1 6.3

We have a set S of size n and we will pick r elements.

$n = /S/ = 3$

$S = \{a, b, c\}$

There are two ways this can go.

Imagine we want 2 of these. What do we want?

$/\{\{a, b\}, \{a, c\}, \{b, c\}\}/ = 3$

If the order matters:

$/\{< a, b >, < b, a >, < a, c >, < c, a >, < b, c >, < c, b >\}/ = 6$

Choose r from n elements.

If the order of picking matters: *permutation*. If not: *combination*.

Runners

We have 100 runners. What is the number of winning permutations for first, second and third place? Everybody finishes, and there are no ties.

$100 \cdot 99 \cdot 98$

Number of ways for the top 20:

$P(100, 20) = \frac{100!}{(100-20)!}$

Number of permutations of length r from a set of n elements:

$$P(n, r) = \frac{n!}{(n-r)!}$$

Students

$n = 4$ students

$S = \{a, b, c, d\}$

Take 2 students to a competition.

$C(4, 2) = /\{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}/ = 6$

There is no trivial way to "fill in the blanks" as in previous examples.

Trying to to $4 \cdot 3 \cdot 2 \cdot 1$ would result in $4!/2!$, but this has to be "un-permuted" to avoid double-counting.

Final result: $\frac{4!}{2! \cdot 2!}$

Marathon

There are 40000 runners, and 100 will be invited to an event. The order of invites does not matter.

$C(40000, 100) = \frac{40000!}{(40000-100)! \cdot 100!}$

Number of combinations of length r from a set of n elements:

$$C(n,r) = \frac{n!}{(n-r)! \cdot r!}$$

Note: $C(100,99) = C(100,1)$ (edge case)

Bit strings

Number of bit strings of length 8 having exactly six 1s.

$n = 8,\ r = 6$

$C(8,6) = \frac{8!}{(8-6)! \cdot 6!}$ (the order of picking the bits themselves does not matter)

Substrings

We have the alphabet $\{A, B, C, D, E, F, G, H\}$ and we want to form strings of length 8 that have BED as a subtring.

All permutations: 8!

Considering all possible positions of $BED : 6! \cdot 5! = 6!$