The Possibility of Temporal Traces

A Possibilistic Extension of LTL_f

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Abstract

Linear temporal logic over finite traces (LTL_f) has been successfully used to model and reason about processes. However, in its classical form, this logic cannot handle the uncertainty often encountered in real-life scenarios. In this position paper we propose a possibilistic extension of LTL_f, which is able to express uncertainty on traces, while avoiding many of the technicalities of probability theory.

Keywords

linear temporal logic, uncertainty, possibility theory

1. Introduction

The linear temporal logic on finite traces LTL_f is a successful formalism for modelling and reasoning about finite executions, including those representing processes [1, 2]. However, in its classical form, these formalism cannot deal with the uncertainty associated to any real-world domain or the unpredictable actions of the environment. For that reason, there has been a recent push at extending LTL_f with uncertainty measures, mainly focused on probabilities [3, 4]. However, a correct handling of probabilities requires a very detailed analysis, which leads to complex formalisms and often to counter-intuitive results.

In this position paper we change the focus, and consider a different kind of uncertainty measure based on possibility theory [5]. The main formal idea that differentiates possibility theory from probabilities is that a high possibility of an event does not exclude a high possibility of its complement. That is, stating that "*it is completely possible that it rains and also completely possible that it does not rain*" is perfectly reasonable in this formalism, without leading to contradictions. On a technical level, the possibility of an event is less influenced by other events, which usually leads to more efficient reasoning methods.

We introduce a simple possibilistic extension of LTL_f called $\Pi\mathrm{LTL}_f^0$, where axioms are used to constraint the possibility values associated to observable traces. We show how to decide consistency and entailment in this new language using a classical LTL_f reasoner as a back end. In the end we provide a tight (PSpace-complete) complexity bound for these reasoning tasks.

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2. Preliminaries

We start providing the background knowledge necessary for understanding our formalism; namely, the temporal logic LTL$_f$ and the basics of possibility theory.

2.1. LTL$_f$

LTL$_f$ [6] is a (discrete) linear temporal logic over finite traces. Syntactically, it looks exactly like the better known (infinite time) LTL [7, 8]. More precisely, LTL$_f$ *formulas* are constructed from a class $\mathcal{P}$ of propositional *variables* through the syntactic rule

$$\varphi ::= x \mid \neg\varphi \mid \varphi \wedge \varphi \mid \bigcirc\varphi \mid \varphi \,\mathcal{U}\, \varphi,$$

where $x \in \mathcal{P}$. Informally, $\bigcirc\varphi$ expresses that φ should hold in the *next* point in time, while $\varphi \,\mathcal{U}\, \psi$ expresses that φ holds *until* ψ is observed. The important difference with LTL is that time does not run infinitely, but eventually stops; that is, there is a final point in time. All this is formalised through temporal models.

A *temporal model* is a finite sequence of propositional valuations, where as usual a propositional valuation is represented as a set of propositional variables. Informally, a valuation $\mathcal{V} \subseteq \mathcal{P}$ expresses which variables are *true* in it. Let $\mathcal{M} = \mathcal{V}_0\mathcal{V}_1 \cdots \mathcal{V}_n$ be a temporal model. The *satisfiability* relation of a formula φ w.r.t. $\mathcal{M}$ at time $k, 0 \leq k \leq n$ is defined inductively as follows:

- $\mathcal{M}, k \models x$ (where $x \in \mathcal{P}$) iff $x \in \mathcal{V}_k$;
- $\mathcal{M}, k \models \neg\varphi$ iff $\mathcal{M}, k \not\models \varphi$;
- $\mathcal{M}, k \models \varphi \wedge \psi$ iff $\mathcal{M}, k \models \varphi$ and $\mathcal{M}, k \models \psi$;
- $\mathcal{M}, k \models \bigcirc\varphi$ iff $k < n$ and $\mathcal{M}, k+1 \models \varphi$; and
- $\mathcal{M}, k \models \varphi \,\mathcal{U}\, \psi$ iff (i) $\mathcal{M}, k \models \psi$ or (ii) $\mathcal{M}, k \models \varphi$ and $\mathcal{M}, k \models \bigcirc(\varphi \,\mathcal{U}\, \psi)$.

