**CS 325 – Project 3 – Linear Programming**
**Due:** 2/26/17
**Members:** Matthew Yates, Ian O'Leary, Riley Brandau

**Problem 1: Transshipment Model**

| Cost | W1 | W2 | W3 |
|---|---|---|---|
| P1 | X1 | X2 | |
| P2 | X3 | X4 | |
| P3 | X5 | X6 | X7 |
| P4 | | X8 | X9 |

| Cost | R1 | R2 | R3 | R4 | R5 | R6 | R7 |
|---|---|---|---|---|---|---|---|
| W1 | X10 | X11 | X12 | X13 | | | |
| W2 | | | X14 | X15 | X16 | X17 | |
| W3 | | | | X18 | X19 | X20 | X21 |

1-A) **Determine the number of refrigerators to be shipped plants to warehouses and then warehouses to retailers to minimize the cost.**

The optimal minimum cost is $17,100, as follows;

150 – p1 → w1

200 – p2 → w1

250 – p2 → w2

150 – p3 → w2

100 – p3 → w3

150 – p4 → w3

Then,

100 – w1 → r1

150 – w1 → r2

100 – w1 → r3

200 – w2 → r4

200 – w2 → r5

150 – w3 → r6

100 – w3 → r7

1-B). **Due to old infrastructure Warehouse 2 is going to close eliminating all of the associated routes. What is the optimal solution for this modified model? Is it feasible to ship all of the refrigerators to either warehouse 1 or 3 and then to the retailers without using warehouse 2? Why or why not?**

It would not be possible to close down Warehouse 2 and still meet the demand given the other constraints. Retailers 5, 6, and 7 require 450 combined units per week; however, without warehouse 2, the supply (P3 and P4) is only 400 units per week since the only other routes for those retailers come from warehouse 3.

1-C). **Instead of closing Warehouse 2 management has decided to keep a portion of it open but limit shipments to 100 refrigerators per week. Is this feasible? If so what is the optimal solution when warehouse 2 is limited to 100 refrigerators?**

The additional constraint to limit Warehouse 2 to 100 shipments per week is feasible; however, the optimal minimal cost increases to $18,300 with the following routes;

150 – p1 → w1

350 – p2 → w1

100 – p2 → w2

250 – p3 → w3

150 – p4 → w3

Then,

100 – w1 → r1

150 – w1 → r2

100 – w1 → r3

150 – w1 → r4

50 – w2 → r4

50 – w2 → r5

150 – w3 → r5

150 – w3 → r6

100 – w3 → r7

1-D) **Formulate a generalized linear programming model for the transshipment problem. Give the objective function and constraints as mathematical formulas.**

Generalized Linear Programming Model

i = warehouse to plant route, j = plant to retailer route

a = warehouse supply, b = retailer demand

Minimize $\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} c_{ijk}x_{ijk}$

Subject To:

$\sum_{i=1}^{m} x_{ij} <= a_i$ for i = 1, 2,.....,n

$\sum_{j=1}^{n} x_{ij} <= b_i$ for j = 1, 2,.....,m

$x_{ij} >= 0$ for i = 1, 2, ......., m and j = 1, 2,...,n

**Problem 2:** A mixture problem. Veronica, the owner of Very Veggie Vegeria is creating a new healthy salad that is low in calories but meets certain nutritional requirements. A salad is any combination of the following ingredients: Tomato, Lettuce, Spinach, Carrot, Smoked Tofu, Sunflower Seeds, Chickpeas, Oil.

Each salad must contain: at least 15g of protein, between 2 and 8g of fat, at least 4g of carbohydrates, no more than 200mg of sodium, and at least be 40% leafy greens by mass.

**2-A) Determine the combination of ingredients that minimizes calories but meets all nutritional requirements.**

**2-B) Veronica realizes that it is also important to minimize the cost associated with the new salad. Unfortunately, some of the ingredients can be expensive. Determine the combination of ingredients that minimizes cost.**

**2-C) Compare the results from part A and B. Veronica's goal is to create a Very Veggie Salad that is both low calorie and low cost. She would like to sell the salad for $5 and still have a profit of at least $3. However, if she can advertise that the salad has under 250 calories then she may be able to sell more.**

**Problem 3:** Solving shortest path problems using linear programming.

The file Project3Problem3.txt contains a list of the edges and weights in a directed graph. Use linear programming to answer the following questions. Include a copy of the linear program code.

