Ryan Plate
DSP Homework 2

1. $H(z) = \frac{z+1}{z-0.5}$

a.) $\frac{z}{z-0.5} + \frac{1}{z-0.5} \quad \frac{z}{z-6} = 6^n u[n]$.

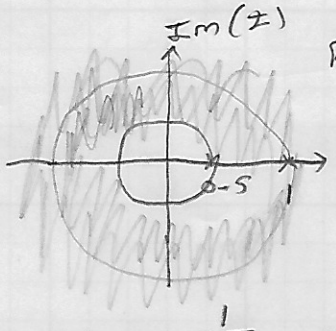
$$-0.5^n u[n] + -0.5^{n-1} u[n-1] = h[n]$$

b.) $\frac{Y(z)}{X(z)} = \frac{z+1}{z-0.5} \rightarrow Y(z)(z-0.5) = X(z)(z+1)$

$Y(z)z - Y(z)0.5 = X(z)z + X(z) \rightarrow y(z) - 0.5z^{-1}Y(z) = X(z) + z^{-1}X(z)$

$$y[n] - 0.5y[n-1] = x[n] + x[n-1]$$

c.)



ROC > 0.5

causal so on right side

d.) $X(z) = \frac{1}{1-0.25z^{-1}}$

$$y(z) = \frac{z+1}{z-0.5}\left(\frac{1}{1-0.25z^{-1}}\right) = z\frac{z+1}{z-0.5}\left(\frac{1}{z-0.25}\right) = \left(\frac{A}{z-0.5} + \frac{B}{z-0.25}\right)z$$

$(z+1) = A(z-0.25) + B(z-0.5)$

$z = 0.5, \quad 1.5 = A(.25) \quad A = 6$

$z = 0.25, \quad 1.25 = B(-0.25), \quad B = -5$

$$y(z) = z\left(\frac{6}{z-0.5} - \frac{5}{z-0.25}\right) = \frac{6z}{z-0.5} - \frac{5z}{z-0.25} = \frac{6}{1-0.5z^{-1}} - \frac{5}{1-0.25z^{-1}}$$

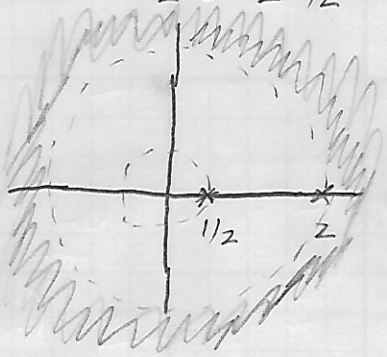
$y[n] = 6 \cdot 0.5^n u[n] - 5 \cdot 0.25^n u[n]$

$$y[n] = 6(0.5^n u[n]) - 5(0.25^n u[n])$$

Problem 2

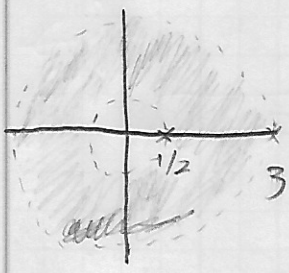
a.) $x[n] = 2^n u[n] + 3(1/2)^n u[n]$

$X(z) \quad \frac{z}{z-2} + \frac{3z}{z-1/2}$



b.) $x[n] = (1/2)^n u[n+1] + 3^n u[-n-1]$

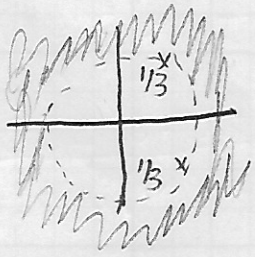
$X(z) = z\left(\frac{z}{z-1/2}\right) - \frac{z}{z-3}$



c.) $x[n] = (1/3)^n \sin(2\pi(1/8)n) u[n] \Rightarrow (1/3)^n \left(\frac{1}{2j} e^{j2\pi(1/8)n} - \frac{1}{2j} e^{-j2\pi(1/8)n}\right) u[n]$

$\rightarrow \frac{1}{2j}(1/3\, e^{j\pi/4})^n - (1/3\, e^{-j\pi/4})^n u[n]$

$X(z) = \frac{1}{2j}\left(\frac{z}{z-(1/3 e^{j\pi/4})}\right) - \frac{1}{2j}\left(\frac{z}{z-(1/3 e^{-j\pi/4})}\right)$



