

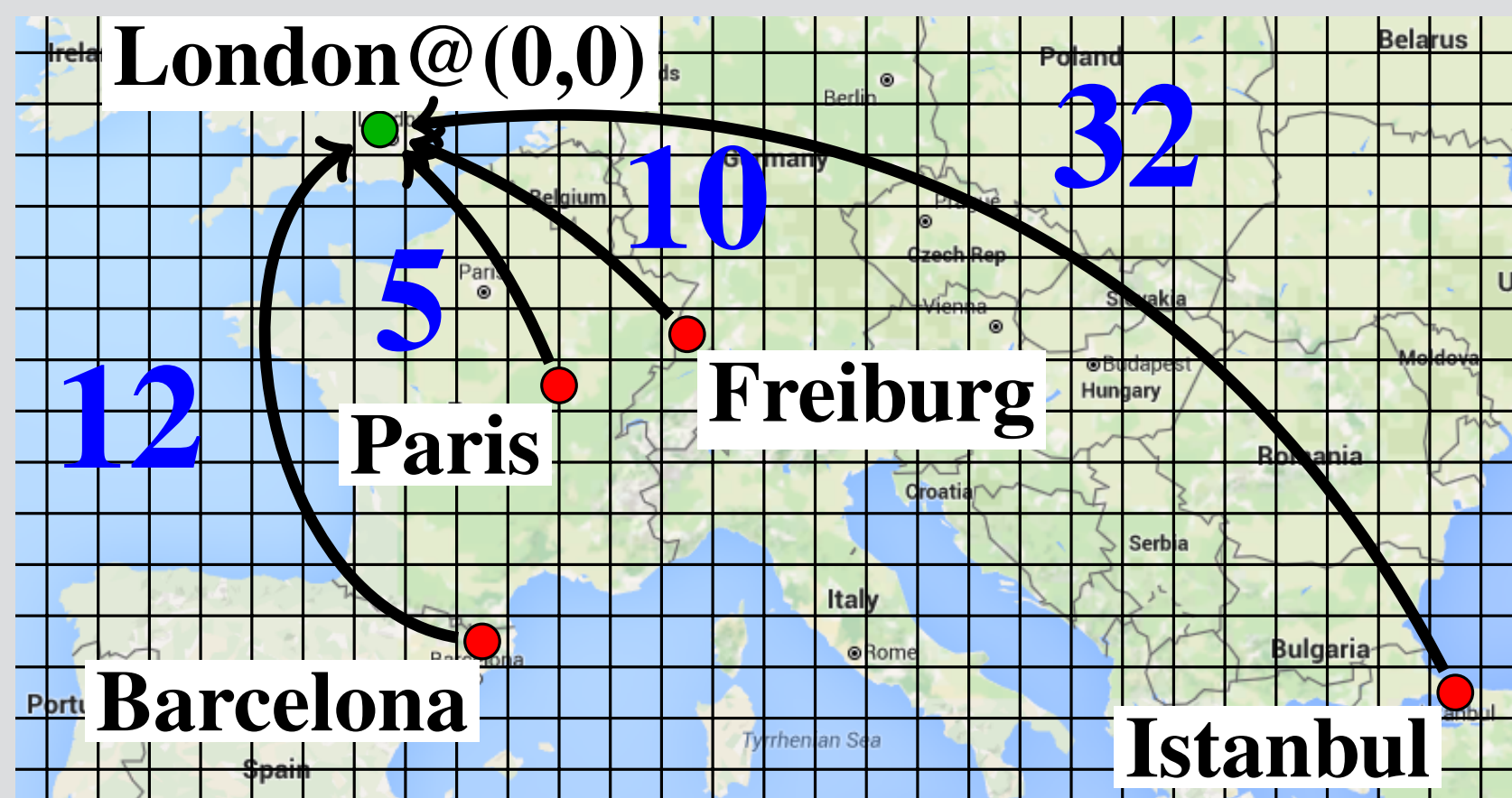
# On the Compilability and Expressive Power of State-Dependent Action Costs

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## Motivation



- Planning with **state-dependent action costs** (SDAC)

### Open Questions!

- How **computationally hard** is planning with SDAC?
- What is the **expressive power** of SDAC?
- Can SDAC simply be **compiled away**?

## Planning with SDAC

An **SDAC planning task** is a 5-tuple  $\langle \mathcal{P}, \mathcal{A}, \mathcal{C}, \mathcal{I}, \mathcal{G} \rangle$ .

- $\mathcal{P}$ : finite set of **propositional atoms**
- $\mathcal{A}$ : finite set of **actions**  $a = \langle pre(a), eff(a) \rangle \in \mathcal{A}$
- $\mathcal{C} : \mathcal{S} \times \mathcal{A} \mapsto \mathbb{N}_0$ : **state-dependent** action cost function
- $\mathcal{I}$ : **initial state** over  $\mathcal{P}$
- $\mathcal{G} \subseteq \mathcal{P}$ : **goal** description

### SDAC Plan

- Applicable **sequence of actions**  $\pi = \langle a_0, \dots, a_n \rangle$
- Starting in  $s_0 = \mathcal{I}$  and resulting in  $s_n \in \mathcal{G}$
- **Plan cost**  $c(\pi) = \sum_{i=0}^{n-1} \mathcal{C}(s_i, a_i)$

## Complexity Results and Compilability

Planning with SDAC is **PSPACE-complete** if the **cost function** is in FPSPACE.

