CRANFIELD UNIVERSITY

R PINQUIE

DEVELOPMENT OF PROGNOSTICS AND HEALTH MANAGEMENT (PHM) DESIGN TECHNOLOGY

SCHOOL OF ENGINEERING
Computational & Software Techniques in Engineering
Computer Aided Engineering

MSc THESIS
Academic Year: 2011 - 2012

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Supervisors: Dr Karl W Jenkins & Dr Jacek Stecki
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This thesis is submitted in partial fulfilment of the requirements for the degree of Master of Science

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ABSTRACT

As the complexity of our technological systems increases, companies have moved from routine maintenance, i.e. replacing components to a schedule, whether or not they needed replacing, to *Condition Based-Maintenance (CBM)*, i.e. regular inspections and replacing components as they wear out, to *Prognostics and Health Management (PHM)*.

PHM includes the process of determining the state of a component to perform its function – *Diagnostics*. It also has the capability to estimate at what time the part will no longer operate within its stated specifications – *Prognostics*. Finally, PHM evaluates the current health of a part based on operational data from sensors at key location on the system to support just in time maintenance actions – *Health Management*.

Firstly, this thesis sets out with detail the validation of the *Reliability Block Diagram (RBD)* tool implemented in MADe software. Additionally, this thesis describes how the MADe Functional Design module has been enhanced in order to enable engineers to analyse the potential failure modes with emphasis on complex engineering structures.

Keywords:

Prognostics and Health Management - Failure Mode, Effects and Criticality Analysis - Risk assessment - Reliability Engineering - Sensor Set Design & Optimisation

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Melbourne, Victoria, August 2012

Romain Pinquié

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LIST OF ABBREVIATIONS

A, B, C, …	Events A, B, C…
CA	Criticality Analysis
CBM	Condition Based Maintenance
F&DT	Fatigue and Damage Tolerance
$F_n(t)$	Probability of failure *(or unreliability)* of the item n
$F_s(t)$	Overall probability of failure *(or unreliability)* of the system
FMEA	Failure Mode and Effects Analysis
FMECA	Failure Mode, Effects and Criticality Analysis
IVHM	Integrated Vehicle Health Management
P	Probability
P(A)	Probability that an event A occurs
PHM	Prognostics and Health Management
RAM	Reliability Availability and Maintainability
RBD	Reliability Block Diagram
$R_n(t)$	Reliability of the component n
$\mathrm{R_s(t)}$	Overall reliability of the system
SHM	Structural Health Monitoring
t	Duration of operation *(or mission time)*
T / TTF	Time To failure

1 Introduction

"For want of a nail, the shoe was lost;
For want of a shoe, the horse was lost;
For want of a horse, the rider was lost;
For want of a rider, the battle was lost;
For want of a battle, the kingdom was lost!"

How would you control the loss of a nail?
Is there more you can do?

Greenfield M. A. (2002), NASA Langley Research Center

PHM Technology is an advanced engineering technology company which has developed a '***M**aintenance **A**ware **D**esign **e**nvironment' (MADe).* MADe is a software that provides engineers with an integrated functional modelling, analysis and decision support solution for the design and sustainment of complex engineering systems - reducing costs and risk. The MADe suite gathers three modules:

- MADe automated Failure Modes, Effects and Criticality Analysis (FMECA)
  ⇒ *Functional design – Failures database – Potential failure modes analysis*
- MADe Reliability Availability and Maintainability (RAM)
  ⇒ *Reliability Block Diagram (RBD) – Availability calculations – Fault Tree*
- MADe Prognostics and Health Management (PHM)
  ⇒ *Sensor Set Design & Optimisation – Fault Detection and Isolation*

Failure Mode, Effects and Criticality Analysis (FMECA) and Reliability Block Diagram (RBD) are both powerful methods frequently used in proactive maintenance, which have been reviewed, improved and automated in MADe package.

The automated FMECA module includes an extensive failures database that removes the language-related problems encountered by FMECA practitioners and proposes a uniform framework for defining functions and failure concepts.

On the other hand, RBD tool is a system graphical representation that enables probabilistic calculations in order to evaluate systems' reliability.

1.1 Purpose and goal

This thesis is divided into two main topics. In a first hand, the purpose is to demonstrate the probabilistic calculations which have been carried out in order to validate the Reliability Block Diagram tool. In other words, the thesis shows the different analytical studies performed to check whether the algorithms have been properly implemented in MADe or not.

The second main part of this thesis presents the enhancement of the standardised taxonomy ensuring that functional and failure concept descriptions for structures are consistent. This is introduce throughout a general mission which was to research how the use of the MADe suite can improve the design of complex engineering structures at all stages of the design process; i.e. how MADe software can assess and manage risks from the conceptual design to the Structural Heath Monitoring (SHM) phase.

1.2 Literature Review

Prognostics and Health Management (PHM) or Integrated Vehicle Health Management (IVHM) is an emerging field bringing together the use of advanced risk assessment techniques and real-time monitoring to increase both reliability and availability of complex systems as well as to improve maintenance decisions.

It is broadly accepted that reliability engineering is a cornerstone for the development of cutting-edge technologies. The beginning of survival analysis was in 1662 with the early work on mortality by John Graunt (Jerenz, 2008). Then, the improvement of the V1 and V2 rockets during the World War II was a breakthrough in reliability engineering (Dhillon, 1999 and McLinn, 2011). Most of reference books introduce the Reliability Block Diagram (RBD) methodology. Although authors clearly present the theory for classic RBD configurations (Dhillon, 1999 and Guangbin, 2007), only some of them deal with standby systems (Way and Ming, 2003). Indeed, several researches are currently undertaken in order to mimic the design of advanced products made up with numerous redundancies. Nowadays, reliability engineering is boosted by the use of stochastic algorithms in order to overcome the complexity due to tedious recursive integrations. Monte-Carlo algorithm is the Rolls Royce of reliability calculations since it can be applied to various system architectures and can also be easily implemented using any general programming language like JAVA, C, C++ or Visual Basic.

The second method to enhance the reliability of a system is to recognise that every part is designed to perform a number of functions. For complex systems, a combination of the functional and hardware approaches should be adopted (MIL-STD-1629A, 1980). The Failure Modes, Effects and Criticality Analysis (FMECA) is the most widely used reliability analysis technique in the initial stage of product development in order to eliminate potential failure modes at the component level. The origins of the FMECA method can be traced back to the early Military Procedure MIL-P-1629 '*Procedures for performing a Failure Mode, Effects and Criticality Analysis'* in 1949. The National Aeronautics and Space Administration (NASA) subsequently used the FMECA on the Apollo program in the 1960s. Initially, the technique was called FMEA, the C in FMECA indicates that the criticality of the failure modes is taken into consideration, however both are now considered as synonyms. Researches with regard to the efficiency of the FMECA method have shown several disadvantages. For instance, Wirth, Bethold, Krämer and Peter highlight that the unconstrained use of natural language and the lack of methodological guideline lead to inconsistent descriptions of systems and functions and to FMECA knowledge that is hardly reusable. The FMECA technique plays a key role in many industries, e.g. military applications, civil aviation, automotive engineering, etc. Nevertheless, its use in the design process of complex engineering structure is blurred. Actually, the reliability of a structure is based on a deterministic design code such as safety margin; the use of conservative material properties; Fatigue & Damage Tolerance (F&DT) analysis; advanced numerical analysis and certification tests (Acar, 2006). Even using such sophisticated theories and tools, catastrophic structural failures remain present. FMECA and the functional approach in general should play a more important role during the conceptual design of complex structures so as to identify, prioritise and eliminate the potential structural failures modes. Furthermore, the analysis of functional failures can be an asset for the achievement of the Structural Health Monitoring (SHM) process as it eases and speeds up the definition of the critical components as well as the selection of sensors.

1.3 Outline of the thesis

- *Chapter 2* presents the theoretical background necessary to understand the probabilistic reliability calculations.
- *Chapter 3* shows the results for most of the classic system configurations such as Series – Parallel – Series-Parallel – Parallel-Series – K-out-of-N and Complex groups. Moreover, at the end of this chapter, i.e. chapter 3-3-9 broaches the subject of the reliability evaluation of standby groups. Analytical and stochastic techniques are described hereafter and are currently for discussion.
- *Chapter 4* discusses the use of the functional modelling approach integrated into MADe with emphasis on structural integrity problems.
- *Chapter 5* summarises the work done through a conclusion but also suggest a list of future improvements and tasks that should be considered.
- *Appendix A* provides the code written in Visual Basic showing how compute the reliability of standby systems using Monte-Carlo stochastic algorithm.

2 Theoretical background

This chapter gives the gist of the reliability engineering theory so as to understand the probabilistic calculations carried out to validate the Reliability Block Diagram (RBD) tool. Readers who are familiar with either the probability or reliability engineering theory might want to start reading from chapter 3, and then focus on the later chapters.

2.1 Reliability definitions

Reliability Block Diagram (RBD): The purpose of the reliability block diagram is to show by concise visual short hand the various block combinations (paths) that result in item success. (MIL-STD-756b 1981)

RBD is a widely used technique to evaluate systems' reliability using probabilistic calculations. RBD is a friendly way to illustrate the architecture of a system as a graphical representation where the components are symbolised by rectangular blocks, which are characterized by their own reliability properties, i.e. Failure Rates and System Mission Time.

Reliability: Schüler (1997) defines the reliability as the probability that the component experiences no failure during the time interval (0,t), given that the component was as good as new at time zero.

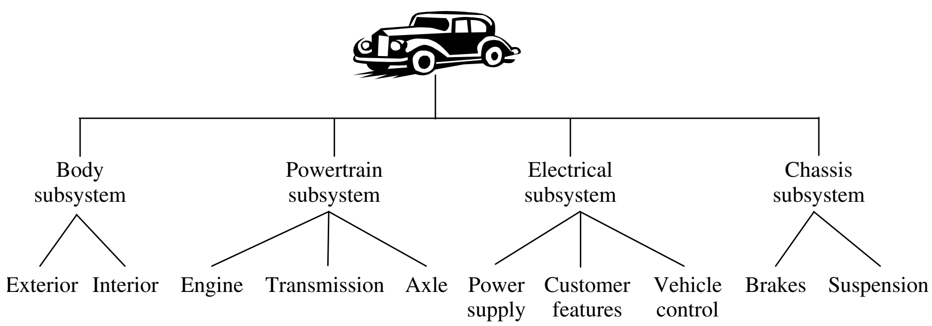


FIGURE 4.1 Hierarchical configuration of a typical automobile

Figure 2-1 A simple *series* Reliability Block Diagram. (Guangbin 2007)

2.2 Mathematical background

Operation of event A and B	Boolean Logic	Mathematics	Engineering
Union	A_1 or A_2	$A_1 \cup A_2$	$A_1 + A_2$
Intersection	A_1 and A_2	$A_1 \cap A_2$	$A_1 . A_2$
Complement	Not A_i	$\overline{A_i}$	$\overline{A_i}$

Table 2-1 List of the main operations

❖ **Union operation**

$$P(\cup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) \quad \textbf{(2-1)}$$

- *Example:* Union of two independent events A and B *(n=2)*.

$$\mathbf{P(\cup_{i=1}^{2} A_i) = \sum_{i=1}^{2} P(A_i) = P(A_1) + P(A_2)} \quad \textbf{(2-2)}$$

❖ **Intersection operation**

$$\mathbf{P(\cap_{i=1}^{n} A_i) = \prod_{i=1}^{n} P(A_i)} \quad \textbf{(2-3)}$$

- *Example:* Intersection of two independent events A and B *(n=2)*.

$$P(\cap_{i=1}^{2} A_i) = \prod_{i=1}^{2} P(A_i) = P(A_1).P(A_2) \quad \textbf{(2-4)}$$

2.3 Reliability engineering background

❖ ***Failure rate λ:*** the small quantity λ(t).dt is the probability that the component experiences a failure between t and t+dt, given that the component has survived to time t. (Schüler,1997)

❖ ***Probability Density Function f(t):*** indicates the failure distribution over the entire time range and represents the absolute failure speed. (Guangbin, 2007)

❖ ***Cumulative Distribution Function F(t):*** is the probability that a product will fail by a specified time t. It is the **probability of failure**. (Guangbin, 2007)

$$F(t) = \Pr(T < t) = \int_{-\infty}^{t} f(t)dt \tag{2-5}$$

❖ ***Reliability R(t):*** The reliability function, also called the survival function, is often interpreted as the population fraction surviving time t. R(t) is the **probability of success**, which is the complement of F(t). (Guangbin, 2007)

$$R(t) = \Pr(T \geq t) = 1 - \Pr(T < t) = 1 - F(t) = \int_{t}^{+\infty} f(t)dt \tag{2-6}$$

2.4 Exponential distribution

This thesis only deals with failure obeying an exponential distribution. So far, only the exponential distribution has been implemented in MADe RBD because it is the most commonly used within the reliability engineering area.

It is assumed that the exponential distribution is characterized by a **constant failure rate *λ*** (Schüler, 1997). Moreover**,** another well-known and interesting property of the exponential distribution is that the probability of a surviving component is independent of what occurred in the past, this is called the *memoryless property*.

❖ ***Probability Density Function f(t):***

$$\boxed{\mathbf{f(t) = \lambda e^{-\lambda t}} \qquad \text{with } t \geq 0} \tag{2-7}$$

❖ ***Cumulative Distribution Function F(t):***

$$F(t) = \int_{0}^{t} f(t)dt = \int_{0}^{t} \lambda e^{-\lambda t}\, dt = \lambda \left[-\frac{e^{-\lambda t}}{\lambda} \right]_{0}^{t} = \lambda \left(-\frac{e^{-\lambda t}}{\lambda} + \frac{1}{\lambda} \right)$$

$$\boxed{\mathbf{F(t) = -e^{-\lambda t} + 1}} \tag{2-8}$$

❖ ***Reliability R(t):***

$$R(t) = 1 - F(t) = 1 — e - \lambda t + 1$$

$$\boxed{\mathbf{R(t) = e^{-\lambda t}}} \tag{2-9}$$

3 RBD Validation

This chapter covers the reliability calculations of the classic configurations which can be encountered. Combinations are treated with detail and include:

- A short paragraph stating the condition of success of the system mission time.
- The analytical calculations and techniques used.
- The comparison between the results found by hand and those of MADe in order to make sure that they match together.

Nevertheless, in order to fit the reality, the reliability evaluation phase may require much more complex models such as redundant systems with standby components. Thus, the general methods introduced hereunder can no longer enable customers to compute the overall reliability of systems including standby components. Currently, the RBD workbench of MADe is limited since it cannot perform such an analysis. Consequently, more advanced techniques such as Laplace transform and the stochastic Monte-Carlo computational method have been approached and are presently for discussion.

3.1 Assumptions

Reliability computation with MADe is limited and based on several assumptions. To date, components obey the following rules:

- The events are independents, i.e. the probability of A_1 is not affected by whether A_2 occurs or not. In other words, the probability of failure (or reliability) of a component A_1 is not affected by the probability of failure (or reliability) of another component A_2.
- Only the exponential failure distribution is considered.
- The duration of operation is obviously equal or superior to zero.
- The components are assumed to be irreparable.
- Items are not sensitive to ageing damage.

3.2 Component Reliability

The different types of groups have been built using a range of four (five for *complex* group) random components. Moreover, each component has a different failure rate in order to avoid any special case.

❖ *Component's definition:*

	Part 1	**Part 2**	**Part 3**	**Part 4**	***Part 5***
Mission Time (t_i) *(hours)*	10	35	200	80	*80*
Failure Rate (λ_i) *(/10^6 hours)*	7843.14	8039.22	9166.67	10000	*10000*
Reliability $R_i(t)$	0.9245655	0.754747	0.1598796	0.449329	*0.449329*

Table 3-1 Definition of components

Using *Eq. 3-9* we obtain the following reliabilities:

- Part 1: $R_1(t) = e^{-\lambda_1 . t_1} = e^{-\left(\frac{7843.14}{10^6}\right).10} = 0.9245655$ **(3-1)**
- Part 2: $R_2(t) = e^{-\lambda_2 . t_2} = e^{-\left(\frac{8039.22}{10^6}\right).35} = 0.7547467$ **(3-2)**
- Part 3: $R_3(t) = e^{-\lambda_3 . t_3} = e^{-\left(\frac{9166.67}{10^6}\right).200} = 0.1598796$ **(3-3)**
- Part 4: $R_4(t) = e^{-\lambda_4 . t_4} = e^{-\left(\frac{10000}{10^6}\right).80} = 0.449329 = R_5(t)$ **(3-4)**

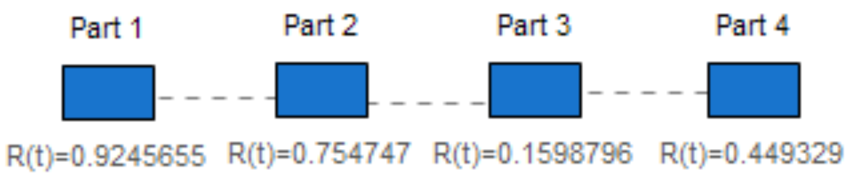


Figure 3-1 MADe Items' reliability

According to the calculations it has been approved that the computation of the reliability at the component level has been well implemented.

