S*: A Heuristic Information-Based Approximation Framework for Multi-Goal Path Finding

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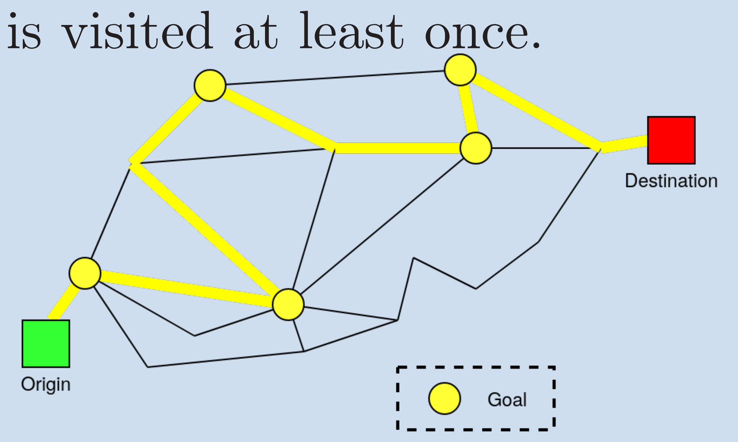
Introduction

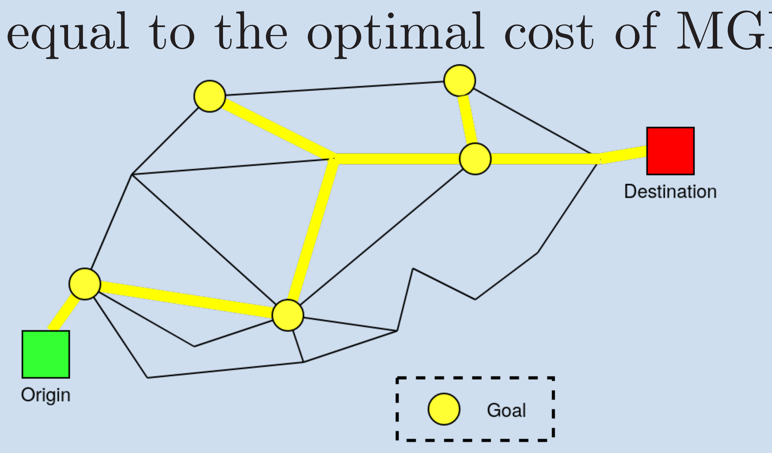
Multi-Goal Path Finding (MGPF) problem:

- **Given:** Graph $G = (V, E)$ with non-negative edge costs, origin $s \in V$, destination $d \in V$, and goals $\bar{T} \subseteq V$.
- **Task:** Find a least-cost path from origin s to destination d such that each goal in $\bar{T}$ is visited at least once.

