

Diplôme d'Ingénieur

**École Supérieure des Technologies
Industrielles Avancées**

Master's Degree Thesis



DEVELOPMENT OF
PROGNOSTICS AND HEALTH MANAGEMENT (PHM)
DESIGN TECHNOLOGY

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Graduation Year: 2012

Internship period: 16 04 2012 – 20 10 2012

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ACKNOWLEDGEMENTS

Firstly, I would like to thank the CEO Chris Stecki, for providing me with the opportunity to undertake my thesis at PHM Technology Pty Ltd.

I would like to express my gratitude to my supervisor, Dr Jacek Stecki for his guidance and countless helpful suggestions throughout my thesis. Even though his three-month business trip in Europe challenged my quality of being fully autonomous on my project, I was lucky enough to receive advice from him *via* Internet calls.

I am also grateful to my supervisors in '*École Supérieure des Technologies Industrielles Avancées*', Jean Roch Guiress, Oliver Patrouix and Pascal Bagieu for their attention and making sure that my mission took place under the best possible conditions.

I also wish to thank my colleagues Dr Shanna Rudov-Clark, Cameron Weick and Adrian Lyons for their genuine support during my internship.

Special thanks to Dr Jean Esterle from '*Université Bordeaux 1*' and '*Ecole Supérieure des Technologies Industrielles Avancées*', who gave me the precious advice to use Laplace transform to solve the recursive integration to calculate the reliability of standby redundant systems.

Finally, my special thanks go to Professor Ian Jennions and Dr Octavian Niculita, who gave me an overview on Prognostics and Health Management research projects carried out at the Integrated Vehicle Health Management Centre of Cranfield University.

Melbourne, Victoria, August 2012

Romain Pinquié

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LIST OF ABBREVIATIONS

A, B, C, ...	Event A, B, C...
CA	Criticality Analysis
CBM	Condition Based Maintenance
FDI	Fault Detection and Isolation
F&DT	Fatigue and Damage Tolerance
$F_n(t)$	Probability of failure (<i>or unreliability</i>) of the item n
$F_s(t)$	Overall probability of failure (<i>or unreliability</i>) of the system
FMEA	Failure Mode and Effects Analysis
FMECA	Failure Mode, Effects and Criticality Analysis
IVHM	Integrated Vehicle Health Management
LRU	Line Replaceable Unit
OEM	Original Equipment Manufacturer
P	Probability
P(A)	Probability that an event A occurs
PHM	Prognostics and Health Management
RAM	Reliability Availability and Maintainability
RBD	Reliability Block Diagram
$R_n(t)$	Reliability of the component n
$R_s(t)$	Overall reliability of the system
SHM	Structural Health Monitoring
t	Duration of operation (<i>or mission time</i>)
T or TTF	Time To Failure
TOC	Total Ownership Costs

1 Introduction

*"For want of a nail, the shoe was lost;
For want of a shoe, the horse was lost;
For want of a horse, the rider was lost;
For want of a rider, the battle was lost;
For want of a battle, the kingdom was lost!"*

*How would you control the loss of a nail?
Is there more you can do?*

Greenfield M. A. (2002), NASA Langley Research Center

The two basic components of Prognostics and Health Management (PHM) are:

- ❖ Prognostics - predictive diagnostics to determine the remaining life or time span of proper operation of a component.
- ❖ Health Management - the capability to make appropriate decisions about maintenance actions based on diagnostics prognostics information, available resources and operational demand.

PHM requires the tasks of fault failure detection and isolation (FDI), enhanced diagnostics, material condition assessment, performance monitoring, life tracking and residual life estimation can be achieved during operation of the system.

Failure Mode, Effects and Criticality Analysis (FMECA) and Reliability Block Diagram (RBD) are both powerful methods frequently used in proactive maintenance, which have been reviewed, improved and automated in MADe package.

The automated FMECA module includes an extensive failures database that removes the language-related problems encountered by FMECA practitioners and proposes a uniform framework for defining functions and failure concepts.

On the other hand, RBD tool is a system graphical representation that enables probabilistic calculations in order to evaluate the system's reliability using components' failure rates based on either tests or historical feedbacks.

1.1 Purpose and goal

This thesis is divided into two main topics. In a first hand, the purpose is to demonstrate the probabilistic calculations which have been carried out in order to validate the Reliability Block Diagram tool. In other words, the thesis shows the different analytical studies performed to check whether the algorithms have been properly implemented in MADe or not.

The second main part of this thesis presents the enhancement of the standardised taxonomy ensuring that functional and failure concept descriptions for structures are consistent. This is introduce throughout a general mission which was to research how the use of the MADe suite can improve the design of complex engineering structures at all stages of the design process; i.e. how MADe software can assess and manage risks from the conceptual design to the Structural Heath Monitoring (SHM) phase.

1.2 Literature Review

Prognostics and Health Management (PHM) or Integrated Vehicle Health Management (IVHM) is an emerging field bringing together the use of advanced risk assessment techniques and real-time monitoring to increase both reliability and availability of complex systems as well as to improve maintenance decisions.

It is broadly accepted that reliability engineering is a cornerstone for the development of cutting-edge technologies. The beginning of survival analysis was in 1662 with the early work on mortality by John Graunt (Jerenz, 2008). Then, the improvement of the V1 and V2 rockets during the World War II was a breakthrough in reliability engineering (Dhillon, 1999 and McLinn, 2011). Most of reference books introduce the Reliability Block Diagram (RBD) methodology. Although authors clearly present the theory for classic RBD configurations (Dhillon, 1999 and Guangbin, 2007), only some of them deal with Standby systems (Way and Ming, 2003). Indeed, several researches are currently undertaken in order to mimic the design of advanced products made up with numerous redundancies. Nowadays, reliability engineering is boosted by the use of stochastic algorithms in order to overcome the complexity due to tedious recursive integrations. Monte-Carlo algorithm is the Rolls Royce of reliability calculations since it can be applied to various system architectures and can also be easily implemented using any general programming language like JAVA, C, C++ or Visual Basic.

The second method to enhance the reliability of a system is to recognize that every part is designed to perform a number of functions. For complex systems, a combination of the functional and hardware approaches should be adopted (MIL-STD-1629A, 1980). The Failure Modes, Effects and Criticality Analysis (FMECA) is the most widely used reliability analysis technique in the initial stage of product development in order to eliminate potential failure modes at the component level. The origins of the FMECA method can be traced back to the early Military Procedure MIL-P-1629 '*Procedures for performing a Failure Mode, Effects and Criticality Analysis*' in 1949. The National Aeronautics and Space Administration (NASA) subsequently used the FMECA on the Apollo program in the 1960s. Initially, the technique was called FMEA, the C in FMECA indicates that the criticality of the failure modes is taken into consideration, however both are now considered as synonyms. Researches with regard to the efficiency of the FMECA method have shown several disadvantages. For example, Wirth, Bethold, Krämer and Peter highlight that the unconstrained use of natural language and the lack of methodological guideline lead to inconsistent descriptions of systems and functions and to FMECA knowledge that is hardly reusable. The FMECA technique plays a key role in many industries, e.g. military applications, civil aviation, automotive engineering, etc. Nevertheless, its use in the design process of complex engineering structure is blurred. Actually, the reliability of a structure is based on a deterministic design code such as safety margin; the use of conservative material properties; Fatigue & Damage Tolerance (F&DT) analysis; advanced numerical analysis and certification tests (Acar, 2006). Even using such a sophisticated theories and tools, catastrophic structural failures remain present. FMECA and the functional approach in general should play a more important role during the conceptual design of complex structures so as to identify, prioritise and eliminate the potential structural failures modes. Furthermore, the analysis of functional failures can be an asset for the achievement of the Structural Health Monitoring (SHM) process as it eases and speeds up the definition of the critical components as well as the selection of sensors.

1.3 Outline of the thesis

- *Chapter 2* presents the theoretical background necessary to understand the probabilistic reliability calculations.

- *Chapter 3* shows the results for most of the classic system configurations such as Series – Parallel – Series-Parallel – Parallel-Series – K-out-of-N and Complex groups. Moreover, at the end of this chapter, i.e. chapter 4-3-9 broaches the subject of the reliability evaluation of Standby groups. Some analytical and stochastic techniques are described hereafter and are currently for discussion.
- *Chapter 4* discusses the use of the functional modelling approach integrated into MADe with emphasis on structural integrity problems.
- *Chapter 5* summarises the work done through a conclusion but also suggest a list of future improvements and tasks that should be considered.
- *Appendix A* provides the code written in Visual Basic showing how compute the reliability of Standby systems using Monte-Carlo stochastic algorithm.
- *Appendix B* corresponds to a technical white paper that has been written to demonstrate how MADe can be used to carry out a FMECA for structural integrity problems. Additionally, another white paper will be shortly published for the PHM module. Both documents have been requested by Cassidian a subsidiary of the EADS corporation during the international PHM conference in Dresden, Germany.

2 PHM Technology

2.1 PHM Technology's History

A key innovation and requirement for the JSF Project was the mandating of PHM in order to deliver massive gains in reliability, safety, and cost control. In 2004 PHMT was approached by the JSF Joint Project Office (JPO) and asked to develop MADe to commercial-ready standard in return for funding of 1.75m US\$. PHM Technology (PHMT) successfully delivered the Aerospace PHM Software tool to the JPO in April 2009 and was proud to have the JSF Project as its first commercial customer.

2.2 Executive Summary

PHMT is an advanced engineering company that has created a decision support tool for designing and analysing complex engineering systems that complements the product lifecycle management approach.

The **Maintenance Aware Design** environment software (MADe) is a significant process improvement and cost risk mitigation tool for engineers responsible for system design, operation and maintenance of safety & mission critical systems.

2.3 Customers and Pipeline

PHMT is targeting Original Equipment Manufacturers as well as their supply chains developing increasingly complex mission and safety-critical systems by leveraging the virtual mandating of MADe for platforms or projects via supply chains. OEMs are increasingly assuming financial risk (known as Performance Based Contracting) based on the Total Ownership Costs (TOC) of systems where maintenance and logistics can represent 40-60% of total TOC.

Licenses are currently generated through direct sales. Current users in the EU, UK and US include the NASA, Sikorsky, Boeing, US Army – AMRDEC & AMSAA, JSF JPO, IVHM and CALCE. Early sales are expected with the Royal Australian Navy, Pratt & Whitney as well as BAE. Strategic relationships have been established in the UK with the Cranfield Integrated Vehicle Health Management centre (funded by BAE, Boeing,

& Rolls-Royce) and in the US with the Centre for Advanced Life Cycle Engineering ‘CALCE’ (funded by EADS, Lockheed Martin, & US DoD) and key US DoD engineering support contractors.

❖ Reference sites

MADe is currently being used or evaluated by the following organisations:

- Finland: Department of Intelligent Hydraulics and Automation (IHA), Tampere University of Technology (TUT), Tampere
- Italy: IMAMOTER - Institute for Agricultural and Earth-moving Machines, Italian National Research Council, Ferrara
- Poland: Institute of Computing Science, Faculty of Mechanical Engineering, Cracow University, Cracow
- UK: Centre for Integrated Vehicle Health Management, Cranfield University, Cranfield
- US: Centre for Life Cycle Engineering (CALCE), University of Maryland, Maryland
- US: Joint Project Office, Joint Strike Fighter program, Washington
- US: Palo Alto Research Centre (PARC), Palo Alto, California

2.4 PHM Technology Personnel

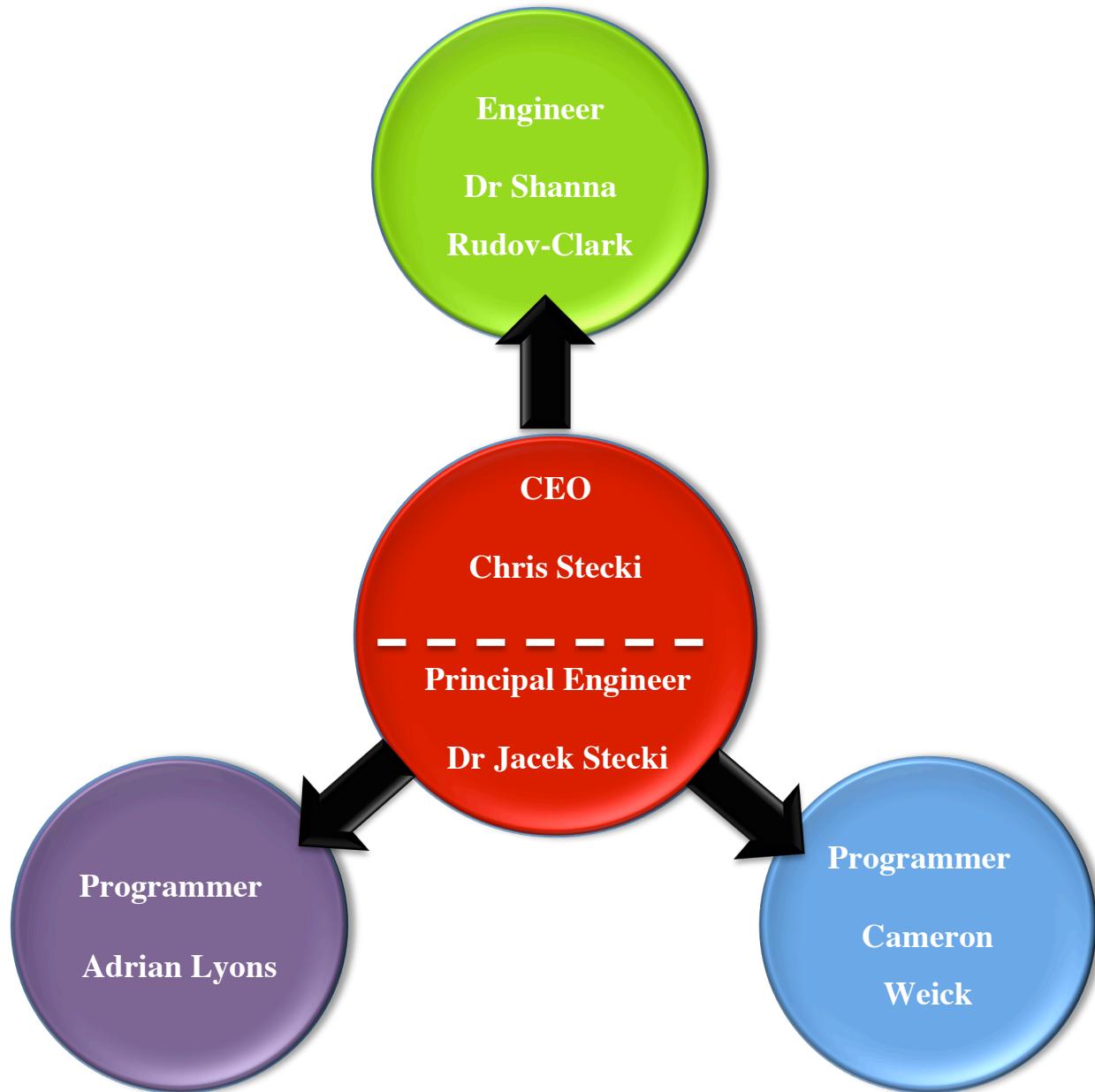


Figure 2-1 PHM Technology Personnel

Key Personnel:



Chris Stecki: is CEO and co-founder of PHM Technology Pty Ltd and has responsibility for corporate and commercialisation activities including strategic sales and marketing, channel development in the defence and aerospace sectors since 2006. He has co-authored a number of technical papers relating to PHM and the advanced techniques developed to support MADe and is a frequent presenter at industry conferences around the world.

CV :

- CEO of PHMT (since 2006)
- CEO of Swish Group Limited (2001-2002)
- Executive Director of Critical Mass Corporate (1997-2001)
- Associate Director of Rhys Capital (1993-1997)



Dr Jack Stecki: Principal Engineer and co-founder of PHM Technology, and originator of the MADE technology. He has over 35 years industrial, research and academic experience in a range of industries including aerospace; defence, mining, offshore, power generation and subsea. He has published over 100 technical papers and presented at many technical and industrial conferences around the world since 1980.



Shoshanna Rudov-Clark: (PhD) is an aerospace engineer at PHM Technology. She has conducted research in the area of composite materials for aerospace applications within the Department of Aerospace Engineering at RMIT-University (Melbourne, and has worked in the area of aircraft structural integrity for the Defence Science and Technology Organisation (Melbourne).

2.5 PHMT's Solution: MADE suite

MADE is the only set of tools providing the integrated technical, functionality and productivity requirements for the development and assessment of PHM, as evidenced by the JSF Project. MADE uses functional analysis, simulation of dynamic behaviour of engineering systems, data fusion techniques and Artificial Intelligence (AI) technology (neural networks, fuzzy logic, genetic algorithms, expert systems) to accurately model system behaviours, determine the criticality of specific failures at all levels throughout the system and then uses this information to design sensor combinations that offer required FDI for PHM capability.

The Maintenance Aware Design environment ('MADE') is an integrated suite of data generation and decision support tools designed to meet the practical engineering requirements of design, RAM and PHM.

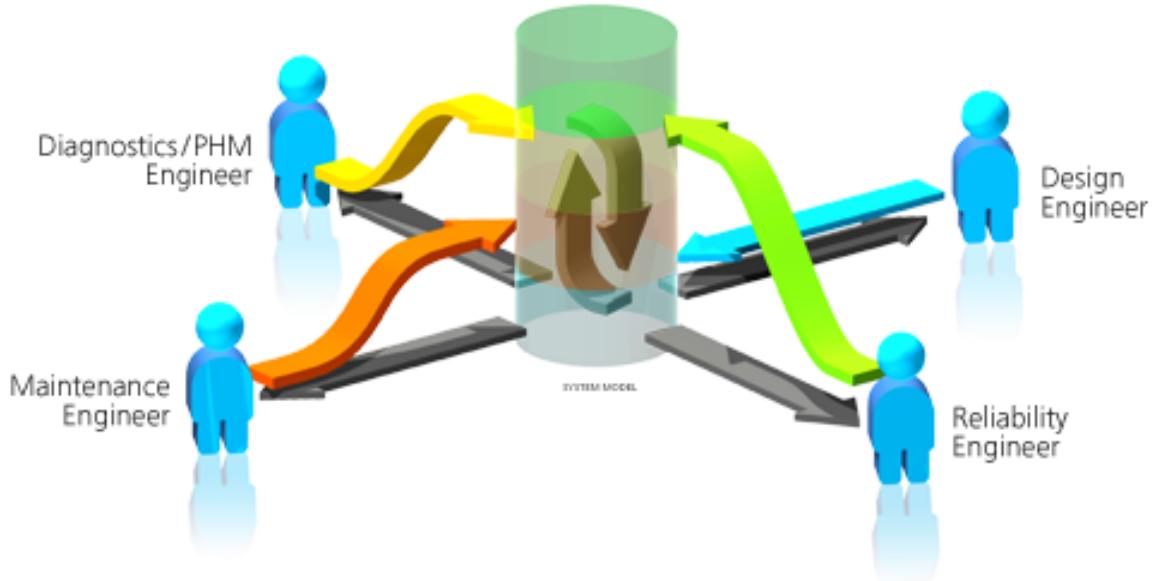


Figure 2-2 THE PHM's Solution MADe suite

The specific capability relevant to each potential user group has been segmented into separate modules, the current modules in the MADe suite are:

- **MADe**
- **MADe for RAM (Reliability, Availability, Maintainability)**
- **MADe for PHM (Prognostics and Health Management)**

❖ **MADe**

MADe is used by system designers to model systems and their behaviour in order to identify and report on potential functional, monitoring and maintenance issues in system design.

Engineers use MADe to model parts, components, sub-systems and systems in order to identify potential functional issues in the system design (including FMEA). MADe can be used at all stages of the design process (concept, functional, hardware, technology refresh, upgrade) and handles complex, integrated, multi-domain systems (mechanical, hydraulic, electronic)

❖ **MADe RAM**

MADe RAM is used by reliability, availability and maintainability engineers (RAM) to assess systems in order to determine system level criticality and reliability and to identify potential issues in system design (including FMECA).

MADe RAM handles complex integrated multi-domain systems (mechanical, hydraulic, electronic) and is designed to be used by both Original Equipment Manufacturers (OEM) (system level analysis) and the members of their supply chain (component, LRU, sub-system analysis).

❖ **MADe PHM**

MADe PHM is used by diagnostics, prognostics and health management engineers (PHM) to optimise the design of condition monitoring and PHM systems. MADe PHM handles complex integrated multi-domain systems (mechanical, hydraulic, electronic) and is designed to be used by both OEM's and the members of their supply chain.

3 Theoretical background

This chapter gives the gist of the reliability engineering theory so as to understand the probabilistic calculations carried out to validate the Reliability Block Diagram (RBD) tool. Readers who are familiar with either the probability or reliability engineering theory might want to start reading from chapter 3, and then focus on the later chapters.

3.1 Reliability definitions

Reliability Block Diagram (RBD): The purpose of the reliability block diagram is to show by concise visual short hand the various block combinations (paths) that result in item success. (MIL-STD-756b 1981)

RBD is a widely used technique to evaluate systems' reliability using probabilistic calculations. RBD is a friendly way to illustrate the architecture of a system as a graphical representation where the components are symbolised by rectangular blocks, which are characterized by their own reliability properties, i.e. Failure Rates and System Mission Time.

Reliability: Schüler (1997) defines the reliability as the probability that the component experiences no failure during the time interval $(0,t)$, given that the component was as good as new at time zero.

