EFFICIENT SIMPLE LIABILITY ASSIGNMENT RULES: A COMPLETE CHARACTERIZATION

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Abstract. In this paper we consider a general class of rules called *simple liability assignment rules* under which the assignment of liabilities for losses arising out of interactions involving negative externalities can be coupled for some combinations of the levels of nonnegligence of the interacting parties and decoupled for other combinations, and explore the possibility of efficient assignment of liabilities in the presence of decoupling. The main result of the paper establishes that a simple liability assignment rule is efficient if and only if its structure is such that (i) whenever one party is negligent and the other is not then the negligent party is made to bear the full loss and the nonnegligent party bears none; (ii) whenever both parties are negligent they are made to together bear at least the full loss; and (iii) whenever both parties are nonnegligent they are made to together bear at most the full loss. Thus it follows that the assignment of liabilities under an efficient rule has to be coupled only when one party is negligent and the other is not and hence decoupling liability is not inconsistent with efficiency.

Keywords: Simple liability assignment rule, efficiency, Nash equilibrium, decoupling.
JEL Classification: K10, D6

1. Introduction

The problem of inefficiencies arising out of the presence of negative externalities has been extensively studied in economics and it is well known that imposition of appropriately designed taxes is one solution to the problem.[1] The most important among the alternative solutions is the legal remedy of assignment of liabilities between interacting parties. Courts from across the world are routinely required to decide on matters relating to assignment of liabilities for accidental losses due to negative externalities. A variety of rules are used by courts for this purpose.Which of these rules invariably induce the involved parties to make efficient choices is a key question addressed in the economic analysis of law of torts.

Assignment of liabilities under such rules could be coupled in the sense that the liabilities of all parties taken together add up to the total loss or decoupled in the sense that the liabilities of all parties taken together add upto more or less than the total loss.[2] While efficiency properties of rules under which the assignment of liabilities is always coupled have been studied extensively the rules under which the assignment is not always coupled have received very little attention. It is generally believed that decoupling can yield better outcomes by invariably achieving efficiency

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[1]The problem of designing appropriate taxes to deal with negative externalities was formally analysed by Pigou[16].

[2]Liabilities could be decoupled when (i) a part of the amount imposed on the injurer is allocated to some state or court administered fund or (ii) the victim is compensated out of some special fund set up for this purpose. In the US several states have split recovery statues which necessitate compulsory state sharing of damages. Victims are often compensated by the state if the injurer is insolvent and judgement-proof. See Polinsky and Che[17] and Sharkey[20].

not only in contexts where coupling can do the same but also in some contexts where no coupled rule is efficient.[3] Contrary to this, Jain[10] demonstrates the inconsistency of decoupled liability and efficiency even in contexts for which efficient coupled rules exist. This demonstration, however, is based on the analysis of only a small subclass of the class of all rules under which the assignment of liabilities is always decoupled. Needless to say that the limited applicability of this contrary claim leaves room for further exploration of the possibility of efficient assignment of liabilities in the presence of decoupling and such an exploration assumes its significance from the importance of determining the validity of the general belief about decoupling yielding better outcomes. This paper is a step in that direction. We study the efficiency properties of a class of rules under which the assignment of liabilities is not always coupled and demonstrate that the contrary claim about the inconsistency of decoupling and efficiency is not robust.[4] Our results also indicate that the validity of the belief about achieving efficiency by decoupling liability when coupled rules are all inefficient is also questionable.

We exploit the framework which is now standard in the significant literature on the efficient assignment of liabilities.[5] In this framework the analysis is usually done in the context of interactions between two risk-neutral parties who are strangers to one another. It is assumed that the loss, in case of accident, falls on one of the parties called the victim. The other party is referred to as the injurer. It is assumed that the expected accident loss from this interaction depends on the care levels of the two parties and the social objective is to minimize the total social costs which are defined as the sum of costs of care of the parties involved and the expected accident losses. The assignment of liabilities for the losses in case of occurrence of accident is usually based on the levels of nonnegligence of the victim and the injurer where nonnegligence of a party is defined with respect to a legally specified total social costs minimizing due care level. If there is a legally specified due care for a party and the party chooses a care level which is less than the due care specified for the party then the party is called negligent, otherwise the party is nonnegligent. The level of nonnegligence of a party is either 0 (indicating that the party is negligent) or 1 (indicating that the party is nonnegligent).[6] A rule for the assignment of liabilities specifies the portions of the loss, in case of occurrence of accident, that the victim and the injurer have to bear for every possible combination of their levels of nonnegligence. An application of a rule is simply a complete specification of the possible care levels of the two parties, the expected loss function and the due care levels for each of the two parties. Given a rule and an application, expected costs of parties for every possible configuration of care levels can be defined. It is assumed that the parties know about the rule and the application and choose their care levels simultaneously to minimize their expected costs. Thus an application of a given liability rule is a two-player simultaneous move game of complete information. A rule for assignment of liabilities is efficient if and only if for every game that it can induce there is a Nash equilibrium and every Nash equilibrium is total social costs minimizing. In other words, a

[3]This point is further discussed in the conclusion.

[4]Kaur and Kundu[14] demonstrated the existence of such rules by providing an example. In this paper we provide a complete characterization of efficient rules belonging to a class of rules under which the assignment of liabilities is not always coupled.

[5]See Calabresi ([2], [3], [4]), Coase[5], Brown[1], Diamond[6], Posner ([18], [19]), Shavell [21], Jain and Singh[13], Jain([10], [9]), Jain and Kundu[12], Grady ([7], [8]), Kundu and Pal[15] and Kaur and Kundu[14]. Brown was the first to present formal analysis of many of the important rules for assignment of liabilities. The subsequent literature is built upon Brown's formal model. A systematic treatment of the economic analysis of rules for the assignment of liabilities is contained in Jain[11]

[6]The level of non-negligence can be more generally defined to distinguish between varying degrees of negligence of a negligent party and, thus, taking the value 1 or the ratio of her actual level of care to the due care level legally specified for her, whichever is lower.

rule for assignment of liabilities is said to be efficient if and only if it always induces both parties to choose care levels that minimize total social costs.

The assignment of liabilities corresponding to any combination of the levels of nonnegligence of the interacting parties is said to be *coupled* if and only if the interacting parties together are made to bear the full loss and is said to be *decoupled* if and only if the interacting parties together bear less or more than the loss. The standard rules used by courts are all examples of rules under which liabilities are coupled for all combinations of levels of nonnegligence of the parties.[7] The rule under which the injurer pays tax equal to the harm and the victim bears her loss is an example of a rule under which liabilities are decoupled for all combinations of levels of nonnegligence of the parties. It has been established in the literature that while there are efficient rules under which liabilities are always coupled,[8] rules under which liabilities are always decoupled are all inefficient.[9] Thus it appears that the notion of decoupled liability is inconsistent with efficiency.

In this paper we consider a very general class of rules called *simple liability assignment rules* under which the assignment of liabilities can be coupled for some combinations of the levels of nonnegligence of the interacting parties and decoupled for other combinations and explore the possibility of efficient assignment of liabilities in the presence of decoupling. The main result of the paper establishes that a simple liability assignment rule is efficient if and only if its structure is such that (i) whenever one party is negligent and the other is not then the negligent party is made to bear the full loss and the nonnegligent party bears none; (ii) whenever both parties are negligent they are made to together bear at least the full loss; and (iii) whenever both parties are nonnegligent they are made to together bear at most the full loss. Thus it follows that the assignment of liabilities under an efficient rule has to be coupled only when one party is negligent and the other is not and hence decoupling liability is not inconsistent with efficiency.

