

# Unifying Abduction and Deduction through Argumentation

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## Abstract

The Cognitive Argumentation framework is based on the premise that human logical reasoning is fundamentally a process of dialectic argumentation. To achieve a human cognitive form of logical reasoning, Cognitive Argumentation incorporates, within a general and abstract framework of computational argumentation from AI, cognitive principles derived from empirical and theoretical work in Cognitive Science. Concentrating on the case of conditional reasoning, we study how the two modes of reasoning, predictive (deductive) or explanatory (abductive) as used by humans, can be uniformly captured within the Cognitive Argumentation framework. Within this unification of deductive and abductive reasoning we show how the relative weakness of abductive reasoning is reflected in argumentation by the presence of multiple explanatory arguments that conflict with one another. The approach is evaluated using Byrne’s suppression task showing how the whole empirical data concerning both deductive and abductive reasoning cases by the participants is modelled well by Cognitive Argumentation.

## 1. Introduction

Observing the world and from there, inferring new knowledge, is often called abduction whose conclusion is referred to as an explanation for the observation. This differs from deduction as abduction works in reverse, from observations to potential premises. We are interested in investigating whether humans distinguish between these types of inferences, and if so, to understand how the difference comes about.

Although deduction and abduction are formally defined as two different forms of reasoning, there are clearly related as typically abduction is defined by appealing to deduction. We conjecture that in common sense human reasoning they are realized in the same way in terms of reasoning via argumentation. We thus claim that arriving at a deductive or abductive conclusion is carried out by the same process of forming a good quality argument supporting the conclusion arrived at by either form of reasoning.

This unified view of abductive reasoning with that of deductive reasoning can be achieved via the formation of (common sense) knowledge of associations between information, called argument schemes, generating supporting arguments in both a deductive direction but also in the reverse abductive direction. Hence, we have knowledge of argument schemes that associate premise information from which we would deductively arrive at a conclusion, e.g. premise information that can bring about the conclusion of a new state of affairs, and abductive or explanatory argument schemes that associate these two same pieces of information in the reverse order, e.g. that would associate the premise (observation) of a new state of affairs as a premise to support the information that would bring this premise about. For example, we would have a deductive argument scheme “birthday supports having a party” and in addition the abductively explanatory scheme “having a party supports birthday”.

Both of these types of argument schemes, deductive or abductive, are treated as defeasible by argumentation, even the deductive one, since we can have exceptional cases, e.g. where “although it is our birthday we do not have a party”. Yet, deduction is generally stronger than abduction and hence the weakness of abductive reasoning over deduction should manifest itself in their argumentative formulation. This relative weakness of abduction comes about through the fact that typically we can have many different

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explanatory abductive argument schemes based on the same premise observation and that these different argument schemes are considered (based on the principle of Occams razor of simplicity of explanation) as conflicting with each other, thus producing counter-arguments of each other. For example, we can have a second explanatory scheme of “having a party supports anniversary day” and this would form a counter argument to the earlier one of supporting “birthday” from the same premise information of observing that we are “having a party” (and vice-versa). In contrast, although we would also have a second deductive argument scheme of “anniversary day supports having a party” this is not considered to be in conflict with the earlier one and hence the deductive inference that is generated from either of these two argument schemes is not questioned by the other one.

### 1.1. Related Work

We will be validating our approach using the empirical data from the Suppression Task experiment [1]. Although traditionally this experiment is seen as showcasing the non-monotonicity of human reasoning it is also evident that its results related more generally to different aspects of human reasoning. The experiment reports the “suppression” of inference, i.e. the comparative reduction in the percentage of human participants reaching a definite conclusion, in both deductive and abductive mode of reasoning. Although the experiment is old, first carried out in 1989 and repeated several times after that - e.g. in [2], it becomes relevant in today’s AI that has turned again to automating human cognitive reasoning. Indeed, developments in Human-centric or Cognitive AI [3, 4] have shown the importance of synthesis of AI computational theory and models with Cognitive Science and its study of human reasoning.

The “reverse link” between link between abduction and deduction is well known, starting from the pioneering work of Peirce on abduction to many recent works such as the link of abduction to the inverse process of completing a logic program [5]. In this paper, we are further claiming that this “inverse” link can be formalized through argumentation. For example, in inverse planning [6], together with argument schemes capturing the standard “effect axioms” of the causal generation of effects by actions or agent intentions, we can also have abductive explanatory argument schemes in the reverse order, supporting the occurrence of an action based on the premise of some effects of the action or intention that would bring about the occurrence of the action. Similarly, in a theory of mind used to understand the behaviour of people [7], we can have a model of argument schemes which through deduction would allow us to predict peoples actions from their mental-states of beliefs and desires, while mental-state inference is achieved via abduction by inverting the model.

At the foundational level abduction has normally been viewed as *surrogate* to deductive inference, formalized at the meta-level in terms of the underlying deductive inference of a given logical system. In [8] this is characterized as an *external* approach to abduction. Our approach, like the *internal* approach in [8] treats abductive reasoning at the same level as deductive reasoning. Both forms of reasoning are captured in terms of the same process of argumentation. The internal approach of [8] unifies the abduction and deduction within an argumentation-based sequent calculus by extending this with abductive sequents. Therefore the same process of this extended calculus produces the two forms of reasoning. In comparison, whereas this work is interested in the (pure) logical properties of the formulation, e.g. to connect its internal and external approaches, our work puts the emphasis on the cognitive properties of the unification, e.g. encompassing the Occam’s Razor feature of (typically) mutual exclusiveness of different abductive explanations.

It is also important to note that there are many formal models of human reasoning that address more generally the way that humans arrive at conclusions and decisions. Within this terrain of work and amongst those that are based on argumentation a notable approach is that of Bayesian Argumentation [9, 10]. Bayesian Argumentation is based on probability theory to capture arguments for the degree of human belief in statements and conclusions drawn from them and how such beliefs are revised according to Bayes theorem. In comparison our approach of Cognitive Argumentation is complementary to the Bayesian approach where its probability theory gives us a level of quality of the argument schemes and the relative strength between them that we then adopt in Cognitive Argumentation to reason with.

Summarizing, in this paper we will examine and support the following hypotheses:

1. Human reasoning is driven by deductive and explanatory associations between information.
2. These associations are realized in the same way in terms of reasoning via argumentation captured by deductive and explanatory schemes in Computational Argumentation from AI.
3. These two different but related types of associations can have distinct properties: In some cases and often explanatory reasoning is weaker than deductive reasoning, which in Argumentation, is captured by the existence of several 'weaker' explanatory arguments conflicting with each other.