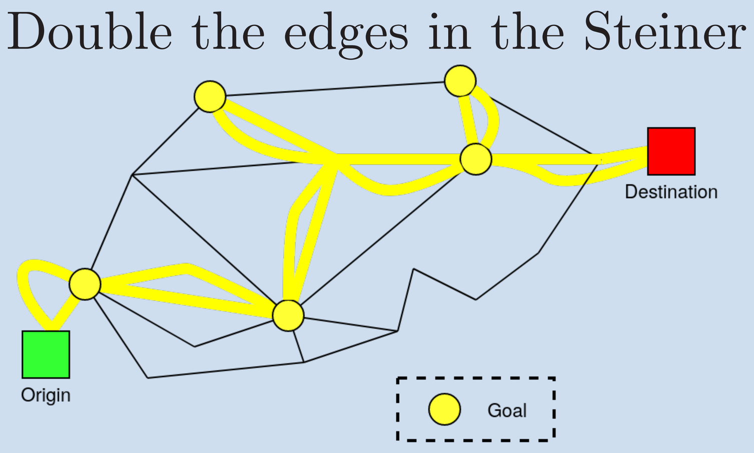


2-Approximation Algorithm

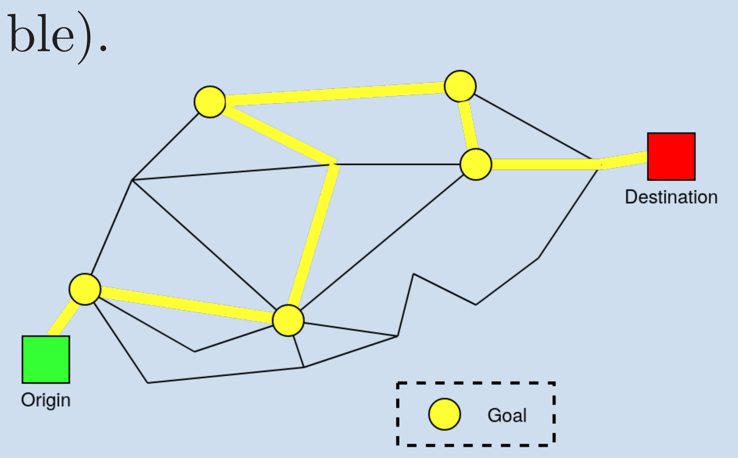
1. Find a Steiner tree spanning the origin, destination, and goals such that the sum of the edge costs in the tree is at most equal to the optimal cost of MGPF.



2. Double the edges in the Steiner tree.



3. Find a feasible path for MGPF by pruning edges and performing shortcuts (if possible).



Simulations Setup

- 4 variants of **S*:** **S*-unmerged** (uni-directional search), **S*-HS** (uses Pohl's *fmin* criteria), **S*-BS** (uses Nicholson's *gmin* criteria), **S*-MM** (uses Meet-in-the-middle criteria).
- Baseline: Kruskal's Algorithm (requires all pairs shortest path).
- Measured number of expanded nodes and computation time.
- 5 grid map environments from MAPF library.
- Heuristics scaled by $w \in [0, 1]$.
- Up to 50 terminals.

Key Contributions

- One way to build the Steiner tree is to use a *Primal-Dual* Algorithm, which generalizes Uni- and Bi-directional search (available for the shortest path problem) when $\bar{T}$ is an empty set.
- Our new framework Steiner* (S*) fuses the Primal-Dual algorithm with heuristic information; therefore, S* generalizes A*/Bi-HS to Steiner tree problems and MGPF.

Table 1

Uni-directional Search	+ Heuristics =	A*
Bi-directional Search	+ Heuristics =	Bi-directional Heuristic Search
Primal-Dual	+ Heuristics =	Steiner* (S*)

Properties satisfied by the S* framework

Property 1 (Shortest Path). *When a path P is declared between a pair of nodes, it is indeed the least-cost path between them.*

Property 2 (Kruskal). *When a path P is added to S_T (Steiner tree solution set), it obey's Kruskal's condition, namely all paths with cost less than cost of P have been considered and no cycles can be induced.*

S*

Initialization:

