MOBIL: General Lane-Changing Model for Car-Following Models

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Abstract

We propose a general model to derive lane-changing rules for discretionary and mandatory lane changes for a wide class of car-following models. Both the utility of a given lane and the risk associated with lane changes is determined in terms of longitudinal accelerations calculated with the microscopic traffic models. This allows for the formulation of compact and general safety and incentive criteria both for symmetric and asymmetric passing rules. Moreover, anticipative elements and the crucial influence of velocity differences of these car-following models are automatically transferred to the lane-changing rules.

While the safety criterion prevents critical lane changes and collisions, the incentive criterion also takes into account the (dis-)advantages of other drivers associated with a lane change via the 'politeness factor'. The parameter allows to vary the motivation for lane-changing from purely egoistic to more cooperative driving behavior. This novel feature allows first to prevent change lanes for a marginal advantage if this obstructs other drivers, and, second, to let a 'pushy' driver induce a lane change of a slower driver ahead in order to be no longer obstructed. This is a common phenomenon for asymmetric passing rules with a dedicated lane for passing.

We apply the model to traffic simulations of cars and trucks with the Intelligent Driver Model (IDM) as underlying car-following model. We study an open system with an on-ramp and investigate the resulting lane-changing rate as a function of the spatial coordinate as well as a function of traffic density.

1 Introduction

In the past, single-lane car-following models have been successfully applied to describe traffic dynamics (Nagatani, 2002; Helbing, 2001). Particularly collective phenomena such as traffic instabilities and the spatiotemporal dynamics of congested traffic can be well understood within the scope of single-lane traffic models. But real traffic consists of different types of vehicles, e.g., cars and trucks. Therefore, a realistic description of heterogeneous traffic streams is only possible within a multi-lane modeling framework allowing faster vehicles to improve their driving conditions by passing slower vehicles. Hence, freeway lane changing has recently received increased attention (Laval and Daganzo, 2006; Hidas, 2005; Coifman et al., 2005; Wei et al., 2000; Brackstone et al., 1998; Nagel et al., 1998). Moreover, since lane-changing maneuvers often act as initial perturbations, it is crucial to understand their impact on the capacity, stability, and breakdown of traffic flows. Particularly near bottleneck sections such as on-ramps and off-ramps, lane changing is often a significant ingredient to trigger a traffic breakdown (provided that the traffic volume is high) (Laval and Daganzo, 2004). Additionally, the drivers' lane-changing behavior has direct influence on traffic safety.

Despite its great significance, lane-changing has by far not been studied as extensively as the longitudinal acceleration and deceleration behavior. One reason for this is the scarcity of reliable data (Hidas and Wagner, 2004; Brackstone and McDonald, 1996). For measuring lane changes, cross-sectional data from detectors are not sufficient and, therefore, only a few empirical studies about lane-changing rates as a function of traffic flow or density are available. Sparmann (1978) investigated the lane-changing rates on a German two-lane *autobahn*. Data for a British motorway were presented by Yousif and Hunt (1995). Recent progress in video tracking methods, however, allows for a collection of high-quality trajectory data from aerial observations (Hoogendoorn et al., 2003; NGSIM, 2006). These 2D data will become more and more available in the future and will allow for a more profound understanding of the microscopic lane-changing processes.

The modeling of lane changes is typically considered as a multi-step process. On a *strategic* level, the driver knows about his or her route in a network, which influences the lane choice, e.g., with regard to lane blockages, on-ramps, off-ramps, or other mandatory merges (Toledo et al., 2005). In the *tactical* stage, an intended lane change is prepared and initiated by advance accelerations or decelerations of the driver, and possibly by cooperation of drivers in the target lane (Hidas, 2005). Finally, in the *operational* stage, one determines if an immediate lane change is both safe and desired (Gipps, 1986). This choice is typically modeled by the use of gap-acceptance models, in which drivers compare the available gaps to the smallest acceptable gap, the critical gap. Critical gaps depend in general on the relative speed of the subject vehicle with respect to those of the lead and the lag vehicles in the adjacent lane and on the type of lane change (Toledo et al., 2003). Most lane-changing models in the literature classify lane changes as either mandatory or discretionary (Gipps, 1986; Yang and Koutsopoulos, 1996; Ahmed, 1999; Toledo et al., 2003; Halati et al., 1997; Skabardonis, 2002). While mandatory changes are performed for strategic reasons, the driver's motivation for discretionary lane changes is a perceived improvement of the driving conditions in the target lane compared to the actual situation.

In this paper, we present a lane-changing model for microscopic car-following models, which describes the *rational decision* to change lanes and, therefore, only deals with the operational decision process. When considering a lane change, we assume that a driver makes a trade-off between the expected own advantage and the disadvantage imposed on other drivers. In particular, our model includes the follower on the target lane in the decision process. For a

driver considering a lane change, the subjective utility of a change increases with the gap to the new leader on the target lane. However, if the velocity of this leader is lower, it may be favorable to stay on the present lane despite of the smaller gap. A criterion for the utility including *both* situations is the difference of the accelerations after and before the lane change. In this work, we therefore propose as utility function considering the difference in vehicle *accelerations* (or *decelerations*) after a lane change, calculated with an underlying microscopic longitudinal traffic model. The higher the acceleration on a given lane, the nearer it is to the 'ideal' acceleration on an empty road and the more attractive it is to the driver. Therefore, the basic idea of our proposed lane-changing model is to formulate the anticipated advantages and disadvantages of a prospective lane change in terms of single-lane accelerations.

Compared to explicit lane-changing model, the formulation in terms of accelerations of a longitudinal model has several advantages. First, the assessment of the traffic situation is transfered to the acceleration function of the car-following model, which allows for a compact and general model formulation with only a small number of additional parameters. In contrast to the classical gap-acceptance approach, critical gaps are not taken into account explicitly. Second, it is ensured that both longitudinal and lane-changing models are consistent with each other. For example, if the longitudinal model is collision-free, the combined models will be accident-free as well. Third, any complexity of the longitudinal model such as anticipation is transfered automatically to a similarly complex lane-changing model. Finally, the braking deceleration imposed on the new follower on the target lane to avoid accidents is an obvious measure for the (lack of) safety. Thus, safety and motivational criteria can be formulated in a unified way.

