A Competitive Analysis of Online Multi-Agent Path Finding*

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Abstract

We study online Multi-Agent Path Finding (MAPF), where new agents are constantly revealed over time and all agents must find collision-free paths to their given goal locations. We generalize existing complexity results of (offline) MAPF to online MAPF. We classify online MAPF algorithms into different categories based on (1) controllability (the set of agents that they can plan paths for at each time) and (2) rationality (the quality of paths they plan) and study the relationships between them. We perform a competitive analysis for each category of online MAPF algorithms with respect to commonly-used objective functions. We show that a naive algorithm that routes newly-revealed agents one at a time in sequence achieves a competitive ratio that is asymptotically bounded from both below and above by the number of agents with respect to flowtime and makespan. We then show a counter-intuitive result that, if rerouting of previously-revealed agents is not allowed, any rational online MAPF algorithms, including ones that plan optimal paths for all newly-revealed agents, have the same asymptotic competitive ratio as the naive algorithm, even on 2D 4-neighbor grids. We also derive constant lower bounds on the competitive ratio of any rational online MAPF algorithms that allow rerouting. The results thus provide theoretical insights into the effectiveness of using MAPF algorithms in an online setting for the first time.

Online MAPF

Problem: Find collision-free paths for a stream of m agents on a connected undirected graph.

- Agent a_i: start vertex s_i, goal vertex g_i, and release time r_i. Agent a_i appears at r_i and can be added to G at any time (only) since then. Agent a_i is removed from G upon arriving at g_i at *arrival time* $t_i^{(g)}$.
- Agents need to avoid collisions only when on G.

A *plan* for a set $\mathcal{A}$ of agents consists of a path π_i assigned to each agent $a_i \in \mathcal{A}$.

Objective functions:

1. The *flowtime* $\sum_{a_i \in \mathcal{A}} (t_i^{(g)} - r_i)$ is the sum of the *service times* $t_i^{(g)} - r_i$.
2. The *makespan* $\max_{a_i \in \mathcal{A}} t_i^{(g)}$ is the maximum of the arrival times of all agents in $\mathcal{A}$.
3. The *latency* $\sum_{a_i \in \mathcal{A}} (t_i^{(g)} - r_i - dist_i)$ is the sum of the differences between the service times $t_i^{(g)} - r_i$ and the sortest path distances $dist_i$ from s_i to g_i of all agents in $\mathcal{A}$.

Previous theoretical results are only for Offline MAPF (see survey paper [2]):

- All agents have release time 0.
- Agents remain on G (= need to avoid collisions) all the time.

Complexity Results

Theorem. *For any $\epsilon > 0$, it is NP-hard to find a $4/3 - \epsilon$-approximate solution to online MAPF for makespan minimization, even if all agents are known a priori.*

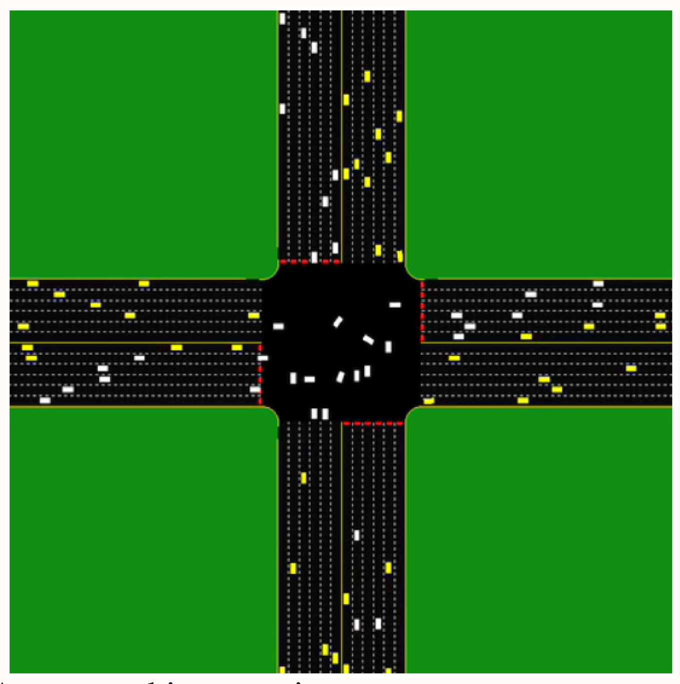
Corollary. *It is NP-hard to find an optimal solution to online MAPF for flowtime minimization, even if all agents are known a priori.*

Results are generalization of [1]:

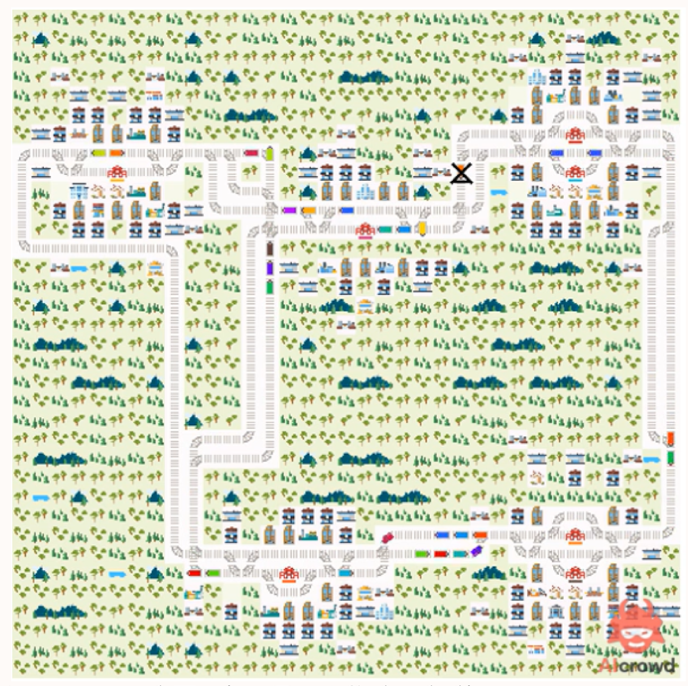
- Same reduction from a specialized NP-complete version of SAT.
- Insight: An (Offline) MAPF instance can be viewed as an online MAPF instance where all agents have release time 0.