3.3 System Reliability

3.3.1 Series group

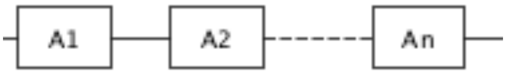


Figure 3-2 Series group

- **Reliability condition:**

All components must to be up for the system works.

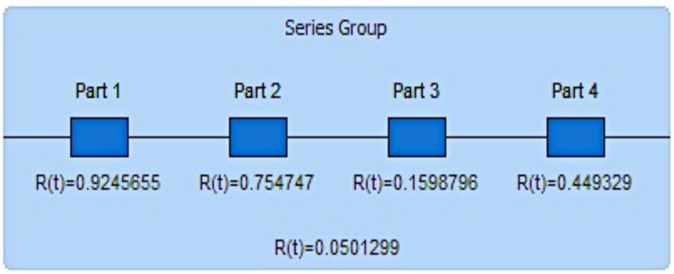
- **Analytical calculation:**

$$
\begin{aligned}
Rs\ (t) &= P(\cap_{i=1}^{n} A_i) \\
&= P(A_1).P(A_2).P(A_3).\ldots.P(A_n) \\
&= R_1.R_2.R_3.\ldots.R_n \\
&= \prod_{i=1}^{n} \mathbf{R_i}
\end{aligned}
\qquad \textbf{(3-5)}
$$

For the n = 4 components defined §3.2 we obtain:

$$
Rs\ (t) = \prod_{i=1}^{4} R_i = R_1.R_2.R_3.R_4 \cong \mathbf{0.0501299} \qquad \textbf{(3-6)}
$$

- **MADe Series group:**



Rs (t) = 0.0501299

Figure 3-3 MADe Series group

3.3.2 Parallel group

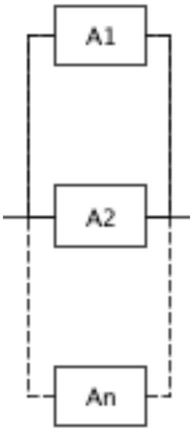


Figure 3-4 Parallel group

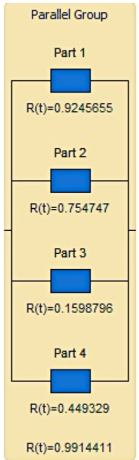


Figure 3-5 MADe Parallel group

Rs (t) = 0.9914411

❖ **Reliability condition:**

All components must be failed for the system fails.

❖ **Analytical calculation:**

$$\begin{aligned} Rs\,(t) &= 1 - Fs\,(t) \\ &= 1 - P(\cap_{i=1}^{n} \bar{A}_i) \\ &= 1 - P(\bar{A}_1).\,P(\bar{A}_2).\,\ldots.\,P(\bar{A}_n) \\ &= 1 - F_1.\,F_2.\,\ldots.\,F_n \\ &= 1 - (1 - R_1).\,(1 - R_2).\,\ldots.\,(1 - R_n) \\ &= \mathbf{1} - \prod_{\mathbf{i=1}}^{\mathbf{n}} (\mathbf{1} - \mathbf{R_i}) \end{aligned} \qquad \textbf{(3-7)}$$

$$\begin{aligned} Rs\,(t) &= 1 - \prod_{i=1}^{4} (1 - R_i) \\ &= 1 - (1 - R_1)\ (1 - R_2)\ (1 - R_3)\ (1 - R_4)\ \cong \mathbf{0.9914411} \end{aligned} \qquad \textbf{(3-8)}$$

3.3.3 Combined group

A *combined* group is a blend of either parallel groups in series, i.e. *Series-Parallel group*, or series groups in parallel, *i.e. Parallel-Series group*. These types of combinations are evaluated by using a reduction technique that consists in calculating the equivalent reliability for each series or parallel group in order to simplify the system in a basic series or parallel configuration.

3.3.3.1 Series-Parallel group

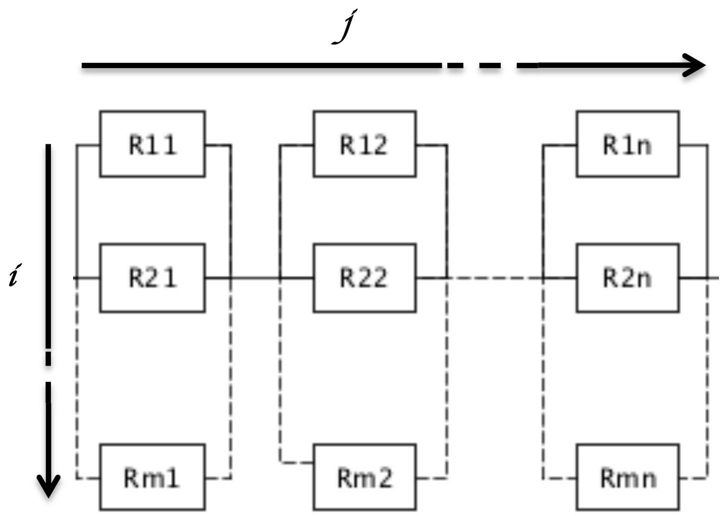


Figure 3-6 Series-Parallel group

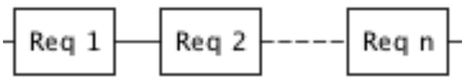


Figure 3-7 Series-Parallel group equivalent

❖ **Reliability condition:**

As a combination of parallel groups in series, at least one parallel group must be failed for the system stop functioning.

❖ **Analytical calculation:**

Since the system in Figure 3-7 is identical to a series group (*Fig. 3-2)*. Thus, the reliability can be calculated based on the formula of a series group (*Eq. 3-5 & 3-6)*.

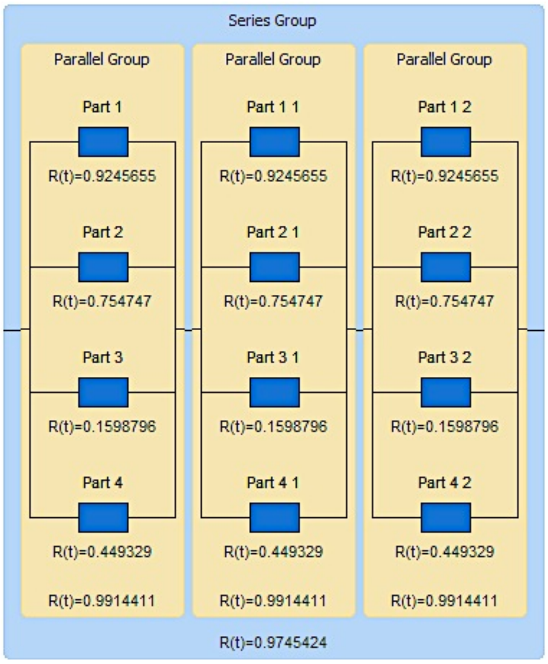
$$\mathrm{Rs}\ (t) = \prod_{j=1}^{n}\left[1-\prod_{i=1}^{m}(1-R_{ij})\right] \text{ using } 1-\prod_{i=1}^{m}(1-R_{ij}) = R_{eqj} \text{ then,}$$

$$= \prod_{j=1}^{n} R_{eqj} \qquad \textbf{(3-9)}$$

$$Rs\ (t) = \prod_{j=1}^{3} R_{eqj} \qquad \textbf{(3-10)}$$

$$= R_{eq1}.R_{eq2}.R_{eq3} \cong \mathbf{0.9745424}$$

- **MADe Series-Parallel group**



Rs (t) = 0.9745424

Figure 3-8 MADe Series-Parallel group

3.3.3.2 Parallel-Series group

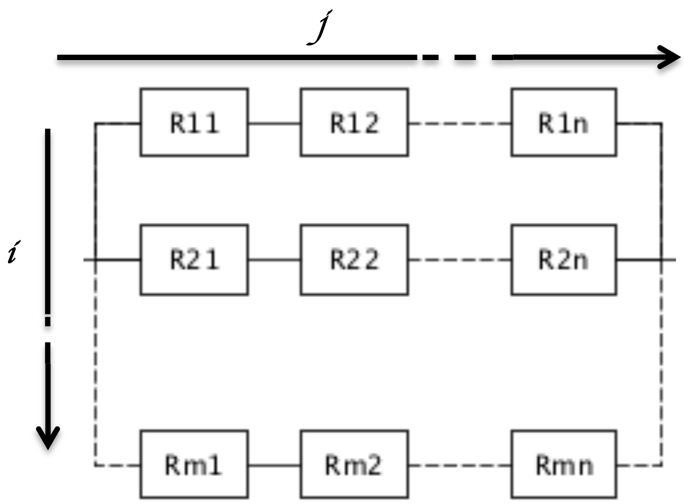


Figure 3-9 Parallel-Series group

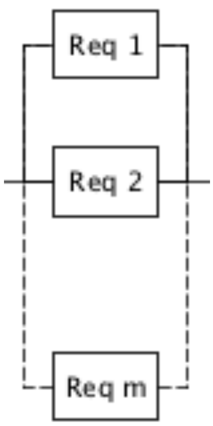


Figure 3-10 Parallel-Series group equivalent

❖ **Reliability condition:**

Conversely, here is a combination of series groups in parallel configuration, thus at least one series group must be up for the system works.

❖ **Analytical calculation:**

Using the same method as above, the system can be reduced to a parallel group *(Fig. 3-4)*. Hence, the reliability of the system can be calculated using the equation of a classic parallel system *(Eq. 3-7 and Eq. 3-17).*

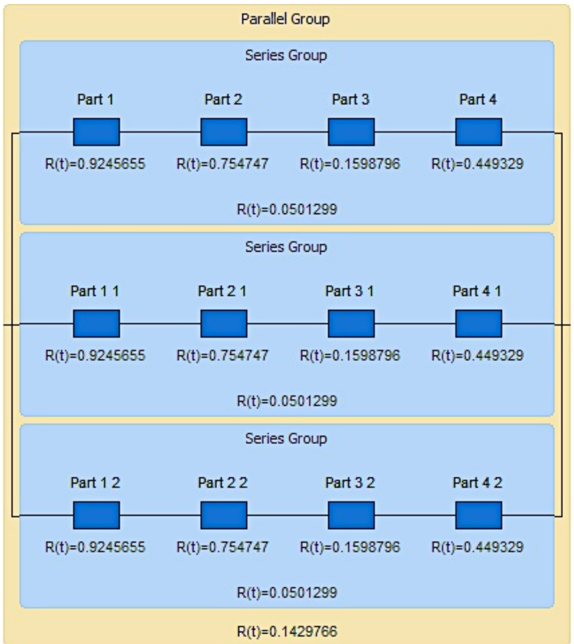
$$\mathrm{Rs}\ (\mathrm{t}) = 1 - \prod_{i=1}^{m}\left[1 - \prod_{j=1}^{n} R_{ij}\right] \text{ using } \prod_{j=1}^{n} R_{ij} = R_{eqi} \text{ then,}$$

$$= 1 - \prod_{i=1}^{m} 1 - R_{eqi} \tag{3-11}$$

$$\mathrm{Rs}\ (\mathrm{t}) = 1 - \prod_{i=1}^{3} 1 - R_{eqi}$$

$$= 1 - (1 - R_{eq1})(1 - R_{eq2})(1 - R_{eq3}) \cong \mathbf{0.1429766} \tag{3-12}$$

❖ **MADe Parallel-Series group:**



Rs (t) = $\mathbf{0.1429766}$

Figure 3-11 MADe Parallel-Series group

3.3.4 K-out-of-N group

- **Reliability condition:**

A K-out-of-N group is a parallel group (§3-3-2) with n independent components which functions if and only if at least K components function. The easiest method to calculate the overall reliability of the system is to use a true table in order to list all the possible combinations.

- **Analytical calculation:**

TRUE TABLE:

- ***F_i : Component i is failed***
- ***R_i : Component i is up***

Part 1	Part 2	Part 3	Part 4
F_1	F_2	F_3	F_4
F_1	F_2	F_3	R_4
F_1	F_2	R_3	F_4
F_1	F_2	R_3	R_4
F_1	R_2	F_3	F_4
F_1	R_2	F_3	R_4
F_1	R_2	R_3	F_4
F_1	R_2	R_3	R_4
R_1	F_2	F_3	F_4
R_1	F_2	F_3	R_4
R_1	F_2	R_3	F_4
R_1	F_2	R_3	R_4
R_1	R_2	F_3	F_4
R_1	R_2	F_3	R_4
R_1	R_2	R_3	F_4
R_1	R_2	R_3	R_4

- $Rs_{(1-\text{out}-\text{of}-4)}(t) = F_1F_2F_3R_4 + F_1F_2R_3F_4 + F_1F_2R_3R_4 + F_1R_2F_3F_4 + F_1R_2F_3R_4 + F_1R_2R_3F_4 + F_1R_2R_3R_4 + R_1F_2F_3F_4 + R_1F_2F_3R_4 + R_1F_2R_3F_4 + R_1F_2R_3R_4 + R_1R_2F_3F_4 + R_1R_2F_3R_4 + R_1R_2R_3F_4 + R_1R_2R_3R_4 = Rs_{(\text{parallel})}$

$\Rightarrow \mathbf{Rs_{(1-out-of-4)}(t) \cong 0.9914411}$

- $Rs_{(2-\text{out}-\text{of}-4)}(t) = F_1F_2R_3R_4 + F_1R_2F_3R_4 + F_1R_2R_3F_4 + F_1R_2R_3R_4 + R_1F_2F_3R_4 + R_1F_2R_3F_4 + R_1F_2R_3R_4 + R_1R_2F_3F_4 + R_1R_2F_3R_4 + R_1R_2R_3F_4 + R_1R_2R_3R_4$

$\Rightarrow \mathbf{Rs_{(2-out-of-4)}(t) \cong 0.449329}$

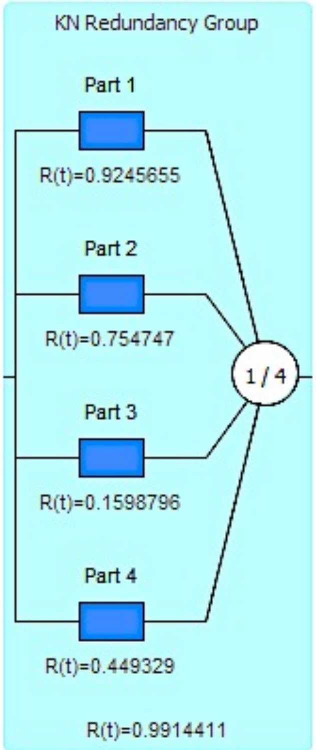
- $Rs_{(3-\text{out}-\text{of}-4)}(t) = F_1R_2R_3R_4 + R_1F_2R_3R_4 + R_1R_2F_3R_4 + R_1R_2R_3F_4 + R_1R_2R_3R_4$

$\Rightarrow \mathbf{Rs_{(3-out-of-4)}(t) \cong 0.3953634}$

- $Rs_{(4-\text{out}-\text{of}-4)}(t) = R_1R_2R_3R_4 = Rs_{(\text{series})}$

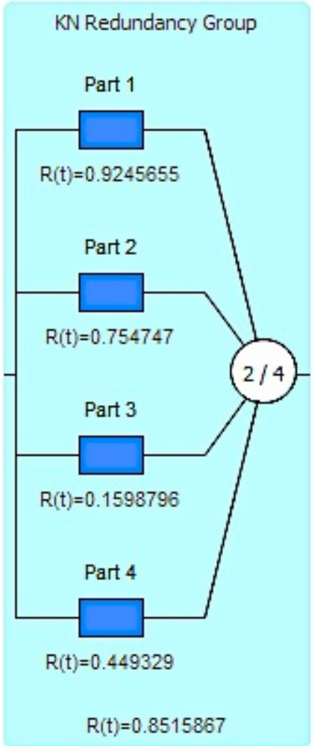
$\Rightarrow \mathbf{Rs_{(4-out-of-4)}(t) \cong 0.0501299}$

❖ **MADe K-out-of-N group:**



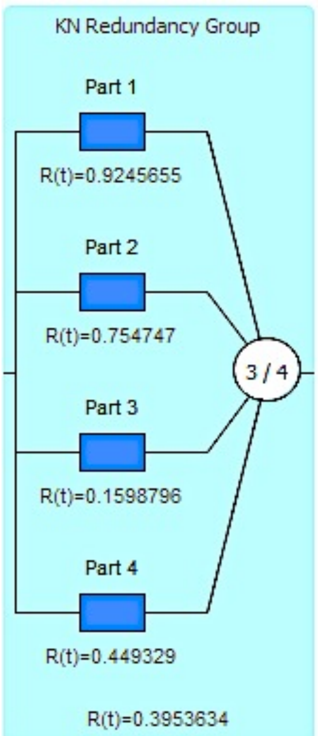
Rs ≅ 0.9914411

Figure 3-12 MADe Active Redundancy "1-out-of-4"



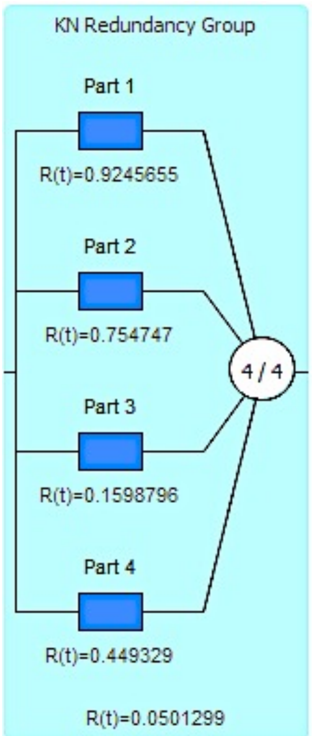
Rs ≅ 0.449329

Figure 3-13 MADe Active Redundancy "2-out-of-4"



Rs ≅ 0.3953634

Figure 3-14 MADe Active Redundancy "3-out-of-4"



Rs ≅ 0.0501299

Figure 3-15 MADe Active Redundancy "4-out-of-4"

3.3.5 Complex group

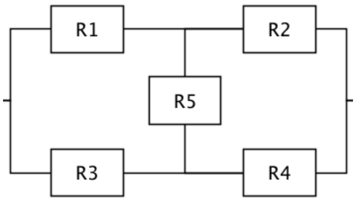


Figure 3-16 Complex group

- **Reliability condition:**

A system is *complex*, when it can be defined neither as a series nor as a parallel configuration.