Apart from using accelerations as utility functions, the main novel feature of the proposed lane-changing model consists in taking into account the (dis-) advantage of the followers via a *politeness parameter*. By adjusting this parameter, the motivations for lane-changing can be varied from purely egoistic to a more altruistic behavior. Particularly, there exists a value where lane changes are only carried out if this increases the combined accelerations of the lane-changing driver and all affected neighbors. This strategy can be paraphrased by the acronym *'Minimizing Overall Braking Induced by Lane Changes'* (MOBIL). In the following, we will refer to our concept by this acronym, regardless of the value of the politeness parameter. Notice that all lane-changing models cited before assume egoistic behavior. By the politeness factor, we can model two common lane-changing patterns. First, most drivers do not change lanes for a marginal advantage if this obstructs other drivers in addition to a common safety condition. Second, in countries with asymmetric lane-changing rules, 'pushy' drivers may induce a lane change of a slower driver in front of them in order to the slower lane to be no longer obstructed on the faster lane, which is dedicated to passing.

Our paper is organized as follows: In Sec. 2, we formulate the lane-changing model MOBIL both for symmetric ('US') and asymmetric ('European') passing rules. In Sec. 3, we apply the MOBIL rules and simulate multi-lane traffic in combination with the Intelligent Driver Model (IDM) as underlying longitudinal car-following model (Treiber et al., 2000). Finally, in Sec. 4, we will conclude with a discussion of the proposed model, future research directions, and a generalization of the MOBIL concept to other traffic-related decision processes, e.g., when approaching traffic lights.

2 The lane-changing model MOBIL

Most time-continuous microscopic single-lane traffic models describe the motion of single 'vehicle-driver units' α as a function of their own velocity v_α, the bumper-to-bumper distance
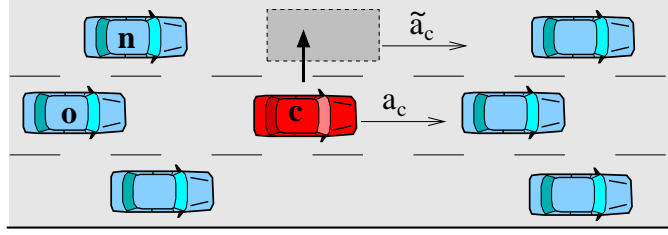
Figure 1: Sketch of the nearest neighbors of a central vehicle c considering a lane change to the left. The new and old successors are denoted by n and o, respectively. Accelerations after a possible change are denoted with a tilde.

s_α to the front vehicle $(\alpha - 1)$ and the relative velocity $\Delta v_\alpha = v_\alpha - v_{\alpha-1}$. The acceleration of these car-following models is of the general form

$$a_\alpha := \frac{dv_\alpha}{dt} = a(s_\alpha, v_\alpha, \Delta v_\alpha). \tag{1}$$

Some examples are the model of Gipps (1986), the 'optimal velocity model' (Bando et al., 1995), the 'intelligent driver model' (Treiber et al., 2000), or the 'velocity difference model' (Helly, 1959; Jiang et al., 2001). Moreover, a generalization to models taking into account more than one predecessor (Belexius, 1968; Lenz et al., 1999; Treiber et al., 2006), or to models with explicit reaction time, is straightforward.

A specific lane change, e.g., from the center lane to the median lane as shown in Fig. 1 depends generally on the two following vehicles on the present and the target lane, respectively. In order to formulate the lane-changing criteria, we use the following notation: For a vehicle c considering a lane change, the successive vehicles on the target and present lane are represented by n and o, respectively. The acceleration a_c denotes the acceleration of vehicle c on the actual lane, while $\tilde{a}_c$ refers to the situation on the target lane, i.e., to the new acceleration of vehicle c on the target lane. Likewise, $\tilde{a}_o$ and $\tilde{a}_n$ denote the acceleration of the old and new followers after the lane change of vehicle c.

2.1 Safety criterion

Like other lane-changing models (Gipps, 1986), we distinguish between an incentive to change lanes and safety constraints. The safety criterion checks the possibility of executing a lane change (gap acceptance) by considering the effect on the upstream vehicle on the target lane. Formulated in terms of longitudinal accelerations, our safety criterion guarantees that, after the lane change, the deceleration of the successor $\tilde{a}_n$ on the target lane does not exceed a given safe limit b_{safe}, i.e.,

$$\tilde{a}_n \geq -b_{\text{safe}}. \tag{2}$$

Although formulated as a simple inequality, this condition contains all the information provided by the longitudinal car-following model via the acceleration $\tilde{a}_n(t)$ typically depending on the gap, the velocity and, eventually, on the approaching rate, cf. Eq. (1). In particular, if the longitudinal model has a built-in sensitivity with respect to *velocity differences*, this essential dependence is transfered to the lane-changing decisions. In this way, larger gaps between the following vehicle in the target lane and the own position are required to satisfy the safety constraint if the following vehicle is faster than the own speed. In contrast, lower values for the gap are allowed if the following vehicle is slower. Compared to conventional gap-acceptance

models this approach depends on gaps only indirectly, via the dependence on the longitudinal acceleration. The assessment of the situation in terms of accelerations allows for the compact formulation.

Moreover, by formulating the criterion in terms of safe braking decelerations of the longitudinal model, crashes due to lane changes are *automatically* excluded. For realistic longitudinal models, b_{safe} should be well below the maximum possible deceleration b_{max}, which is about $9\,\text{m/s}^2$ on dry road surfaces. Notice that the maximum safe deceleration b_{safe} prevents accidents even in the case of totally selfish drivers as long as its value is not greater than the maximum possible deceleration b_{max} of the underlying longitudinal model.

