Approximate bi-criteria search by efficient representation of subsets of the Pareto-optimal frontier

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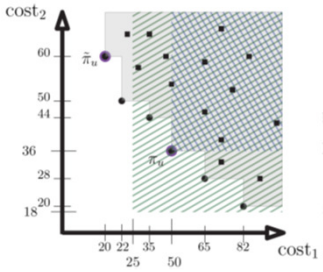
(approximate) Bi-criteria shortest path

- **Input**: a graph G two cost functions c_1, c_2 over the edges and start and goal vertices on the graph (and an approximation factor ε)
- **Task:** compute the (ε - approximate) Pareto-optimal frontier
  - **(approximate) Pareto-optimal frontier:** set of paths from start to goal that
    - (i) are not dominated by any other path and
    - (ii) collectively (ε-) dominate any other path



Each path is a 2D point according to the two cost functions

The set of all paths that are dominate and ε-dominated by π_u are shown in blue and green, respectively

(approximate) Pareto-optimal frontier: (purple) black dots

- **Motivation:** Planning of power-transmission lines , transporting hazardous material in order to balance between minimizing the travel distance and the risk of exposure for residents and more.

Background & Related work

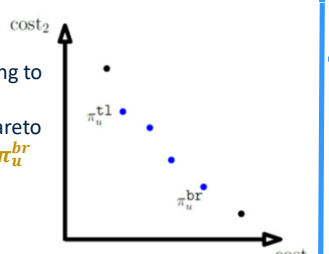
- Computing the Pareto-optimal frontier is NP−hard [Serafini 1987]
- Existing algorithms either try to
  - Efficiently compute the Pareto-optimal frontier [Hernandez et al. 2020] -> may returns a solution whose size is exponential in the input size [Ehrgott 2005]
  - Relax the problem and only compute an approximation of this set [Breugem et al. 2017] -> are often slower in practice than exact approaches

Contribution

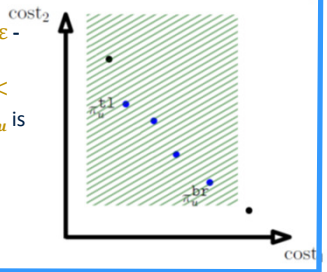
- Our key insight is that the Pareto-optimal frontier can be approximated using **pairs of paths**.
- This simple observation allows us to run a best-first search while efficiently and effectively pruning away intermediate solutions in order to obtain an approximation of the Pareto frontier for any given approximation factor.

Path pairs

- A **path pair** (π_u^{tl}, π_u^{br}) corresponding to two points on the Pareto frontier of vertex u represents the subset of Pareto frontier solutions between π_u^{tl} and π_u^{br}

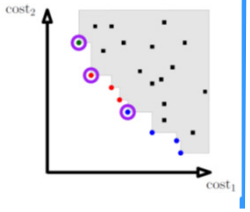


- A path pair (π_u^{tl}, π_u^{br}) is said to be ε - **bounded** if for any path π_u on the Pareto frontier such that $c\_1(\pi_u^{tl}) < c_1(\pi_u) < c_1,(\pi_{br})$ it holds that π_u is ε - dominated by π_u^{tl} and π_u^{br}



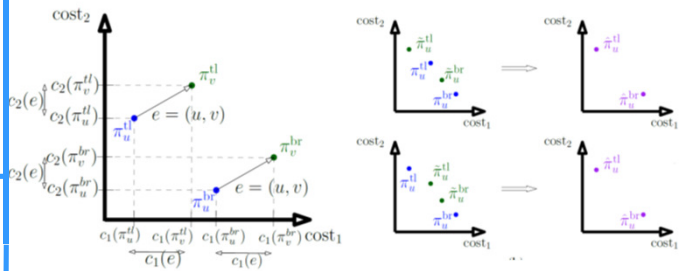
Algorithmic approach - PPA*

- We run an A*-like algorithm with path pairs as nodes
- When possible (resultant path pair is ε-bounded), we **merge** path pairs
- Solution returned contains **one** path in each path pair



Operations on path pairs

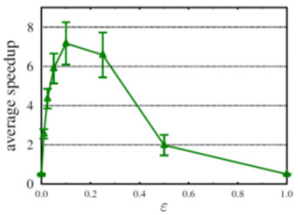
- **Extending** a path pair
- **Merging** two path pairs

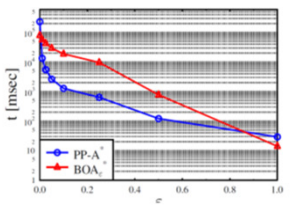


Evaluation

- Evaluation performed on roadmaps from the 9th DIMACS Implementation Challenge : Shortest path
- PPA* was compared with BOA*$_\varepsilon$ an adaptation of BOA* [Hernandez et al. 2020]
- All code publicly available at https://github.com/CRL-Technion/path-pair-graph-search

North East (NE) 1,524,453 states, 3,897,636 edges						
ε	avg t		min t		max t	
	PP-A*	BOA*	PP-A*	BOA*	PP-A*	BOA*
0	192.6	59.5	0.04	0.02	2,4189.9	592.6
0.01	13.1	68.3	0.03	0.01	111.6	600.9
0.025	5.6	57.3	0.02	0.01	46.9	510.9
0.05	2.7	40.8	0.02	0.01	22.6	345.1
0.1	1.3	25.8	0.02	0.01	9.0	229.8





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