Online Hedge Reservation for Diverse Plans and Competitive Analysis

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Summary

We investigate the plan reservation problem with diverse plans in mobile networks with competitive analysis as the data volume is not known until an app is used. The pricing scheme includes: 1) Pay-as-you-go (PAYG) payment; 2) All-in-one plan: an upfront fee is charged to cover data volume of a period of time; and 3) Directional plan: an upfront fee is charged to cover data volume of a specific app for a period of time. We propose the Online Hedge Reservation (OHR) Algorithm and prove that it achieves $e^{\beta}/(e^{\beta}-1)$ competitive ratio when each plan is valid till the end of each calendar month and $2e^{\beta}/(e^{\beta}-1)$ competitive ratio when each plan is valid for a full month, where β is the ratio of prices of the directional plans and the all-in-one plan. This is an exciting neat extension of the competitive ratio $e/(e-1)$ of the classic ski-rental problem [1].

Introduction

Sponsored data plan (SDP) or directional plan is a win-win-win solution to **internet service providers (ISPs)**, **content providers (CPs)**, **end users**.

In the **plan reservation problem** which is considered in this paper, the decision-maker need to consider the following payment options:

- *PAYG (pay-as-you-go)*: pay for on-demand usages
- *All-in-one plan*: covers all network traffic by paying an up-front fee
- *Directional plan*: covers network traffic for a single app by paying an up-front fee

It is unclear how to purchase them in the *temporal dimension* as a *single* user since the user's future data traffic is usually non-predictable.

Online Hedge Reservation (OHR) Algorithm

We have two randomized threshold:

- γ_d: if PAYG cost of app i $\geq \gamma_d$, reserve the directional plan for the app i.
- γ_o: if the overall PAYG cost $\geq \gamma_o$, reserve the all-in-one plan.

Pay the task by PAYG otherwise. The randomized thresholds γ_d and γ_o following a pre-set distribution.

The probability mass functions (PMFs) of γ_d and γ_o are:

$$\mathbb{P}(\gamma_d = i) \triangleq P_i^{(d)} = \begin{cases} 1, & \text{if } i = C_d, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

$$\mathbb{P}(\gamma_o = i) \triangleq P_i^{(o)} = \begin{cases} aq^{i-1}, & \text{if } i \in [1, C_d - 1], \\ \frac{C_o}{2C_o-1} aq^{C_d-2}, & \text{if } i = C_o, \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

where $q \triangleq \frac{C_o}{C_o-1}$ and $a \triangleq (\frac{1-q^{C_d-1}}{1-q} + \frac{C_o}{2C_o-1} q^{C_d-2})^{-1}$.

Proof Scheme

Lemma 1 ↘ Lemma 5 (link between non-extensive and extensive) ↘

Lemma 3 ⟶ Theorem 1 (non-extension mode) ⟶ Theorem 2 (extension mode)

Lemma 2 ↗ Lemma 4 ↗

Competitive Analysis

Through lemma 1 and 2, we build pruned sequence s_1 from the general input s. And we bound the competitive ratio of s_1 by lemma 3. Given γ_d and γ_o, the upper bound of the deterministic competitive ratio is

$$\frac{\text{ALG}^{(n)}(s_1, \gamma_d, \gamma_o)}{\text{OPT}^{(n)}(s_1)} \leq \frac{2C_o + C_d - 1}{\gamma_d}. \tag{3}$$

Based on (3), we have theorem 1 which states the competitive ratio of the OHR Algorithm in the non-extension mode is

