Interpretation-based Provenance

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Abstract

Provenance refers to the task of identifying and tracing the axiomatic origins of a consequence from a knowledge base. Most current approaches to provenance are limited to inexpressive logics mainly due to the difficulty in dealing with complex constructors like negations. We surpass this issue through a general notion of provenance for interpretation-based semantics.

Keywords

semiring provenance, expressive logics, linear temporal logic, non-standard reasoning

1. Introduction

Over the last years, the question of tracing the provenance of a consequence has been addressed for a wide variety of knowledge representation languages. In essence, *provenance* refers to the task of tracing the "origin" of a given consequence, among the pieces of knowledge available and, perhaps, the rules available in the language for manipulating this knowledge. *Semiring provenance* uses an algebraic structure with two operators—called a semiring—to represent the full provenance information of a consequence. In a nutshell, the "product" expresses which combinations of axioms produce the consequence, while the "addition" represents different possible derivations of the same result: if a consequence can be derived from the facts f_1 and f_2 or from the facts f_3, f_4, and f_5, its provenance will be expressed as $(f_1 \otimes f_2) \oplus (f_3 \otimes f_4 \otimes f_5)$. Originating from the database community [1, 2], the same problem has been recently studied in many other areas; e.g., [3, 4, 5, 6, 7].

Existing definitions of semiring provenance all rely (sometimes implicitly) on a notion of "proof." The idea is that the provenance information does not only indicate *which* pieces of knowledge are responsible for a consequence, but also *how* they interact through the derivation process. But for this, the derivation procedure needs to be explicitised. However, such a view becomes difficult to apply in knowledge representation languages for which a natural definition of a proof is unavailable, or difficult to handle within the semiring operations. One example of such a language is the linear temporal logic LTL_f [8], which underlies Declare [9], a declarative language for specifying process models. For this discussion paper, we assume that the reader is familiar with the basic notions of LTL_f and Declare.

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When a process or even an execution of this process disagrees with a Declare specification expressed as a set of temporal constraints, one should understand which constraints are violated; that is, the provenance of this disagreement. But, as mentioned already, LTL$_f$ (and hence Declare) has no notion of proof; this makes it difficult to provide a definition of provenance. To surpass this issue, a new definition of provenance, which is based on interpretations instead of proofs, was proposed in [10]. In a nutshell, the provenance of a consequence introduced in [10] describes the combinations of axioms *excluding* all possible *countermodels*, which explains (indirectly) why the consequence holds. This yields a sound notion of provenance suitable for LTL$_f$, and any KR language with interpretation-based semantics, which coincides with "classical" provenance where the comparison is possible.

The scope of this paper is to briefly recall and discuss this new approach to provenance, with a particular focus on its applicability to LTL$_f$ and process modelling in general.

2. Semirings

A *semiring* is an algebraic structure $\mathbb{S} = (S, \oplus, \otimes, \mathbf{0}, \mathbf{1})$ where $\oplus$ and $\otimes$ are two associative binary operators over S, called the *addition* and *product*, respectively, such that $\oplus$ is commutative and has neutral element $\mathbf{0}$; $\mathbf{1}$ is the neutral element of $\otimes$; and $\otimes$ distributes over $\oplus$ on both sides. As usual, $\otimes$ has precedence over $\oplus$. A semiring is *commutative* if $\otimes$ is commutative, $\oplus$-idempotent if $s \oplus s = s$ for all $s \in S$, and $\otimes$-idempotent if $s \otimes s = s$ for all $s \in S$. $\mathbb{S}$ is *idempotent* if it is both, $\oplus$- and $\otimes$-idempotent.

