Computational Complexity of Computing Symmetries in Finite-Domain Planning

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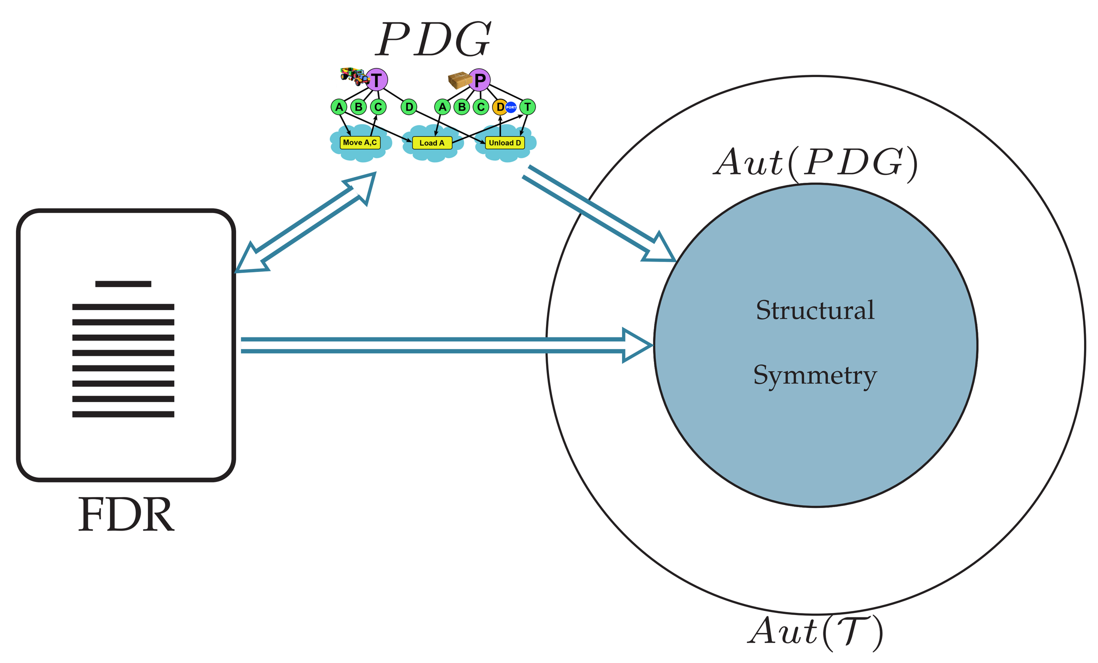
Language for Planning: FDR

An FDR Planning task is 5-tuple $\langle V, A, cost, I, G\rangle$:

- V: finite set of multi-valued (finite-domain) state variables
- A: finite set of actions of form $\langle$pre, eff$\rangle$ (preconditions/effects; partial variable assignments)
- $cost : A \mapsto \mathbb{R}^{0+}$ captures action cost
- I: initial state (variable assignment)
- G: goal description (partial variable assignment)

Symmetry Groups (FDR)



