PROBLEM-SOLVING FLOWCHARTS

FINDING & CLASSIFYING CRITICAL POINTS

1. Compute $\nabla f(x)$
2. Set $\nabla f(x) = 0$
3. Solve for x^*
4. Compute Hessian H
5. Evaluate H at x^*
6. Check det(H), eigenvalues
7. Classify: min/max/saddle
8. Verify answer

TESTING CONVEXITY

Is f twice differentiable?

YES: Compute $\nabla^2 f(x)$

Check $\nabla^2 f(x) \geq 0$ for all x

All eigenvalues ≥ 0?

YES → CONVEX!

NO: Use definition or

first-order condition

TESTING POSITIVE DEFINITENESS

Choose ONE method:

Method 1: Eigenvalues

Find all λ_i

All $\lambda_i > 0$? → PD

Method 2: Sylvester

Compute minors $M_1, M_2, \ldots$

All $M_i > 0$? → PD

Method 3: Definition

Check $x^TAx > 0$ $\forall x \neq 0$

True? → PD

MATRIX MULTIPLICATION GUIDE

Matrix × Matrix: AB

$(m \times n)(n \times p) = (m \times p)$

Row of A · Column of B

Matrix × Vector: Ax

$(m \times n)(n \times 1) = (m \times 1)$

Each row · vector x

Vector^T × Matrix: x^TA

$(1 \times n)(n \times m) = (1 \times m)$

Vector · each column

Quadratic Form: x^TAx

$(1 \times n)(n \times n)(n \times 1) = \text{scalar}$