Increasing the value for b_{safe} generally leads to stronger perturbations due to individual lane changes. But the braking reaction of the follower on the target lane is always limited by the value of b_{safe}. This is relevant in traffic simulations due to the fact that performing a lane change implies a discontinuous change in the input parameters for the acceleration function for the new follower.

2.2 Incentive criterion for symmetric ('US') lane-changing rules

The *incentive criterion* typically determines if a lane change improves the individual local traffic situation of a driver. In our model, we generalize the incentive criterion to include the immediately affected neighbors as well. The *politeness factor* p determines to which degree these vehicles influence the lane-changing decision. For symmetric overtaking rules, we neglect differences between the lanes and propose the following incentive condition for a lane-changing decision of the driver of vehicle c:

$$\underbrace{\tilde{a}_c - a_c}_{\text{driver}} + p\Big(\underbrace{\tilde{a}_n - a_n}_{\text{new follower}} + \underbrace{\tilde{a}_o - a_o}_{\text{old follower}} \Big) > \Delta a_{\text{th}}. \tag{3}$$

The first two terms denote the advantage (utility) of a possible lane change for the driver him- or herself, where $\tilde{a}_c$ refers to the new acceleration for vehicle c after a prospective lane change. The considered lane change is favorable if the driver can accelerate more, i.e., go faster in the new lane. The third term with the *politeness factor* p is the main innovation in our model. It denotes the total advantage (acceleration gain or loss, if negative) of the two immediately affected neighbors, weighted with p. Finally, the switching threshold Δa_{th} on the right-hand side of Eq. (??) models a certain inertia and prevents lane changes if the overall advantage is only marginal compared to a 'keep lane' directive. In summary, the incentive criterion is fulfilled if the own advantage (acceleration gain) is higher than the weighted sum of the disadvantages (acceleration losses) of the new and old successors and the threshold Δa_{th}.[1] Notice that the threshold a_{th} influences the lane-changing behavior *globally*, while the politeness parameter affects the *local* lane-changing behavior depending on the involved neighbors.

The generalization to traffic on more than two lanes per direction is straightforward. If, for a vehicle on a center lane, the incentive criterion is satisfied for both neighboring lanes, the change is performed to the lane where the incentive is larger.

Since the disadvantages of other drivers and the own advantage are balanced via the politeness factor p, the lane-changing model contains typical strategical features of classical game

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[1] Notice that Eq. (??) contains *both* the possibility *and* the incentive to change lanes. Even for a politeness $p = 0$, the condition makes sure that lane changes only take place if the distance to the next downstream vehicle in the target lane is large enough to prevent a collision. Otherwise, the acceleration function would result in an boundless braking deceleration. Therefore, this scenario is automatically excluded when comparing it to the actual traffic situation in the current lane. However, for reasons of numerical efficiency, the safety criterion (2) can be initially applied to $\tilde{a}_c$ as well.

theory. The value of p can be interpreted as the degree of altruism. It can vary from $p = 0$ (for selfish lane-hoppers) to $p > 1$ for altruistic drivers who do not change if that would deteriorate the overall traffic situation considering the followers, while they would perform even disadvantageous lane changes if this improves the situation of the followers sufficiently. By setting $p < 0$, even malicious drivers could be modeled, who accept own disadvantages in order to thwart others. In the special case $p = 1$ and $\Delta a_{\mathrm{th}} = 0$, the incentive criterion simplifies to

$$\tilde{a}_c + \tilde{a}_n + \tilde{a}_o > a_c + a_n + a_o. \tag{4}$$

Thus, lane changes are only performed when they increase the sum of accelerations of all involved vehicles, which corresponds to the concept of *'Minimizing Overall Braking Induced by Lane Changes'* (MOBIL) in the ideal sense. In this case, no additional safety constraint is needed since a braking maneuver in order to avoid an accident would be automatically excluded by Eq. (4) as long as the advantage in terms of the acceleration is lower than the disadvantage in terms of the braking deceleration. Therefore, the 'ideal' MOBIL strategy corresponding to $p = 1$ has no free parameters and might therefore be considered as a 'minimal model' for lane-changing decisions. In Sec. 3, we investigate the rate of lane changes (per km and hour) that is primarily determined by the politeness factor p.

2.3 Incentive criterion for asymmetric ('European') passing rules

In most European countries, the driving rules for lane usage are restricted by legislation. We now formulate an asymmetric lane-changing criterion for two-lane freeways and assume, without loss of generality, that the right lane is the default lane, i.e., a 'keep-right' directive is implemented. A reformulation for left-oriented traffic describing, e.g., traffic rules in the UK as well as generalizations to more than two lanes are straightforward. Specifically, we assume the following 'European' traffic rules:

(i) *Passing rule*: Passing on the right-hand lane is forbidden, unless traffic flow is congested, in which case the symmetric rule (??) applies. We treat any vehicle driving at a velocity below some suitably specified velocity v_{crit} as driving in bound or congested traffic, e.g., $v_{\mathrm{crit}} = 60\,\mathrm{km/h}$.

(ii) *Lane usage rule*: The right lane is the default lane. The left lane should only be used for the purpose of overtaking.

We have implemented the passing rule by replacing the longitudinal dynamics on the right-hand lane by the condition

$$a_c^{\mathrm{eur}} = \begin{cases} \min(a_c, \tilde{a}_c) & \text{if } v_c > \tilde{v}_{\mathrm{lead}} > v_{\mathrm{crit}}, \\ a_{\mathrm{c}} & \text{otherwise}, \end{cases} \tag{5}$$

where $\tilde{a}_c$ corresponds to the acceleration on the left lane and $\tilde{v}_{\mathrm{lead}}$ denotes the velocity of the front vehicle on the left-hand lane. The passing rule influences the acceleration on the right-hand lane only (i) if there is no congested traffic ($\tilde{v}_{\mathrm{lead}} > v_{\mathrm{crit}}$), (ii) if the front vehicle on the left-hand lane is slower ($v_c > \tilde{v}_{\mathrm{lead}}$), and (iii) if the acceleration $\tilde{a}_c$ for following this vehicle would be lower than the single-lane acceleration a_c in the actual situation. Notice that the condition $v_c > \tilde{v}_{\mathrm{lead}}$ prevents that vehicles on the right-hand lane brake whenever they are passed.

