

# EXPECTIMINIMAX CHEATSHEET

Game Trees: Adversaries + Uncertainty | AI Search Algorithms

## What is Expectiminimax?

An extension of Minimax that handles BOTH adversarial opponents AND random chance events.

Used in games like Backgammon: you roll dice (chance) and play against an opponent (adversary).

Three node types alternate in the game tree: MAX (you) -> CHANCE (randomness) -> MIN (opponent).

## The Three Node Types



### MAX Node

YOUR turn - pick BEST

$$U(s) = \max Q(s,a)$$

"Me - I want MAX!"



### MIN Node

OPPONENT picks WORST

$$U(s) = \min Q(s,a)$$

"Monster - hurts me"



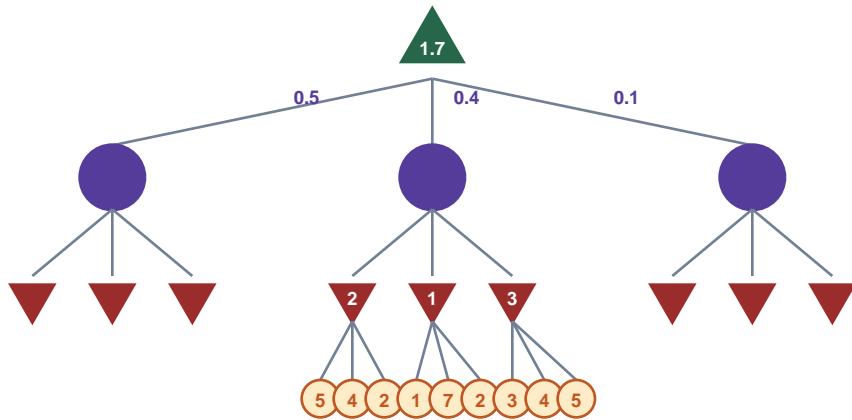
### CHANCE Node

RANDOM weighted average

$$Q = \sum P(s') * U(s')$$

"Chaos - dice roll"

## Worked Example (from class slide)



### Step-by-Step Calculation:

**Step 1: MIN nodes** pick smallest child value

Left MIN:  $\min(5,4,2) = 2$

Mid MIN:  $\min(1,7,2) = 1$

Right MIN:  $\min(3,4,5) = 3$

**Step 2: CHANCE node** weighted average

$$0.5 \times 2 + 0.4 \times 1 + 0.1 \times 3$$

$$= 1.0 + 0.4 + 0.3 = 1.7$$

**Step 3: MAX node** picks highest = 1.7

## Mnemonic: "Me, Chaos, Monster"

**Me = MAX**

**Chaos = CHANCE**

**Monster =**

Layers: Me -> Chaos -> Monster -> Chaos -> Me ...

Also: "MaCMIN" = Max, Chance, MIN

## Quick Algorithm Comparison

**Minimax:** MAX <-> MIN (Chess)

**Expectimax:** MAX <-> CHANCE (Dice solitaire)

**Expectiminimax:** MAX <-> CHANCE <-> MIN (Backgammon)

# PROBLEMS, SOLUTIONS & PSEUDOCODE

## Key Problems

### 1. Exponential Blowup

Extra CHANCE layers = much deeper tree.  $O((b^*n)^d)$

### 2. No Alpha-Beta Pruning

Can't skip branches - need ALL children for averages

### 3. Utility Scale Sensitivity

Nonlinear transforms change decisions (unlike Minimax)

### 4. Need Probability Model

Must know  $P(\text{outcome})$ . Wrong model = bad play

## Solution Strategies

### 1. Depth-Limited Search

Stop early + evaluation function to estimate

### 2. Monte Carlo Sampling

Sample random outcomes instead of enumerating all

### 3. Star1/Star2 Pruning

Special pruning using bounds on chance values

### 4. Forward Pruning

Ignore very low-probability branches

## Pseudocode

```
def value(state):
    if state is terminal: return utility(state)
    if agent(state) == MAX: return max_value(state)
    if agent(state) == MIN: return min_value(state)
    if agent(state) == EXP: return exp_value(state)

def max_value(state):           # Agent's turn
    v = -infinity
    for s' in successors(state):
        v = max(v, value(s'))
    return v

def min_value(state):           # Opponent's turn
    v = +infinity
    for s' in successors(state):
        v = min(v, value(s'))
    return v

def exp_value(state):          # Chance event
    v = 0
    for s' in successors(state):
        p = probability(s')
        v += p * value(s')
    return v
```

## Formula Quick Reference

Node Type	Symbol	Operation	Formula	Analogy
MAX	Triangle Up	max(children)	$U(s) = \max Q(s,a)$	Your best move
MIN	Triangle Down	min(children)	$U(s) = \min Q(s,a)$	Opponent's best
CHANCE	Circle	weighted avg	$Q = \sum P(s') * U(s')$	Dice / luck
Terminal	Leaf	known value	$V(s) = \text{utility}$	Game over

## Exam Tips & Key Takeaways

Always work BOTTOM-UP: leaves  $\rightarrow$  MIN/MAX  $\rightarrow$  CHANCE  $\rightarrow$  repeat upward to root.

CHANCE = expected value =  $\text{Sum}(\text{probability} \times \text{child})$ . Don't pick min or max here!

Expectiminimax IS sensitive to monotonic utility transforms. Minimax is NOT.

Alpha-beta doesn't work directly here. Use Monte Carlo / Star pruning instead.