Note that despite the recursion on the last point, the semantics of $\varphi \,\mathcal{U}\, \psi$ is well defined and in particular $\mathcal{M}, n \models \varphi \,\mathcal{U}\, \psi$ iff $\mathcal{M}, n \models \psi$. This means that the until operator is satisfied if there is a point in the future (before the end of the trace) where ψ holds and in between φ is always satisfied.

We say that $\mathcal{M}$ satisfies the formula φ (denoted $\mathcal{M} \models \varphi$) iff $\mathcal{M}, 0 \models \varphi$. A formula is *satisfiable* if there is a model that satisfies it. A set of formulas Γ is *consistent* iff there is a model that satisfies all the formulas in Γ. From now on, we will often refer to temporal models also as *traces*.

LTL$_f$ has been successfully used to represent processes, by introducing constraints on the traces which can be observed [2]. Usually, these process models are constructed as a set of LTL$_f$ formulas. One common approach to obtain these formulas for existing processes is to mine the trace logs for repeating patterns [9, 10]. Yet, due to the uncertainty of the real world and potential writing mistakes, it is not uncommon to produce inconsistent models in this manner.

2.2. Possibility Theory

Possibility theory [5] is an alternative to probability theory developed for handling some kinds of uncertainty by measuring the *possibility* and *necessity* of events. Similarly to probability theory, each event is assigned a degree in $[0, 1]$, but possibility measures are typically easier to manipulate.

Formally, given a set Ω of *outcomes*, a *possibility measure* is a function $\pi : 2^{\Omega} \to [0, 1]$ which satisfies the following three properties:

1. $\pi(\emptyset) = 0$;
2. $\pi(\Omega) = 1$; and
3. for any two disjoint sets $U, V \subseteq \Omega$, $\pi(U \cup V) = \max\{\pi(U), \pi(V)\}$.

The subsets of Ω (that is, sets of outcomes) are often called *events*. Equivalently, one can define a possibility measure as a function $\pi : \Omega \to [0, 1]$, which is later extended to events by setting $\pi(U) = \sup_{\omega \in U} \pi(\omega)$ for all $U \subseteq \Omega$.

One can immediately see the main difference between possibility and probability measures: for any set $U \subseteq \Omega$ a possibility measure π is such that $\max\{\pi(U), \pi(\Omega \setminus U)\} = 1$. In other words, whenever an event has a possibility strictly smaller than 1, its complement *must* have possibility 1. To emphasise the dissimilarity, we also note that the constant function that maps every non-empty event to 1 (that is, $\pi(U) = 1$ for all $U \subseteq \Omega \setminus \{\emptyset\}$) is a possibility measure. This refers to a completely agnostic situation, where anything is possible. Note also that if an event U is finite, one can verify that $\pi(U) \geq p$ for some $p \in [0, 1]$ simply by finding an outcome $\omega \in U$ with $\pi(\omega) \geq p$.

Possibility measures implicitly define a dual *necessity measure* $N : 2^{\Omega} \to [0, 1]$ given by $N(U) = 1 - \pi(\Omega \setminus U)$ for all $U \subseteq \Omega$. We introduce this function here for completeness, but focus on possibility measures only for the rest of the paper.

3. ΠLTL$_f^0$

We now introduce a possibilistic extension of LTL$_f$. Taking inspiration from the simple probabilistic PLTL$_f^0$ [3], we define ΠLTL$_f^0$, which defines a possibility measure over the class of all temporal models, based on a set of constraints which limit the possibility of observing certain types of behaviour.

Definition 1. *A ΠLTL$_f^0$* knowledge base *(KB) is a finite set of constraints of the form $\langle \varphi \geq p \rangle$ or $\langle \varphi \leq p \rangle$ where φ is an LTL$_f$ formula and $p \in [0, 1]$. A* possibilistic model *is a pair $\mathfrak{I} = (\mathfrak{M}, \pi)$ where $\mathfrak{M}$ is a set of temporal models and π is a possibility measure over $\mathfrak{M}$. $\mathfrak{I}$ is* finite *iff $\mathfrak{M}$ is finite.*

Given a possibilistic model $\mathfrak{I} = (\mathfrak{M}, \pi)$, the possibility *of an LTL$_f$ formula φ w.r.t. $\mathfrak{I}$ is defined as $\pi(\varphi) := \pi(\{\mathcal{M} \in \mathfrak{M} \mid \mathcal{M} \models \varphi\})$. $\mathfrak{I}$* satisfies *the KB $\mathcal{K}$ (denoted as $\mathfrak{I} \models \mathcal{K}$) iff for every constraint $\langle \varphi \bowtie p \rangle \in \mathcal{K}$, it holds that $\pi(\varphi) \bowtie p$. $\mathcal{K}$ is* consistent *iff there is at least one possibilistic model that satisfies it.*