The paper is organized as follows: The model is presented in Section 2. All definitions and assumptions are stated here and are illustrated with appropriate examples. Section 3 contains the main result of the paper in the form of Theorem 1 and also contains the intermediate results

[7]The rules of *no liability*, *strict liability*, *negligence*, *strict liability with the defence of contributory negligence* and *negligence with the defence of contributory negligence* are among the rules which are most widely used by courts and extensively analyzed in the literature. The assignment of liabilities under these rules are as follows:

(i) *No liability*: the victim always bears the entire loss and injurer bears nothing.
(ii) *Strict liability*: the injurer is always liable for the entire loss and victim bears nothing.
(iii) *Negligence*: if the injurer is negligent then she has to bear the entire loss and victim bears nothing; if the injurer is not negligent then she bears nothing and the entire loss is borne by the victim.
(iv) *Strict liability with the defence of contributory negligence*: if the victim is negligent then she has to bear the entire loss and injurer bears nothing; if the victim is not negligent then she bears nothing and the entire loss is borne by the injurer.
(v) *Negligence with the defence of contributory negligence*: if the injurer is negligent and the victim is not then she has to bear the entire loss and victim bears nothing; otherwise she bears nothing and the entire loss is borne by the victim.

[8]Rules under which the assignment of liabilities are always coupled are called liability rules. Jain and Singh[13] establishes that a liability rule is efficient if and only if it satisfies the condition of negligence liability. The condition of negligence liability requires that whenever one party is negligent and the other is not the negligent party should bear the entire loss and the nonnegligent party should bear none of the loss.

[9]According to Shavell[22] (footnote 8 in p. 147), the rule under which the injurer pays tax equal to the harm and the victim bears her losses, efficiency would obtain even when both care and activity levels can be varied: 'However, fully optimal behavior can readily be induced with tools other than liability rules. For example, if injurers have to pay the state for harm caused and victims bear their own losses, both victims and injurers will choose levels of care and of activity optimally'. Jain[10] analyses the set of all rules under which the assignment of liabilities is either coupled always or decoupled always and demonstrates that every rule under which the liabilities are always decoupled is inefficient even in the case of fixed activity levels.

(Propositions 1-6) which are used to prove the theorem. Section 4 concludes the paper with a discussion on the implications of the results.

2. Model

We consider interactions, between two parties (generically called party i where $i \in \{1, 2\}$) assumed to be strangers to each other, which can result in an accidental harm falling on party 1. We'll refer to party 1 as the victim and party 2 as the injurer. It is assumed that the probability of accident and the magnitude of harm in case of an accident depend on the levels of non-negative care that the parties might choose to take. Let $a_i \geq 0$ be the index of the level of care taken by party i and let $A_i = \{a_i \mid a_i \geq 0$ be the index of some feasible level of care which can be taken by party $i\}$. We assume that

$0 \in A_i$. (A1)

We denote by $c_i(a_i)$ the cost to party i of care level a_i. Let $C_i = \{c_i(a_i) \mid a_i \in A_i\}$. We assume

$c_i(0) = 0$. (A2)

We also assume that

c_i is a strictly increasing function of a_i. (A3)

In view of (A2) and (A3) it follows that $(\forall c_i \in C_i)(c_i \geq 0)$.

A consequence of (A3) is that c_i itself can be taken to be an index of the level of care taken by party i.

Let $\pi : C_1 \times C_2 \mapsto [0, 1]$ denote the probability of occurrence of accident and $H : C_1 \times C_2 \mapsto \Re_+$ the loss in case of occurrence of accident. Let $L : C_1 \times C_2 \mapsto \Re_+$ be defined as: $L(c_1, c_2) = \pi(c_1, c_2)H(c_1, c_2)$ for all $(c_1, c_2) \in C_1 \times C_2$. L is thus the expected loss due to accident.

We assume:

π and H are non-increasing in c_1 and c_2. (A4)

(A4) implies that L is non-increasing in c_1 and c_2.

We define the total social cost of the interaction between the two parties, $T : C_1 \times C_2 \mapsto \Re_+$, as: $T(c_1, c_2) = c_1 + c_2 + L(c_1, c_2)$ for all $(c_1, c_2) \in C_1 \times C_2$. Let $M = \{(c_1', c_2') \in C_1 \times C_2 \mid (\forall (c_1, c_2) \in C_1 \times C_2)[T(c_1', c_2') \leq T(c_1, c_2)\}$. Thus M is the set of all costs of care profiles (c_1', c_2') which are total social cost minimizing. It will be assumed that:

C_1, C_2, π and H are such that M is nonempty. (A5)

2.1. Negligence.

Let $(c_1^*, c_2^*) \in M$. Given c_1^*, c_2^*, we define functions $p_i : C_i \mapsto \{0, 1\}$as follows:

$$\begin{aligned} p_i(c_i) &= 0 \quad \text{if } c_i < c_i^* \\ &= 1 \quad \text{otherwise.} \end{aligned}$$

If there is a legally specified due care level for party i then c_i^* used in the definition of p_i would be taken to be identical with the legally specified due care level for i. If no due care level is legally specified for party i then c_i^* used in the definition of p_i can be taken to be any $c_i^* \in C_i$ subject to the requirement that $(c_1^*, c_2^*) \in M$. Thus in all cases, for each party i, c_i^* would denote the legally binding due care level for party i whenever the idea of legally binding due care level for party i is applicable.[10]

$p_i(c_i)$ would be interpreted as the level of nonnegligence of party i. $p_i(c_i) = 1$ would mean that party i has taken at least the due care and $p_i(c_i) = 0$ would mean that party i has taken less

[10]Thus, implicitly it is being assumed that the legally specified due care levels are in all cases consistent with the objective of total social cost minimization.

than due care. If $p_i(c_i) = 1$, party i would be called nonnegligent; and if $p_i(c_i) = 0$, party i would be called negligent.[11]

2.2. **Simple Liability Assignment Rule.** A simple liability assignment rule is a function $g : \{0,1\}^2 \mapsto [0,1]^2$, such that: $g(p_1, p_2) = (x_1, x_2)$, where $0 \leq x_1 \leq 1$ is the portion of loss to be borne by the victim and $0 \leq x_2 \leq 1$ is the portion of loss to be borne by the injurer. In other words, a simple liability assignment rule is a rule which specifies the portions of the loss, in case of accident, to be borne by each of the two parties on the basis of who is negligent and who is not.

A simple liability assignment rule g is called (i) a simple liability rule iff $x_1 + x_2 = 1$ for all $(p_1, p_2) \in \{0,1\}^2$ and (ii) a simple hybrid liability rule iff $x_1 + x_2 = k$ for all $(p_1, p_2) \in \{0,1\}^2$; where $k \in \Re_+$. Thus under a simple hybrid liability rule the sum of x_1 and x_2 is same for all combinations of $(p_1, p_2) \in \{0,1\}^2$. But under a simple liability assignment rule, in general, the sum of x_1 and x_2 can vary across combinations of $(p_1, p_2) \in \{0,1\}^2$. Note that the class of all simple liability rules is a proper subclass of the class of all simple hybrid liability rules and the class of all simple hybrid liability rules is a proper subclass of the class of all simple liability assignment rules.[12]

Throughout the paper we shall denote $L(c_1^*, c_2^*)$ by L^*, $L(\overline{c}_1, \overline{c}_2)$ by $\overline{L}$, $p_i(\overline{c}_i)$ by $\overline{p}_i$, $x_i(0,0)$ by x_i^0, $x_i(1,1)$ by x_i^1 and $x_i(\overline{p}_1, \overline{p}_2)$ by $\overline{x}_i$.

