LM-Cut and Operator Counting Heuristics for Optimal Numeric Planning with Simple Conditions

Ryo Kuroiwa, Alexander Shleyfman, Chiara Piacentini, Margarita P. Castro, and J. Christopher Beck (University of Toronto)

Numeric Planning with Simple Conditions

- State s contains numeric variables, and $s[v]$ is the value of variable v
- Goals and preconditions of actions can be numeric conditions, which are linear inequalities using numeric variables
- The effect of action a on v: $v \mathrel{+}= k^a_v$ (constant increase/decrease)

  - $s = \{ v = 0 \}$, goal: $\{ v \geq 3 \}$
  - Action a_1: pre(a_1) = {}, eff(a_1) = $\{ v \mathrel{+}= 1 \}$
  - Action a_2: pre(a_2) = $\{ v \geq 1 \}$, eff(a_2) = $\{ v \mathrel{+}= 2 \}$

  Optimal plan: $\langle a_1, a_2 \rangle$
  Optimal cost: $h^*(s) = 2$

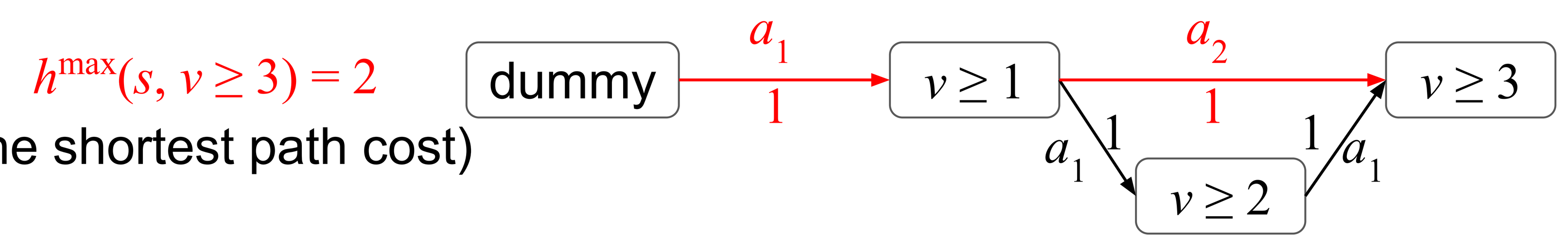
Example task

$h^{\max}$ in Numeric Planning

$$h^{\max}(s, v \geq w) = \min_{a \in \text{supp}(v \geq w)} h^{\max}(s, \text{pre}(a) \cup \{ v \geq w - k^a_v \}) + \text{cost}(a)$$

where $\text{supp}(v \geq w) = \{ a \in A \mid k^a_v > 0 \}$

- Generalizing $h^{\max}$ to consider condition $v \geq w$ (v: variable, w: constant)
- Achieving $v \geq w - k^a_v$ and pre(a) is needed to achieve $v \geq w$ applying a
- **Computing $h^{\max}$ is NP-hard** (Proposition 2)



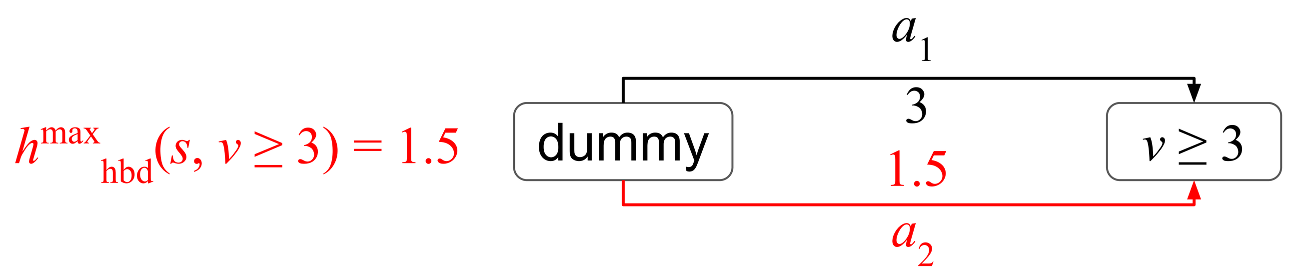
Justification Graph (JG) of $h^{\max}$

$h^{\max}_{\text{hbd}}$: Polynomial $h^{\max}$ (Scala et al. 2016)

$$h^{\max}_{\text{hbd}}(s, v \geq w) = \min_{a \in \text{supp}(v \geq w)} h^{\max}_{\text{hbd}}(s, \text{pre}(a)) + \min_{a \in \text{supp}(v \geq w)} \text{m}_a(s, v \geq w)\, \text{cost}(a)$$

where $\text{m}_a(s, v \geq w) = (w - s[v]) / k^a_v$ (# of repetition of a to achieve $v \geq w$ from s)

