25.03.25

Causality: $\Delta_i = Y_i(1) - Y_i(0)$

missing at least one of them

Estimand: $\tau = ATE \triangleq E[Y(1) - Y(0)]$

Estimator: $\hat{\tau}_{DM} = \frac{1}{n_1}\sum_{W_i=1} Y_i - \frac{1}{n_0}\sum_{W_i=0} Y_i$

① $\hat{\tau} \to_p \tau$ WLLN ② $\sqrt{n}(\hat{\tau} - \tau) \Rightarrow N(0, V)$

sooth consistency; CLT

(i) i.i.d ; (ii) RCT $Y_i(1), Y_i(0) \perp\!\!\!\perp W_i$

(iii) SUTVA

gold standard

Interating LR more efficience than DM

RCT violated: (a) Unconfoundedness + $Y_i(1), Y_i(0) \perp\!\!\!\perp W_i \mid X_i$

(b) $\eta \le e(x) \le 1-\eta$ overlapping + $e(x) = Pr[W_i = 1 \mid X_i = x]$

SUTVA

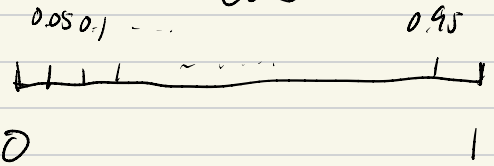
$X_i \in \{x_1, x_2, \cdots x_k\}$

$$\hat{\tau}_k = \frac{1}{n_{k,1}} \sum_{\substack{w_i=1 \\ x_i=x_k}} Y_i - \frac{1}{n_{k,0}} \sum_{\substack{w_i=0 \\ x_i=x_k}} Y_i \longrightarrow \tau_k = \mathbb{E}[Y_i(1) - Y_i(0) \mid X_i = x_k]$$

$$\hat{\tau}_{STRAT} = \sum_{k=1}^{K} \frac{n_{k,1}+n_{k,0}}{n} \hat{\tau}_k \longrightarrow \tau = ATE$$

$\hat{e}(\cdot)$

$e(x) = P[W_i = 1 \mid X_i = x]$ PSTRATI



$(\hat{\tau}_{IPW} - \tau) \sim n^{-\frac{1}{2}}$ $\quad$ $(\hat{\tau}^*_{IPW} - \tau) \sim n^{-\frac{1}{2}}$

$$\hat{\tau}_{IPW} - \tau = \hat{\tau}_{IPW} - \hat{\tau}^*_{IPW} + \hat{\tau}^*_{IPW} - \tau$$

$\hat{\tau}_{IPW} - \hat{\tau}^*_{IPW}$: $o(n^{-\frac{1}{2}})$

$\hat{\tau}^*_{IPW} - \tau$: $\sim n^{-\frac{1}{2}}$, CLT