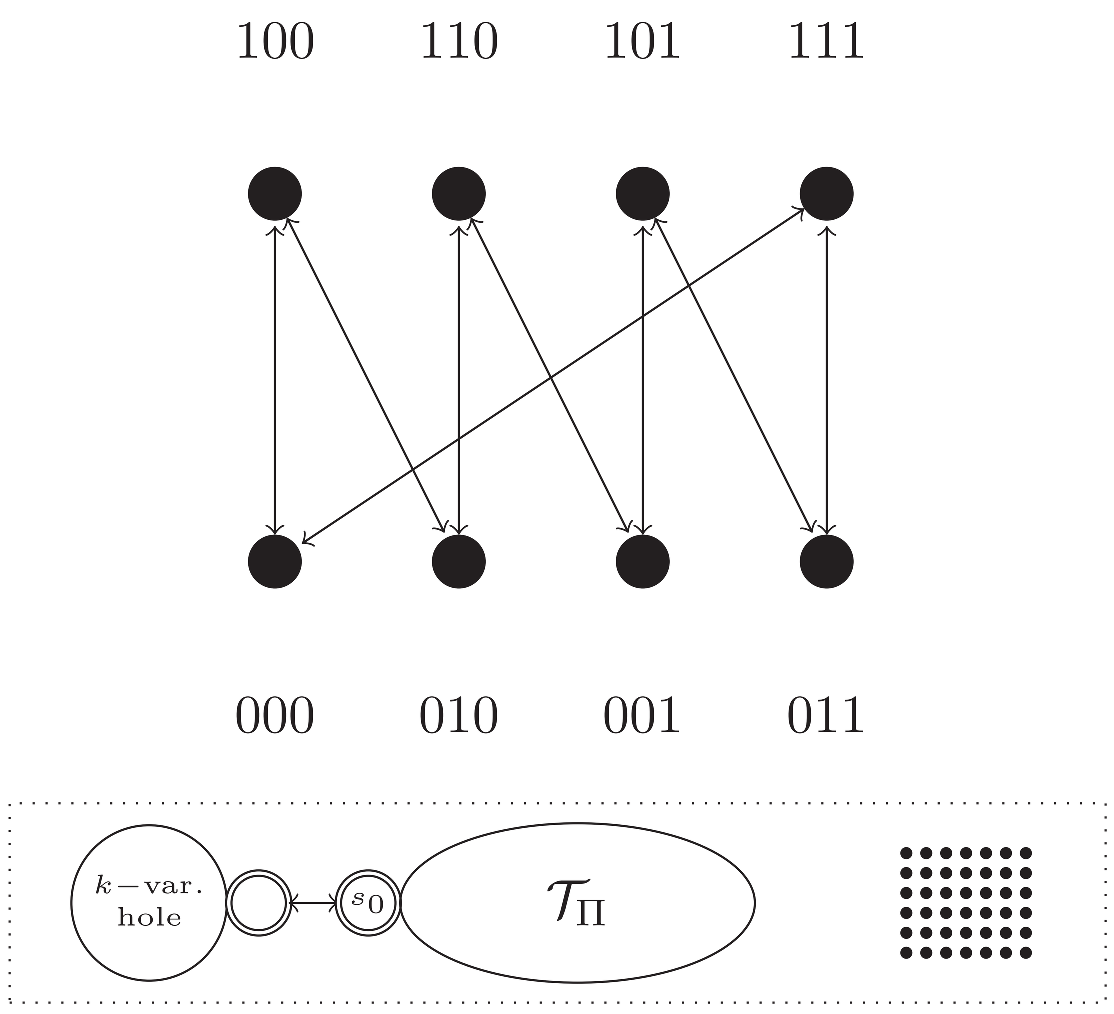
Equivalent Classes

- CGI – GI for connected undirected graphs;
- DGI – GI for directed graphs;
- AGEN– returns a set of generators for the graphs automorphism group;
- CAGEN – AGEN for colored graphs..

1. GI $=_p$ CGI $=_p$ DGI – Zemlyachenko et al. (1985)
2. GI $=_p$ IMAP $=_p$ AGEN
   Mathon[a] (1979)
3. GI $=_p$ CAGEN
   Ghosh and Kurur[b] (2014)

[a] The proof is incomplete.
[b] The provided proof sketch is wrong, and we have a counter example.