In the context of provenance, one often considers commutative idempotent semirings. A typical example of such a semiring is the bounded distributive lattice $\mathbb{L} = (L, \vee, \wedge, \mathbf{0}, \mathbf{1})$, where $\vee$ and $\wedge$ are the *join* and *meet* operators of L, and $\mathbf{0}$ and $\mathbf{1}$ are its least and greatest elements, respectively. $\mathbb{S}$ is *distributive* if both operations distribute over each other. Distributive lattices have the additional property of being *annihilating*: for any two elements $s, t \in S$, it holds that $s \otimes (s \oplus t) = s = s \oplus (s \otimes t)$. Importantly, other distributive idempotent semirings (which are not lattices) exist, and other types of semiring may be of interest in different contexts [11].

From now on, unless otherwise specified, $\mathbb{S}$ refers to an arbitrary–but fixed–distributive commutative idempotent (dci) semiring over the set S, $\mathbb{S} = (S, \oplus, \otimes, \mathbf{0}, \mathbf{1})$, and s to an element of S.

3. Interpretation-based Provenance

The basic idea behind provenance is to keep track of the different combinations of axioms in a knowledge base which yield a consequence of interest. In logic-based KR languages, this is usually instantiated to mirror how (derived) consequences are combined to yield new (previously implicit) consequences. One important disadvantage of this approach is that provenance semantics must be defined anew for each logical formalism, and it is not obvious how to define the provenance in settings where consequences are not cumulative in this sense. As mentioned already, LTL$_f$ is one such language without natural consequence-based reasoning mechanisms. We recall the general notion of provenance based on interpretation semantics proposed in [10].

Definition 1 (KR language). *A* KR language *is a triple* $(\mathfrak{T}, \mathfrak{I}, \models)$ *where* $\mathfrak{T}$ *is a potentially infinite set of* axioms*,* $\mathfrak{I}$ *is a class of* interpretations*, and* $\models \subseteq \mathfrak{I} \times \mathfrak{T}$ *is the binary* satisfiability relation*, usually expressed in infix notation. A* knowledge base *(KB) is a finite set of axioms* $\mathcal{T} \subseteq \mathfrak{T}$*. The interpretation* $I \in \mathfrak{I}$ *is a* model *of the KB* $\mathcal{T}$ *iff* $I \models \alpha$ *for all* $\alpha \in \mathcal{T}$*. An axiom* $\alpha \in \mathfrak{T}$ *is a* consequence *of a KB* $\mathcal{T}$ *(denoted* $\mathcal{T} \models \alpha$*) iff* $I \models \alpha$ *holds for every model* I *of* $\mathcal{T}$*.*

This definition guarantees that the consequence relation between KBs and axioms is monotonic; i.e., adding more axioms to a KB can never remove a consequence. An example KR language is the triple $(\Phi, \mathfrak{I}, \models)$ where Φ is the set of all LTL$_f$ formulas, $\mathfrak{I}$ is the class of all temporal interpretations (or finite traces) , and $\models$ is the usual satisfiability relation from LTL$_f$ (see [8]). Unsatisfiability of a KB in this language corresponds to checking whether the axiom $\bot := x \wedge \neg x$ is a consequence.

We now fix an arbitrary KR language $(\mathfrak{T}, \mathfrak{I}, \models)$, and assume that every axiom is annotated with an element of the semiring $\mathbb{S}$. The provenance of a consequence traces the combinations of axioms that yield it, as formalised next.

Definition 2 (provenance). *Let* $\mathbb{S}$ *be a semiring and* $(\mathfrak{T}, \mathfrak{I}, \models)$ *a KR language. An* annotated KB *is a pair* $(\mathcal{T}, \mathsf{lab})$ *where* $\mathcal{T} \subseteq \mathfrak{T}$ *and* $\mathsf{lab} : \mathcal{T} \to S$*. The* provenance *of* $\alpha \in \mathfrak{T}$ *w.r.t. the annotated KB* $(\mathcal{T}, \mathsf{lab})$ *is* $\mathsf{Prov}_{\mathcal{T}}(\alpha) := \bigotimes_{I \in \mathfrak{I}, I \not\models \alpha} \bigoplus_{\beta \in \mathcal{T}, I \not\models \beta} \mathsf{lab}(\beta)$*.*

Traditionally, provenance is defined as an addition of products of labels referring to combinations of axioms that *yield* a consequence. Definition 2 follows a dual approach. For α to be a consequence, any interpretation I that *does not* satisfy α must be excluded from the set of models; this is expressed by the outer product. The addition $\bigoplus_{\beta \in \mathcal{T}, I \not\models \beta} \mathsf{lab}(\beta)$ intuitively expresses that at least one axiom violated by I is needed in $\mathcal{T}$ to exclude I from the class of models.