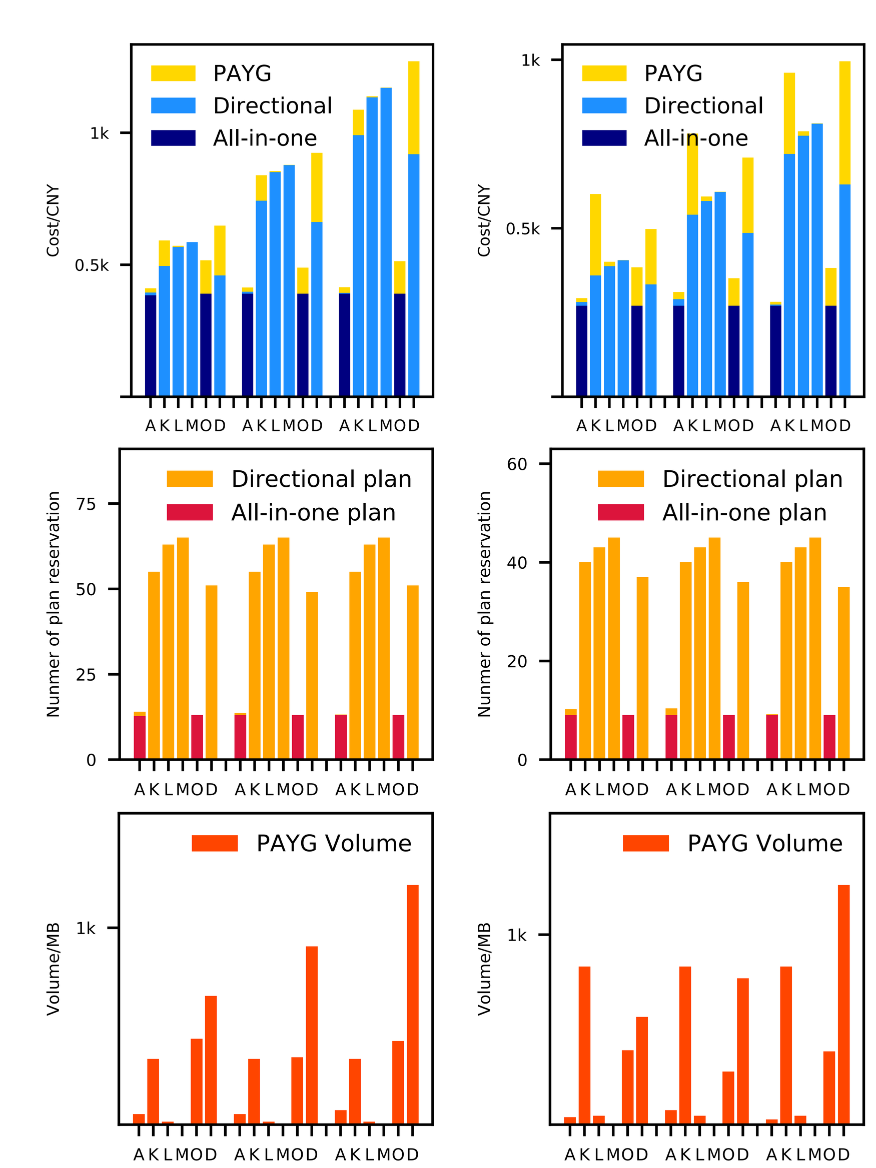
$$\frac{\mathbb{E}[\text{ALG}^{(n)}(s)]}{\text{OPT}^{(n)}(s)} \leq \frac{e^{\beta}}{e^{\beta}-1}, \tag{4}$$

for any sequence s, where $\beta = \frac{C_d-1}{C_o-1} \simeq \frac{C_d}{C_o}$.

For extension mode, the competitive ratio of the OHR Algorithm is

$$\frac{\mathbb{E}[\text{ALG}^{(e)}(s)]}{\text{OPT}^{(e)}(s)} \leq \frac{2e^{\beta}}{e^{\beta}-1}. \tag{5}$$

Evaluation



(a) Non-Extension, $T = 0.5$ week. (b) Extension, $T = 0.5$ week.

Figure 1:Performance comparison.

Evaluation-Cond.

The trace-driven experiment results are shown in Fig. 1. Our proposed the OHR Algorithm is marked as [A] and the rest are benchmarks. Each data point is averaged over 10 rounds of simulation to show the average performance.

Figs. 1(a) shows the results for the non-extension mode. It shows that OHR performs the best. Benchmarks [K], [L], and [M] try to use historical experiences. However, as the users preference changes drastically, they does not perform well. Figs. 1(b) shows the simulation result of the extension mode. Compared to the non-extension mode, the overall costs are slightly smaller. Still, the results show a similar trend as the non-extension mode.

Discussion and Future work

In some real-world instances, a data cap may be employed to limit the overall data volume in a period of time. In this case, our model still works for a wide range of situations when the data cap is sufficiently large, and users do not exceed the cap.

References

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