Ablative Cooling Documentation

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1 Principles of Ablative Cooling

The ablative material protects the outer wall through several processes - by acting as a thermal insulator, undergoing an endothermic reaction, convecting heat towards the wall with the gaseous results of that reaction, and improving insulative properties of the inner-wall gas boundary layer. In this analysis, we assume that the latter two effects are ignorable to within our degree of safety.

The endothermic reaction that occurs is the well-known process of pyrolysis. Pyrolysis is the irreversible decomposition of a complex material at high temperatures. One example of this process is the transformation of wood into charcoal.

We wrote a one-dimensional time-incremented simulator for the thermal situation described above in python. The simulator is motivated by the following equations.

$$\frac{dT}{dt} = \alpha \frac{d^2 T}{dx^2} - \frac{H_{vap} \rho \frac{dV_p}{dt}}{V_p \rho c_p} \tag{1}$$

The first term on the right of the equality should be familiar as the heat transfer due to thermal diffusion. In this case, our alpha is variable, as the material properties vary with temperature and also due to the chemical reaction. In all following situations where material properties are referenced, they remain dependent upon temperature and the degree of decomposition.

The heat transfer due to this chemical reaction is encompassed in the second term. Here, the numerator defines the energy of decomposition, and the denominator transforms that energy into the resulting change in temperature. V_p is the volume fraction of the polymer present in each cell, and each cell is split into polymer, char, fiber, and gas.

In order to find dV_p, the rate of reaction, we use a first-order Arrhenius approximation.

$$J_p = A_0 \frac{V_p}{V_0} e^{\frac{-E_a}{RT}} \tag{2}$$

Where J_p is the reaction rate coefficient, A_0 is the rate constant, E_a is the activation energy, R the ideal gas constant, and T the temperature. The term $\frac{V_p}{V_0}$ represents the fraction of the total volume of a cell that is still in the polymer phase.

We can use this rate coefficient directly to find out the change in the polymer phase, and the consequent change in the fraction of the char phase. The second calculation requires knowing the gassification coefficient Γ of the reaction, which is just the fraction of mass that is transformed into gaseous products.

$$\rho_p \frac{dV_p}{dt} = -J_p \tag{3}$$

$$\rho_c \frac{dV_c}{dt} = J_p(1 - \Gamma) \tag{4}$$

Knowing the cell-wise operations that must be done, we can now write a full 1-d time discretized simulation of the thermal situation in the engine by vectorizing all the equations in the spatial dimension, and iterating over the process in the time dimension.
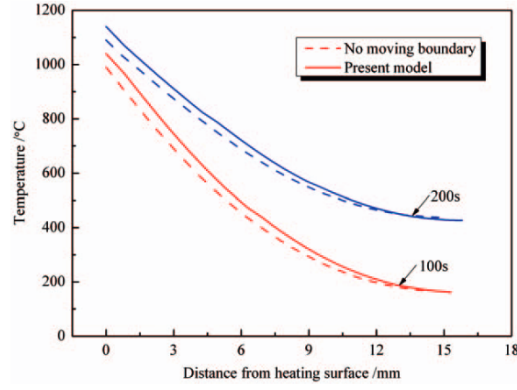
The results of the simulation are compared against Shi et. al. for identical material and heating situations.[1]

[1]Shengbo Shi et al. "Modeling of one-dimensional thermal response of silica-phenolic composites with volume ablation". In: *Journal of Composite Materials* 47 (Aug. 2013), pp. 2219–2235. DOI: 10.1177/0021998312454907.
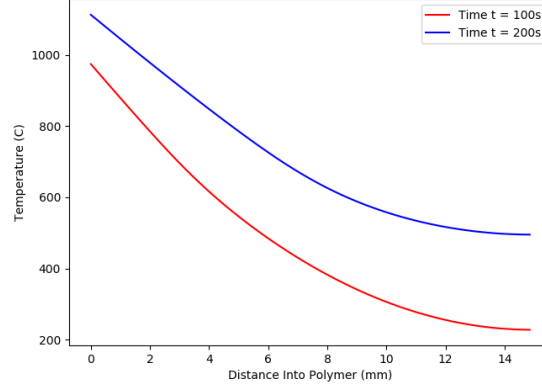
(a) Results from our work

(b) Results from [1]

Figure 1: Comparison of works

There are many differences between our methods. Shi et. al. takes thermal expansion of the material into account, likewise the actual effects of gassifi- cation such as pressure increase inside the pores of the material, as well as heat being transfered by that gas, and radiation from the material.