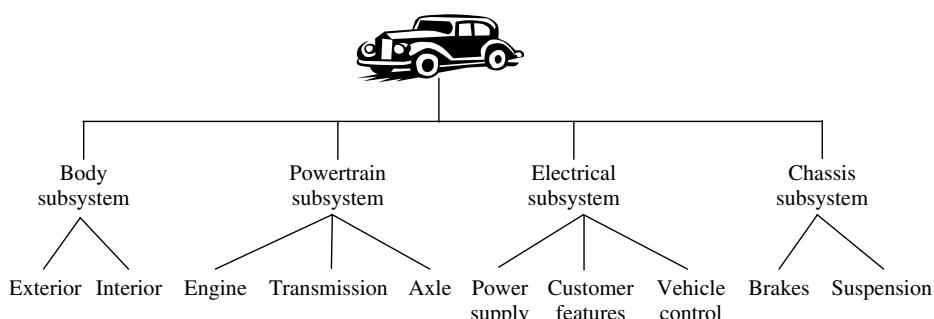


FIGURE 4.1 Hierarchical configuration of a typical automobile



Figure 3-1 A simple Series Reliability Block Diagram. (Guangbin 2007)

3.2 Mathematical background

Operation of event A and B	Boolean Logic	Mathematics	Engineering
Union	A ₁ or A ₂	A ₁ ∪ A ₂	A ₁ + A ₂
Intersection	A ₁ and A ₂	A ₁ ∩ A ₂	A ₁ .A ₂
Complement	Not A _i	\bar{A}_i	\bar{A}_i

Table 3-1 List of the main operations

❖ Union operation

$$P(\cup_{i=1}^n A_i) = \sum_{i=1}^n P(A_i) \quad (3-1)$$

- Example: Union of two independent events A and B ($n=2$).

$$P(\cup_{i=1}^2 A_i) = \sum_{i=1}^2 P(A_i) = P(A_1) + P(A_2) \quad (3-2)$$

❖ Intersection operation

$$P(\cap_{i=1}^n A_i) = \prod_{i=1}^n P(A_i) \quad (3-3)$$

- Example: Intersection of two independent events A and B ($n=2$).

$$P(\cap_{i=1}^2 A_i) = \prod_{i=1}^2 P(A_i) = P(A_1).P(A_2) \quad (3-4)$$

3.3 Reliability engineering background

- ❖ **Failure rate λ :** the small quantity $\lambda(t).dt$ is the probability that the component experiences a failure between t and $t+dt$, given that the component has survived to time t . (Schüler, 1997)
- ❖ **Probability Density Function $f(t)$:** indicates the failure distribution over the entire time range and represents the absolute failure speed. (Guangbin, 2007)

- ❖ **Cumulative Distribution Function $F(t)$:** is the probability that a product will fail by a specified time t . It is the probability of failure. (Guangbin, 2007)

$$F(t) = \Pr(T < t) = \int_{-\infty}^t f(t)dt \quad (3-5)$$

- ❖ **Reliability $R(t)$:** The reliability function, also called the survival function, is often interpreted as the population fraction surviving time t . $R(t)$ is the probability of success, which is the complement of $F(t)$. (Guangbin, 2007)

$$R(t) = \Pr(T \geq t) = 1 - \Pr(T < t) = 1 - F(t) = \int_t^{+\infty} f(t)dt \quad (3-6)$$

3.4 Exponential distribution

This thesis only deals with failure obeying an exponential distribution. To date, only the exponential distribution has been implemented in MADe RBD because it is the most commonly used within the reliability engineering area.

It is assumed (Schüler, 1997) that the exponential distribution is characterized by a constant failure rate λ . Moreover, another well-known and interesting property of the exponential distribution is that the probability of a surviving component is independent of what occurred in the past, this is called the *memoryless property*.

- ❖ **Probability Density Function $f(t)$:**

$$f(t) = \lambda e^{-\lambda t} \quad \text{with } t \geq 0 \quad (3-7)$$

- ❖ **Cumulative Distribution Function $F(t)$:**

$$F(t) = \int_0^t f(t)dt = \int_0^t \lambda e^{-\lambda t} dt = \lambda \left[-\frac{e^{-\lambda t}}{\lambda} \right]_0^t = \lambda \left(-\frac{e^{-\lambda t}}{\lambda} + \frac{1}{\lambda} \right) \quad (3-8)$$

$$F(t) = -e^{-\lambda t} + 1$$

- ❖ **Reliability $R(t)$:**

$$R(t) = 1 - F(t) = 1 - e^{-\lambda t} + 1 \quad (3-9)$$

$$R(t) = e^{-\lambda t}$$

4 RBD Validation

This chapter covers the reliability calculations of the classic configurations which can be encountered. Combinations are treated with detail and include:

- A short paragraph stating the condition of success of the system mission time.
- The analytical calculations and techniques used.
- The comparison between the results found by hand and those of MADe in order to make sure that they match together.

Nevertheless, in order to fit the reality, the reliability evaluation phase may require much more complex models such as redundant systems with standby components. Thus, the general methods introduced hereunder can no longer enable customers to compute the overall reliability of systems including standby items. Currently, the RBD workbench of MADe is limited since it cannot perform such an analysis. Consequently, more advanced techniques such as *Laplace transform* and the stochastic *Monte-Carlo* computational method have been approached and are presently for discussion.

4.1 Assumptions

Computation of system's reliability with MADe is limited and based on several assumptions. So far, components obey the following rules:

- The events are independents, i.e. the probability of A_1 is not affected by whether A_2 occurs or not. In other words, the probability of failure (or reliability) of a component A_1 is not affected by the probability of failure (or reliability) of another component A_2 .
- Only the exponential failure distribution is considered.
- The duration of operation is obviously equal or superior to zero.
- The components are assumed to be irreparable.
- Items are not sensitive to ageing damage.

4.2 Component Reliability

The different types of groups have been built using a range of four (five for Complex group) random components. Moreover, each component has a different failure rate so as to avoid any special case.

- ❖ Component's definition:

	Part 1	Part 2	Part 3	Part 4	Part 5
Mission Time (t_i) (hours)	10	35	200	80	80
Failure Rate (λ_i) ($/10^6$ hours)	7843.14	8039.22	9166.67	10000	10000
Reliability $R_i(t)$	0.9245655	0.754747	0.1598796	0.449329	0.449329

Table 4-1 Definition of components

Using eq. 3-9 we obtain the following reliabilities:

- Part 1: $R_1(t) = e^{-\lambda_1 \cdot t_1} = e^{-(\frac{7843.14}{10^6}) \cdot 10} = 0.9245655$ (4-1)

- Part 2: $R_2(t) = e^{-\lambda_2 \cdot t_2} = e^{-(\frac{8039.22}{10^6}) \cdot 35} = 0.7547467$ (4-2)

- Part 3: $R_3(t) = e^{-\lambda_3 \cdot t_3} = e^{-(\frac{9166.67}{10^6}) \cdot 200} = 0.1598796$ (4-3)

- Part 4: $R_4(t) = e^{-\lambda_4 \cdot t_4} = e^{-(\frac{10000}{10^6}) \cdot 80} = 0.449329 = R_5(t)$ (4-4)

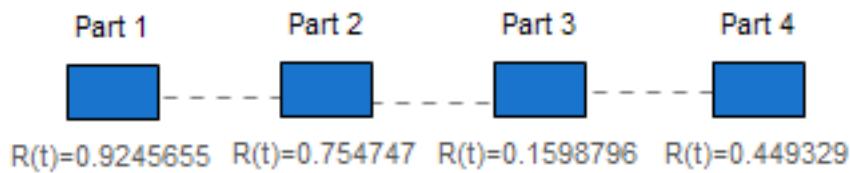


Figure 4-1 MADe Items' reliability

According to the calculations it has been approved that the computation of the reliability at the component level has been well implemented.

4.3 System Reliability

4.3.1 Series group



Figure 4-2 Series group

❖ **Reliability condition:**

All components must be up for the system works.

❖ **Analytical calculation:**

$$\begin{aligned}
 R_s(t) &= P(\cap_{i=1}^n A_i) \\
 &= P(A_1) \cdot P(A_2) \cdot P(A_3) \cdot \dots \cdot P(A_n) \\
 &= R_1 \cdot R_2 \cdot R_3 \cdot \dots \cdot R_n \\
 &= \prod_{i=1}^n R_i
 \end{aligned} \tag{4-5}$$

For the $n = 4$ components defined § 4.2 we obtain:

$$R_s(t) = \prod_{i=1}^4 R_i = R_1 \cdot R_2 \cdot R_3 \cdot R_4 \cong 0.0501299 \tag{4-6}$$

❖ **MADe Series group:**

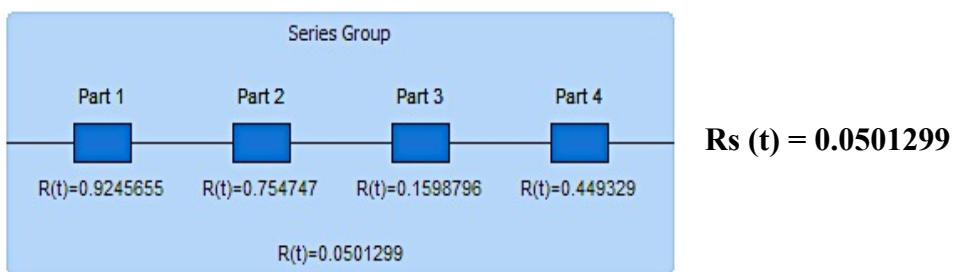


Figure 4-3 MADe Series group

4.3.2 Parallel group

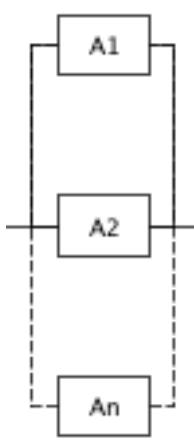


Figure 4-4 Parallel group

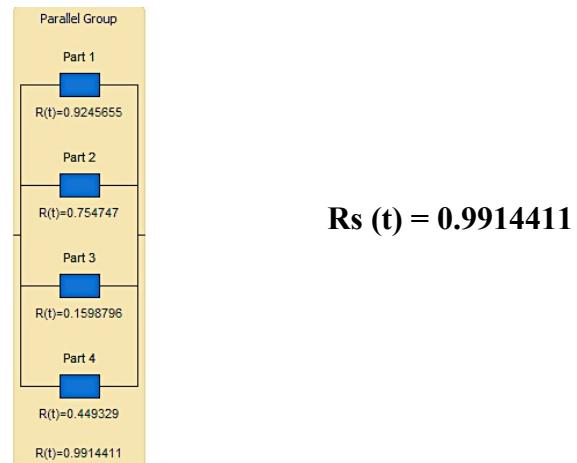


Figure 4-5 MADE Parallel group

❖ Reliability condition:

All components must be failed for the system fails.

❖ Analytical calculation:

$$\begin{aligned}
 R_s(t) &= 1 - F_s(t) \\
 &= 1 - P(\cap_{i=1}^n \bar{A}_i) \\
 &= 1 - P(\bar{A}_1) \cdot P(\bar{A}_2) \cdot \dots \cdot P(\bar{A}_n) \\
 &= 1 - F_1 \cdot F_2 \cdot \dots \cdot F_n \\
 &= 1 - (1 - R_1) \cdot (1 - R_2) \cdot \dots \cdot (1 - R_n) \\
 &= 1 - \prod_{i=1}^n (1 - R_i)
 \end{aligned} \tag{4-7}$$

$$\begin{aligned}
 R_s(t) &= 1 - \prod_{i=1}^4 (1 - R_i) \\
 &= 1 - (1 - R_1) (1 - R_2) (1 - R_3) (1 - R_4) \cong 0.9914411
 \end{aligned} \tag{4-8}$$

4.3.3 Combined group

A combined group is a blend of either parallel groups in series, i.e. *Series-Parallel group*, or series groups in parallel, i.e. *Parallel-Series group*. These types of combinations are evaluated by using reduction techniques that consists in calculating the equivalent reliability for each series or parallel group in order to simplify the system in a basic series or parallel configuration.

4.3.3.1 Series-Parallel group

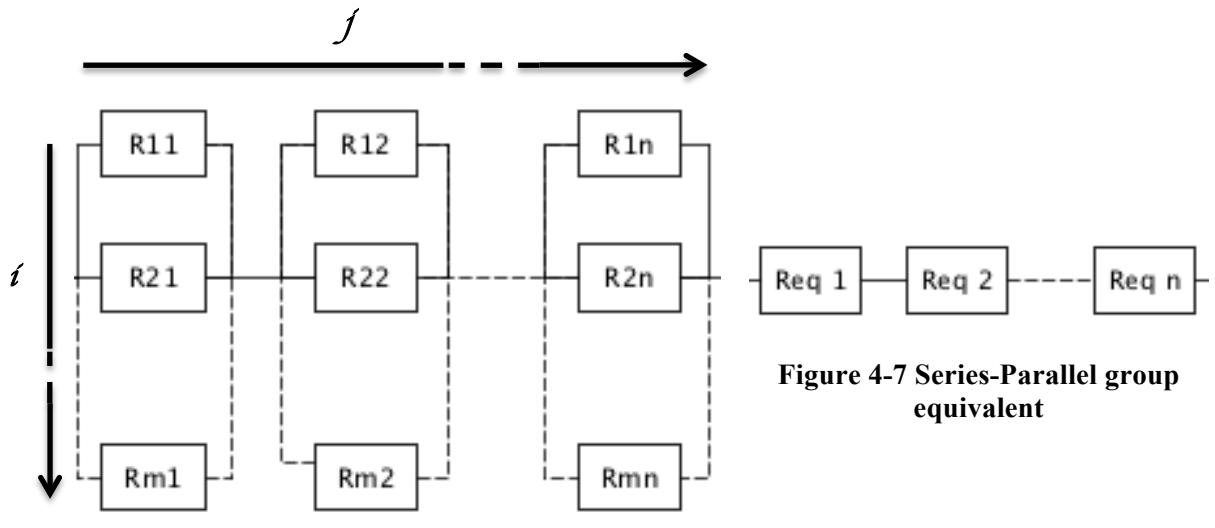


Figure 4-7 Series-Parallel group equivalent

❖ Reliability condition:

As a combination of parallel groups in series, at least one parallel group must be failed for the system stop functioning.

❖ Analytical calculation:

Since the system in figure 4-6 is identical to a series group (Fig. 4-7). Thus, the reliability can be calculated based on the formula of a series group (Eq. 4-5 & 4-6).

$$\begin{aligned}
 R_s(t) &= \prod_{j=1}^n \left[1 - \prod_{i=1}^m (1 - R_{ij}) \right] \text{ using } 1 - \prod_{i=1}^m (1 - R_{ij}) = R_{eqj} \text{ then,} \\
 &= \prod_{j=1}^n R_{eqj}
 \end{aligned} \tag{4-9}$$

$$R_s(t) = \prod_{j=1}^3 R_{eqj}$$

$$= R_{eq1} \cdot R_{eq2} \cdot R_{eq3} \cong 0.9745424$$
(4-10)

❖ **MADe Series-Parallel group:**

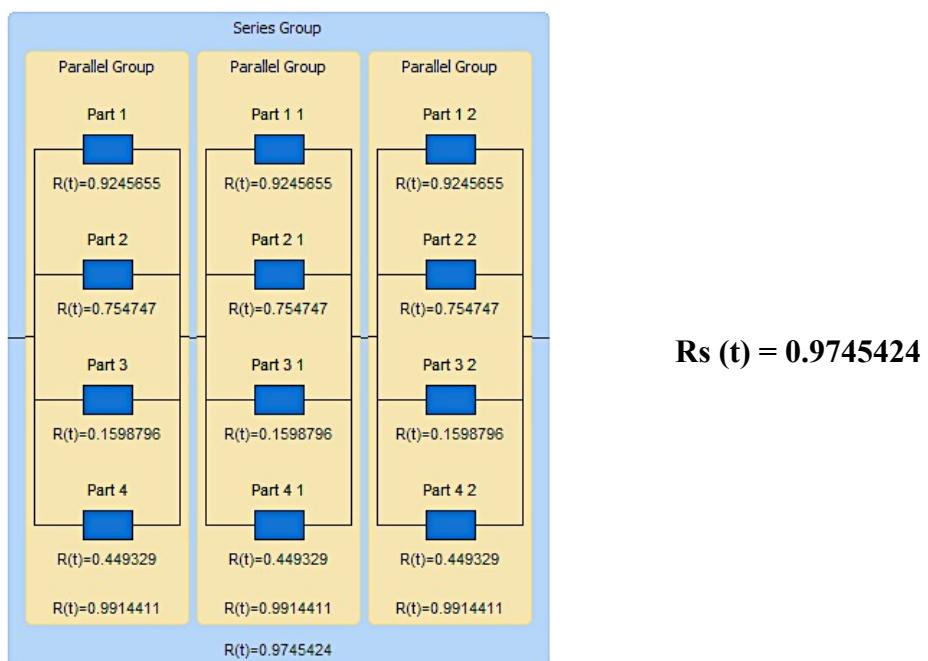


Figure 4-8 MADe Series-Parallel group

4.3.3.2 Parallel-Series group

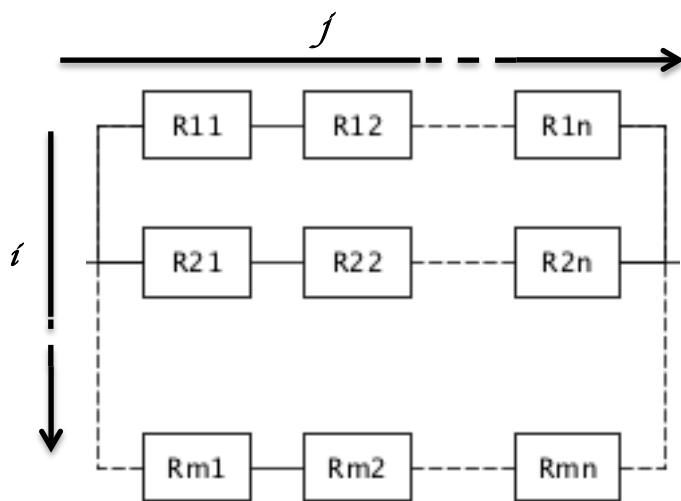


Figure 4-9 Parallel-Series group

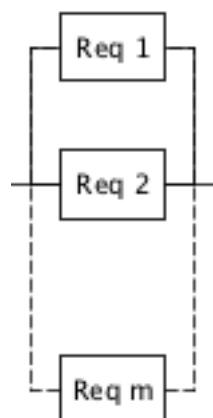


Figure 4-10 Parallel-Series group equivalent

❖ Reliability condition:

Conversely, here is a combination of series groups in parallel configuration, thus at least one series group must be up for the system works.

❖ Analytical calculation:

Using the same method as above, the system can be reduced to a parallel group (*Fig. 4-4*). Hence, the reliability of the system can be calculated using the equation of a classic parallel system (*Eq. 4-7 & Eq. 4-8*).

$$R_s(t) = 1 - \prod_{i=1}^m \left[1 - \prod_{j=1}^n R_{ij} \right] \text{ using } \prod_{j=1}^n R_{ij} = R_{eqi} \text{ then,} \quad (4-11)$$

$$= 1 - \prod_{i=1}^m 1 - R_{eqi}$$

$$R_s(t) = 1 - \prod_{i=1}^3 1 - R_{eqi} \quad (4-12)$$

$$= 1 - (1 - R_{eq1})(1 - R_{eq2})(1 - R_{eq3}) \cong \mathbf{0.1429766}$$

❖ MADe Parallel-Series group:

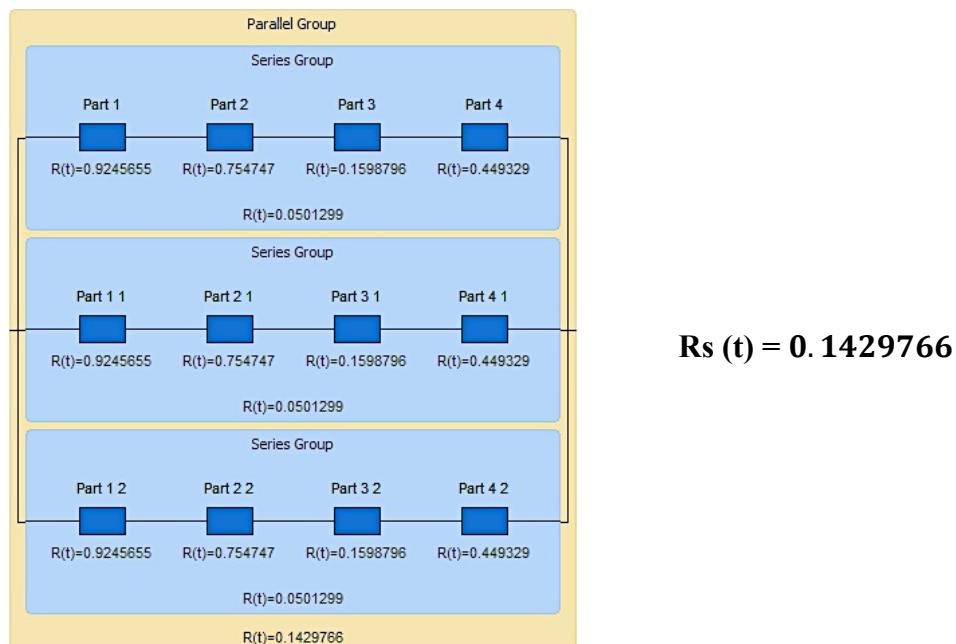


Figure 4-11 MADe Parallel-Series group

4.3.4 K-out-of-N group

❖ Reliability condition:

A K-out-of-N group is a parallel group (§4-3-2) with n independent components which functions if and only if at least K components function. The easiest method to calculate the overall reliability of the system is to use a true table in order to list all the possible combinations.

