TU WIEN Informatics
Christian Doppler Laboratory for Artificial Intelligence and Optimization for Planning and Scheduling

Constraint-based Modeling for Scheduling Paint Shops in the Automotive Supply Industry

Felix Winter and Nysret Musliu
Contact: winter@dbai.tuwien.ac.at

Published in *ACM Transactions on Intelligent Systems, March 2021*

dbai Databases and Artificial Intelligence Group
Christian Doppler Laboratory for Artificial Intelligence and Optimization for Planning and Scheduling
DBAI, TU Wien

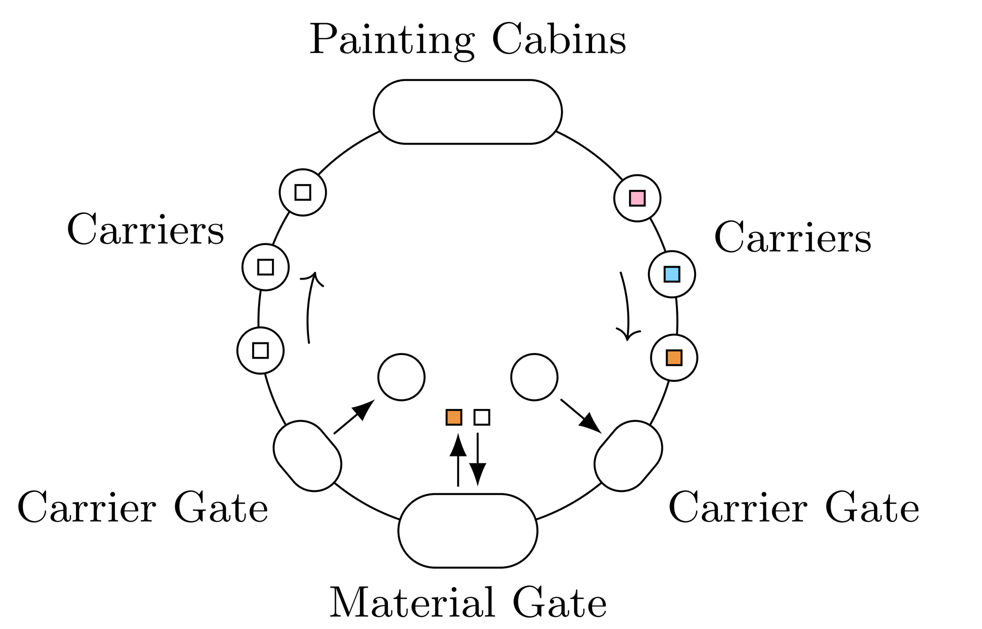
Problem Statement

- **Paint Shop Scheduling in the Automotive Supply Industry**
  - Large number of items are painted every day
  - Great potential to minimize waste and setup times
- **Finding good schedules is challenging**
  - Many restrictions and parameters have to be considered
  - Items need to be allocated on customized carrier devices
- **Goals**
  - Minimize color changes in production sequence
  - Minimize number of required carrier device changes

Contribution

- **Two modeling approaches**
  - Direct Model
  - Deterministic Finite Automata (DFA) Model
- **Experimental evaluation**
  - Benchmark instances based on real life scenarios
  - Experiments with state of the art MIP and CP solvers
  - We provide previously unknown optimal solutions for 7 instances
- **Complexity Analysis**
  - We show that the decision variant is NP-complete

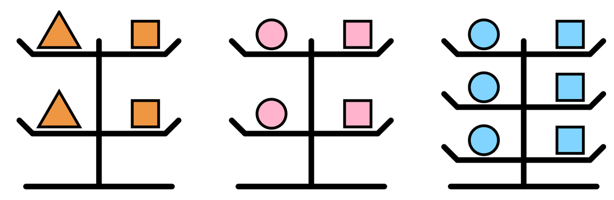
Paint Shop Scheduling in the Automotive Supply Industry



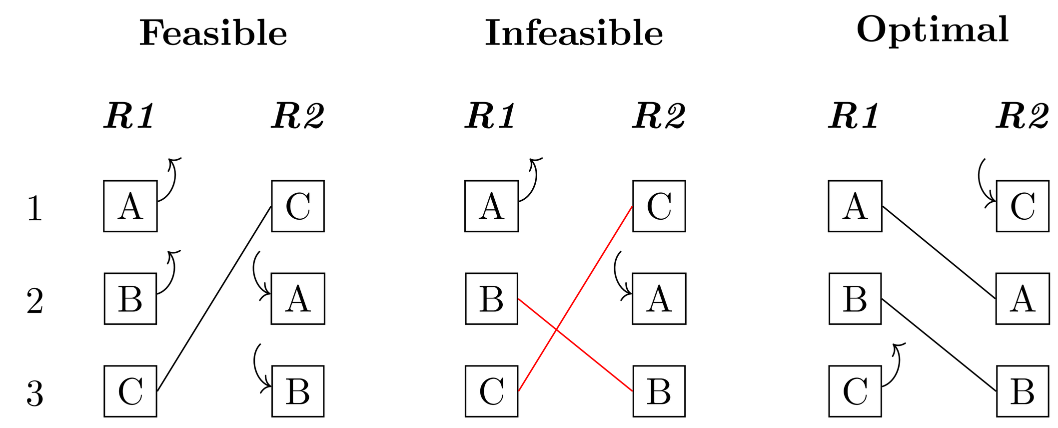
During production, a number of carrying devices will be inserted onto a circular system of conveyor belts which will then transport unpainted material pieces to the painting cabins.

