

Problem: Calculate compound interest.

Assume  $P_0 = 10,000$ , interest rate 11%.

First technique: Recurrence Equation

$$P_n = P_{n-1} \times 1.11$$

Big O:  $O(n)$  ( $n$  is the number of years)

$$P_1 = P_0 \times 1.11$$

$\vdots$

$$P_{30} = P_{29} \times 1.11$$

$$P_{100} = P_0 \times 1.11^{100}$$

"Shortcut" for  $\sum_{i=1}^n i$  with  $O(1)$  complexity (closed form)

Inverting the order of terms of the sum and adding them to the original sum, we will get  $n$  elements. Each element is going to equal  $n + 1$ . Since we added the same sum in the opposite order, we have to divide the final result by 2.

So, a formula for this kind of sum is  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$

## 1 Section 3. Algorithms

procedure max( $a_1, a_2, a_3, \dots, a_n$  : integers)

    max :=  $a_1$

    for 1:= 2 to  $n$

        if  $a_i > \text{max}$  then

            max :=  $a_i$

    Big O is  $O(n)$

function exponential( $b$ : positive integer,  $a$ : non-negative integer)

    if  $a = 0$  then

        result := 1

    else

        result :=  $b \cdot \text{exponential}(b, a - 1)$

    return result

    Big O is  $O(a)$

function fastExpo( $b$ : positive integer,  $a$ : non-negative integer)

    if  $a = 0$  then

        result := 1

    else if  $a \% 2 = 0$  then

        half := fastExpo( $b, a/2$ )

        result := half · half

    else

        result :=  $b \cdot \text{fastExpo}(b, a-1)$

    return result

Example:  $3^{12} = 3^6 \cdot 3^6 = g \cdot g$

Big O is  $O(\log n)$

The fastExpo function is an example of "divide-and-conquer".

There are also, for instance, dynamic programming (example: longest common subsequence), and greedy algorithm.