- **Hardness**: reduction of planning with unit costs
- **Membership**: nondeterministic Turing Machine
- **Same** complexity  $\nRightarrow$  **same** expressive power
  - Metric: **conciseness** of **compilation**

### Compilation Schemes

- **Translation** from one formalism to another
  - SDAC tasks  $\rightsquigarrow$  **constant cost** tasks
- Preservation of
  - polynomial **task size**
  - **plan existence** and **plan cost**

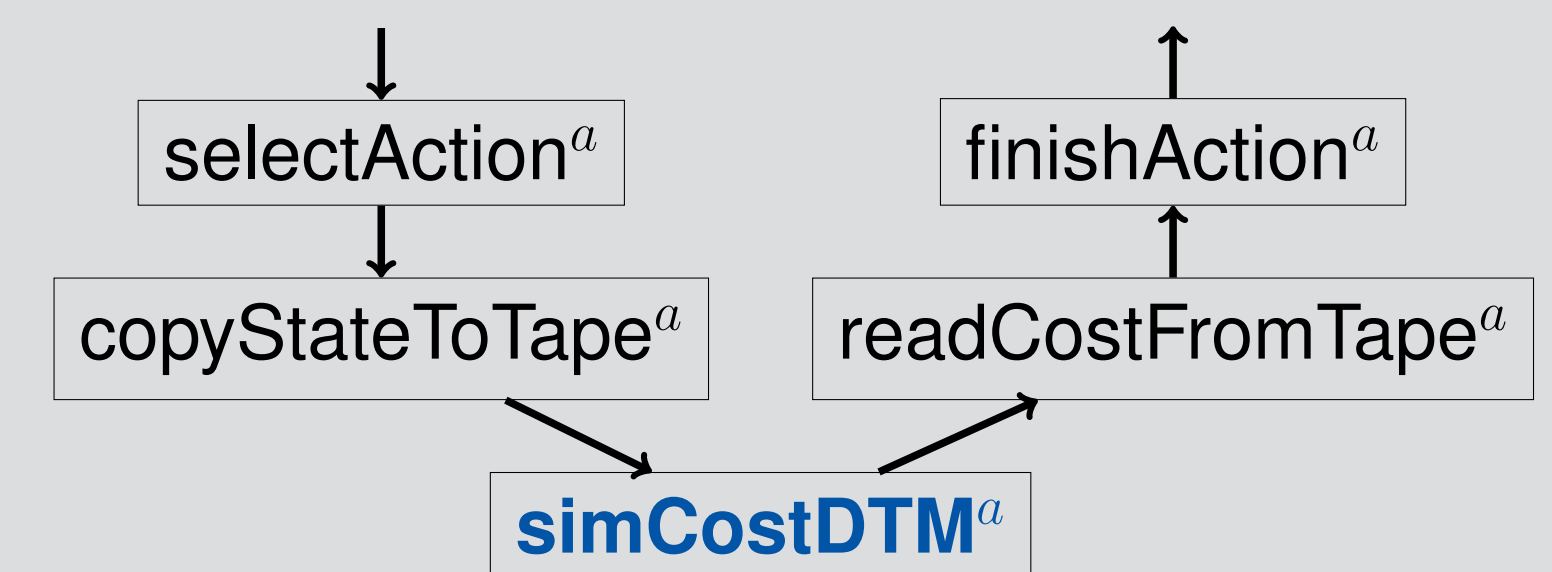
Is it **possible** to preserve the **plan length exactly**, **linearly**, or **polynomially**?

## Compilability – Possibility Results

- Compile **each action**  $a \in \mathcal{A}$  with cost  $\mathcal{C}(a, s)$
- **Simulate** a **DTM** computing  $\mathcal{C}(a, s)$  within the task
- **Size** of new task is bounded by **space of the DTM**

- Cost function in **FP**  $\rightsquigarrow$  **polynomial** plan length
- Cost function in **FPSPACE**  $\rightsquigarrow$  **any** plan length

Compilation of Action  $a$ :



## Impossibility Results

- SDAC tasks with **one action**  $a$
- **Cost** of  $a$  **encodes instance** of
  - Parity (FP) or QBF (PSPACE)

With compilation **restrictions**:

- $\rightsquigarrow$  **Contradiction**
- $\rightsquigarrow$  **PH collapses at 3rd level**

## Overview

|              |         | Desired plan length preservation                         |                                       |                                       |
|--------------|---------|--|---------------------------------------|---------------------------------------|
|              |         | linearly   | polynomially                          | don't care                            |
| C complexity | FP      | <b>impossible</b>  | <b>possible</b> using DTM compilation |                                       |
|              | FPSPACE | <b>impossible</b> (unless PH collapses at the 3rd level) |                                       | <b>possible</b> using DTM compilation |