Computability

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Deterministic and non-deterministic finite automata

Define the two types of finite automata. Are the classes of languages they define the same? Explain and justify your answer.

Finite Automata

$(Q, \Sigma, q_0, A, \delta)$
Q is a finite set of *states*;
Σ is a finite *input alphabet*;
$q_0 \in Q$ is the *initial* state;
$A \subseteq Q$ is the set of *accepting* states;
δ is the *transition* function.

Deterministic & Nondeterministic FA

DFA

$\delta : Q \times \Sigma \to Q$

For $q \in Q$ and $\sigma \in \Sigma$ then $\delta(q, \sigma)$ denotes the state transition from q on input σ.

NFA

$\delta : Q \times (\Sigma \cup \{\Lambda\}) \to 2^Q$

For $q \in Q$ and $\sigma \in (\Sigma \cup \{\Lambda\})$ then $\delta(q, \sigma)$ denotes the set of states the NFA can move to from q on input σ.

Eliminating Λ-transistions

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$M_1 = (Q, \Sigma, q_0, A_1, \delta_1)$

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$A_1 = \begin{cases} A \cup \{q_0\} & \text{if } \Lambda \in L \\ A & \text{if not} \end{cases}$
For every $q \in Q$ and every $\sigma \in \Sigma$ then $\delta_1(q, \sigma) = \delta^*(q, \sigma)$

DFA accepting the same language as an NFA

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$\delta_1(q, \sigma) = \bigcup\{\delta(p, \sigma) | p \in q\}$

The End