These hypotheses 1 to 3 will guide the structure of this paper. Section 2 will informally introduce human conditional reasoning that motivates hypothesis 1. After that, Section 3 addresses hypothesis 2 and shows how these associations are realized in Argumentation. In Section 4, we will illustrate the distinct 'behavior' of these two associations that are put forward by hypothesis 3 and validate the conjecture of these three hypotheses by showing how a unified argumentative model of deduction and abduction captures well the complete empirical data from the suppression task [1].

## 2. Conditional Reasoning

Both reasoning forms of deduction and abduction operate based on some given knowledge. We come to deductive conclusions that follow from some prior knowledge and similarly we generate abductive explanations of observations according to some accepted model of the world that we are observing. In common sense human reasoning one important form of knowledge is that of *conditionals*. In this section, we will consider some basic characteristics of conditionals stemming from their use in human reasoning.

Humans make assumptions while reasoning, many of which are not necessarily valid under (formal) classical logic. We will call such assumptions *cognitive principles* of Human Reasoning [11]. We will propose canonical associations for conditionals based on prediction and explanatory associations and examine how Byrne's [12] distinction between different types of conditions influence them.

### 2.1. Deductive and Explanatory Schemes from Conditions

Let us first review, through an example, the different forms of conditionals based either on a sufficient or a necessary condition.

Consider the sentence: *If I need milk, then I will buy milk.* ( $need \leadsto buy$ ).

The condition *I need milk* can be understood as **sufficient**, in the sense that if the condition holds, then this forms a support for the consequent, *I will buy milk*, to hold as well (modus ponens). On the other hand, the negation of the condition, *I don't need milk*, seems to be a plausible support for the negation of the consequent, *I will not buy milk* (denying the antecedent). Thus the condition can also be understood as **necessary** for the consequence to hold.

Now consider also: *If my mother asks me to get her milk, then I will buy milk.* ( $asks \leadsto buy$ )

Both conditions in ( $need \leadsto buy$ ) and ( $asks \leadsto buy$ ) are separately sufficient for the consequence to hold. However, now the negation of either of these conditions alone is not enough to conclude the negation of the consequence, *I will not buy milk*. Only the negation of both conditions together, gives sufficient support to conclude the negation of the consequence. Therefore, individually the conditions in ( $need \leadsto buy$ ) and ( $asks \leadsto buy$ ) are not necessary conditions. Now that there is a second way to bring about the consequent, the condition *I need milk* has lost its (possibly) necessary property.

Let us assume that, in addition to ( $need \leadsto buy$ ) and ( $asks \leadsto buy$ ), we are given the following conditional: *If I have enough money, then I will buy milk.* ( $money \leadsto buy$ ) By ( $money \leadsto buy$ ) we are made aware of the possibility that even in the case where, *I need milk* or *my mother asks me to get her milk*, I might not buy milk, because possibly *I don't have enough money*. Having enough money is a clear necessary condition for the consequent: without it the consequent cannot hold, i.e. *I cannot buy milk*, no matter what other (sufficient) conditions might hold at the time. Also in comparison with the above cases we might consider this a **strong necessary** condition in the sense that it is very unlikely for this to

lose its necessary property. On the other hand, the condition of ( $money \leadsto buy$ ) cannot be considered as a sufficient condition: even if *I have enough money*, I might not buy milk.

The distinction between the two different types of conditions, sufficient and necessary, is significant when we consider **explanations** of the consequent and its negation. Assume that we are given the information that *I did not buy milk*. ( $\overline{buy}$ ) It is reasonable that, given ( $need \leadsto buy$ ) and ( $asks \leadsto buy$ ) (without ( $money \leadsto buy$ )), to conclude the negation of the condition of both conditionals, namely that *I did not need milk and my mother did not ask me to get her milk*. Adding the conditional ( $money \leadsto buy$ ) in the context of reasoning does not extend this conjunction but results in a disjunctive addition of the negation of the new (necessary) condition: Either (*I do not need milk and my mother does not ask me to get her milk*) or (*I do not have enough money*). Hence the observation of the negation of the consequent can be **explained** by the negation of a necessary condition (e.g. *I do not have enough money*) or by assuming that there is “no reason” for the consequent to hold, resulting in a more complex explanation, namely that none of the sufficient conditions can hold.

In contrast, if we are given the positive information that a consequent holds, e.g. *I buy milk*, then this can be simply **explained** by any one of the sufficient conditions for the consequent, e.g. either by *I need milk* or by *my mother asks me to get her milk*. It is important to note that typically we will not consider that two such sufficient conditions, together, form an explanation. In fact, we typically consider that different explanations are incompatible with each other, except perhaps in very exceptional cases where many different reasons can hold together. Hence we will only accept one, either *I need milk* or *my mother asks me to get her milk* to explain the consequent *I buy milk* but not both together. Similarly, when we are explaining the negation of the consequent, e.g. *I did not buy milk*, we will only accept one of the explanations, either *I do not have enough money* or there is “no reason”, i.e. *I did not need milk and my mother did not ask me to get her milk*.

Hence different explanations are in general considered to be in tension with each other. They are competing or contrasting alternatives as implied for example by the maxim of “Inference to the best explanation” (see e.g. [13, 14]). The process of explanation is not merely to find why something holds but also why this is indeed the reason for holding and not for some other reason. In [15, 16] a cognitive principle of **explanatory discounting** is identified which assumes that alternative explanations are in conflict with each other so that support for one explanation results in diminishing support, thus counter-support, against alternative explanations.

We note that depending on the nature of the condition, sufficient or necessary, we can draw further conclusions in, what we will call, a **secondary mode** of predictive or explanatory reasoning. Observing the negation of the consequent can lead us to the prediction of the negation of any of its sufficient conditions. We refer to this as secondary since the conditional is not used in its canonical form of “if ... then ...” but in a transformed form of the contra-positive.

Finally, we note that a necessary condition cannot be considered as a possible explanation for the consequent holding. Prediction of several necessary conditions from the same consequent, as opposed to different explanations, do not compete with each other [17] and hence they can hold together when we are given that the consequent holds. They always hold and hence they do not offer any *discriminatory information* between alternatives as we would require from explanations.

## 2.2. Canonical Associations of Condition and Consequence

Based on the above analysis, Table 1 summarizes the canonical associations of different types of conditions with a consequent under prediction and abduction or explanation. These associations will correspond to argument schemes that will form the basis for the argumentative reasoning that we will consider for the unification of deduction and abduction. We establish the following rule associations<sup>1</sup> between a condition and a consequent. These are read from the table by associating the given fact labeling any of the last 4 columns with the statement appearing below in the column.

<sup>1</sup>Associations are written with  $\leadsto$  instead of  $\rightarrow$  to emphasize their defeasible nature.

