Ryan Platte
DSP Homework 2

1. $H(z) = \frac{z+1}{z-0.5}$

a.) $\frac{z}{z-0.5} + \frac{1}{z-0.5}$ $\quad \frac{z}{z-6} = 6^n u[n]$

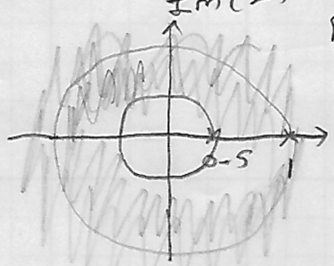
$$\boxed{-0.5^n u[n] + -0.5^{n-1} u[n-1] = h[n]}$$

b.) $\frac{Y(z)}{X(z)} = \frac{z+1}{z-0.5} \rightarrow Y(z)(z-0.5) = X(z)(z+1)$

$Y(z)z - Y(z)0.5 = X(z)z + X(z) \rightarrow y(z) - 0.5z^{-1}Y(z) = X(z) + z^{-1}X(z)$

$$\boxed{y[n] - 0.5y[n-1] = x[n] + x[n-1]}$$

c.)



ROC > 0.5

causal so on right side

d.) $X(z) = \frac{1}{1-0.25z^{-1}}$

$y(z) = \frac{z+1}{z-0.5}\left(\frac{1}{1-0.25z^{-1}}\right) = z\frac{z+1}{z-0.5}\left(\frac{1}{z-0.25}\right) = \left(\frac{A}{z-0.5} + \frac{B}{z-0.25}\right)z$

$(z+1) = A(z-0.25) + B(z-0.5)$

$z = 0.5,\quad 1.5 = A(.25) \quad A = 6$

$z = 0.25,\quad 1.25 = B(-0.25),\quad B = -5$

$y(z) = z\left(\frac{6}{z-0.5} - \frac{5}{z-0.25}\right) = \frac{6z}{z-0.5} - \frac{5z}{z-0.25} = \frac{6}{1-0.5z^{-1}} - \frac{5}{1-0.25z^{-1}}$

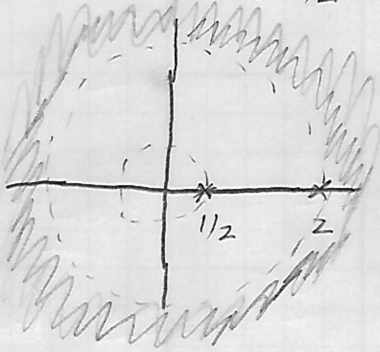
$y[n] = 6 \cdot 0.5^n u[n] - 5 \cdot 0.25^n u[n]$

$$\boxed{y[n] = 6(0.5^n u[n]) - 5(0.25^n u[n])}$$

Problem 2:

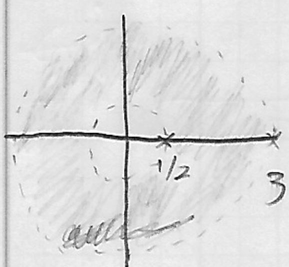
a.) $x[n] = 2^n u[n] + 3(1/2)^n u[n]$

$x(z)\ \frac{z}{z-2} + \frac{3z}{z-1/2}$



b.) $x[n] = (1/2)^n u[n+1] + 3^n u[-n-1]$

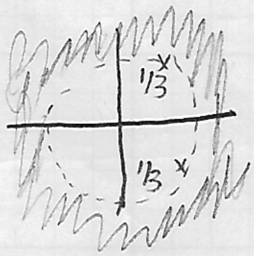
$x(z) = z\left(\frac{z}{z-1/2}\right) - \frac{z}{z-3}$



c.) $x[n] = (1/3)^n \sin(2\pi(1/8)n)\, u[n] \Rightarrow (1/3)^n \left(\frac{1}{2j} e^{j2\pi(1/8)n} - \frac{1}{2j} e^{-j2\pi(1/8)n}\right) u[n]$

$\rightarrow \frac{1}{2j}\left(1/3\, e^{j\pi/4}\right)^n - \left(1/3\, e^{-j\pi/4}\right)^n u[n]$