Example 1. *Consider the following simple liability assignment rules:*

(i) g_1 *(The standard negligence rule): If the injurer is negligent then she has to bear the entire loss and victim bears nothing; if the injurer is not negligent then she bears nothing and the entire loss is borne by the victim.*

$$\begin{aligned} g_1(p_1, p_2) &= (0,\ 1) \quad \text{if} \quad p_2 = 0 \\ &= (1,\ 0) \quad \text{otherwise.} \end{aligned}$$

(ii) g_2 *(The rule under which the injurer pays tax equal to the harm and the victim bears her losses): The injurer and the victim both bear the full loss individually.*

$$g_2(p_1, p_2) = (1,\ 1) \quad \text{for all } (p_1, p_2) \in \{0,1\}^2.$$

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[11] The level of nonnegligence can be more generally formalized as $p_i : C_i \mapsto [0,1]$ such that:

$$\begin{aligned} p_i(c_i) &= \tfrac{c_i}{c_i^*} \quad \text{if } c_i < c_i^* \\ &= 1 \quad \text{otherwise.} \end{aligned}$$

Note that this formulation distinguishes between varying degrees of negligence of a negligent party. In this paper we will restrict ourselves to the less general formulation given in the text above.

[12] A rule for the assignment of liabilities can be more generally formalized as a liability assignment rule. A liability assignment rule is a function $f : [0,1]^2 \mapsto [0,1]^2$, such that: $f(p_1, p_2) = (x_1, x_2)$, where x_1 is the proportion of loss to be borne by the victim and x_2 is the proportion of the loss to be borne by the injurer. In other words, a liability assignment rule is a rule which specifies, for every possible configuration of the levels of nonnegligence of the two parties, the proportions of the loss, in case of accident, to be borne by each of the two parties. Thus, in general, a liability rule distinguishes between varying degrees of negligence of a negligent party and this distinction can matter in the assignment of liabilities. A simple liability assignment rule, however, makes no distinction between varying degrees of negligence of a negligent party.

Corresponding to every simple liability assignment rule g there is a class of liability assignment rules $F(g) = \{f \mid f = g$ for all $(p_1, p_2) \in \{0,1\}^2\}$. Any two rules in this class can differ in their assignment of liabilities only if $p_1 \in (0,1)$ or $p_2 \in (0,1)\}$. If $f \in F(g)$ is such that $(\forall p_1, p_2 \in [0,1])[p_1 < 1 \to f(p_1, p_2) = f(0, p_2)$ and $p_2 < 1 \to f(p_1, p_2) = f(p_1, 0)]$ then the assignment of liabilities under f is identical to that under g.

A liability assignment rule f is called (i) a liability rule iff $x_1 + x_2 = 1$ for all $(p_1, p_2) \in \{0,1\}^2$ and (ii) a hybrid liability rule iff $x_1 + x_2 = k$ for all $(p_1, p_2) \in \{0,1\}^2$; where $k \in \Re_+$.

(iii) g_3: *If the injurer is not negligent then she bears nothing and the entire loss is borne by the victim; if the injurer is negligent and the victim is not then the injurer has to bear the entire loss and victim bears nothing; and if both are negligent then each one has to individually bear the entire loss.*

$$\begin{aligned} g_3(p_1, p_2) &= (1,\ 1) \quad \text{if} \quad p_1, p_2 = 0 \\ &= (0,\ 1) \quad \text{if} \quad p_1 = 1, p_2 = 0 \\ &= (1,\ 0) \quad \text{if} \quad p_2 = 1. \end{aligned}$$

Note that g_1 is a simple liability rule; g_2 is a simple hybrid liability rule (with $k = 2$) but g_3 is not a simple hybrid liability rule.

2.3. **Application of a Rule.** Let ω be any specification of C_1, C_2, π, H and $(c_1^*, c_2^*) \in M$. Let Ω denote the set of all ωs which satisfy assumptions (A1) - (A5).
Let g be any simple liability assignment rule and $\omega \in \Omega$ be any application of g. If the victim chooses c_1, the injurer chooses c_2 and the accident occurs then the actual loss will be $H(c_1, c_2)$. According to g the victim will be made to bear $x_1(p_1(c_1), p_2(c_2))H(c_1, c_2)$ and the injurer will be liable for $x_2(p_1(c_1), p_2(c_2))H(c_1, c_2)$. $E_1 : C_1 \times C_2 \mapsto \Re_+$ defined as: $E_1(c_1, c_2) = c_1 + x_1(p_1, p_2)L(c_1, c_2)$ for all $(c_1, c_2) \in C_1 \times C_2$ is the expected cost to the victim and $E_2 : C_1 \times C_2 \mapsto \Re_+$ defined as: $E_2(c_1, c_2) = c_2 + x_2(p_1, p_2)L(c_1, c_2)$ for all $(c_1, c_2) \in C_1 \times C_2$ is the expected cost to the injurer. We assume that for all $(c_1, c_2), (c_1', c_2') \in C_1 \times C_2$, the victim considers (c_1, c_2) to be at least as good as (c_1', c_2') iff $E_1(c_1, c_2) \leq E_1(c_1', c_2')$ and the injurer considers (c_1, c_2) to be at least as good as (c_1', c_2') iff $E_2(c_1, c_2) \leq E_2(c_1', c_2')$. Thus an application of a simple liability assignment rule is a two-player simultaneous move game in which the strategies are the feasible levels of care and the payoffs are the expected costs.[13] We shall denote the game induced by simple liability assignment rule g in application ω by (g, w) and, whenever possible, represent it by the corresponding payoff matrix.

2.4. **Efficiency.** A simple liability assignment rule, g, is said to be efficient for ω iff (i) $(\exists (c_1, c_2) \in C_1 \times C_2)[(c_1, c_2)$ is a Nash equilibrium of $(g, \omega)]$ and (ii) $(\forall (c_1, c_2) \in C_1 \times C_2)[$If (c_1, c_2) is a Nash equilibrium of (g, ω) then $(c_1, c_2) \in M]$.[14] A simple liability assignment rule, g, is said to be efficient for Ω iff it is efficient for every $\omega \in \Omega$. In other words, a simple liability rule, g, is said to be efficient for Ω iff for every application $\omega \in \Omega$ (i) there exists a Nash equilibrium of the game (g, ω) and (ii) every Nash equilibrium of (g, ω) minimizes T.

Example 2. *Let $C_1 = C_2 = \{0, 10, 20\}$ and let the expected loss function, $L(c_1, c_2)$ be as given in 2.1.*

		c_2		
		0	10	20
	0	100	80	75
c_1	10	80	60	55
	20	75	55	42

TABLE 2.1. Application ω_1

[13]It has to be noted that the payoffs are non-positive.
[14]Only pure strategy Nash equilibria are considered in this paper.

Note that $M = \{(10, 10)\}$*. Let* $(c_1^*, c_2^*) = (10, 10)$*. Thus* C_1*,* C_2 *given above and* $L(c_1, c_2)$ *specified as in table constitutes an application in* Ω*.*[15] *We denote this application as* ω_1*.* $(c_1^*, c_2^*) = (10, 10)$ *implies that* $p_1(0) = 0, p_1(10) = p_1(20) = 1$ *and* $p_2(0) = 0, p_2(10) = p_2(20) = 1$

Consider ω_1 *as an application of* g_1*. The payoff matrix of* (g_1, ω_1) *is given in Table2.2 below:*

		c_2: 0	10	20
	0	$(\underline{0}, 100)$	$(80, \underline{10})$	$(75, 20)$
c_1	10	$(10, 80)$	$(\underline{70}, \underline{10})$	$(65, 20)$
	20	$(20, 75)$	$(75, \underline{10})$	$(\underline{62}, 20)$

TABLE 2.2. (g_1, ω_1)

$(10, 10)$*, the unique total social cost minimizing configuration of care levels, is also the unique Nash equilibrium of* (g_1, ω_1)*. Therefore* g_1 *is efficient for* ω_1*.*[16]
Now consider ω_1 *as an application of* g_2*. The payoff matrix of* (g_2, ω_1) *is given in Table2.3 below:*

		c_2: 0	10	20
	0	$(100, 100)$	$(80, \underline{90})$	$(75, 95)$
c_1	10	$(\underline{90}, 80)$	$(\underline{70}, \underline{70})$	$(65, 75)$
	20	$(95, 75)$	$(75, 65)$	$(\underline{62}, \underline{62})$

TABLE 2.3. (g_2, ω_1)

$(20, 20)$ *is a Nash equilibrium of* (g_2, ω_1) *but* $(20, 20) \notin M$*. Therefore* g_2 *is not efficient for* ω_1*.*

Finally consider ω_1 *as an application of* g_3*.The payoff matrix of* (g_3, ω_1) *is given in Table2.4 below:*

		c_2: 0	10	20
	0	$(100, 100)$	$(80, \underline{10})$	$(75, 20)$
c_1	10	$(\underline{10}, 80)$	$(\underline{70}, \underline{10})$	$(65, 20)$
	20	$(20, 75)$	$(75, \underline{10})$	$(\underline{62}, 20)$

TABLE 2.4. (g_3, ω_1)

$(10, 10)$*, the unique total social cost minimizing configuration of care levels, is also the unique Nash equilibrium of* (g_3, ω_1)*. Therefore* g_3 *is efficient for* ω_1*.*[17]

[15]We shall, simply, refer to a table representing expected loss function as an application.
[16]Theorem 1 establishes that g_1 is efficient for every application in Ω.
[17]Theorem 1 establishes that g_3 is efficient for every application in Ω.

