1 1.7: Intro to Proofs

Technique	Assume	Show
direct proof	p	q
proof by contraposition	$\neg q$	$\neg p$
vacuous proof	(nothing)	$\neg p$ (p is always false)
trivial proof	(nothing)	q(q is always true)
proof by contradiction	$p \wedge \neg q$	?

Prove p $\rightarrow$ q.
1) Assume p is true.
2) Work in definitions. Do math, do logic.
3) q is then also true.

Definition. An integer n is even if there exists an integer k where $n = 2k$

Definition. An integer n is odd if there exists an integer k where $n = 2k + 1$

Direct proof: The sum of two even integers is even.
$p \rightarrow q$ a, b $\in \mathbb{Z}$
p: a and b are even
q: a + b is even.
a and b are even (assume p)
a = 2k, k $\in \mathbb{Z}$ (by definition even)
b = 2k', $k' \in \mathbb{Z}$ (by definition even)
a+b = 2k + 2k' (add LHS, RHS)
a+b = 2(k+k') (factor)
Note: by definition of even, a+b is even, having form a+b = 2k' where k" = k + k', k" $\in \mathbb{Z}$
Integers are closed with respect to addition.
Conclusion: I've shown that assuming p leads to q being true, hence $p \rightarrow q$.

Direct proof: If n is odd, then n^2 is odd.
Proof by contraposition: If n is an integer and $3n + 2$ is odd, then n is odd.
$p \rightarrow q$
$p : 3n + 2$ is odd
$q : n$ is odd ($n \in \mathbb{Z}$)
$\neg$ (n is odd) (assume $\neg q$)
n is even (by definition of even and odd)
$n = 2k, k \in \mathbb{Z}$ (by definition of even)
$3n + 2$ Let's examine this.
$3(2k) + 2$ Substitute for n.
$2(3k + 1)$ Factor
Notice this is $\neg p$, the fact than $3n + 2$ is even.
Matching the format of $2k'$, where $k' = 3k + 1, k' \in \mathbb{Z}$
Conclusion: since $\neg q$ leads to $\neg p$, I've shown $p \rightarrow q$.