Problem 5

- For stability magnitude of poles <1
- To reject 0.06 samples/sec set 0's at $F=0.06$
- For Peaks at 0.3 set poles to $F=0.3$
- DC Gain of 1 means at $z=1$ $H(z)=1$

$$H(z)=\frac{(z-e^{j2\pi(0.06)})(z-e^{-j2\pi-(0.06)})}{(z-0.9e^{j2\pi(0.3)})(z-0.9e^{j2\pi-(0.3)})}$$

Top $= z^2-2\cos(2\pi(0.06))z+1$

Bottom $= z^2-1.8\cos(2\pi(0.3))z+.81$

so $Y(z)(1-1.8\cos(2\pi(0.3))z^{-1}+.81z^{-2}=X(z)(1-2\cos(2\pi(0.06))z^{-1}+z^{-2})$

$\rightarrow y[n]-1.8\cos(2\pi(0.3))y[n-1]+0.81y[n-2]$

$=$

$x[n]-2\cos(2\pi(0.06)x[n-1]+x[n-2]$

y coeffs $[1 \quad -1.8\cos(2\pi(0.3)) \quad 0.81]$

x coeffs $[1 \quad -2\cos(2\pi(0.06)) \quad 1]$

Problem 7

$$X(z) = \frac{5z^2 - 0.7z}{z^2 - 0.9z + 0.81} \qquad |z| > 0.9$$

a) using PFE

$$p = 0.9e^{j\pi/3} \qquad X(z) = \frac{5z - 0.7}{(z-p)(z-p^*)}z \Rightarrow \left(\frac{A}{z-p} + \frac{A^*}{z-p^*}\right)z$$

$$5z - 0.7 = A(z-p^*) + A^*(z-p)$$

for $z = p \Rightarrow 5p - .7 = A(p-p^*) \Rightarrow A\left(\frac{5p-0.7}{p-p^*}\right) = 2.5 - j0.4443$

$\Downarrow$

$2.6905e^{-j(.3785)}$

for $z = p^*$ we will obtain conjugate of $z = p$

so $2.5 + j0.4443 \Rightarrow 2.6905e^{j(.3785)}$

$$H(z) = z\left(\frac{2.6905e^{-j(.3785)}}{z - 0.9e^{j\pi/3}} + \frac{2.6905e^{j(.3785)}}{z - 0.9e^{-j\pi/3}}\right)$$

$$h[n] = z\left((2.69)(0.9)^n \cos\left(\tfrac{\pi}{3}n\right)\right)u[n]$$

b) $$\frac{5z^2 - 0.7z}{z^2 - 0.9z + 0.81} \rightarrow 5\left(\frac{1 - 0.14z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}\right)$$

$r = 0.9 \quad w_0 = \frac{\pi}{3}, \quad 0.9\sin\left(\frac{\pi}{3}\right) \approx 0.779$

$$\Rightarrow 5\left(\underbrace{\left(\frac{1 - 0.14z^{-1} - 0.31z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}\right)}_{0.9^n \cos\left(\frac{\pi}{3}n\right)u[n]} + \frac{0.31}{0.779}\underbrace{\left(\frac{0.779z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}\right)}_{0.9^n \sin\left(\frac{\pi}{3}n\right)u[n]}\right)$$

$$\therefore h[n] = 5(0.9)^n \cos\left(\tfrac{\pi}{3}n\right)u[n] + 1.989(0.9)^n \sin\left(\tfrac{\pi}{3}n\right)u[n]$$

c) $$\frac{5}{0.779z^{-1}}\left(\frac{0.779z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}\right) - \frac{0.7}{0.779}\left(\frac{0.779z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}\right)$$

$$h[n] = 6.418(.9)^{n+1}\sin\left(\tfrac{\pi}{3}n + 1\right)u[n+1] - 0.898(0.9)^n \sin\left(\tfrac{\pi}{8}n\right)u[n]$$

