

Contract-based Software Development

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Overview

1 Loop Invariants

Loop Invariant

```
// { Q }  
// S0  
// { P }  
while(B) {  
    // { P ∧ B }  
    // S  
    // { P }  
}  
// { P ∧ ¬ B ⇒ R }
```

Loop Invariant: Example

Algorithm for summing integers in a array.

$$a[0] + a[1] + \dots a[N - 1] = (\sum i | 0 \leq i < N : a[i])$$

Loop Invariant: Example

```
// { 0 ≤ N }  
int n = 0;  
int s = 0;  
// { s = (Σ i | 0 ≤ i < n : a[i]) }  
while (n != N) {  
    // { s = (Σ i | 0 ≤ i < n : a[i]) ∧ n ≠ N }  
    s = s + a[n];  
    n = n + 1;  
    // { s = (Σ i | 0 ≤ i < n : a[i]) }  
}  
// { s = (Σ i | 0 ≤ i < N : a[i]) ∧ n = N }
```

Loop Invariant: Example proof

Basis: $n = 1$

$$a[0] = (\sum i | 0 \leq i < 1 : a[i])$$

Inductive step: $n + 1$

$$a[0] + a[1] + \dots + a[n-1] + a[n] = (\sum i | 0 \leq i < n + 1 : a[i])$$

Loop Invariant: Example proof

```
while (n != N) {  
    s = s + a[n];  
    // { s = ( $\sum i \mid 0 \leq i < n + 1 : a[i]$ ) }  
    n = n + 1;  
}
```

Loop Invariant: Termination

Function T such that loop execution ends when $T = 0$.
 $T = N - n$ for the example.

The End

“Testing shows the presence, not the absence of bugs.”
— *Edsger W. Dijkstra*