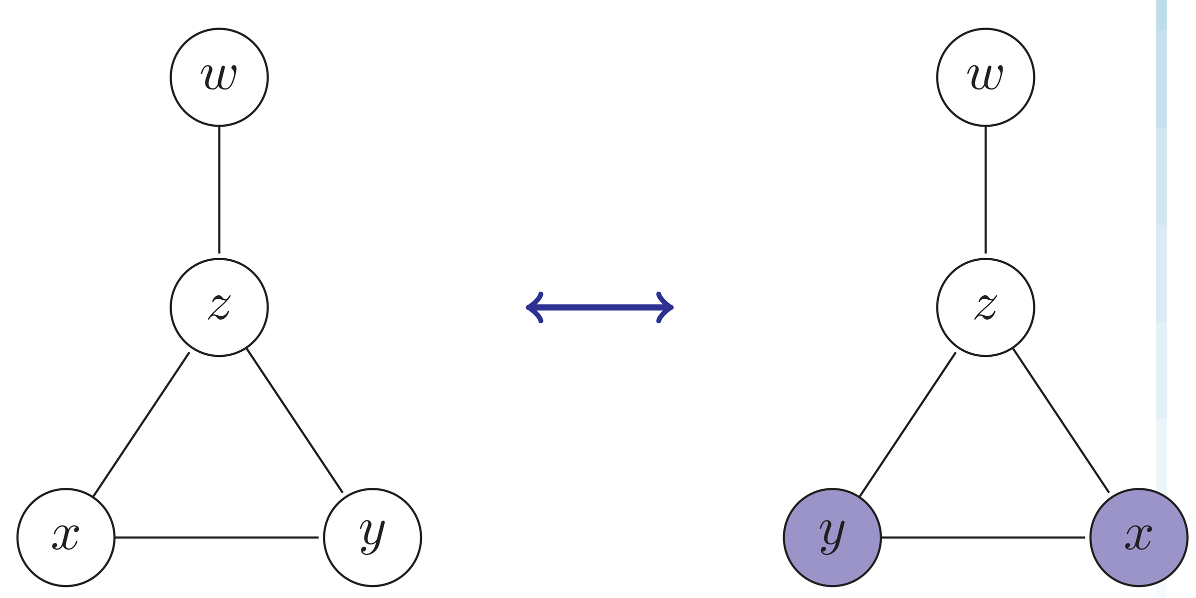
Adding Cycles



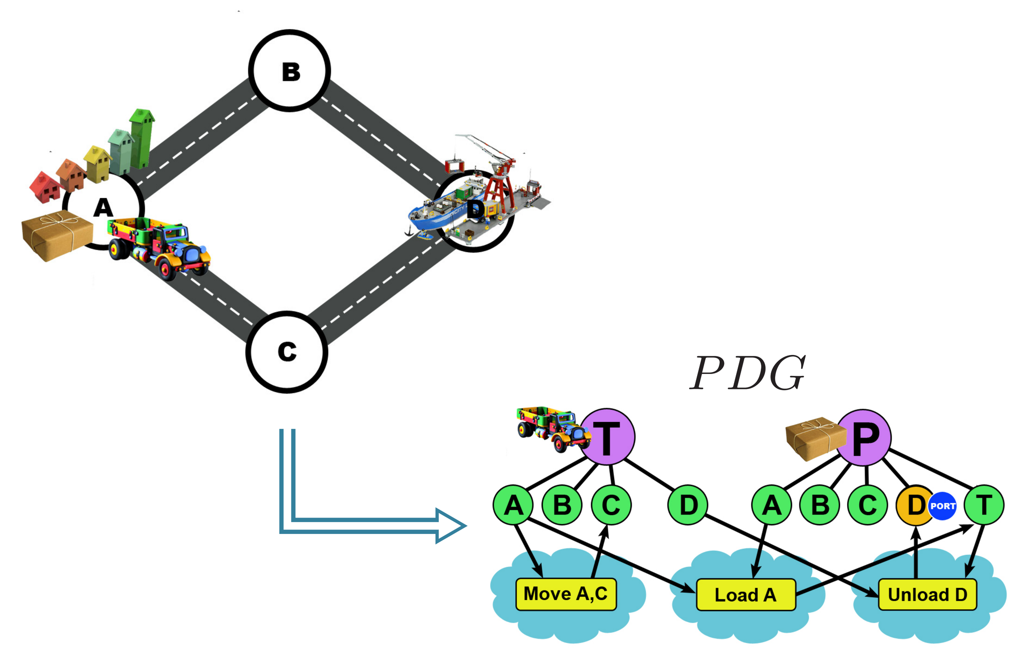
The state-transition graph of structure $\mathcal{T}_\Pi$ with an k-var. hole attached at $(b_1, \ldots, b_m)$. $s_0 = (b_1, \ldots, b_m, 0, \ldots, 0, 0)$ The dots to the right represent mixed states.