2.5. **Decoupled assignment of liabilities.** Let g be a simple liability assignment rule and let $(p_1, p_2) \in \{0,1\}^2$. The assignment of liabilities under g at (p_1, p_2) is said to be coupled if and only if $x_2(p_1, p_2)H = H - x_1(p_1, p_2)H$ and to be decoupled if and only if $x_2(p_1, p_2)H \neq H - x_1(p_1, p_2)H$. In other words, the assignment of liabilities corresponding to any combination of the levels of nonnegligence of the interacting parties is said to be coupled if and only if the liability imposed on the injurer is equal to the payment to the victim and to be decoupled if the two amounts are unequal. Thus, if the assignment of liabilities is coupled then the interacting parties parties together are made to bear the full loss and if the assignment of liabilities is decoupled then the interacting parties together bear less or more than the total loss.

Example 3. *Consider the simple liability assignment rules given in Example 1. Note that the assignment of liabilities under g_1 is always coupled, under g_2 it is always decoupled and under g_3 it is is decoupled if and only if both parties are negligent.*

Remark 1. *The liability assignment under a simple liability rule is such that the victim and the injurer together bear the exact amount of the loss. Thus, by definition, the assignment of liabilities under a simple liability rule is always coupled. The same is not true about a simple hybrid liability rule or a simple liability assignment rule in general. While the assignment of liabilities under any simple hybrid liability rule which does not belong to the class of simple liability rules is always decoupled; the assignment of liabilities under a simple liability assignment rule can be coupled for some configurations of nonnegligence of the parties and decoupled for other configurations.*

2.6. **Conditions on simple liability assignment rules.** A simple liability assignment rule g satisfies condition

1. N1 if and only if $x_1(0, 1) = 1$ and $x_2(1, 0) = 1$.
2. N2 if and only if $x_1(1, 0) = 0$ and $x_2(0, 1) = 0$.
3. N3 if and only if $x_1^0 + x_2^0 \geq 1$.
4. N4 if and only if $x_1^1 + x_2^1 \leq 1$.

In other words, a simple liability assignment rule satisfies condition N1 if and only if it is such that whenever one party is negligent and the other is not then the negligent party bears the entire loss; N2 if and only if it is such that whenever one party is negligent and the other is not then the nonnegligent party bears none of the loss; N3 if and only if it is such that whenever both parties are negligent then they together bear no less than the entire loss; and N4 if and only if it is such that whenever both parties are nonnegligent then they together bear no more than the entire loss.

Example 4. *Consider the simple liability assignment rules given in Example 1. Note that g_1 and g_3 satisfy N1-N4, g_2 satisfies only N1 and N3.*

Remark 2. *Suppose g is a simple liability rule. Then g satisfies N1 if and only if it satisfies N2.*

Remark 3. *Every simple liability rule g satisfies N3 and N4.*

Remark 4. *A simple hybrid liability rule g for which $k \neq 1$ cannot satisfy all four of the above conditions. If g satisfies N1 then it violates N2 and N4. If g satisfies N2 then it violates N1 and N3. If g satisfies N3 then it violates N4.*

3. Results

In this section we present the main result of the paper which states that an efficient simple liability assignment rule is completely characterized by the set of conditions N1-N4. The characterization result follows from 6 intermediate results which are presented in the form of Propositions 1-6. Propositions 1 and 2 establish that conditions N1-N4 together are sufficient for the efficiency of a simple liability assignment rule. Proposition 3, 4, 5 and 6 establish the necessity of N1, N2, N3 and N4 respectively.

Proposition 1. *If a simple liability assignment rule g satisfies N1 then (c_1^*, c_2^*) is a Nash equilibrium of (g, ω) for any $\omega \in \Omega$.*

Proof. Let simple liability assignment rule g satisfy N1. Consider any application $\omega \in \Omega$ of g. Suppose (c_1^*, c_2^*) is not a Nash equilibrium of (g, ω). This implies that, there exists $c_1' \in C_1$ such that $E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$ or there exists $c_2' \in C_2$ such that $E_2(c_1^*, c_2') < E_2(c_1^*, c_2^*)$. Note that $E_1(c_1^*, c_2^*) = c_1^* + x_1^1 L^* \leq c_1^* + L^*$.
Suppose, there exists $c_1' \in C_1$ such that $E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$. Take any $c_1' < c_1^*$. This, in view of the fact that g satisfies N1, implies $E_1(c_1', c_2^*) = c_1' + L(c_1', c_2^*)$. Therefore, $E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$ implies $T(c_1', c_2^*) < T(c_1^*, c_2^*)$. Now take any $c_1' > c_1^*$. This and $E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$ imply $c_1' + x_1^1 L(c_1', c_2^*) < c_1^* + x_1^1 L^*$. $c_1' > c_1^*$, by (A4), also implies $L(c_1', c_2^*) \leq L^*$ which implies $(1 - x_1^1) L(c_1', c_2^*) \leq (1 - x_1^1) L^*$. $(1 - x_1^1) L(c_1', c_2^*) \leq (1 - x_1^1) L^*$ and $c_1' + x_1^1 L(c_1', c_2^*) < c_1^* + x_1^1 L^*$ together imply $T(c_1', c_2^*) < T(c_1^*, c_2^*)$. Thus, in all cases, $E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$ implies $T(c_1', c_2^*) < T(c_1^*, c_2^*)$ which contradicts the fact that $(c_1^*, c_2^*) \in M$. Therefore, there does not exist $c_1' \in C_1$ such that $E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$.
By an analogous argument it can be shown that there does not exist $c_2' \in C_2$ such that $E_2(c_1^*, c_2') < E_2(c_1^*, c_2^*)$.
Hence, (c_1^*, c_2^*) is a Nash equilibrium of (g, ω). □

Proposition 1 establishes that, if a simple liability assignment rule, g, satisfies condition N1 then for any application ω in Ω, (c_1^*, c_2^*) is a Nash equilibrium of (g, ω). The proof of the proposition involves showing that no unilateral deviation from (c_1^*, c_2^*) can be profitable. Consider a unilateral deviation by party i from c_i^* to $c_i' < c_i^*$. By condition N1 the entire increase in expected loss has to be borne by party i and, in view of the fact that (c_1^*, c_2^*) minimizes total social costs, this has to be at least as large as the decrease in her cost of the care. Therefore, such a deviation cannot be profitable. Now consider a unilateral deviation by party i from c_i^* to $c_i' > c_i^*$. Note that the liability share of party i does not change due to the deviation and this, in view of the fact that (c_1^*, c_2^*) minimizes total social costs, implies that the increase in her cost of care must be more than the decrease in her share of the expected loss. Therefore, such a deviation cannot be profitable.

