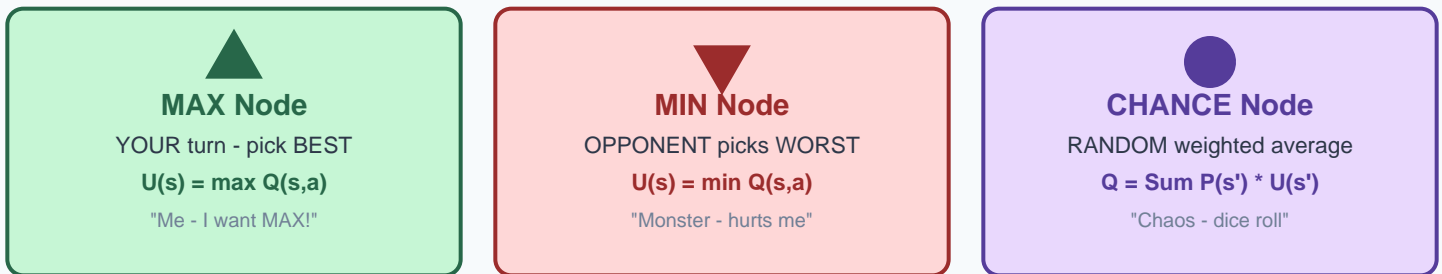


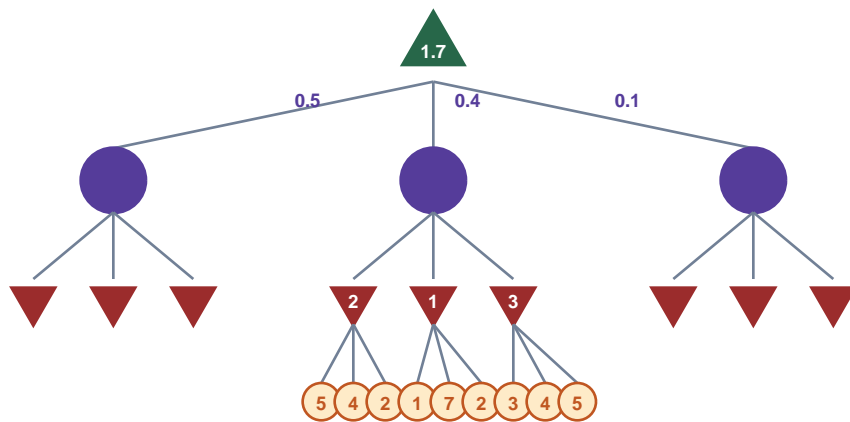
Game Trees: Adversaries + Uncertainty | AI Search Algorithms

An extension of Minimax that handles BOTH adversarial opponents AND random chance events.
Used in games like Backgammon: you roll dice (chance) and play against an opponent (adversary).
Three node types alternate in the game tree: MAX (you) -> CHANCE (randomness) -> MIN (opponent).

The Three Node Types



Worked Example (from class slide)



Step-by-Step Calculation:

Step 1: MIN nodes pick smallest child value

Left MIN: $\min(5,4,2) = 2$

Mid MIN: $\min(1,7,2) = 1$

Right MIN: $\min(3,4,5) = 3$

Step 2: CHANCE node weighted average

$$0.5 \times 2 + 0.4 \times 1 + 0.1 \times 3$$
$$= 1.0 + 0.4 + 0.3 = 1.7$$

Step 3: MAX node picks highest = 1.7

Mnemonic: "Me, Chaos, Monster"

Me = MAX

Chaos = CHANCE

Monster =

Layers: Me -> Chaos -> Monster -> Chaos -> Me ...

Also: "MaCMIN" = Max, Chance, MIN

Quick Algorithm Comparison

Minimax: MAX <-> MIN (Chess)

Expectimax: MAX \leftrightarrow CHANCE (Dice solitaire)

Expectiminimax: MAX \leftrightarrow CHANCE \leftrightarrow MIN (Backgammon)

PROBLEMS, SOLUTIONS & PSEUDOCODE

Key Problems

1. Exponential Blowup

Extra CHANCE layers = much deeper tree. $O((b \cdot n)^d)$

2. No Alpha-Beta Pruning

Can't skip branches - need ALL children for averages

3. Utility Scale Sensitivity

Nonlinear transforms change decisions (unlike Minimax)

4. Need Probability Model

Must know $P(\text{outcome})$. Wrong model = bad play

Solution Strategies

1. Depth-Limited Search

Stop early + evaluation function to estimate

2. Monte Carlo Sampling

Sample random outcomes instead of enumerating all

3. Star1/Star2 Pruning

Special pruning using bounds on chance values

4. Forward Pruning

Ignore very low-probability branches

Pseudocode

```
def value(state):
    if state is terminal: return utility(state)
    if agent(state) == MAX: return max_value(state)
    if agent(state) == MIN: return min_value(state)
    if agent(state) == EXP: return exp_value(state)

def max_value(state):          # Agent's turn
    v = -infinity
    for s' in successors(state):
        v = max(v, value(s'))
    return v

def min_value(state):          # Opponent's turn
    v = +infinity
    for s' in successors(state):
        v = min(v, value(s'))
    return v

def exp_value(state):           # Chance event
    v = 0
    for s' in successors(state):
        p = probability(s')
        v += p * value(s')
    return v
```

Formula Quick Reference

Node Type	Symbol	Operation	Formula	Analogy
MAX	Triangle Up	$\max(\text{children})$	$U(s) = \max Q(s,a)$	Your best move
MIN	Triangle Down	$\min(\text{children})$	$U(s) = \min Q(s,a)$	Opponent's best
CHANCE	Circle	weighted avg	$Q = \sum P(s') \cdot U(s')$	Dice / luck
Terminal	Leaf	known value	$V(s) = \text{utility}$	Game over

Exam Tips & Key Takeaways

Always work BOTTOM-UP: leaves \rightarrow MIN/MAX \rightarrow CHANCE \rightarrow repeat upward to root.

CHANCE = expected value = $\sum(\text{probability} \times \text{child})$. Don't pick min or max here!

Expectiminimax IS sensitive to monotonic utility transforms. Minimax is NOT.

Alpha-beta doesn't work directly here. Use Monte Carlo / Star pruning instead.