Table of Contents


```
%Ryan Plante
%DSP Homework 2
%14 February 2018
```

Question 1

```
%See paper
```

Question 2

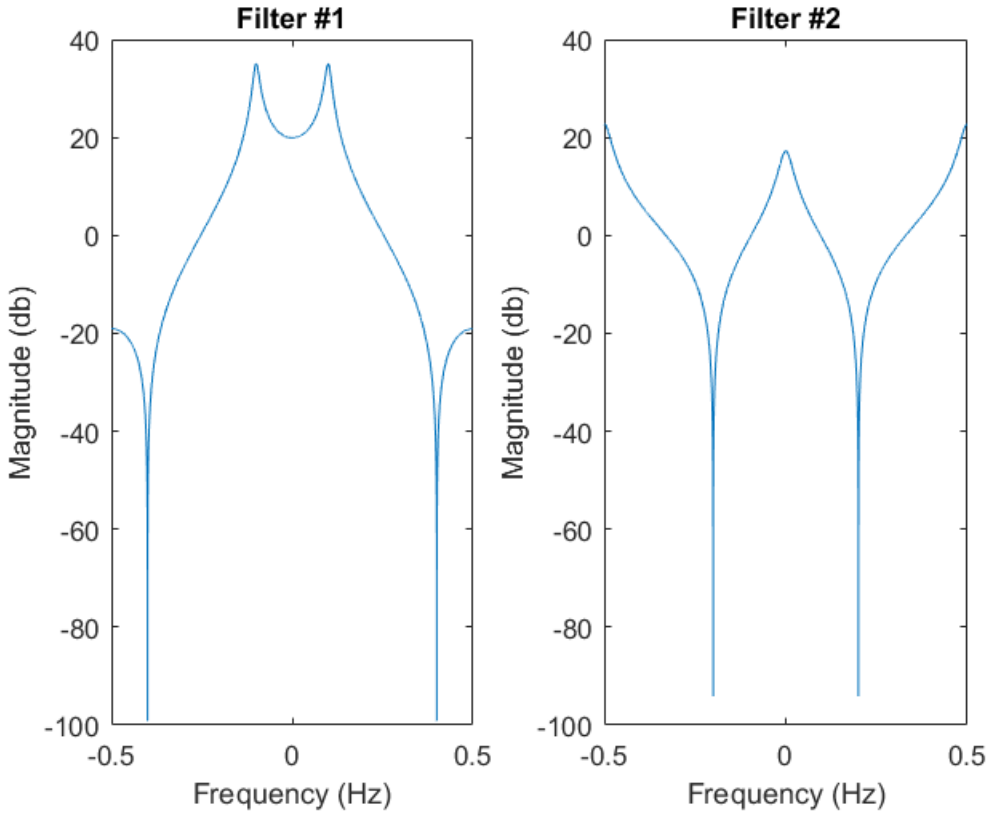
```
%See paper
```

Question 3

```
%Filter 1

%Declare Difference Eq Coefficients
a1 = [1 -1.5371 0.9025];
b1 = [1 1.6180 1];

%frequency array, ~10000 point resolution
f = -0.5:.0001:0.5;

z = exp(i*2*pi*f); %input

%Evaluate H(z) = Y(z)/X(z) at all points of Z
hz1 = polyval(b1, z) ./polyval(a1, z);
hz1db = 20*log10(hz1); %convert from magnitude to db
subplot(1,2,1);
plot(f, hz1db)
xlabel('Frequency (Hz)');
ylabel('Magnitude (db)')
title('Filter #1');
```

```
%Filter 2
a2 = [1 0 -0.81];
b2 = [1 -0.618 1];

hz2 = polyval(b2, z) ./polyval(a2, z);
hz2db = 20*log10(hz2);
subplot(1,2,2);
plot(f, hz2db)
xlabel('Frequency (Hz)');
ylabel('Magnitude (db)')
title('Filter #2');
```