- **Analytical calculation:**

DECOMPOSITION APPROACH

Key element (5) functioning

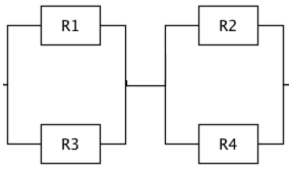


Figure 3-17 Complex group with key up

If the key element is always functioning, then the system is equivalent to a Series-Parallel group *(fig. 3-5).*

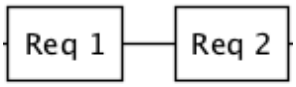


Figure 3-18 Equivalent complex group with key up

$$\mathbf{R_{eq1} = 1 - \prod_{\substack{i=1 \\ i\neq 2}}^{3} 1 - R_i \cong 0.9366259}$$

$$R_{eq2} = 1 - \prod_{\substack{i=2 \\ i\neq 3}}^{4} 1 - R_i \cong 0.8649463$$

$$Rs_{(key\ functioning)} = \prod_{i=1}^{2} R_{eq\ i}$$

$$\Rightarrow \mathbf{Rs_{(key\ functioning)}(t) \cong 0.8101311}$$

Key element (5) failed

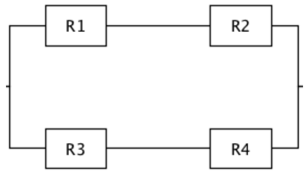


Figure 3-19 Complex group with key failed

Conversely, if the key part is always failed, then it is a Parallel-Series group *(fig. 3-8).*

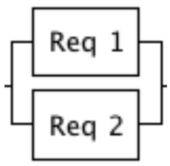


Figure 3-20 Equivalent complex group with key failed

$$R_{eq1} = \prod_{i=1}^{2} R_i \cong 0.697813$$

$$R_{eq2} = \prod_{i=3}^{4} R_i \cong 0.0718386$$

$$Rs_{(key\ failed)} = 1 - \prod_{i=1}^{2} 1 - R_{eq\ i}$$

$$\Rightarrow \mathbf{Rs_{(key\ failed)}(t) \cong 0.7195217}$$

<u>The reliability of the whole system is:</u>

Rs (t) = P(part good | key good).P(key good) + P(part good | key failed).P(key failed)

Given the:

- Reliability of the key element (5) ≡ $R_5 \cong 0.449329$
- Probability of failure of the key element (5) ≡ $F_5 = 1 - R_5 \cong 0.550671$

Therefore the reliability of the system is

$$\textbf{Rs (t)} = \textbf{Rs}_{(key\ functioning)}\mathbf{R_5} + \textbf{Rs}_{(key\ failed)}\mathbf{F_5}$$

$$= 0.\ 8101311 * 0.449329 + 0.7195217 * 0.550671 \qquad \textbf{(3-13)}$$

$$\Rightarrow \textbf{Rs(t)} \cong \mathbf{0.7602351}$$

- **MADe Complex group:**

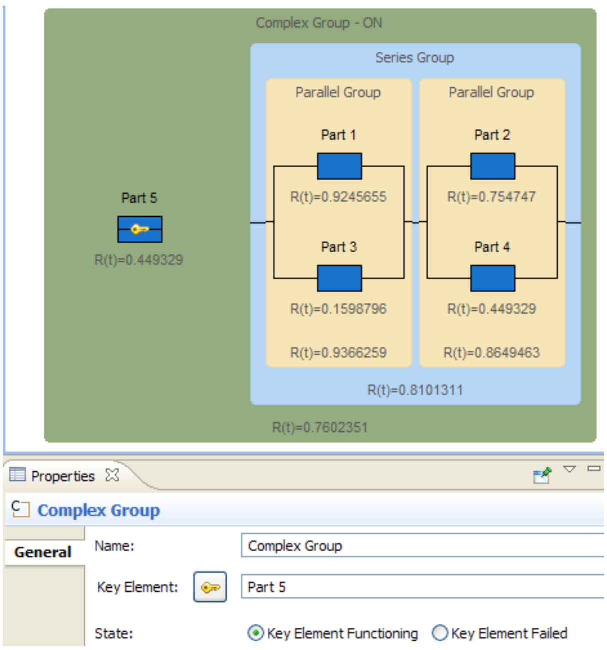


Figure 3-21 MADe Complex group with key element functioning

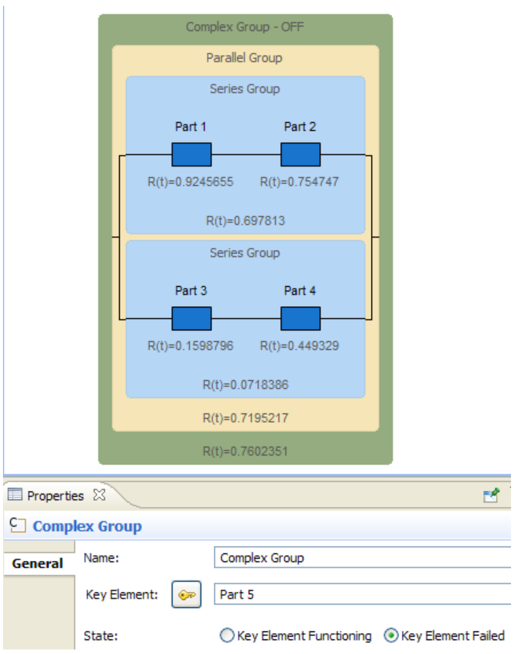


Figure 3-22 MADe Complex group with key element failed

$$\Rightarrow \text{Rs}_{(key\ functioning)} \cong 0.8101311 \qquad \Rightarrow \text{Rs}_{(key\ failed)} \cong 0.7195217$$

$$\Rightarrow \textbf{Rs(t)} \cong \mathbf{0.7602351}$$

3.3.6 Summary of results

The following table summarises all the results (overall reliability system) calculated by hand as well as using MADe.

Configuration	***Series***	***Parallel***	***Series-Parallel***	***Parallel-Series***
Analytical	0.0501299	0.9914411	0.9745424	0.1429766
MADe	*0.0501299*	*0.9914411*	*0.9745424*	*0.1429766*

K/N with K=1	***K/N with K=2***	***K/N with K=3***	***K/N with K=4***	***Complex***
0.9914411	0.44939329	03953634	0.0501299	0.7602351
0.9914411	*0.44939329*	*03953634*	*0.0501299*	*0.7602351*

We notice that the analytical results match with those of MADe, therefore the Reliability Bloc Diagram tool has been validated.

Prior to deal with more complex configurations it is interesting to notice that the use of redundant components can significantly improve the reliability of a system. Conversely, as we can see on the histogram hereunder, designs based on series configurations tremendously increase the probability of failure of the system.

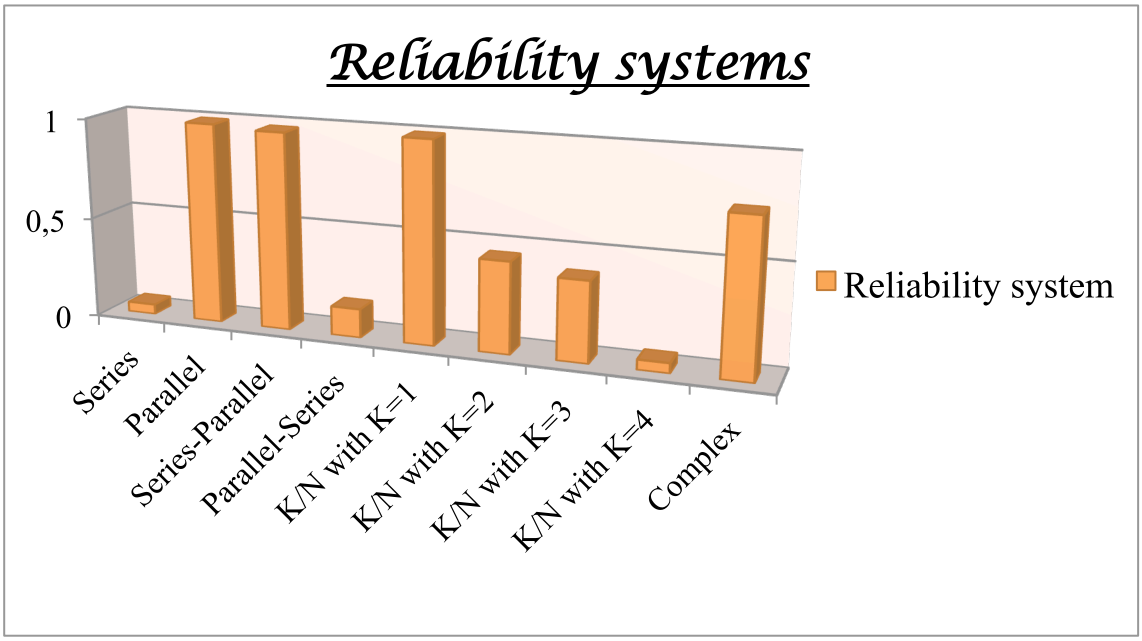


Figure 3-23 Reliability system of the different combinations

3.3.7 Redundant group

In the previous chapter, it has been acknowledged that the use of redundant items can improve the reliability of a product. Nevertheless, sometimes, critical systems could require a probability of failure extremely low. In order to improve the reliability of the system, a common technique is to add standby (non-operating) components in parallel to one active (operating) component.

In standby redundancy the components are set to have two states: an active state and a standby state. Components in standby redundancy have two failure distributions, one for each state. When in the standby state, they have a quiescent (or dormant) failure distribution and when operating, they have an active failure distribution.

- In the case that both quiescent and active rates of failure are the same, the units are in a simple parallel configuration (also called a *hot standby* configuration, § 3.3.2).
- When the rate of failure of the standby component is lower in quiescent mode than in active mode, this is called a *warm standby* configuration. This type of redundancy is out of scope with respect to this thesis but might be considered as a future work.
- When the rate of failure of the standby component is zero in quiescent mode (i.e. the component cannot fail when in standby) that is called a *cold standby* configuration.

Standby components can be switched on thanks to a switching device as soon as the operating component failed. Nevertheless, the switching device may fail before the functional item, then the component in standby cannot be turned on and so the system fails. Since the switching device is never one hundred per cent reliable, there are two types of redundancy evaluation techniques.

- The first solution is to make the assumption that the switching device never fails, and the switching is instantaneous between components – *Perfect switching.*
- On the other hand, we can consider and evaluate the probability of failure of the switching device – *Imperfect switching.*

Redundant system can no longer be evaluated by using the friendly formulae used up to this point. To solve this type of problem it is necessary to come back to the original definitions of the reliability theory *(Eq. 2-5 and Eq. 2-6)*.

3.3.7.1 Cold redundancy

3.3.7.1.1 Perfect switching

Perfect switching means that when the active part fails, the standby components are instantaneously switched on and the probability of failure of the switching device is equal to zero.

The reliability of a cold standby redundant system with perfect switching can be literally defined as follows:

Reliability = The system's lifetime is longer than the mission time T

$$\Leftrightarrow \mathbf{Pr(T \geq t) = \int_t^{+\infty} f(t)dt}$$

Two possible scenarios can lead to a success event:

1. The active component survives beyond the mission time T
2. The active component fails at time X_1 ($0 \leq X_1 < T$) and the nth standby component functions for a time period longer than ($T - \sum_{i=1}^{n-1} X_i$).

❖ **Recursion for N standby components:**

- Components 1 fails after a working duration of X_1:

  $\Leftrightarrow$ n = 1: $0 \leq X_1 < T$

- Component 2 fails after a working duration of X_2:

  $\Leftrightarrow$ n = 2: $0 \leq X_2 < T - X_1$

- Component 3 fails after a working duration of X_3:

  $\Leftrightarrow$ n = 3: $0 \leq X_3 < T - X_1 - X_2$

- Component N-1 fails after a working duration of X_{N-1}:

  $\Leftrightarrow$ n = N: $0 \leq X_{N-1} < T - X_1 - X_2 - \ ... - \ X_{N-2} \equiv 0 \leq X_N < T - \sum_{i=1}^{N-2} X_i$

- Component N operates properly for a time period longer than $T - \sum_{i=1}^{N-1} X_i$

Given:

- λ_1: Failure rate of the active part
- λ_i: Failure rate of the standby parts i
- T: Mission time
- X_i: Lifetime of the part i

Hence,

- R_1 : Reliability of the active (operating) component
- R_i : Reliability of the standby (non-operating) components

$\Rightarrow$ $R_i = e^{-\lambda_i * T_i}$ *(Eq. 2-9)*

The reliability of the cold standby redundant system with perfect switching and exponential distribution requires integration over all times as follows:

$$Rs(t) = P\left(\sum_{i=1}^{N} X_i > T\right) = R_1(T) + \int_0^T f_1(X_1)R_2(T - X_1)\, dX_1$$

$$+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2)R_3(T - X_1 - X_2)\, dX_2\, dX_1$$

$$+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \int_0^{T-X_1-X_2} f_3(X_3)R_4(T - X_1 - X_2 - X_3)dX_3dX_2dX_1$$

$$+ \dots$$

$$+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \dots \int_0^{T-\sum_{i=1}^{N-2} X_i} f_{N-1}(X_{N-1})R_N\left(T - \sum_{i=1}^{N-1} X_i\right) dX_{N-1} \dots dX_2\, dX_1$$

<u>Where $f_n(X_u)\, dX_u$ is the probability of the part n failed at an unknown time X_u.</u>

Using:

- $\boldsymbol{f_n(X_u) = \lambda_n e^{-\lambda_n X_u}}$ *(Probability density function of item n)*
- $\boldsymbol{R_n(X_u) = e^{-\lambda_n X_u}}$ *(Reliability of item n)*

Hence,

$$Rs(t) = e^{-\lambda_1 T} + \int_0^T \lambda_1 e^{-\lambda_1 X_1} e^{-\lambda_2 (T-X_1)} dX_1$$

$$+ \int_0^T \lambda_1 e^{-\lambda_1 X_1} \int_0^{T-X_1} \lambda_2 e^{-\lambda_2 X_2} e^{-\lambda_3 (T-X_1-X_2)} dX_2 dX_1$$

$$+ \int_0^t \lambda_1 e^{-\lambda_1 X_1} \int_0^{T-X_1} \lambda_2 e^{-\lambda_2 X_2} \int_0^{T-X_1-X_2} \lambda_3 e^{-\lambda_3 X_3} e^{-\lambda_4 (T-X_1-X_2-X_3)} dX_3\, dX_2 dX_1$$

$$+ \ldots$$

$$+ \int_0^t \lambda_1 e^{-\lambda_1 X_1} \int_0^{T-X_1} \lambda_2 e^{-\lambda_2 X_2} \int_0^{T-X_1-X_2} \lambda_3 e^{-\lambda_3 X_3} \ldots$$

$$\int_0^{T-\sum_{i=1}^{N-2} X_i} \lambda_{N-1} e^{-\lambda_{N-1} X_{N-1}} e^{-\lambda_{N-1}} \left(T - \sum_{i=1}^{N-1} X_i \right) dX_{N-1} \ldots dX_3 dX_2 dX_1$$

Way (2003, p.133) provides the general equation:

$$\Rightarrow \boldsymbol{Rs\ (t) = \sum_{j=1}^{N} e^{-\lambda_j T} \left(\prod_{i \neq j} \frac{\lambda_i}{\lambda_i - \lambda_j} \right)} \quad \textbf{(3-14)}$$

3.3.7.1.2 Imperfect switching

In most cases, it should be required to consider the reliability of the switching device for more accuracy. Thus, the reliability of a cold standby redundant system with imperfect switching can be modelled with either a *probability of success of switching*, or by seeing the *switching device as a component* with its own lifetime distribution.

