Lecture 12

Planning - IV - Sampling-based Planning

Course Logistics

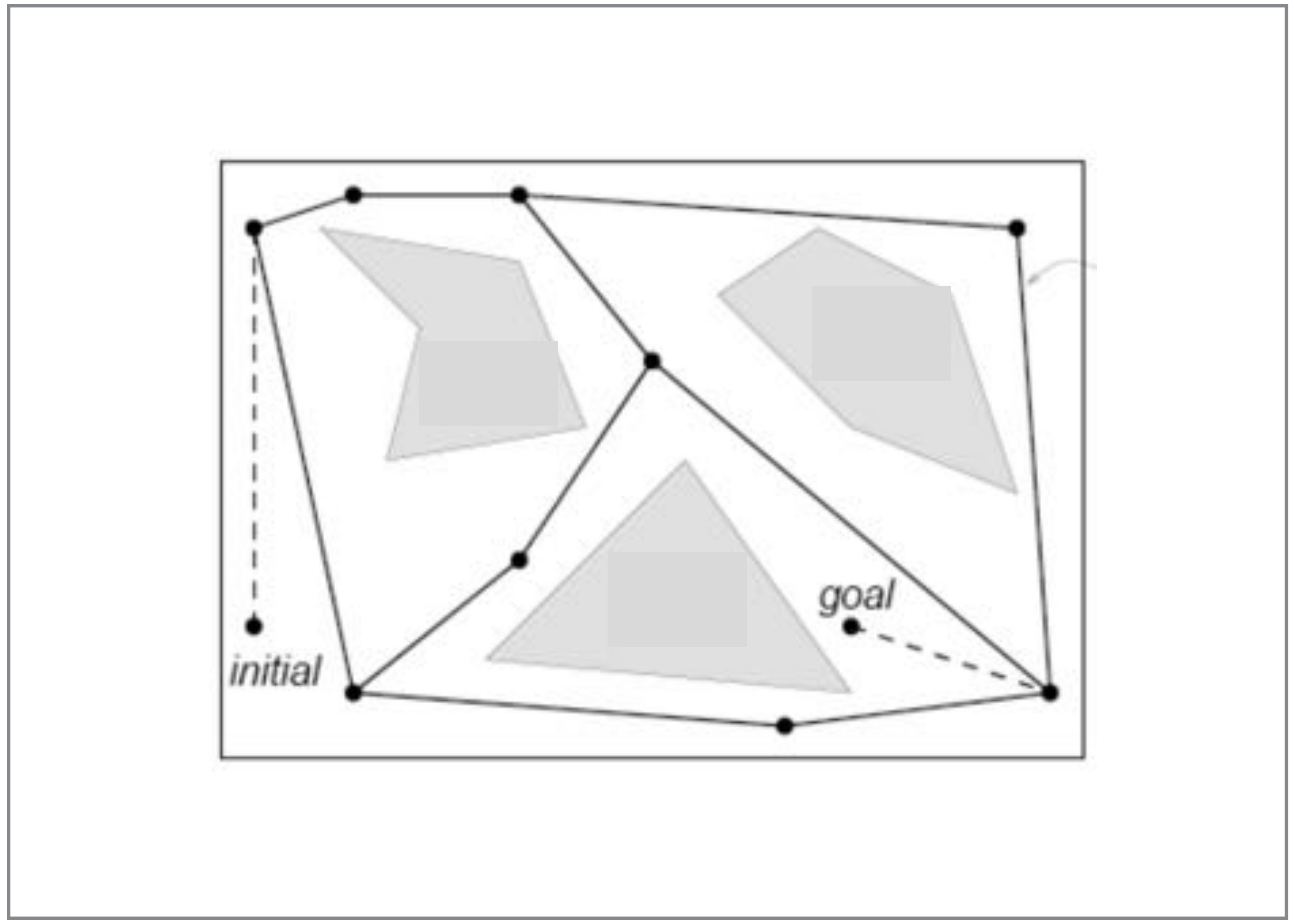
- **Quiz 10 was posted today and was due before the lecture.**
- Project 3 will be posted today 10/11 and will be due 10/25.