❖ Analytical calculation:

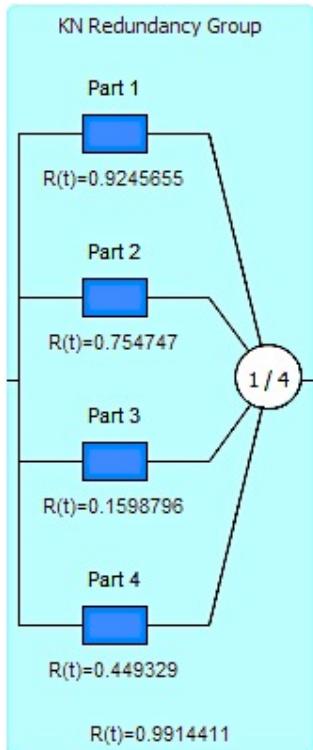
TRUE TABLE:

■ F_i : Component i is failed

■ R_i : Component i is up

Part 1	Part 2	Part 3	Part 4	▪ $Rs_{(1\text{-out-of-}4)}(t) =$
F_1	F_2	F_3	F_4	$F_1F_2F_3R_4 + F_1F_2R_3F_4 + F_1F_2R_3R_4 + F_1R_2F_3F_4 + F_1R_2F_3R_4 + F_1R_2R_3F_4 + F_1R_2R_3R_4 + R_1F_2F_3F_4 + R_1F_2F_3R_4 + R_1F_2R_3F_4 + R_1F_2R_3R_4 + R_1R_2F_3F_4 + R_1R_2F_3R_4 + R_1R_2R_3F_4 + R_1R_2R_3R_4$
F_1	F_2	F_3	R_4	
F_1	F_2	R_3	F_4	
F_1	F_2	R_3	R_4	
F_1	R_2	F_3	F_4	$= Rs_{(\text{parallel})}$
F_1	R_2	F_3	F_4	$\Rightarrow Rs_{(1\text{-out-of-}4)}(t) \cong 0.9914411$
F_1	R_2	F_3	R_4	▪ $Rs_{(2\text{-out-of-}4)}(t) =$
F_1	R_2	R_3	F_4	$F_1F_2R_3R_4 + F_1R_2F_3R_4 + F_1R_2R_3F_4 + F_1R_2R_3R_4 + R_1F_2F_3R_4 + R_1F_2R_3F_4 + R_1F_2R_3R_4 + R_1R_2F_3R_4 + R_1R_2R_3F_4 + R_1R_2R_3R_4$
F_1	R_2	R_3	R_4	
R_1	F_2	F_3	F_4	
R_1	F_2	F_3	R_4	$\Rightarrow Rs_{(2\text{-out-of-}4)}(t) \cong 0.449329$
R_1	F_2	R_3	F_4	▪ $Rs_{(3\text{-out-of-}4)}(t) =$
R_1	F_2	R_3	R_4	$F_1R_2R_3R_4 + R_1F_2R_3R_4 + R_1R_2F_3R_4 + R_1R_2R_3F_4 + R_1R_2R_3R_4$
R_1	R_2	F_3	F_4	
R_1	R_2	F_3	R_4	$\Rightarrow Rs_{(3\text{-out-of-}4)}(t) \cong 0.3953634$
R_1	R_2	R_3	F_4	▪ $Rs_{(4\text{-out-of-}4)}(t) = R_1R_2R_3R_4 = Rs_{(\text{series})}$
R_1	R_2	R_3	R_4	$\Rightarrow Rs_{(4\text{-out-of-}4)}(t) \cong 0.0501299$

❖ MADe K-out-of-N group:



$$Rs \cong 0.9914411$$

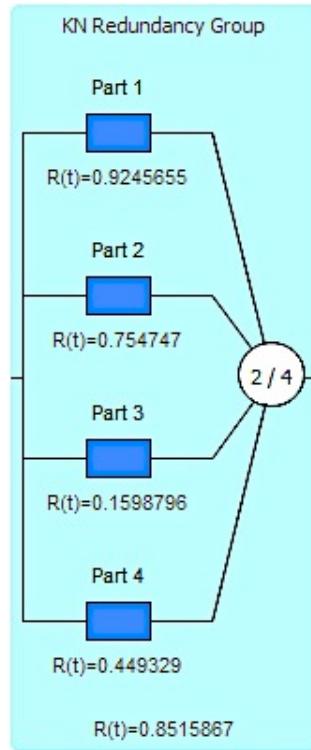


Figure 4-13 MADe Active Redundancy “2-out-of-4”

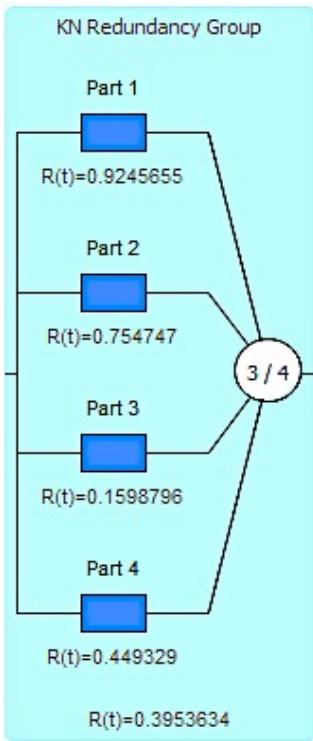


Figure 4-14 MADe Active Redundancy “3-out-of-4”

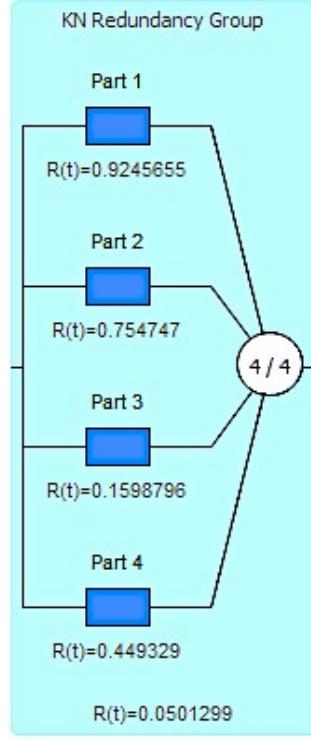


Figure 4-15 MADe Active Redundancy “4-out-of-4”

4.3.5 Complex group

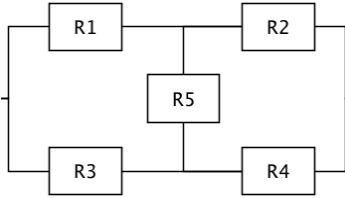


Figure 4-16 Complex group

❖ **Reliability condition:**

A system is complex, when it can be defined neither as a series nor as a parallel configuration.

❖ **Analytical calculation:**

DECOMPOSITION APPROACH

Key element (5) functioning

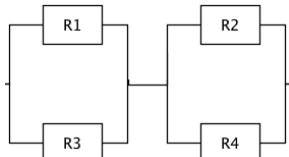


Figure 4-17 Complex group with key up

If the key element is always functioning, then the system is equivalent to a Series-Parallel group (fig. 4-6).



Figure 4-18 Equivalent complex group with key up

$$R_{eq1} = 1 - \prod_{\substack{i=1 \\ i \neq 2}}^3 (1 - R_i) \cong 0.9366259$$

$$R_{eq2} = 1 - \prod_{\substack{i=2 \\ i \neq 3}}^4 (1 - R_i) \cong 0.8649463$$

$$Rs_{(key \ functioning)} = \prod_{i=1}^2 R_{eq \ i}$$

$$\Rightarrow Rs_{(key \ functioning)}(t) \cong 0.8101311$$

Key element (5) failed

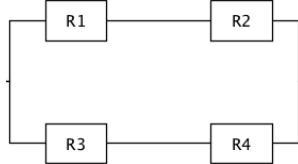


Figure 4-19 Complex group with key failed

Conversely, if the key part is always failed, then it is a Parallel-Series group (fig. 4-9).

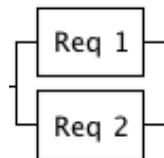


Figure 4-20 Equivalent complex group with key failed

$$R_{eq1} = \prod_{i=1}^2 R_i \cong 0.697813$$

$$R_{eq2} = \prod_{i=3}^4 R_i \cong 0.0718386$$

$$Rs_{(key \ failed)} = 1 - \prod_{i=1}^2 (1 - R_{eq \ i})$$

$$\Rightarrow Rs_{(key \ failed)}(t) \cong 0.7195217$$

The reliability of the whole system is:

$$Rs(t) = P(\text{part good} | \text{key good}) \cdot P(\text{key good}) + P(\text{part good} | \text{key failed}) \cdot P(\text{key failed})$$

Given the:

- Reliability of the key element (5) $\equiv R_5 \cong 0.449329$
- Probability of failure of the key element (5) $\equiv F_5 = 1 - R_5 \cong 0.550671$

Therefore the reliability of the system is

$$\begin{aligned} Rs(t) &= Rs_{(\text{key functioning})} R_5 + Rs_{(\text{key failed})} F_5 \\ &= 0.8101311 * 0.449329 + 0.7195217 * 0.550671 \quad (4-13) \\ &\Rightarrow Rs(t) \cong 0.7602351 \end{aligned}$$

❖ MADe Complex group:

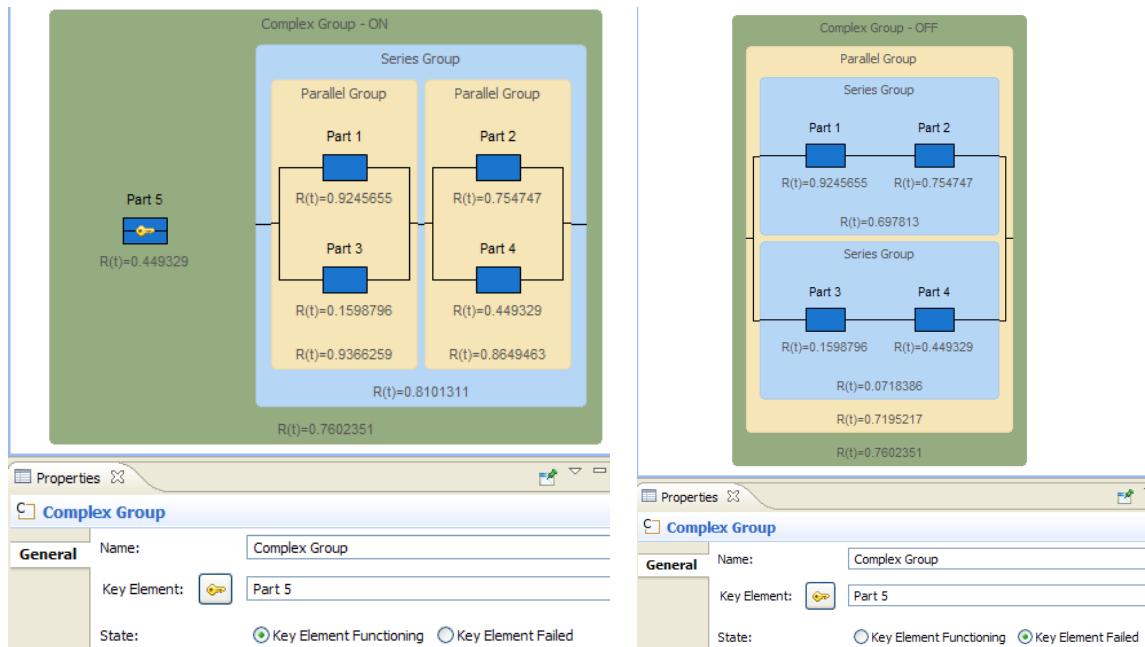


Figure 4-21 MADe Complex group with key element functioning

Figure 4-22 MADe Complex group with key element failed

$$\Rightarrow Rs_{(\text{key functioning})} \cong 0.8101311 \qquad \Rightarrow Rs_{(\text{key failed})} \cong 0.7195217$$

$$\Rightarrow Rs(t) \cong 0.7602351$$

4.3.6 Summary of results

The following table summarises all the results (overall reliability system) calculated by hand as well as using MADe.

Configuration	<i>Series</i>	<i>Parallel</i>	<i>Series-Parallel</i>	<i>Parallel-Series</i>
Analytical	0.0501299	0.9914411	0.9745424	0.1429766
MADe	0.0501299	0.9914411	0.9745424	0.1429766

<i>K/N with K=1</i>	<i>K/N with K=2</i>	<i>K/N with K=3</i>	<i>K/N with K=4</i>	<i>Complex</i>
0.9914411	0.44939329	03953634	0.0501299	0.7602351
0.9914411	0.44939329	03953634	0.0501299	0.7602351

We notice that the analytical results match with those of MADe, therefore the Reliability Bloc Diagram tool has been validated.

Prior to deal with more complex configurations it is interesting to notice that the use of redundant components can significantly improve the reliability of a system. Conversely, as we can see on the histogram hereunder, designs based on series configurations tremendously increase the probability of failure of the system.

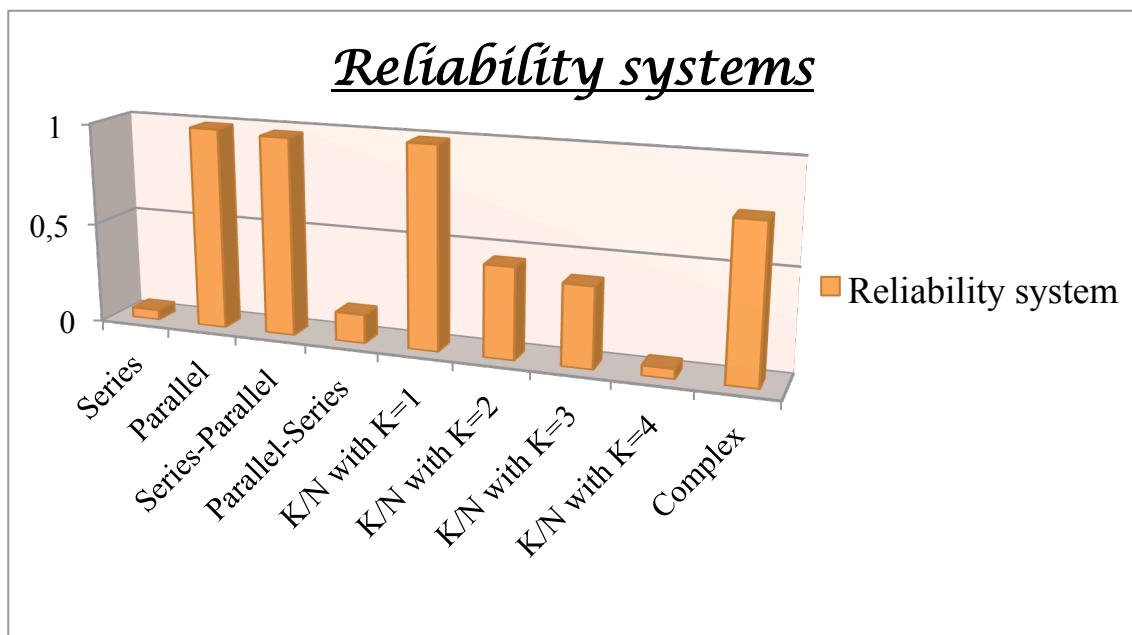


Figure 4-23 Reliability system of the different combinations

4.3.7 Redundant group

In the previous chapter, it has been acknowledged that the use of redundant items can improve the reliability of a product. Nevertheless, sometimes, critical systems could require a probability of failure extremely low. In order to improve the reliability of the system, a common technique is to add standby (non-operating) components in parallel to one active (operating) component.

In standby redundancy the components are set to have two states: an active state and a standby state. Components in standby redundancy have two failure distributions, one for each state. When in the standby state, they have a quiescent (or dormant) failure distribution and when operating, they have an active failure distribution.

- In the case that both quiescent and active rates of failure are the same, the units are in a simple parallel configuration (also called a *hot standby* configuration, § 4.3.2).
- When the rate of failure of the standby component is lower in quiescent mode than in active mode, this is called a *warm standby* configuration. This type of redundancy is out of scope with respect to this thesis but might be considered as a future work.
- When the rate of failure of the standby component is zero in quiescent mode (i.e. the component cannot fail when in standby) that is called a *cold standby* configuration.

Standby components can be switched on thanks to a switching device as soon as the operating component failed. Nevertheless, the switching device may fail before the functional item, then the component in standby cannot be turned on and so the system fails. Since the switching device is never one hundred per cent reliable, there are two types of redundancy evaluation technique.

- The first solution is to make the assumption that the switching device never fails, and the switching is instantaneous between components – *Perfect switching*.
- On the other hand, we can consider and evaluate the probability of failure of the switching device – *Imperfect switching*.

Redundant system can no longer be evaluated by using the friendly formulae used up to this point. To solve this type of problem it is necessary to come back to the original definitions of the reliability theory (*eq. 3-5 and 3-6*).

4.3.8 Cold redundancy

4.3.8.1 Perfect switching

Perfect switching means that when the active part fails, the standby components are instantaneously switched on and the probability of failure of the switching device is equal to zero.

The reliability of a cold standby redundant system with perfect switching can be literally defined as follows:

Reliability = The system's lifetime is longer than the mission time T

$$\Leftrightarrow \Pr(T \geq t) = \int_t^{+\infty} f(t) dt$$

Two possible scenarios can lead to a success event:

1. The active component survives beyond the mission time T
2. The active component fails at time X_1 ($0 \leq X_1 < T$) and the nth standby component functions for a time period longer than $(T - \sum_{i=1}^{n-1} X_i)$.

❖ **Recursion for N standby components:**

- Components 1 fails after a working duration of X_1 :

$$\Leftrightarrow n=1: 0 \leq X_1 < T$$

- Component 2 fails after a working duration of X_2 :

$$\Leftrightarrow n=2: 0 \leq X_2 < T - X_1$$

- Component 3 fails after a working duration of X_3 :

$$\Leftrightarrow n=3: 0 \leq X_3 < T - X_1 - X_2$$

- Component $N-1$ fails after a working duration of X_{N-1} :

$$\Leftrightarrow n=N: 0 \leq X_{N-1} < T - X_1 - X_2 - \dots - X_{N-2} \equiv 0 \leq X_N < T - \sum_{i=1}^{N-2} X_i$$

- Component N operates properly for a time period longer than $T - \sum_{i=1}^{N-1} X_i$



Given:

- λ_1 : Failure rate of the active part
- λ_i : Failure rate of the standby parts i
- T: Mission time
- X_i : Lifetime of the part i

Hence,

- R_1 : Reliability of the active (operating) component
 - R_i : Reliability of the standby (non-operating) components
- $$\Rightarrow R_i = e^{-\lambda_i * T_i} \text{ (Eq. 3-9)}$$

The reliability of the cold standby redundant system with perfect switching and exponential distribution requires integration over all times as follows:

$$\begin{aligned}
 RS(t) &= P\left(\sum_{i=1}^N X_i > T\right) = R_1(T) + \int_0^T f_1(X_1) R_2(T - X_1) dX_1 \\
 &+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) R_3(T - X_1 - X_2) dX_2 dX_1 \\
 &+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \int_0^{T-X_1-X_2} f_3(X_3) R_4(T - X_1 - X_2 - X_3) dX_3 dX_2 dX_1 \\
 &+ \dots \\
 &+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \dots \int_0^{T-\sum_{i=1}^{N-2} X_i} f_{N-1}(X_{N-1}) R_N\left(T - \sum_{i=1}^{N-1} X_i\right) dX_{N-1} \dots dX_2 dX_1
 \end{aligned}$$

Where $f_n(X_u) dX_u$ is the probability of the part n fails at an unknown time X_u .

Using:

- $f_n(X_u) = \lambda_n e^{-\lambda_n X_u}$ (*Probability density function of item n*)
- $R_n(X_u) = e^{-\lambda_n X_u}$ (*Reliability of item n*)

Hence,

$$\begin{aligned}
Rs(t) = & e^{-\lambda_1 T} + \int_0^T \lambda_1 e^{-\lambda_1 X_1} e^{-\lambda_2(T-X_1)} dX_1 \\
& + \int_0^T \lambda_1 e^{-\lambda_1 X_1} \int_0^{T-X_1} \lambda_2 e^{-\lambda_2 X_2} e^{-\lambda_3(T-X_1-X_2)} dX_2 dX_1 \\
& + \int_0^t \lambda_1 e^{-\lambda_1 X_1} \int_0^{T-X_1} \lambda_2 e^{-\lambda_2 X_2} \int_0^{T-X_1-X_2} \lambda_3 e^{-\lambda_3 X_3} e^{-\lambda_4(T-X_1-X_2-X_3)} dX_3 dX_2 dX_1 \\
& + \dots \\
& + \int_0^t \lambda_1 e^{-\lambda_1 X_1} \int_0^{T-X_1} \lambda_2 e^{-\lambda_2 X_2} \int_0^{T-X_1-X_2} \lambda_3 e^{-\lambda_3 X_3} \dots \\
& \int_0^{T-\sum_{i=1}^{N-2} X_i} \lambda_{N-1} e^{-\lambda_{N-1} X_{N-1}} e^{-\lambda_{N-1}} \left(T - \sum_{i=1}^{N-1} X_i \right) dX_{N-1} \dots dX_3 dX_2 dX_1
\end{aligned}$$

Way (2003, p.133) gives the general equation:

$$\Rightarrow Rs(t) = \sum_{j=1}^N e^{-\lambda_j t} \left(\prod_{i \neq j} \frac{\lambda_i}{\lambda_i - \lambda_j} \right) \quad (4-14)$$

4.3.8.2 Imperfect switching

In most cases, it should be required to consider the reliability of the switching device for more accuracy. Thus, the reliability of a cold standby redundant system with imperfect switching can be modelled with either a *probability of success of switching*, or by seeing the *switching device as a component* with its own lifetime distribution.

❖ Probability of success of switching (p)

After an extensive literature research, a few people (Lenz and Rhodin, 2011; Way, 2003) approach this problem, however nobody provides a general formula that could be implemented. Consequently, another method of calculation based on Laplace Transform has been used in order to solve the recursive integration.

The two scenarios leading to a success of the mission time are:

1. The active component survives beyond the mission time T
2. The active component fails at time X_1 ($0 \leq X_1 < T$), the $n-1$ switch occurs without any problem and the n th standby component functions for a time period longer than $(T - \sum_{i=1}^{n-1} X_i)$.

$$\begin{aligned}
Rs(t) = & R_1(T) + p \int_0^T f_1(X_1) R_2(T - X_1) dX_1 \\
& + p^2 \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) R_3(T - X_1 - X_2) dX_2 dX_1 \\
& + p^3 \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \int_0^{T-X_1-X_2} f_3(X_3) R_4(T - X_1 - X_2 - X_3) dX_3 dX_2 dX_1 \\
& + \dots \\
& + p^{n-1} \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \dots \int_0^{T-\sum_{i=1}^{N-2} X_i} f_{N-1}(X_{N-1}) R_N \left(T - \sum_{i=1}^{N-1} X_i \right) dX_{N-1} \dots dX_2 dX_1
\end{aligned}$$

To simplify the expression we can write the equation as follows:

$$\begin{aligned}
Rs(t) = & R_1 + p * F_1 * R_2 + \dots + p^{n-1} * F_1 * F_2 * \dots * F_{n-1} * R_n \\
\Rightarrow \textbf{Rs } (\textbf{t}) = & \sum_{j=1}^n p^{j-1} F_1 * F_2 * \dots * F_{j-1} * R_j
\end{aligned} \tag{4-15}$$

The first term of the sum is by convention equal to $p^0 R_1 = R_1$.

The function f_i is defined over $[0, +\infty[$ and can be integrated over $[0, M] \forall M > 0$.

