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PLM:

$$\begin{cases} Y = D\cdot\theta_0 + g_0(X) + U & E[U\mid D, X] = 0 \\ D = m_0(X) + V & E[V\mid X] = 0 \end{cases}$$

X: Observable Confounders

D: Treatment Variable

θ_0: parameter of interest

$\eta_0 = (g_0, m_0)$: nuisance parameter can be learned by ML

$$\left(E\|g_0(X) - \hat{g}_0(X)\|^2\right)^{\frac{1}{2}} = o(n^{-\frac{1}{4}})$$

$$\left(E\|m_0(X) - \hat{m}_0(X)\|^2\right)^{\frac{1}{2}} = o(n^{-\frac{1}{4}})$$

Plug-in: $\boxed{\text{Train} \mid \text{Estimate}}$

$Y \sim D\hat{\theta} + \hat{g}_0(X)$

$Y - \hat{g}_0(X) \sim \hat{a} + D\cdot\hat{\theta}_0$ OLS

$\sqrt{n}\,|\hat{\theta}_0 - \theta_0| \to +\infty$

Train	Estimate
$Y \sim D\tilde{\theta}_0 + \hat{g}_0(x)$	$\check{\theta}_0 = \left(\frac{1}{n}\sum \hat{V}_i D_i\right)^{-1} \frac{1}{n}\sum \hat{V}_i (Y_i - \hat{g}_0(x))$
$D \sim \hat{m}_0(x)$	$\hat{V}_i = D_i - \hat{m}_0(X_i)$

$$\sqrt{n}(\check{\theta}_0 - \theta_0) \Rightarrow N_s(0, V_{DML})$$

$$\psi(W, \theta, \eta) = (g(1,x) - g(0,x)) + \frac{D(Y - g(1,x))}{m(x)} - \frac{(1-D)(Y - g(0,x))}{1 - m(x)} - \theta$$

$$\partial_\eta \mathbb{E}\psi \Big|_{(\theta=\theta_0, \eta=\eta_0)} = 0 \quad (NO)$$

$$\eta = (g(1,x), g(0,x), m(x))$$

$$\mathbb{E}\,\partial_\eta \psi = \mathbb{E}\begin{bmatrix} 1 + \frac{-D}{m(x)} \\ -1 - \frac{-(1-D)}{1-m(x)} \\ \frac{-D(Y - g(1,x))}{m^2(x)} + \frac{(1-D)(Y - g(0,x))}{(1-m(x))^2} \end{bmatrix} \Bigg|_{(g_0, m_0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$