- $h^{\max}_{\text{hbd}}$ does not introduce additional conditions such as $v \geq w - k^a_v$ and is computed in polynomial time
- $h^{\max}_{\text{hbd}}$ decouples preconditions and effects to ensure the admissibility: LB of cost to make actions applicable + LB of cost to achieve $v \geq w$

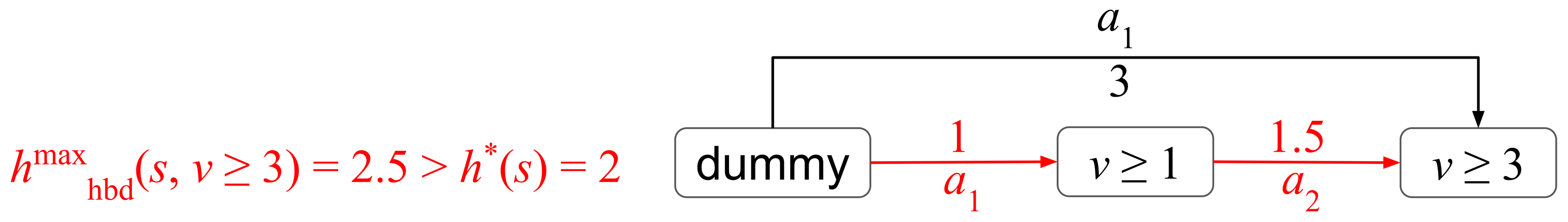


Justification Graph (JG) of $h^{\max}_{\text{hbd}}$

$h^{\max}_{\text{cri}}$: Polynomial but Inadmissible $h^{\max}$

$$h^{\max}_{\text{cri}}(s, v \geq w) = \min_{a \in \text{supp}(v \geq w)} h^{\max}_{\text{cri}}(s, \text{pre}(a)) + \text{m}_a(s, v \geq w)\, \text{cost}(a)$$

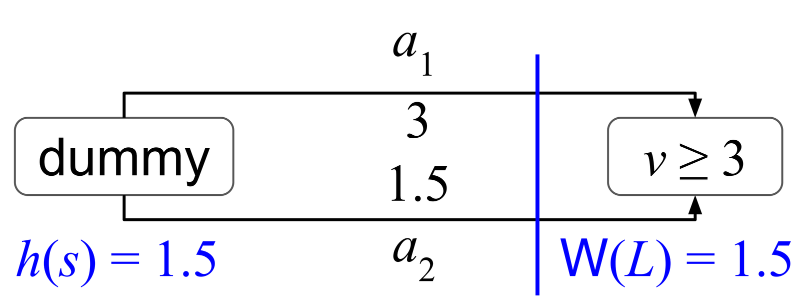
- $h^{\max}_{\text{cri}}$ does not decouple preconditions and effects and is inadmissible



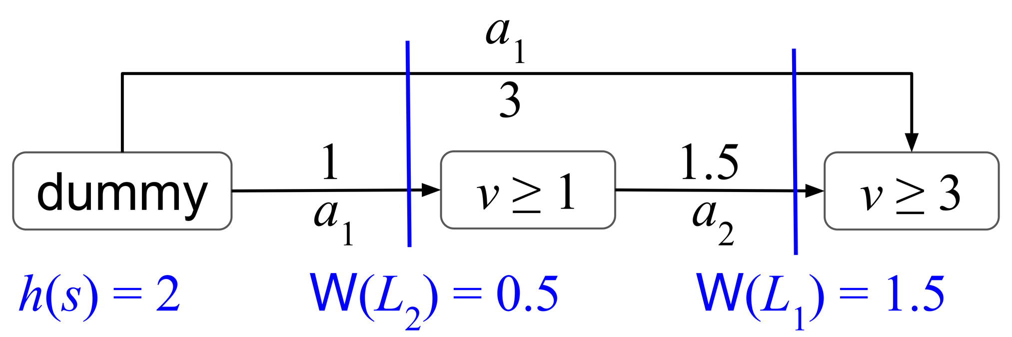
Justification Graph (JG) of $h^{\max}_{\text{cri}}$