Proposition 2. *Suppose simple liability assignment rule g satisfies N1-N4 and let $\omega \in \Omega$ be any application of g. If $(\bar{c}_1, \bar{c}_2)$ is a Nash equilibrium of (g, ω) then $(\bar{c}_1, \bar{c}_2) \in M$.*

Proof. Let simple liability assignment rule g satisfy conditions N1-N4. Consider any application $\omega \in \Omega$ of g. Suppose $(\bar{c}_1, \bar{c}_2)$ is a Nash equilibrium of (g, ω). This implies $E_1(\bar{c}_1, \bar{c}_2) \leq E_1(c_1^*, \bar{c}_2)$ and $E_2(\bar{c}_1, \bar{c}_2) \leq E_2(\bar{c}_1, c_2^*)$ and, therefore, $E_1(\bar{c}_1, \bar{c}_2) + E_2(\bar{c}_1, \bar{c}_2) \leq E_1(c_1^*, \bar{c}_2) + E_2(\bar{c}_1, c_2^*)$ which further implies

$$\bar{c}_1 + \bar{c}_2 + (\bar{x}_1 + \bar{x}_2)\bar{L} \leq c_1^* + c_2^* + x_1(1, \bar{p}_2) L(c_1^*, \bar{c}_2) + x_2(\bar{p}_1, 1) L(\bar{c}_1, c_2^*) \tag{2.1}$$

Suppose $\bar{c}_1 < c_1^*$ and $\bar{c}_2 < c_2^*$. Then $x_1(1, \bar{p}_2) = x_1(1, 0) = x_2(\bar{p}_1, 1) = x_2(0, 1) = 0$ by N2 and $\bar{x}_1 + \bar{x}_2 = x_1^0 + x_2^0 \geq 1$ by N3. Thus $T(\bar{c}_1, \bar{c}_2) \leq \bar{c}_1 + \bar{c}_2 + (\bar{x}_1 + \bar{x}_2)\bar{L}$ and $c_1^* + c_2^* + x_1(1, \bar{p}_2) L(c_1^*, \bar{c}_2) + x_2(\bar{p}_1, 1) L(\bar{c}_1, c_2^*) = c_1^* + c_2^* \leq T(c_1^*, c_2^*)$. Therefore, (2.1) implies $T(\bar{c}_1, \bar{c}_2) \leq T(c_1^*, c_2^*)$.

Suppose $\bar{c}_1 < c_1^*$ and $\bar{c}_2 \geq c_2^*$. Then $x_2(\bar{p}_1, 1) = x_2(0, 1) = 0$ by N2; $\bar{x}_1 + \bar{x}_2 = x_1(0, 1) + x_2(0, 1) = 1$ by N1 and N2; $L(c_1^*, \bar{c}_2) \leq L^*$ by (A4); and $x_1(1, \bar{p}_2) = x_1^1 \leq 1$. Thus $T(\bar{c}_1, \bar{c}_2) = \bar{c}_1 + \bar{c}_2 +$

$(\overline{x}_1+\overline{x}_2)\overline{L}$ and $c_1^*+c_2^*+x_1(1,\overline{p}_2)L(c_1^*,\overline{c}_2)+x_2(\overline{p}_1,1)L(\overline{c}_1,c_2^*) = c_1^*+c_2^*+x_1^1L(c_1^*,\overline{c}_2) \leq T(c_1^*,c_2^*)$. Therefore, (2.1) implies $T(\overline{c}_1,\overline{c}_2) \leq T(c_1^*,c_2^*)$.

Suppose $\overline{c}_1 \geq c_1^*$ and $\overline{c}_2 < c_2^*$. Then $x_1(1,\overline{p}_2) = x_1(1,0) = 0$ by N2; $\overline{x}_1+\overline{x}_2 = x_1(1,0)+x_2(1,0) = 1$ by N1 and N2; $L(\overline{c}_1,c_2^*) \leq L^*$ by (A4); and $x_2(\overline{p}_1,1) = x_2^1 \leq 1$. Thus $T(\overline{c}_1,\overline{c}_2) = \overline{c}_1+\overline{c}_2+(\overline{x}_1+\overline{x}_2)\overline{L}$ and $c_1^*+c_2^*+x_1(1,\overline{p}_2)L(c_1^*,\overline{c}_2)+x_2(\overline{p}_1,1)L(\overline{c}_1,c_2^*) = c_1^*+c_2^*+x_2^1L(\overline{c}_1,c_2^*) \leq T(c_1^*,c_2^*)$. Therefore, (2.1) implies $T(\overline{c}_1,\overline{c}_2) \leq T(c_1^*,c_2^*)$.

Finally, suppose $\overline{c}_1 \geq c_1^*$ and $\overline{c}_2 \geq c_2^*$. Then $\overline{x}_1 = x_1(1,\overline{p}_2) = x_1^1$, $\overline{x}_2 = x_2(\overline{p}_1,1) = x_2^1$ and $L(c_1^*,\overline{c}_2), L(\overline{c}_1,c_2^*), \overline{L} \leq L^*$ by A4. Therefore, (2.1) implies $\overline{c}_1+\overline{c}_2+(x_1^1+x_2^1)\overline{L} \leq c_1^*+c_2^*+(x_1^1+x_2^1)L^*$ which, in view of N4, implies $T(\overline{c}_1,\overline{c}_2) \leq T(c_1^*,c_2^*)$.

Thus in all cases (2.1) implies $T(\overline{c}_1,\overline{c}_2) \leq T(c_1^*,c_2^*)$. This, in view of the fact that $(c_1^*,c_2^*) \in M$, implies $(\overline{c}_1,\overline{c}_2) \in M$. □

Proposition 2 establishes that if a simple liability assignment rule satisfies conditions N1-N4 then for any application ω in Ω, every Nash equilibrium of (g,ω) minimizes total social costs. The intuition underlying the proof of the proposition can be explained as follows: Let g satisfy conditions N1-N4. Consider any $\omega \in \Omega$, and take any $(\overline{c}_1,\overline{c}_2)$ which is a Nash equilibrium of (g,ω). $(\overline{c}_1,\overline{c}_2)$ is a Nash equilibrium implies that $E_1(\overline{c}_1,\overline{c}_2) \leq E_1(c_1^*,\overline{c}_2)$ and $E_2(\overline{c}_1,\overline{c}_2) \leq E_2(\overline{c}_1,c_2^*)$. Note that $E_1(c_1^*,\overline{c}_2) = c_1^*+x_1(1,\overline{p}_2)L(c_1^*,\overline{c}_2)$. If $\overline{c}_2 < c_2^*$ then $x_1(1,\overline{p}_2) = x_1(1,0) = 0$ by condition N2; and if $\overline{c}_2 \geq c_2^*$ then $x_1(1,\overline{p}_2) = x_1^1$. Also, if $\overline{c}_2 \geq c_2^*$ then $L(c_1^*,\overline{c}_2) \leq L^*$. Therefore $x_1(1,\overline{p}_2)L(c_1^*,\overline{c}_2) \leq x_1^1L^*$ and this implies that the expected costs of the victim at (c_1^*,c_2^*) is at least as large as her expected costs at $(\overline{c}_1,\overline{c}_2)$. An analogous argument shows that expected costs of the injurer at (c_1^*,c_2^*) is at least as large as her expected costs at $(\overline{c}_1,\overline{c}_2)$. Therefore the sum of expected costs of the two parties at (c_1^*,c_2^*) is at least as large as the sum of their expected costs at $(\overline{c}_1,\overline{c}_2)$. By condition N4 total social costs (c_1^*,c_2^*) is at least as large as the sum of expected costs of the two parties at (c_1^*,c_2^*). If $\overline{c}_1 < c_1^*$ or $\overline{c}_2 < c_2^*$ then $\overline{x}_1+\overline{x}_2 \geq 1$ by condition N1, N2 and N3 and consequently the sum of expected costs at $(\overline{c}_1,\overline{c}_2)$ is at least as large as the total social cost at $(\overline{c}_1,\overline{c}_2)$. Therefore if $\overline{c}_1 < c_1^*$ or $\overline{c}_2 < c_2^*$ then total social costs at (c_1^*,c_2^*) is at least as large as the total social costs at $(\overline{c}_1,\overline{c}_2)$. If $\overline{c}_1 \geq c_1^*$ and $\overline{c}_2 \geq c_2^*$ then $\overline{x}_1+\overline{x}_2 = x_1^1+x_2^1 \leq 1$ by condition N4 and $\overline{L} \leq L^*$ by (A4) and consequently total social costs at (c_1^*,c_2^*) is at least as large as the total social costs at $(\overline{c}_1,\overline{c}_2)$. This establishes that the total social costs at (c_1^*,c_2^*) is at least as large as the total social costs at any Nash equilibrium and this, in view of the fact that (c_1^*,c_2^*) minimizes total social costs implies that every Nash equilibrium is total social cost minimizing .