Table 1

Predictions &amp; explanations from factual information and the type of condition.

Type	Applicable Principle	Abbrev	Given Fact			
			cond	$\overline{\text{cond}}$	consq	$\overline{\text{consq}}$
sufficient	sufficient prediction	(suff_p)	consq	-	-	-
	sufficient explanation	(suff_e)	-	-	cond	-
	sec. sufficient prediction	(sec_suff_p)	-	-	-	$\overline{\text{cond}}$
	sec. sufficient explanation	(sec_suff_e)	-	-	-	$\overline{\text{cond}}$
necessary	necessary prediction	(necc_p)	-	$\overline{\text{consq}}$	-	-
	sec. necessary prediction	(sec_necc_p)	-	-	cond	-
	necessary explanation	(necc_e)	-	-	-	$\overline{\text{cond}}$
	exogenous explanation	(exo)	-	-	exo(consq)	exo( $\overline{\text{consq}}$ )

1. **Predictions:**  $\text{cond} \rightsquigarrow \text{consq}$  (suff\_p) and  $\overline{\text{cond}} \rightsquigarrow \overline{\text{consq}}$  (necc\_p) are the canonical predictive association for a sufficient condition and a necessary condition, respectively.
2. **Explanations:**  $\overline{\text{consq}} \rightsquigarrow \overline{\text{cond}}$  (necc\_e) and  $\text{consq} \rightsquigarrow \text{cond}$  (suff\_e) are the canonical explanatory association for a necessary condition and sufficient condition, respectively. Note that these explanatory associations are the reverse of the predictive ones.
3. **Secondary Associations:**
  - a) **Secondary Predictions:**  $\overline{\text{consq}} \rightsquigarrow \overline{\text{cond}}$ (sec\_suff\_p) and  $\text{consq} \rightsquigarrow \text{cond}$ (sec\_necc\_p) are secondary associations (which correspond to the *contrapositive* of the prediction association) for the sufficient and the necessary condition, respectively.
  - b) **Secondary Explanations:**  $\overline{\text{consq}} \rightsquigarrow \overline{\text{cond}}$ (sec\_suff\_e) is the secondary explanatory association for a sufficient condition.
  - c) **Exogenous Explanations:** These associations link the consequent (positive or negative) with an exogenous explanation (represented by  $\text{exo}(\text{consq})$  and  $\text{exo}(\overline{\text{consq}})$  respectively),  $\text{consq} \rightsquigarrow \text{exo}(\text{consq})$  and  $\overline{\text{consq}} \rightsquigarrow \text{exo}(\overline{\text{consq}})$ (exo\_e). They are supported by psychological experiments (e.g. [17]) which show that humans are sometimes likely to come up with alternative causes that are *not appearing* within the given context to account for an observation.
4. **Strength of Associations:** Predictive associations from necessary conditions (necc\_p) are stronger than conflicting associations from sufficient conditions (suff\_p). This reflects the strength of a pragmatic *disabling condition* over a motivational *enabling condition* for the same consequent.
5. **Incompatibility:** Explanatory associations are typically incompatibly exclusive. For example, if there is more than one explanatory sufficient condition for the consequence then they are incompatible with each other and of equal strength. Table 2 provides a summary of the (in)compatibility of explanations. Note that exogenous explanations are by their nature in conflict with other explanations: people introduce them only when they are in doubt about other explanations.

Table 1 shows canonical predictions and explanations that we can draw for different types of conditions. These canonical predictions and explanations are not meant to necessarily represent definite conclusions but rather that they are plausible conclusions that are cognitively admissible in human reasoning.

### 3. Cognitive Argumentation

The framework of Cognitive Argumentation (CA) is built by synthesizing cognitive principles from Cognitive Science and Philosophy within the framework of computational argumentation in AI, [18, 19, 20]. The task is to understand the cognitive principles into a concrete computational form so that they can be reflected in computational argumentation. We review the basic components of the formal framework of CA. Details are found in [21, 11].



Table 2

Overview of the (in)compatibility of explanations given the observation.

Observation	(In)compatibility of explanations
consq	Explanations from sufficient conditions mutually incompatible
$\overline{\text{consq}}$	Explanations from sufficient conditions mutually compatible
$\overline{\text{consq}}$	Explanations from necessary conditions mutually incompatible
	Also incompatible with explanations from sufficient conditions
consq	Exogenous explanation incompatible with any other explanation
$\overline{\text{consq}}$	Exogenous explanation incompatible with any other explanation

### 3.1. Argumentation Theory

An **argumentation theory or model** within CA consists of a triple  $\mathcal{A}_{\mathcal{L}} = \langle \mathcal{A}s, \mathcal{C}, \succ \rangle$  where  $\mathcal{A}s$  is a set of argument schemes,  $\mathcal{C}$  is a conflict relation in the language of the framework and  $\succ$  is a binary strength relation on  $\mathcal{A}s$ . Argument schemes were introduced as stereotypical reasoning patterns that are typically non-deductive [22, 23]. Formally, an **argument scheme**,  $as \in \mathcal{A}s$ , is a tuple of the form  $as = (\text{pre}, \text{pos})$  where the **premises**  $\text{pre}$  and **position**  $\text{pos}$  are (sets of) statements in the language of discourse  $\mathcal{L}$ . Using an argument scheme  $as = (\text{pre}, \text{pos})$  we can construct an **individual argument** that **supports** the position  $\text{pos}$  based on the premises  $\text{pre}$ . An **argument**  $\Delta$ , is then a set of individual arguments that are grouped together as a coalition to support a position (e.g. a conclusion) we are interested in. The **conflict relation**  $\mathcal{C}$  in  $\mathcal{A}_{\mathcal{L}} = \langle \mathcal{A}s, \mathcal{C}, \succ \rangle$  specifies when arguments conflict with each other and is used to give the notion of attack between two arguments.  $\Delta'$  **attacks** or is a **counterargument** of  $\Delta$ , iff together these arguments have a conflict under  $\mathcal{C}$ , e.g. when  $\Delta$  supports  $Q$  and  $\Delta'$  supports  $\bar{Q}$ , the negation or complement of  $Q$ . The **strength relation**  $\succ$  in  $\mathcal{A}_{\mathcal{L}} = \langle \mathcal{A}s, \mathcal{C}, \succ \rangle$  captures the relative strength among arguments. Given two argument schemes  $as$  and  $as'$ ,  $as \succ as'$  means that arguments constructed from  $as$  are **stronger** than arguments constructed from  $as'$ . This gives a notion of **defense** between conflicting arguments. Informally, argument  $\Delta$  defends against  $\Delta'$  only when its arguments are at least as strong as those of  $\Delta'$ .