Example 3. *Let* $\mathcal{T} := \{y, \bigcirc y, \Box\neg y, \neg y\}$ *be an annotated LTL$_f$ KB with the axioms labelled as* $s_1, \ldots, s_4$*, respectively. It is easy to verify that* $\mathcal{T}$ *is unsatisfiable. To compute the provenance for this unsatisfiability, we multiply over all temporal interpretations (since none satisfies* $\bot$*) the addition of the labels of the axioms they violate.*

If the semiring is idempotent, it then suffices to partition the class of all interpretations $\mathfrak{I}$ *by the axioms they violate. Representatives of the equivalence classes defined by this partition are:*

- y, y *which yields* $s_3 \oplus s_4$*;*
- $y, \neg y$ *which yields* $s_2 \oplus s_3 \oplus s_4$*;*
- $\neg y, y$ *which yields* $s_1 \oplus s_3$*;*
- $\neg y, \neg y$ *which yields* $s_1 \oplus s_2$*; and*
- $\neg y, \neg y, y$ *which yields* $s_1 \oplus s_2 \oplus s_3$*.*

$\mathsf{Prov}_{\mathcal{T}}(\bot)$ *is the product of those additions. If* $\mathbb{S}$ *is annihilating, this is equivalent to the expression* $(s_1 \oplus s_2) \otimes (s_1 \oplus s_3) \otimes (s_3 \oplus s_4)$ *which, under distributivity, becomes* $(s_1 \otimes s_3) \oplus (s_1 \otimes s_4) \oplus (s_2 \otimes s_3)$*. Note that the sets of axioms corresponding to these products* $(\{y, \Box\neg y\}, \{y, \neg y\}$*, and* $\{\bigcirc y, \Box\neg y\})$ *characterise the minimal unsatisfiable sub-KBs; also known as* minimal unsatisfiable cores *in temporal logic [12]. Indeed, provenance w.r.t. annihilating dci semirings yields the class of all minimal sets of axioms entailing the consequence (see Theorem 4).*

As this example suggests, our definition of provenance traces the axioms responsible for a consequence. In knowledge representation, a *justification* of a consequence w.r.t. a KB as a (subset-) minimal sub-KB that entails the consequence [13]. If the underlying semiring is annihilating, the provenance of a consequence can be obtained by operating over the labels of the axioms appearing in the justifications [14]. Thus, our notion of provenance generalises justification-based explanations.

Theorem 4. *Let $\mathcal{T}$ be a KB and α a consequence, and $\mathsf{Just}_{\mathcal{T}}(\alpha)$ the class of all justifications of α w.r.t. $\mathcal{T}$. If $\mathbb{S}$ is an annihilating dci semiring, then $\mathsf{Prov}_{\mathcal{T}}(\alpha) = \bigoplus_{\mathcal{J} \in \mathsf{Just}_{\mathcal{T}}(\alpha)} \bigotimes_{\beta \in \mathcal{J}} \mathsf{lab}(\beta)$.*

The simplicity of Example 3 may be misleading; it is in general not obvious how to compute the provenance of a consequence for an arbitrary KR language, nor even for LTL_f. In fact, a direct application of Definition 2 is made impossible from the fact that the class $\mathfrak{I}$ of interpretations is potentially infinite. In Example 3 we had to resort to a partition of $\mathfrak{I}$ into equivalence classes defined by the axioms each interpretation violates. In more general settings, or as the number of axioms grows, it is not obvious how to apply this approach.