The convolution $f * g$ is defined for $t > 0$ by

$$(f * g)(t) = \int_0^{+\infty} f(x)g(t - x)dx \tag{4-16}$$

Using $\mathcal{L}(f)$ for the Laplace transform of a function f , which is defined as follows:

$$\mathcal{L}(f)(z) = \int_0^{+\infty} f(t)e^{-tz}dt \tag{4-17}$$

Also it can be demonstrated that the Laplace transform is linear, hence:

$$\mathcal{L}(f * g)(z) = \mathcal{L}(f)(z)\mathcal{L}(g)(z) \tag{4-18}$$

Then the Laplace transform of Eq. 4-15 yields:

$$\mathcal{L}(R_{cold-standby})(z) = \sum_{j=1}^n p^{j-1} \mathcal{L}(f_1)(z) \dots \mathcal{L}(f_{j-1})(z) \mathcal{L}(R_j)(z) \tag{4-19}$$

Since $R_i(t) = e^{-\lambda_i t}$ and $f_i(t) = -\lambda_i e^{-\lambda_i t}$ then using Laplace transform we obtain,

$$\mathcal{L}(R_i)(z) = \frac{1}{z + \lambda_i} \text{ and } \mathcal{L}(f_j)(z) = \frac{\lambda_i}{z + \lambda_i} \quad (4-20)$$

Therefore, Eq. 4-19 becomes,

$$\mathcal{L}(R_{\text{cold-standby}})(z) = \sum_{j=1}^n p^{j-1} \frac{\lambda_1 \dots \lambda_{j-1}}{(z + \lambda_1) \dots (z + \lambda_j)} \quad (4-21)$$

By carrying out a partial fraction expansion using,

$$\frac{1}{(z + \lambda_1) \dots (z + \lambda_j)} = \sum_{i=1}^j \frac{\mu_{i,j}}{z + \lambda_i} \quad (4-22)$$

Multiplying Eq. 4-22 by $z + \lambda_i$ and substituting $z = -\lambda_i$, we obtain for $i \leq j$,

$$\mu_{i,j} = \frac{1}{\prod_{\substack{1 \leq k \leq j \\ k \neq i}} (\lambda_k - \lambda_i)} \quad (4-23)$$

Then by injecting Eq. 4-23 into Eq. 4-22 and Eq. 4-22 into Eq. 4-21 we get,

$$\begin{aligned} \mathcal{L}(R_{\text{cold-standby}})(z) &= \sum_{j=1}^n p^{j-1} \lambda_1 \dots \lambda_{j-1} \sum_{i=1}^n \frac{1}{(z + \lambda_i) \prod_{\substack{1 \leq k \leq j \\ k \neq i}} (\lambda_k - \lambda_i)} \\ &= \sum_{i=1}^n \left[\sum_{j=i}^n p^{j-1} \frac{\prod_{1 \leq k \leq j-1} \lambda_k}{\prod_{\substack{1 \leq k \leq j \\ k \neq i}} (\lambda_k - \lambda_i)} \right] \frac{1}{z + \lambda_i} \end{aligned} \quad (4-24)$$

Finally, reading Laplace from ‘s’ domain to ‘t’ domain (*or right to left*) we obtain the general formula:

$$R_{\text{cold-standby}}(t) = \sum_{i=1}^n \left[\sum_{j=i}^n p^{j-1} \frac{\prod_{1 \leq k \leq j-1} \lambda_k}{\prod_{\substack{1 \leq k \leq j \\ k \neq i}} (\lambda_k - \lambda_i)} \right] e^{-\lambda_i t} \quad (4-25)$$

In order to check the general (Eq. 4-25), two examples ($n = 1$ and $n = 2$) have been expanded and compared to Rhodin’s examples (2011).

- Example for n=2:

$$\begin{aligned} R_{\text{cold-standby}}(t) &= \left[p^0 + p \frac{\lambda_1}{\lambda_2 - \lambda_1} \right] e^{-\lambda_1 t} + p \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 t} \\ &= e^{-\lambda_1 t} - p \frac{\lambda_1}{\lambda_1 - \lambda_2} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \end{aligned}$$

- Example for n=3:

$$\begin{aligned} R_{\text{cold-standby}}(t) &= \left[1 + p \frac{\lambda_1}{\lambda_2 - \lambda_1} + p^2 \frac{\lambda_1 \lambda_2}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)} \right] e^{-\lambda_1 t} \\ &+ \left[p \frac{\lambda_1}{\lambda_2 - \lambda_1} + p^2 \frac{\lambda_1 \lambda_2}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_2)} \right] e^{-\lambda_2 t} + p^2 \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)} e^{-\lambda_3 t} \end{aligned}$$

Both examples confirmed that the results provided in the recent studies of Lenz and Rhodin (2011). Therefore, the general equation Eq. 4-25) seems to be verified.

❖ Switching device as a component

Another method to model a cold standby redundant group with imperfect switching is to consider the switching device as a component itself with its own probability of failure.

Once again, two scenarios are possible:

1. The active component survives beyond the mission time T
2. The active component fails at time X_1 ($0 \leq X_1 < T$), and both the switching device as well as the nth standby component function for a time period longer than $(T - \sum_{i=1}^{n-1} X_i)$.

Using R_{sd} for the reliability of the switching device, we can develop the integrals recursions as demonstrated in the previous cases.

$$\begin{aligned} R_s(t) &= R_1(T) + \int_0^T f_1(X_1) R_{sd}(X_1) R_2(T - X_1) dX_1 \\ &+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) R_{sd}(X_1 + X_2) R_3(T - X_1 - X_2) dX_2 dX_1 \end{aligned}$$

$$\begin{aligned}
& + \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \int_0^{T-X_1-X_2} f_3(X_3) R_{sd}(X_1 + X_2 + X_3) R_4(T - X_1 - X_2 \\
& \quad - X_3) dX_3 dX_2 dX_1 \\
& + \dots \\
& + \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \dots \int_0^{T-\sum_{i=1}^{N-2} X_i} f_{N-1}(X_{N-1}) R_{sd} \left(\sum_{i=1}^{N-1} X_i \right) R_N \left(T \right. \\
& \quad \left. - \sum_{i=1}^{N-1} X_i \right) dX_{N-1} \dots dX_2 dX_1
\end{aligned}$$

This equation is the same as for Cold Standby redundancy with perfect switching (cf. above: § 4.3.10.1), with one more component, the switching device. Such calculations can be performed using symbolic computations, e.g. Mupad, Maple, Matlab, etc. However, classic computers cannot solve integrals directly, therefore, numerical integration methods must be developed. Monte-Carlo integration is a powerful method for computing the value of complex integrals using probabilistic techniques and is briefly presented in the next chapter.

4.3.9 Monte-Carlo Integration

Using the Cold Standby redundancy with perfect switching model as a case study, it has been shown in section 4.3.10.1 that the overall reliability is equal to:

$$\begin{aligned}
R_s(t) = P \left(\sum_{i=1}^N X_i > T \right) = R_1(T) + \int_0^T f_1(X_1) R_2(T - X_1) dX_1 \\
+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) R_3(T - X_1 - X_2) dX_2 dX_1 \\
+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \int_0^{T-X_1-X_2} f_3(X_3) R_4(T - X_1 - X_2 - X_3) dX_3 dX_2 dX_1 \\
+ \dots \\
+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \dots \int_0^{T-\sum_{i=1}^{N-2} X_i} f_{N-1}(X_{N-1}) R_N \left(T - \sum_{i=1}^{N-1} X_i \right) dX_{N-1} \dots dX_2 dX_1
\end{aligned}$$

Instead of going through the time-consuming multidimensional numerical integration, this problem and most of the other system configurations can be solved using Monte-Carlo. Monte-Carlo is a stochastic algorithm based on experience involving n trials to

determine the mean of a random variable x . With a theoretical approach and using the *central limit theorem* (cf. Appendix A.1) Monte-Carlo algorithm can be written as follows:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (4-26)$$

The reliability of the system is the probability that the system experiences no failure during the time interval $(0,t)$, given that the system was as good as new at time zero. In other words, a system is reliable if its lifetime is longer than the mission time T .

Thus, the idea is to compute the Time To Failure (TTF or lifetime) of the system using random variables and as a ‘hit-or-miss’ technique, for each iteration we check whether the survival time of the system is longer than the mission time or not. Finally, the reliability of the system is the sum of all successes divided by the total number of trials.

We remind:

- ❖ That the condition of success for a Cold Standby redundancy with perfect switching is:

- \Rightarrow The active component fails at time X_1 ($0 \leq X_1 < T$) and the nth standby component functions for a time period longer than $(T - \sum_{i=1}^{n-1} X_i)$.

- ❖ The reliability of a component ‘i’ at the time ‘ t_i ’ is $R_i(t_i) = e^{-\lambda_i t_i}$ with λ_i the failure rate of component ‘i’.

$$\Rightarrow t_i = -\frac{\ln(R_i)}{\lambda_i}$$

Algorithm:

```
/* Initialise the number of success */

Nb_of_success = 0;

/* Run Monte-Carlo simulation for n trials */

For i = 1 to Number_of_trials do

{

    /* Initialise the system lifetime */

    System_lifetime = 0;

    /* Compute the system's lifetime by adding the random time to failure of
       each component, i.e.  $TTF_{system} = \sum_{i=1}^{\# items} TTF_i$  with  $TTF_i = -\frac{\ln(R_i)}{\lambda_i}$ 
       and  $R_i$  a random number uniformly distributed between 0 and 1 */

    For i = 1 to Number_of_components

        System_lifetime = System_lifetime - TTFi;
```

/* Check whether the system lifetime is longer than the Mission Time or not. If it is true then the counter of success is incremented by 1 */

```
    If (System_lifetime > Mission_Time)

        then Nb_of_failure = Nb_of_success + 1;

    }
```

/* Compute the reliability of the system as the ratio of number of success to number of trials */

```
Reliability = Nb_of_success / Number_of_trials;
```

This code has been developed in Visual Basic (cf. Appendix A.2) and provides the same results as symbolic calculations ($Rs = 0.4485$ calculating integrals with Matlab and $Rs \approx 0.45$ with Monte-Carlo 5000 iterations for 3 components). This shows that the reliability of redundant systems can be evaluated using stochastic methods like for instance Monte-Carlo algorithm. Future works might involve the development of advanced Monte-Carlo numerical methods for Cold and Warm standby configurations with both perfect and imperfect switching devices.

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5 Functional Modelling for structural integrity problems

Failures of structures are considered as the most catastrophic as well as the most arduous to prevent since they often occur without any warning. Causes of structural failures can be classified into four categories: the material is subject to an environment beyond its design domain; an inappropriate choice for the design and operating conditions; the material, to start with, is defective; or the design itself is wrong (Ramachandran, 2005). According to this classification, it becomes obvious to assess potential failure modes throughout the design and manufacturing process as well as during the operational phase. Afterwards precautions can be taken to avoid introduction of deleterious defects, advanced analyses as well as appropriate physical testing can be carried out to accurately evaluate the most critical and the most likely failures.

Adapted from the international standards, the Functional Design module implemented in MADe has proved to be an asset to carry out FMECA for dynamic systems. However, MADe does not allow the functional modelling of structural integrity problems, this is something under research. Thus, this chapter introduces a new technique to assess potential structural failures. *Functional Block Diagram – Functional Taxonomy – Failure Diagram and Failure Taxonomy – Failure Propagation – Criticality Analysis and PHM Design and Optimisation*, with emphasis on structure are presented hereafter.

5.1 FMECA Definitions

Functional failure mode: describes the way an item fails to fulfil its function.

Failure Mode and Effects Analysis (FMEA): a procedure by which each potential failure mode in a system is analysed to determine the results or effects thereof on the system and to classify each potential failure mode according to its severity. (MIL-STD-1629A 1980, p4)

Criticality Analysis (CA): a procedure by which each potential failure mode is ranked according to the combined influence of severity and probability of occurrence. (MIL-STD-1629A 1980, p3)

N.B: The different examples and models shown hereafter are based on the current version of MADe. All the improvements with regard to structural modelling have not been implemented yet, thereby the case studies are not consistent. Nevertheless, readers must keep in mind that the numerous enhancements presented throughout this chapter shall lead to a much more relevant functional design of structures.

5.2 Functional Block Diagram

A functional model is used to represent a system and its elements in terms of their functions. A popular functional model is the Functional Block Diagram (FBD) technique, which presents the system as a breakdown of its major functions and provides the ability for tracing failure mode effects through all levels of indenture. The functional modelling of a complex structure using MADe software is based on five levels of indentures which are defined hereunder.



Figure 5-1 Functional Block Diagram with a single level of indenture

5.2.1 Indenture levels

The highest level of indenture is the ‘System level’ and corresponds to the MADe project itself.

❖ Subsystem level

A Subsystem is an element that is made up of *Components* and/or other *Subsystems*, e.g. a wing box made up of spars, ribs and panels. *Subsystems* can be assigned Functions.

❖ Component level

A *Component* is an element that is made up of *Parts* and *Pairs*, e.g. a stiffened panel made up of beam and shell parts. *Components* can be assigned Functions.

❖ Pair level

A *Pair* is the connection between a *Part* and another *Part/Component/Subsystem*, e.g. a stiffener made up of two beam parts welded together. A *Pair* defines how the two elements are joined, and can be assigned functions so that it may be included in Functional Analysis.

❖ Part level

A *Part* is the lowest level element on a System Structure Diagram. *Parts* in MADE cannot be directly modelled functionally. Instead they are joined with other *Parts* to form *Pairs*.

❖ Feature

A *Feature* is a particular property of a *Part*, e.g. a hole, a thread, *etc.* The use of *Features* is important since at the Failure Diagram stage, the mechanism of failure can be applied either to the part or to the feature. For example, fillet or a hole should be set as *Features* since discontinuities in structures introduce stress concentrations.

5.2.2 Structural Parts

The definition of the *Part* indenture level should enable engineers to model any sort of physical structure. The language used in the Finite Element discipline is the most general to model a structure, thus the structural part definitions are related thereof. The seven basic structural components are stored in a library so as to quickly drag and drop them or user can create new designs and share them among the project team members.

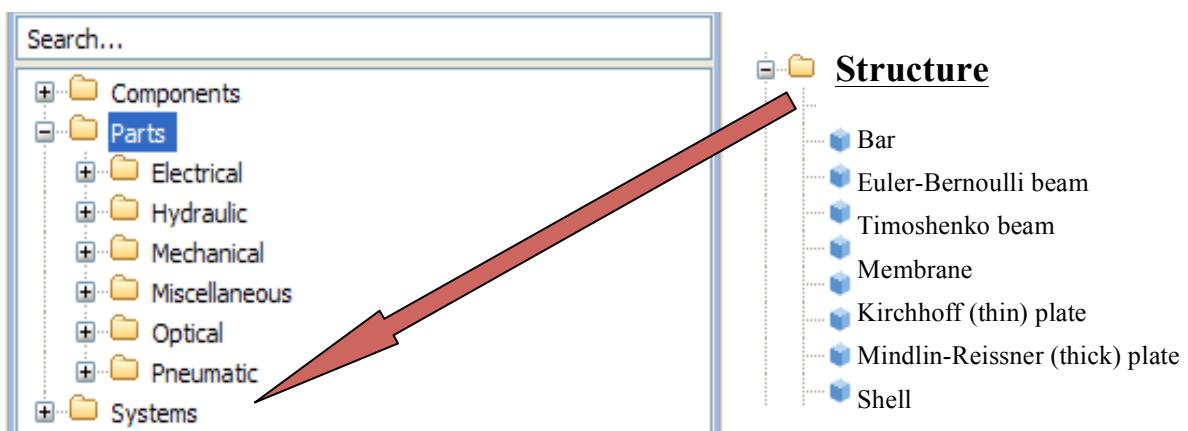


Figure 5-2 MADE Library of *structural parts*

5.2.3 Joint Parts

Joint is a connection between two structural parts. Joint parts have been classified as *Fastened joints (removable)*; *Riveted joints*; *Welded joints and Bonded joints*. It cannot be acceptable to have one component, i.e. one functional block, standing for one joint part, e.g. one block for one rivet. Therefore, an analogy has been drawn between physical joints and MADe joint parts.

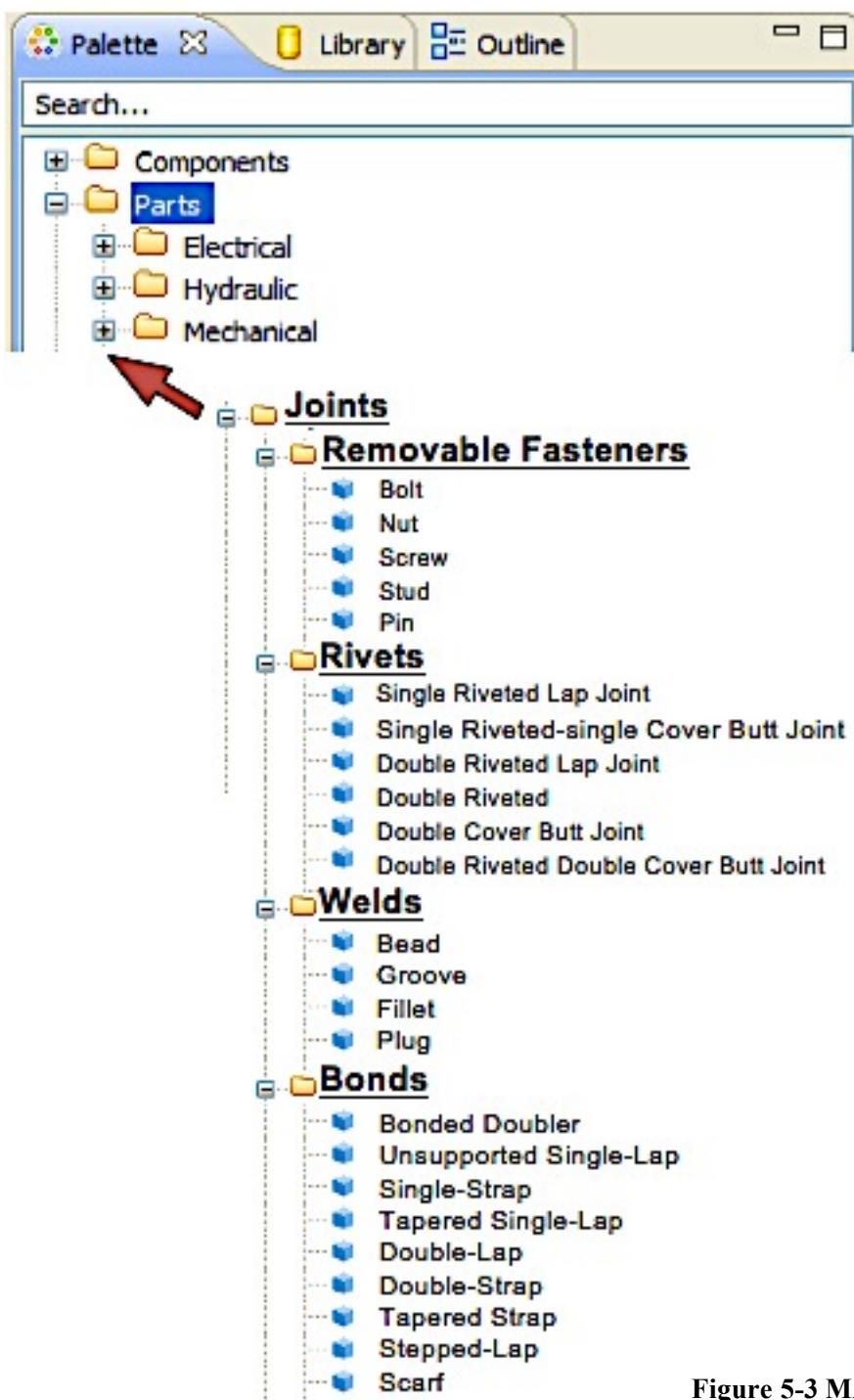


Figure 5-3 MADe Library of *joint parts*

5.3 Function Definitions

Although FMECA is a valuable source of know-how for companies, the challenge of a consistent functional definition could be a nightmare that sometimes has dissuaded engineers for using a functional approach (Wirth; Berthold; Krämer and Peter). This issue has been removed thanks to the development of function and failure taxonomies.

The first taxonomy is used to state item's function and is constructed as follows:

- **Function verb**, e.g. *transmit, reduce*: the action of the item on the input flows.
- **Flow noun**, e.g. *fluid, mechanical energy*: the material, energy or signal that is being acted upon.
- **Flow property**, e.g. *bulk modulus, rotational velocity*: the property of the flow that is required to fulfil the performance specifications of the item.

Every item has a function that can be defined by using the extensive database of MADe. For example, the function of a Rib component (Figure 5-1) is to *transmit* the air loads from the skin to the spars.

- ▲ **Function verb – *Transmit***: *To move an energy or signal from one place to another. (Synonyms: Advance; Conduct; Convey; Drop; Deliver).*

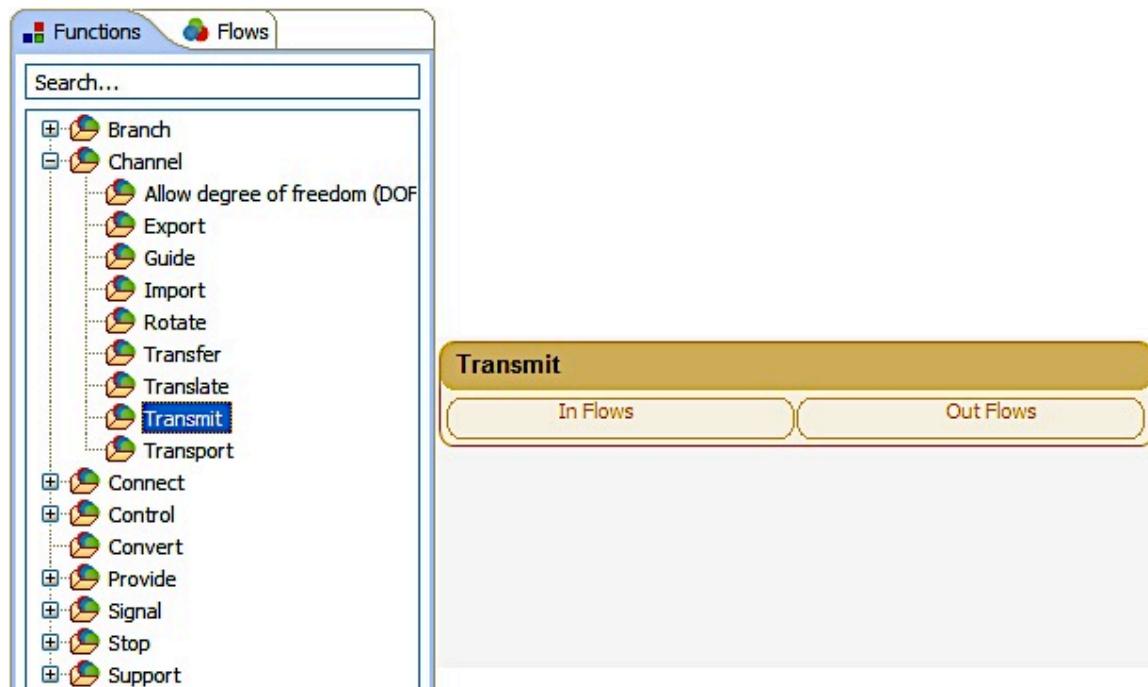


Figure 5-4 Definition of the function *Transmit*

The other objective of the Functional Block Diagram is to illustrate the interrelationships between functional entities of a system and provides a functional flow sequence. In MADe, a functional flow completes the definition of functions by specifying the *noun* of the flow as well as its *property*.

