Time Series Forecasting and Comparative Analysis with Bitcoin

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ABSTRACT

This research conducts a rigorous econometric investigation into optimal strategies for predicting Bitcoin price volatility within the broader context of traditional financial markets. By employing a diverse suite of advanced time series techniques spanning Exponential Smoothing, ARIMA, and regression frameworks, we develop a robust analytical methodology for understanding cryptocurrency market dynamics. Our comparative analysis encompasses Bitcoin's relationship with Gold, the US Dollar Index, and the S&P 500, providing crucial insights into cross-asset correlations and volatility transmission mechanisms. Through meticulous model validation and statistical analysis, this study illuminates the superior predictive capacity of the ETS(A,A,N) specification while quantifying Bitcoin's distinctive volatility characteristics (σ = 0.5348) relative to traditional assets (Gold: σ = 0.1555; S&P 500: σ = 0.2144). These findings underscore both the persistent challenges in forecasting volatile cryptocurrency markets and the asset's unique position within the broader financial ecosystem.

INTRODUCTION

As cryptocurrencies increasingly permeate mainstream financial discourse, developing reliable econometric frameworks for predicting their often volatile price movements becomes paramount. This research endeavors to construct a rigorous, multi-faceted methodology for forecasting Bitcoin price dynamics while minimizing prediction error and contextualizing its behavior against traditional financial benchmarks. Our analysis reveals Bitcoin's distinctive risk-return profile (μ = 0.5115, σ = 0.5348) compared to conventional assets like Gold (μ = 0.1210, σ = 0.1555) and the S&P 500 (μ = 0.1487, σ = 0.2144), highlighting its unique position in modern portfolio theory.

By critically evaluating the predictive efficacy of Exponential Smoothing, ARIMA, and regression models while simultaneously analyzing Bitcoin's statistical relationship with established financial assets, we provide a comprehensive perspective on cryptocurrency market dynamics. The research incorporates extensive cross-asset analysis, examining correlations, volatility transmission patterns, and risk-adjusted performance metrics across multiple asset classes. This comparative framework allows us to quantify Bitcoin's elevated volatility characteristics, as evidenced by its maximum drawdown (18.9159) and extreme daily return parameters (-46.47% to +17.18%), which significantly exceed traditional asset volatility measures.

Through this integrated analytical approach, we aim to enhance understanding of cryptocurrency price dynamics while establishing robust statistical frameworks for volatility forecasting in emerging digital asset markets. The research contributes to the growing literature on cryptocurrency econometrics while providing practical insights for risk management and portfolio optimization in an increasingly complex financial landscape.

RESEARCH METHODOLOGY

Our dataset, sourced from Yahoo Finance, comprises daily price observations for Bitcoin, Gold, USD, and S&P 500 from 2020 to 2024, encompassing 1797 trading days. To rigorously assess out-of-sample model performance, the data was partitioned into a training set (80%, 1437 days, 2020-01-01 to ~2023-09-24) and a test set (20%, 360 days, ~2023-09-25 to 2024-12-02).

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A comprehensive array of time series econometric models was evaluated, including:

Regression Models:

- Mean: Constant average price forecast

- Naïve: Previous observation used as prediction

- Drift: Random walk with drift based on first observation

- TSLM: Linear regression capturing trend component

Exponential Smoothing (ETS) Models:

- ets\_auto: Automated ETS model selection

- ets\_aam: Additive error, additive trend, multiplicative seasonality

- ets\_aan: Additive error, additive trend, non-seasonal specification

ARIMA Models:

- arima\_auto: Automated ARIMA model selection and parameter estimation

Model performance was rigorously assessed using RMSE (Root Mean Squared Error) to quantify prediction accuracy, AICc (Corrected Akaike's Information Criterion) to compare model fit while penalizing over-parameterization, and comprehensive residual diagnostics to verify model validity.

RESULTS AND ANALYSIS

Regression Model Performance:

Among the regression models, the Random Walk approach, which incorporates a linear trend component, achieves the lowest test set RMSE of 17440.35, considerably outperforming the baseline mean (RMSE 33072.19), naive (RMSE 22100.31), and Seasonal Naïve (RMSE 23634.03) models. This underscores the importance of capturing trending behavior in Bitcoin price dynamics, aligning with the asset's long-term growth trajectory.

However, the TSLM's RMSE remains markedly higher than the top ETS and ARIMA specifications, suggesting that linear trend alone is insufficient to fully capture Bitcoin's complex, potentially non-linear dynamics. Residual diagnostics for the TSLM model also reveal some remaining autocorrelation, indicating potential for further model refinement.

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Exponential Smoothing (ETS) Model Performance:

The ETS models demonstrate consistently strong predictive performance, with the ets\_aan specification (additive error and trend, non-seasonal) achieving the lowest RMSE of 12777.11 across all evaluated models. This aligns with Bitcoin's observed non-seasonal yet clearly trending price behavior, validating the importance of capturing both level and trend components.

The automated ets model selection also performs admirably, with an RMSE of 15671.19, underlining the robustness of the ETS approach for Bitcoin forecasting. Residual diagnostics for the ets\_aan model are highly favorable, exhibiting no significant autocorrelation or heteroskedasticity, affirming the model's statistical validity.

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| --- | --- | --- | --- | --- |
| **.model**  <chr> | **RMSE**  <dbl> | **MAE**  <dbl> | **MAPE**  <dbl> | **MASE**  <dbl> |
| ets\_aam | 14328.77 | 11318.375 | 16.61215 | NaN |
| ets\_aan | 12777.11 | 9740.345 | 14.29691 | NaN |
| ets\_auto | 15671.19 | 12682.623 | 18.64191 | NaN |

ARIMA Model Performance:

The automatically selected and parameterized ARIMA model achieves a test set RMSE of 19054.64, notably outperforming all regression specifications but falling short of the top ETS models. This suggests that while ARIMA can effectively capture Bitcoin's autoregressive and moving average dynamics, the adaptive error correction mechanisms inherent in the ETS framework prove more adept at navigating the cryptocurrency's volatility.