Symmetries

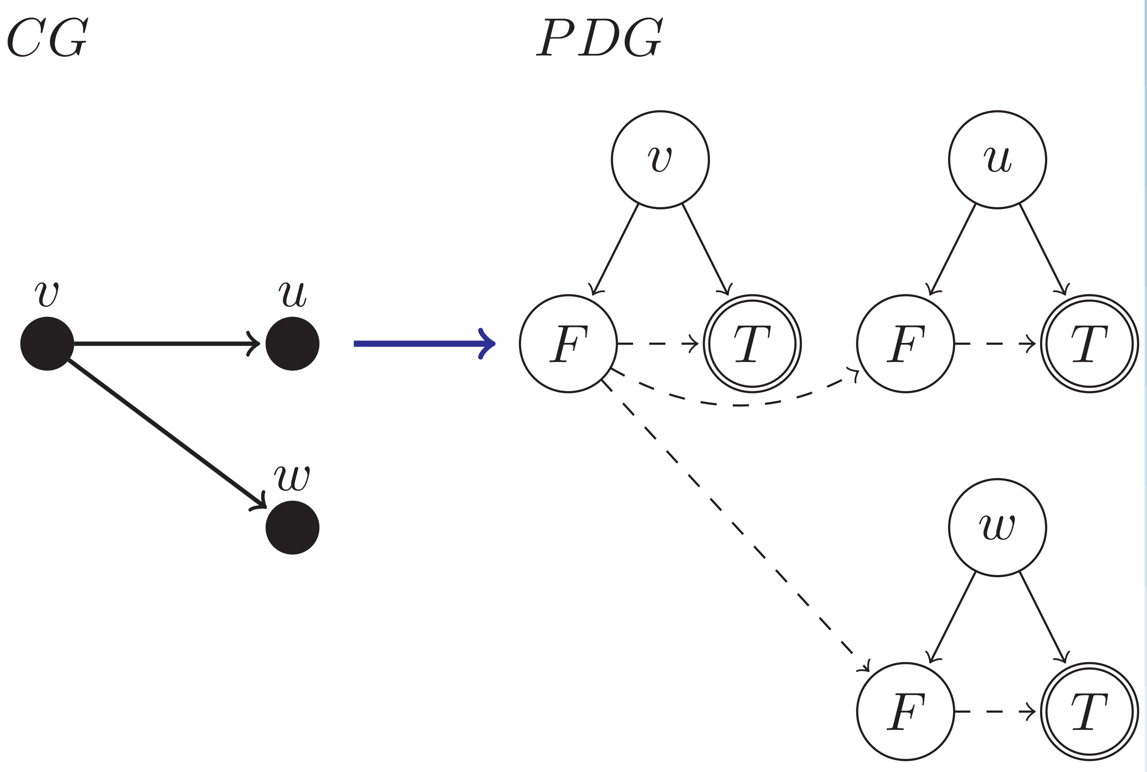
Let $\langle N_1, E_1\rangle$ and $\langle N_2, E_2\rangle$ be two graphs. The map $\rho : N_1 \to N_2$ is an isomorphism if ρ is an edge-preserving bijection. If $N_1 = N_2$ ρ is an automorphism.



