1 5.1: Mathematical induction (continued)

Example. $P(n) : 3|(n^3 - n)\forall n,\ n \in \mathbb{Z}^+$

Proof by math induction.

Base step, $P(1)$:

Let's look at $P(1) : 3|(1^3 - 1)$

$3|(1 - 1)$ (simplify)

$3|0$ (simplify)

By definiton, 3 divides 0, since $3 \cdot 0$ yields 0. Therefore, $P(1)$ is true.

Inductive step: prove $P(k) \to P(k+1), k \in \mathbb{Z}^+$.

Let's look at $P(k+1)$.

$P(k+1) : 3|[(k+1)^3 - (k+1)]$

$3|(k^3 + 3k^2 + 3k + 1) - (k+1)$ Expand

$3|(k^3 + 3k^2 + 3k - k)$ Regroup

$3|(k^3 - k + 3k^2 + 3k)$ Regroup

$3|[(k^3 - k) + 3(k^2 + k)]$ Factor 3

Note that by our inductive hypothesis, $P(k)(k^3 - k)$ is divisible by 3. We conclude that $3(k^2 + k)$ is divisible by 3 since it is a multiple of 3, $k^2 + k \in \mathbb{Z}$.

The sum of two values divisible by 3 yields a sum divisible by 3.

I've shown $P(k) \to P(k+1)$.

Conclusion: since the base step and inductive step are both true, $P(n)$ is true $\forall n \in \mathbb{Z}^+$, by the principle of mathematical induction.

Tiling

Problem. $P(n)$: let $n \in \mathbb{Z}^+$. Show that every 2 by 2 board with one square removed can be tiled by right triangles.

Proof by mathematical induction.

Base case. $P(1)$: 2 by 2 board can be tiled having any one square removed as shown below.

Base case is complete.

Inductive step. $P(k) \to P(k+1), k \in \mathbb{Z}^+$

Let's look at a $2^{k+1} \times 2^{k+1}$ board.

2 5.2: Strong induction

Recall the "ladder" from the mathematical induction section.

With strong induction, it is possible to use more than 1 previous result to reach a desired next result.