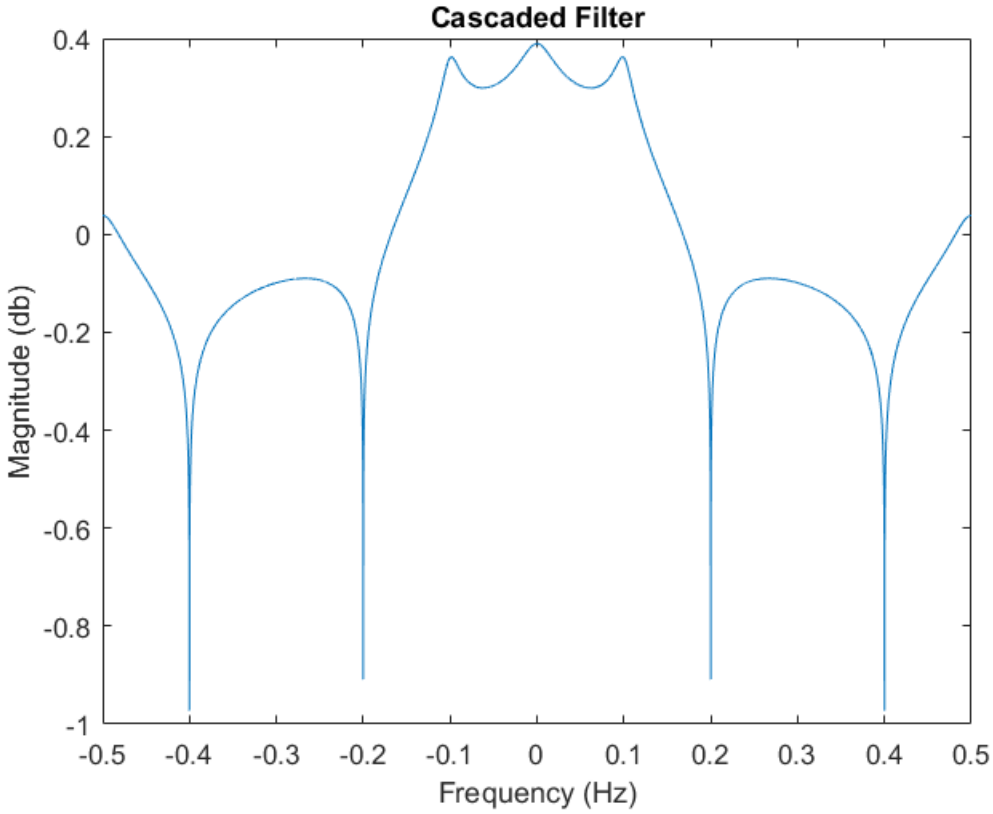
```
Warning: Imaginary parts of complex X and/or Y arguments ignored
Warning: Imaginary parts of complex X and/or Y arguments ignored
```



Question 4

```
%a
casc =(hz1db+hz2db); %cascade the two filters together
gain = 1/abs(max(casc)) %find value needed for max 1db gain

%b
cascadeWithGain = gain*casc; %apply our gain
figure(2)
plot(f,cascadeWithGain)
xlabel('Frequency (Hz)');
```

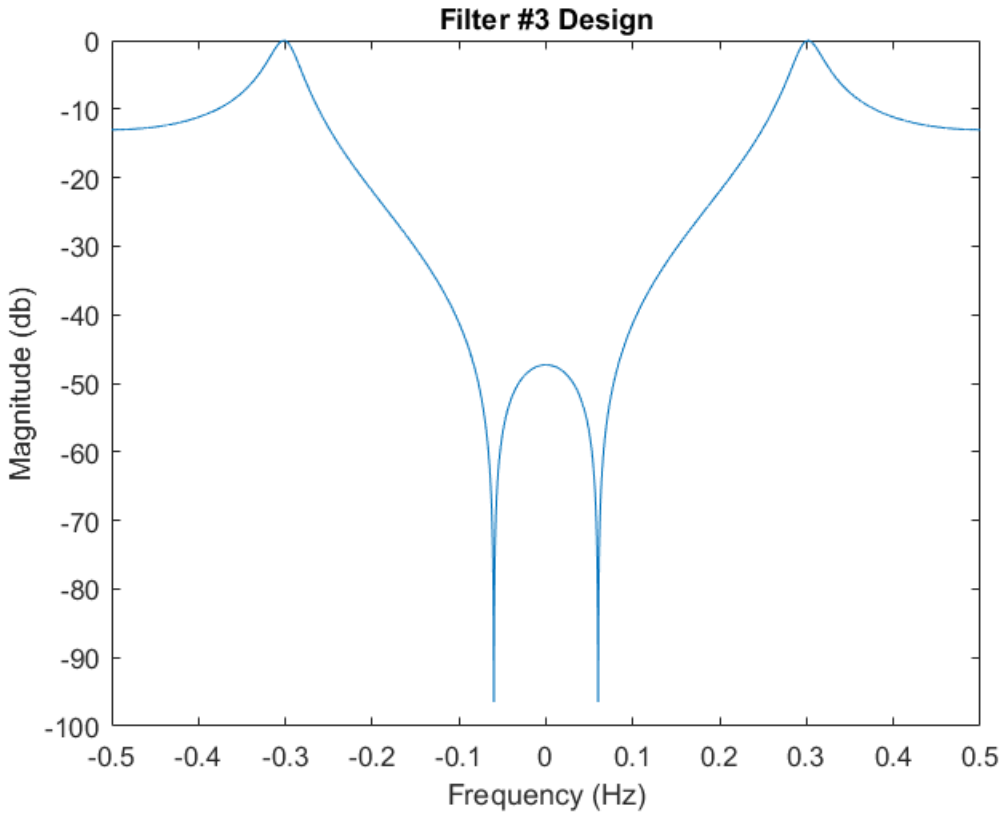
```
ylabel('Magnitude (db)');
title('Cascaded Filter');
%c
x = [0 0 0 0 0 0 0 0 3 1.5 -1.8 0.2 0 0 0 0 0 0 0 0]; %input array of
 20 values
stage1 = filter(b1, a1, x*gain) %output of stage one
stage2 = filter(b2, a2, stage1) %output of stage two


gain =

    0.0105

Warning: Imaginary parts of complex X and/or Y arguments ignored

stage1 =

  Columns 1 through 7

         0         0         0         0         0         0         0

  Columns 8 through 14

         0    0.0314    0.1150    0.1863    0.1700    0.0776   -0.0320

  Columns 15 through 20

   -0.1192   -0.1544   -0.1297   -0.0600    0.0248    0.0923


stage2 =

  Columns 1 through 7

         0         0         0         0         0         0         0

  Columns 8 through 14

         0    0.0314    0.0955    0.1722    0.2471    0.2984    0.2902

  Columns 15 through 20

    0.2198    0.1223    0.0245   -0.0352   -0.0479   -0.0116
```