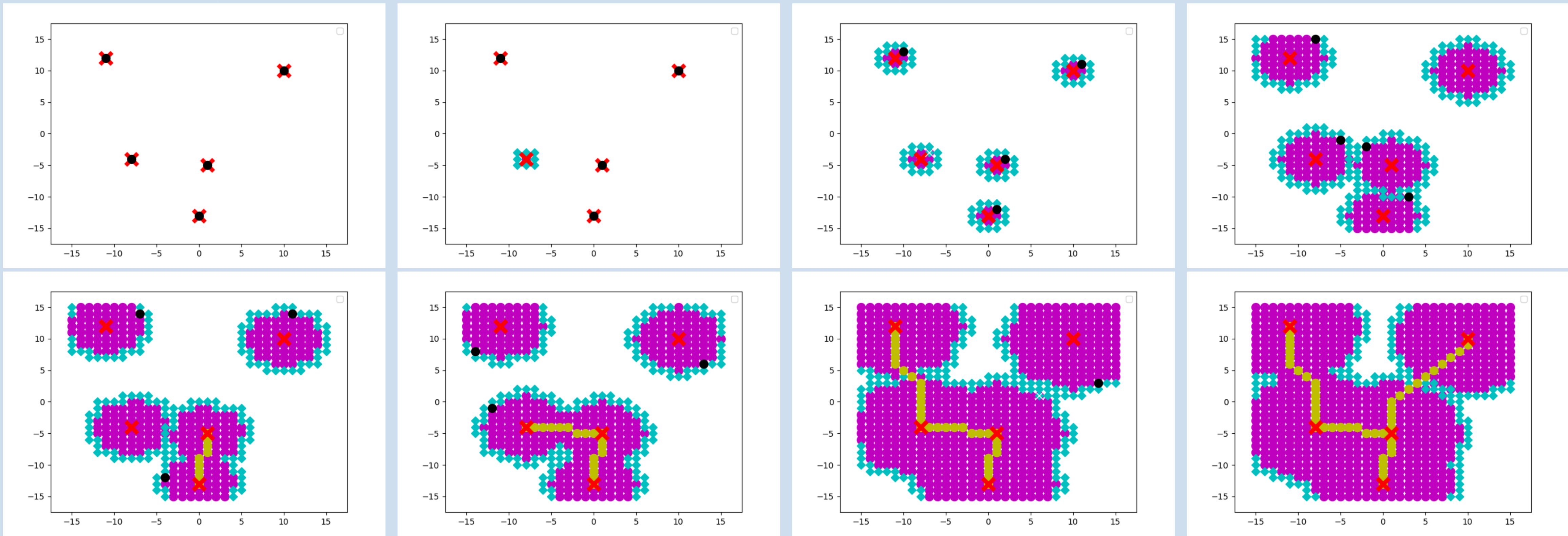
$G = (V, E)$ \\Input graph

$T = \{s, d\} \cup \bar{T}$ \\Does not distinguish between origin, destination, or goals; called *terminals*

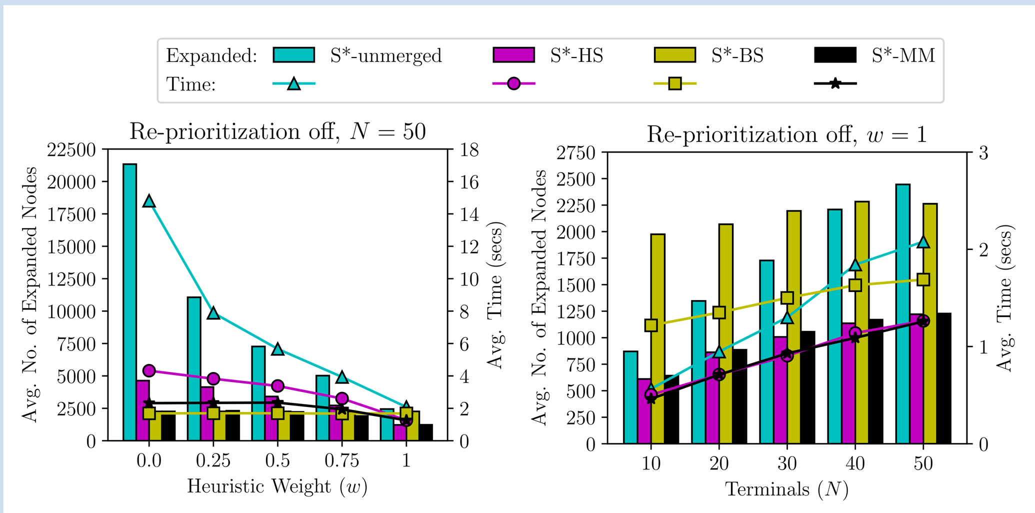
$S_T = \emptyset$ \\Steiner tree solution set

Create $|T|$ components \\a data structure that behaves like an A* search starting from each terminal

Main loop (Repeat until Steiner tree is obtained): Nominate, Update, Path, and Tree



Results



- S*-unmerged does not scale as well as S*-merged.
- S*-BS performs well when heuristics are weak.
- S*-HS performs well when heuristics are strong.
- S*-MM beats S*-BS when heuristics are strong, but loses slightly when heuristics are weak. Overall best performer.

Conclusion

- A new approximation framework for the MGPF problem which successfully:
  - combines the Primal-Dual algorithm with heuristic-based search for a **faster** steiner tree construction.
  - generalizes the uni- and bi-directional search procedures to the MGPF.
- Future Work:
  - Apply other bi-directional termination criteria (aside from fmin, gmin, and meet-in-the-middle).
  - Generalize framework to other multi-robot routing problems.
  - Consider asymmetric edge weights.