The 'keep-right' directive of the lane-usage rule is implemented by a constant bias Δa_{bias} in addition to the threshold Δa_{th}. Furthermore, we neglect the disadvantage of the successor in

the right lane in Eq. (**??**), because the left lane has priority. Explicitly speaking, the resulting asymmetric incentive criterion for lane changes from left (L) to right (R) reads

$$L \rightarrow R : \; \tilde{a}_c^{\mathrm{eur}} - a_c + p\left(\tilde{a}_o - a_o\right) > \Delta a_{\mathrm{th}} - \Delta a_{\mathrm{bias}}. \tag{6}$$

Moreover, the incentive criterion for a lane change from right (R) to left (L) is given by

$$R \rightarrow L : \; \tilde{a}_c - a_c^{\mathrm{eur}} + p\left(\tilde{a}_n - a_n\right) > \Delta a_{\mathrm{th}} + \Delta a_{\mathrm{bias}}. \tag{7}$$

Again, the quantities with a tilde refer to the new situation after a prospective lane change. While the parameter Δa_{bias} is small, it clearly has to be larger than the threshold Δa_{th}. Otherwise, the switching threshold would prevent changes to the right-hand lane even on an empty road.

Neglecting the follower on the right-hand lane leads to a different interpretation of the politeness parameter p than for the symmetric rule. Via the politeness factor p, a driver on the right lane considering a lane change to the left takes into account the *disadvantage* measured in terms of the braking deceleration for the approaching vehicle on the target lane. This can prevent the considered lane change even if the lane change is not critical, which is assured by the safety criterion (2). This feature of the MOBIL lane-changing model reflects realistically a far-seeing and anticipative driving behavior commonly observed for asymmetric passing rules. Furthermore, taking into account only the follower of the faster (left) lane via the politeness factor p applies a selective *dynamic pressure* on slow vehicles driving on the left lane in order to let fast vehicles pass on the left lane, which is a frequently observed behavior on European freeways, particularly on Germany freeways with their broad distribution of desired velocities. Notice that the safety criterion prevents a critical lane change to the slower lane.

3 Application to multi-lane traffic simulations

Let us now apply the MOBIL concept to simulate two-lane freeway traffic with an on-ramp as merging zone. Since the rules are formulated in a model-independent way based on longitudinal accelerations, we have to specify the underlying microscopic traffic model. In the following, we will use the Intelligent Driver Model (IDM) (Treiber et al., 2000), which is a simple car-following model with descriptive parameters (Treiber, 2006).

The IDM acceleration $\dot{v}$ of each vehicle α is a continuous function of the velocity v_α, the net distance gap s_α, and the velocity difference Δv_α to the leading vehicle:

$$\dot{v}_\alpha = a \left[1 - \left(\frac{v_\alpha}{v_0}\right)^4 - \left(\frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha}\right)^2 \right]. \tag{8}$$

This expression is a superposition of the acceleration $\dot{v}_{\mathrm{free}}(v) = a[1 - (v/v_0)^4]$ on a free road and the braking deceleration $\dot{v}_{\mathrm{int}}(s, v, \Delta v) = -a(s^*/s)^2$, when vehicle α comes too close to the vehicle ahead. The deceleration term depends on the ratio between the effective 'desired minimum gap'

$$s^*(v, \Delta v) = s_0 + vT + \frac{v\Delta v}{2\sqrt{ab}} \tag{9}$$

and the actual gap s_α. The minimum distance s_0 in congested traffic is significant for low velocities only. The main contribution in stationary traffic is the term vT, which corresponds to following the leading vehicle with a constant 'safe time gap' T. The last term is only active in non-stationary traffic and implements an 'intelligent' driving behavior including a

braking strategy that, in nearly all situations, limits braking decelerations to the 'comfortable deceleration' b. Notice that the IDM guarantees crash-free driving.

The lane-changing behavior does not only depend on the lane-changing and the car-following model, but also on the heterogeneity of the vehicle-driver units. Particularly, for identical vehicle-driver units, a stationary state would be reached soon. To avoid this artifact, we have introduced heterogeneity by implementing two types of vehicles. The slower 'trucks' differ in their reduced desired velocity $v_0 = 80\,\mathrm{km/h}$ compared to the faster 'cars' ($v_0 = 120\,\mathrm{km/h}$). Apart from different desired velocities for the vehicle type, we have also uniformly distributed the desired velocity with a variation of $\pm 20\%$ for each single vehicle in order to increase the degree of heterogeneity. For our simulations, we use the following IDM parameters: The time gap is set to $T = 1.2\,\mathrm{s}$, the maximum acceleration to $a = 1.5\,\mathrm{m/s^2}$, the desired deceleration to $b = 2\,\mathrm{m/s^2}$, and the minimum distance to $s_0 = 2\,\mathrm{m}$. Furthermore, the vehicle length is assumed to be 4 m for cars and 12 m for trucks. In addition, we assumed a truck fraction of 20%. The values of the MOBIL parameters used in the simulations are listed in Table 1.

The incentive criterion is evaluated in each numerical update step in the simulation, i.e., the drivers continuously check their incentives. If a lane change is favorable *and* safe, the lane change is performed immediately and the transition from the present lane to the target lane is neglected. Notice that the acceleration will be discontinuous for the considered vehicle, and also for the old and new successors. However, as the velocity is given by integrating the acceleration, the velocities of all vehicles (and the accelerations of all other vehicles not directly involved in the lane change) remain continuous. We checked the simulation results for the different numerical update steps $\Delta t = 0.25, 0.1$ and $0.01\,\mathrm{s}$ and found only a marginal quantitative difference with respect to lane-changing rates. Furthermore, the multi-lane model combination of IDM and MOBIL is mathematically consistent in the sense that the numerical results for a limited simulation period converge in the limit $\Delta t \to 0\,\mathrm{s}$. For the following simulations, we used an explicit numerical update of $\Delta t = 0.25\,\mathrm{s}$.