According to this definition, a model $\mathfrak{I}$ may describe infinitely many traces (with an adequate measure over them). Yet, for consistency, it is only important how these traces behave over the formulas in the KB. Indeed, given a possibilistic model $\mathfrak{I} = (\mathfrak{M}, \pi)$ and a KB $\mathcal{K}$, we can define an equivalence relation $\sim$ over $\mathfrak{M}$ where $\mathcal{M} \sim \mathcal{M}'$ iff for every φ appearing in $\mathcal{K}$ it holds that $\mathcal{M} \models \varphi$ iff $\mathcal{M}' \models \varphi$. This induces a new possibilistic model $\mathfrak{I}_\sim = (\mathcal{M}/\!\sim, \pi_\sim)$ where $\mathcal{M}/\!\sim$ is the quotient set of $\mathcal{M}$ w.r.t. $\sim$ and for every equivalence class $[\varphi]_\sim$ we have $\pi_\sim([\varphi]_\sim) := \sup_{\psi\sim\varphi} \pi(\psi)$. It is a simple exercise to verify that $\mathfrak{I} \models \mathcal{K}$ iff $\mathfrak{I}_\sim \models \mathcal{K}$ and, moreover, that $\mathcal{M}/\!\sim$ contains finitely many equivalence classes; in fact, at most 2^n, where n is the number of constraints in $\mathcal{K}$. This yields our first theorem; namely, that ΠLTL_f^0 has the finite-model property.

Theorem 2. *If a ΠLTL_f^0 KB is consistent, then there is a finite possibilistic model that satisfies it.*

The construction used to prove this theorem suggests also a way to effectively decide consistency: one can try to construct a temporal interpretation satisfying each of the 2^n combinations of formulas in $\mathcal{K}$ and assign an adequate possibility degree according to the constraints. Yet, this may require us to verify many unnecessary cases. Indeed, note that all constraints of the form $\langle \varphi \geq p \rangle$ can be treated independently (and similarly for $\langle \varphi \leq p \rangle$); for instance the KB $\mathcal{K} = \{\langle x \geq 0.9 \rangle, \langle \neg x \geq 0.9 \rangle\}$ is consistent although there is no trace that satisfies both formulas. The main thing to consider is thus whether upper and lower bounds contradict each other.

We partition the KB $\mathcal{K}$ into two classes, $\mathcal{K}^+$ which contains all the constraints of the form $\langle \varphi \geq p \rangle$ (from now on *positive constraints*) and $\mathcal{K}^-$ containing the constraints of the form $\langle \varphi \leq p \rangle$ (*negative constraints*). A positive constraint requires the *existence* of a trace (satisfying a formula) with a possibility larger or equal to some value; dually, negative constraints express that *all* such traces must have a low possibility degree. If any positive constraint is given by a contradictory formula, then $\mathcal{K}$ is inconsistent; likewise, if any negative constraint uses a tautology $\mathcal{K}$ is inconsistent. We say that $\mathcal{K}$ is *trivial* if any of these cases occur. For the non-trivial cases, it suffices to consider pairwise combinations of formulas to check consistency.

Theorem 3. *A non-trivial ΠLTL_f^0 KB $\mathcal{K}$ is inconsistent iff there exist $\langle \varphi_1 \geq p_1 \rangle \in \mathcal{K}^+$ and $\langle \varphi_2 \leq p_2 \rangle \in \mathcal{K}^-$ such that (i) $p_2 < p_1$ and (ii) $\varphi_1 \wedge \neg\varphi_2$ is unsatisfiable.*

