neg(x \in A) \vee \neg(x \in B)\}$ (De Morgan's for logic)
$= \{x \mid (x \notin A) \vee (x \notin B)\}$ (definition of $\notin$)
$= \{x \mid (x \in \bar{A}) \vee (x \in \bar{B})\}$ (definition of complement)
$= \{x \mid x \in \bar{A} \cup \bar{B}\}$ (definition of $\cup$)
$= \bar{A} \cup \bar{B}$
$\therefore$ It is true, as shown, that $A \bar{\cap} B = \bar{A} \cup \bar{B}$.

Membership table
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ Distributive property
Table indicates if x belongs to set $A, B, C \ldots$

A	B	C	$(B \cup C)$	$A \cap (B \cup C)$	$A \wedge B$	$A \wedge C$	$(A \cap B) \cup (A \cap C)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

Since for every category of elements the result is the same, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
$\cap$ is like multiply.
$\cup$ is like add.

2 Functions

Function f from A to B (non-empty sets): $f : A \to B$
Examples:

f : characters$\to$grades (domain to co-domain)
(The function above is not one-to-one and not onto.)
abs : $\mathbb{Z} \to \mathbb{Z}$ (range: $\mathbb{Z}^{\geq 0}$)

When a function is one-to-one and onto, we can find an inverse function f^{-1}.
f is one-to-one $\leftrightarrow \forall a \forall b (a \neq b \to f(a) \neq f(b))$, with $a, b \in$ domain of f.
f is onto if $\forall y \exists x (f(x) = y)$, with $x \in$ domain of f, $y \in$ co-domain of f.

In relational databases:
$f : A \to B \subseteq A \times B$
$A = \{1, 2\}$
$B = \{10, 20, 30\}$
$A \times B = \{(1, 10), (1, 20), (1, 30), (2, 10), (2, 20), (2, 30)\}$ (all theoretically possible transitions; Cartesian product.)
We can define the "real" transitions to be $\{(1, 10), (2, 10)\}$, using the function.