

25.04.01

When RCT is missing

$$\hat{\tau}_{AIPW} = \frac{1}{n} \left(\sum_{i=1}^n \hat{\mu}_1(x_i) - \hat{\mu}_0(x_i) \right) + \frac{w_i(Y_i - \hat{\mu}_1(x_i))}{\hat{e}(x_i)} - \frac{(1-w_i)(Y_i - \hat{\mu}_0(x_i))}{1 - \hat{e}(x_i)}$$

$$\hat{\mu}_1(x) \approx E[Y_i | W_i=1, X_i=x]$$

$$\hat{\mu}_0(x) \approx E[Y_i | W_i=0, X_i=x]$$

$$[E(\hat{\mu}_j(x) - \mu_j(x))^2]^{\frac{1}{2}} = o(n^{-\frac{1}{4}})$$

$$\sqrt{n}(\hat{\tau}_{AIPW} - \tau) \Rightarrow N(0, V)$$

$$[\hat{\tau}_{AIPW} - \Phi^{-1}(1 - \frac{\alpha}{2}) \hat{V}_{AIPW}, \hat{\tau}_{AIPW} + \Phi^{-1}(1 - \frac{\alpha}{2}) \hat{V}_{AIPW}]$$

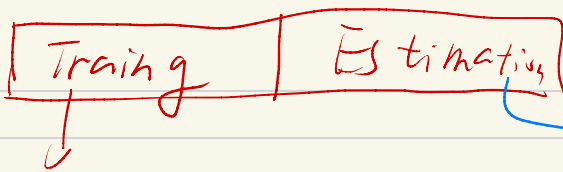
AIPW is "efficient", \therefore sample splitting

① Unconfoundedness
 $Y_i(0), Y_i(1) \perp W_i | X_i$

② Overlapping
 $\eta \leq e(x) = P[W_i=1 | X_i=x] \leq 1 - \eta$

③ SUTVA.
 $Y_i = Y_i(W_i)$

④ $(E[e(x) - e(x)]^2)^{\frac{1}{2}} = o(n^{-\frac{1}{4}})$

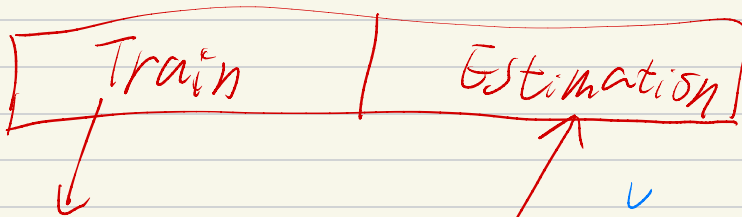


$$Y - \hat{g}_0(x) \stackrel{\text{as}}{\sim} a + D$$

$$\text{Train } Y \sim \hat{\theta}_0' D + \hat{g}_0(x)$$

Plug in Estimator

DM (Estimator for PLM)



$$Y \sim \hat{\theta}_0' D + \hat{g}_0(x)$$

$$D \sim \hat{m}_0(x); \hat{V} = D - \hat{m}_0(x)$$

$$\hat{\theta}_{0, \text{PLM}} = \left(\frac{1}{n} \sum D_i \hat{V}_i^2 \right)^{-1} \left(\frac{1}{n} \sum \hat{V}_i^2 (Y_i - \hat{g}_0(x_i)) \right)$$