Question 5

```
%See paper for derivation of coefficients
a = [1 -1.8*cos(2*pi*(0.3)) 0.81];
b = [1 -2*cos(2*pi*(0.06)) 1];

%Evaluate H(z) = Y(z)/X(z) at all points of Z
hz3 = polyval(b, z) ./ polyval(a, z);
gain2 = 1./abs(max(hz3)) %gain of 0 db
hz3 = 20*log10(hz3*gain2); %convert to db, add in gain
figure(3)
plot(f, hz3)
xlabel('Frequency (Hz)');
ylabel('Magnitude (db)');
title('Filter #3 Design');
```

```
gain2 =

    0.0726

Warning: Imaginary parts of complex X and/or Y arguments ignored
```



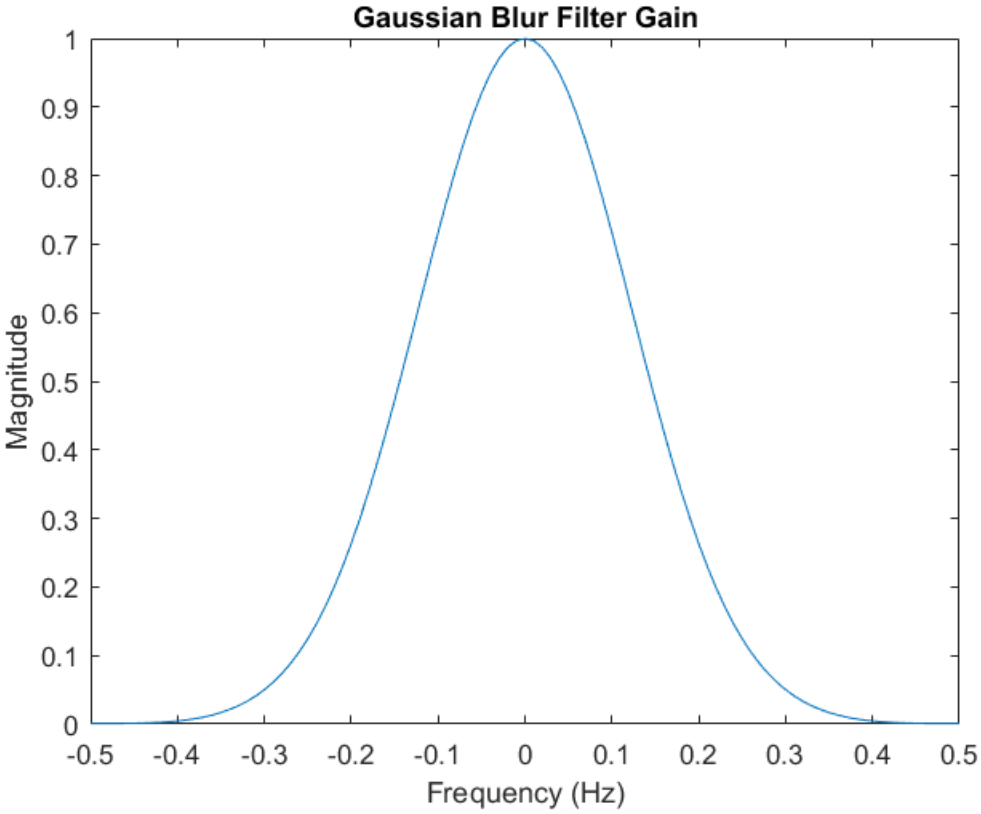
Question 6

```
clear
```

A

```
M = 4;
sigma = 1.3;
G = 1; %placeholder value

%ii
%compute sum of impulse response to find DC gain
sum = 0;
for n = -M:M
  sum = sum + exp(-0.5*(n/sigma)^2);
end

G = 1/sum %gain coefficient for DC gain of 1

%iii
f= (-0.5:0.01:0.5); %frequency, 100 samples
window = (-M:1:M); %window in which filter is non-zero

%define our filter symbolically
syms z freq n
```

```
h(n) = piecewise((n >= -M) & (n <= M), G*exp(-0.5*(n/sigma)^2),0);
H(z) = 0*z;
for i = -M:M
  H(z) = H(z) + h(i)*(z^-i);
end

%substitute our exponential in for z
H(freq) = subs(H(z), z, exp(1j*2*pi*freq));
%plot our filter gain
figure(4)
plot(f, abs(double(H(f))))
xlabel('Frequency (Hz)');
ylabel('Magnitude')
title('Gaussian Blur Filter Gain');

%iv
% Load the image, and convert to doubles for processing.
p1 = double(imread('Cavvy_bw.jpg'));

%calculate number of rows in image array
rows = size(p1(:,1));

%for all rows in image create new image processed through gaussian
 blur
%filter
for i= 1:rows
  p2(i,:) = conv(p1(i,:), double(h(window)));
end
figure(5)
%orignal image
imshow(p1,[0 255])
title('Original Image');

figure(6)
%gaussian blur processed image
imshow(p2,[0 255])
title('Gaussian Blur Filtered Image');


G =

    0.3070
```


Gaussian Blur Filter Gain
Magnitude
Frequency (Hz)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5