Approaches to motion planning

- Bug algorithms: Bug[0-2], Tangent Bug
- Graph Search (fixed graph)
    - Depth-first, Breadth-first, Dijkstra, A-star, Greedy best-first
- **Sampling-based Search (build graph):**
    - **Probabilistic Road Maps, Rapidly-exploring Random Trees**
- Optimization and local search:
    - Gradient descent, Potential fields, Simulated annealing, Wavefront

Roadmaps

Roadmap over geolocations



Roadmap over robot configurations

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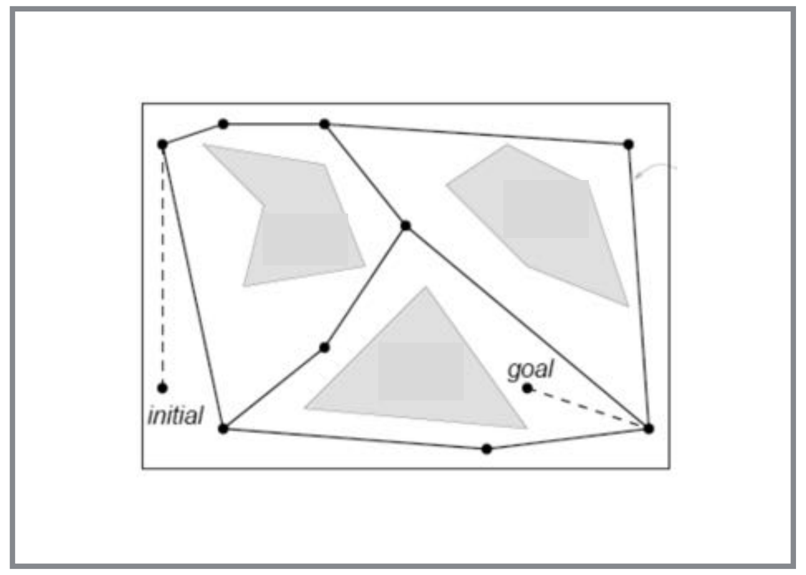
Roadmaps



- Graph search assumed C-space as a fixed uniform grid
    - finite set of discretized cells
- How does this scale beyond planar navigation?
    - curse of dimensionality
- Roadmaps are a more general notion of graphs in C-space