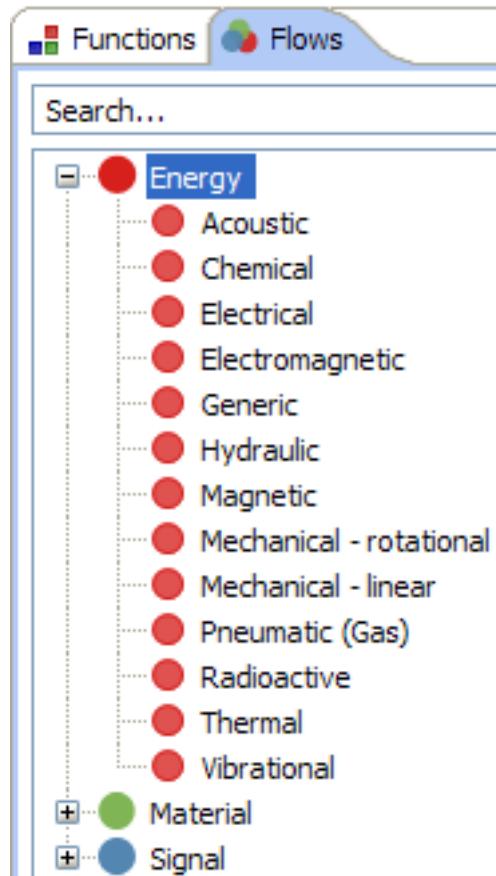


Figure 5-5 Energy flows properties

Using the example of the Rib, the air loads are assumed to be *Mechanical Forces*, hence:

- ▲ **Flow noun - *Mechanical***
- ▲ **Flow property - *Force***

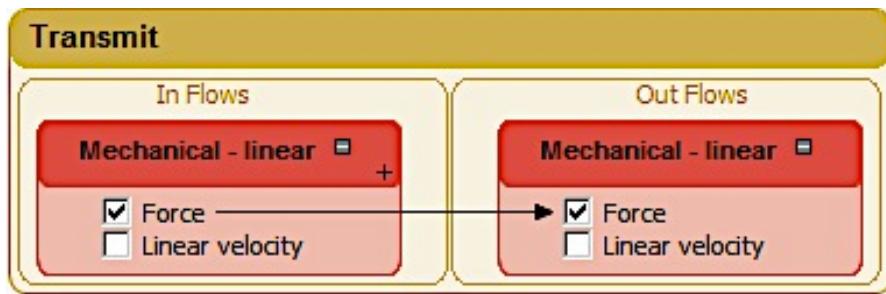


Figure 5-6 Definition of the function *Transmit Mechanical Force*

The causal connection between the component's input and output flows defines how the component responds to the 'erroneous' input flow. This connection possesses a positive polarity which means that an increase of the input property will increase the output property.

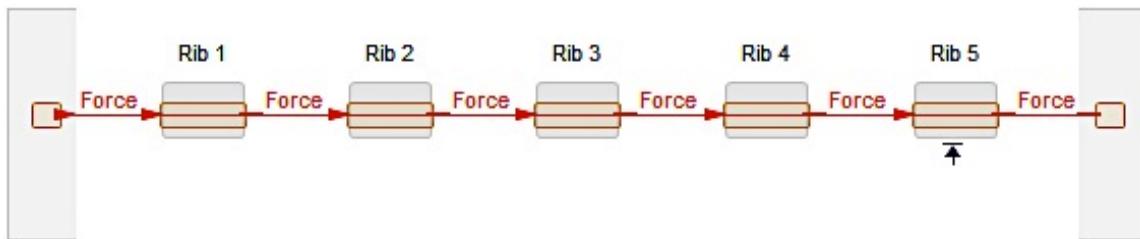


Figure 5-7 Functional Block Diagram and Functional Flows

As we can see above (Figure 5-7), MADe has a limited capability for structural flows modelling. Thus, the functional taxonomy has been enhanced with several flow nouns as well as flow properties:

Function Verb	Flow noun		Flow property
Resist	Energy	<i>Structural – Static Stresses</i>	<i>Tensile stress</i> <i>Compressive stress</i> <i>Shear stress</i> <i>Bending stress</i> <i>Torsion stress</i> <i>Combined bending and direct stress</i> <i>Hydrostatic (volumetric) stress</i> <i>Combined bending and twisting stress</i>
	Energy	<i>Structural – Dynamic Stresses</i>	<i>Impact stress</i>
	Energy	<i>Structural – Fatigue Stresses</i>	<i>Alternating/Reversing stress</i> <i>Repeated stress</i> <i>Combined steady and alternating stress</i> <i>Random stress</i>
	Energy	<i>Structural – Thermal stress</i>	<i>Thermal stress</i>
Limit	Energy	<i>Structural – Strains</i>	<i>Normal Strain</i> <i>Shear Strain</i> <i>Volumetric Strain</i>
	Energy	<i>Structural – Displacements</i>	<i>Displacement</i> <i>Deflection</i> <i>Angle of torsion</i>
Transmit	Energy	<i>Structural – Static Concentrated Loads</i>	<i>Tensile force</i> <i>Compressive force</i> <i>Shear force</i> <i>Bending moment</i> <i>Torsional moment</i>
	Energy	<i>Structural – Static Distributed Loads</i>	<i>Uniform load</i> <i>Triangular load</i> <i>Parabolic load</i> <i>Exponential load</i> <i>Sine load</i> <i>Cosine load</i>
	Energy	<i>Structural – Inertia loads</i>	<i>Gravity load</i> <i>Spinning load</i>
	Energy	<i>Structural – Dynamic Periodic Loads</i>	<i>Harmonic High cyclic load</i> <i>Harmonic Low cyclic load</i> <i>Non-Harmonic High cyclic load</i> <i>Non-Harmonic Low cyclic load</i>
	Energy	<i>Structural – Dynamic Aperiodic Loads</i>	<i>Transient load</i>
	Energy	<i>Structural – Impact Loads</i>	<i>Impact load</i>
	Energy	<i>Structural – Pressure</i>	<i>Pressure</i>

Table 5-1 Functional Taxonomy for structural modelling

5.4 Failure Definitions

The second taxonomy consists of providing engineers with a database of numerous events, i.e. potential Causes; Mechanisms; Faults and Symptoms of functional failure modes. The failure of a component in MADe is expressed in terms of an unacceptable increase or decrease of the output flow property. In our case study, the failure mode would exist when the output flow, *Mechanical Force*, increases or decreases.

Failure Diagram enables the engineer to map the sequence of events that result in a failure mode, and forms the starting point for automated processing of the Failure Concept Map for functional FMEA. The failure diagram is also used to calculate the criticality of each causal path from initial failure cause to final end-effect of the failure mode. A sequence of event, also known as Failure Concept, includes:

- **Cause:** abnormal state of input, loading or environment that initiates the failure process.
- **Mechanism:** physical process of degradation of the item.
- **Fault:** physically degraded state of a system element that renders it unable to completely fulfil its function. It is the result of a failure cause, and a failure mechanism.
- **Symptom:** detectable symptoms generated by the failure process, e.g. change to the appearance, behaviour, etc.

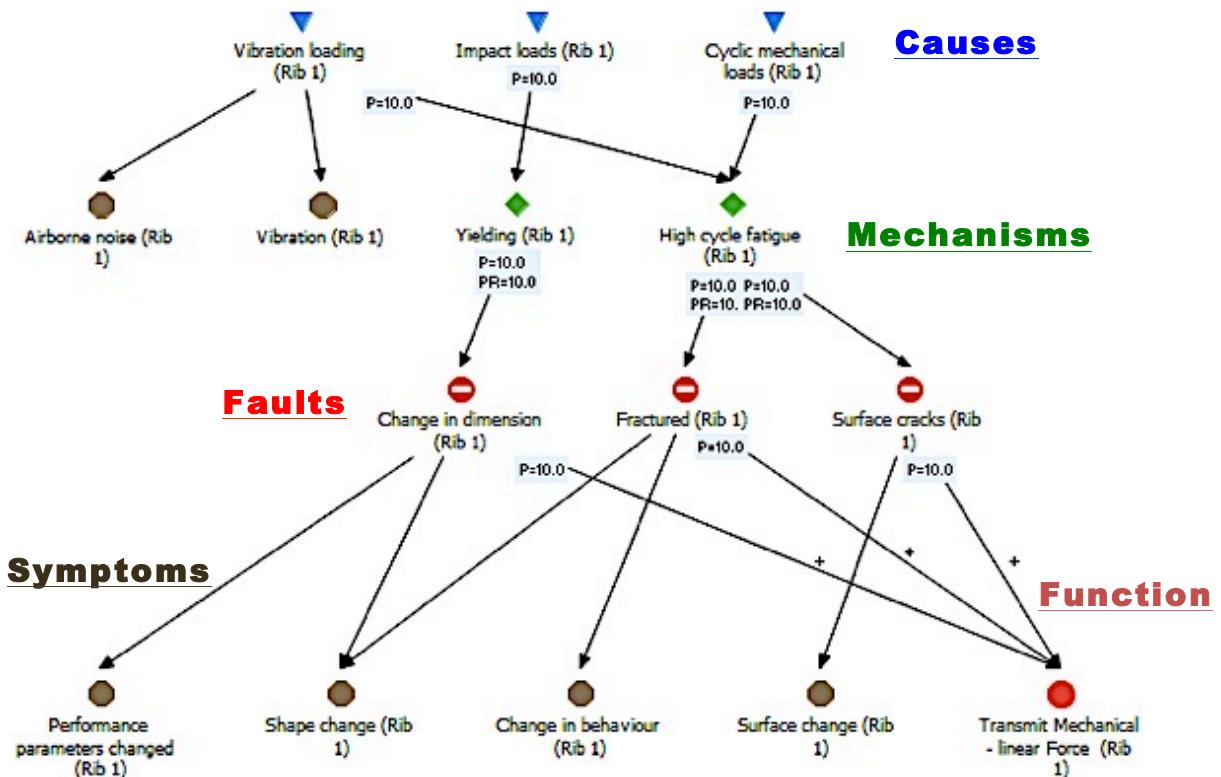


Figure 5-8 Example of a Failure Diagram for Component ‘Rib 1’

This Failure Diagram has been designed with the purpose to avoid needlessly overcrowding of the model, thus only two potential mechanisms of failure have been considered:

- ▲ **High cycle fatigue:** it is well known that the assessment of fatigue damage is the keystone of structural integrity. In this case, the cause of the fatigue mechanism can either be a *Vibration loading* or a *Cyclic loading*. This mechanism of failure will inevitably leads to different faults such as *Surface cracks* and sometimes the *Fracture* of the structural components. In addition, the failure can be detected according to different symptoms, e.g. *Surface cracks* and/or a *Surface change* can be noticed during inspections.
- ▲ **Yielding:** corresponds to the second failure mechanism impacting the strength of the material. The yielding of the rib may for example occur after a collision during the taxi phase, i.e. an *Impact load*. As a result of the collision, the structure *Changes in dimension* and can be detected by a *Shape change* and/or a *Change of the performance parameters*.

All potential faults (red icons) directly affect the function of the component. In order to specify whether a fault impacts 'positively' or 'negatively' the functional flow, a causal connection must be specified (+ or -, cf. symbol "+" of the arrow connection on Figure 5-8). In other words, a causal connection with a positive polarity "+" means that an increase of the related Causes, i.e. *Cyclic mechanical loads* or *Vibration loading* will increase the flow property, i.e. *Force*.

Even if MADe includes a few Causes and Mechanisms of structural failures, most of them are missing. Thus, the standardised taxonomy ensuring that the failure concept description for structures is consistent has also been improved.

Failures of structure are in general either due to loadings or environmental damages. Environmental causes have already been implemented in MADe; consequently, only mechanical loads are treated. After a literature review, a list of potential causes of structural failure has been created.

Cause Group	Cause of failure
Static Concentrated Loads	<i>Tensile force</i> <i>Compressive force</i> <i>Shear force</i> <i>Bending moment</i> <i>Torsional moment</i>
Static Distributed Loads	<i>Uniform load</i> <i>Triangular load</i> <i>Parabolic load</i> <i>Exponential load</i> <i>Sine load</i> <i>Cosine load</i>
Inertia loads	<i>Gravity load</i> <i>Spinning load</i>
Dynamic Periodic Loads	<i>Harmonic High cyclic load</i> <i>Harmonic Low cyclic load</i> <i>Non-Harmonic High cyclic load</i> <i>Non-Harmonic Low cyclic load</i>
Dynamic Aperiodic Loads	<i>Transient load</i>
Impact Loads	<i>Impact load</i>
Pressure	<i>Pressure</i>

Table 5-2 Potential Causes of structural failures

Furthermore, structures are physically degraded either by an environmental process (already in MADE) or by an overloading. Some mechanisms of structural failures are present in the software (*blue ones*) and they are listed in three different groups: plastic deformations, elastic deformations and fractures. Nevertheless, new mechanisms have been constructed so as to provide the user with more relevant design.

Mechanism Group	Mechanism of failure
Fracture	<i>Tensile fracture</i> <i>Compressive fracture</i> <i>Shear fracture</i> <i>Bending fracture</i> <i>Torsion fracture</i> <i>Combined bending and tensile fracture</i> <i>Combined bending and compression fracture</i> <i>Combined torsion and tensile fracture</i> <i>Combined torsion and compression fracture</i> <i>Combined bending and torsion fracture</i> <i>Resonance fracture</i>
Elastic deformation	<i>Tensile deformation</i> <i>Compressive deformation</i> <i>Shear deformation</i> <i>Bending deformation</i> <i>Torsion deformation</i> <i>Combined bending and tensile deformation</i> <i>Combined bending and compression deformation</i> <i>Combined torsion and tensile deformation</i> <i>Combined torsion and compression deformation</i> <i>Combined bending and torsion deformation</i>
Plastic deformation	<i>Tensile yielding</i> <i>Compressive yielding</i> <i>Shear yielding</i> <i>Bending yielding</i> <i>Torsion yielding</i> <i>Combined bending and tensile yielding</i> <i>Combined bending and compression yielding</i> <i>Combined torsion and tensile yielding</i> <i>Combined torsion and compression yielding</i> <i>Combined bending and torsion yielding</i>

Table 5-3 Potential Mechanisms of structural failures

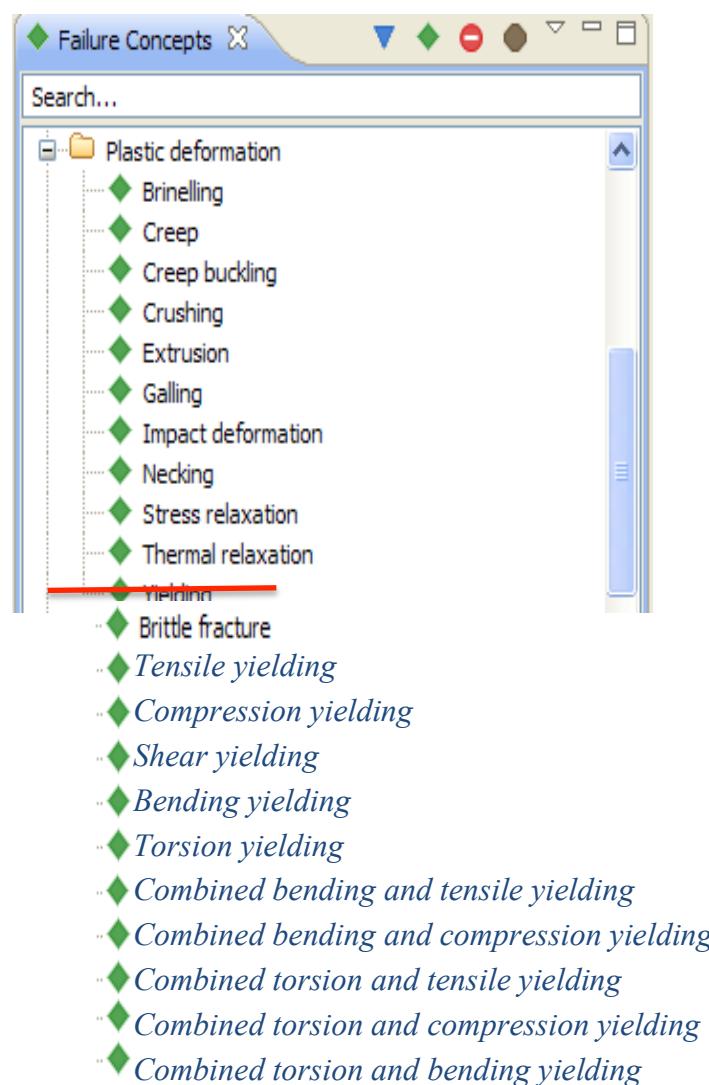


Figure 5-9 Failure mechanism of structures corresponding to a plastic deformation

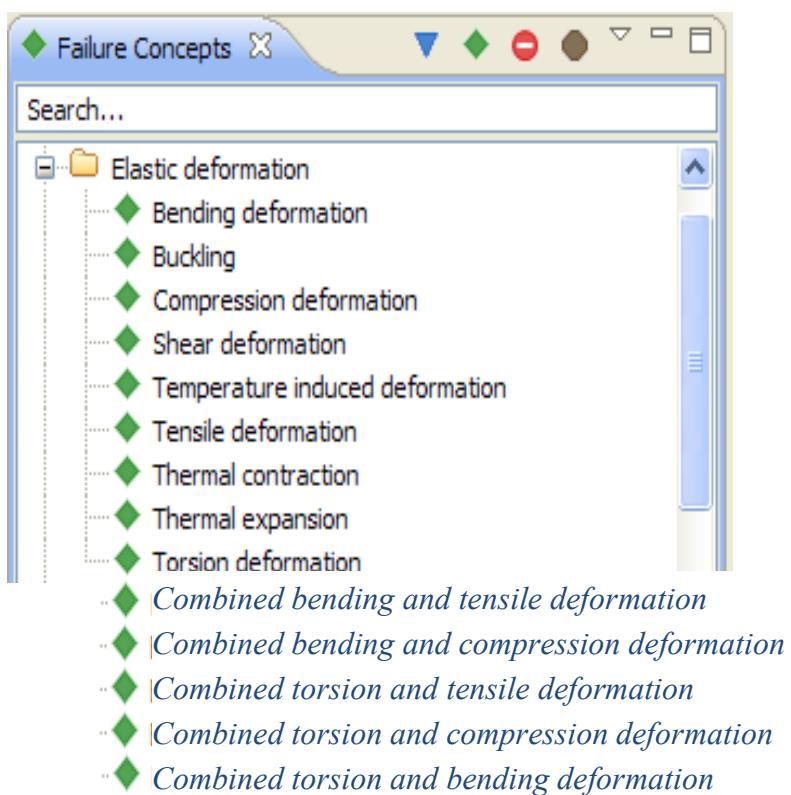


Figure 5-10 Failure mechanisms of structures corresponding to an elastic deformation

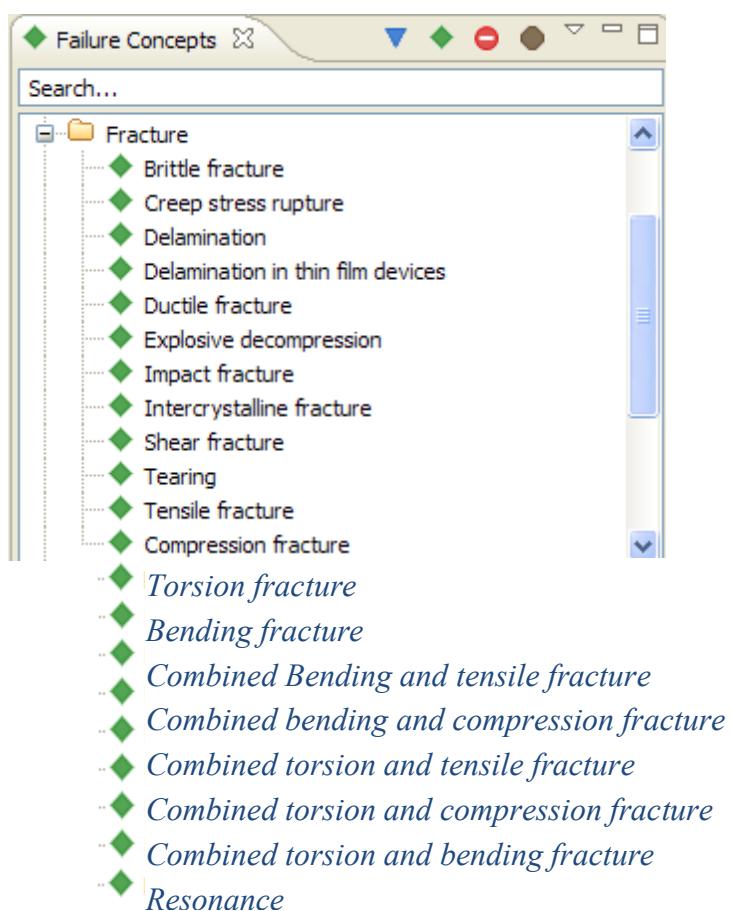


Figure 5-11 Failure mechanisms of structures corresponding to a fracture

5.5 ‘What-if’ Analysis & Failure propagation

Looking at the design process of complex structures we see three main iterations:

- Customers’ requirements are not met at the conceptual design phase and so corrective actions must be taken.
- Structural weaknesses acknowledged during the Finite Element Analysis as well as at the Fatigue & Damage Tolerance stage require modifications of the detailed design.
- Finally, the physical tests may show anticipated structural failure due to a fault in the design or during the manufacturing. Thus, once again, it is necessary to back out in the process.