Proof. [$\Leftarrow$] If $\langle \varphi_1 \geq p_1 \rangle \in \mathcal{K}$ then any model satisfying $\mathcal{K}$ must have a trace $\mathcal{M}$ such that $\mathcal{M} \models \varphi_1$ and $\pi(\mathcal{M}) \geq p_1 > p_2$. To ensure that this models satisfies $\langle \varphi_2 \leq p_2 \rangle$ it must be the case that $\mathcal{M} \not\models \varphi_2$ (recall that if $\mathcal{M} \models \varphi_2$ then $\pi(\varphi_2) \geq \pi(\mathcal{M}) > p_2$) but that is impossible since $\varphi_1 \wedge \neg\varphi_2$ is unsatisfiable. Hence, $\mathcal{K}$ is inconsistent.
[$\Rightarrow$] If for every pair $\langle \varphi_1 \geq p_1 \rangle \in \mathcal{K}^+$ and $\langle \varphi_2 \leq p_2 \rangle \in \mathcal{K}^-$ such that $p_2 < p_1$, $\varphi_1 \wedge \neg\varphi_2$ is satisfiable then we construct a possibilistic model as follows: for each $\langle \varphi \geq p \rangle \in \mathcal{K}^+$, for each $\langle \psi \leq q \rangle \in \mathcal{K}^-$ with $q < p$ construct a model that satisfies $\varphi \wedge \neg\psi$ and assign it degree 1. If there is no negative constraint, simply select a satisfying model of φ. For each $\langle \psi \leq q \rangle \in \mathcal{K}^-$ construct a satisfying model of $\neg\psi$ and assign it degree 1. It is easy to see that this model satisfies $\mathcal{K}$, and hence $\mathcal{K}$ is consistent. □

This means that ΠLTL_f^0 KB consistency can be decided quite efficiently, simply by making an at most quadratic number of calls to a standard LTL_f reasoner. Indeed, if $\mathcal{K}$ has at m positive constraints and n negative constraints, then one needs to satisfiability of $m + n$ formulas (to guarantee non-triviality) and at most $m \cdot n$ formulas to verify the conditions of Theorem 3.

Since each such check can be made using only polynomial space, we overall obtain that $\Pi\mathrm{LTL}_f^0$ KB consistency is PSpace-complete (just as in classical LTL$_f$).

Interestingly, if the LTL$_f$ satisfiability algorithm is modular (as in the case of automata-based procedures) this consistency method can be implemented quite efficiently, as it is based on mixing and matching $m + n$ different formulas, which are fixed at the input. Moreover, not all combinations of positive and negative constraints are relevant.

Corollary 4. *Let* $m_{\mathcal{K}} := \max_{\langle\varphi\geq p\rangle\in\mathcal{K}^+} p$ *and* $n_{\mathcal{K}} := \min_{\langle\varphi\leq p\rangle\in\mathcal{K}^-} p$*. If* $\mathcal{K}$ *is nontrivial and* $m_{\mathcal{K}} \leq n_{\mathcal{K}}$ *then* $\mathcal{K}$ *is consistent.*

In particular, nontrivial KBs without any positive or without any negative constraints are always consistent.

As usual in logical languages, consistency is only one of the many reasoning problems that can be considered. Another important problem, specially in the context of reasoning, is that of *entailment*. We say that the KB $\mathcal{K}$ entails the constraint $\langle\varphi \bowtie p\rangle$ ($\mathcal{K} \models \langle\varphi \bowtie p\rangle$) iff every possibilistic model that satisfies $\mathcal{K}$ is such that $\pi(\varphi) \bowtie p$. This problem can be reduced to consistency as well.

Theorem 5. *Let* $\mathcal{K}$ *be a* ΠLTL_f^0 *KB,* φ *an* LTL_f *formula, and* $p \in [0, 1]$*. (i)* $\mathcal{K} \models \langle\varphi \geq p\rangle$ *iff* $\mathcal{K} \cup \{\langle\varphi < p\rangle\}$ *is inconsistent and (ii)* $\mathcal{K} \models \langle\varphi \leq p\rangle$ *iff* $\mathcal{K} \cup \{\langle\varphi > p\rangle\}$ *is inconsistent.*

4. Conclusions

We have introduced a possibilistic extension of the linear temporal logic on finite traces LTL$_f$, as an alternative manner to deal with uncertainty. In our definition, all the uncertainty is expressed at the beginning of the execution, with the help of a KB that constraints the possibility of observing different kinds of traces. Within this logic, we show how to perform the basic reasoning tasks of deciding consistency of a KB and entailment of a formula. Interestingly, these can be solved through standard techniques from classical LTL$_f$.

This is just a position paper, which aims at setting the bases of the new formalism and attracting attention to it. In future work we plan to further extend the ideas, in particular to allow for a possibilistic constructor native to the logical language and not relegated to the KB only. Dealing with such a constructor will require a more dedicated analysis of the semantics.

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