

1 非线性滞后

1.1 交叉基展开

$$f(x, \tau) = \sum_{v, \ell} \beta_{v\ell} s_{v(x)} h_{\ell}(\tau) \quad (1)$$

- basisvar: $[n, v_x]$, basislag: $[L, v_\ell]$
- v_x : 非线性影响基函数个数;
- v_ℓ : 滞后基函数个数;

如果是 bspline, 需要指定 knots 节点, 以及基函数的个数。

1.2 考虑滞后时间

$$\eta_t = \sum_{\tau=0}^L f(x_{t-\tau}, \tau), \tau \in [0, 1, \dots, L] \quad (2)$$
$$\eta_t = \sum_{\tau=0}^L \sum_{v=0}^{v_x} \sum_{\ell=0}^{v_\ell} \beta_{v\ell} s_v(x_{t-\tau}) h_{\ell}(\tau)$$

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1 x = basisvar[, v] # 第i个基函数
2 mat <- as.matrix(Lag(basisvar[, v], seqlag(lag), group = group))
3 # mat: {X[n, t], X[n, t-1], ..., X[n, t-L]}, [n, L+1]
4
5 for (l in seq(length = vl)) {
6   ck <- basislag[, l] # [L+1]
7   crossbasis[, vl * (v - 1) + l] <- mat %*% ck # [n, L+1] %*% [L+1, 1] = [n]
8 }
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2 矩阵形式

上式写成, 矩阵形式

$$S(x) = [s_1(x), \dots, s_{v_x}(x)]^T, \in \mathbb{R}^{[n, v_x]} \quad (3)$$

$$H(\tau) = [h_1(\tau), \dots, h_{v_\ell}(\tau)] \in \mathbb{R}^{v_\ell}, \tau \in [0, \dots, L] \quad (4)$$

$$B \in \mathbb{R}^{v_x \times v_\ell}, \beta = \text{vec}(B) \in \mathbb{R}^{v_x \times v_\ell} \quad (5)$$

$$f(x, \tau) = S(x)^T B H(\tau) = [H(\tau) \otimes S(x)]^T \beta \quad (6)$$

$$\eta_t = \sum_{\tau=0}^L [H(\tau) \otimes S(x)]^T \beta \quad (7)$$