25.04.15

{ ① Unconfoundedness
② Overlapping
③ SUTVA

DML: $\psi(W, \theta, \eta)$

- Data
- parameter of interest ATE
- nuisance parameters learned w. ML
  $\|\hat{\eta} - \eta_0\|_2 = o(n^{-\frac{1}{4}})$

(a) zero-order moment condition

$$E\psi(W, \theta_0, \eta_0) = \vec{0}$$

(b) Neyman Orthogonality

$$\partial_\eta E\psi(W, \theta, \eta)\Big|_{(\theta_0, \eta_0)} = \vec{0}$$

$$\Updownarrow$$

$$E\left[\partial_\eta \psi\Big|_{(\theta_0, \eta_0)}\right] = 0$$

Train | Estimation

$\hat{\eta}_0$

$$\sum_i \frac{1}{n} \psi(W_i, \hat{\theta}_0, \hat{\eta}_0) = 0$$

$$\Downarrow$$

$\hat{\theta}_0$

$$\sqrt{n}(\hat{\theta}_0 - \theta_0)$$

$$\Rightarrow N(0, V_{DML})$$

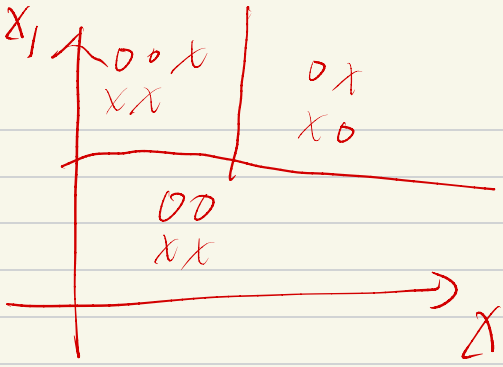
$$\frac{1}{N}\sum \psi(W_i, \hat{\theta}_0, \hat{\eta}_0)^2$$

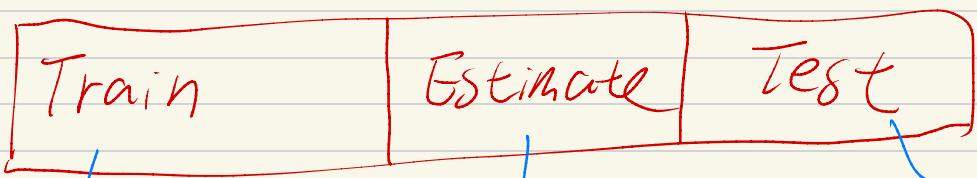
$\tau = ATE$

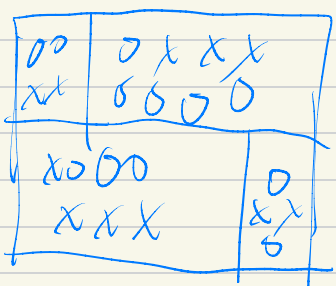
$\theta_0 = \tau = ATE$

$(\mu_1, \mu_0, e)$

$$\psi_{AIPW} = \mu_1(\cdot) - \mu_0(\cdot) + \frac{D(Y - \mu_1(\cdot))}{e(\cdot)} - \frac{(1-D)(Y - \mu_0(\cdot))}{1 - e(\cdot)} - \tau$$



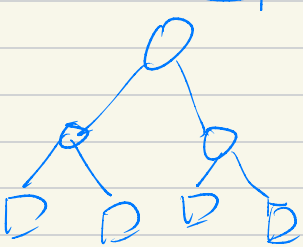



$$\hat{y}_{leaf}^{treat} = \frac{1}{|leaf|} \sum_i y_{i,treat}^{est}$$

$$\sum_i (y_i - \hat{y}_{leaf})^2$$

$$\hat{y}_{leaf}^{control} = \frac{1}{|leaf|} \sum_i y_{i,control}^{est}$$



GRF + DML

$$\begin{cases} Y = \theta(x) \cdot D + g_0(x) + U \\ D = m_0(x) + V \end{cases}$$

| Training | Estimation |
|---|---|

(i): $\hat{a}(\cdot)$

(ii): $\hat{g}_0(\cdot)$

(iii): $\hat{m}_0(\cdot)$

$$\Rightarrow \sum_i \hat{a}(X_i)\psi^2(W_i, \hat{\theta}_0, \hat{\eta}_0)$$

is minimized at $\hat{\theta}_0$

$$\psi = \left(Y - \theta(x) \cdot D - g_0(x)\right) \cdot \left(D - m_0(x)\right)$$