Original Image

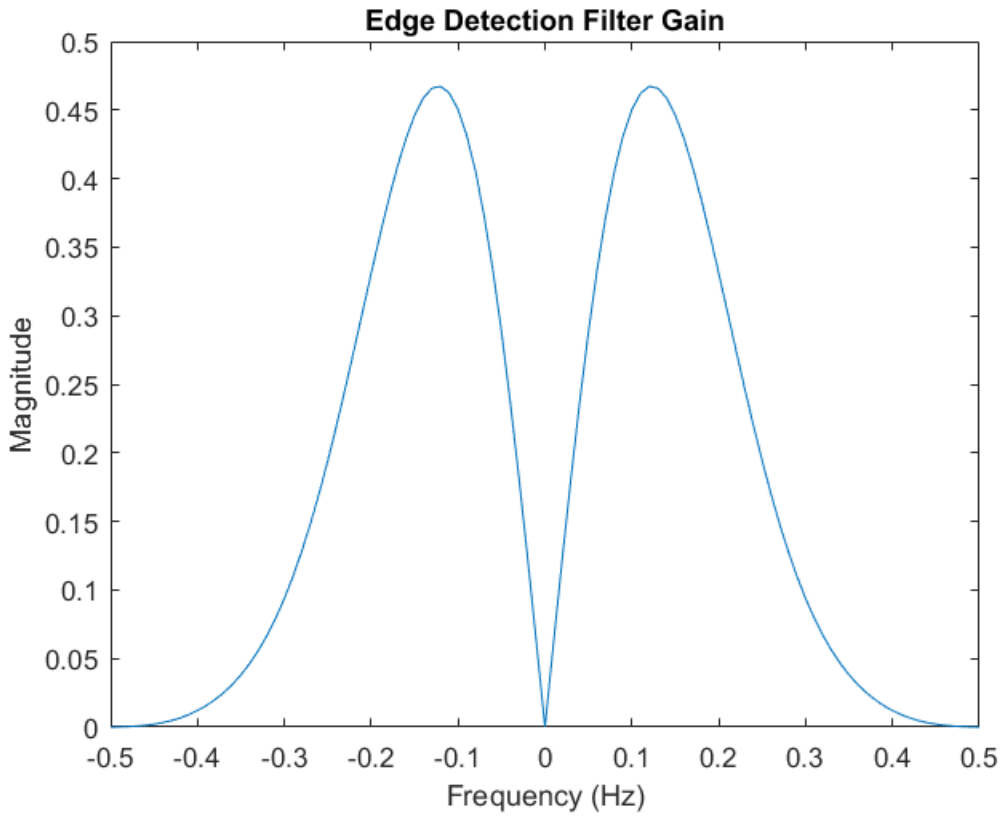
Gaussian Blur Filtered Image

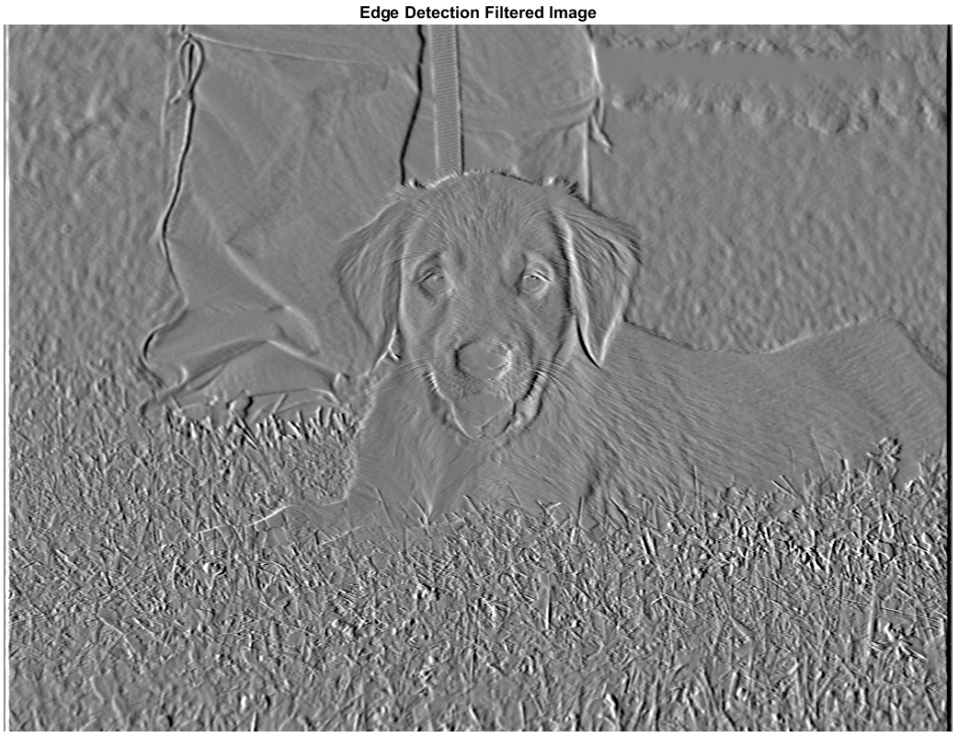
B

```
%define our edge detection filter
h2(n) = piecewise((n >= -M) & (n <= M), -G*(n/(sigma^2))*exp(-0.5*((n/
sigma)^2)),0);

%i
%once again find our filter gain
sum = 0;
for i = -M:M
  sum = sum + (i/(sigma^2))*exp(-0.5*((i/sigma)^2));
end
%create new gain coefficient
G = -1/sum

H(z) = 0*z;
for i= -M:M
  H(z) = H(z) + h2(i)*(z^-i);
end

%substitue our exponential in for z
H(freq) = subs(H(z), z, exp(1j*2*pi*freq));

%plot filter gain vs frequency
figure(7)
plot(f, abs(double(H(f))))
```

```
xlabel('Frequency (Hz)');
ylabel('Magnitude')
title('Edge Detection Filter Gain');

%ii
%for all rows in image create new image processed through edge
 detection
%filter
for i= 1:rows
  p3(i,:) = conv(p1(i,:), double(h2(window)));
end

figure(8)
%edge detection
imshow(p3, [-20 20])
title('Edge Detection Filtered Image');
```

```
G =

   7.7900e+15
```





Question 7

```
%See paper
```

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