Every time that the design is reviewed it is imperative to analyse the impact of the modifications on the whole structure so as to identify which other parts need to be considered. However, in the case of complex structures, it might be difficult to identify all the physical interactions among the components. One solution is to carry out a ‘What-if’ analysis by injecting a failure in the model and then to look-over the responses.

In MADe, a failure mode is the result of the deviation, i.e. *increase* or *decrease*, of the output flow. For example, if the output flow increases, then the failure mode is *High Mechanical Force*. Conversely, if the output flow decreases, then the failure mode is *Low Mechanical Force*. In a more realistic approach, the failure of a rib would be that the *Force* increases beyond the design criteria of the limit state analysis and then the rib could not *Transmit* the air loads to the spars.

The automated analysis consists in simulating all potential failure modes mapped in the failure diagrams by activating every failure causes in turn. Then, failures are propagated through the system by Fuzzy Cognitive Map analysis and reported in a Propagation Table showing the next and end-effects of a failure mode.

For example, the Propagation Table shown hereunder presents the evolution of the five failure modes discussed up to now.

Component	Flow Property	Failure	Rib 1...	Rib 2...	Rib 3...	Rib 4...	Rib 5...
Rib 1	Force (Mechanical - linear)	↑ High...	↑ High				
Rib 2	Force (Mechanical - linear)	↑ High...		↑ High	↑ High	↑ High	↑ High
Rib 3	Force (Mechanical - linear)	↑ High...			↑ High	↑ High	↑ High
Rib 4	Force (Mechanical - linear)	↑ High...				↑ High	↑ High
Rib 5	Force (Mechanical - linear)	↑ High...					↑ High

Figure 5-12 Propagation Table for the case study

We can notice that a failure, or modification of the component, *Rib 1*, impacts all other parts. However, changes in the component, *Rib 3*, only affect the *Rib 4* and *5*.

5.6 Criticality Analysis

The purpose of the Criticality Analysis (CA) is to rank each potential failure mode identified by the FMEA according to the combined influence of Severity Classification, its Probability of Occurrence as well as its Difficulty of Detection (RAC, 1993). Afterwards, the most critical potential failure can be monitored in real time, which is called Structural Health Monitoring and this is discussed later.

Using MADe, structural engineers can define the level of criticality by using different techniques such as:

- ▲ **Risk Priority Number (RPN)** = Difficulty of detection * Occurrence * Severity

Each criterion is ranked between 1 and 10 with 10 corresponding to the worst case, i.e. Very High Difficulty of Detection, Occurrence and Severity. For example, the figure 5-13 shows the Criticality definition of the potential cause of failure, *Cyclic Mechanical Loads*. A rib is permanently exposed to fluctuating loads, thus the Occurrence parameter has been set to 9 (Very high occurrence), and as the detection of fatigue loading remains plenty of uncertainties, the Difficulty of Detection has also been defined equal to 9.

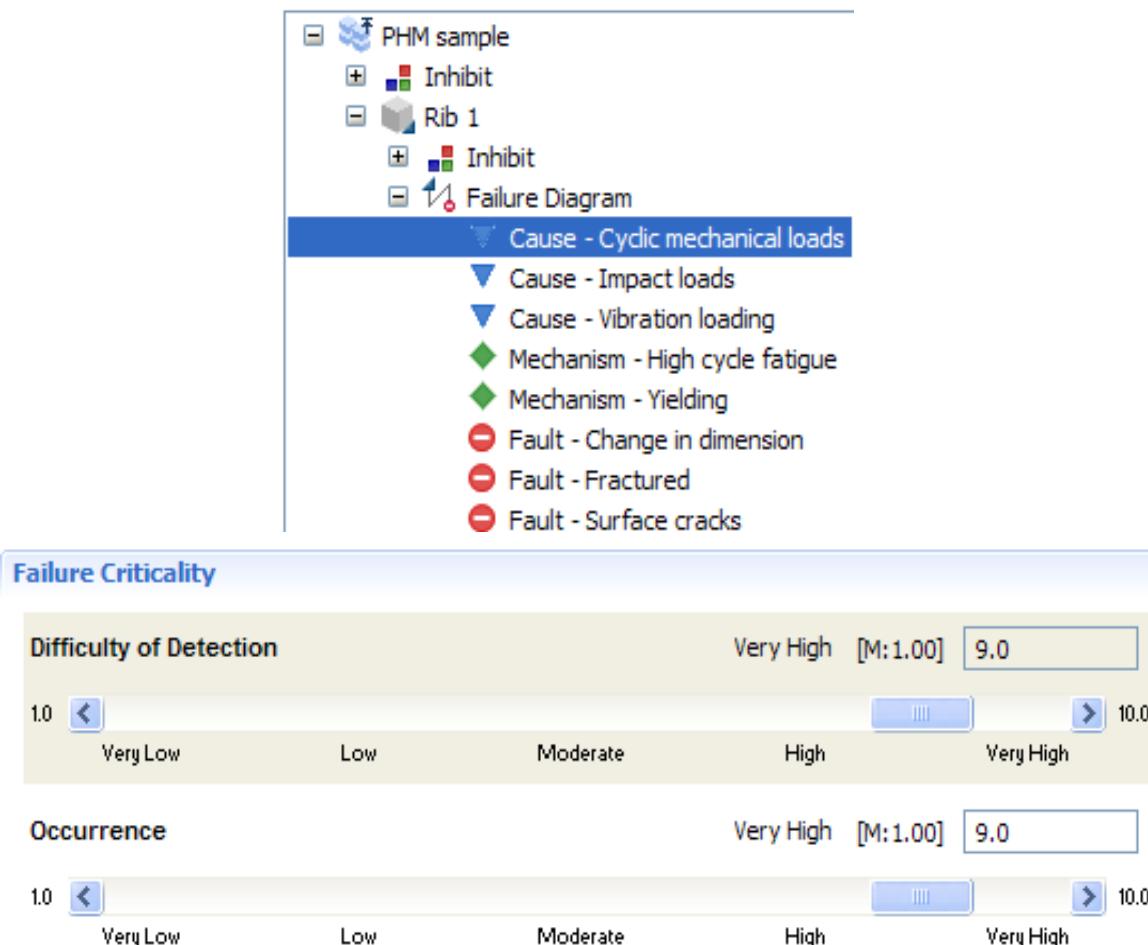


Figure 5-13 Criticality definition of a potential Cause ‘Cyclic mechanical loads’

- ▲ **Fuzzy Criticality:** is a more advanced technique that enables engineers to define the criticality levels by doing a trade-off between Difficulty of Detection and Apparent Occurrence. The Fuzzy Criticality tool integrates a range of rules and memberships accurately defined via explicit graphical definitions (Figure 5-14) which simplify and guide users.

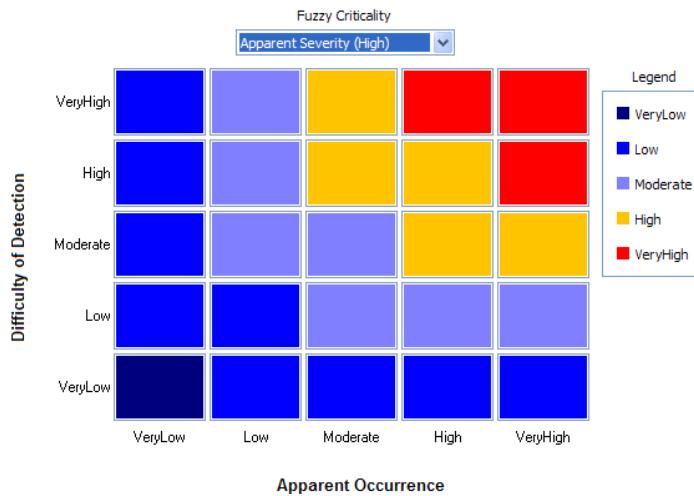


Figure 5-14 Example of a Fuzzy Criticality rule

Criticality evaluation plays a key role in the functional analysis since during the sensor set optimisation, the engineer can specify a threshold so as to only consider the most likely functional failure modes, thereby monitoring the most critical parts.

5.7 FMECA Reporting

Information are printed in a report so as to discuss potential failures during technical meetings. Several standard templates are provided like for instance the MIL-STD-16291. Nevertheless, users can customise the report as desired.

ITEM ID N/A		NEXT INDENTURE LEVEL PHM sample								
ITEM NAME Rib 1		SYSTEM NAME PHM sample								
OPERATION MODE N/A		REFERENCE DRAWING N/A								
FUNCTIONAL N/A NARRATIVE		PHYSICAL N/A DESCRIPTION								
Item Name	Function	Cause	Failure Mechanism	Effects			Criticality			
				Local	Next	End	O	S	D	RPN
Rib 1	Transmit	Vibration loading (Rib 1)	High cycle fatigue (Rib 1)	Transmit Mechanical - linear Force High (Rib 1)	N/A	Transmit Mechanical - linear Force High (Rib 5)	9.0	10.0	7.0	630
Rib 1	Transmit	Cyclic mechanical loads (Rib 1)	High cycle fatigue (Rib 1)	Transmit Mechanical - linear Force High (Rib 1)	N/A	Transmit Mechanical - linear Force High (Rib 5)	9.0	10.0	9.0	810

Figure 5-15 Example of FMECA report

5.8 PHM Design and Optimisation

Structural Health Monitoring (SHM) is a breakthrough in the safety and reliability assessment of complex structures. Real time monitoring of the condition state of a structure provides engineers with accurate data that enable to diagnostic faults and prognostic the remaining useful life of critical structural components so as to schedule just in time maintenance tasks. However, the quality and quantity of structural damage detections depends directly on the number of sensors as well as their locations. This section discusses how MADe PHM module can assist structural engineers to answer questions such as: How many sensors are required? Which critical parts have to be monitored? Which data need to be collected? As a decision-making tool, it also enables engineers to not only optimise the design of sensors but also to compare the sensor sets according to their cost and weight.

The PHM workbench is based on the Functional Design. Indeed, the MADe PHM module analyses the functional flows and focuses on the Symptoms of the potential failure modes. Running a sensor analysis we can specify which type(s) of flow property, e.g. *Force*, we want to monitor.

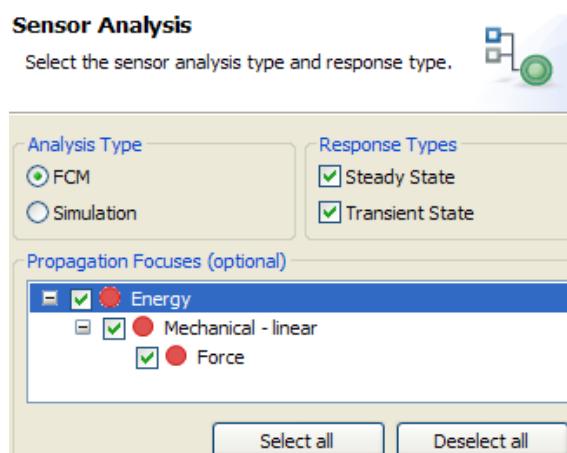


Figure 5-16 Analyse of the functional flow '*Mechanical Force*'

Then, MADe automatically generates various initial sensor sets including the coverage percentage of potential failure modes, the added-weight, the cost of the sensors and further meaningful information. A sensor set is a list of sensors that are individually assigned (linked) to a particular modelling block (subsystem, component, part) of a MADe model.

Name		Location	Type	# of Sensors	Coverage	Possible Coverage	Cost	Weight (g)
Sensor set	PHM sample	FCM	4		100%	100%	\$0.00	0.00

Figure 5-17 Sensor Set automatically designed

5.8.1 Sensor Discrimination

For every sensor set in the system, we need to generate Diagnostic Sets for it. A diagnostic set is a set of sensors which can be used to identify a given failure. A functional failure mode might have more than one diagnostic set, but we only choose the one which is the subset of the given sensor set. Comparing the responses of each failure mode provided by the propagation table (Figure 5-12), we generate MAXTERMS.

Responses of components to each failure must be checked against responses to other failures. Thus, for ‘n’ failure modes, the XOR logic rules are applied to each pair of functional failure modes $F(i), F(j)$, where $i = 1 \dots n, j = 1 \dots n$ with $j > i$. If the symptoms are different, then $G(i,j) = 1$, if the symptoms are the same then $G(i,j) = 0$. The MAXTERM $G(i,j)$ is therefore a binary text string. If a MAXTERM is a null set, the two failure modes i and j cannot be discriminated and are identified as an ‘ambiguity group’ which is reported in the analysis results.

Example:

This example shows how the diagnostic algorithm can generate sensor sets for Structural Health Monitoring. The study uses the propagation table of the case study, and in a first hand, only the first two failure modes $F(1)$ and $F(2)$ have been considered.

Component		Flow Property	Failure	Rib 1...	Rib 2...	Rib 3...	Rib 4...	Rib 5...
Rib 1	Rib 2	Force (Mechanical - linear)	High...	High	High	High	High	High
Rib 1	Rib 2	Force (Mechanical - linear)	High...	High	High	High	High	High

Figure 5-18 The first two failure modes of the case study

The reduced propagation table can be rewritten as follows:

F(1)	High	High	High	High	High
F(2)	\emptyset	High	High	High	High

Table 5-4 The first two failure modes of the case study

Comparing both failure mode effects, we can generate the MAXTERM G(1,2):

Using binary values and XOR rules we get,

F(1)	1	1	1	1	1
F(2)	0	1	1	1	1
F(1) XOR F(2) = G(1,2)	1	0	0	0	0

Reminder: XOR logic rules

XOR	
00	0
01	1
10	1
11	0

Table 5-5 MAXTERM G(1,2)

Table 5-6 XOR logic rules

Doing the same thing for all possible combinations we obtain MAXTERMS:

G(i,j)	MAXTERM	G(i,j)	MAXTERM
G(1,2)	10000	G(2,4)	01110
G(1,3)	11000	G(2,5)	00100
G(1,4)	11100	G(3,5)	00110
G(1,5)	11110	G(4,5)	00010
G(2,3)	01100		

Table 5-7 MAXTERMS for all potential functional failure modes

We notice that there is no 'ambiguity group', i.e. none MAXTERM set is equal to zero. Therefore, sensor sets provide 100% coverage of potential functional failure modes.

5.8.2 Sensor Minimisation

The results of the discrimination process are then carried over to the minimisation process. The minimisation process is used to reduce the amount of sensors needed for the model to a bare minimum and generate a list of prime applicants that should provide full coverage of sensors. MINTERMS are sets containing the minimum number of

symptoms required to discriminate between failure modes. This is achieved by intersecting the MAXTERMS.

The following table lists the MAXTERMS generated before (Figure 4-7) and now we will see how proceed with the minimisation technique.

G(i,j)	X1 (Rib 1)	X2 (Rib 2)	X3 (Rib 3)	X4 (Rib4)	X5 (Rib 5)
G(1,2)	1	0	0	0	0
G(1,3)	1	1	0	0	0
G(1,4)	1	1	1	0	0
G(1,5)	1	1	1	1	0
G(2,3)	0	1	1	0	0
G(2,4)	0	1	1	1	0
G(2,5)	0	0	1	0	0
G(3,5)	0	0	1	1	0
G(4,5)	0	0	0	1	0

Table 5-8 MAXTERMS for all potential functional failure modes

Thus, intersecting the MAXTERMS sets we obtain,

$$F(X_1, X_2, X_3, X_4) =$$

$$X_1 \cdot (X_1 + X_2) \cdot (X_1 + X_2 + X_3) \cdot (X_1 + X_2 + X_3 + X_4) \cdot (X_2 + X_3) \cdot (X_2 + X_3 + X_4) \cdot X_3 \cdot (X_3 + X_4) \cdot X_4$$

Using the classic rules of Boolean algebra:

- *Idempotent 1: $A+A = A$*
- *Absorption 1: $A \cdot (A+B) = A$*
- *Idempotent 2: $A \cdot A = A$*
- *Absorption 2: $A+(A \cdot B) = A$*

$$\text{Therefore, after calculations, } F(X_1, X_2, X_3, X_4) = X_1 \cdot X_2 \cdot X_3 \cdot X_4$$

This example ends up with a single MINTERM $X_1 \cdot X_2 \cdot X_3 \cdot X_4$ corresponding to the minimal sensor set. Therefore, the optimised sensor set includes one sensor on X_1 , X_2 , X_3 and X_4 . This may seem quite obvious in this easy case study, however, readers must imagine the same analysis for a structure decomposed into several levels of indentures with numerous sorts of flow and function, so as to really see the benefits of MADe PHM.

5.8.3 Sensor Allocation

Exact sensor type selection is then automated according to the response of the component. Any exact sensor matching is achieved by the user through selective sensor selection. For example, selecting the function *Transmit Mechanical Force*, the type of sensor is automatically filtered and only three categories remain appropriated.

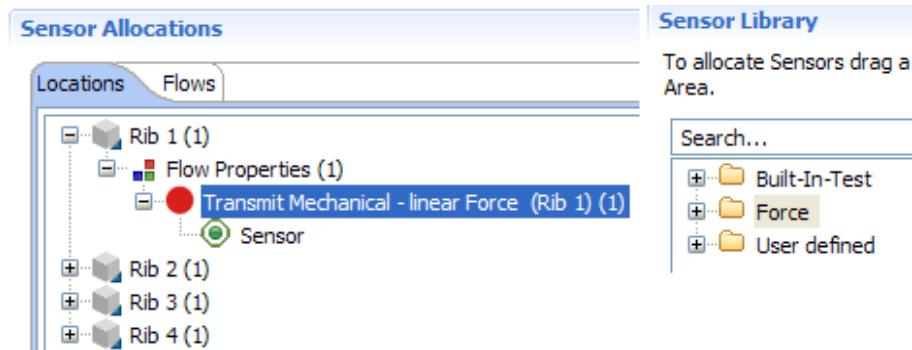


Figure 5-19 Auto-filtering of the suitable sensors

Then, the user can replace the initial sensor by a more suitable one, like for instance the use of a *Strain Gauge* to monitor the *Mechanical Force* of the Component 'Rib 1'.

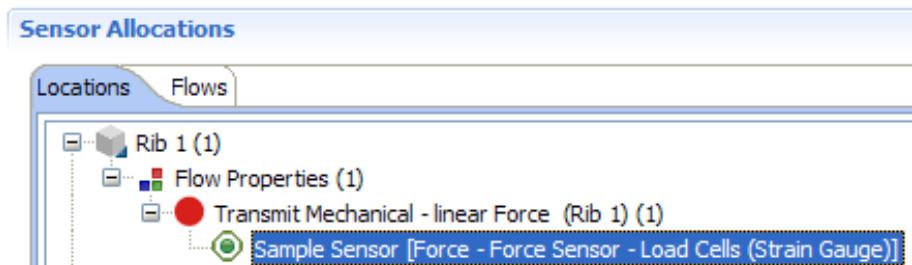


Figure 5-20 Selection of a Strain Gauge for the Mechanical Load flow property

Finally, the design is updated including sensor set details and sensor locations.

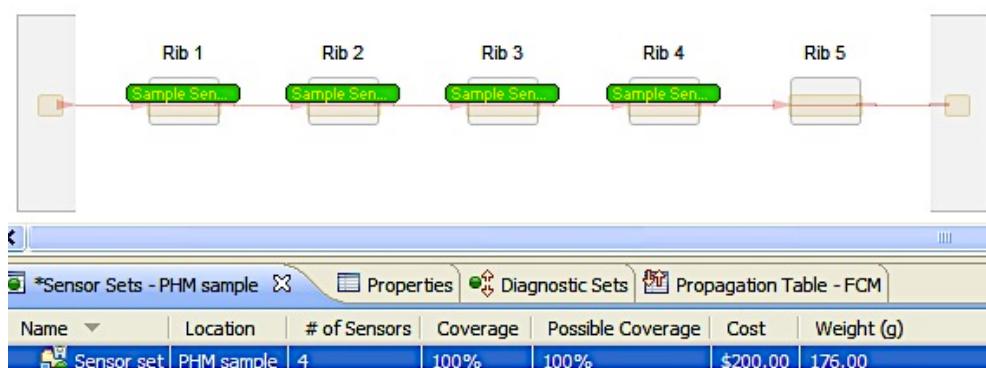


Figure 5-21 Sensor set design

5.8.4 Sensor Details

Details relative to the sensor, which are used to calculate the cost and weight of the sensor sets (Figure 5-17), are input by the user in agreement with the sensor manufacturers' offers.

Name: Sample Sensor [Force - Force Sensor -	
Serial: N/A	
Vendor: N/A	
Vendor Number: N/A	
Parameters	
Height: 0.0	
Width: 0.0	
Depth: 0.0	
Weight (g): 88.0	
Cost (\$): 100.00	
Sensitivity (%): 0.0	
Replacement Time (min): 0.0	
Accuracy (%): 0.0	
Repeatability: 0.0	
Other	
Operating Environment: -50 to 120 °C	
Constraints: -50 to 120 °C	
Contact: <input type="checkbox"/>	
Single Functionality: <input checked="" type="checkbox"/>	
Description: N/A	

Figure 5-22 Sensor details

MADe PHM already includes a generous library of sensors that can be improved in case of specific applications. The creation of a customised sensor is based on the type of functional symptoms which need to be detected as well as the type of functional flow which required monitoring. Thus, MADe has an unlimited capability, which can answer most of the structural engineers' requirements. Moreover, all new sensor designs can be stored in the library, so as to be either reused in future projects or shared among the project team members.