Reasoning via argumentation is normally based on a normative criterion of **acceptability** of arguments. We will consider a particular notion of acceptability, called admissibility. Formally, an argument  $\Delta$  is **admissible** in  $\mathcal{A}_{\mathcal{L}}(\mathcal{S})$  iff  $\Delta$  is conflict-free (under  $\mathcal{C}$ ) and  $\Delta$  defends against all its counter-arguments attacking  $\Delta$ . Hence an admissible set of arguments is a coalition in which there exists arguments which are strong enough to counter-attack, i.e. defend against, any argument that attacks it.

We can then define what is a conclusion of argumentative reasoning as follows. We say that a statement  $L$  is an **acceptable** or a **credulous conclusion** of a given argumentation framework  $\mathcal{A}_{\mathcal{L}}$ , iff there exists an admissible argument  $\Delta$  in  $\mathcal{A}_{\mathcal{L}}$  that supports  $L$ .  $L$  is a **skeptical conclusion** of  $\mathcal{A}_{\mathcal{L}}$  iff  $L$  is a credulous conclusion of  $\mathcal{A}_{\mathcal{L}}$ , and  $\bar{L}$  is not a credulous conclusion of  $\mathcal{A}_{\mathcal{L}}(\mathcal{S})$ , i.e. there is no admissible argument supporting  $\bar{L}$ . Credulous and skeptical conclusions represent **plausible** and **definite** conclusions, respectively. Given the semantic definition of a plausible conclusion, the actual reasoning process to find such conclusions follows a natural **dialectic argumentation process**. This consist of the following basic steps (see e.g. [24, 25] for technical details): **(step 1)** Construct a root argument supporting a conclusion of interest, **(step 2)** consider a counterargument against the root argument, **(step 3)** find a defense argument against the counterargument, **(step 4)** check that this defense argument is not in conflict with the root argument, **(step 5)** add this defense argument to the root argument, and repeat from (step 2), i.e. consider another counterargument to the now extended root argument.

### 3.2. Conditional Reasoning in Cognitive Argumentation

In order to apply the argumentative reasoning to model human reasoning we also need a second orthogonal condition of the arguments involved, namely that these are **grounded** on the perceived (current) state of the environment. Human reasoning is carried out under a current state of information that the environment gives or makes us aware of. This information can be captured by a **cognitive state**  $\mathcal{S} = (\mathcal{F}, \mathcal{A})$  where  $\mathcal{F}$  is a *set of facts* provided by the environment and  $\mathcal{A}$  is an *awareness set* of propositions which the

environment has awoken as relevant in the current reasoning. The first element consists of explicit factual information that the environment provides to the reasoner while the second consists of the propositions that the reasoner is made aware of by the current environment. We can assume,  $\mathcal{F} \subseteq \mathcal{A}$ . An admissible argument  $\Delta$  is then also required to be **grounded** on the current cognitive state, i.e. that all arguments in  $\Delta$  need to eventually be based on information that refers to the cognitive state. Reasoning under argumentation is then carried out by considering grounded and admissible arguments that support a concluding statement.

To ground the reasoning we can introduce two new argument schemes in  $\mathcal{A}s$ . The **fact scheme**:  $\text{fact}(L) = (\emptyset, L) \in \mathcal{A}s$ , applied for any statement  $L \in \mathcal{F}$  of the current cognitive state  $\mathcal{S} = (\mathcal{F}, \mathcal{A})$ . Similarly, we have a **hypothesis scheme**:  $\text{hyp}(A) = (\emptyset, A) \in \mathcal{A}s$  and  $\text{hyp}(\bar{A}) = (\emptyset, \bar{A}) \in \mathcal{A}s$ , for any proposition,  $A$ , in  $\mathcal{S}$ . The conflict relation  $\mathcal{C}$  is then also extended by: (1) hypotheses schemes are weaker than any other opposing scheme and (2) fact schemes are stronger than any other opposing scheme.

The dialectic argumentation process for constructing admissible arguments can be given a tree structure and illustrated as such by figures of growing trees of attacking and defending arguments. For the above example this is shown on the left of Figure 1. In these dialectic reasoning figures, (temporarily) admissible arguments are highlighted in gray and non-admissible arguments are in white.  $\uparrow$  shows attacks between arguments, i.e. arguments that are in conflict.  $\Uparrow$  shows *strong* defenses, i.e. attacks that cannot be defended against by the argument they are defending against. In many cases these strong defenses determine the (final) acceptability of arguments.

Let us now analyse an example of the dialectic argumentative reasoning process for conditional reasoning as illustrate in Figure 1. Assume that in the milk example of the previous section the cognitive cognitive state is  $\mathcal{S}' = (\{\text{need}\}, \{\text{need}, \text{asks}, \text{buy}, \text{money}\})$ . The position of interest is *buy*. In **Step 1** we construct a root argument,

$$\Delta_{\text{need} \leadsto \text{buy}}^{\text{need}} = \{\text{fact}(\text{need}), \text{suff\_p}(\text{need} \leadsto \text{buy})\}, \text{ supporting } \text{buy}$$

In **Step 2**, we check whether there are counterarguments against this argument. We can construct the following counterargument:

$$\Delta_{\text{money} \leadsto \text{buy}}^{\text{money}} = \{\text{hyp}(\overline{\text{money}}), \text{necc\_p}(\overline{\text{money}} \leadsto \overline{\text{buy}})\},$$

which is grounded in  $\mathcal{S}'$ , because *money*  $\in \mathcal{A}'$ . Can we find (**Step 3**) a defense against  $\Delta_{\text{money} \leadsto \text{buy}}^{\text{money}}$ ?

Note that  $\Delta_{\text{need} \leadsto \text{buy}}^{\text{need}}$  cannot itself defend against  $\Delta_{\text{money} \leadsto \text{buy}}^{\text{money}}$ , because  $\text{necc\_p}(\overline{\text{money}} \leadsto \overline{\text{buy}})$  is stronger than  $\text{suff\_p}(\text{need} \leadsto \text{buy})$ . Nevertheless, a defense is given by the hypothesis argument  $\text{hyp}(\text{money}) = (\emptyset, \text{money})$ , which we can add (**Step 4** and **Step 5**) to  $\Delta_{\text{need} \leadsto \text{buy}}^{\text{need}}$ .

This new extended argument is denoted as  $\Delta_{\text{need} \leadsto \text{buy}, m}^{\text{need}}$ . Returning to **Step 2** we look for other counterarguments. Such a counterargument is given by the hypothesis argument,  $\text{hyp}(\overline{\text{buy}}) = (\emptyset, \overline{\text{buy}})$ , which is trivially defended against with the root argument,  $\Delta_{\text{need} \leadsto \text{buy}}^{\text{need}}$ . In other words, there is no need to find a different defense argument and extend further the current root argument in **Step 1**.