- **<u>Probability of success of switching (p)</u>**

After an extensive literature research, a few people (Lenz and Rhodin, 2011; Way, 2003) approach this problem, however nobody provides a general formula that could be implemented. Consequently, another method of calculation based on Laplace Transform has been used in order to solve the recursive integration.

The two scenarios leading to a success of the mission time are:

1. The active component survives beyond the mission time T
2. The active component fails at time X_1 ($0 \leq X_1 < T$), the n-1 switch occurs without any problem and the nth standby component functions for a time period longer than (T-$\sum_{i=1}^{n-1} X_i$).

$$Rs(t) = R_1(T) + \ p\int_0^T f_1(X_1)R_2(T - X_1)\ dX_1$$

$$+ p^2 \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2)R_3(T - X_1 - X_2)\ dX_2dX_1$$

$$+ p^3 \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \int_0^{T-X_1-X_2} f_3(X_3)R_4(T - X_1 - X_2 - X_3)dX_3dX_2dX_1$$

$$+ \ \dots$$

$$+ p^{n-1} \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \ \dots \int_0^{T-\sum_{i=1}^{N-2} X_i} f_{N-1}(X_{N-1})R_N\left(T - \sum_{i=1}^{N-1} X_i\right) dX_{N-1} \dots dX_2dX_1$$

To simplify the expression we can write the equation as follows:

$$Rs(t) = \ R_1 + pF_1 * R_2 + \cdots + p^{n-1} * F_1 * F_2 * \dots * F_{n-1} * R_n$$

$$\Rightarrow \boldsymbol{Rs\ (t)} = \ \sum_{j=1}^{n} p^{j-1}F_1 * F_2 * \dots * F_{j-1} * R_j \qquad \textbf{(3-15)}$$

The first term of the sum is by convention equal to $p^0R_1 = R_1$.

The function f_i is defined over $[0, +\infty[$ and can be integrated over $[0, \mathrm{M}]\ \forall\ \mathrm{M} > 0$.

The convolution $f * g$ is defined for $t > 0$ by

$$(f * g)(t) = \int_0^{+\infty} f(x)g(t - x)dx \qquad \textbf{(3-16)}$$

Using $\mathcal{L}(f)$ for the Laplace transform of a function f, which is defined as follows:

$$\mathcal{L}(f)(z) = \int_0^{+\infty} f(t)e^{-tz}dt \qquad \textbf{(3-17)}$$

Also it can be demonstrated that the Laplace transform is linear, hence:

$$\mathcal{L}(f * g)(z) = \mathcal{L}(f)(z)\mathcal{L}(g)(z) \qquad \textbf{(3-18)}$$

Then the Laplace transform of Eq. 3-15 yields:

$$\mathcal{L}(R_{cold-standby})(z) = \sum_{j=1}^{n} p^{j-1}\, \mathcal{L}(f_1)(z) \dots \mathcal{L}(f_{j-1})(z)\mathcal{L}(R_j)(z) \qquad \textbf{(3-19)}$$

Since $R_i(t) = e^{-\lambda_i t}$ and $f_i(t) = -\lambda_i e^{-\lambda_i t}$ then using Laplace transform we obtain,

$$\mathcal{L}(R_i)(z) = \frac{1}{z+\lambda_i} \; and \; \mathcal{L}(f_j)(z) = \frac{\lambda_i}{z+\lambda_i} \tag{3-20}$$

Therefore, Eq. 3-19 becomes,

$$\mathcal{L}(R_{cold-standby})(z) = \sum_{j=1}^{n} p^{j-1} \frac{\lambda_1 \dots \lambda_{j-1}}{(z+\lambda_1)\dots(z+\lambda_j)} \tag{3-21}$$

By carrying out a partial fraction expansion using,

$$\frac{1}{(z+\lambda_1)\dots(z+\lambda_j)} = \sum_{i=1}^{j} \frac{\mu_{i,j}}{z+\lambda_i} \tag{3-22}$$

Multiplying Eq. 3-22 by $z+\lambda_i$ and substituting $z = -\lambda_i$, we obtain for $i \leq j$,

$$\mu_{i,j} = \frac{1}{\prod_{\substack{1\leq k\leq j \\ k\neq i}} (\lambda_k - \lambda_i)} \tag{3-23}$$

Then by injecting the Eq. 3-23 into Eq. 3-22 and Eq. 3-22 into Eq. 3-21 we get,

$$\begin{aligned}\mathcal{L}(R_{cold-standby})(z) &= \sum_{j=1}^{n} p^{j-1} \; \lambda_1 \dots \lambda_{j-1} \sum_{i=1}^{n} \frac{1}{(z+\lambda_i)\prod_{\substack{1\leq k\leq j \\ k\neq i}}(\lambda_k - \lambda_i)} \\ &= \sum_{i=1}^{n}\left[\sum_{j=i}^{n} p^{j-1} \frac{\prod_{1\leq k\leq j-1}\lambda_k}{\prod_{\substack{1\leq k\leq j \\ k\neq i}}(\lambda_k - \lambda_i)}\right]\frac{1}{z+\lambda_i}\end{aligned} \tag{3-24}$$

Finally, reading Laplace from 's' domain to 't' domain *(or right to left)* we obtain the general formula:

$$\boxed{\boldsymbol{R_{cold-standby}(t)} = \sum_{i=1}^{n}\left[\sum_{j=i}^{n} \boldsymbol{p^{j-1}} \frac{\prod_{1\leq k\leq j-1}\boldsymbol{\lambda_k}}{\prod_{\substack{1\leq k\leq j \\ k\neq i}}(\boldsymbol{\lambda_k} - \boldsymbol{\lambda_i})}\right]\boldsymbol{e^{-\lambda_i t}}} \tag{3-25}$$

In order to check the general equation (Eq. 3-25), two examples (n = 1 and n = 2) have been expanded and compared to Rhodin's examples (2011).

- *Example for n=2:*

$$R_{cold-standby}(t) = \left[p^{0} + p\frac{\lambda_1}{\lambda_2 - \lambda_1}\right] e^{-\lambda_1 t} + p\frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 t}$$
$$= e^{-\lambda_1 t} - p\frac{\lambda_1}{\lambda_1 - \lambda_2}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

- *Example for n=3:*

$$R_{cold-standby}(t) = \left[1 + p\frac{\lambda_1}{\lambda_2 - \lambda_1} + p^2\frac{\lambda_1\lambda_2}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)}\right] e^{-\lambda_1 t}$$
$$+ \left[p\frac{\lambda_1}{\lambda_2 - \lambda_1} + p^2\frac{\lambda_1\lambda_2}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_2)}\right] e^{-\lambda_2 t} + p^2\frac{\lambda_1\lambda_2}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)} e^{-\lambda_3 t}$$

Both examples confirmed the results documented in the recent studies of Lenz and Rhodin (2011). Therefore, the general equation (Eq. 3-25) seems to be verified.

❖ **Switching device as a component**

Another method to model a cold standby redundant group with imperfect switching is to consider the switching device as a component itself with its own probability of failure.

Once again, two scenarios are possible:

1. The active component survives beyond the mission time T
2. The active component fails at time X_1 ($0 \leq X_1 < T$), and both the swithing device as well as the nth standby component functions for a time period longer than ($T-\sum_{i=1}^{n-1} X_i$).

Using R_{sd} for the reliability of the switching device, we can develop the recursions of integrals as in the previous case.

$$Rs(t) = R_1(T) + \int_0^T f_1(X_1) R_{sd}(X_1) R_2(T - X_1)\, dX_1$$
$$+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) R_{sd}(X_1 + X_2) R_3(T - X_1 - X_2)\, dX_2\, dX_1$$

$$+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \int_0^{T-X_1-X_2} f_3(X_3) R_{sd}(X_1 + X_2 + X_3) R_4(T - X_1 - X_2 - X_3) dX_3\, dX_2 dX_1$$

$$+ \ \ldots$$

$$+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \ \ldots \int_0^{T-\sum_{i=1}^{N-2} X_i} f_{N-1}(X_{N-1}) R_{sd}\left(\sum_{i=1}^{N-1} X_i\right) R_N\left(T - \sum_{i=1}^{N-1} X_i\right) dX_{N-1} \ldots dX_2 dX_1$$

This equation is the same as for Cold Standby redundancy with perfect switching (cf. above: § 3.3.10.1), with one more component, the switching device. Such calculations can be performed using symbolic computations, e.g. Mupad, Maple, Matlab, etc. However, classic computers cannot solve integrals directly, therefore, numerical integration methods must be developed. Monte-Carlo integration is a powerful method for computing the value of complex integrals using probabilistic techniques and is briefly presented in the next chapter.

3.3.8 Monte-Carlo Integration

Using the Cold Standby redundancy with perfect switching model as a case study, it has been shown in the section 3.3.9.1.1 that the overall reliability is equal to:

$$Rs(t) = P\left(\sum_{i=1}^{N} X_i > T\right) = R_1(T) + \int_0^T f_1(X_1) R_2(T - X_1)\, dX_1$$

$$+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) R_3(T - X_1 - X_2)\, dX_2\, dX_1$$

$$+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \int_0^{T-X_1-X_2} f_3(X_3) R_4(T - X_1 - X_2 - X_3) dX_3 dX_2 dX_1$$

$$+ \ \ldots$$

$$+ \int_0^T f_1(X_1) \int_0^{T-X_1} f_2(X_2) \ \ldots \int_0^{T-\sum_{i=1}^{N-2} X_i} f_{N-1}(X_{N-1}) R_N\left(T - \sum_{i=1}^{N-1} X_i\right) dX_{N-1} \ldots dX_2\, dX_1$$

Instead of going through the time-consuming multidimensional numerical integration, this problem and most of the other system configurations can be solved using Monte-Carlo. Monte-Carlo is a stochastic algorithm based on experience involving n trials to determine the mean of a random variable x. With a theoretical approach and using the *central limit theorem* (cf. Appendix A.1) Monte-Carlo algorithm can be written as follows:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{3-26}$$

The reliability of the system is the probability that the system experiences no failure during the time interval (0,t), given that the system was as good as new at time zero. In other words, a system is reliable if its lifetime is longer than the mission time T**.**

Thus, the idea is to compute the Time To Failure (TTF or lifetime) of the system using random variables and as a 'hit-or-miss' technique, for each iteration we check whether the survival time of the system is longer than the mission time or not. Finally, the reliability of the system is the sum of all successes divided by the total number of trials.

We remind:

- That the condition of success for a Cold Standby redundancy with perfect switching is:
  - ⇒ The active component fails at time X_1 ($0 \leq X_1 < T$) and the nth standby component functions for a time period longer than ($T-\sum_{i=1}^{n-1} X_i$).
- The reliability of a component 'i' at the time 't_i' is $R_i(t_i) = e^{-\lambda_i . t_i}$ with λ_i the failure rate of component 'i'.
  - ⇒ $t_i = -\frac{\ln(R_i)}{\lambda_i}$

Algorithm:

```
/* Initialise the number of success */
Nb_of_ success = 0;
/* Run Monte-Carlo simulation for n trials */
For i = 1 to Number_of_trials do
{
    /* Initialise the system lifetime */
    System_lifetime = 0;

    /* Compute the system's lifetime by adding the random time to failure of
    each component, i.e. TTF_system = Σ_{i=1}^{# items} TTF_i with TTF_i = − ln(R_i)/λ_i
    and Ri a random number uniformly distributed between 0 and 1 */
    For i = 1 to Number_of_components
        System_lifetime = System_lifetime – TTF_i;

    /* Check whether the system lifetime is longer than the Mission Time or
    not. If it is true then the counter of success is incremented by 1 */
    If (System_lifetime > Mission_Time)
        then Nb_of_failure = Nb_of_success + 1;
}
/* Compute the reliability of the system as the ratio of number of success to
number of trials */
Reliability = Nb_of_ success / Number_of_trials;
```

This code has been developed in Visual Basic (cf. Appendix A.2) and provides the same results as symbolic calculations (Rs = 0.4485 calculating integrals with Matlab and Rs ≅ 0.45 with Monte-Carlo 5000 iterations for 3 components). This shows that the reliability of redundant systems can be evaluated using stochastic methods like for instance Monte-Carlo algorithm. Future works might involve the development of advanced Monte-Carlo numerical methods for Cold and Warm standby configurations with both perfect and imperfect switching devices.

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4 Functional Modelling for structural integrity problems

Failures of structures are considered as the most catastrophic as well as the most arduous to prevent since they often occur without any warning. Causes of structural failures can be classified into four categories: the material is subject to an environment beyond its design domain; an inappropriate choice for the design and operating conditions; the material, to start with, is defective; or the design itself is wrong (Ramachandran, 2005). According to this classification, it becomes obvious to assess potential failure modes throughout the design and manufacturing process as well as during the operational phase. Afterwards precautions can be taken to avoid introduction of deleterious defects, advanced analyses as well as appropriate physical testing can be carried out to accurately evaluate the most critical and the most likely failures.

Adapted from the international standards, the Functional Design module implemented in MADe has proved to be an asset to carry out FMECA for dynamic systems. However, MADe does not allow the functional modelling of structural integrity problems, this is something under research. Thus, this chapter introduces a new technique to assess potential structural failures. *Functional Block Diagram – Functional Taxonomy – Failure Diagram and Failure Taxonomy – Failure Propagation – Criticality Analysis and PHM Design and Optimisation,* with emphasis on structure are presented hereafter.

4.1 FMECA Definitions

<u>*Functional failure mode:*</u> describes the way an item fails to fulfil its function.

<u>*Failure Mode and Effects Analysis (FMEA):*</u> a procedure by which each potential failure mode in a system is analysed to determine the results or effects thereof on the system and to classify each potential failure mode according to its severity. (MIL-STD-1629A 1980, p4)

<u>*Criticality Analysis (CA):*</u> a procedure by which each potential failure mode is ranked according to the combined influence of severity and probability of occurrence. (MIL-STD-1629A 1980, p3)

N.B: The different examples and models shown hereafter are based on the current version of MADe. All the improvements with regard to structural modelling have not been implemented yet, thereby the case studies are not consistent. Nevertheless, readers must keep in mind that the numerous enhancements presented throughout this chapter shall lead to a much more relevant functional design of structures.

4.2 Functional Block Diagram

A functional model is used to represent a system and its elements in terms of their functions. A popular functional model is the Functional Block Diagram (FBD) technique, which presents the system as a breakdown of its major functions and provides the ability for tracing failure mode effects through all levels of indenture. The functional modelling of a complex structure using MADe software is based on five levels of indentures which are defined hereunder.

Figure 4-1 Functional Block Diagram with a single level of indenture

4.2.1 Indenture levels

The highest level of indenture is the 'System level' and corresponds to the MADe project itself.

- **Subsystem level**

A Subsystem is an element that is made up of *Components* and/or other *Subsystems*, e.g. a wing box made up of spars, ribs and panels. *Subsystems* can be assigned Functions.

- **Component level**

A *Component* is an element that is made up of *Parts* and *Pairs*, e.g. a stiffened panel made up of beam and shell parts. *Components* can be assigned Functions.

❖ **Pair level**

A *Pair* is the connection between a *Part* and another *Part/Component/Subsystem,* e.g. a stiffener made up of two beam parts welded together. A *Pair* defines how the two elements are joined, and can be assigned functions so that it may be included in Functional Analysis.

❖ **Part level**

A *Part* is the lowest level element on a System Structure Diagram. *Parts* in MADe cannot be directly modelled functionally. Instead they are joined with other *Parts* to form *Pairs*.

❖ **Feature**

A *Feature* is a particular property of a *Part*, e.g. a hole, a thread, *etc.* The use of *Features* is important since at the Failure Diagram stage, the mechanism of failure can be applied either to the part or to the feature. For example, fillet or a hole should be set as *Features* since discontinuities in structures introduce stress concentrations.

4.2.2 Structural Parts

The definition of the *Part* indenture level should enable engineers to model any sort of physical structure. The language used in the Finite Element discipline is the most general to model a structure, thus the structural part definitions are related thereof. The seven basic structural components are stored in a library so as to quickly drag and drop them, or user can create new designs and share them among the project team members.