When evaluating the MOBIL accelerations for the old and new followers, one has, in principle, the freedom to evaluate the accelerations using the own model parameter set or that of the respective successors. Clearly, using the driving parameters of the followers is in line with the reasoning behind MOBIL, although they are not directly observable by the driver initiating a lane change. However, strong clues are given to the driver both by the vehicle type (truck, family car, sports car) and by the past driving style. Therefore, we evaluate all MOBIL accelerations with the model parameters of the respective successors.

3.1 Spatial distribution of the lane-changing rate

In this section, we apply the proposed lane-changing model to the simulations of discretionary and mandatory lane changes. To this end, we have simulated a two-lane road section of 10 km length with open boundary conditions. For an open system, the inflow at the upstream boundary is the natural control parameter. The inflow at the upstream boundary has been kept constant at 1000 vehicles/h/lane. Furthermore, we have assumed an on-ramp (merging length 300 m) at the location $x = 7.5$ km with a constant inflow of 500 vehicles/h.

The *mandatory merge* from the on-ramp to the right lane of the freeway is modeled by a 'virtual vehicle' standing at the end of the merging lane. Due to the imposed deceleration to avoid a collision, the attractiveness of the merging lane automatically decreases, and, consequently the incentive to merge to the freeway increases, when approaching the standing vehicle. To favor lane-changing in this situation, we assume an egoistic behavior for the merging vehicle in the weaving lane by setting $p = 0$.

Table 1: Parameters of the MOBIL lane-changing model. The politeness parameter p of the incentive criterion mainly determines the lane-changing rate. The changing threshold Δa_{th} prevents lane changes of marginal advantage. For $p < 1$, the maximum safe deceleration b_{safe} serves as additional safety criterion. The value of b_{safe} is chosen considerably below the physically possible maximum deceleration of about $9\,\text{m/s}^2$ on dry roads. In the case of asymmetric ('European') lane-changing rules, the additional bias Δa_{bias} models a preferred lane-usage of the default lane. The values are used in the simulations in combination with the Intelligent Driver Model (IDM). Notice that lane-changing properties and, consequently, the values depend on the respective longitudinal traffic model.

Parameter	Value
Politeness Factor p	$0 \ldots 1$
Changing threshold Δa_{th}	$0.1\ \text{m/s}^2$
Maximum safe deceleration b_{safe}	$4\ \text{m/s}^2$
Bias for right lane Δa_{bias}	$0.3\ \text{m/s}^2$

The impact of the on-ramp on the lane-changing behavior with symmetric rules is displayed in a spatiotemporal diagram of the lane-changing events in Fig. 2 (upper row) for a politeness factor $p = 1$. The displayed lane-changing events from the right to the left lane, and from the left to the right lane express clearly the inhomogeneity of the road section. As expected, the local lane-changing rate is increased near the on-ramp located at $x = 7.5\,\text{km}$. The on-ramp induces a locally strongly increased activity of *discretionary lane changes* from the right to the left lane, while the number of lane changes from the left to the right is reduced. Since vehicles merge from the on-ramp to the right lane of the freeway, the right lane becomes less attractive for vehicles on the freeway upstream the merging zone. Therefore, the incentive to change to the left lane is locally increased.

This observation is displayed in the distribution of lane-changing events as a function of space. The diagrams in the lower row of Fig. 2 show the lane-changing rates for simulations with politeness factors $p = 0$ and $p = 1$. The lane-changing rate measures the performed lane changes per kilometer and hour. The simulations for different values of p show the same form of the spatial distribution: The lane-changing rate is nearly homogeneously distributed up- and downstream the on-ramp. In a range of about $500\,\text{m}$ around the center of the on-ramp at $x = 7.5\,\text{km}$, the number of lane changes to the left lane is increased by approximately a factor of 4, while the changes to the right lane are slightly reduced. This demonstrates the strong dependence of the lane-changing behavior due to spatial inhomogeneities of the road section. The relative increase is even higher for 'polite' drivers ($p = 1$) compared to the simulated 'egoistic' behavior referring to $p = 0$. Notice that the lane-changing rate is slightly increased downstream the on-ramp because of the increased traffic density, see the following section.

