Computability

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Turing machines as a model universal computer

Give a formal definition of a Turing Machine. Describe the language accepted by a Turing machine and what it means that a Turing machine is total.

Turing Machine

$T = (Q, \Sigma, \Gamma, q_0, \delta)$

Q, a finite set of states
Σ, the input alphabet $(\Sigma \subseteq \Gamma)$
Γ, the tape alphabet $(\Delta \notin \Gamma)$
q_0, the initial state $(q_0 \in Q)$
δ, the transition function

$$\delta : Q \times (\Gamma \cup \{\Delta\}) \rightarrow (Q \cup \{h_a, h_r\}) \times (\Gamma \cup \{\Delta\} \times \{R, L, S\}$$

Language accepted by a TM

If $x \in \Sigma$ then x is accepted by T if

$$q_0 \Delta x \vdash_T^* w h_a y$$

$L \subseteq \Sigma^*$ is accepted by T if $L = L(T)$ where

$$L(T) = \{x \in \Sigma^* \,|\, x \text{ is accepted by } T\}$$

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Feed T' to T.

The End