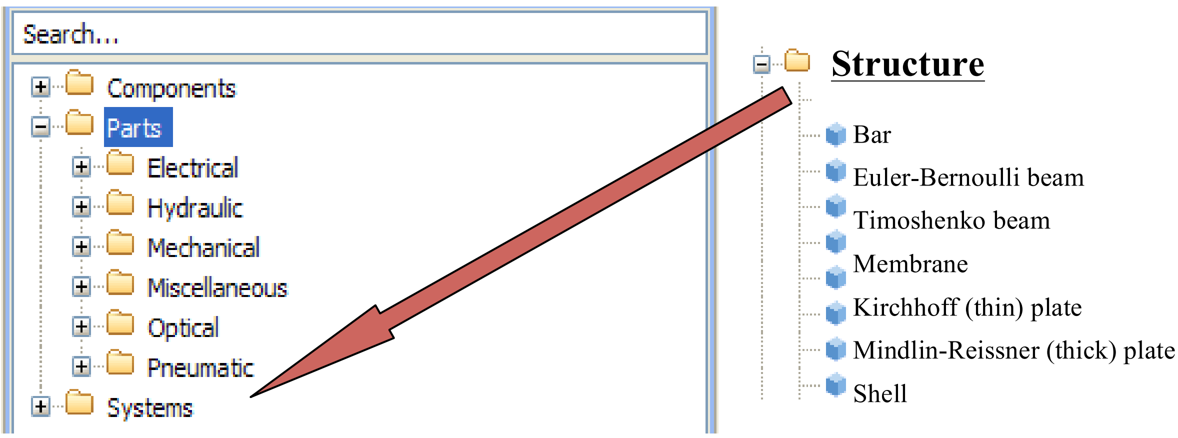


Figure 4-2 MADe Library of *structural parts*

4.2.3 Joint Parts

Joint is a connection between two structural parts. Joint parts have been classified as *Fastened joints (removable); Riveted joints; Welded joints and Bonded joints*. It cannot be acceptable to have one component, i.e. one functional block, standing for one joint part, e.g. one block for one rivet. Therefore, an analogy has been drawn between physical joints and MADe joint parts.

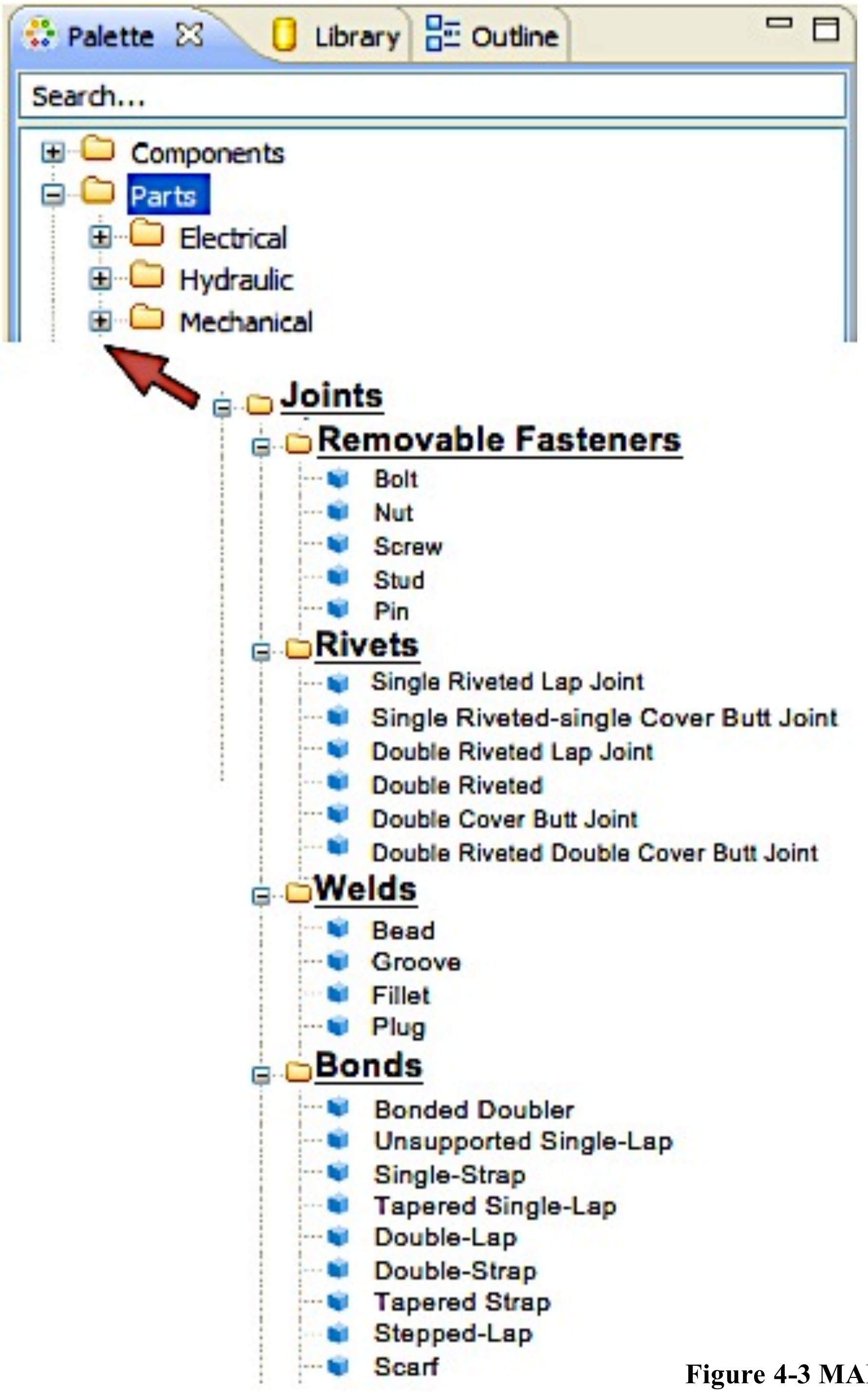


Figure 4-3 MADe Library of *joint parts*

4.3 Function Definitions

Although FMECA is a valuable source of know-how for companies, the challenge of a consistent functional definition could be a nightmare that sometimes has dissuaded engineers for using a functional approach (Wirth; Berthold; Krämer and Peter). This issue has been removed thanks to the development of function and failure taxonomies.

The first taxonomy is used to state item's function and is constructed as follows:

- **Function verb**, *e.g. transmit, reduce:* the action of the item on the input flows.
- **Flow noun**, *e.g. fluid, mechanical energy*: the material, energy or signal that is being acted upon.
- **Flow property**, *e.g. bulk modulus, rotational velocity*: the property of the flow that is required to fulfil the performance specifications of the item.

Every item has a function that can be defined by using the extensive database of MADe. For example, the function of a Rib component (Figure 4-1) is to *transmit* the air loads from the skin to the spars.

- **Function verb – *Transmit***: *To move an energy or signal from one place to another. (Synonyms: Advance; Conduct; Convey; Drop; Deliver).*

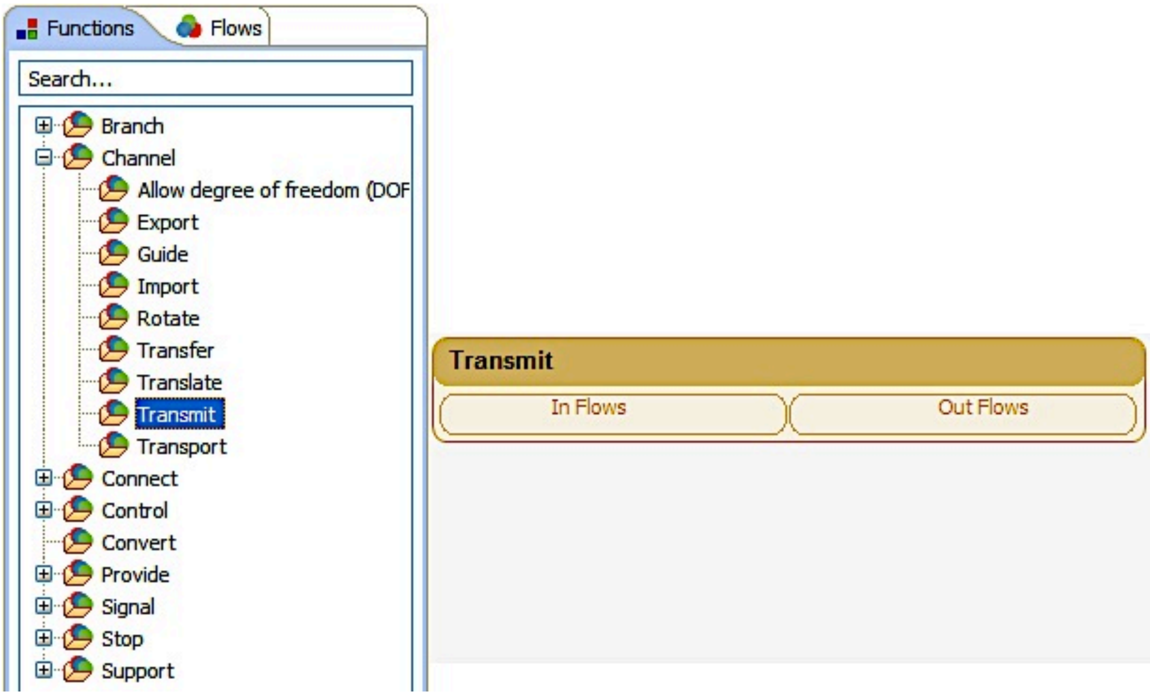


Figure 4-4– Definition of the function *Transmit*

The other objective of the Functional Block Diagram is to illustrate the interrelationships between functional entities of a system and provides a functional flow sequence. In MADe, a functional flow completes the definition of functions by specifying the *noun* of the flow as well as its *property*.

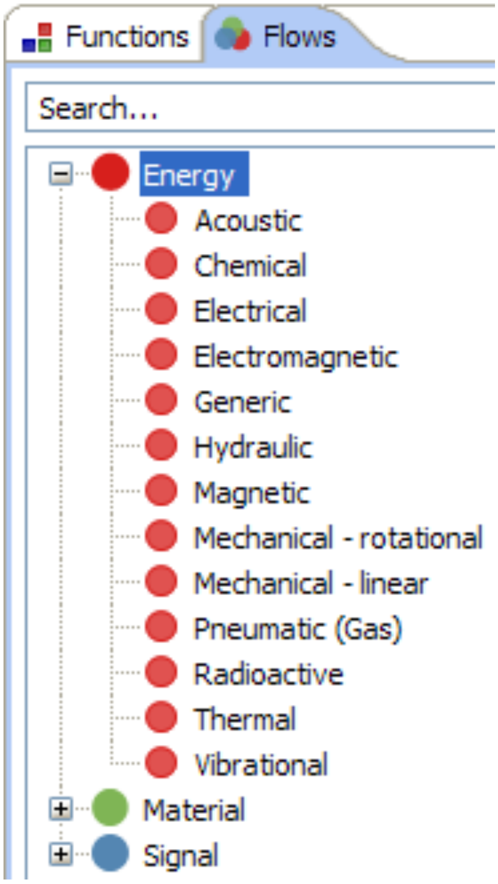


Figure 4-5 Energy flows properties

Using the example of the Rib, the air loads are assumed to be *Mechanical Forces,* hence:

- **Flow noun - *Mechanical***
- **Flow property - *Force***

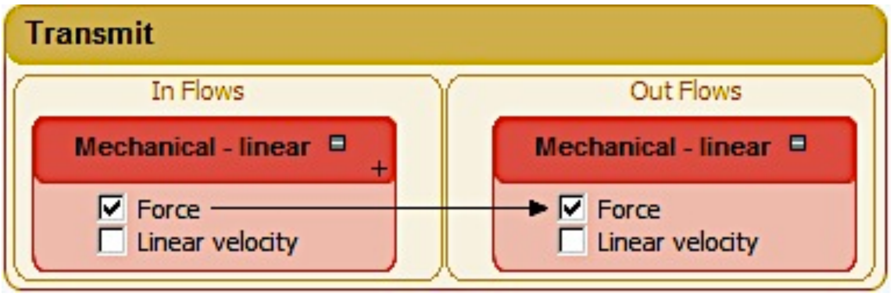


Figure 4-6 Definition of the function *Transmit Mechanical Force*

The causal connection between the component's input and output flows defines how the component responds to the 'erroneous' input flow. This connection possesses a positive polarity which means that an increase of the input property will increase the output property.

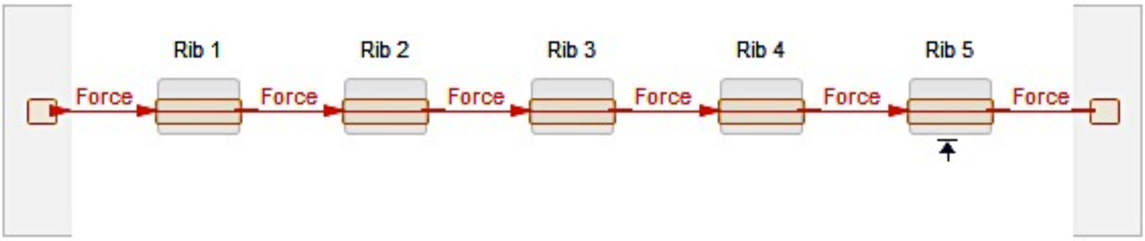


Figure 4-7 Functional Block Diagram and Functional Flows

As we can see above (Figure 4-6), MADe has a limited capability for structural flows modelling. Thus, the functional taxonomy has been enhanced with several flow nouns as well as flow properties:

Function Verb	Flow noun		Flow property
Resist	*Energy*	*Structural – Static Stresses*	*Tensile stress* *Compressive stress* *Shear stress* *Bending stress* *Torsion stress* *Combined bending and direct stress* *Hydrostatic (volumetric) stress* *Combined bending and twisting stress*
	Energy	*Structural – Dynamic Stresses*	*Impact stress*
	Energy	*Structural – Fatigue Stresses*	*Alternating/Reversing stress* *Repeated stress* *Combined steady and alternating stress* *Random stress*
	Energy	*Structural – Thermal stress*	*Thermal stress*
Limit	*Energy*	*Structural – Strains*	*Normal Strain* *Shear Strain* *Volumetric Strain*
	Energy	*Structural – Displacements*	*Displacement* *Deflection* *Angle of torsion*
Transmit	*Energy*	*Structural – Static Concentrated Loads*	*Tensile force* *Compressive force* *Shear force* *Bending moment* *Torsional moment*
	Energy	*Structural –Static Distributed Loads*	*Uniform load* *Triangular load* *Parabolic load* *Exponential load* *Sine load* *Cosine load*
	Energy	*Structural – Inertia loads*	*Gravity load* *Spinning load*
	Energy	*Structural – Dynamic Periodic Loads*	*Harmonic High cyclic load* *Harmonic Low cyclic load* *Non-Harmonic High cyclic load* *Non-Harmonic Low cyclic load*
	Energy	*Structural – Dynamic Aperiodic Loads*	*Transient load*
	Energy	*Structural – Impact Loads*	*Impact load*
	Energy	*Structural – Pressure*	*Pressure*

Table 4-1 Functional Taxonomy for structural modelling

4.4 Failure Definitions

The second taxonomy consists of providing engineers with a database of numerous events, i.e. potential Causes; Mechanisms; Faults and Symptoms of functional failure modes. The failure of a component in MADe is expressed in terms of an unacceptable increase or decrease of the output flow property. In our case study, the failure mode would exists when the output flow, *Mechanical Force*, increases or decreases.

Failure Diagram enables the engineer to map the sequence of events that result in a failure mode, and forms the starting point for automated processing of the Failure Concept Map for functional FMEA. The failure diagram is also used to calculate the criticality of each causal path from initial failure cause to final end-effect of the failure mode. A sequence of event, also known as Failure Concept, includes:

- **Cause:** abnormal state of input, loading or environment that initiates the failure process.
- **Mechanism:** physical process of degradation of the item.
- **Fault:** physically degraded state of a system element that renders it unable to completely fulfil its function. It is the result of a failure cause, and a failure mechanism.
- **Symptom:** detectable symptoms generated by the failure process, e.g. change to the appearance, behaviour, etc.