3.2 Lane-changing rate

Let us now investigate the lane-changing rate as a function of the traffic density. A method to measure locally the lane-changing rate and the traffic density in a microscopic simulation is as follows: The road is divided into subsections, e.g., of length $\Delta x = 1\,\text{km}$, and time is divided into time intervals of durations $\Delta t = 1\,\text{min}$. For each spatiotemporal element $\Delta x \Delta t$ obtained in this way, the number n of lane changes and the average density ρ is determined.
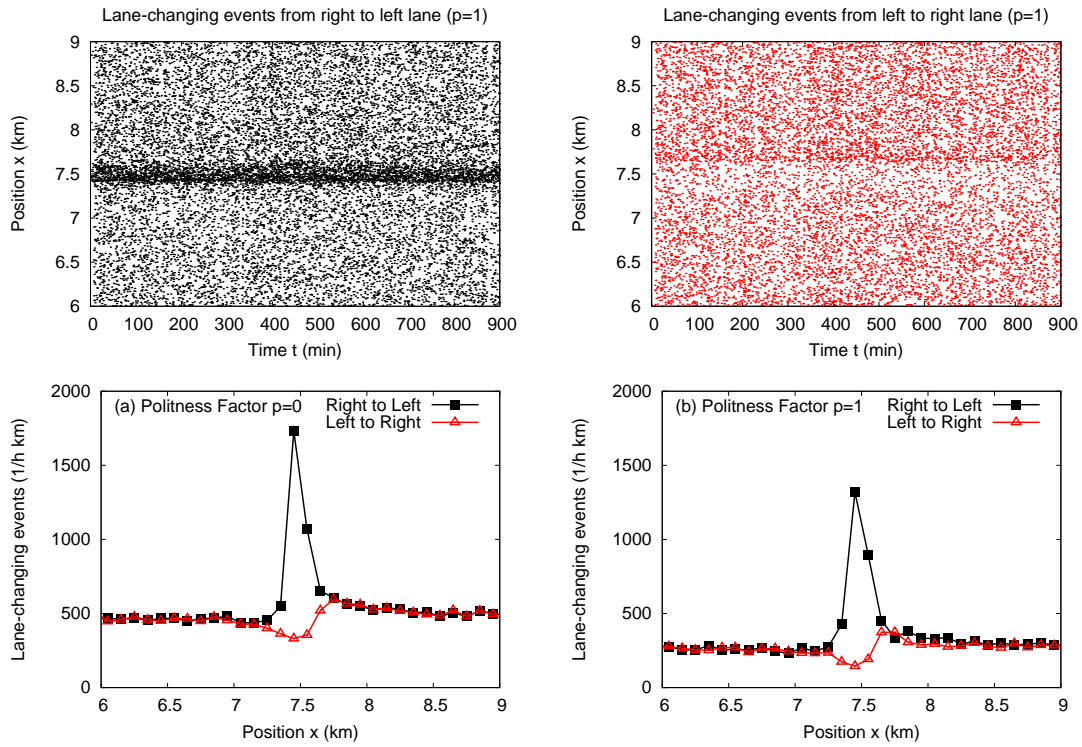
Figure 2: (Upper row) Spatiotemporal diagrams of the lane-changing events from the right to the left lane and vice versa for symmetric lane-changing rules and a politeness factor of $p = 1$. Each lane change is displayed as a dot. The traffic demands are 1000 vehicles/h/lane on the main road and 500 vehicles/h on the on-ramp. (Lower row) Distributions of the lane-changing rate as a function of space for $p = 0$ and $p = 1$.
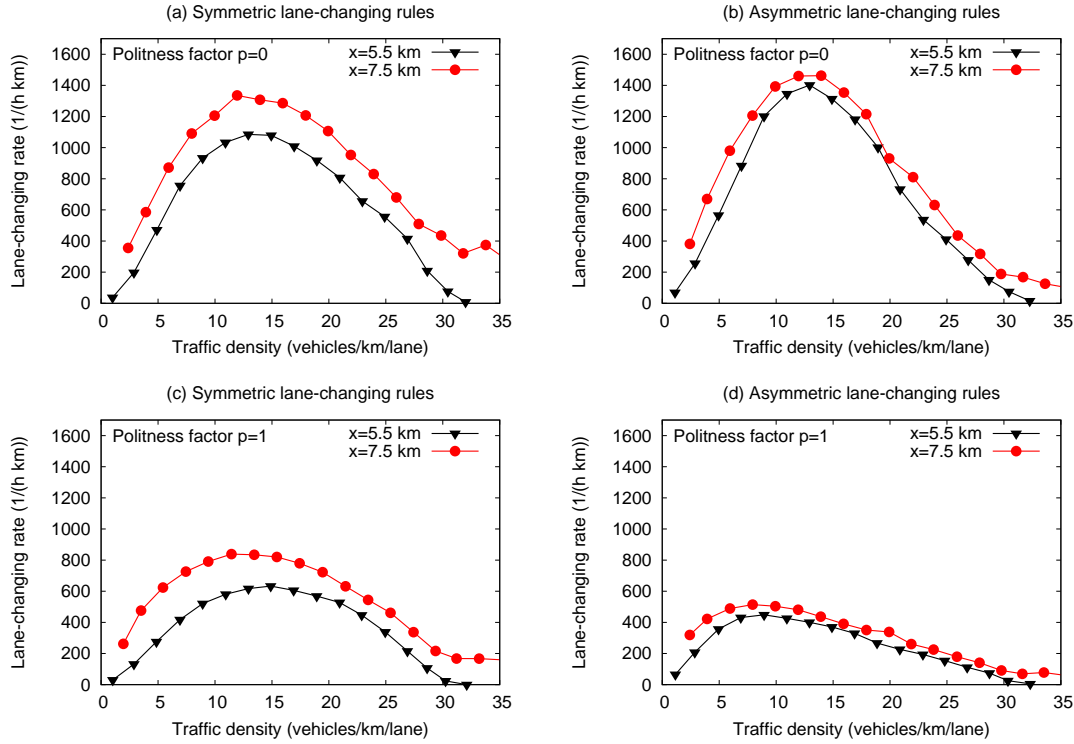
Figure 3: Lane-changing rates for symmetric ('US') and asymmetric ('European') lane-changing rules as a function of traffic density. In the simulations, the politeness parameter has been considered for values of $p = 0$ and $p = 1$. Furthermore, the diagrams show the lane-changing rates measured in the road sections $[5, 6]$ km and $[7, 8]$ km of the 10 km road stretch. As shown in the diagrams, the maximum lane-changing rate is mainly determined by the politeness factor. An on-ramp at location $x = 7.5$ km with a merging zone of 300 m increases the lane-changes locally.

The lane-changing rate is then given by

$$r(\rho) = \frac{n}{\Delta x \Delta t}. \tag{10}$$

Finally, we average over all lane-changing rates belonging to the same density interval. Taking different values of Δx, Δt, or $\Delta \rho$ have not changed the results qualitatively.

We have run multiple simulations of the scenario presented in the previous section with inflows varying from 100 vehicles/h/lane up to 1800 vehicles/h/lane and a constant ramp flow of $Q_{rmp} = 500$ vehicles/h. The resulting lane-changing rates for politeness factors of $p = 0$ and $p = 1$ and for symmetric and asymmetric lane-changing rules are shown in Fig. 3. The results for the considered road sections around $x = 5.5$ km and $x = 7.5$ km of length 1 km displayed in Fig. 3 show the following characteristics:

- The lane-changing rates increase for traffic densities 1 km/lane $< \rho < 10$/km/lane. A more detailed analysis revealed a quadratic slope at the origin for small densities.