On the right of Figure 1 the same process for the non-admissibility of  $\Delta_{\text{need} \leadsto \text{buy}}^{\text{need}}$  is shown when the cognitive state contains as a fact  $\overline{\text{money}}$ . It shows that there is no defense against the strong attack of  $\Delta_{\text{money} \leadsto \text{buy}}^{\overline{\text{money}}} = \{\text{fact}(\overline{\text{money}}), \text{necc\_p}(\overline{\text{money}} \leadsto \overline{\text{buy}})\}$ .

#### 4. Deduction and Abduction in the Suppression Task

We will now apply the argumentation framework, developed in the previous section, to the well-known study of human conditional reasoning called the suppression task [1]. We will see how within the same argumentation framework containing predictive and explanatory argument schemes we can model well the whole empirical data, whether these are cases of deductive or abductive reasoning.

The experimental setting of the suppression task was as follows. Three groups of participants were asked to derive conclusions given variations of a set of conditionals (and factual information). Group I

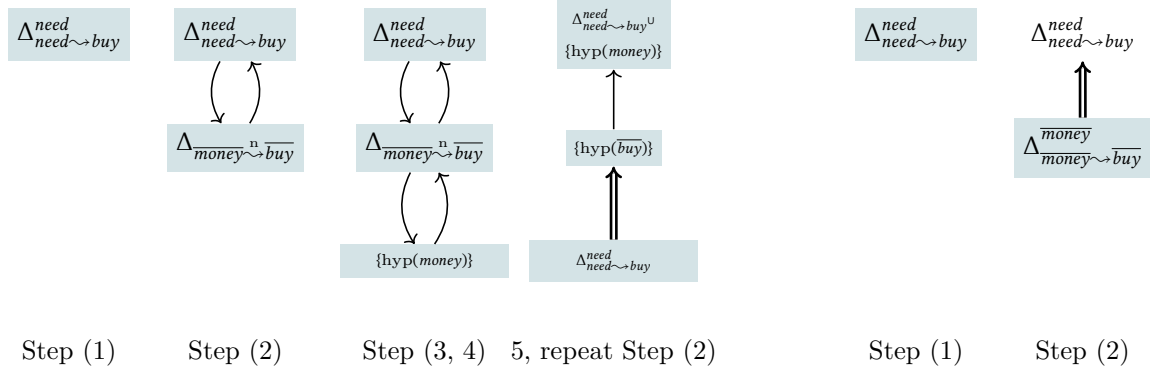


Figure 1: Dialectic argumentation process showing that *buy* is an acceptable conclusion given  $\mathcal{S}' = (\{need\}, \{need, asks, buy, money\})$  (left) and showing that *buy* is not an acceptable conclusion given  $\mathcal{S}'' = (\{need, \overline{money}\}, \{need, asks, buy, money\})$  (right).

Table 3

Summary of the twelve cases and the corresponding suppression effects denoted in gray.

Group	Conditional(s)	Essay ( $e$ )	No essay ( $\bar{e}$ )	Library ( $\ell$ )	No library ( $\bar{\ell}$ )
I	$e \rightsquigarrow \ell$	library (96%)	no library (46%)	essay (71%)	no essay (92%)
II	$e \rightsquigarrow \ell, t \rightsquigarrow \ell$	library (96%)	no library (4%)	essay (13%)	no essay (96%)
III	$e \rightsquigarrow \ell, o \rightsquigarrow \ell$	library (38%)	no library (63%)	essay (54%)	no essay (33%)

was given the following conditional knowledge:<sup>2</sup>

*If she has an essay to finish, then she will study late in the library.* ( $e \rightsquigarrow \ell$ )

In addition to the above conditional for Group I, Group II was given the following conditional:

*If she has a textbook to read, then she will study late in the library.* ( $t \rightsquigarrow \ell$ )

Group III received, together with the conditional of Group I, additionally the following conditional:

*If the library stays open, then she will study late in the library.* ( $o \rightsquigarrow \ell$ )

For each group, four different **cases of reasoning** were considered by combining their conditional knowledge with one of the following factual information: *She has an essay to finish* ( $e$ ), *She will study late in the library* ( $\ell$ ), *She does not have an essay to finish* ( $\bar{e}$ ) or *She will not study late in the library*  $\bar{\ell}$ .

The participants were asked what **necessarily** had to follow in each case based on their conditional knowledge and the fact of the case. For each case of a given factual information they were asked if some other statement followed. For example, in the first case where the factual information is “She has an essay to finish” they were asked whether “She will study late in the library”. They would answer by choosing one of following: *She will study late in the library*, *She will not study late in the library* or *She may or may not study late in the library*.

The study in [1] then reported the experimental results for twelve cases as summarized in Table 3. This table shows for each group (column 1), the conditional information they received (column 2) together with the factual information for each of the four cases (column 3 to 6). In each row we can see the percentage of responses by the participants in the group corresponding to the row of the table. Those in gray are the responses demonstrating the suppression effect, namely when in Groups II or III we observe a significant reduction of the percentage of the participants choosing the same majority answer as in Group I. For example, the majority’s responses in Group II diverges in two cases from the majority’s

<sup>2</sup>The participants received the natural language sentences but not the abbreviated notation on the right hand side.



Table 4  
Argument Schemes in each group of the Suppression Task.

Argument Scheme/Name	Group I	Group II	Group III
sufficient prediction/ suff_p	$e \rightsquigarrow \ell$	$e \rightsquigarrow \ell, t \rightsquigarrow \ell$	$e \rightsquigarrow \ell$
necessary prediction/ necc_p	$(\bar{e} \rightsquigarrow \bar{\ell})$		$\bar{o} \rightsquigarrow \bar{\ell}, (\bar{e} \rightsquigarrow \bar{\ell})$
secondary necessary prediction/ sec_necc_p ( $\ell \rightsquigarrow e$ )			$(\ell \rightsquigarrow e), \ell \rightsquigarrow o$
secondary sufficient prediction/ sec_suff_p	$\bar{\ell} \rightsquigarrow \bar{e}$	$\bar{\ell} \rightsquigarrow \bar{e}, \bar{\ell} \rightsquigarrow \bar{t}$	$\bar{\ell} \rightsquigarrow \bar{e}$
sufficient explanation/ suff_e	$\ell \rightsquigarrow e$	$\ell \rightsquigarrow e, \ell \rightsquigarrow t$	$\ell \rightsquigarrow e$
necessary explanation/ necc_e	$(\bar{\ell} \rightsquigarrow \bar{e})$		$(\bar{\ell} \rightsquigarrow \bar{e}), \bar{\ell} \rightsquigarrow \bar{o}$
secondary sufficient explanation/ sec_suff_e	$\bar{\ell} \rightsquigarrow \bar{e}$	$\bar{\ell} \rightsquigarrow \bar{e}, \bar{\ell} \rightsquigarrow \bar{t}$	$\bar{\ell} \rightsquigarrow \bar{e}$
exogenous explanation/ exo_e	$\ell \rightsquigarrow \text{exo}(\ell), \bar{\ell} \rightsquigarrow \text{exo}(\bar{\ell})$	$\ell \rightsquigarrow \text{exo}(\ell), \bar{\ell} \rightsquigarrow \text{exo}(\bar{\ell})$	$\ell \rightsquigarrow \text{exo}(\ell), \bar{\ell} \rightsquigarrow \text{exo}(\bar{\ell})$