Proposition 3. *If a simple liability assignment rule g is efficient then it satisfies N1.*

Proof. Let g be a simple liability assignment rule which violates N1. This implies that $x_1(0,1) < 1$ or $x_2(1,0) < 1$. Suppose $x_1(0,1) < 1$. Choose $\delta_1 > 0$. $\delta_1 x_1(0,1) < \delta_1$. Choose u_1 such that $\delta_1 x_1(0,1) < u_1 < \delta_1$ and choose $u_2, \delta_2 > 0$.
Let $C_1 = \{0,u_1\}$ and $C_2 = \{0,u_2\}$ and let the expected loss function, L, be as given in Table 3.1 :
Table 3.2 gives the total social costs for every possible configuration of costs of care. Here $M = \{(u_1,u_2)\}$. Let $(c_1^*,c_2^*) = (u_1,u_2)$. This implies that $p_1(0) = 0, p_1(u_1) = 1$ and $p_2(0) = 0, p_2(u_1) = 1$. Note that $\{0,u_1\}, \{0,u_2\}, L$ and (u_1,u_2) constitute an application in Ω. We shall refer to this application as ω_3.
$E_1(0,u_2) = \delta_1 x_1(0,1), E_1(u_1,u_2) = u_1$.
$E_1(0,u_2) - E_1(u_1,u_2) = \delta_1 x_1(0,1) - u_1 < 0$. This implies that the unique total social cost

$c_1 \backslash c_2$	0	u_2
0	$u_2+\delta_1+\delta_2$	δ_1
u_1	$u_2+\delta_2$	0

TABLE 3.1. ω_3: Expected loss

$c_1 \backslash c_2$	0	u_2
0	$u_2+\delta_1+\delta_2$	$u_2+\delta_1$
u_1	$u_1+u_2+\delta_2$	u_1+u_2

TABLE 3.2. ω_3: Total Social Costs

minimizing profile of care levels, (u_1, u_2), is not a Nash equilibrium of (g, ω_3) and hence g is inefficient for ω_3. Therefore, if $x_1(0,1) < 1$ then g is not efficient.
By an analogous argument it can be shown that if $x_2(1,0) < 1$ then g is not efficient.
Therefore if g is efficient then it satisfies N1. □

Proposition 3 states that if a simple liability assignment rule is efficient then it satisfies condition N1. Our proof establishes the contrapositive by showing the existence of inefficient applications for every possible violation of condition N1. We consider an arbitrary simple liability assignment rule g which is such that when the victim is negligent and the injurer is not the victim is made to bear less than the full loss (i.e., $x_1(0,1) < 1$) and construct an application for which g is inefficient. Our application is such that (i) there are only two possible levels of care for each party, (ii) the profile of care levels in which both parties take positive care is the only one minimizing total social costs, (iii) the expected loss at the total social costs minimizing profile of care levels is zero, and (iv) for the victim taking no care is a profitable unilateral deviation from the profile of care levels that minimizes total social costs. Thus the unique total social cost minimizing profile of care levels is not a Nash equilibrium and hence g is inefficient for the application so constructed. Note that, the victim's unilateral deviation from the profile of care levels which minimizes total social costs increases expected loss by more than the reduction in victim's cost of care and such a deviation can be profitable for the victim only if the victim does not have to bear the entire increase in expected loss which is what $x_1(0,1) < 1$ implies. A similar construction is possible for any simple liability assignment rule which is such that when the injurer is negligent and the victim is not the injurer is made to bear less than the full loss.

Proposition 4. *If a simple liability assignment rule g is efficient then it satisfies N2.*

Proof. Let g be a simple liability assignment rule which violates N2. This implies that $x_1(1,0) > 0$ or $x_2(0,1) > 0$. Suppose $x_1(1,0) > 0$. Let $\delta > 0$. Note that $0 < \delta x_1(1,0) \leq \delta$. Choose positive numbers u_1, u_2, L^0, θ such that

(i) $\theta < u_2 < u_1 < \delta x_1(1,0)$
(ii) $\theta + \delta < L^0 < u_2 + \delta$

Let $C_1 = \{0, u_1\}$ and $C_2 = \{0, u_2\}$ and let the expected loss function, L, be specified as in Table 3.3
Table 3.4 gives the total social costs for every possible configuration of costs of care.
$u_2 < u_1 < \delta$ and $\theta < u_1$ imply $M = \{(0, u_2)\}$. Let $(c_1^*, c_2^*) = (0, u_2)$. This implies that $p_1(0) = p_1(u_1) = 1$ and $p_2(0) = 0, p_2(u_1) = 1$. Note that $\{0, u_1\}, \{0, u_2\}, L$ and $(0, u_2)$ constitute

		c_2: 0	u_2
c_1	0	L^0	$L^0 - \delta$
	u_1	$L^0 - \delta$	$L^0 - \delta - \theta$

TABLE 3.3. ω_4: Expected loss

		c_2: 0	u_2
c_1	0	L^0	$u_2 + L^0 - \delta$
	u_1	$u_1 + L^0 - \delta$	$u_1 + u_2 + L^0 - \delta - \theta$

TABLE 3.4. ω_4: Total Social Costs

an application in Ω. We shall refer to this application as ω_4.
$E_1(u_1, 0) = u_1 + x_1(1, 0)(L^0 - \delta)$ and $E_1(0, 0) = x_1(1, 0)L^0$.
$E_2(u_1, 0) = x_2(1, 0)(L^0 - \delta), E_2(u_1, u_2) = u_2 + x_2(1, 1)(L^0 - \delta - \theta)$.
$E_1(u_1, 0) - E_1(0, 0) = [u_1 + x_1(1, 0)(L^0 - \delta)] - x_1(1, 0)L^0 = u_1 - \delta x_1(1, 0) < 0$.
$E_2(u_1, 0) - E_2(u_1, u_2) = x_2(1, 0)(L^0 - \delta) - [u_2 + x_2(1, 1)(L^0 - \delta - \theta)] \leq L^0 - \delta - u_2 < 0$.
Therefore $(u_1, 0)$, a profile of care levels which does not minimize total social costs, is a Nash equilibrium of (g, ω_4) and hence g is inefficient for ω_4. Therefore, if $x_1(1, 0) > 0$ then g is not efficient.
By an analogous argument it can be shown that if $x_2(0, 1) > 0$ then g is not efficient.
Therefore if g is efficient then it satisfies N2.

