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When RCT is missing

① Unconfoundedness
$Y_i(1), Y_i(0) \perp\!\!\!\perp W_i \mid X_i$

② Overlapping
$\eta \le e(x) = P[W_i = 1 \mid X_i = x] \le 1-\eta$

③ SUTVA
$Y_i = Y_i(W_i)$

④ $\left(\mathbb{E}[\hat{e}(x) - e(x)]^2\right)^{\frac{1}{2}} = o(n^{-\frac{1}{4}})$

$$\hat{\tau}_{AIPW} = \frac{1}{n}\left(\sum_{i=1}^{n} \hat{\mu}_1(X_i) - \hat{\mu}_0(X_i) + \frac{W_i(Y_i - \hat{\mu}_1(X_i))}{\hat{e}(X_i)} - \frac{(1-W_i)(Y_i - \hat{\mu}_0(X_i))}{1-\hat{e}(X_i)}\right)$$

$$\hat{\mu}_1(x) \approx \mathbb{E}[Y_i \mid W_i = 1, X_i = x]$$

$$\hat{\mu}_0(x) \approx \mathbb{E}[Y_i \mid W_i = 0, X_i = x]$$

$$\left[\mathbb{E}\left(\hat{\mu}_j(X) - \mu_j(X)\right)^2\right]^{\frac{1}{2}} = o(n^{-\frac{1}{4}})$$

$$\sqrt{n}\left(\hat{\tau}_{AIPW} - \tau\right) \Rightarrow N(0, V)$$

$$\left[\hat{\tau}_{AIPW} - \Phi^{-1}\left(1-\frac{\alpha}{2}\right)\hat{V}_{AIPW},\ \hat{\tau}_{AIPW} + \Phi^{-1}\left(1-\frac{\alpha}{2}\right)\hat{V}_{AIPW}\right]$$

AIPW is "efficient" ; Sample splitting

Training	Estimation

Train $Y \sim \hat{\theta}_0^1 \cdot D + \hat{g}_0(X)$

$Y - \hat{g}_0(X) \overset{\text{OLS}}{\sim} \alpha + D$

Plug in Estimator

DML Estimator for PLM

Train	Estimation

$Y \sim \hat{\theta}_0^1 D + \hat{g}_0(\cdot)$

$D \sim \hat{m}_0(\cdot)$; $\hat{V} = D - \hat{m}_0(X)$

$$\check{\theta}_{0,DML} = \left(\frac{1}{n}\sum_i D_i \hat{V}_i\right)^{-1} \left(\frac{1}{n}\sum_i \hat{V}_i \left(X_i - \hat{g}_0(X_i)\right)\right)$$