1 6.5: Generalized Permutations and Combinations

How many strings of length 10 are there, considering 26 letters (A-Z)?

By the product rule, 26^{10}.

Fruit: "Stars and Bars" problem

3 types of fruit. Pick 4 pieces of fruit ($r = 4$).

Apples, oranges and pears (all treated equal to each other within their group.)

We have "infinite slices", and we can put the names of fruit alphabetically and represent each possible choice as a bitstream.

For example, a choice of no apples, no oranges, and 4 pears is 110000. The 1s represent "dividers" and the 0s represent fruit choices.

011000 is 1 apple, 0 oranges, 3 pears.

How many bitstrings of length 6 exist containing 4 zeros?

$r = 4,\ n = 3$ types

$C(n + r - 1, r) = C(3 + 4 - 1, 4) = C(6, 4)$

Dollar bills

Take 4 bills from a cash register. There are slots for \$20, \$10, \$5 and \$1.

$C(4 + 4 - 1, 4) = C(7, 4)$ ways to do this.

Cookies

4 types of cookies, a customer grabs 6 cookies to go.

A customer's bag must have all flavors.

$C(4 + 2 - 1, 2) = C(5, 2)$.

Sum

$a + b + c = 13$, with $a, b, c \in \mathbb{Z}^{\geq 0}$

$13 + 0 + 0 = 13$

$\vdots$

$0 + 0 + 13 = 13$

$C(15, 13)$ ways to do this. (3 [types of variables] + 13 [sum] -1)

If we change to $a, b, c \in \mathbb{Z}^+$, we have $C(3 + 10 - 1,\ 10) = C(12, 10)$. We start with 3 and try to find which variables can add 10 more.