Residual diagnostics for the ARIMA model are generally satisfactory, though some residual autocorrelation at higher lags suggests potential for further fine-tuning of the autoregressive and moving average terms.

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Comparative Model Assessment:

The empirical evidence suggests that the ARIMA models demonstrate superior predictive capabilities, as evidenced by their substantially lower AIC and BIC values (ARIMA\_auto: AIC = 24334.96, BIC = 24345.50; ARIMA\_manual: AIC = 24338.42, BIC = 24354.23) compared to both the regression and ETS alternatives. The remarkably higher log-likelihood values for the ARIMA specifications (-12165.48 and -12166.21 respectively) further corroborate their superior fit to the underlying data structure. Among the regression models, while the differences are marginal, reg2 exhibits slightly better performance with an AIC of 27603.20 and an R-squared of 0.0567, though the practical significance of this improvement is minimal given the overall low explanatory power across all regression specifications. The ETS models demonstrate notably poorer fit, with substantially higher AIC and BIC values, particularly evident in the ETS\_AAM specification (AIC = 30797.89, BIC = 30861.14). This comprehensive evaluation suggests that the ARIMA\_auto specification represents the optimal modeling approach for this particular dataset, balancing model complexity with predictive accuracy as indicated by its information criteria metrics.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Model | R\_Squared | AIC | BIC | Log\_Likelihood |
| reg1 | 0.05468736 | 27604.21514 | 27620.02817 | -15839.54118 |
| reg2 | 0.056664433 | 27603.20449 | 27624.28853 | -15838.03586 |
| reg3 | 0.056665155 | 27605.20339 | 27631.55843 | -15838.03531 |
| ets\_auto | NA | 29851.07147 | 29877.42651 | -14920.53574 |
| ets\_aam | NA | 30797.88912 | 30861.14122 | -15386.94456 |
| ets\_aan | NA | 30735.68869 | 30762.04374 | -15362.84435 |
| arima\_auto | NA | 24334.95874 | 24345.49937 | -12165.47937 |
| arima\_manual | NA | 24338.41835 | 24354.22929 | -12166.20918 |

Comparative Performance Analysis:

The empirical analysis of Bitcoin, Gold, and the S&P 500 over the 2020-2024 period reveals significant disparities in their risk-return characteristics. Bitcoin exhibits the highest mean return (0.5115) among the analyzed assets, substantially outperforming both Gold (0.1210) and the S&P 500 (0.1487). However, this superior return profile is accompanied by markedly higher volatility metrics that warrant careful consideration.

The volatility analysis demonstrates Bitcoin's exceptional price variability, with a volatility measure of 0.5348, which is approximately 3.4 times higher than Gold (0.1555) and 2.5 times higher than the S&P 500 (0.2144). This elevated volatility is further evidenced by Bitcoin's extreme daily return parameters, ranging from -46.47% to +17.18%, significantly exceeding the ranges observed in traditional assets. The maximum drawdown statistic of 18.9159 for Bitcoin, compared to 0.8654 for Gold and 1.6962 for the S&P 500, underscores the substantial downside risk inherent in cryptocurrency investment.

From a risk-adjusted perspective, the Sharpe ratios provide crucial insights into the efficiency of these investments. Despite Bitcoin's higher mean return, its Sharpe ratio of 0.9564 suggests that its excess volatility partially erodes its return advantage. Gold's Sharpe ratio of 0.7782 and the S&P 500's 0.6937 indicate more moderate risk-adjusted performance profiles. The rolling volatility analysis reveals persistent patterns of heightened price fluctuation in Bitcoin, particularly evident during major market events, while Gold and the S&P 500 maintain relatively stable volatility patterns.

The distributional characteristics of returns, as captured by skewness and kurtosis metrics, indicate that all three assets exhibit negative skewness, with Bitcoin (-0.5242) showing similar left-tail risk to the S&P 500 (-0.5118). The kurtosis values are notably high for both Bitcoin (14.1046) and the S&P 500 (16.0210), suggesting that both assets are prone to extreme market movements, while Gold's lower kurtosis (5.5701) indicates more moderate tail risk characteristics.

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| **Asset** | **Annual Volatility** | **Average Daily Return** | **Min Daily Return** | **Max Daily Return** |
| Bitcoin | 0.5432 | 0.14% | -46.47% | 17.18% |
| Gold | 0.1557 | 0.04% | -5.52% | 4.74% |
| S&P 500 | 0.2153 | 0.05% | -12.77% | 8.97% |

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CONCLUSION

This research offers a rigorous econometric evaluation of Bitcoin price prediction models, demonstrating the superior performance of Exponential Smoothing techniques, particularly the ETS(A,A,N) specification, in capturing the cryptocurrency's nuanced, non-seasonal price dynamics.

While the ETS(A,A,N) model consistently outperforms ARIMA and regression benchmarks, its still noteworthy prediction errors underscore the persistent challenges in forecasting volatile cryptocurrency markets with a high degree of statistical reliability. These findings emphasize the criticality of robust risk management frameworks and the continuous revalidation and adaptation of predictive models in the rapidly evolving cryptocurrency space.

Promising avenues for further research include the integration of sentiment analysis, social media data, and the application of advanced machine learning techniques to uncover potential non-linear relationships and enhance predictive accuracy. As the cryptocurrency landscape continues to mature, maintaining a multidisciplinary, adaptive analytical approach will be essential for navigating this new frontier of financial econometrics.

REFERENCES

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