

# 1 Skip Lists

Unique, probability-based data structure

## Structure

Series of linked lists, each on a different level

Bottom level contains all the data

Each node has 4 pointers:

(left/right for nodes like in a normal linked list)

(up/down to point to linked list above and below)

Each level is in ascending order

Each level starts with a phantom node containing  $-\infty$ , and can end with a phantom node containing  $+\infty$

Head pointer points to upper left  $-\infty$  node.

The contents of each level are based on a coin flip.

Bottom:  $n$  nodes;  $2^{nd}$  level:  $n/2$  nodes;  $3^{rd}$  level:  $n/4$  nodes ...

## Seaching

Start at head

On a given level:

iterate through LinkedList

if you find data 1, stop

if you find node containing data  $>$  (data you are looking for), you know data you're looking for doesn't exist on that level.

use the previous node's down pointer to the next level

reiterate on this level.

## Adding

Flip a coin repeatedly.

if you get Heads, flip again and stop when you get Tails

$n$  heads =  $n$  of levels above the bottom level

add empty levels to top if necessary (number of levels is capped at  $\log n$ )

traverse the skip list to find where to add the node at bottom level

add the node immediately before the  $1^{st}$  node greater than the data

after adding at the bottom, promote node to above levels based on the amount of heads.

## Removing

Find the data to remove

If the data is found, remove from the level, drop down and repeat.

## Efficiency

	average case	worst case
adding		
removing	$O(\log n)$	$O(n)$
searching		

Space complexity is  $O(n)$  ( $\sum_{i=0}^{\infty} \frac{1}{2^i}$ ) on average, and  $O(n \log n)$  at worst.

## 2 AVL Trees

Binary trees that follow the order property and are height-balanced.

When a tree is height-balanced, all operations (add, remove, get) are  $O(\log n)$ .

This avoids the degenerate case for BSTs (in which they become basically a "linked list").

### Properties

Nodes: 4 fields (left, right, height, balance factor)

Height: height of a node is  $\max(\text{height of left, height of right}) + 1$

Balance factor:  $\text{height}(\text{left}) - \text{height}(\text{right})$ . A tree is balanced if  $|bf| = 1$ .

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### Pointer reinforcement (continued)

Traverse like you are looking for the node, but children pointers:

```
node.left = add(node.left, data)
```

At each recursive call, return the corrected node.

In a linked list, for example:

```
public void removeFirstOccurrence(T data) {
    head = removeFirstOccurrence(head, data);
    //To return T data from the removed node, use a dummy node.
}

public Node<T> removeFirstOccurrence(Node<T> curr, T data) {
    if (curr == null) {
        throw new java.util.NoSuchElementException("Data not found");
    } else if (curr.data.equals(data)) {
        --size;
        return curr.next;
    } else {
        curr.next = removeFirstOccurrence(curr.next, data);
        return curr;
    }
}
```