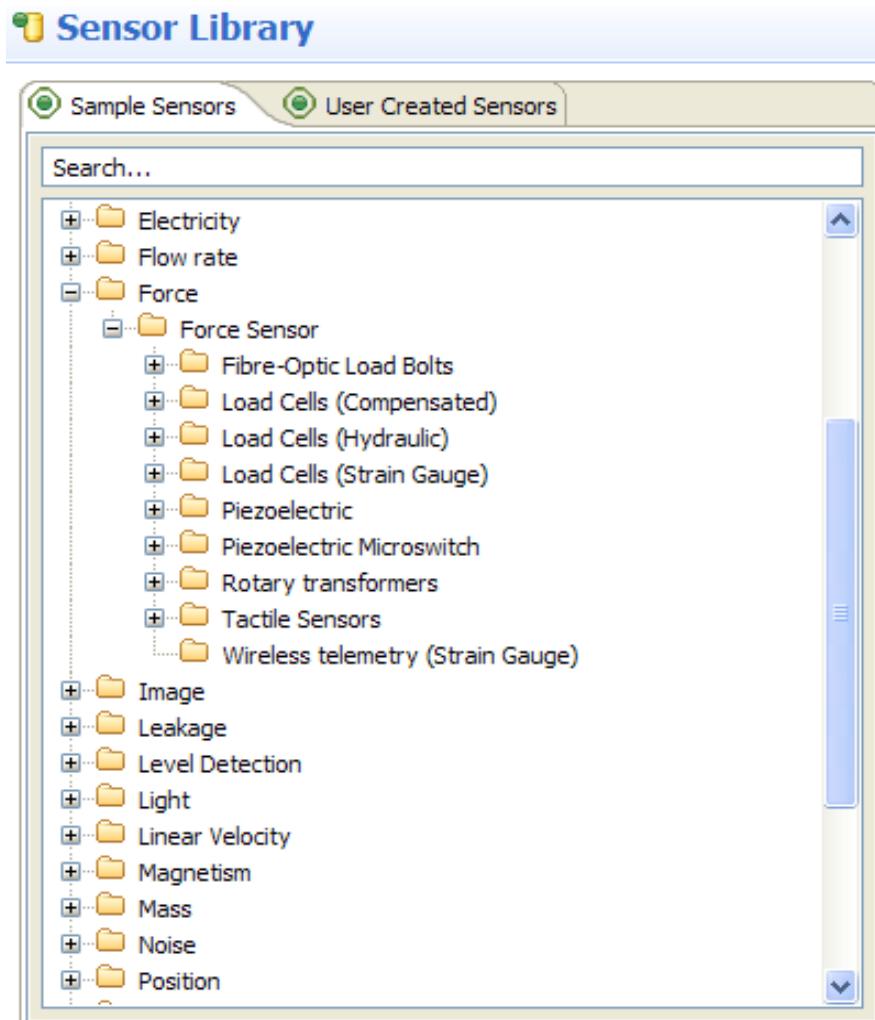


Figure 5-23 MADE sensor library

5.8.5 Sensor design reporting

MADE can generate a report including a summary of the sensors' details:

- Total (& unit) % coverage
- Total (& unit) weight
- Total (& unit) cost
- Number of particular type sensors

The reports can be customised according to engineers' needs.

Sensor Set Overview

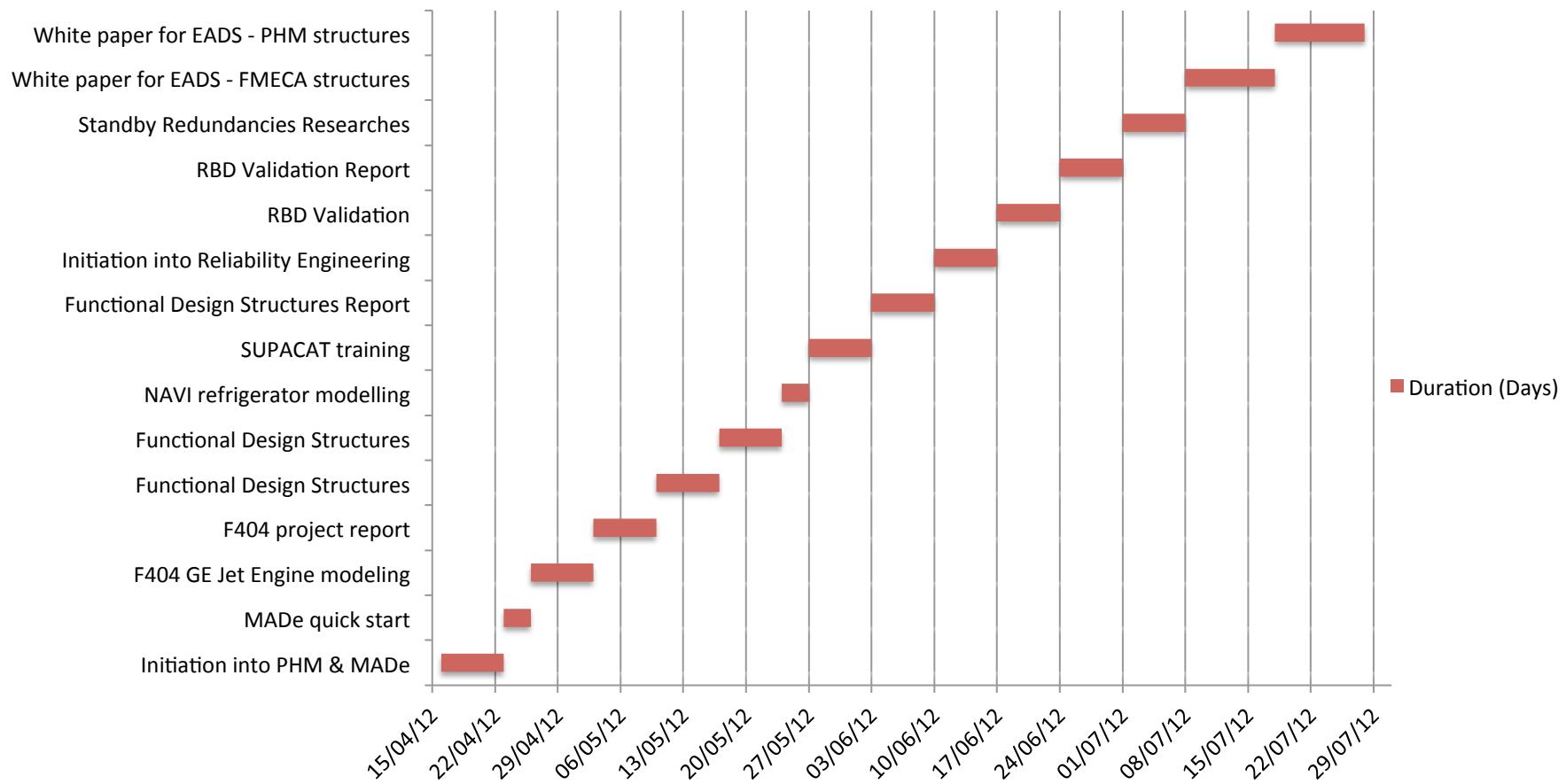
Analysis Type	<u>FCM</u>
Depth	<u>Single</u>
Number of Sensors	<u>4</u>
Total Coverage	<u>100%</u>
Total Cost	<u>\$200.00</u>
Total Weight	<u>176.0 g</u>

Sensor set (Assigned) - Sensor Detail

Sensor	Location	Monitoring	Coverage	Weight	Cost
Sample Sensor [Force - Force Sensor - Piezoelectric]	Rib 4	Transmit Mechanical - linear Force (Rib 4)	100%	88.0 g	\$100.00
Sample Sensor [Force - Force Sensor - Piezoelectric]	Rib 3	Transmit Mechanical - linear Force (Rib 3)	100%	88.0 g	\$100.00
Sample Sensor [Force - Force Sensor - Load Cells (Strain Gauge)]	Rib 2	Transmit Mechanical - linear Force (Rib 2)	100%	0.0 g	\$0.00
Sample Sensor [Force - Force Sensor - Load Cells (Strain Gauge)]	Rib 1	Transmit Mechanical - linear Force (Rib 1)	100%	0.0 g	\$0.00
Totals			100%	176.0 g	\$200.00

Figure 5-24 Example of Sensor Design Report

6 Project Management



As we can see, neither deadlines nor milestones have been imposed. Tasks were not scheduled since the company used to work according to customers' requirements. Indeed, the feedbacks coming back from target clients as well as current users must be quickly fulfilled so as to convince them to either invest in MADe licences or to carry on working with PHM Technology. This original philosophy of project management is mainly due to the fact that the company is still young and need to gain the confidence of customers in order to get more market shares and so more money which can be reinvested to increase the capability of MADe.

7 Conclusions and Further Work

7.1 Conclusions

This thesis has outlined two different approaches for risk assessment and management. Firstly, It has been demonstrated that probabilistic calculations using historic component failure rates can be performed for reliability evaluation. Reliability of classic designs such as Series, Parallel and combinations of both can be easily calculated by hand or using classic computational tools (Excel, symbolic computation, programming language). However, the limited capability of MADe software to compute the reliability of Standby redundant systems shows the necessity to move from special closed form results to numerical integration methods based on stochastic algorithms, so as to provide engineers with more flexibility. Monte-Carlo algorithm has been briefly introduced for a Cold Standby configuration and this powerful method should be investigated in further details since it can compute the reliability of any system configuration without paying attention to the challenging recursive integrations.

As a complement to the hardware approach, functional modelling as a means of possibly enhance the assessment of potential functional failure modes of complex systems has already found the breakthrough. However, such a technique remains blurred within the structural engineering field. The strength of MADe Functional Design is to provide engineers with both standardised taxonomies for function and failure concept definitions. This thesis has been written with the purpose to demonstrate the potential use of MADe for functional modelling of complex structures. The library of MADe has been reviewed, so that engineers can use common structural parts.

Additionally, both taxonomies used to define functions and failures with emphasis on structural integrity problems have been constructed and are currently for discussion. On the other hand, the latest development of state-of-the-art built-in sensors through mushrooming computation power enable remarkable real-time monitoring of structures, also known as Structural Health Monitoring (SHM). SHM allows collecting data with respect to the external loads as well as the current health of the structure, so as to prognostic the remaining useful life of the structural components and schedule just in time maintenance tasks. Through a case study, this thesis has introduced the capability of the MADE PHM module to facilitate and optimise the design of sensor sets. Even if the research has outlined the opportunity to use MADE PHM for SHM technique, the use of functional analysis' results as a foundation for sensor set design reduces the possibility to illustrate the outcomes with a relevant example. Nevertheless, MADE PHM will be more persuasive as soon as the functional taxonomy of structure will be implemented.

7.2 Further Work

7.2.1 Reliability evaluation

Although this thesis has covered a fair part of the basics in the reliability engineering, further developments and researches can be carried, so as to obtain more realistic reliability evaluations. Here are some interesting improvements which should be considered.

7.2.1.1 Standby redundant systems

Monte-Carlo algorithms should be studied, optimised and implemented in MADE in order to evaluate cold redundancies including N standby components with perfect and imperfect switching. In addition to the cold redundancy configuration, warm redundancy (also known as load sharing, i.e. component failure rate in active state is slightly different from standby state) may be developed so as to fit the reality as much as possible (Way, 2003). The famous Markov Chain Monte-Carlo (MCMC) has been suggested as a future improvement and should be shortly investigated.

7.2.1.2 Lifetime distributions

So far, MADe only used the exponential probability distribution. Sometimes such a lifetime distribution may be inappropriate and not tends to justify the choice of the model. Thus, other models such as Weibull, Extreme Value, Lognormal and Gamma distributions should be implemented.

7.2.1.3 Dependent lifetime

Even if most of the reliability analysis techniques assume that the components are independent on each other, in some cases, this assumption is no longer valid. For example, the failure of a bar in a truss increases the load on the other bars, and this may impact their lifetime.

7.2.1.4 Repairable items

Many references devote several chapters to repairable systems. Consider components as repairable tremendously increase the complexity of calculations. Indeed, when the system is repaired, it may be in the same state as a time zero, or better, or worse. Furthermore, ageing of the system may play an important role in the evaluation of repairable systems. This area should be investigated since most of the systems of our everyday life are repairable.

7.2.2 Risks assessment and management methods

In addition to the FMECA, several risk assessment techniques such as SIL, LOPA and HAZOP will be implemented in MADe so as to enable users to use the method which matches as much as possible the reality of his systems or process. These techniques might play an important role to manage structural failures.

7.2.2.1 HAZOP

Hazard and Operability Study (HAZOP) is also a structured and systematic technique based on *guide-words* and enable multi-disciplinary team (*HAZOP team*) to assess potential failure modes during a set of meetings. It is an examination of a work procedure to identify hazards and causes for operation problems. (Rausand, 2005).

7.2.2.2 SIL & LOPA

The Safety Integrity Level (SIL) is a discrete level for specifying the safety integrity requirements of the safety function to be allocated to the safety-related systems (M. A. Lundteigen, 2007). This type of evaluation is often combined with the Layer Of Protection Analysis (LOPA), which evaluates risks by analysing *initiating causes* leading to a *process deviation*, which again may lead to an impact event that may result in an *end-consequence*. Then risks are managed by using protective layers.

7.3 Feeling

This last experience as a trainee was both rewarding and difficult. Indeed, risk assessment; Condition Based Maintenance; Reliability Centered Maintenance; Prognostics and Health Management were not part of my skills before joining PHM Technology. Therefore, I have learnt a lot from this work placement, in particular in the probabilistic reliability engineering area as well as in the use of a functional approach to assess and manage potential risk of failures throughout the product's lifecycle. In addition to the technical benefits, it has also been the opportunity to meet various interesting people working for companies internationally established.

Nevertheless, several factors have made this internship difficult. Firstly, at the beginning of the mission, I have been struggling for a long time to understand and memorise the numerous acronyms as well as the technical vocabulary of PHM. Moreover, as the only two engineers were often out of the office it has been difficult for me to gain from their experience. The fact that my supervisor was overseas from roughly one month after the beginning of my internship to mid-September, in order to attend international conferences has significantly affected the progress of my researches since only Skype chats and emails were our means of communication.

Ancora Imparo... I am still learning.

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APPENDICES

Appendix A Monte Carlo for Standby Redundancies

A.1 Central limit theorem

The distribution of the sum $X = \sum_{i=1}^n X_i$ of a large number n of statistically independent random variables X_1, X_2, \dots, X_n approaches a Gaussian (normal) distribution with increasing the number of random variables, if none of the random variables dominates the distribution of the sum (Todinov, 2005).

A.2 Visual Basic code for Cold Standby Redundancy

```
Private Sub cmdCompute_Click()
    Call ComputeSystemReliability
End Sub

Public Sub ComputeSystemReliability()
    'Variables'
    Dim i As Long, CompCount As Long, Iteration As Long, MaxIterations As Long, SuccessCount As Long
    Dim TotalLife As Double, Mission_Time As Double, sum As Double, SystemReliability As Double
    Dim FailureRate() As Single, Coeff() As Double
    With Worksheets("Sheet1")
        'Read & Store # components, # iterations and the mission time in cells(row,column)'
        CompCount = .Cells(2, 5)
        MaxIterations = .Cells(3, 5)
        Mission_Time = .Cells(4, 5)
        'Change array size according to # components'
        ReDim FailureRate(CompCount) As Single
        'Error message if # components or # iterations is smaller than 1'
        If CompCount < 1 Or MaxIterations < 1 Then
            MsgBox "Error in data": Exit Sub
        End If
        'Read & Store components' failure rates'
        For i = 2 To 2 + CompCount - 1
            FailureRate(i - 1) = .Cells(i, 2)
        Next i
    End With
    'Initialise the number of failure'
    SuccessCount = 0

    ' Run Monte-Carlo'
    For Iteration = 1 To MaxIterations
        TotalLife = 0
        ' Compute system's lifetime'
        For i = 2 To 2 + CompCount - 1
            TotalLife = TotalLife - Log(Rnd()) / FailureRate(i - 1)
        Next i
        ' Check whether system's lifetide is longer than the mission time'
        If TotalLife > Mission_Time Then SuccessCount = SuccessCount + 1
    Next Iteration
    ' Compute the reliability of the system'
    SystemReliability = SuccessCount / MaxIterations
    'Print the value in the cell(row,column)'
    Worksheets("Sheet1").Cells(6, 4) = "System Reliability = " & Format(SystemReliability, "0.#####")
End Sub
```

	A	B	C	D	E
1	Component #	Failure Rate			
2	0	0,05	Number of components =	3	
3	1	0,04	Number of Iterations =	500	
4	2	0,03	Survival Time =	75	
5					
6				System Reliability = 0.438	
7					
8					
9					
10			Compute System Reliability		
11					

Appendix B Technical White Paper



Design of complex engineering structures with MADe automated Failure Modes and Effects Analysis

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Abstract

Reliability assessment, Condition Based Maintenance and Structural Health Monitoring cannot be achieved properly without being aware of all potential failure modes which may occur throughout the life cycle of the structure. The most popular method for the identification and detection of potential failures is the *Failure Modes, Effects and Criticality Analysis (FMECA)*, which is based on a functional approach.

This paper discusses the opportunity to carry out an automated FMECA in a ‘Maintenance Aware Design environment’ with emphasis on structural engineering failures.

Keywords: MADe; PHM; Functional Modelling; Structures

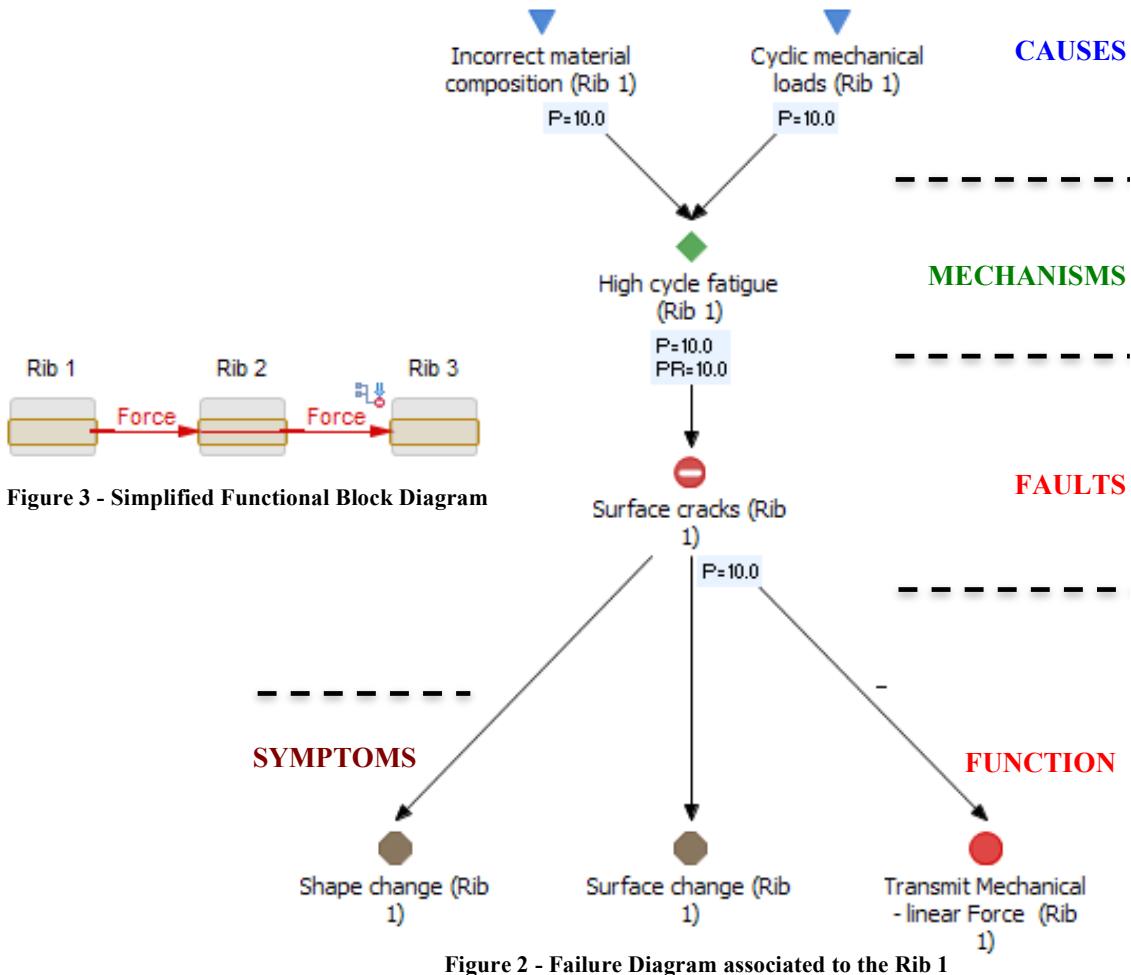
I. Introduction

Failures of structures are considered as the most catastrophic as well as the most arduous to prevent since they often occur without any warning. Indeed, most of the experience acquired by structural engineers comes from the investigation of failures and accidents. Causes of structural failures can be classified into four categories: the material is subject to an environment beyond its design domain; an inappropriate choice for the design and operating conditions; the material, to start with, is defective; or the design itself is wrong (Ramachandran, 2005). According to this classification, it becomes necessary to assess the potential failure modes throughout the design and manufacturing process and during the operational phase.

MADe turns out to be an efficient decision making tool so as to not only assess the potential failures but also to simulate the propagation of failures across the structure.

II. MADe Functional Modelling

According to the report AD 2012-0013 [1] from the European Aviation Safety Agency (EASA), several cracks have been found in the rib feet of the Airbus A380 wing box. EADS confirmed that cracks were caused by a combination of fault design and improper manufacturing process inducing stresses affecting fatigue life. This failure shows that whatever the accuracy of the risk assessment method is, if a probable failure mode slips through the net, then the safety could be compromised and a tremendous loss of money arises. In the same reference, the *EASA* has ordered Detailed Visual Inspections of the aeroplanes, which in addition to the safety issue involve considerable repair costs for the manufacturer as well as unexpected downtimes for the airline. MADe does not directly help to solve this type of physical failure, however its functional modelling approach gives engineers the awareness of all potential failure modes. Afterwards precautions can be taken to avoid introduction of deleterious defects, maintenance department are aware of which sub-systems and/or parts need to be inspected and advanced analyses as well as appropriate physical testing can be carried out to evaluate the most critical and the most likely failure modes.



<u>Function</u>	<u>Cause</u>	<u>Failure Mechanism</u>	<u>Effects</u>		
			<u>Local</u>	<u>Next</u>	<u>End</u>
Transmit	Incorrect material composition (Rib 1)	High cycle fatigue (Rib 1)	Transmit Mechanical - linear Force Low (Rib 1)	N/A	Transmit Mechanical - linear Force Low (Rib 3)
Transmit	Cyclic mechanical loads (Rib 1)	High cycle fatigue (Rib 1)	Transmit Mechanical - linear Force Low (Rib 1)	N/A	Transmit Mechanical - linear Force Low (Rib 3)

Figure 1 - FMEA Report

1. Functional Block Diagram

In the last few years, MADe has proved to be an asset as it eases and speeds up the execution of FMECA for dynamic systems. FMECA is an early warning or preventive engineering technique used at the component level to define, identify, and eliminate known and/or potential failures, problems, and errors from the design before they reach the customer (MIL-STD-1629A, 1980). The standard FMD-91 (RAC, 1991) states a functional approach as the method to recognise that every item or combination of items is designed to perform a number of functions. The way a component fails to fulfil its intended function is called a functional failure mode is (MIL-STD-1629A, 1980).