<u>**Part A)**</u> **What are the lengths of the shortest paths from vertex a to all other vertices.**

a) LINDO CODE:
b) MAX Dm // change this to vertex you want to find shortest path to from A

a. ST:
b. Da = 0
c. Db – Da <= 2
d. Dc – Da <= 3
e. Dd – Da <= 8
f. Dh – Da <= 9
g. Dc – Db <= 5
h. Df – Db <= 4
i. De – Db <= 7
j. Da – Db <= 4
k. Db – Dc <= 5
l. Di – Dc <= 11
m. Df – Dc <= 4
n. Dd – Dc <= 10
o. Dg – Dc <= 9
p. Df – Dd <= 1
q. Dg – Dd <= 2
r. Da – Dd <= 8
s. Dh – De <= 5
t. Dc – De <= 4
u. Di – De <= 10
v. Di – Df <= 2
w. Dg – Df <= 2
x. Dj – Dg <= 8
y. Dk – Dg <= 12
z. Dd – Dg <= 2
aa. Di – Dh <= 5
bb. Dk – Dh <= 10
cc. Dm – Di <= 12
dd. Da – Di <= 20
ee. Dj – Di <= 2
ff. Dk – Di <= 6
gg. Di – Dj <= 2
hh. Dk – Dj <= 4
ii. Dl – Dj <= 5
jj. Dh – Dk <= 10
kk. Dm – Dk <= 10
ll. Dm – Dl <= 2

END

c) SHORTEST PATHS
   a. A to B: 2
   b. A to C: 3
   c. A to D: 8
   d. A to E: 9
   e. A to F: 6
   f. A to G: 8
   g. A to H: 9
   h. A to I: 8
   i. A to J: 10
   j. A to K: 14
   k. A to L: 15
   l. A to M: 17

**<u>Part B)</u> If a vertex z is added to the graph for which there is no path from vertex a to vertex z, what will be the result when you attempt to find the lengths of shortest paths as in part a).**

a) The distance from vertex a to all the other vertices would remain the same, but the distance from vertex a to vertex z is infeasible. Say we added vertex Z and connect it to Vertex M by one directed edge leading from Z to M with a weight of 2 (the weight doesn't really matter). While Z is able to reach M, it is unreachable for all the other vertices. If we add Z and connect it to any of the other vertices, say I, using one directed edge from Z to I, Z is still unreachable by any of the vertices, but Z would now be able to reach any of the other vertices in the graph.

**<u>Part C)</u> What are the lengths of the shortest paths from each vertex to vertex m. How can you solve this problem with just one linear program?**

a) SHORTEST PATHS
   a. A to M: 17
   b. B to M: 15
   c. C to M: 15
   d. D to M: 12
   e. E to M: 19
   f. F to M: 11
   g. G to M: 14

h. H to M: 14
i. I to M: 9
j. J to M: 7
k. K to M: 10
l. L to M: 2
m. M to M: 0

**b) How can you solve this problem with just one linear program?**

a. We could use a SIMPLEX algorithm that loops through each vertex, and returns the shortest path from that vertex to vertex M. Or we could reverse the direction of all the edges and find the shortest path from M to each vertex.
   i. Maximize $\sum_v d_v$
      1. (note: I tried to place the v under the summation symbol, but it ended up placing it to the right for some reason)
   ii. Subject To:
      1. $d_s = 0$
      2. $d_v - d_s \leq \ell_{u \to v}$ for every edge u→v

<u>Part D)</u> **Suppose that all paths must pass through vertex i. How can you calculate the length of the shortest path from any vertex x to vertex y that pass through vertex i (for all x,y ∈ V)? Calculate the lengths of these paths for the given graph. (Note for some vertices x and y it may be impossible to pass through vertex i).**

a) You can calculate the shortest path from x to y that passes through I by taking the shortest path from x to I + the shortest path from I to y.

b) A to I to X
   a. A to I to A = 8+20 = 28
   b. A to I to B = 8+22 = 30
   c. A to I to C = 8+23 = 31
   d. A to I to D = 8+28 = 36
   e. A to I to E = 8+29 = 37
   f. A to I to F = 8+26 = 32
   g. A to I to G = 8+28 = 36
   h. A to I to H = 8+16 = 24
   i. A to I to I = 8+0 = 8
   j. A to I to J = 8+2 = 10
   k. A to I to K = 8+6 = 14
   l. A to I to L = 8+7 = 15
   m. A to I to M = 8+9 = 17

c) B to I to X
   a. B to I to A = 6+20 = 26
   b. B to I to B = 6+22 = 28
   c. B to I to C = 6+23 = 29
   d. B to I to D = 6+28 = 34
   e. B to I to E = 6+29 = 35
   f. B to I to F = 6+26 = 32
   g. B to I to G = 6+28 = 34
   h. B to I to H = 6+16 = 22
   i. B to I to I = 6+0 = 6
   j. B to I to J = 6+2 = 8
   k. B to I to K = 6+6 = 12
   l. B to I to L = 6+7 = 13
   m. B to I to M = 6+9 = 15