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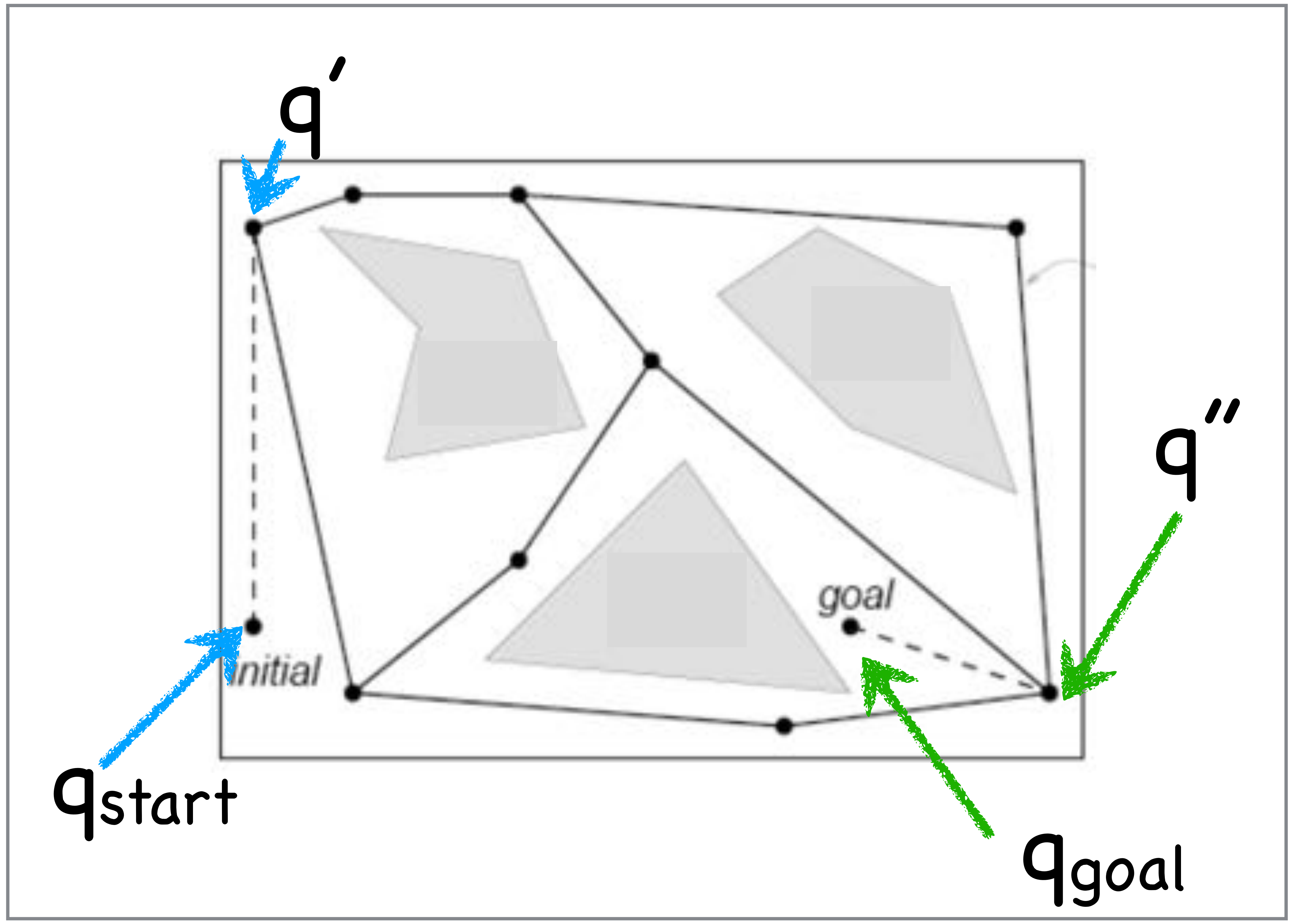
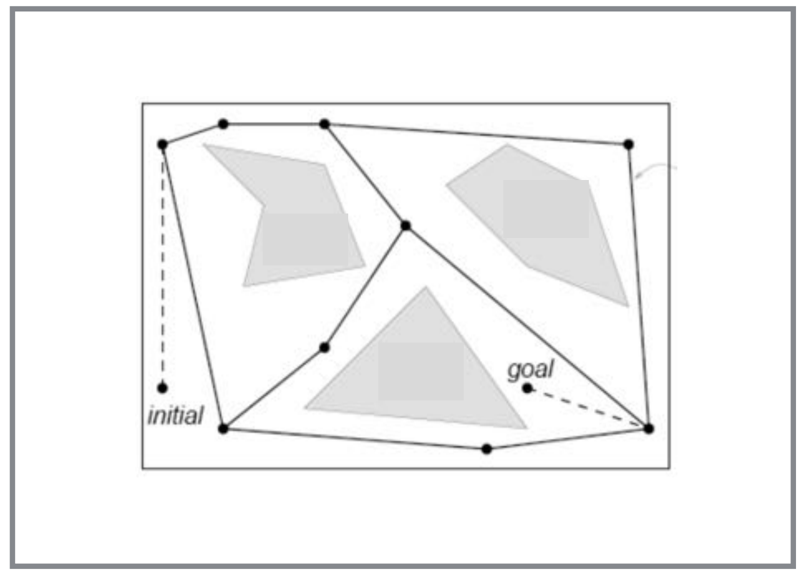
Roadmap Definition

- A roadmap *RM* is a union of curves s.t. all start and goal points in C-space (Q_{free}) can be connected by a path



- Roadmap properties:
  - **Accessibility**: There is a path from $q_{start} \in Q_{free}$ to some $q' \in RM$
  - **Departability**: There is a path from $q'' \in RM$ to $q_{goal} \in Q_{free}$
  - **Connectivity**: there exists a path in *RM* between q' and q''

Basic Roadmap Planner



1) Build the roadmap *RM* as graph $G(V,E)$

   V: nodes are "valid" in C-space in Q_{free}

   - a configuration q is valid if it is not in collision and within joint limits

   E: an edge $e(q_1,q_2)$ connects two nodes if a free path connects q_1 and q_2

   - all configurations along edge assumed to be valid

2) Connect start and goal configurations to *RM* at q' and q'', respectively

3) Find path in *RM* between q' and q''

How to build a roadmap?