Planning Description Graph (PDG)

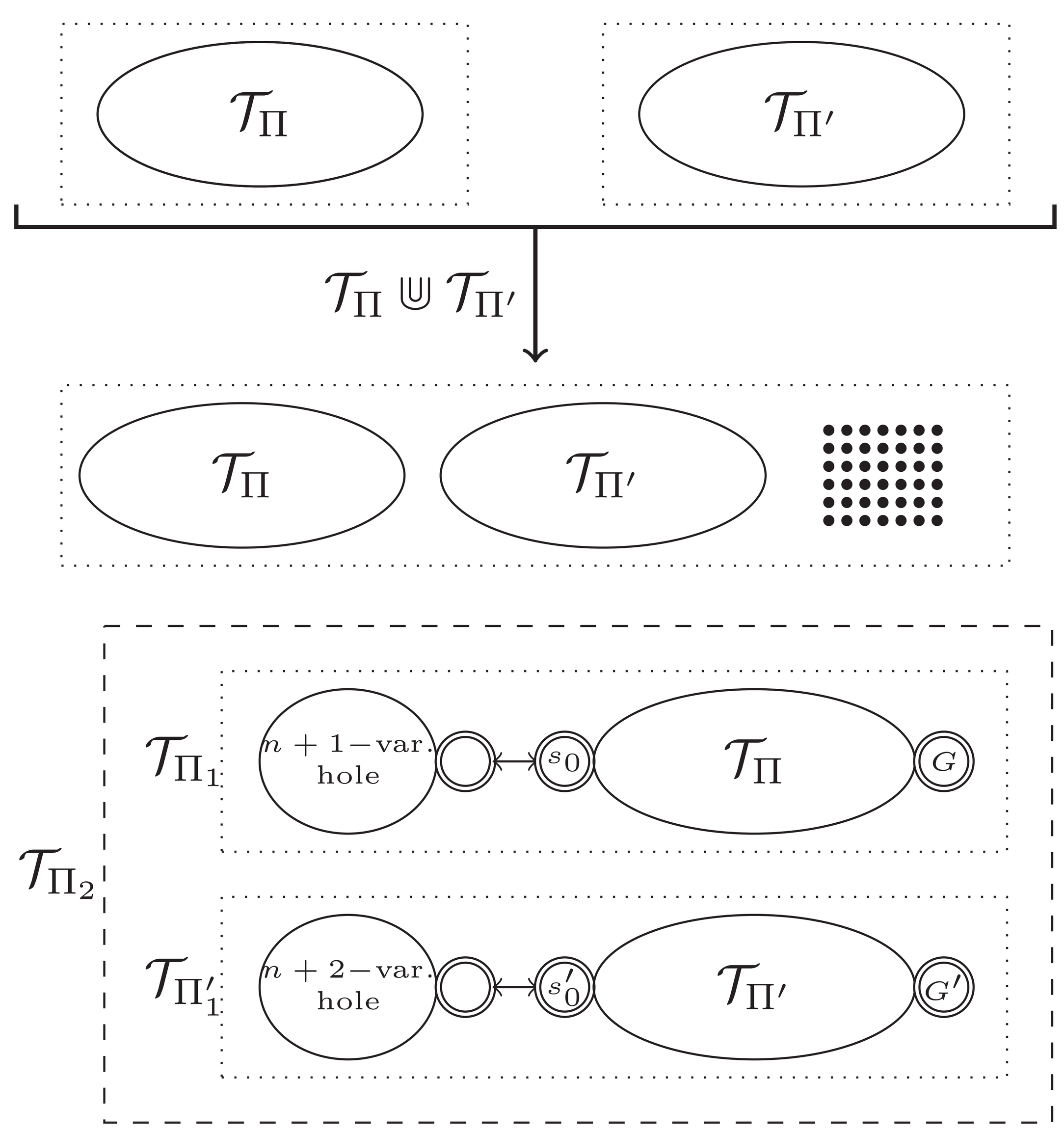


$Aut(PDG)$ – Lower Bound



Constructing a PDG from a directed graph while preserving the automorphism group.

Planning Tasks: Two to One



The state-transition graph of the direct union of structures $\mathcal{T}_\Pi$ and $\mathcal{T}_{\Pi'}$. The dots to the right represent mixed states.

Results

Let Π be a planning task.

Computing the structural symmetries group of Π is GI-complete.

If Π has a bounded domain and a bounded-degree causal grpah than the structural symmetries group of Π can be computed in poly-time.