□

Proposition 4 states that if a simple liability assignment rule is efficient then it satisfies condition N2. Our proof establishes the contrapositive by showing the existence of inefficient applications for every possible violation of condition N2. We consider an arbitrary simple liability assignment rule g which is such that when the injurer is negligent and the victim is not the victim is made to bear a positive portion of the full loss (i.e., $x_1(1, 0) > 0$) and then construct an application for which g is inefficient. Our application is such that (i) there are only two possible levels of care for each party, (ii) unilateral deviations from the configuration in which neither party takes care reduces total social costs, (iii) the configuration of care levels in which the victim takes no care and the injurer takes positive care is the only one minimizing total social costs, and (iv) the configuration of care levels in which the victim taking positive care and the injurer takes no care is a Nash equilibrium. Thus there is a Nash equilibrium which is different from the unique total social cost minimizing profile of care levels and hence g is inefficient for the application so constructed. Note that, the victim's unilateral deviation from the profile of care levels in which she takes positive care and the injurer takes no care increases expected loss by more than the decrease in victim's cost of care and such a deviation may not be profitable for the victim only if she has to bear some of the increase in expected loss which is what follows from $x_1(1, 0) > 0$. A similar construction is possible for any simple liability assignment rule which is such that when the victim is negligent and the injurer is not the injurer is made to bear a positive portion of the full loss.

Proposition 5. *If a simple liability assignment rule g is efficient then it satisfies N3.*

Proof. Let g be a simple liability assignment rule such that $x_1^0 + x_2^0 < 1$. Choose a positive number v. $x_1^0 v, x_2^0 v \leq [x_1^0 + x_2^0]v < v$. Let

(i) $u_1 = [x_1^0 + \frac{1-x_1^0-x_2^0}{4}]v$
(ii) $u_2 = [x_2^0 + \frac{1-x_1^0-x_2^0}{4}]v$
(iii) $v_1 = [x_1^0 + \frac{1-x_1^0-x_2^0}{2}]v$
(iv) $v_2 = [x_2^0 + \frac{1-x_1^0-x_2^0}{2}]v$

Note that $u_1 < v_1, u_2 < v_2$ and $v_1 + v_2 = v$. Let $C_1 = \{0, u_1\}$ and $C_2 = \{0, u_2\}$ and let the expected loss function L be specified as as in Table 3.5

c_1 \ c_2	0	u_2
0	$v_1 + v_2$	v_1
u_1	v_2	0

TABLE 3.5. ω_5: Expected loss

Table 3.6 gives the total social costs for every possible configuration of costs of care.

c_1 \ c_2	0	u_2
0	$v_1 + v_2$	$u_2 + v_1$
u_1	$u_1 + v_2$	$u_1 + u_2$

TABLE 3.6. ω_5: Total Social Costs

$u_1 < v_1$ and $u_2 < v_2$ implies that $M = \{(u_1, u_2)\}$. Let $(c_1^*, c_2^*) = (u_1, u_2)$. This implies that $p_1(0) = 0, p_1(u_1) = 1$ and $p_2(0) = 0, p_2(u_2) = 1$. Note that $\{0, u_1\}, \{0, u_2\}, L$ and (u_1, u_2) constitute an application in Ω. We shall refer to this application as ω_5.
$E_1(u_1, 0) = u_1 + x_1(1,0)v_2$ and $E_1(0,0) = x_1^0[v_1 + v_2]$.
$E_2(0, u_2) = u_2 + x_2(0,1)v_1$, and $E_2(0,0) = x_2^0[v_1 + v_2]$.
$E_1(u_1, 0) - E_1(0,0) = u_1 + x_1(1,0)v_2 - x_1^0[v_1 + v_2] = x_1(1,0)v_2 + \frac{1-x_1^0-x_2^0}{4}v > 0$
$E_2(0, u_2) - E_2(0,0) = u_2 + x_2(0,1)v_1 - x_2^0[v_1 + v_2] = x_2(0,1)v_1 + \frac{1-x_1^0-x_2^0}{4}v > 0$.
Therefore (0, 0), a profile of care levels which does not minimize total social costs, is a Nash equilibrium of (g, ω_5) and hence g is inefficient for this application.

□

Proposition 5 establishes that condition N3 is necessary for efficiency of a simple liability assignment rule. To prove the proposition we consider an arbitrary simple liability assignment rule, g, which is such that when both parties are negligent they together bear less than the total loss (i.e., $x_1^0 + x_2^0 < 1$) and then construct an application for which g is inefficient. Our application is such that (i) there are only two possible levels of care for each party, (ii) unilateral deviation from taking no care, by either party, always reduces total social costs, and (iii) unilateral deviation from the configuration in which both parties take no care is profitable for neither. Thus the configuration of care levels in which neither of the two parties take care is a Nash equilibrium but it does not minimize total social costs. Note that: a unilateral deviation from the configuration in which both take no care decreases expected loss by more than the increase in the deviating party's cost of care but such a deviation may still not be profitable for her if she does not get the full benefit of the reduction in expected loss which is what $x_1^0 + x_2^0 < 1$ implies in view of the fact $x_1(1,0), x_2(0,1) \leq 1$.

Proposition 6. *If a simple liability assignment rule g is efficient then it satisfies N4.*

Proof. Let g be an efficient simple liability assignment rule. Suppose $x_1^1 + x_2^1 > 1$. This implies that $x_1^1, x_2^1 > 0$. Choose a positive number β. Note that $0 < \frac{x_1^1}{x_1^1+x_2^1}\beta < \beta x_1^1 \leq \beta$ and $0 < \frac{x_2^1}{x_1^1+x_2^1}\beta < \beta x_2^1 \leq \beta$. Choose $u_1, u_2, \epsilon_1, \epsilon_2, \theta_1, \theta_2, \delta_1$ and δ_2 such that

(i) $u_1, u_2, \epsilon_1, \epsilon_2 > 0$

(ii) $\frac{x_1^1}{x_1^1+x_2^1}\beta < \delta_1 < \theta_1 < \beta x_1^1$

(iii) $\frac{x_2^1}{x_1^1+x_2^1}\beta < \delta_2 < \theta_2 < \beta x_2^1$.

Let $C_1 = \{0, u_1, u_1 + \theta_1\}$ and $C_2 = \{0, u_2, u_2 + \theta_2\}$ and let the expected loss function L be specified as in Table 3.7

		c_2: 0	u_2	$u_2 + \theta_2$
c_1	0	$u_1 + \theta_1 + \epsilon_1 + u_2 + \theta_2 + \epsilon_2$	$u_1 + \theta_1 + \epsilon_1 + \theta_2$	$u_1 + \theta_1 + \epsilon_1 + \delta_2$
	u_1	$\theta_1 + u_2 + \theta_2 + \epsilon_2$	$\theta_1 + \theta_2$	$\theta_1 + \delta_2$
	$u_1 + \theta_1$	$\delta_1 + u_2 + \theta_2 + \epsilon_2$	$\delta_1 + \theta_2$	$\delta_1 + \delta_2 - \beta$

TABLE 3.7. ω_6: Expected loss

Table 3.8 gives the total social costs for every possible configuration of costs of care.

		c_2: 0	u_2	$u_2 + \theta_2$
c_1	0	$u_1 + \theta_1 + \epsilon_1 + u_2 + \theta_2 + \epsilon_2$	$u_1 + \theta_1 + \epsilon_1 + u_2 + \theta_2$	$u_1 + \theta_1 + \epsilon_1 + u_2 + \theta_2 + \delta_2$
	u_1	$u_1 + \theta_1 + u_2 + \theta_2 + \epsilon_2$	$u_1 + \theta_1 + u_2 + \theta_2$	$u_1 + \theta_1 + u_2 + \theta_2 + \delta_2$
	$u_1 + \theta_1$	$u_1 + \theta_1 + \delta_1 + u_2 + \theta_2 + \epsilon_2$	$u_1 + \theta_1 + \delta_1 + u_2 + \theta_2$	$u_1 + \theta_1 + \delta_1 + u_2 + \theta_2 + \delta_2 - \beta$

TABLE 3.8. ω_6: Total Social Costs

$\frac{x_1^1}{x_1^1+x_2^1}\beta < \delta_1$ and $\frac{x_2^1}{x_1^1+x_2^1}\beta < \delta_2$ imply $\beta < \delta_1 + \delta_2$ and therefore $M = \{(u_1, u_2)\}$. Let $(c_1^*, c_2^*) = (u_1, u_2)$. Thus $p_1(0) = 0, p_1(u_1) = p_1(u_1 + \theta_1) = 1$ and $p_2(0) = 0, p_2(u_2) = p_2(u_2 + \theta_2) = 1$. Note that $\{0, u_1, u_1 + \theta_1\}, \{0, u_2, u_2 + \theta_2\}, L$ and (u_1, u_2) constitute an application in Ω. We shall refer to this application as ω_6.