How to build a roadmap?

2 Approaches

2 Approaches to Roadmaps

Deterministic:

complete algorithms

- Visibility Graph
  - trace lines connecting obstacle polygon vertices
- Voronoi Planning
  - trace edges equidistant from obstacles

Probabilistic:

C-space sampling

- Probabilistic Roadmap (PRM)
  - sample and connect vertices in graph for multiple planning queries
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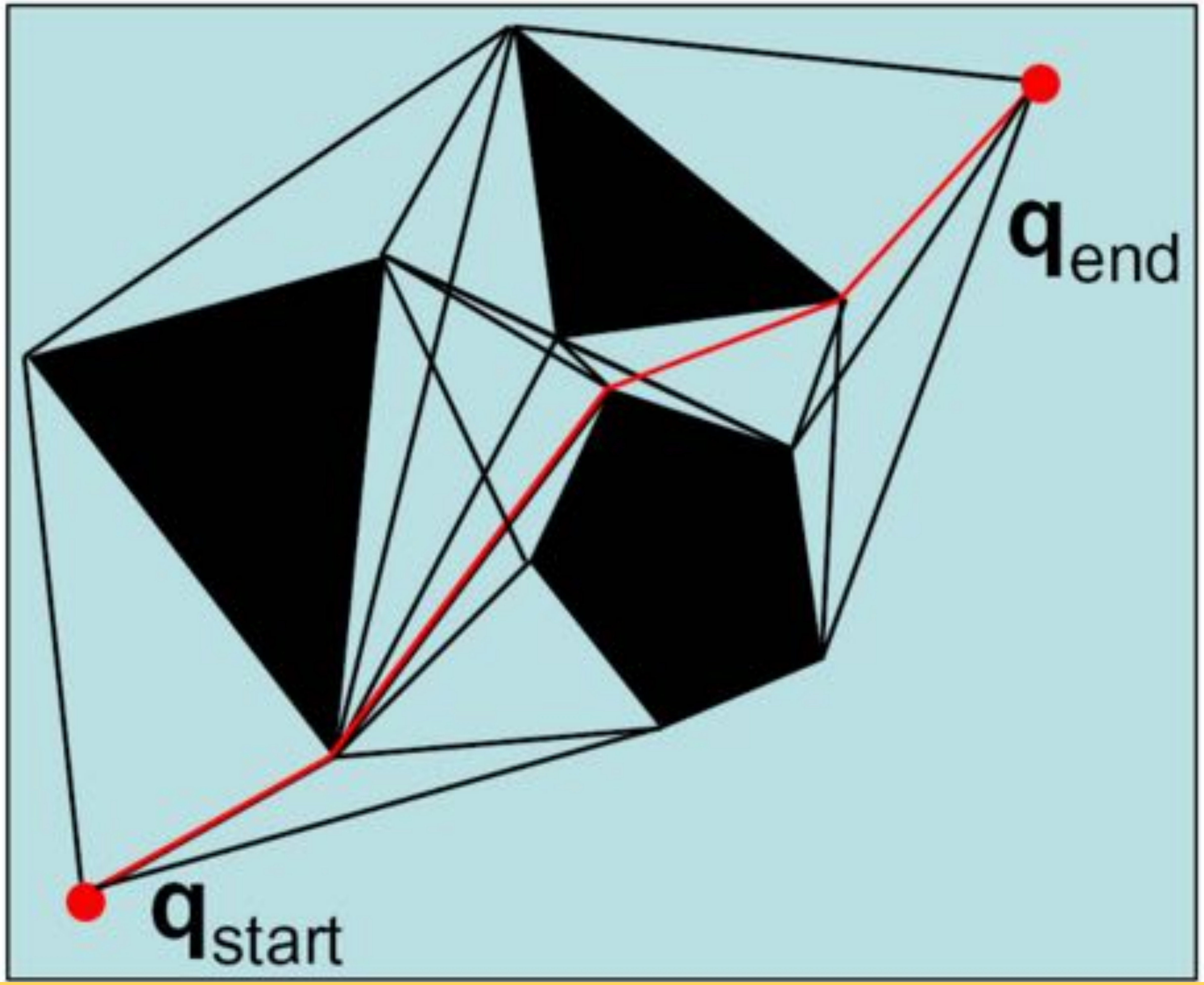