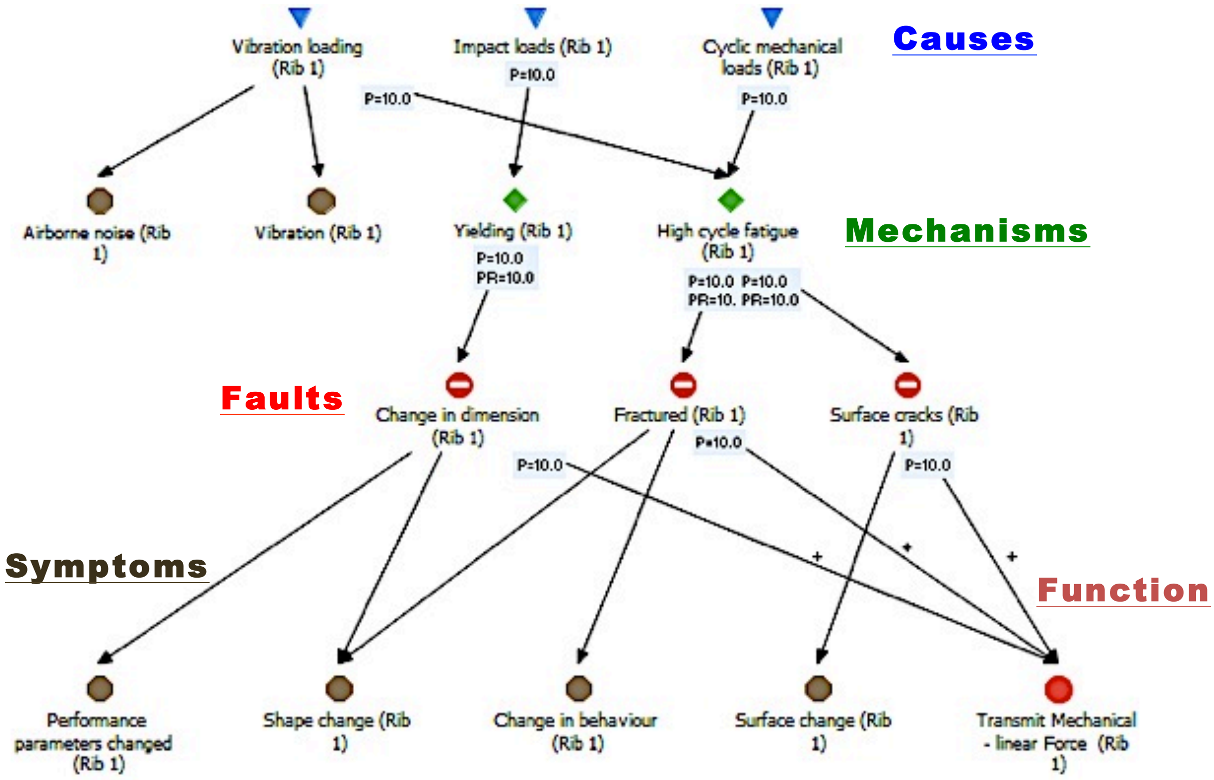


Figure 4-8 Example of a Failure Diagram related to the Component 'Rib 1'

This Failure Diagram has been designed with the purpose to avoid needlessly overcrowding of the model, thus only two potential mechanisms of failure have been considered:

- **High cycle fatigue:** it is well known that the assessment of fatigue damage is the keystone of structural integrity. In this case, the cause of the fatigue mechanism can either be a *Vibration loading* or a *Cyclic loading*. This mechanism of failure will inevitably leads to different faults such as *Surface cracks* and sometimes the *Fracture* of the structural components. In addition, the failure can be detected according to different symptoms, e.g. *Surface cracks* and/or a *Surface change* can be noticed during inspections.

- **Yielding:** corresponds to the second failure mechanism impacting the strength of the material. The yielding of the rib may for example occur after a collision during the taxi phase, i.e. an *Impact load.* As a result of the collision, the structure *Changes in dimension* and can be detected by a *Shape change* and/or a *Change of the performance parameters.*

All potential faults (red icons) directly affect the function of the component. In order to specify whether a fault impacts 'positively' or 'negatively' the functional flow, a causal connection must be specified (+ or -, cf. symbol "+" of the arrow connection on Figure 4-8). In other words, a causal connection with a positive polarity "+" means that an increase of the related Causes, i.e. *Cyclic mechanical loads* or *Vibration loading* will increase the flow property, i.e. *Force.*

Even if MADe includes a few Causes and Mechanisms of structural failures, most of them are missing. Thus, the standardised taxonomy ensuring that the failure concept description for structures is consistent has also been improved.

Failures of structure are in general either due to loadings or environmental damages. Environmental causes have already been implemented in MADe; consequently, only mechanical loads are treated. After a literature review, a list of potential causes of structural failure has been created.

Cause Group	Cause of failure
Static Concentrated Loads	***Tensile force*** ***Compressive force*** ***Shear force*** ***Bending moment*** ***Torsional moment***
Static Distributed Loads	***Uniform load*** ***Triangular load*** ***Parabolic load*** ***Exponential load*** ***Sine load*** ***Cosine load***
Inertia loads	***Gravity load*** ***Spinning load***
Dynamic Periodic Loads	***Harmonic High cyclic load*** ***Harmonic Low cyclic load*** ***Non-Harmonic High cyclic load*** ***Non-Harmonic Low cyclic load***
Dynamic Aperiodic Loads	***Transient load***
Impact Loads	***Impact load***
Pressure	***Pressure***

Table 4-2 Potential Causes of structural failures

Furthermore, structures are physically degraded either by an environmental process (already in MADe) or by an overloading. Some mechanisms of structural failures are present in the software *(blue ones)* and they are listed in three different groups: plastic deformations, elastic deformations and fractures. Nevertheless, new mechanisms have been constructed so as to provide the user with more relevant design.

Mechanism Group	Mechanism of failure
Fracture	*Tensile fracture* *Compressive fracture* *Shear fracture* *Bending fracture* *Torsion fracture* *Combined bending and tensile fracture* *Combined bending and compression fracture* *Combined torsion and tensile fracture* *Combined torsion and compression fracture* *Combined bending and torsion fracture* *Resonance fracture*
Elastic deformation	*Tensile deformation* *Compressive deformation* *Shear deformation* *Bending deformation* *Torsion deformation* *Combined bending and tensile deformation* *Combined bending and compression deformation* *Combined torsion and tensile deformation* *Combined torsion and compression deformation* *Combined bending and torsion deformation*
Plastic deformation	*Tensile yielding* *Compressive yielding* *Shear yielding* *Bending yielding* *Torsion yielding* *Combined bending and tensile yielding* *Combined bending and compression yielding* *Combined torsion and tensile yielding* *Combined torsion and compression yielding* *Combined bending and torsion yielding*

Table 4-3 Potential Mechanisms of structural failures

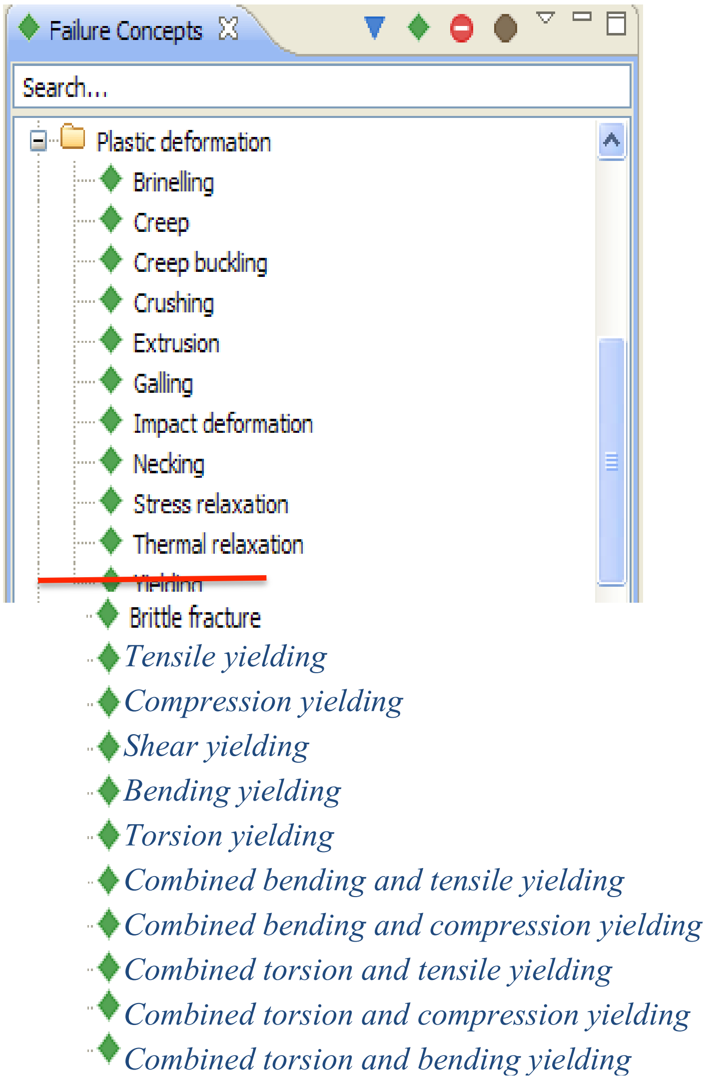


Figure 4-9 Failure mechanism of structures corresponding to a plastic deformation

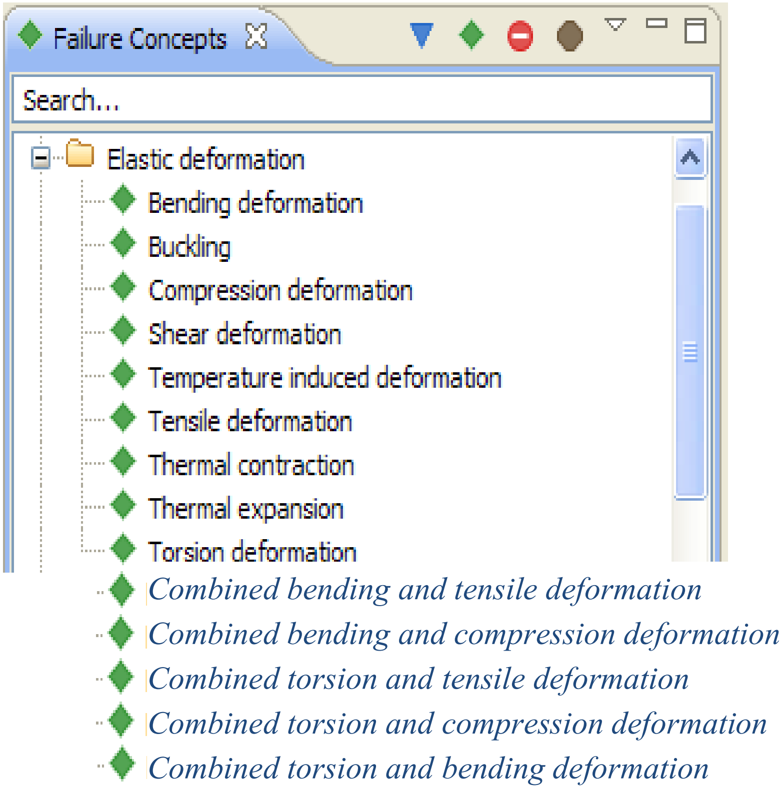


Figure 4-10 Failure mechanisms of structures corresponding to an elastic deformation

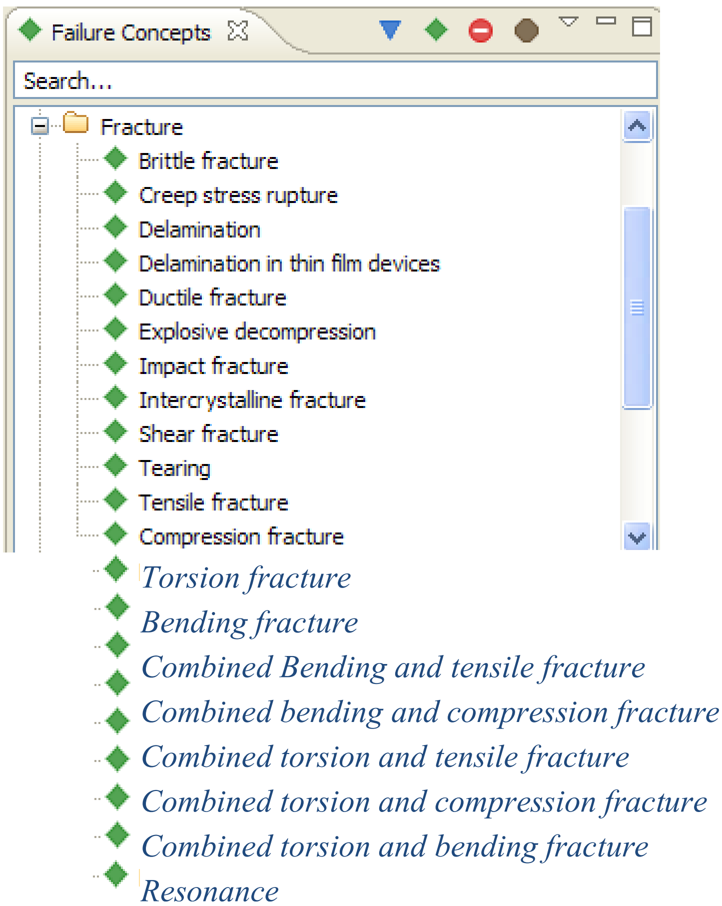


Figure 4-11 Failure mechanisms of structures corresponding to a fracture

4.5 *'What-if'* Analysis & Failure propagation

Looking at the design process of complex structures we see three main iterations:

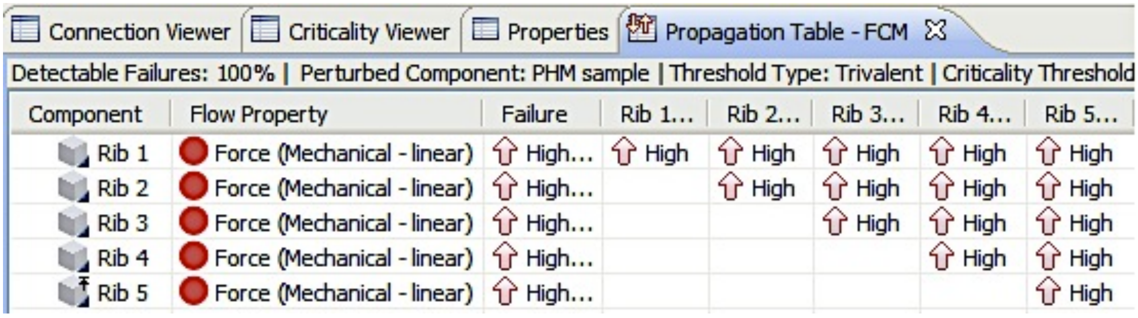
- Customers' requirements are not meet at the conceptual design phase and so corrective actions must be taken.
- Structural weaknesses acknowledged during the Finite Element Analysis as well as at the Fatigue & Damage Tolerance stage require modifications of the detailed design.
- Finally, the physical tests may show anticipated structural failure due to a fault in the design or during the manufacturing. Thus, once again, it is necessary to back out in the process.

Every time that the design is reviewed it is imperative to analyse the impact of the modifications on the whole structure so as to identify which other parts need to be considered. However, in the case of complex structures, it might be difficult to identify all the physical interactions among the components. One solution is to carry out a *'What-if'* analysis by injecting a failure in the model and then to look-over the responses.

In MADe, a failure mode is the result of the deviation, i.e. *increase* or *decrease*, of the output flow. For example, if the output flow increases, then the failure mode is *High Mechanical Force.* Conversely, if the output flow decreases, then the failure mode is *Low Mechanical Force.* In a more realistic approach, the failure of a rib would be that the *Force* increases beyond the design criteria of the limit state analysis and then the rib could not *Transmit* the air loads to the spars.

The automated analysis consists in simulating all potential failure modes mapped in the failure diagrams by activating every failure causes in turn. Then, failures are propagated through the system by Fuzzy Cognitive Map analysis and reported in a Propagation Table showing the next and end-effects of a failure mode.

For example, the Propagation Table shown hereunder presents the evolution of the five failure modes discussed up to now.



Connection Viewer | Criticality Viewer | Properties | Propagation Table - FCM

Detectable Failures: 100% | Perturbed Component: PHM sample | Threshold Type: Trivalent | Criticality Threshold

Component	Flow Property	Failure	Rib 1...	Rib 2...	Rib 3...	Rib 4...	Rib 5...
Rib 1	Force (Mechanical - linear)	High...	High	High	High	High	High
Rib 2	Force (Mechanical - linear)	High...		High	High	High	High
Rib 3	Force (Mechanical - linear)	High...			High	High	High
Rib 4	Force (Mechanical - linear)	High...				High	High
Rib 5	Force (Mechanical - linear)	High...					High

Figure 4-12 Propagation Table for the case study

We can notice that a failure, or modification of the component, *Rib 1,* impacts all other parts. However, changes in the component, *Rib 3*, only affect the *Rib 4* and *5.*

4.6 Criticality Analysis

The purpose of the Criticality Analysis (CA) is to rank each potential failure mode identified by the FMEA according to the combined influence of Severity Classification, its Probability of Occurrence as well as its Difficulty of Detection (RAC, 1993). Afterwards, the most critical potential failure can be monitored in real time, which is called Structural Health Monitoring and this is discussed later.
Using MADe, structural engineers can define the level of criticality by using different techniques such as:

- **Risk Priority Number (RPN)** = Difficulty of detection * Occurrence * Severity

Each criterion is ranked between 1 and 10 with 10 corresponding to the worst case, i.e. Very High Difficulty of Detection, Occurrence and Severity. For example, the Figure 4-13 shows the Criticality definition of the potential cause of failure, *Cyclic Mechanical Loads*. A rib is permanently exposed to fluctuating loads, thus the Occurrence parameter has been set to 9 (Very high occurrence) and as the detection of fatigue loadings remains plenty of uncertainties, the Difficulty of Detection has also been defined equal to 9.