Numeric LM-Cut

1. $h(s) := 0$; repeat 2. and 3.
2. Construct a JG where nodes are numeric conditions and propositions, and edges are created so that the shortest path cost from s to $v \geq w$ is equal to $h^{\max}_{\text{hbd}}(s, v \geq w)$ / $h^{\max}_{\text{cri}}(s, v \geq w)$
3. Find cut L separating s from the goal conditions having weight $\text{W}(L) = \min_{a \in L} \text{m}_a(s, v > w)\, \text{cost}(a) > 0$
   $h(s) \mathrel{+}= \text{W}(L)$ if L exists, and return $h(s)$ otherwise
   For action a in L, $\text{cost}(a) \mathrel{-}= \text{W}(L) / \text{m}^L_a$
   where m^L_a is the minimum $\text{m}_a(s, v \geq w)$ in L



LM-cut based on $h^{\max}_{\text{hbd}}$



LM-cut based on $h^{\max}_{\text{cri}}$

Theoretical Results

Admissibility

	$h^{\max}$	LM-cut
hbd	✓ admissible (Scala et al. 2016)	✗ inadmissible (Proposition 3)
cri	✗ inadmissible (Example 1)	✓ admissible (Theorem 1)

Dominance

classical	LM-cut dominates $h^{\max}$ (Helmert and Domshlak 2009)
numeric	LM-cut (cri) and $h^{\max}_{\text{hbd}}$ are incomparable (Proposition 4)

Operator-Counting (OC) Constraints

Classical LM-cut Constraints (Bonet and Helmert 2010)

$$\sum_{a \in L} X_a \geq 1$$

Numeric LM-cut Constraints (Theorem 2)

$$\sum_{a \in L} \frac{X_a}{\text{m}^L_a} \geq 1$$

- OC constraints are satisfied by any plan (Pommerening et al. 2014)
  X_a: the number of action a in a plan
- L: cut obtained by the LM-cut, m^L_a: the minimum $\text{m}_a(s, v \geq w)$ in L
- Generalized to numeric planning (Piacentini et al. 2018)

Experimental Evaluation

- A* with admissible heuristics, 30 min time limit, 4 GB memory limit
- $\hat{h}^{\text{rmax}}_{\text{hbd+}}$, $h^{\text{gen}}_{\text{hbd}}$ (Scala et al. 2020): subgoaling-based heuristics
- $h^{\text{lm+}}_{\text{hbd}}$ (Scala et al. 2017): numeric landmark heuristic
- SEQ (Bonet 2013; Piacentini et al. 2018): state equation constraints (OC)
- h^C_{IP} (Piacentini et al. 2018): delete-relaxation constraints + SEQ using IP

domain	$\hat{h}^{\text{rmax}}_{\text{hbd+}}$	$h^{\text{lm+}}_{\text{hbd}}$	$h^{\text{gen}}_{\text{hbd}}$	LM-cut	h^C_{IP}	SEQ (LP)	LM-cut (LP)	LM-cut + SEQ (LP)
Counters (60)	16	18	**51**	18	39	**60**	18	**60**
Gardening (114)	**75**	**75**	74	**75**	77	75	75	**78**
Sailing (80)	34	26	14	**54**	**64**	12	54	52
Depots (20)	5	3	1	**7**	1	6	**7**	**7**
Rovers (20)	**4**	**4**	2	**4**	2	**4**	**4**	**4**
Satellite (20)	1	1	1	**2**	1	1	**2**	**2**
Total (364)	165	157	173	**190**	214	188	190	**233**

- LM-cut is empirically better than $\hat{h}^{\text{rmax}}_{\text{hbd+}}$ which is based on $h^{\max}_{\text{hbd}}$
- The LM-cut and SEQ constraints are complementary
- The OC heuristic using LM-cut and SEQ constraints is state-of-the-art

Conclusion

- Generalizing admissible heuristics to numeric planning is interesting; straightforward approaches yield NP-hardness or inadmissibility
- Future work: classifying existing admissible heuristics and analyzing the relationships such as dominance