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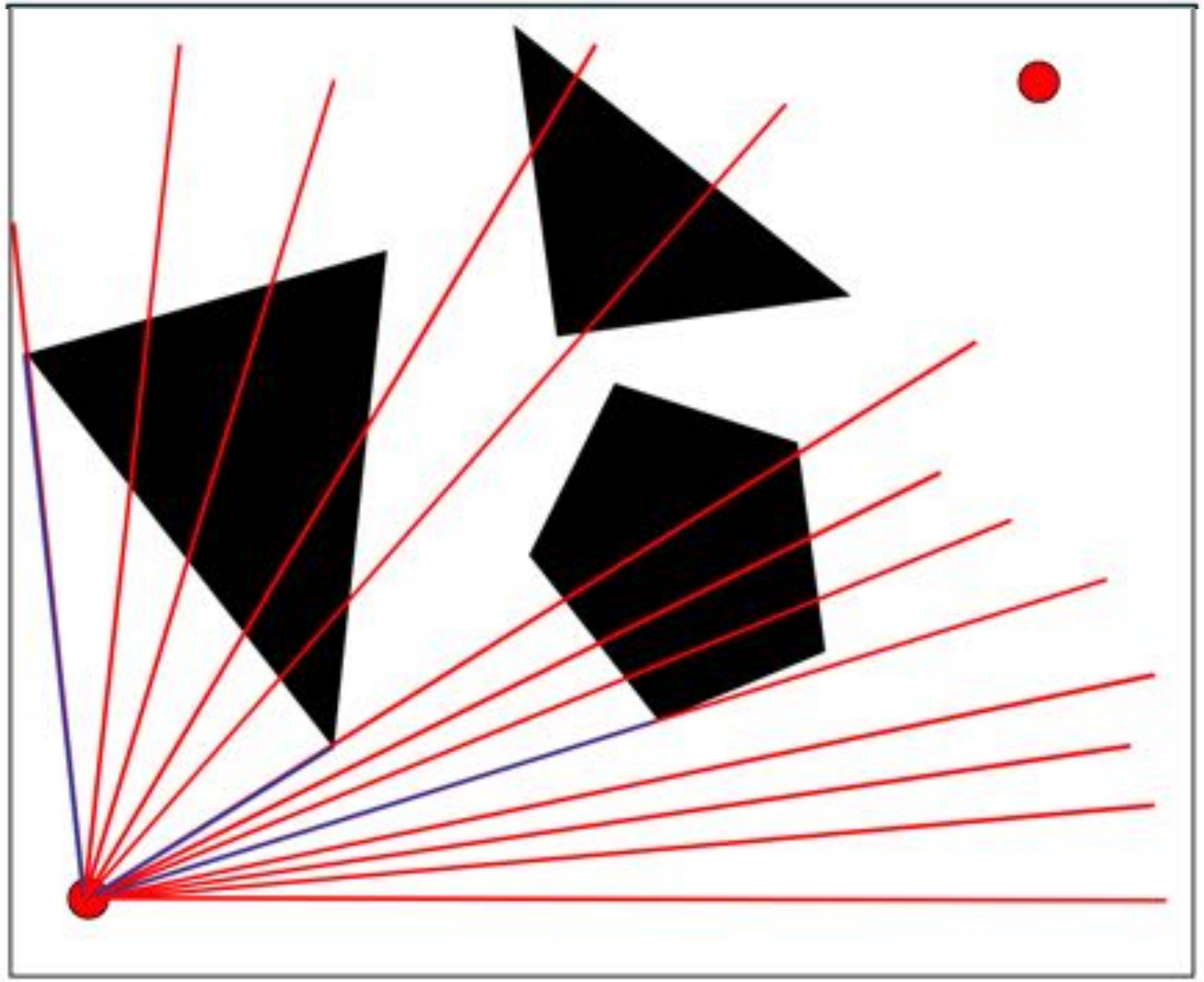