Given two states s and s' in the transition graph $\mathcal{T}_\Pi$ of Π, determining whether s and s' are symmetric is PSPACE-complete.

Graph Isomorphism Problem

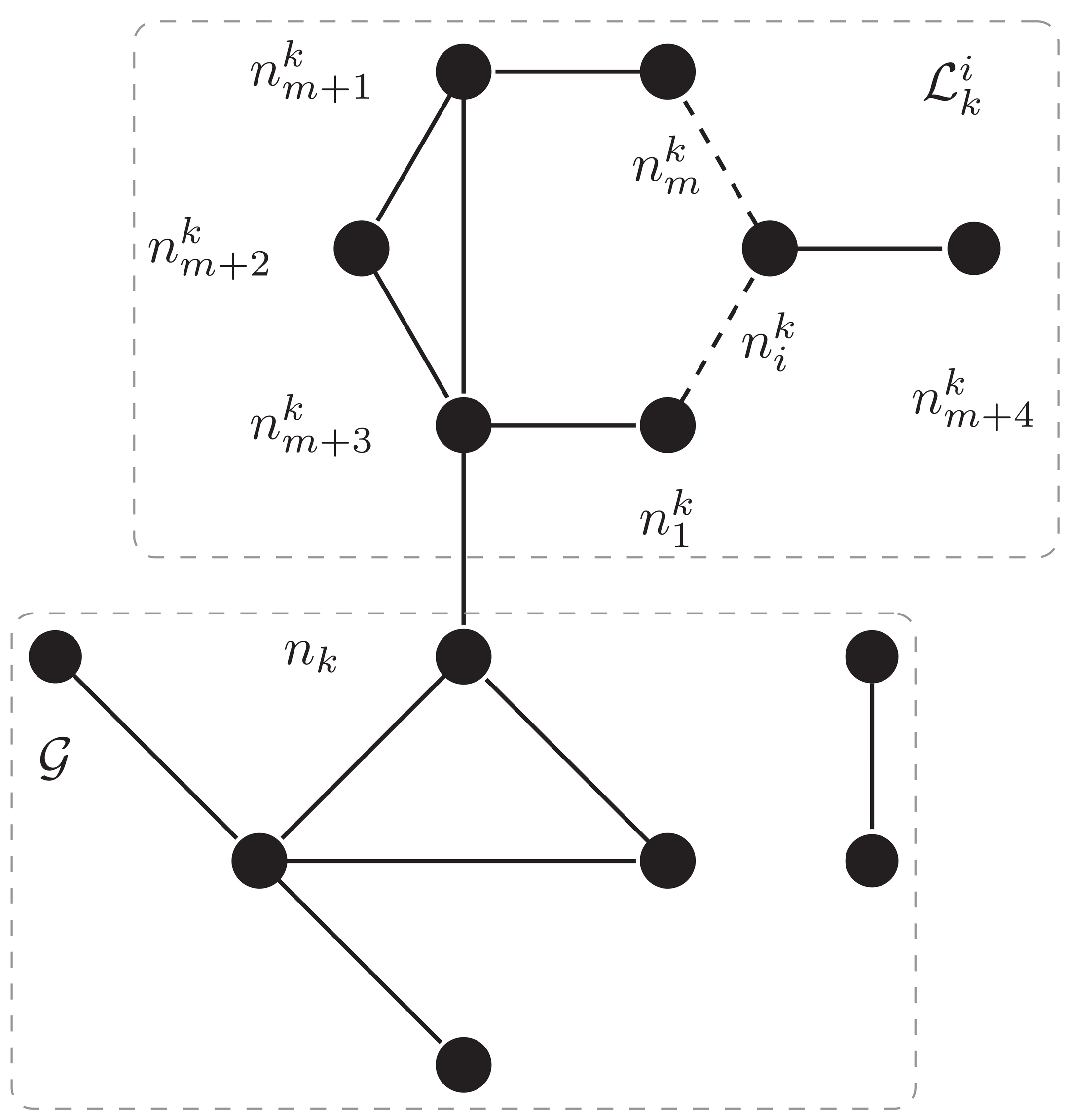
The **graph isomorphism problem** (GI) is the computational problem of determining whether two finite graphs are isomorphic.