	R1	*R2*	*R3*
1	A1	A2	C1
2	A1	A2	C2
3	A2	C1	C3
4	B1	B2	B1
5	B2	B3	B2

An example painting schedule for three rounds. Each column represents the scheduled carrier sequences scheduled within a single round.



Three carriers carrying painted items in different color and shape configurations.



Three options to reuse carrier devices between two consecutive rounds that schedule three different carrier types.

Modeling the Paint Shop Scheduling Problem

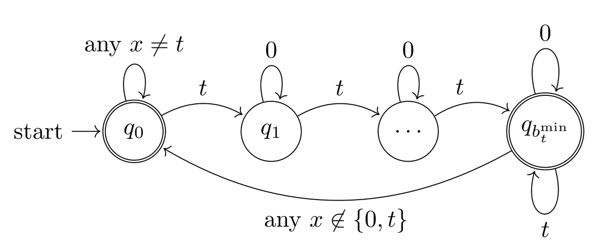
Decision Variables

- $x_{i,j}$: Carrier in round i at position j
- $c_{i,j}$: Color used in round i at position j

Hard Constraints

- Due dates
- Forbidden carrier sequences
- Forbidden color sequences
- Min/Max consecutive carriers with same type
- Min/Max carriers per round
- Carrier availability

Sequence constraints can be modeled with DFAs:



DFA accepting only sequences that schedule carriers of type t at least $b_t^{\min}$ times consecutively

Objective Function

- $R = \{1, \ldots, n\}$: Rounds to schedule
- $S = \{1, \ldots, s\}$: Positions in each round
- $sc_r, \forall r \in \{1, \ldots, n-1\}$: Number of carrier changes after each round
- $cc_r, \forall r \in R$: Number of color changes in each round

$$\mathit{minimize} \sum_{r \in \{0,\ldots,n-1\}} sc_r^2 + \sum_{r \in R} cc_r^2$$

- **Aim:** Minimize the number of carrier changes (sc) and color change costs (cc).
- Squared sums of changes per round are used to encourage an even distribution of changes over the scheduling horizon.

Modeling Carrier Changes

We propose a modeling approach that introduces variables to store the positions of all reused carriers:

$$kept^1_{i,j} \in \{0, \ldots, s\}, \forall i \in \{0, \ldots, r-1\}, j \in S$$
$$kept^2_{i,j} \in \{0, \ldots, s\}, \forall i \in R, j \in S$$

Feasible

x	$kept^1_{1,x}$	$kept^2_{2,x}$
1	1	3
2	0	0
3	0	0

Infeasible

x	$kept^1_{1,x}$	$kept^2_{2,x}$
1	3	3
2	1	2
3	0	0

Optimal

x	$kept^1_{1,x}$	$kept^2_{2,x}$
1	2	1
2	3	2
3	0	0

The tables show the $kept^1_{i,j}, kept^2_{i,j}$ variable values that correspond to the three options to reuse carriers between two consecutive rounds shown in the example above.

Complexity Analysis

Decision variant of the problem is NP-complete

- Can we find a schedule with costs $<= k$?
- Proof via reduction from set covering
  - We create demands for each element in universe
  - Create a single carrier configuration for each set
  - Find schedule for a single round

Experimental Results & Conclusion

- Direct model did not produce competitive results compared to DFA based model
- The best results for instances 1–24 using MiniZinc with Chuffed, Gurobi and Cplex compared with the best known upper bounds produced by Local Search (LS).
- Best result are shown in bold face, a * denotes optimal solutions.
- Exact methods can solve small to medium sized instances.

	Chuffed	Gurobi	Cplex	LS
I1	**775***	**775***	776	930.9
I2	**842***	**842***	**842***	1015.5
I3	**961***	**961***	2761	971.6
I4	**918***	967	12920	1100.8
I5	**530***	**530***	11085	551.7
I6	**842***	—	1933	863.5
I7	**844***	904	—	912.1
I8	**1237***	—	—	1529.5
I9	**975***	—	—	1406.3
I10	**964**	—	—	1029.9
I11	—	—	—	**4471.5**
I12	—	—	—	**4917.9**

	Chuffed	Gurobi	Cplex	LS
I13	—	—	—	**62816.5**
I14	—	—	—	**91587.3**
I15	—	—	—	**136675.8**
I16	—	—	—	**180608.1**
I17	—	—	—	**297230.8**
I18	—	—	—	**526878**
I19	—	—	—	**460643.5**
I20	—	—	—	**839361.1**
I21	—	—	—	**841710.7**
I22	—	—	—	**1524201.9**
I23	—	—	—	**1641116.1**
I24	—	—	—	**2542131.3**

Acknowledgment: The financial support by the Austrian Federal Ministry for Digital and Economic Affairs, the National Foundation for Research, Technology and Development and the Christian Doppler Research Association is gratefully acknowledged.