$x(z) = \frac{1}{2j}\ \frac{z}{z-(1/3\, e^{j\pi/4})} - \frac{1}{2j}\ \frac{z}{z-(1/3\, e^{-j\pi/4})}$



Problem 5

- For stability magnitude of poles < 1
- To reject 0.06 samples/sec set 0's at $F = 0.06$
- For peaks at 0.3 set poles to $F = 0.3$
- DC Gain of 1 means at $z = 1$ $H(z) = 1$

$$H(z) = \frac{\left(z - e^{j2\pi(0.06)}\right)\left(z - e^{-j2\pi(0.06)}\right)}{\left(z - 0.9\, e^{j2\pi(0.3)}\right)\left(z - 0.9\, e^{-j2\pi(0.3)}\right)}$$

$$\text{Top} = z^2 - 2\cos(2\pi(0.06))\, z + 1$$

$$\text{Bottom} = z^2 - 1.8\cos(2\pi(0.3))\, z + .81$$

so $Y(z)\left(1 - 1.8\cos(2\pi(0.3))\, z^{-1} + .81\, z^{-2}\right) = X(z)\left(1 - 2\cos(2\pi(0.06))\, z^{-1} + z^{-2}\right)$

$$\rightarrow \; y[n] - 1.8\cos(2\pi(0.3))\, y[n-1] + 0.81\, y[n-2]$$
$$=$$
$$x[n] - 2\cos(2\pi(0.06)\, x[n-1] + x[n-2]$$

y coeffs $[\,1 \quad -1.8\cos(2\pi(0.3)) \quad 0.81\,]$

x coeffs $[\,1 \quad -2\cos(2\pi(0.06)) \quad 1\,]$

Problem 7

$$X(z) = \frac{5z^2 - 0.7z}{z^2 - 0.9z + 0.81} \qquad |z| > 0.9$$

a) using PFE

$$p = 0.9e^{j\pi/3} \qquad X(z) = \frac{5z - 0.7}{(z-p)(z-p^*)} z \rightarrow \left(\frac{A}{z-p} + \frac{A^*}{z-p^*}\right) z$$

$$5z - 0.7 = A(z-p^*) + A^*(z-p)$$

For $z = p \rightarrow 5p - .7 = A(p-p^*) \Rightarrow A\left(\frac{5p-0.7}{p-p^*}\right) = 2.5 - j0.4443$

$\Downarrow$

$2.6905e^{-j(.3785)}$

For $z = p^*$ we will obtain conjugate of $z = p$

so $2.5 + j0.4443 \Rightarrow 2.6905e^{j(.3785)}$

$$H(z) = z\left(\frac{2.6905e^{-j(.3785)}}{z - 0.9e^{j\pi/3}} + \frac{2.6905e^{j(.3785)}}{z - 0.9e^{-j\pi/3}}\right)$$

$$h[n] = z\left((2.69)(0.9)^n \cos\left(\frac{\pi}{3}n\right)\right)u[n]$$

b)

$$\frac{5z^2 - 0.7z}{z^2 - 0.9z + 0.81} \rightarrow 5\left(\frac{1 - 0.14z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}\right)$$

$r = 0.9 \quad w_0 = \frac{\pi}{3}, \quad 0.9\sin\left(\frac{\pi}{3}\right) \approx 0.779$

$$\Rightarrow 5\left(\underbrace{\left(\frac{1 - 0.14z^{-1} - 0.31z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}\right)}_{0.9^n \cos\left(\frac{\pi}{3}n\right)u[n]} + \frac{0.31}{0.779}\underbrace{\left(\frac{0.779z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}\right)}_{0.9^n \sin\left(\frac{\pi}{3}n\right)u[n]}\right)$$

$$\therefore h[n] = 5(0.9)^n \cos\left(\frac{\pi}{3}n\right)u[n] + 1.989(0.9)^n \sin\left(\frac{\pi}{3}n\right)u[n]$$

c)

$$\frac{5}{0.779z^{-1}}\left(\frac{0.779z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}\right) - \frac{0.7}{0.779}\left(\frac{0.779z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}\right)$$

$$h[n] = 6.418(.9)^{n+1}\sin\left(\frac{\pi}{3}n + 1\right)u[n+1] - 0.898(0.9)^n \sin\left(\frac{\pi}{3}n\right)u[n]$$