d) C to I to X
   a. C to I to A = 6+20 = 26
   b. C to I to B = 6+22 = 28
   c. C to I to C = 6+23 = 29
   d. C to I to D = 6+28 = 34
   e. C to I to E = 6+29 = 35
   f. C to I to F = 6+26 = 32
   g. C to I to G = 6+28 = 34
   h. C to I to H = 6+16 = 22
   i. C to I to I = 6+0 = 6
   j. C to I to J = 6+2 = 8
   k. C to I to K = 6+6 = 12

l. C to I to L = 6+7 = 13
m. C to I to M = 6+9 = 15

e) D to I to X
   a. D to I to A = 3+20 = 23
   b. D to I to B = 3+22 = 25
   c. D to I to C = 3+23 = 26
   d. D to I to D = 3+28 = 31
   e. D to I to E = 3+29 = 32
   f. D to I to F = 3+26 = 29
   g. D to I to G = 3+28 = 31
   h. D to I to H = 3+16 = 19
   i. D to I to I = 3+0 = 3
   j. D to I to J = 3+2 = 5
   k. D to I to K = 3+6 = 9
   l. D to I to L = 3+7 = 10
   m. D to I to M = 3+9 = 12
f) E to I to X
   a. E to I to A = 10+20 = 30
   b. E to I to B = 10+22 = 32
   c. E to I to C = 10+23 = 33
   d. E to I to D = 10+28 = 38
   e. E to I to E = 10+29 = 39
   f. E to I to F = 10+26 = 36
   g. E to I to G = 10+28 = 38
   h. E to I to H = 10+16 = 26
   i. E to I to I = 10+0 = 10
   j. E to I to J = 10+2 = 12
   k. E to I to K = 10+6 = 16
   l. E to I to L = 10+7 = 17
   m. E to I to M = 10+9 = 19
g) F to I to X
   a. F to I to A = 2+20 = 22
   b. F to I to B = 2+22 = 24
   c. F to I to C = 2+23 = 25
   d. F to I to D = 2+28 = 30
   e. F to I to E = 2+29 = 31
   f. F to I to F = 2+26 = 28
   g. F to I to G = 2+28 = 30
   h. F to I to H = 2+16 = 18
   i. F to I to I = 2+0 = 2
   j. F to I to J = 2+2 = 4
   k. F to I to K = 2+6 = 8
   l. F to I to L = 2+7 = 9
   m. F to I to M = 2+9 = 11
h) G to I to X
   a. G to I to A = 5+20 = 25
   b. G to I to B = 5+22 = 27
   c. G to I to C = 5+23 = 28
   d. G to I to D = 5+28 = 33
   e. G to I to E = 5+29 = 34
   f. G to I to F = 5+26 = 31
   g. G to I to G = 5+28 = 33
   h. G to I to H = 5+16 = 21
   i. G to I to I = 5+0 = 0
   j. G to I to J = 5+2 = 7
   k. G to I to K = 5+6 = 11
   l. G to I to L = 5+7 = 12
   m. G to I to M = 5+9 = 14
i) H to I to X
   a. H to I to A = 5+20 = 25
   b. H to I to B = 5+22 = 27
   c. H to I to C = 5+23 = 28
   d. H to I to D = 5+28 = 33
   e. H to I to E = 5+29 = 34
   f. H to I to F = 5+26 = 31
   g. H to I to G = 5+28 = 33
   h. H to I to H = 5+16 = 21
   i. H to I to I = 5+0 = 5
   j. H to I to J = 5+2 = 7
   k. H to I to K = 5+6 = 11
   l. H to I to L = 5+7 = 12
   m. H to I to M = 5+9 = 14
j) I to I to X
   a. I to I to A = 0+20 = 20
   b. I to I to B = 0+22 = 22
   c. I to I to C = 0+23 = 23
   d. I to I to D = 0+28 = 28
   e. I to I to E = 0+29 = 29
   f. I to I to F = 0+26 = 26
   g. I to I to G = 0+28 = 28
   h. I to I to H = 0+16 = 16
   i. I to I to I = 0+0 = 0
   j. I to I to J = 0+2 = 2
   k. I to I to K = 0+6 = 6
   l. I to I to L = 0+7 = 7
   m. I to I to M = 0+9 = 9
k) J to I to X
   a. J to I to A = 2+20 = 22
   b. J to I to B = 2+22 = 24
   c. J to I to C = 2+23 = 25
   d. J to I to D = 2+28 = 30
   e. J to I to E = 2+29 = 31

