Computability

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Regular and Non-regular Languages

How can a language be shown to be regular? How can a language be shown to be not regular? Is the language $\{a^n b^n | n > 0\}$ regular? Explain and justify your answer.

Regular Languages

If Σ is an alphabet, then the set of regular languages is defined as: $\mathcal{R}$:

1. $\emptyset \in \mathcal{R}$
2. For every $a \in \Sigma$, $\{a\} \in \mathcal{R}$
3. For any L_1 and L_2 in $\mathcal{R}$,

$$L_1 \cup L_2 \in \mathcal{R}$$

$$L_1 L_2 \in \mathcal{R}$$

$$L_1^* \in \mathcal{R}$$

Pumping Lemma

$L \subseteq \Sigma^*$
$M = (Q, \Sigma, q_0, A, \delta)$
$n = |Q|$

For every $x \in L$ where $|x| \geq n$, $x = uvw$ and

1. $|uv| \leq n$,
2. $|v| > 0$ (or $v \neq \Lambda$),
3. $\forall_{i \geq 0} | uv^i w \in L$

must be true.

A not regular language

$\{a^n b^n | n > 0\}$ is not regular.

Proof:
Assume a FA M with $n = |Q|$, $x = a^n b^n$ and $|x| \geq n$.
There should be a u, v and w such that $x = uvw$.

1. $|uv| \leq n$
2. $|v| > 0$, $v = a^k$ where $k > 0$
3. With $i > 1$ then $uv^i w \in L \rightarrow a^{i+k} b^i \in L$ is a contradiction because $i + k \neq i$.

The End