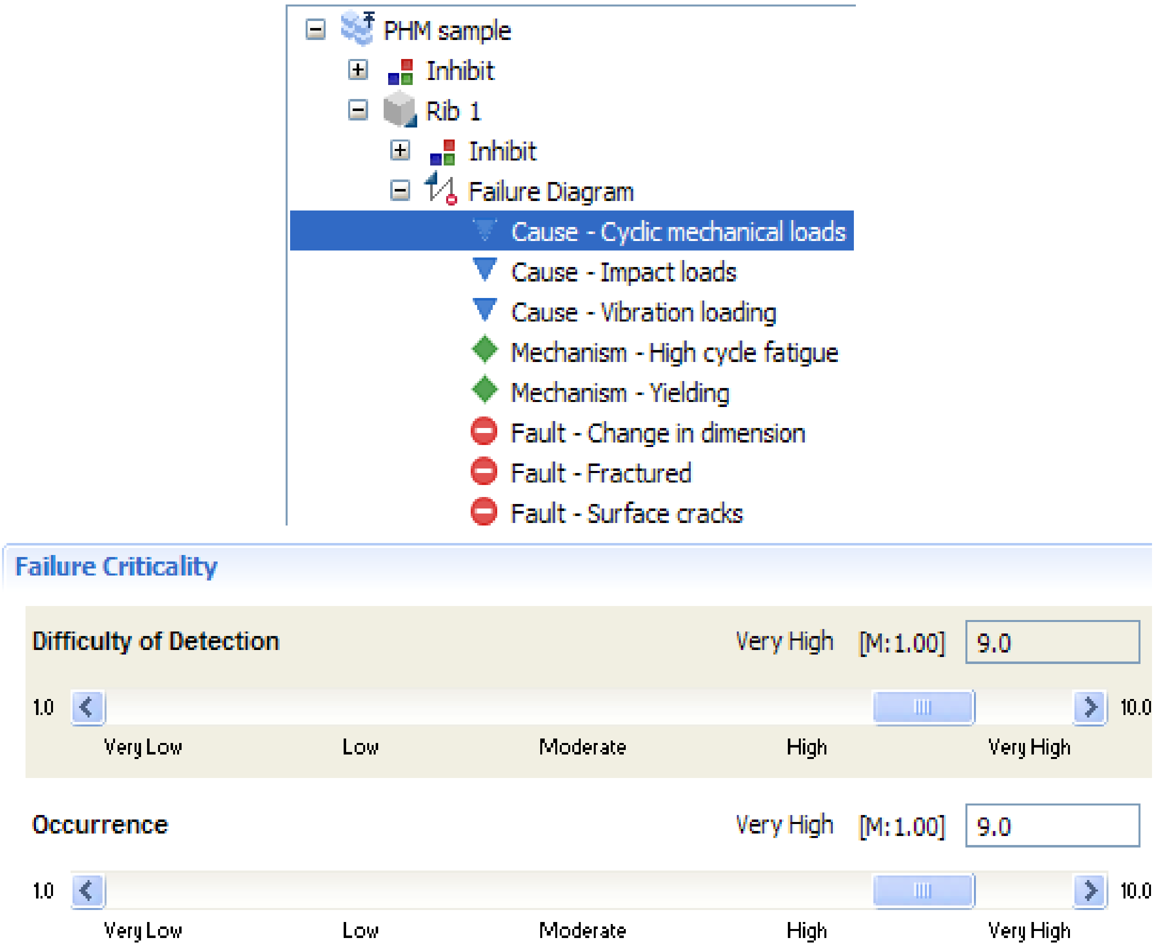


Figure 4-13 Criticality definition of a potential Cause *'Cyclic mechanical loads'*

- **Fuzzy Criticality:** is a more advanced technique that enables engineers to define the criticality levels by doing a trade-off between Difficulty of Detection and Apparent Occurrence. The Fuzzy Criticality tool integrates a range of rules and memberships accurately defined via explicit graphical definitions (Figure 4-14) which simplify and guide users.

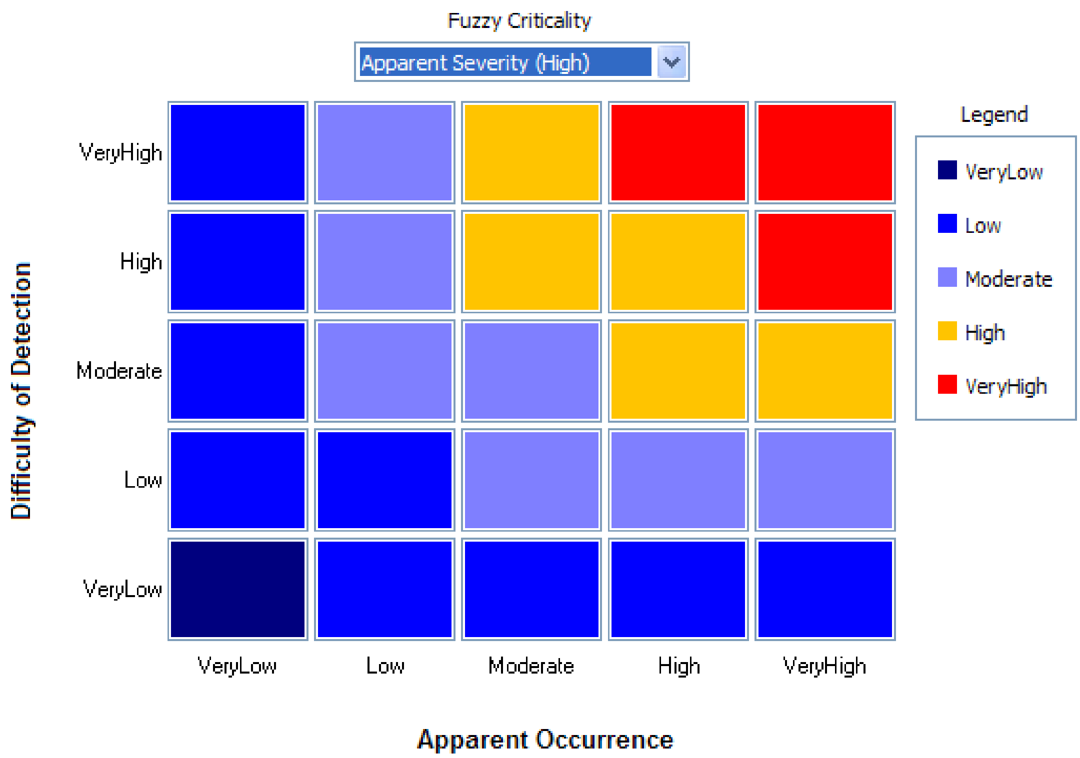


Figure 4-14 Example of a Fuzzy Criticality rule

Criticality evaluation plays a key role in the functional analysis since during the sensor set optimisation, the engineer can specify a threshold so as to only consider the most likely functional failure modes, thereby monitoring the most critical parts.

4.7 FMECA Reporting

Information are printed in a report in order to discuss potential failures during technical meetings. Several standard templates are provided like for instance the MIL-STD-16291. Nevertheless, users can customise the report as desired.

ITEM ID	N/A	**NEXT INDENTURE LEVEL**	PHM sample
ITEM NAME	Rib 1	**SYSTEM NAME**	PHM sample
OPERATION MODE	N/A	**REFERENCE DRAWING**	N/A
FUNCTIONAL NARRATIVE	N/A	**PHYSICAL DESCRIPTION**	N/A

Item Name	Function	Cause	Failure Mechanism	Effects			Criticality			
				Local	Next	End	O	S	D	RPN
Rib 1	Transmit	Vibration loading (Rib 1)	High cycle fatigue (Rib 1)	Transmit Mechanical - linear Force High (Rib 1)	N/A	Transmit Mechanical - linear Force High (Rib 5)	9.0	10.0	7.0	630
Rib 1	Transmit	Cyclic mechanical loads (Rib 1)	High cycle fatigue (Rib 1)	Transmit Mechanical - linear Force High (Rib 1)	N/A	Transmit Mechanical - linear Force High (Rib 5)	9.0	10.0	9.0	810

Figure 4-15 Example of FMECA report

4.8 PHM Design and Optimisation

Structural Health Monitoring (SHM) is a breakthrough in the safety and reliability assessment of complex structures. Real time monitoring of the condition state of a structure provides engineers with accurate data that enable to diagnostic faults and prognostic the remaining useful life of critical structural components so as to schedule just in time maintenance tasks. However, the quality and quantity of structural damage detections depends directly on the number of sensors as well as their locations. This section discusses how MADe PHM module can assist structural engineers to answer questions such as: How many sensors are required? Which critical parts have to be monitored? Which data need to be collected? As a decision-making tool, it also enables engineers to not only optimise the design of sensors but also to compare the sensor sets according to their cost and weight.

The PHM workbench is based on the Functional Design. Indeed, the MADe PHM module analyses the functional flows and focuses on the Symptoms of the potential failure modes. Running a sensor analysis we can specify which type(s) of flow property, e.g. *Force*, we want to monitor.

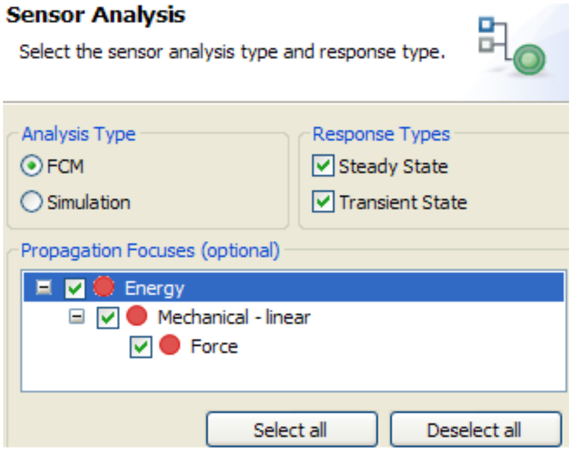


Figure 4-16 Analyse of the functional flow *'Mechanical Force'*

Then, MADe automatically generates various initial sensor sets including the coverage percentage of potential failure modes, the added-weight, the cost of the sensors and further meaningful information. A sensor set is a list of sensors that are individually assigned (linked) to a particular modelling block (subsystem, component, part) of a MADe model.

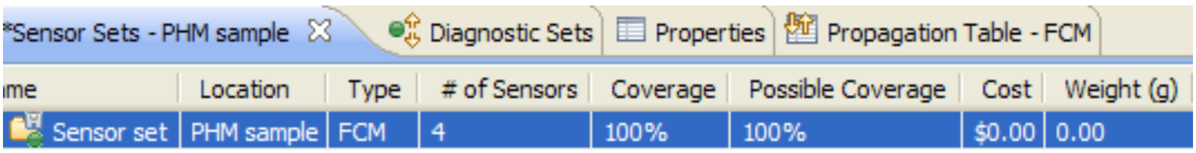


Figure 4-17 Sensor Set automatically designed

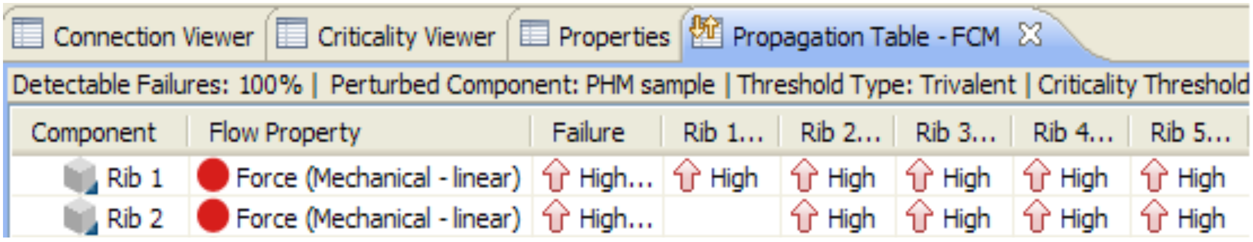
4.8.1 Sensor discrimination

For every sensor set in the system, we need to generate Diagnostic Sets for it. A diagnostic set is a set of sensors which can be used to identify a given failure. A functional failure mode might have more than one diagnostic set, but we only choose the one which is the subset of the given sensor set. Comparing the responses of each failure mode provided by the propagation table (Figure 4-12), we generate MAXTERMS.

Responses of components to each failure must be checked against responses to other failures. Thus, for 'n' failure modes, the XOR logic rules are applied to each pair of functional failure modes F(i), F(j), where i = 1….n, j = 1….n with j > i. If the symptoms are different, then G(i,j) = 1, if the symptoms are the same then G(i,j) = 0. The MAXTERM G(i,j) is therefore a binary text string. If a MAXTERM is a null set, the two failure modes i and j cannot be discriminated and are identified as an 'ambiguity group' which is reported in the analysis results.

<u>*Example:*</u>

This example shows how the diagnostic algorithm can generate sensor sets for Structural Health Monitoring. The study uses the propagation table of the case study, and in a first hand, only the first two failure modes *F(1)* and *F(2)* have been considered.


Connection Viewer | Criticality Viewer | Properties | Propagation Table - FCM

Detectable Failures: 100% | Perturbed Component: PHM sample | Threshold Type: Trivalent | Criticality Threshold

Component	Flow Property	Failure	Rib 1...	Rib 2...	Rib 3...	Rib 4...	Rib 5...
Rib 1	Force (Mechanical - linear)	High...	High	High	High	High	High
Rib 2	Force (Mechanical - linear)	High...		High	High	High	High

Figure 4-18 The first two failure modes of the case study

The reduced propagation table can be rewritten as follows:

F(1)	High	High	High	High	High
F(2)	Ø	High	High	High	High

Table 4-4 The first two failure modes of the case study

Comparing both failure mode effects, we can generate the MAXTERM G(1,2).

Using binary values and XOR rules we get,

F(1)	1	1	1	1	1
F(2)	0	1	1	1	1
F(1) XOR F(2) = G(1,2)	1	0	0	0	0

Table 4-5 MAXTERM G(1,2)

Reminder: XOR logic rules

XOR	
00	0
01	1
10	1
11	0

Table 4-6 XOR logic rules

Doing the same thing for all possible combinations we obtain MAXTERMS

G(i,j)	MAXTERM	G(i,j)	MAXTERM
G(1,2)	10000	G(2,4)	01110
G(1,3)	11000	G(2,5)	00100
G(1,4)	11100	G(3,5)	00110
G(1,5)	11110	G(4,5)	00010
G(2,3)	01100		

Table 4-7 MAXTERMS for all potential functional failure modes

We notice that there is no 'ambiguity group', i.e. none MAXTERM set is equal to zero. Therefore, sensor sets provide 100% coverage of potential functional failure modes.

4.8.2 Sensor Minimisation

The results of the discrimination process are then carried over to the minimisation process. The minimisation process is used to reduce the amount of sensors needed for the model to a bare minimum and generate a list of prime applicants that should provide full coverage of sensors. MINTERMS are sets containing the minimum number of symptoms required to discriminate between failure modes. This is achieved by intersecting the MAXTERMS.

The following table lists the MAXTERMS generated before (Figure 4-7) and now we will see how proceed with the minimisation technique.

G(i,j)	X1 *(Rib 1)*	X2 *(Rib 2)*	X3 *(Rib 3)*	X4 *(Rib4)*	X5 *(Rib 5)*
G(1,2)	1	0	0	0	0
G(1,3)	1	1	0	0	0
G(1,4)	1	1	1	0	0
G(1,5)	1	1	1	1	0
G(2,3)	0	1	1	0	0
G(2,4)	0	1	1	1	0
G(2,5)	0	0	1	0	0
G(3,5)	0	0	1	1	0
G(4,5)	0	0	0	1	0

Table 4-8 MAXTERMS for all potential functional failure modes

Thus, intersecting the MAXTERMS sets we obtain,

F(X1, X2, X3, X4) =

X1.(X1+X2).(X1+X2+X3).(X1+X2+X3+X4).(X2+X3).(X2+X3+X4).X3.(X3+X4).X4

Using the classic rules of Boolean algebra:

- *Idempotent 1: A+A = A*
- *Absorption 1: A.(A+B) = A*
- *Idempotent 2: A.A = A*
- *Absorption 2: A+(A.B) = A*

Therefore, after calculations: F(X1,X2,X3,X4) = X1.X2.X3.X4

This example ends up with a single MINTERM X1.X2.X3.X4 corresponding to the minimal sensor set. Therefore, the optimised sensor set includes one sensor on X1, X2, X3 and X4. This may seems quite obvious in this easy case study, however, readers must imagine the same analysis for a structure decomposed into several level of indentures with numerous sorts of flow and function, so as to really see the benefits of MADe PHM.

4.8.3 Sensor Allocation

Exact sensor type selection is then automated according to the response of the component. Any exact sensor matching is achieved by the user through selective sensor selection. For example, selecting the function *Transmit Mechanical Force*, the type of sensor is automatically filtered and only three categories remain appropriated.