Table 5  
(In)compatibility of explanatory argument schemes depending in the Suppression Task.

Observation	(In)compatibility of explanations
$\ell$	suff_e( $\ell \rightsquigarrow e$ ) and suff_e( $\ell \rightsquigarrow t$ ) are incompatible
$\bar{\ell}$	sec_suff_e( $\bar{\ell} \rightsquigarrow \bar{e}$ ) and sec_suff_e( $\bar{\ell} \rightsquigarrow \bar{t}$ ) are compatible, but incompatible with necc_e( $\bar{\ell} \rightsquigarrow \bar{e}$ ) and necc_e( $\bar{\ell} \rightsquigarrow \bar{o}$ )
$\bar{\ell}$	necc_e( $\bar{\ell} \rightsquigarrow \bar{e}$ ) and necc_e( $\bar{\ell} \rightsquigarrow \bar{o}$ ) are incompatible, also incompatible with sec_suff_e( $\bar{\ell} \rightsquigarrow \bar{e}$ ) and sec_suff_e( $\bar{\ell} \rightsquigarrow \bar{t}$ )
$\ell$	exo( $\ell$ ) is incompatible with any other explanation for $\ell$
$\bar{\ell}$	exo( $\bar{\ell}$ ) is incompatible with any other explanation for $\bar{\ell}$

responses in Group I: When participants received the information, that *She does not have an essay to finish* ( $\bar{e}$ ), only 4% concluded that *She will not study late in the library* ( $\bar{\ell}$ ), and when they received the information that *She will study late in the library*, only 13% concluded that *She has an essay to finish*.

From this table we can also see the connection of the study to deduction and abduction. We can see that the first and third cases, whose factual information is  $e$  or  $\bar{e}$ , are cases of deductive reasoning where participants would reason to see if they can derive that “Library” holds on not. On the other hand, the second and fourth cases, whose factual information is  $\ell$  or  $\bar{\ell}$ , are cases of abductive reasoning where participants would try to explain the given factual information or more specifically to examine if “having an essay to finish” or not, forms an explanation for the given factual information.

The task is then to apply the framework of Cognitive Argumentation for human reasoning to show how this can uniformly capture the experimental results in all twelve cases, accounting for the suppression effect as well as the variation of responses within each group. By doing so we would have captured both forms of deductive and abductive human reasoning via argumentation.

#### 4.1. Cognitive Adequacy of CA in the Suppression Task

To model the human reasoning in the suppression task within the CA framework we will assume that each participant reasons with an argumentation theory that contains a subset of conditional argument schemes presented in Sections 2 and 3. This subset of “conditional argument schemes” differs between the different groups reflecting the different conditionals knowledge given to each group.

Table 4 shows the subset of these arguments for each of the three groups. The table also shows how this subset of argument schemes may differ amongst humans depending on their understanding of a condition as sufficient, necessary or both. For Group I and III the condition *She has an essay to finish* can be interpreted both as sufficient and necessary. For some part of the population this may only be a sufficient condition in which case the associations shown in parentheses in the columns for Groups I and

III will not apply. In Group II, *She has an essay to finish* is no longer considered as necessary due to the presence of a second sufficient condition of *She has a textbook to read*. They are both considered in the whole population of Group II only as sufficient conditions. Table 5 shows, following Table 2 on the incompatibility of explanations, the conflict relation between the various explanatory argument schemes in the Suppression Task argumentation theory. Note, for example, in the first row that the two explanatory schemes based on the observation of  $l$  are incompatible despite the fact that a student would typically have both the tasks of writing an essay or reading a text book. The incompatibility of the explanatory schemes though does not concern what would hold in general, but rather it says that for any particular case of going to the library there is typically only one reason for this.

In the suppression task experiments, participants are asked to select between three alternatives about a (natural language) statement  $L$ : (i)  $L$  holds, (ii)  $L$  does not hold and (iii)  $L$  may or  $L$  may not hold. We can see that the first two options refer to a definite conclusion for or against  $L$ , while the third option refers to a plausible conclusion about  $L$  or its complement. It is thus important to note that the experimental conditions encourage the participants to think and reason both about definite and plausible conclusions. Given that Cognitive Argumentation contains both forms of definite and plausible conclusions, this allows us to set up a criterion of evaluation of the **cognitive adequacy**. This criterion will be based on comparing the percentage of the participants' responses for a position (e.g. for *She will study late in the library* ( $\ell$ )) with the existence of admissible arguments for the position and/or its complement. In particular, we will examine if in each case of the experiment: (i) there is an admissible argument for the position asked but not for its complement (i.e. we have a skeptical definite conclusion), or whether (ii) there is an admissible argument for that position and for its complement (i.e. we have a credulous plausible conclusion).

This distinction will then be required to qualitatively correspond to variations in the observed percentage of answers within the population across the three groups but also the variation within each group as follows: (i) if the population agrees on a position overwhelmingly, then the position asked should follow in a skeptical, definite way. On the other hand, (ii) if there is no overwhelming majority then there should exist admissible arguments for both the position and its negation, i.e. they both should follow credulously as plausible conclusions.