- The maximum lane-changing rates are reached for intermediate densities. The maximum is located between 10/km/lane (for $p = 1$ and asymmetric rules), and 15/km/lane (other cases).

- The peak value depends strongly on the value of the politeness parameter. For $p = 0$, the maximum lane-changing rate is about 1100/h/km (1400/h/km) for symmetric (asymmetric) rules. For $p = 1$, the maximum lane-changing rate is only 600 (450) vehicles/h/km approximately. Further simulations show that already a positive value $p > 0$ reduces the maximum number of lane changes significantly.

- With increasing density, velocity differences between neighboring lanes are reduced. Thus, the lane-changing rates decrease.

- For density values around 30 vehicles/km/lane, the lane-changing rates on the homogeneous road section around $x = 5.5\,\text{km}$ are negligible because changing lanes is no more profitable or possible due to a lack of suitable gaps. This could be attributed to a 'moving like a solid block' effect proposed by Helbing and Huberman (1998).

- The curves of the lane-changing rates measured at the homogeneous road section located around $x = 5.5\,\text{km}$ and the section at $x = 7.5\,\text{km}$ including the merging area show similar shapes. Due to the slower vehicles merging from the on-ramp to the freeway, the lane-changing rate is systematically shifted to higher values (cf. the previous section). Notice that, for high traffic densities, the lane-changing rate does not drop to zero. There are still about 100–200 lane changes per hour and kilometer. This agrees with the findings in Sparmann (1978).

The politeness parameter p is the most important parameter determining the lane-changing rate. However, let us discuss the influence of the other MOBIL parameters as well. The lane-changing threshold Δa_{th} influences the peak of the curve weakly, but does not change $r(\rho)$ qualitatively. For example, increasing Δa_{th} from 0.1 to $0.3\,\text{m/s}^2$ reduces the maximum number of lane changes by approximately 100/h/km. Moreover, the influence of the maximum safe deceleration b_{safe} is negligible within a reasonable range of braking accelerations from $-8\,\text{m/s}^2$ to $-b$ as the IDM braking strategy limits braking decelerations to the 'comfortable deceleration' b in nearly all situations Treiber et al. (2000). Notice that for the special case $p = 1$, the safety criterion is even dispensable as discussed in Sec. 2.1.

Let us finally discuss the mean velocities as a function of traffic density corresponding to the lane-changing rates shown in Fig. 3 of the open system. In the simulations, we have implemented 'virtual' cross sections in order to aggregate the data with 1-min sampling intervals mimicking real-world double-loop detector measurements. For each sample interval, we have recorded the lane-resolved traffic flow Q_i and determined the arithmetic velocity averages V_i. The density ρ was calculated by the hydrodynamic relation $Q = \rho V$ from the lane-averaged quantities $Q = \sum_{i=1}^{L} Q_i$ and $V = \sum_{i=1}^{L} (Q_i V_i)/Q$ for the road consisting of $L = 2$ lanes. To facilitate the discussion, we have suppressed the fluctuations in the velocities occurring in the 1-min data by averaging over all data belonging to the same density class (of class width $\Delta \rho = 2\,\text{/km/lane}$).

Figure 4 shows the velocities of the left and the right lane measured with a detector located at $x = 5\,\text{km}$ for symmetric and asymmetric lane-changing rules and for politeness settings $p = 0$ and $p = 1$. In the simulations, traffic is always free with speeds about $V \geq 65\,\text{km/h}$. For symmetric lane-changing rules without any bias, the velocity is primarily synchronized in all lanes for all densities due to the lack of any lane preference, see Fig. 4(a). In contrast, the difference of average velocities in different lanes in free traffic, Fig. 4(b), is a consequence of explicitly asymmetric lane-changing rules modeled by the parameter a_{bias} in combination with the passing rule (5). The initially equally distributed trucks are mostly found on the
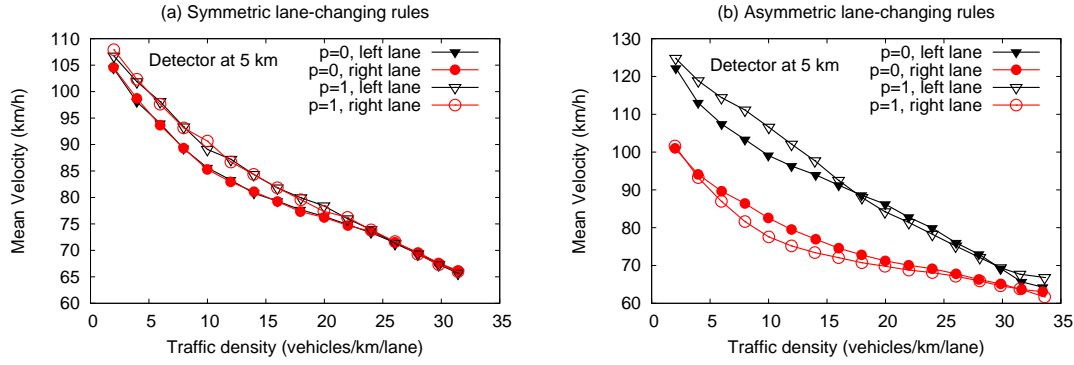
Figure 4: Lane-resolved velocities averaged from double-loop detector samples of 1 min at a cross section $x = 5\,\mathrm{km}$ for symmetric (a) and asymmetric (b) MOBIL rules. The simulation results correspond to the lane-changing rates shown in Fig. 3 for politeness factors $p = 0$ and $p = 1$. Notice the different vertical scale in both diagrams.

right-most lane. The separation results in a different velocity-density relation for the fast (left) lane and the slow (right) lane as shown in Fig. 4(b). For both lane-changing scenarios, the velocity differences decrease with increasing traffic density.

The influence of the politeness factor p leads to the following findings:

- For symmetric lane-changing behavior, the 'altruistic' lane-changing behavior corresponding to $p = 1$ increases the mean speed of *both* lanes for traffic densities of about $\rho \leq 20\,/\mathrm{km/lane}$. Therefore, the suppression of disadvantageous lane changes for the direct environment (Eq. (4)) improves the overall traffic performance. In contrast, an 'egoistic' lane-changing behavior ($p = 0$) results (in average) in higher travel times.