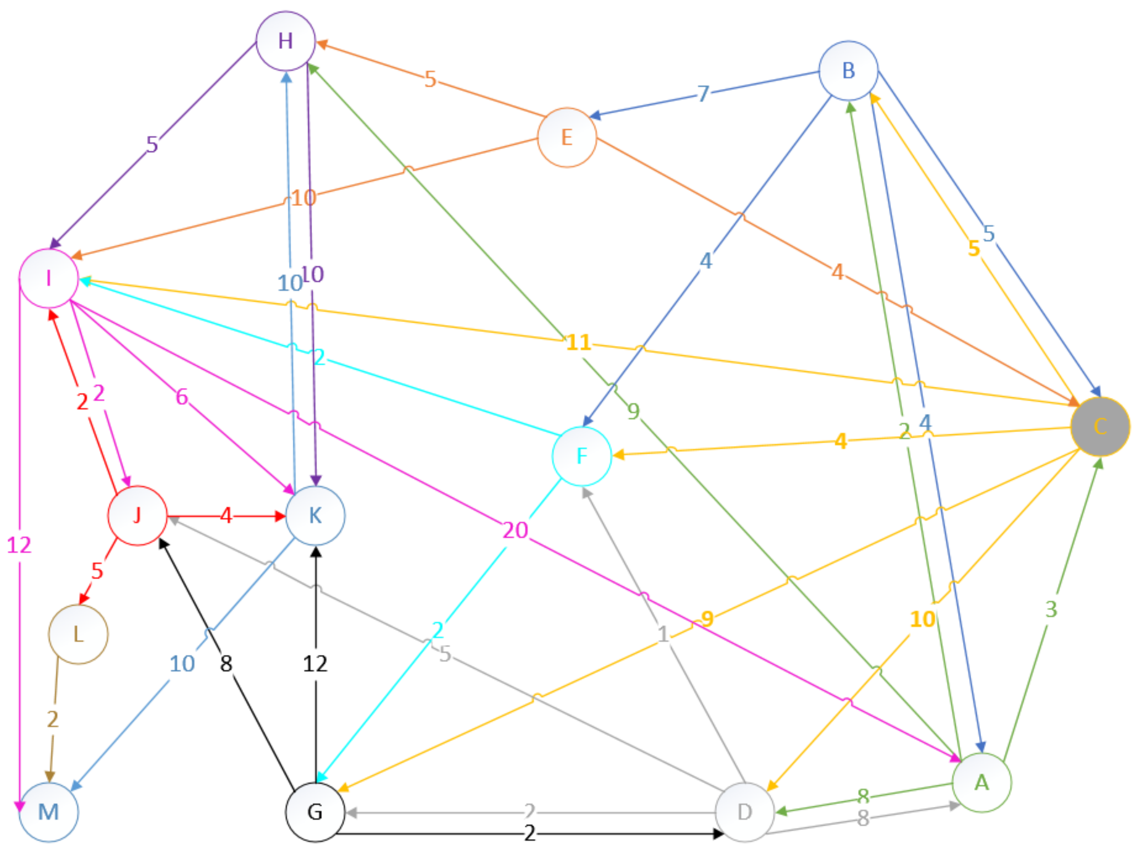
f. J to I to F = 2+26 = 28
g. J to I to G = 2+28 = 30
h. J to I to H = 2+16 = 18
i. J to I to I = 2+0 = 2
j. J to I to J = 2+2 = 4
k. J to I to K = 2+6 = 8
l. J to I to L = 2+7 = 9
m. J to I to M = 2+9 = 11

l) K to I to X
  a. K to I to A = 15+20 = 35
  b. K to I to B = 15+22 = 37
  c. K to I to C = 15+23 = 38
  d. K to I to D = 15+28 = 43
  e. K to I to E = 15+29 = 44
  f. K to I to F = 15+26 = 41
  g. K to I to G = 15+28 = 43
  h. K to I to H = 15+16 = 31
  i. K to I to I = 15+0 = 15
  j. K to I to J = 15+2 = 17
  k. K to I to K = 15+6 = 21
  l. K to I to L = 15+7 = 22
  m. K to I to M = 15+9 = 24

m) L to I to X
  a. L is unable to reach any vertices through I, since it only has one directed edge leading to another vertex, M, and M has no directed edges leading to any other vertices.

n) M to I to X
  a. M is unable to reach any vertices through I, since it doesn’t have any directed edges to any other vertices

<u>Visual Interpretation of the Graph</u>



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