For a graph with n vertices

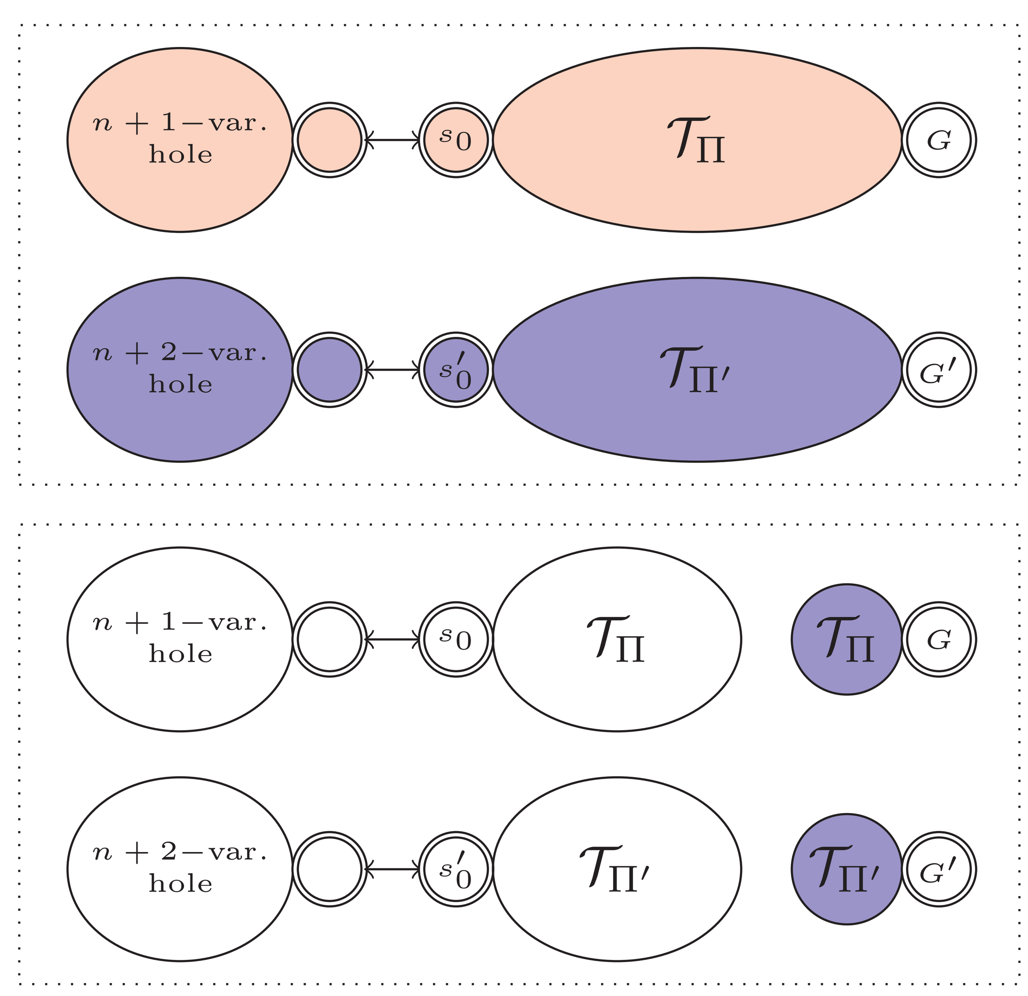
1. $2^{O(\sqrt{n}\log^2 n)}$ – Babai (1983), Babai & Luks (1983)
2. $2^{O(\sqrt{n\log n})}$ – Zemlyachenko et al. (1985)

Quasi-polynomial claim Babai announced an algorithm with running time $2^{O(\log n)^3}$.

$Aut(PDG)$ – Upper Bound



Proof



G is symmetric to G' $\iff$ Π is unsolvable.