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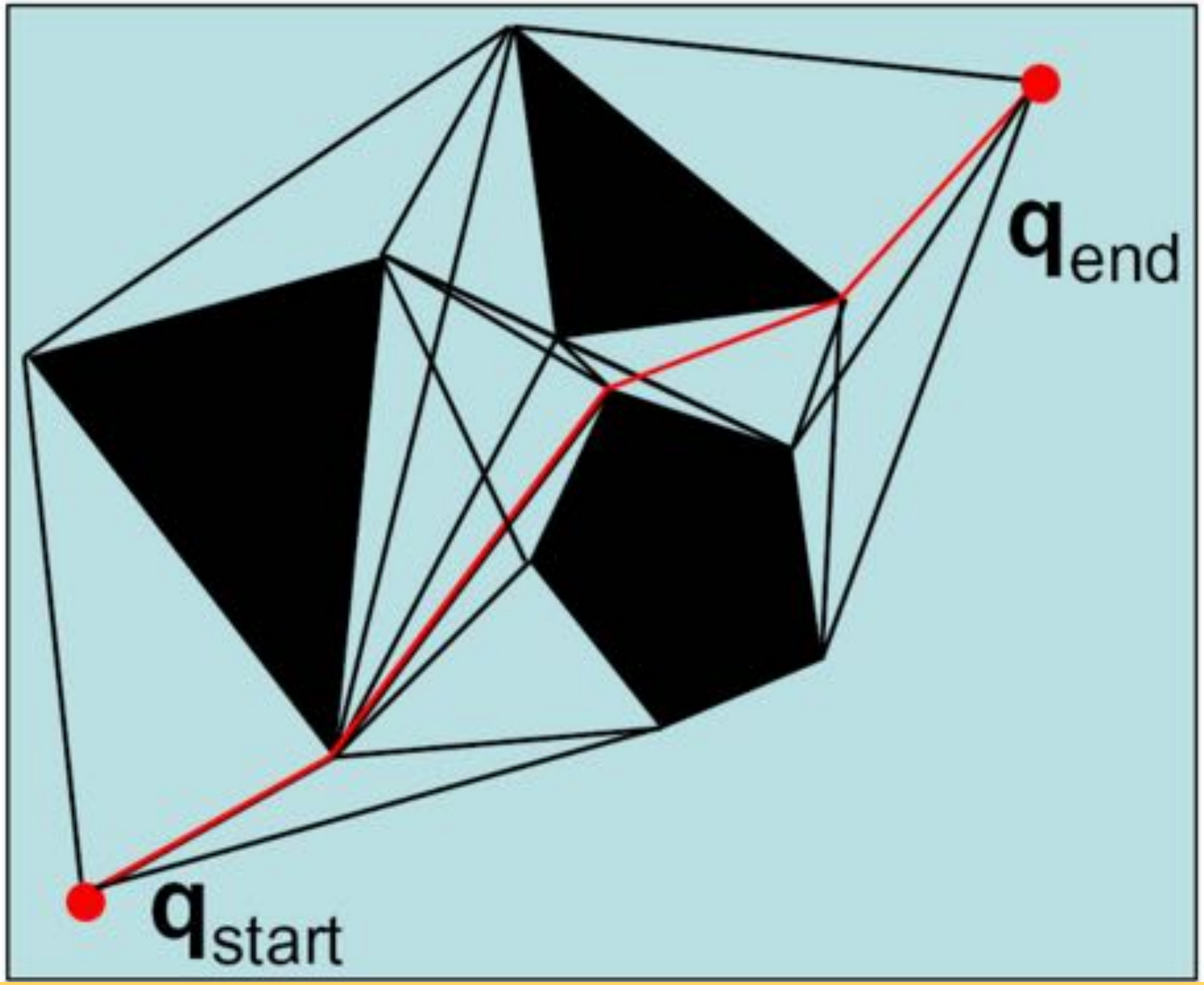

