1 7.1: Finite Probability

Experiment: process yielding one of a set of possible outcomes.

Sample space: set of possible outcomes

Event space: subset of sample space

Socks

4 orange socks, 5 blue socks in a drawer.

What is the probability of retrieving an orange sock?

P(orange) $= \frac{4}{9} = \frac{/E/}{/S/}$

Dice

Roll 2 dice (6-sided)

Rolling $< 1, 2 >$ is different than rolling $< 2, 1 >$. There are 36 possible outcomes.

The possible pairs that add up to 7 are $< 1, 6 >, < 2, 5 >, < 3, 4 >, < 4, 3 >, < 5, 2 >, < 6, 1 >$. There are 6 favourable outcomes.

P(total $= 7) = \frac{6}{36} = \frac{1}{6}$

The possible pair that adds up to 12 is $< 6, 6 >$.

P(total $= 12) = \frac{1}{36}$

Lottery

First prize: pick 4 correct digits. Probability is $\frac{1}{10^4}$ (10 ways for each digit)

Second prize: pick 3 out of 4 correct digits (value and placement).

There are 9 ways to "miss" each digit, and 4 possible positions for each digit. Probability is $\frac{36}{10^4}$

Lottery (again)

Let S be the set of positive integers from 1 to 40, just like 40 distinct, numbered balls.

We will get a subset of 6 numbers with no replacement. To win the game, you must have the 6 correct numbers.

The probability to win is $\frac{1}{C(40,6)} \approxeq 0.00000026$.

If replacement was allowed, this would become a Stars and Bars problem.

The probability would be $\frac{1}{C(40+6-1,6)} = \frac{1}{C(45,6)}$

Poker

52 cards: 13 ranks $\{2, 3, 4, 5, ..., 10, J, Q, K, A\}$ $\times$ 4 suits {Spades, Hearts, Clubs, Diamonds}.

Probability("4 of a kind" given 5-hand card):

$$\frac{C(13,1) \cdot C(4,4) \cdot C(48,1)}{C(52,5)} = \frac{13 \cdot 48}{C(52,5)}$$

"Full House": 3 of a kind and 2 of a kind (notice that $3Q + 2K \neq 3K + 2Q$)

$$\frac{C(13,1) \cdot C(4,3) \cdot C(12,1) \cdot C(4,2)}{C(52,5)}$$

An alternative way to write the numerator is $P(13,2) \cdot C(4,3) \cdot C(4,2)$.

Number sequence

Pick 5 numbers from [1, 50], with no replacement, and order mattering.

Probability: $\frac{1}{P(50,5)}$