Applications

Automated warehouse.
[Source: Alibaba Cainiao and Amazon]



Automated intersection management.
[Source: LARG, UT Austin]

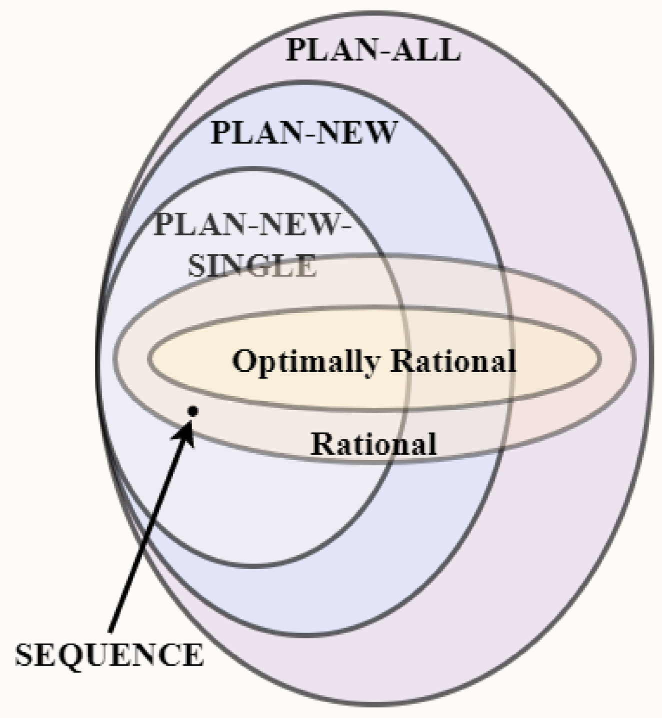


Automated Train (Re-)Scheduling.
[Source: Flatland Challenge]

Categorizations of Online MAPF Algorithms

Two orthogonal categories:

1. **Controllability** (≈ Which agents can be planned):
   - **PLAN-NEW-SINGLE** plans for only one newly-revealed agent at a time.
   - **PLAN-NEW** can plan for all newly-revealed agents.
   - **PLAN-ALL** can (re-)plan for all (previously- and newly-)revealed agents.
2. **Rationality** (≈ Effectiveness of the planner):
   - **SEQUENCE:**
     – Dummy baseline algorithm.
     – Routes one newly-revealed agent at a time in sequence – no two agents on the graph at the same time.
   - **Rational** algorithms:
     – Conceptual class of algorithms.
     – Flowtime and makespan are no worse than certain analytical bounds (to make sure they are asymptotically no worse than ones by SEQUENCE) at each release time.
     – Resulting solutions are guaranteed to be asymptotically no worse than ones by SEQUENCE.
   - **Optimally-rational** algorithms:
     – An optimal planner is employed.



Competitiveness Results

Controllability			PLAN-NEW-SINGLE		PLAN-NEW		PLAN-ALL	
Objective Function	Competitive Ratio Bounds	SEQUENCE	Rational	Optimally Rational	Rational	Optimally Rational	Rational	Optimally Rational
flowtime	upper	$\mathcal{O}(m)$	$\mathcal{O}(m)$					
	lower (even on 2D grids)	$\Omega(m)$					4/3	
makespan	upper	$\mathcal{O}(m)$	$\mathcal{O}(m)$					
	lower (even on 2D grids)	$\Omega(m)$					3/2	
latency	(even on 2D grids)	∞						

Summary

- Two orthogonal categorizations that cover all existing online MAPF algorithms.
- First theoretical (complexity and competitive) analysis of online MAPF.
- Counter-intuitive competitiveness results.
  – Disallowing rerouting (PLAN-NEW/PLAN-NEW-SINGLE): Planning for multiple agents is asymptotically (only) as effective as SEQUENCE, even with an optimal planner.
  – Allowing rerouting (PLAN-ALL): Can potentially result in high effectiveness, indicated by the gap between the upper and lower bounds on the competitive ratio.

References

[1] H. Ma, C. Tovey, G. Sharon, T. K. S. Kumar, and S. Koenig. Multi-agent path finding with payload transfers and the package-exchange robot-routing problem. In *AAAI*, pages 3166–3173, 2016.

[2] Roni Stern, Nathan Sturtevant, Ariel Felner, Sven Koenig, Hang Ma, Thayne Walker, Jiaoyang Li, Dor Atzmon, Liron Cohen, T. K. Satish Kumar, Eli Boyarski, and Roman Bartak. Multi-agent pathfinding: Definitions, variants, and benchmarks. In *SoCS*, pages 151–159, 2019.