Despite these differences, we have found close agreement with [1]. We also find that one of our prior assumptions was confirmed to be valid. The heat convection from the gas moving inward towards the engine was shown to have little effect on the temperatures far from the reaction front. The reaction front, in the analyses of our engine, did not reach even halfway

through the material in our burn time, therefore this assumption had little effect on the outer-wall temperature, which is what we really care about from a cooling perspective.

We have not yet confirmed the effects of a changing boundary layer, and due to the complexity of the problem, it is unlikely that we will formulate correlations for it.

These results are for a strictly one-dimensional heat transfer situation, where each element is of equal volume and surface area. This is not the case in our engine, where the ablative layer is defined by a complex toroidal region.

To begin modelling our engine, we split it longitudinally into rectangular toroidal regions, which would each individually undergo simulation due to a changing heat transfer situation. Furthermore, we placed a normalized correction factor into the α matrix that pushed heat transfer outwards more. We also included the change in volume as one travels raidally outward from the center of the engine.

The heat transferred at each station down the length of the engine is motivated by the following equation.

$$Q = h_g(T_{aw} - T^0) \tag{5}$$

Since each station has different values of h_g and T_{aw}, we were essentially forced into this discretization.

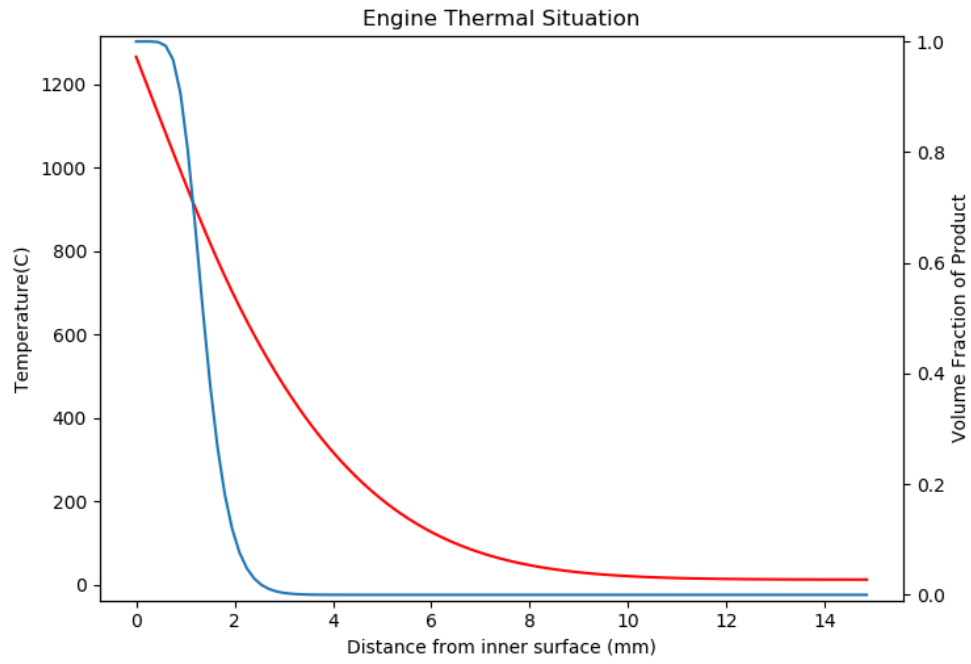Here are some example results for an engine we were considering.

Figure 2: Engine simulation with $h_g = 2000 \frac{W}{m^2 K}$, $T_{aw} = 2500K$, inner radius $= 4mm$