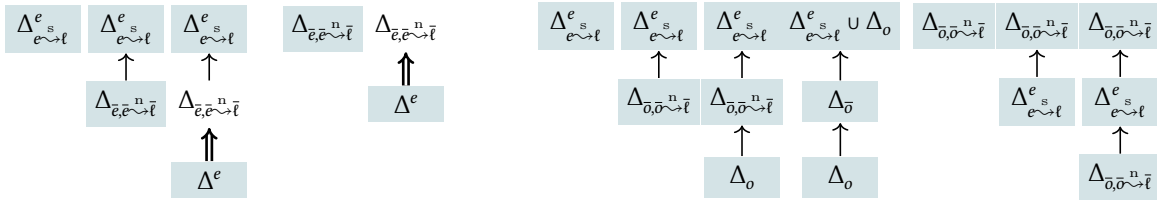
## 4.2. Deductive and Abductive Reasoning in the Suppression Task

We will now examine in each of the four cases of the suppression task experiment how we can capture, within the framework of cognitive argumentation, the reported experimental results. The cognitive state of the participants in different groups has the same factual information in anyone of the four cases, but differs in the awareness part. For example, in the first case, participants in Group I, are assumed to be only aware of  $e$  and  $\ell$ . Thus, their cognitive state is  $\mathcal{S}_1 = (\{e\}, \{e, \ell\})$ . For Group II the cognitive state is  $\mathcal{S}_2 = (\{e\}, \{e, t, \ell\})$  whereas for Group III it is  $\mathcal{S}_3 = (\{e\}, \{e, \ell, o\})$ . Note that the factual information is the same for each group across all cases.

In each of the four cases we will introduce the main arguments that can be constructed for and against the property that is asked and show which ones are admissible by analyzing the relevant dialectic argumentation processes. These will be illustrated by figures that show how the various arguments attack and defend each other, as introduced in Section 3.

### 4.2.1. She has an essay to finish

In the first case all three groups were given the factual information, *She has an essay to finish* ( $e$ ), and were asked whether *She will study late in the library* ( $\ell$ ). Figure 2 gives step-by-step the dialectic construction of the main (i.e. stronger and more cognitively plausible) arguments for  $\ell$  and  $\bar{\ell}$  in Group I (left, middle left) and Group III (middle right, right). The (strongest) argument supporting  $\ell$  is given by combining the *fact scheme* for  $e$  together with the *sufficient prediction scheme* for  $\ell$  (Figure 2, left):  $\Delta_{e \rightsquigarrow \ell}^e = \{\text{fact}(e), \text{suff\_p}(e \rightsquigarrow \ell)\}$ . It is easy to recognize this as a *modus ponens* reasoning form expressed here in an argumentation perspective. For supporting  $\bar{\ell}$  the main argument is constructed by applying the *necessary prediction scheme* for  $\bar{\ell}$  with the hypothesis scheme for  $\bar{e}$  (Figure 2, middle left):



For Group III, differently from Group I and Group II, we can now construct another strong argument for  $\bar{\ell}$  based on the hypothesis prediction scheme for  $\bar{o}$  together with the *necessary prediction scheme* for  $\bar{\ell}$  (Figure 2, right):  $\Delta_{\bar{o}, \bar{o} \rightsquigarrow \bar{\ell}}^n = \{\text{hyp}(\bar{o}), \text{necc\_p}(\bar{o} \rightsquigarrow \bar{\ell})\}.$

Summing up, we see that in Group III both  $\ell$  and  $\bar{\ell}$ , are plausible (credulous) conclusions. This then accounts for the observed suppression effect, as here only 38% concluded that *She will study late in the library*. It is likely, that these participants constructed only the argument  $\Delta_{e \rightsquigarrow \ell}^e$  for  $\ell$  and hence concluded definitely  $\ell$ . It seems likely that the other 62% of the participants were able to construct both  $\Delta_{e \rightsquigarrow \ell}^e$  and  $\Delta_{\bar{e}, \bar{e} \rightsquigarrow \bar{\ell}}^n$  and hence were not (skeptically) sure that  $\ell$  held.

We will only briefly describe this second case referring the reader to [11] for details. The given fact is *She does not have an essay to finish* ( $\bar{e}$ ). Participants were asked whether *She will study late in the library* ( $\ell$ ). For groups I and III about half of the participants give the definite answer of “No”. To account for this we can separate the participants in two subgroups, those who understand  $e$  as sufficient and necessary for  $\ell$  and those who under it only as a sufficient condition. For the first subgroup we have a strong argument supporting  $\bar{\ell}$  given by  $\Delta_{\bar{e} \rightsquigarrow \bar{\ell}}^{\bar{e}} = \{\text{fact}(\bar{e}), \text{necc\_p}(\bar{e} \rightsquigarrow \bar{\ell})\}$ . This argument attacks any possible argument for  $\ell$  and cannot be defended against in any way. Hence  $\bar{\ell}$  is a skeptical definite conclusion that would lead to the definite answer of “No”. For the other subgroup we cannot construct this strong argument for  $\bar{\ell}$  as we do not have the  $\text{necc\_p}(\bar{e} \rightsquigarrow \bar{\ell})$  scheme. This makes it possible to construct admissible arguments for either  $\ell$  or  $\bar{\ell}$  and hence  $\bar{\ell}$  is not a skeptical definite conclusion for these participants, thus not choosing

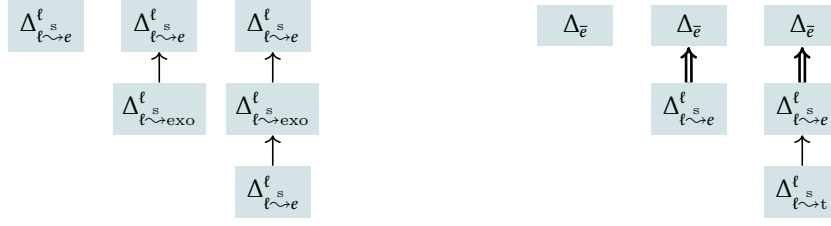


Figure 3: Construction for  $e$  for all groups (left) and for  $\bar{e}$  for Group II (right).

the definite answer of “No”.

Lets us now consider Group II where a significant suppression effect is observed. In this group, participants are additionally made aware of *She might (not) have a textbook to read*, where *She has a textbook to read* ( $t$ ) is a sufficient condition for  $\ell$ . This also means that  $e$  cannot be understood as a necessary condition for  $\ell$  anymore. As a result, with the absence of  $\text{necc\_p}(\bar{e} \leadsto \bar{\ell})$  we cannot construct a strong argument for  $\bar{\ell}$ . Consequently, the majority is more likely to construct, in the way we saw above for Groups I and III, admissible arguments for either conclusion,  $\ell$  and  $\bar{\ell}$ , and thus both follow credulously with no definite answer to choose.

#### 4.4. She will study late in the library

In this third case, all groups were asked whether *She has an essay to finish* ( $e$ ), given the factual information that *She will study late in the library* ( $\ell$ ). The reasoning therefore of the participants is closer to explanatory rather than predictive. We will see that we can model this reasoning with the same process of argumentative reasoning where now the arguments that we consider come from explanatory argument schemes for  $e$  and  $\bar{e}$  rather than predictive schemes, as in the previous cases, for  $\ell$  or  $\bar{\ell}$ .