$E_1(0, u_2 + \theta_2) = x_1(0,1)[u_1 + \theta_1 + \epsilon_1 + \delta_2] = u_1 + \theta_1 + \epsilon_1 + \delta_2$ by Proposition 3; $E_1(u_1, u_2 + \theta_2) = u_1 + x_1^1(\theta_1 + \delta_2)$ and $E_1(u_1 + \theta_1, u_2 + \theta_2) = u_1 + \theta_1 + x_1^1(\delta_1 + \delta_2 - \beta)$.
$E_2(u_1 + \theta_1, 0) = x_2(1,0)[u_2 + \theta_2 + \epsilon_2 + \delta_1] = u_2 + \theta_2 + \epsilon_2 + \delta_1$ by Proposition 3; $E_2(u_1 + \theta_1, u_2) = u_2 + x_2^1(\delta_1 + \theta_2)$ and $E_2(u_1 + \theta_1, u_2 + \theta_2) = u_2 + \theta_2 + x_2^1(\delta_1 + \delta_2 - \beta)$.

$E_1(u_1 + \theta_1, u_2 + \theta_2) - E_1(0, u_2 + \theta_2) = u_1 + \theta_1 + x_1^1(\delta_1 + \delta_2 - \beta) - [u_1 + \theta_1 + \epsilon_1 + \delta_2] = x_1^1(\delta_1 - \beta) - \epsilon_1 - (1 - x_1^1)\delta_2 < 0$.
$E_1(u_1 + \theta_1, u_2 + \theta_2) - E_1(u_1, u_2 + \theta_2) = u_1 + \theta_1 + x_1^1[\delta_1 + \delta_2 - \beta] - u_1 - x_1^1(\theta_1 + \delta_2) =$

$\theta_1 + x_1^1(\delta_1 - \theta_1) - x_1^1\beta < \theta_1 - x_1^1\beta < 0.$
$E_2(u_1 + \theta_1, u_2 + \theta_2) - E_2(u_1 + \theta_1, 0) = u_2 + \theta_2 + x_2^1(\delta_1 + \delta_2 - \beta) - [u_2 + \theta_2 + \epsilon_2 + \delta_1] = x_2^1(\delta_2 - \beta) - \epsilon_2 - (1 - x_2^1)\delta_1 < 0.$
$E_2(u_1 + \theta_1, u_2 + \theta_2) - E_2(u_1 + \theta_1, u_2) = u_2 + \theta_2 + x_2^1[\delta_1 + \delta_2 - \beta] - u_2 - x_2^1(\theta_2 + \delta_1) = \theta_2 + x_2^1(\delta_2 - \theta_2) - x_2^1\beta < \theta_2 - x_2^1\beta < 0.$
Thus $(u_1+\theta_1, u_2+\theta_2)$, which does not belong to M, is a Nash equilibrium of this game. Therefore, g is not efficient.

□

Proposition 6 establishes that condition N4 is necessary for efficiency of a simple liability assignment rule. The proof of the proposition is by contradiction. We consider an arbitrary simple liability assignment rule g which is efficient. We assume that g is such that when both parties are nonnegligent they together bear more than the total loss (i.e., $x_1^1 + x_2^1 > 1$) and then construct an application for which g is inefficient. Our application is such that (i) there are only three possible levels of care for each party, (ii) the unique total social costs minimizing configuration neither involves the highest care nor 0 care by any of the two parties, and (iii) the configuration in which both parties take more than due care is a Nash equilibrium under g. Note that, for both parties unilateral deviation to due care from the configuration in which both take more than due care may not be profitable only if both bear a positive share of the loss which is what $x_1^1 + x_2^1 > 1$ implies in view of the fact $x_1^1, x_2^1 \leq 1$. For both parties unilateral deviation to no care from the configuration in which both take more than due care is also not profitable because g, being efficient, satisfies N_1.

Theorem 1. *A simple liability assignment rule g is efficient if and only if it satisfies conditions N1 - N4.*

Proof. Immediate from Propositions 1-6. □

Corollary 1. *If g is a simple liability rule then it is efficient if and only if it satisfies N1.*

Proof. Immediately follows from Theorem 1 in view of the observations made in Remarks 2 and 3. □

Corollary 2. *Every hybrid simple liability rule with $k \neq 1$ is inefficient.*

Proof. Immediately follows from Theorem 1 in view of the observations made in Remark 4. □

4. Conclusion

In the standard tort model used to analyse the efficiency properties of rules for the assignment of liabilities for accidental losses, the level of activities are held fixed and the parties choose their care levels to minimize their expected costs. Shavell[21] demonstrates that no coupled liability rule can achieve efficiency when the interacting parties can choose their activity levels as well as their care levels. The reason for the impossibility of achieving efficiency both in terms of care levels and activity levels is as follows: in the determination of nonnegligence of a party activity levels are not taken into account and therefore under a coupled liability rule both parties cannot be made to internalize the costs of their choice of activity levels.[18] Shavell[22] suggested a solution to the problem by introducing the possibility of decoupling in the form of a rule under which injurers have to pay the state for harm caused and victims bear their own losses. The idea behind such a rule was to make both parties internalize the costs of their choice of care

[18]According to Shavell ‘the reason, in essence, is that for injurers to choose their correct level of activity they must bear activity losses, whereas for victim to choose their correct level of activity they too must bear activity losses. Yet injurers and victims cannot each bear accidents losses.’

as well as activity. To analyze the idea more systematically Jain[10] introduced the notion of a hybrid liability rule and demonstrated that the rule proposed by Shavell (and any other rule under which the liabilities are always decoupled) is not efficient even when activity levels are held fixed. He argues that efficient assignment of liabilities not only requires internalization of the harm by both parties but also the closure of the externality with respect to the interacting parties. Rules under which the liabilities are always coupled automatically satisfy the closure property by apportioning the full loss between the victim and the injurer and therefore efficiency for such rules depend on whether or not the parties are induced to internalize the harm. On the other hand, rules under which the liabilities are always decoupled necessarily lead to violation of the closure property and therefore result in inefficiency.

In contrast to Jain[10] we focus on a class of rules called simple liability assignment rules under which, in general, the assignment of liabilities can be coupled for some combinations of the levels of nonnegligence of the interacting parties and decoupled for other combinations. We restrict ourselves to the standard model and provide a complete characterization of simple liability assignment rules which always result in efficient care levels. Our result clearly demonstrates that liabilities under an efficient rule have to be coupled only when one party is negligent and the other is not and therefore decoupled liability is not inconsistent with efficient care.Thus, it follows that, closure of the externality with respect to the interacting parties is not necessary for efficient assignment of liabilities. It also follows from our result that under any simple liability assignment rule which results in efficient care levels both parties can be individually made to bear the full loss only if both are negligent. Therefore it appears that under a simple liability assignment rule efficiency cannot be achieved both in terms of care levels and activity levels.

As discussed earlier, simple liability assignment rules do not distinguish between varying degrees of nonnegligence of negligent parties. The analysis contained in this paper can be extended to study efficiency properties of liability assignment rules which distinguish between varying degrees of nonnegligence of negligent parties. It would be interesting to see if the result obtained in this paper holds for liability assignment rules. In this regard we have the following conjecture: Conditions N1-N4, defined appropriately, would turn out to be sufficient for the efficiency of liability assignment rules. It appears that the question of necessity of these conditions for efficient liability rules would require a closer examination.

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