- For asymmetric MOBIL rules, the lane-changing behavior corresponding to $p = 1$ leads to more articulate velocity differences between the lanes. While the speed in the passing (left) lane is higher than in the case for $p = 0$, the slow (right) lane gets slower. Notice that these variations only occur for intermediate traffic densities, i.e., when lane changes lead to interactions between vehicles in neighboring lanes. When a vehicle-driver unit considers to change to the fast lane, the disadvantage of the follower in the target lane is included (and weighted) by the politeness factor. An unselfish driver, therefore, stays in the slower lane to avoid the perturbation of the faster vehicles in the left lane. As a consequence, the lane-changing rate is reduced (see Fig. 3), and velocity differences in the right lane decreases as well.

- However, for symmetric and asymmetric MOBIL rules, the differences between the lane-changing behavior for different p settings disappear for densities $\rho > 20\,/\mathrm{km/lane}$. This result is consistent with the measured lane-changing rate, cf. Fig. 3, as the number of lane changes decrease with increasing density due to a lack of suitable gaps independent from the value of the politeness.

4 Conclusions and outlook

Lane-changing models are an important component of microscopic traffic simulation software. Most of the published and implemented lane-changing models follow a rule-based approach

with different gap-acceptance conditions and, consequently, different lane-changing behavior for various situations. Due to the multiplicity of possible driving conditions associated with discretionary and mandatory lane changes, this approach often tends to lead to complex models with many parameters.

In this paper, we have presented the general concept MOBIL defining lane-changing models for a broad class of car-following models. The basic idea of MOBIL is to measure both the attractiveness of a given lane, i.e., its utility, and the risk associated with lane changes in terms of accelerations. This means, both the incentive criterion and the safety constraint can be expressed in terms of the acceleration function of the underlying car-following model, which allows for an efficient and compact formulation of the lane-changing model with only a small number of additional parameters. As a consequence, the properties of the car-following model, e.g., any dependence on relative velocities or the exclusion of collisions are transfered to the lane-changing behavior. Moreover, the model is able to describe mandatory and discretionary lane changes as well as symmetric and asymmetric lane changing behavior in a unified and consistent way.

As a novel feature, our model takes into account other drivers via a *politeness factor p*. The politeness factor characterizes the degree of 'passive' cooperativeness among drivers, i.e., the subject vehicle makes a decision by considering its effects on other drivers. More specifically, even advantageous lane changes will not be performed if the personal advantage is smaller than the disadvantage to the traffic environment, multiplied by p. Furthermore, a 'pushy' driver is able to initiate a lane change of his or her leader, which is a commonly observed driving behavior in countries with asymmetric lane-changing rules and dedicated passing lanes.

The MOBIL concept has only few parameters and each parameter is associated with an intuitive meaning. The safety criterion is simply described by a critical acceleration threshold b_{safe}. The threshold a_{th} prevents lane-changing yielding only a marginal advantage. For the asymmetric incentive criterion, an additional bias parameter a_{bias} differentiates between default and passing lanes. The optional politeness parameter p weights the accelerations and decelerations of the vehicles directly affected by a lane change. The parameters $b_{\text{safe}}, a_{\text{th}}$ and a_{bias} are given in units of the acceleration and are directly *measurable* quantities. Therefore, the model parameters could be calibrated by using highly resolved trajectory data (Hoogendoorn et al., 2003; NGSIM, 2006). Moreover, the politeness factor can also be empirically tested and measured by comparing the situation before and after the lane change for the affected vehicles.

We have investigated the lane-changing rate by means of simulation in an open system with an on-ramp in combination with the Intelligent Driver Model (IDM) leading to deterministic lane-changing behavior. The lane-changing rate is mainly determined by the politeness factor p, but depends also on the considered location of the road section. As shown in the simulations, the lane-changing rate is locally increased at the location of an road inhomogeneity, which is related to mandatory lane changes. In order to investigate the role of 'critical' lane changes in increasing the breakdown probability, one could vary the safety threshold b_{safe}. A more generic stochastic approach would be based on a car-following model that takes explicitly into account perception errors that lead to a *subjective estimate* for the utility and safety of a lane change (Treiber et al., 2006).

Obviously, research into empirical justification and model calibration and validation is the next step using highly resolved trajectory data (Hoogendoorn et al., 2003; NGSIM, 2006). However, the empirical investigation of lane-changing behavior is even more difficult than the car-following behavior because more vehicles are involved, the situations are more singular due to the overlap of the strategic, tactical, and operational behavior, and, finally, the intra- and inter-driver differences will play even a stronger role as for the longitudinal behavior (Ossen

et al., 2006). The diversity of the drivers could be represented in a microscopic simulation by statistically distributed values for the MOBIL parameters, particularly for the politeness parameter.

Furthermore, extensions of the proposed acceleration-based concept to other traffic-related decision processes are possible as well. For example, when approaching a traffic light that switches from green to yellow, one has to decide whether to stop in front of the signal, or still to pass it. In the framework of MOBIL, the 'stop' decision will be based on the safe braking deceleration b_{safe}. Similar considerations apply when deciding whether it is safe enough to cross an unsignalized intersection (Helbing et al., 2005), to turn into another road in a 'yield situation', or to start an overtaking maneuver on the opposite lane of a two-way rural road.

Finally, we emphasize that MOBIL is meant to represent only the last 'operational' decision whether to immediately perform a lane change or not. In reality, a lane-changing decision also includes strategical and tactical aspects in preparation for this final step, which are relevant particularly for congested traffic and for mandatory lane changes. For example, tactical behavior may involve accelerations (or decelerations) of the own vehicle or of vehicles in the target lane in preparation for a lane change, which corresponds to 'active' cooperation between the drivers. This 'longitudinal-transverse coupling' will be the subject of a forthcoming paper.

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