One of the main results from [10] is the development of a method for computing the provenance of consequences for arbitrary KR languages having automata-based decision processes, with only some minor limitations. That approach originally introduced in [10] is based on automata on *infinite* trees, and hence not directly applicable to LTL_f, where models form finite structures. Yet, the notions can also be adapted to finite trees as well. Briefly, while the original approach was based on Büchi conditions for dealing with infinite constructions, for LTL_f it suffices to consider regular tree automata.

4. Variants and Extensions

Consider once again Example 3. Assuming annihilation, we first computed the provenance to be $(s_1 \oplus s_2) \otimes (s_1 \oplus s_3) \otimes (s_3 \oplus s_4)$. It turns out that each of these additions characterises a so-called *diagnosis*: a minimal set of axioms that, when removed from the KB, cancel the consequence [15]. For instance, if we remove the first two axioms, the resulting KB $\{\Box\neg y, \neg y\}$ becomes satisfiable. This is consistent with the well-known duality between justifications and repairs, observed in different domains [16, 17, 18]. Thus, if rather than *explaining* a consequence through its provenance one was interested in *removing* it, one could use a similar technique to compute this "correction provenance."

For this work, we used as example of a KR language the logic LTL_f. Recently, a probabilistic extension of this logic, called PLTL_f [19] was proposed to deal with uncertainty in business processes, giving birth to ProbDeclare [20]. The semantics of this new logic is based on tree-shaped interpretations, which can be naturally handled by our formalism, and an automata-based decision procedure which can be transformed to compute provenance, is readily available. Thus, we have also introduced the first definition of provenance for this very expressive probabilistic logic.

5. Conclusions

We introduced a semantic notion of provenance, which can be readily applied to arbitrary KR languages with an interpretation-based consequence relation. In contrast to other notions of provenance [21, 22], ours does not depend on an executional consequence relationship between the axioms in a knowledge base, but rather on how they combine to exclude interpretations negating the consequence of interest. Our definition thus allows us to handle provenance in languages where consequence-based reasoning is unavailable, or requires awkward constructions.

At first sight, our definition of provenance may not be very intuitive. It is based on finding the combinations of axioms that guarantee consequences to be entailed, taking into consideration the interpretation-based semantics. Yet, it naturally generalises one of the best known special cases of provenance: axiom pinpointing. It also generalises the cases of weighted reasoning where weights come from a distributive lattice [23]. It is worth mentioning that, while we use axiom pinpointing as a motivation and an example for the formalism, the approach is applicable to provenance over any distributive, idempotent, and commutative semiring.

From a practical point of view, the original work showed that as long as there is an automata-based procedure for deciding a consequence (as is the case in LTL_f and PLTL_f) it is possible to construct a *weighted* automaton which provides the provenance of the consequence. More importantly, computing the provenance is not more expensive than standard reasoning (based on automata emptiness), except for a potential exponential overhead on the number of different transition weights [10]. To showcase the importance and use of our definitions, we instantiated the framework on LTL_f whose "natural" reasoning method is based on automata.

One may consider that limiting the framework to distributive, idempotent, commutative semirings is a very strong restriction. Yet, these assumptions are justified in the context of our framework. Idempotency and commutativity are relatively common requirements in the context of provenance for expressive languages; see e.g., [21]. Indeed, there is some evidence that a lack of idempotency yields to a high complexity, and perhaps even undecidability of provenance-related problems. If the operators are not commutative, then one can construct increasingly long monomials, in detriment to the resource upper bounds derived. Distributivity, on the other hand, is useful for the automata behaviour computation. Indeed, it is known that even for the case of lattice-valued weighted tree automata, if the lattice is not distributive, then its behaviour cannot be computed in polynomial time in general [24].

Some automata-based reasoning methods are not based on the interpretation semantics, but follow a more consequence-based approach of building "proofs" for the derivation of a consequence. Examples of this view are propositional resolution [25] and the completion-based approach for $\mathcal{EL}$ [26]. In future work we will study the connection between these automata-based approaches, and verify whether provenance can be computed efficiently over them.

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