CS 2050 Class Notes for 2018-09-14

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1 Section 2.1: Sets

Unordered collection of elements.

Example: $\{1,2,3\} = \{2,1,3\} = \{1,1,3,2,3,2\}$

$S = \{1,3,7,7,\text{ Mary, Lee}, 3.5, 1\}$
The cardinality of S is 6.
$\mathbb{Z}^+ = \{1,2,3,...\}$
The cardinality of $\mathbb{Z}^+$ is ∞.
$B = \{\mathbb{Z}, \mathbb{Z}^+, \mathbb{R}\}$
The cardinality of B is 3.
$C = \{\{\}, \{1,2\}, 1, 2, \{1,1,2\}, \{\{\}\}\}$
The cardinality of C is 5. The only element not counted is $\{1,1,2\}$, because it is equivalent to $\{1,2\}$.
Note: $1 \neq \{1\}$, and $1 \in \{1\}$
Also, the set that contains the empty set is not equal to the empty set itself.
$D = \{\mathbb{Z}, \mathbb{Z}^+ \cup \{0\} \cup \mathbb{Z}^-\}$
The cardinality of D is 1, because the second element is equivalent to the first.

Set Builder Notation vs. List Notation

$B = \{2,4,6,8,...\}$
$\mathbb{Z}^+ = \{1,2,3,4,...\}$
$B = \{2x \mid x \in \mathbb{Z}^+\}$

$A = \{1,2\}$
$B = \{5,6,7\}$
$A \times B = \{(1,5),(1,6),(1,7),(2,5),(2,6),(2,7)\}$
$A \times B = \{(a,b) \mid a \in A, b \in B\}$

$S = \{2,9,28,65,...\}$
$S = \{x^3 + 1 \mid x \in \mathbb{Z}^+\}$

$A = B \iff \forall x(x \in A \iff x \in B)$

$A \subset B \iff \forall x(x \in A \to x \in B) \land \exists x(x \in B \land x \notin A)$
(Note: the above is a proper subset. A "regular" subset is denoted by $\subseteq$.)
$A \subseteq B \iff \forall x(x \in A \to x \in B)$