Adapted from the international standards, the FMECA methodology has been reviewed, improved and automated in MADe. As recommended in the FMD-91 (RAC, 1991), a complete FMECA includes all levels of indenture, which have been defined in MADe as follows:

❖ **System level:**

The system level is in general the project itself, for example, the modelling of a wing box.

❖ **Subsystem level**

A Subsystem is an element that is made up of Components and/or other Systems, e.g. a wing box made up of skins, ribs and spars assemblies. Systems can be assigned Functions and be modelled dynamically.

❖ **Component level**

A Component is an element that is made up of *Parts* and *Pairs*, e.g. a rib made up of beam and plate *Parts*. Components can be assigned Functions and be modelled dynamically.

❖ **Pair level**

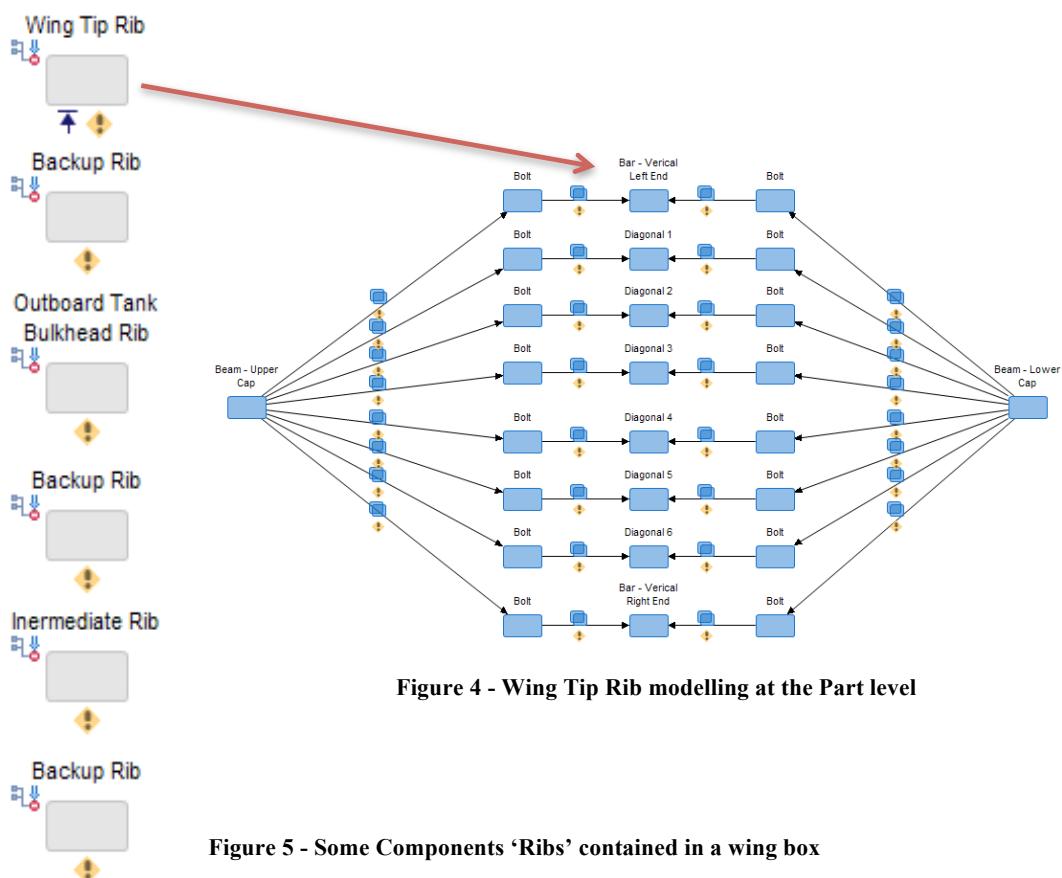
A *Pair* is the connection between a *Part* and another *Part/Component/System*, e.g. stiffener made up of two beams welded together. A *Pair* defines how the two elements are joined, and can be assigned functions so that it may be included in Functional Analysis.

❖ Part level

A *Part* is the lowest level element on a System Structure Diagram. *Parts* cannot be directly modelled dynamically or functionally. Instead they are joined with other Parts to form Pairs. The list of the structural *Parts* is based on FE Analysis and structural joints such as removable fasteners, riveted joints, adhesive bonds and welds (*Appendix A*).

❖ Feature

A *Feature* is a particular property of a Part. For example, it can be a hole or a thread. The definition of feature is important since at the Failure Diagram stage, the mechanism of failure can be applied either to the part or to the feature. A relevant FMEA would define a fillet or a hole as a Feature since it is well known that discontinuities in structure introduce stress concentrations. Ignoring the use of the definition of Features may lead to an irrelevant FMEA.



A. Up-To-Date Function & Failure definitions

Although FMECA is a valuable source of know-how for companies, the challenge of a consistent functional definition could be a nightmare that sometimes dissuaded engineers for using a functional approach to improve the reliability of their products (Wirth; Berthold; Krämer and Peter). This issue has been removed thanks to the development of functions and failures databases. Further details can be found in the conference paper: *Language of FMEA: on the effective use and reuse of FMEA data* (Rudov-Clark and Stecki). In addition to the answer to the language related problems, MADe has been designed with the objective to be quick and intuitive since it uses user-friendly Graphical Unit Interfaces as shown hereafter.

❖ Taxonomies

Two Taxonomies have been developed for MADe. The first one is used to construct functions according to the following methodology:

- **Function verb**, e.g. *transmit, reduce*: the action of the item on the input flows.
- **Flow noun**, e.g. *fluid, mechanical energy*: the material, energy or signal that is being acted upon.
- **Flow property**, e.g. *bulk modulus, rotational velocity*: the property of the flow that is required to fulfil the performance specifications of the item.

The other taxonomy consists of providing engineers with the numerous events, i.e. potential Causes; Mechanisms; Faults and Symptoms of functional failure modes.

❖ Definition of functions

Every item has a function that can be defined by using the extensive MADe database. For example, the function of a Rib component is to *transmit* the air *loads* from the skin to the spars.

- ▲ **Function verb – Transmit**: *To move an energy or signal from one place to another.*
(Synonyms: Advance; Conduct; Convey; Drop; Deliver).

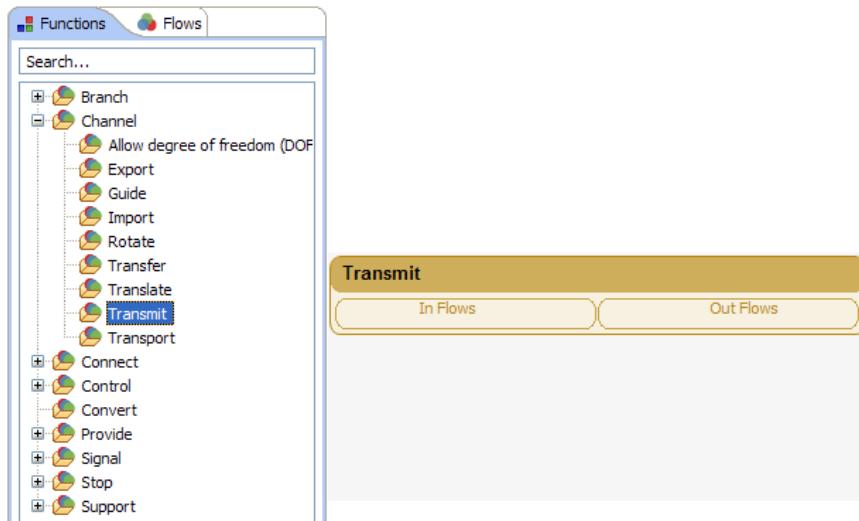


Figure 6 – Definition of the function *Transmit*

The other objective of the Functional Block Diagram is to illustrate the interrelationships between functional entities of a system and provides a functional flow sequence. In MADe, a functional flow completes the definition of functions by specifying the *noun* of the flow as well as its *property*. Using the example of the Rib, the air loads are assumed to be *Mechanical Forces*, hence:

- ▲ Flow noun - *Mechanical*
- ▲ Flow property - *Force*

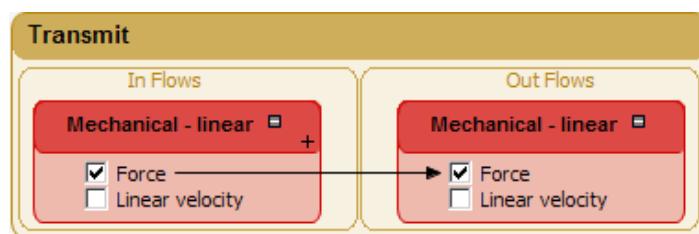


Figure 8 - Definition of the function *Transmit Mechanical Force*

The causal connection between the component's input and output flows defines how the component responds to the 'erroneous' input flow. This connection possesses a positive polarity which means that an increase of the input property will increase output property.



Figure 7 - Functional Block Diagram and Functional Flows

❖ **Definition of functional failure modes**

A Failure Diagram enables engineers to map the sequence of events that result in a failure mode, and forms the starting point for automated processing of the Failure Concept Map for functional FMEA. The failure diagram is also used to calculate the criticality of each causal path from initial failure cause to final end-effect of the failure mode. A sequence of event, also known as Failure Concepts, includes:

- **Cause:** abnormal state of input, loading or environment that initiates the failure process.
- **Mechanism:** physical process of degradation of the item.
- **Fault:** physically degraded state of a system element that renders it unable to completely fulfil its function. It is the result of a failure cause, and a failure mechanism.
- **Symptom:** detectable symptoms generated by the failure process, e.g. change to the appearance, behaviour, etc.

The figure 9 shows how a Cause of a potential failure mode can be easily defined. In this case, the failure mode is the fatigue of the structure. So, it is assumed that the system may fail due a *High Cyclic Fatigue* mechanism of failure caused by either *Cyclic Mechanical Loads* or an *Incorrect Material Composition*. The fault of this failure mode is the formation of *Surface cracks* which disturb the function *Transmit Mechanical Force*. The failure mode can be detected by pointing out a change of shape known as *Symptom*.

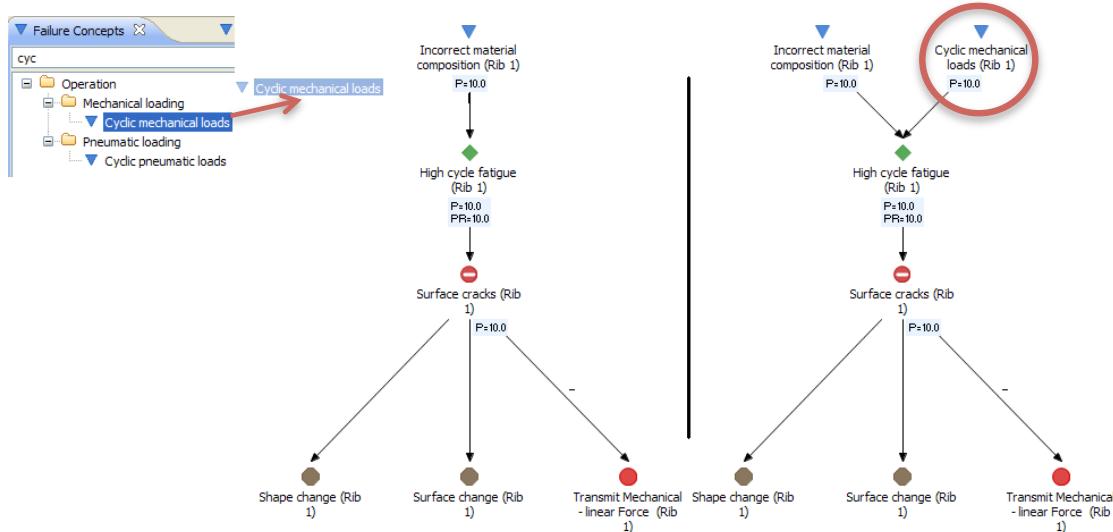


Figure 9 - Failure Diagram modelling - 'drag and drop' of a Cause from the failure database

MADe allows modelling numerous types of plastic deformation; fatigue; fracture; material decomposition and degradation as well as corrosion and wear. All the events used to design a Failure Diagram are also stored in a library that makes their research easier.

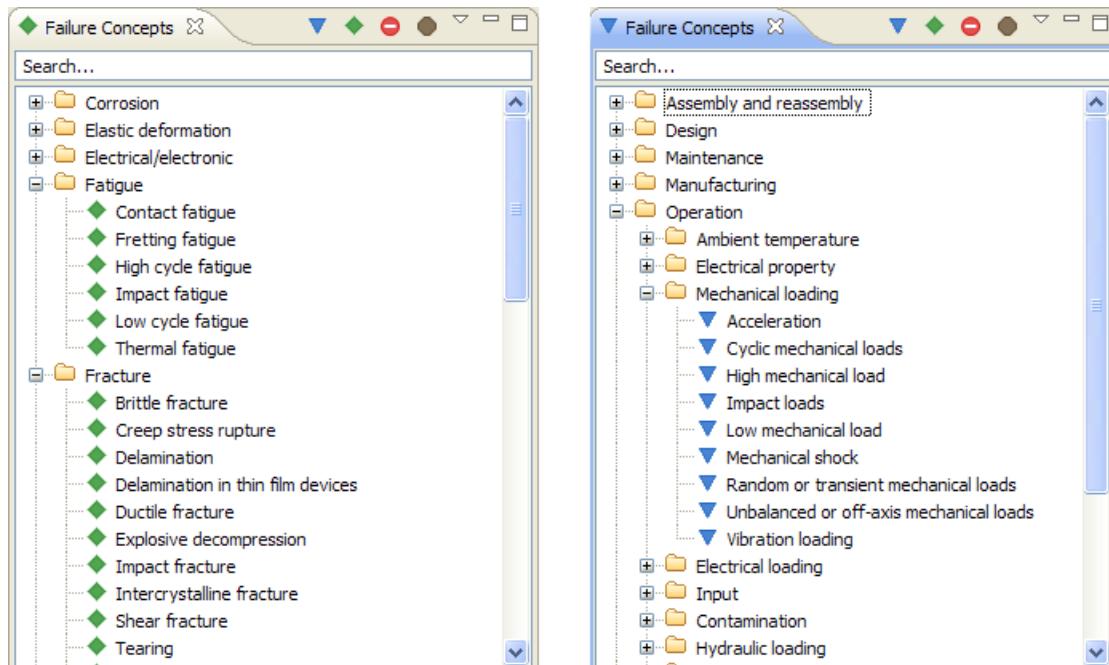


Figure 8 - Overview of the Mechanisms of failure and Causes databases.

B. Automated analysis

So far, the FMECA method was carried out using unfriendly spreadsheets. The RAC (1991) argues that the first FMECA to be performed should be updated throughout the design iteration process or as corrective actions are implemented. Moreover, if we look at the design process of complex structures we see three main iterations:

- Customers' requirements are not met at the conceptual design phase and so corrective actions must be taken.
- Structural weaknesses acknowledged during the Finite Element Analysis as well as at the Fatigue & Damage Tolerance stage require modifications of the detailed design.
- Finally, the physical tests may show anticipated structural failure due to a fault in the design or during the manufacturing. Thus, once again, it is necessary to back out in the process.

Thus, it is necessary to update the models throughout the life cycle of the structure. This is the reason why MADe integrates:

- A fault propagation table that enables to see how a failure of a specific component impacts on the system and what are its consequences. The failure of a component in MADe is expressed in terms of an unacceptable increase or decrease of the output flow property. Moreover, this fault propagation technique play a key role to evaluate the functional interactions among the components and so to aid for defining which parts need to be reviewed according to the design modifications. In the following example we carry out a 'What-if' analysis by injecting a failure, i.e. we simulate an increase of the *Mechanical Force* property and by looking at the Propagation Table we read the failure responses and notice that the failure injected will also increase the functional flow of the Component 'Rib 2' and 'Rib 3'.

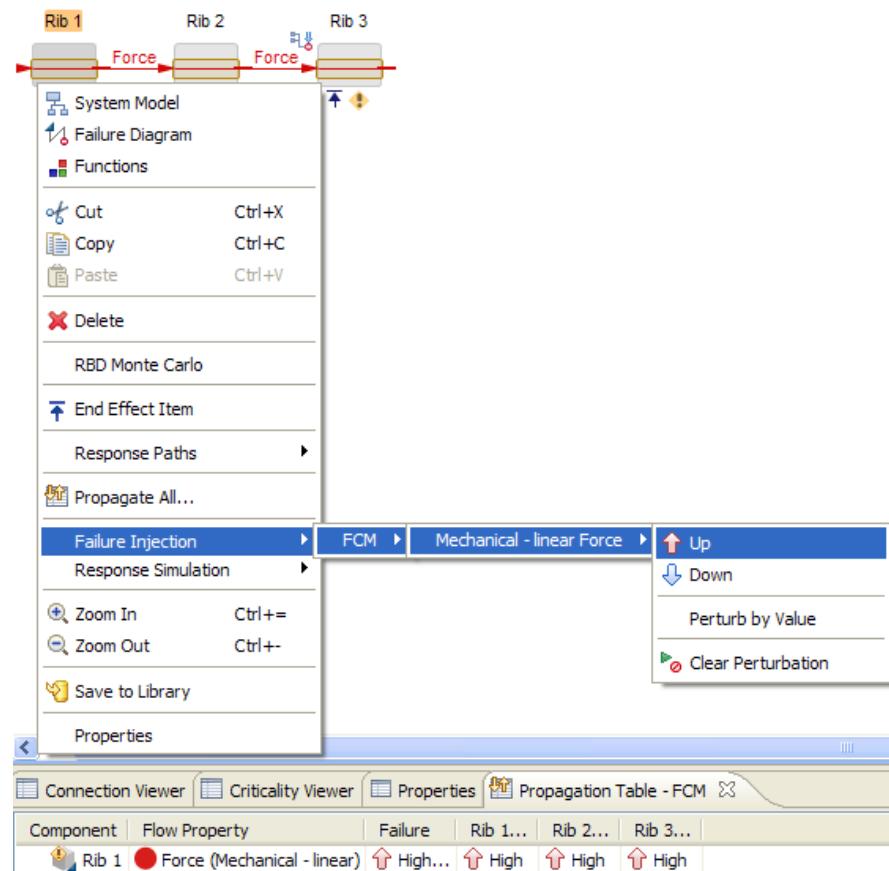


Figure 9 - Failure Injection and Propagation across the system

- A library that enables engineers to store their designs, reuse them in new projects or share them among the project team.

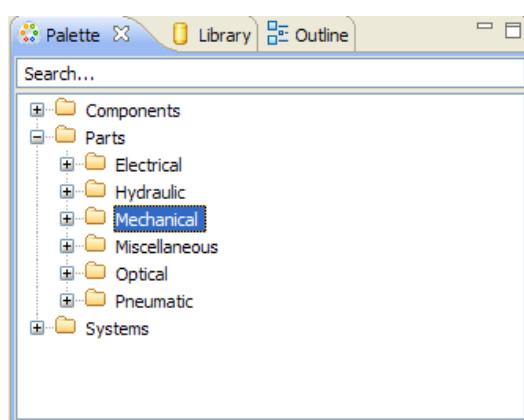


Figure 10 - MADe library

- MADe is also a decision making tool. The level of risk of the functional design is assessed via a Risk Priority Number (RPN = Severity*Occurrence*Capacity of Detection) calculated for each potential failure. The trade-off between safety and risk facilitates the choice between two distinct conceptual designs. Furthermore, the definitions of all terms (parts; functions; causes; mechanisms; etc.) have not been hazardously defined. Indeed, their definitions fit the designations use in the design of structures in order to force structural engineers to think about technical assumptions while the functional modelling.

C. Reporting

Once the FMECA has been carried out, engineer can report all sorts of information with regard to the potential functional failure modes. User can either creates a standard FMEA report using the MIL-STID-1629A as a template or he can include the results of the criticality analysis by picking one template among: Crisp RPN number, Criticality Matrix, Failure Assessment Index or Fuzzy Criticality methods.

ITEM ID N/A				NEXT INDENTURE LEVEL	fmeca_structure					
ITEM NAME Rib 1				SYSTEM NAME	fmeca_structure					
OPERATION MODE N/A				REFERENCE DRAWING	N/A					
FUNCTIONAL N/A NARRATIVE				PHYSICAL DESCRIPTION	N/A					
Item Name	Function	Cause	Failure Mechanism	Effects		Criticality				
				Local	Next	End	O	S	D	Crisp RPN
Rib 1	Transmit	Incorrect material composition (Rib 1)	High cycle fatigue (Rib 1)	Transmit Mechanical - linear Force Low (Rib 1)	N/A	Transmit Mechanical - linear Force Low (fmeca_structure)	10.0	10.0	10.0	1000
Rib 1	Transmit	Cyclic mechanical loads (Rib 1)	High cycle fatigue (Rib 1)	Transmit Mechanical - linear Force Low (Rib 1)	N/A	Transmit Mechanical - linear Force Low (fmeca_structure)	10.0	10.0	10.0	1000

Figure 11 – Example of FMECA Crisp RPN report

III. Conclusion

Be aware of all potential failure modes which may occur is the concept to identify which structural parts and/or subsystems need more or less attention in term of design, manufacturing, health monitoring and inspection. The functional approach, FMECA, has been for thirty years a broadly accepted method to identify and assess the likely failures in spite of its language related problems, unfriendly design and lack of flexibility for updating the existing models. All these disadvantages are taken off with the MADe Functional Design.

ABSTRACT

As the complexity of our technological systems increases, companies have moved from routine maintenance, i.e. replacing components to a schedule, whether or not they needed replacing, to *Condition Based-Maintenance (CBM)*, i.e. regular inspections and replacing components as they wear out, to *Prognostics and Health Management (PHM)*.

PHM includes the process of determining the state of a component to perform its function – *Diagnostics*. It also has the capability to estimate at what time the part will no longer operate within its stated specifications – *Prognostics*. Finally, PHM evaluates the current health of a part based on operational data from sensors at key location on the system to support just in time maintenance actions – *Health Management*.

Firstly, this thesis sets out with detail the validation of the *Reliability Block Diagram (RBD)* tool implemented in MADe software. Additionally, the thesis describes how the MADe Functional Design module has been enhanced in order to enable engineers to analyse the potential functional failure modes with emphasis on complex engineering structures.

Keywords:

Prognostics and Health Management - Failure Mode, Effects and Criticality Analysis - Risk assessment - Reliability Engineering - Sensor Set Design Optimisation

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