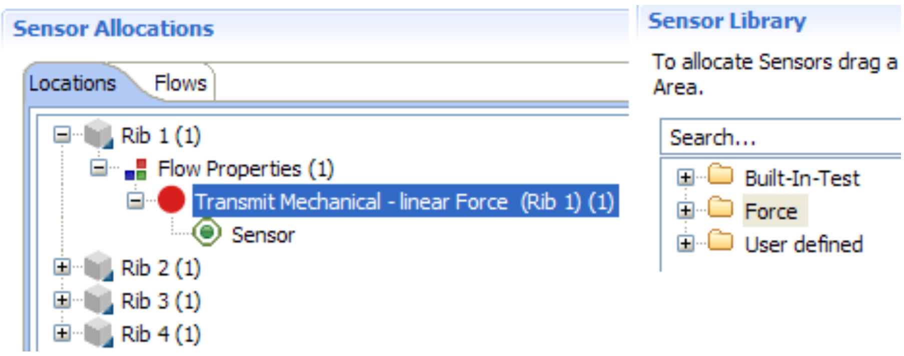


Figure 4-19 Auto-filtering of the suitable sensors

Then, the user can replace the initial sensor by a more suitable one, like for instance the use of a *Strain Gauge* to monitor the *Mechanical Force* of the Component 'Rib 1'.

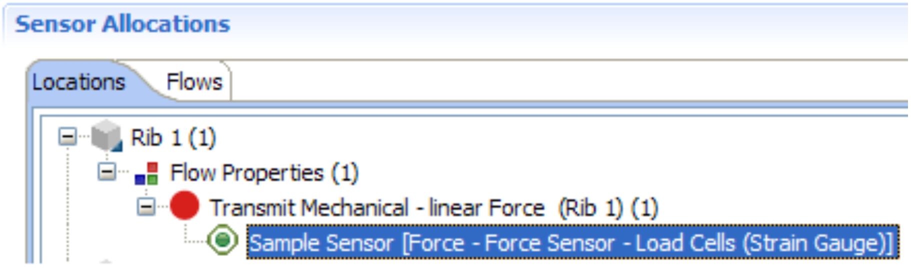


Figure 4-20 Selection of a Strain Gauge for the Mechanical Load flow property

Finally, the design is updated including sensor set details and sensor locations.

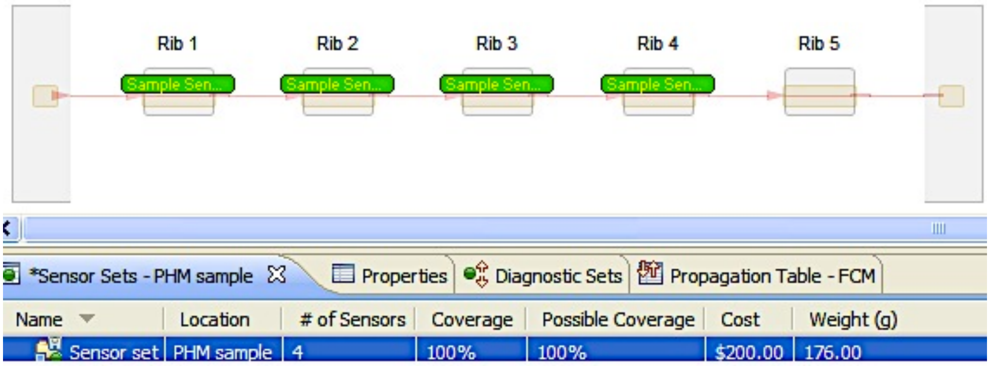


Figure 4-21 Sensor set design

4.8.4 Sensor Details

Details relative to the sensor, which are used to calculate the cost and weight of the sensor sets (Figure 4-21), are input by the user in agreement with the sensor manufacturers' offers.

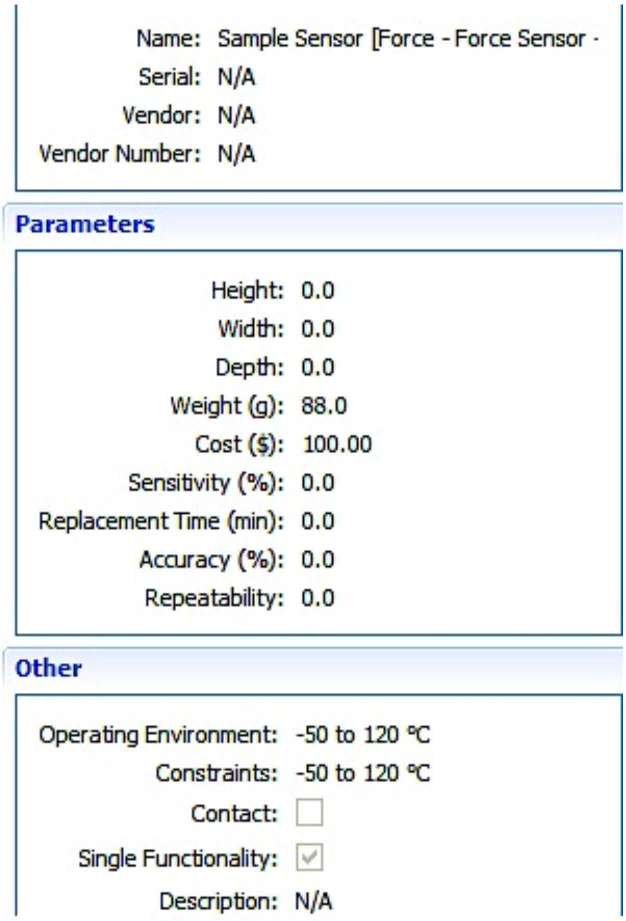


Figure 4-22 Sensor details

MADe PHM already includes a generous library of sensors that can be improved in case of specific applications. The creation of a customised sensor is based on the type of functional symptoms which need to be detected as well as the type of functional flow which required monitoring. Thus, MADe has an unlimited capability, which can answer most of the structural engineers' requirements. Moreover, all new sensor designs can be stored in the library, so as to be either reused in future projects or shared among the project team members.

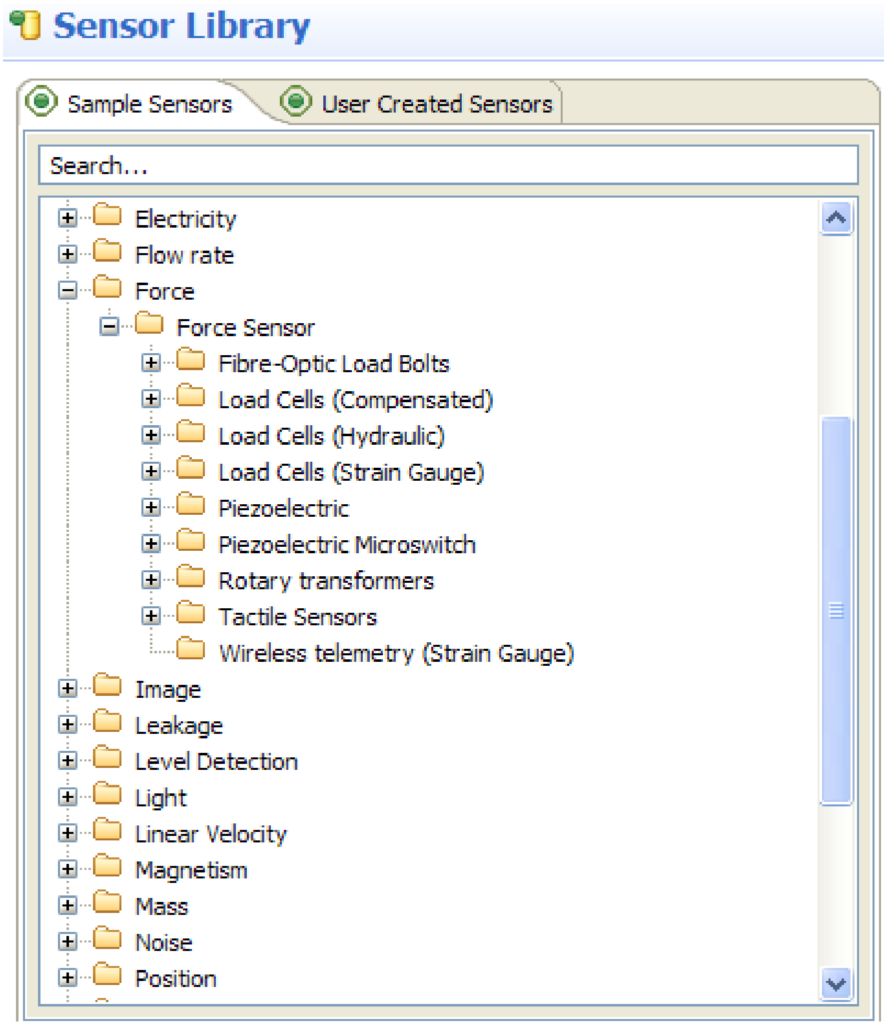


Figure 4-23 MADe sensor library

4.8.5 PHM Reporting

MADe can generate a report including a summary of the sensors' details:

- Total (& unit) % coverage
- Total (& unit) weight
- Total (& unit) cost
- Number of particular type sensors

The reports can also be customised according to engineers' requirements.

Sensor Set Overview

Analysis Type	FCM
Depth	Single
Number of Sensors	4
Total Coverage	100%
Total Cost	$200.00
Total Weight	176.0 g

Sensor set (Assigned) - Sensor Detail

Sensor	Location	Monitoring	Coverage	Weight	Cost
Sample Sensor [Force - Force Sensor - Piezoelectric]	Rib 4	Transmit Mechanical - linear Force (Rib 4)	100%	88.0 g	$100.00
Sample Sensor [Force - Force Sensor - Piezoelectric]	Rib 3	Transmit Mechanical - linear Force (Rib 3)	100%	88.0 g	$100.00
Sample Sensor [Force - Force Sensor - Load Cells (Strain Gauge)]	Rib 2	Transmit Mechanical - linear Force (Rib 2)	100%	0.0 g	$0.00
Sample Sensor [Force - Force Sensor - Load Cells (Strain Gauge)]	Rib 1	Transmit Mechanical - linear Force (Rib 1)	100%	0.0 g	$0.00
Totals			100%	176.0 g	$200.00

Figure 4-24 Example of Sensor Design Report

5 Conclusions and Further Work

5.1 Conclusions

This thesis has outlined two different approaches for risk assessment and management. Firstly, It has been demonstrated that probabilistic calculations using historic component failure rates can be performed for reliability evaluation. Reliability of classic designs such as *series*, *parallel* and *combinations* of both can be easily calculated by hand or using classic computational tools (Excel, symbolic computation, programming language). However, the limited capability of MADe software to compute the reliability of *standby* redundant systems shows the necessity to move from special closed form results to numerical integration methods based on stochastic algorithms, so as to provide engineers with more flexibility. Monte-Carlo algorithm has been briefly introduced for a *cold standby* redundancy and this powerful method should be investigated in further details since it can compute the reliability of any system configuration without paying attention to the challenging recursive integrations.

As a complement to the hardware approach, functional modelling as a means of possibly enhance the assessment of potential functional failure modes of complex systems has already found the breakthrough. However, such a technique remains blurred within the structural engineering field. The strength of MADe Functional Design is to provide engineers with both standardised taxonomies for function and failure concept definitions. This thesis has been written with the purpose to demonstrate the potential use of MADe for functional modelling of complex structures. The library of MADe has been reviewed, so that engineers can use common structural parts. Additionally, both taxonomies used to define functions and failures with emphasis on structural integrity problems have been constructed and are currently for discussion. On the other hand, the latest development of state-of-the-art built-in sensors through mushrooming computation power enable remarkable real-time monitoring of structures, also know as Structural Health Monitoring (SHM). SHM allows collecting data with respect to the external loads as well as the current health of the structure, so as to prognostic the remaining useful life of the structural components and schedule just in time maintenance tasks. Through a case study, this thesis has introduced the capability of the MADe PHM module to facilitate and optimise the design of sensor sets. Even if the research has outlined the opportunity to use MADe PHM for SHM technique, the use of functional analysis' results as a foundation for sensor set design reduces the possibility to illustrate the outcomes with a relevant example. Nevertheless, MADe PHM will be more persuasive as soon as the functional taxonomy of structure will be implemented.

5.2 Further Work

5.2.1 Reliability evaluation

Although this thesis has covered a fair part of the basics in the reliability engineering, further developments and researches can be carried, so as to obtain more realistic reliability evaluations. Here are some interesting improvements which should be considered.

5.2.1.1 Standby redundant systems

Monte-Carlo algorithms should be studied, optimised and implemented in MADe in order to evaluate cold redundancies including N standby components with perfect and imperfect switching. In addition to the cold redundancy configuration, warm redundancy (also know as load sharing, i.e. component failure rate in active state is slightly different from standby state) may be developed so as to fit the reality as much as possible (Way, 2003). The famous Markov Chain Monte-Carlo (MCMC) has been suggested as a future improvement and should be shortly investigated.

5.2.1.2 Further lifetime distributions

So far, MADe only use the exponential probability distribution. Sometimes such a lifetime distribution may be inappropriate and not tends to justify the choice of the model. Thus, other models such as Weibull, Extreme Value, Lognormal and Gamma distributions should be implemented.

5.2.1.3 Dependent lifetime

Even if most of the reliability analysis techniques assume that the components are independent on each other, in some cases, this assumption is no longer valid. For example, the failure of a bar in a truss increases the load on the other bars, and this may impact their lifetime.

5.2.1.4 Repairable items

Many references devote several chapters to repairable systems. Consider components as repairable tremendously increase the complexity of calculations. Indeed, when the system is repaired, it may be in the same state as a time zero, or better, or worse. Furthermore, the ageing of the system may play an important role in the evaluation of repairable systems. This area should be investigated since most of the systems of our everyday life are repairable.

5.2.2 Risks assessment and management methods

In addition to the FMECA, several risk assessment techniques such as SIL, LOPA and HAZOP will be implemented in MADe so as to enable users to use the method which matches as much as possible the reality of his systems or process. These techniques might play an important role to manage structural failures.

5.2.2.1 HAZOP

Hazard and Operability Study (HAZOP) is also a structured and systematic technique based on *guide-words* and enable multi-disciplinary team *(HAZOP team)* to assess potential failure modes during a set of meetings. It is an examination of a work procedure to identify hazards and causes for operation problems, quality problems and delays (Rausand, 2005).

5.2.2.2 SIL & LOPA

The Safety Integrity Level (SIL) is a discrete level for specifying the safety integrity requirements of the safety function to be allocated to the safety-related systems (M. A. Lundteigen, 2007). This type of evaluation is often combined with the Layer Of Protection Analysis (LOPA), which evaluates risks by analysing *initiating causes* leading to a *process deviation,* which again may lead to an impact event that may result in an *end-consequence.* Then risks are managed by using protective layers.

Ancora Imparo… I am steal learning.

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APPENDICES

Appendix A Monte Carlo for Standby Redundancies

A.1 Central limit theorem

The distribution of the sum X = $\sum_{i-1}^{n} X_i$ of a large number n of statistically independent random variables X1, X2,..., Xn approaches a Gaussian (normal) distribution with increasing the number of random variables, if none of the random variables dominates the distribution of the sum (Todinov, 2005).

A.1.1 Visual Basic code for Cold Standby Redundancy

```
Public Sub ComputeSystemReliability()
 'Variables'
 Dim i As Long, CompCount As Long, Iteration As Long, MaxIterations As Long, SuccessCount As Long
 Dim TotalLife As Double, Mission_Time As Double, sum As Double, SystemReliability As Double
 Dim FailureRate() As Single, Coeff() As Double
 With Worksheets("Sheet1")

  'Read & Store # components, # iterations and the mission time in cells(row,column)'
  CompCount = .Cells(2, 5)
  MaxIterations = .Cells(3, 5)
  Mission_Time = .Cells(4, 5)
  'Change array size according to # components'
  ReDim FailureRate(CompCount) As Single

  'Error message if # components or # iterations is smaller than 1'
  If CompCount < 1 Or MaxIterations < 1 Then
   MsgBox "Error in data": Exit Sub
  End If
  'Read & Store components' failure rates'
  For i = 2 To 2 + CompCount - 1
    FailureRate(i - 1) = .Cells(i, 2)
  Next i
 End With
 'Initialise the number of failure'
 SuccessCount = 0

 ' Run Monte-Carlo'
 For Iteration = 1 To MaxIterations
  TotalLife = 0
  ' Compute system's lifetime'
  For i = 2 To 2 + CompCount - 1
    TotalLife = TotalLife - Log(Rnd()) / FailureRate(i - 1)
  Next i
  ' Check whether system's lifetime is longer than the mission time'
  If TotalLife > Mission_Time Then SuccessCount = SuccessCount + 1
 Next Iteration

 ' Compute the reliafility of the system'
 SystemReliability = SuccessCount / MaxIterations
```

	A	B	C	D	E
1	Component #	Failure Rate			
2	0	0,05		Number of components =	3
3	1	0,04		Number of Iterations =	500
4	2	0,03		Survival Time =	75
5					
6				**System Reliability = 0.438**	
7					
8					
9				Compute System Reliability	
10					
11					