We will assume that a significant amount of participants entered into an *explanatory mode* (or diagnostic mode). In this mode these participants tried to explain the factual observation in the context of the information that they are given in each group. Indeed, the form of the conditional information used encourages the process of explanation when participants are given information about the consequent of the conditionals. Nevertheless, it is important to note that reasoning to  $e$  (or  $\bar{e}$ ) can also be carried out assuming that the participants interpret the condition  $e$  as necessary (in addition to sufficient) without any reference to explanatory reasoning. The exposition of this is beyond the scope of this paper<sup>3</sup> where the aim is to show the sufficiency of our approach not its necessity.

Figure 3 (left) shows the reasoning in any groups that renders admissible the following explanatory argument supporting  $e$ :  $\Delta_{\ell \leadsto e}^{\ell} = \{\text{fact}(\ell), \text{suff\_e}(\ell \leadsto e)\}$ . This argument can only be attacked by  $\Delta_{\ell \leadsto \text{exo}}^{\ell} = \{\text{fact}(\ell), \text{exo\_e}(\ell \leadsto \text{exo}(I))\}$ , an argument for an alternative (unknown) explanation constructed via the explanation scheme,  $\text{exo\_e}(I) = (\ell, \text{exo}(I))$ .  $\Delta_{\ell \leadsto e}^{\ell}$  though is strong enough to defend against this attack and thus  $\Delta_{\ell \leadsto e}^{\ell}$  is admissible. Furthermore, strong admissible arguments for  $\bar{e}$  can only be defended against the attack by  $\Delta_{\ell \leadsto e}^{\ell}$  if they contain the alternative explanation argument  $\text{exo\_e}(\ell \leadsto \text{exo}(I))$ . Thus for the majority of participants who did not consider this possibility of some other unknown reason for going to the library they would arrive at the definite answer of “yes” for  $e$ . This conforms well with the empirical data for Groups I and II.

Lets us now consider Group II, where a suppression effect is observed. In this group for most participants the possibility of an alternative explanation, such as  $\text{exo}(I)$ , for the observed fact is made explicit by the explanatory scheme,  $\text{suff\_e}(\ell \leadsto t) = (\ell, t)$ , which they have in their knowledge. Hence we can construct a new argument supporting  $t$ :  $\Delta_{\ell \leadsto t}^{\ell} = \{\text{fact}(\ell), \text{suff\_e}(\ell \leadsto t)\}$ , which conflicts with the argument  $\Delta_{\ell \leadsto e}^{\ell}$  above for  $e$ . They attack each other and are strong enough to defend against each other. Hence as shown in Figure 3 (right),  $\Delta_{\ell \leadsto t}^{\ell}$  can help as a defense to construct an admissible

<sup>3</sup>The interested reader can find the full details of this and its comparison with the explanatory reasoning in [11].

argument supporting  $\bar{e}$ . We therefore now have admissible arguments for both  $e$  and  $\bar{e}$ . Accordingly,  $e$  and  $\bar{e}$  are credulous conclusions for most participants, which reflects well the suppression effect in the second group, as there was no majority (only 13%), that concluded that *She has an essay to finish*.

#### 4.5. She will not study late in the library

As in the second case, we will only present a high-level description of the reasoning for the last case, referring the reader to [11] for details. In this case, all groups were asked whether *She has an essay to finish* ( $e$ ), given the fact that *She will not study late in the library* ( $\bar{\ell}$ ). We will assume again that it is natural for some participants to reason in explanatory mode as the factual information given to them concerns the consequent of the conditional(s) in the general information and the context of reasoning.

In an explanatory mode of reasoning in any one of the three groups we can use, together with the given factual information of  $\bar{\ell}$ , either the explanatory argument scheme,  $\text{necc\_e}(\bar{\ell} \leadsto \bar{e})$ , or  $\text{sec\_suff\_e}(\bar{\ell} \leadsto \bar{e})$ , to construct arguments that support  $\bar{e}$ . Which one we use will depend on whether  $e$  is also understood as a necessary condition for  $\ell$  or not. Both of these are strong arguments and hence admissible. On the other hand, arguments supporting  $e$  can only be defended if the reasoner has another explanation for  $\bar{\ell}$ . This could be an explanation from an unknown reason using the exogenous explanatory argument scheme.

Differently from groups I and II, participants in Group III have a concrete alternative explanation for the given observation, namely that the reason for *She will not study late in the library* is that *library might not be open*. This allows us to construct the argument  $\Delta_{\bar{\ell} \leadsto \bar{o}}^{\bar{\ell}} = \{\text{fact}(\bar{\ell}), \text{necc\_e}(\bar{\ell} \leadsto \bar{o})\}$ .

As  $\text{necc\_e}(\bar{\ell} \leadsto \bar{o})$  is incompatible with the explanatory schemes supporting  $\bar{e}$ , this new argument is able to defend against the above arguments for  $\bar{e}$ . Hence we can construct a new and admissible argument supporting  $e$  with the help of this new argument,  $\Delta_{\bar{\ell} \leadsto \bar{o}}^{\bar{\ell}}$ , as a defending ally against the attacks from the arguments supporting  $\bar{e}$ . Hence we have admissible arguments for both  $\bar{e}$  and  $e$  and so  $\bar{e}$  is a non-definite conclusion. In other words, the suppression effect can be accounted for simply by assuming that a higher proportion of the participants (in comparison with Groups I and II) thought of an alternative explanation, now that they are made explicitly aware of the possible explanation of the library not being open.

## 5. Conclusions

We have seen how human conditional reasoning can be formulated within a framework of dialectic argumentation, called Cognitive Argumentation, where reasoning to conclusions is understood as a process of contemplating between alternatives and the arguments that support them. This framework of Cognitive Argumentation offers a way to unify deductive and abductive reasoning. We have validated this result on Byrne's suppression task. Observed suppression coincides with the loss, in the suppression group, of skeptical argumentative conclusions drawn in the other two groups. In the suppression group admissible arguments exist that support both the conclusion and its complement. It is therefore more likely for participants in the suppression group to consider the conclusion only plausible and hence avoid choosing the conclusion in their answer.

These results stem from two important properties of Cognitive Argumentation: (i) its natural distinction between definite and plausible conclusions via the formal notions of skeptical and credulous conclusions, (ii) its property to adapt to new and different forms of information resulting in a context-sensitive form of reasoning. The results of this paper come to add to earlier studies of Cognitive Argumentation [21, 26] and lend further support to the cognitive adequacy of Cognitive Argumentation.

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