

# Write You Some Proofs for Great Good

<https://github.com/rpeszek/present-proofs-lc19>

<https://github.com/rpeszek/present-proofs-lc19/blob/master/doc/slides.pdf>

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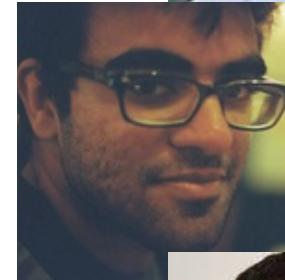
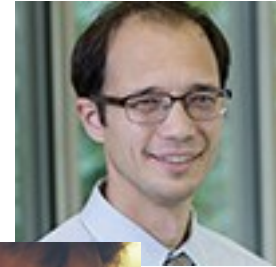
# FP @ Innovation Lab, Holland & Hart

<https://www.meetup.com/Boulder-Haskell-Programmers/>

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# Plan for this talk

Narrow scoped, pragmatic intro to proofs with dependent types

- Type Precision: better List, better Maybe, better Bool, ...
- Mathematics and Software Engineering
  - Type Precision - Gentlemen's agreements
  - Progs == Proofs - Software == has Bugs
  - Dependent types - Implementation Hiding\*
  - Proofs - Maintenance cost\*
- Termination / Totality
- Performance
- Peano Nat vs GHC.TypeLits

# Why proofs?

- Curry - Howard (Why proofs == Why programs)\*
- Formal verification
- Enable `Type Precision` (*focus of this talk*)  
&#;(proofs replace unsafe coercion)

```
data List a =  
    Empty  
    | Cons a (List a)
```





V  
S

```
data Vect (n :: Nat) a where  
    Empty :: Vect 'Z a  
    Cons :: a -> Vect n a -> Vect ('S n) a
```

\*

```
id_sanity :: Vect (1 + n) a -> Vect (n + 1) a  
id_sanity = unsafeCoerce -- ghrrr ☹️
```

# Proofs by who writes them

	Programmers *	Automated Logic Solvers
Work		Free Lunch! 
What can be done	A Lot 	Limited 
Examples	Dependently Typed Languages	Refinement Types LiquidHaskell
	<i>(focus of this talk)</i>	

# Type Precision (*call-side safety*)

```
(!!) :: [a] -> Int -> a
```

```
safeGet :: [a] -> Int -> Maybe a
```

```
-- Liquid:
```

```
{-@ (!! :: x: [a] -> {i:Nat | i < len x} -> a @-}
```

```
(!!) :: [a] -> Int -> a
```

```
-- Dep Typed:
```

```
(!!) :: Vect n a -> Fin n -> a
```

```
(!!!) :: Vect n a -> SNat m -> MaybeB (m < n) a
```

```
(!!!!) : (xs:List a) -> (n:Nat) -> {auto ok: InBounds n xs} -> a
```

```
isElem : DecEq a => (x: a) -> (xs: List a) -> Dec (Elem x xs)
```

- a lie ...
- questionable improvement
- refinements

... still imprecise \*

not in the same boat \*

<https://github.com/rpeszek/present-proofs-lc19/tree/master/src/Motivation>

# Type Precision *(implementation safety)*

(code)

<https://github.com/rpeszek/present-proofs-lc19/blob/master/src/Motivation/Exfer.idr>

# Intro

**(code) Curry-Howard vs Imperative**

<https://github.com/rpeszek/present-proofs-lc19/blob/master/src/Present/AnIntro.hs>

<https://github.com/rpeszek/present-proofs-lc19/blob/master/src/Present/MaybeB.hs>



# Problem in Paradise - Questions

Should type checker know basic algebra?

`a || True <==> True (Bool algebra)`

`a + b == b + a (Nat, Int, Float ... algebra) ...`

Answers:

- Dependently Types Langs: **No** (🤖 *Programmers supply proofs*)
- Refinement Types / LiquidHaskell: **Yes** (♂ *SMT solver does the work*)

# Type Equality

```
data a ~: b where  
  Refl :: a ~: a
```

typecheck **Refl** only if same types

```
test1  = Refl :: 5 ~: 5      -- GOOD  
test10 = Refl :: 4 ~: 5      -- ERR  
  
test2  = Refl :: 2 + 3 ~: 3 + 2 -- GOOD  
test20 :: SNat n1 -> SNat n2 -> n1 + n2 ~: n2 + n1  
test20 _ _ = Refl           -- ERR
```

[\(base\) Data.Type.Equality](#)

# Example Combinators

- library over op semantics
- "pattern-matching on a variable of type  $(a \sim b)$  produces a proof that  $a \sim b$ " - *haddock*

```
sym :: (a ~ b) -> (b ~ a)
```

```
trans :: (a ~ b) -> (b ~ c) -> (a ~ c)
```

```
apply :: (f ~ g) -> (a ~ b) -> (f a ~ g b)
```

```
inner :: (f a ~ g b) -> (a ~ b)
```

```
castWith :: (a ~ b) -> a -> b
```

```
gcastWith :: (a ~ b) -> ((a ~ b) => r) -> r
```

```
gcastWith Refl x = x
```

math

*a bit unexpected ?*

fullfil ~ constraint using ~

[\(base\) Data.Type.Equality](#)

# Proofs - Bool Algebra

(code)

<https://github.com/rpeszek/present-proofs-lc19/blob/master/src/Present/ProofsBoolAlg.hs>

# Better Bool ... Decidability

(code)

```
import Data.Void
data Dec prop = Yes prop |
               No  (prop -> Void)
```



- Programming contradictions
  - -XEmptyCase
- Forced Type Precision
- Programs are proofs! \*

<https://github.com/rpeszek/present-proofs-lc19/blob/master/src/Present/ProofsDecidable.hs>

# Proofs - Nat (Performance)

(code)

```
reverse :: Vect n a -> Vect n a

-- recursive calls
plusCommutative :: SNat left -> SNat right -> ((left + right) :~: (right + left))
plusCommutative left right = case left of
  SZ -> lemma1 right
  (SS k) -> case plusCommutative k right of Refl -> sym (lemma2 right k)
```

unsafeCoerce replacements

- rewrite rules
- proveFast combinators
- CPP

<https://github.com/rpeszek/present-proofs-lc19/blob/master/src/Present/ProofsNatAlg.hs>

# Proofs - working with TypeLits

(code)

- hidden implementation
- Peano approach is slow

<https://github.com/rpeszek/present-proofs-lc19/blob/master/src/Present/WorkingWithTypeLits.hs>

# Some Learning/References

- intro books
  - [Type Driven Development in Idris](#) great book
    - <https://github.com/rpeszek/IdrisTddNotes/wiki>
  - [TAPL](#) great book (*not really dep types but still*)
  - Programming foundation books ([penn/Pierce et al](#), [Wadler](#))
- Haskell projects (with reading references)
  - [singletons](#)
  - [equational-reasoning-in-haskell](#)
  - [liquidhaskell](#)
- blogs
  - [blog.jle.im](#) (Justin Le)
  - [typesandkinds](#) (Richard Eisenberg)
- youtube
  - [Introduction to Agda](#) series by Daniel Peebles published by Edward Kmett