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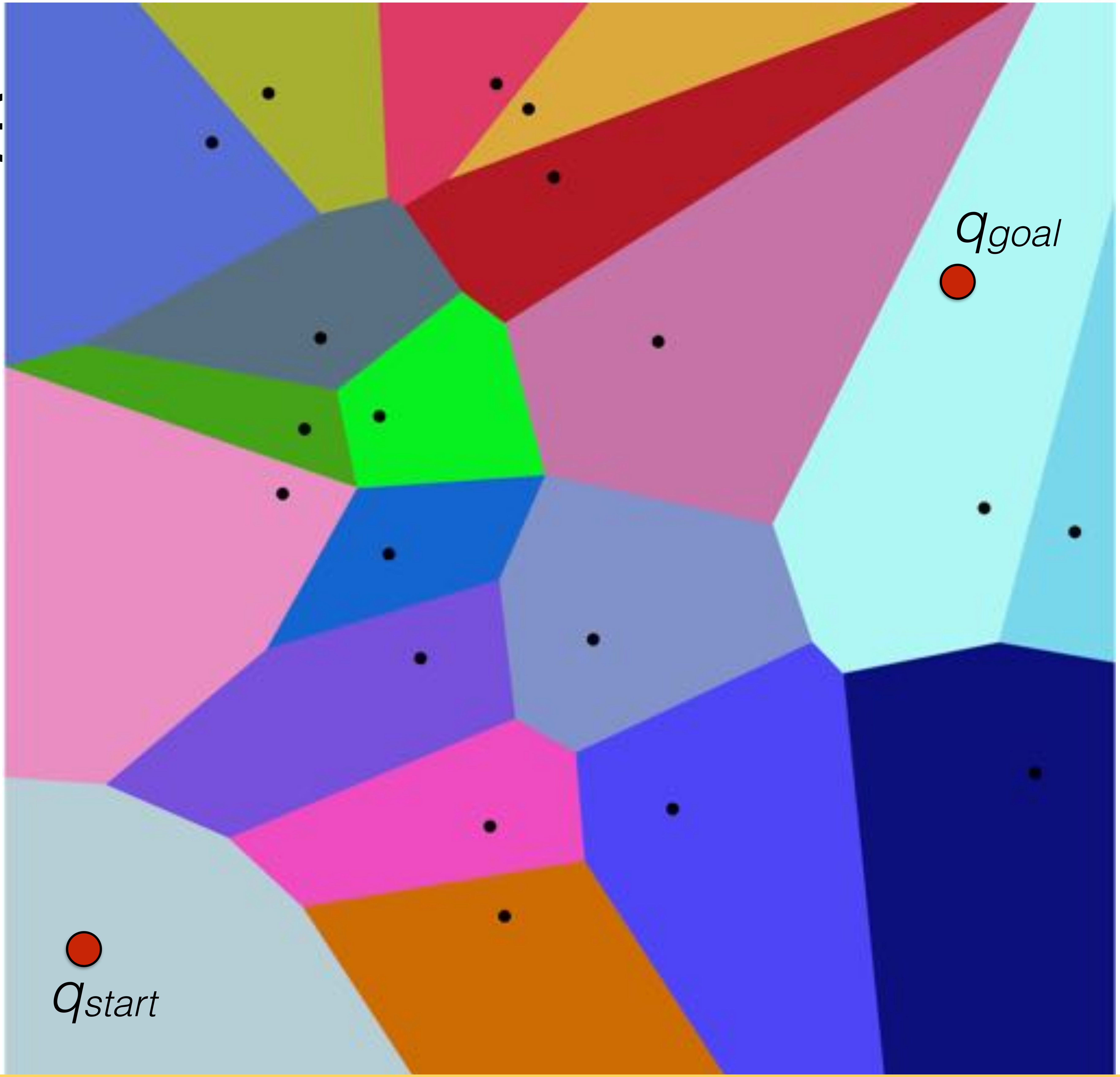

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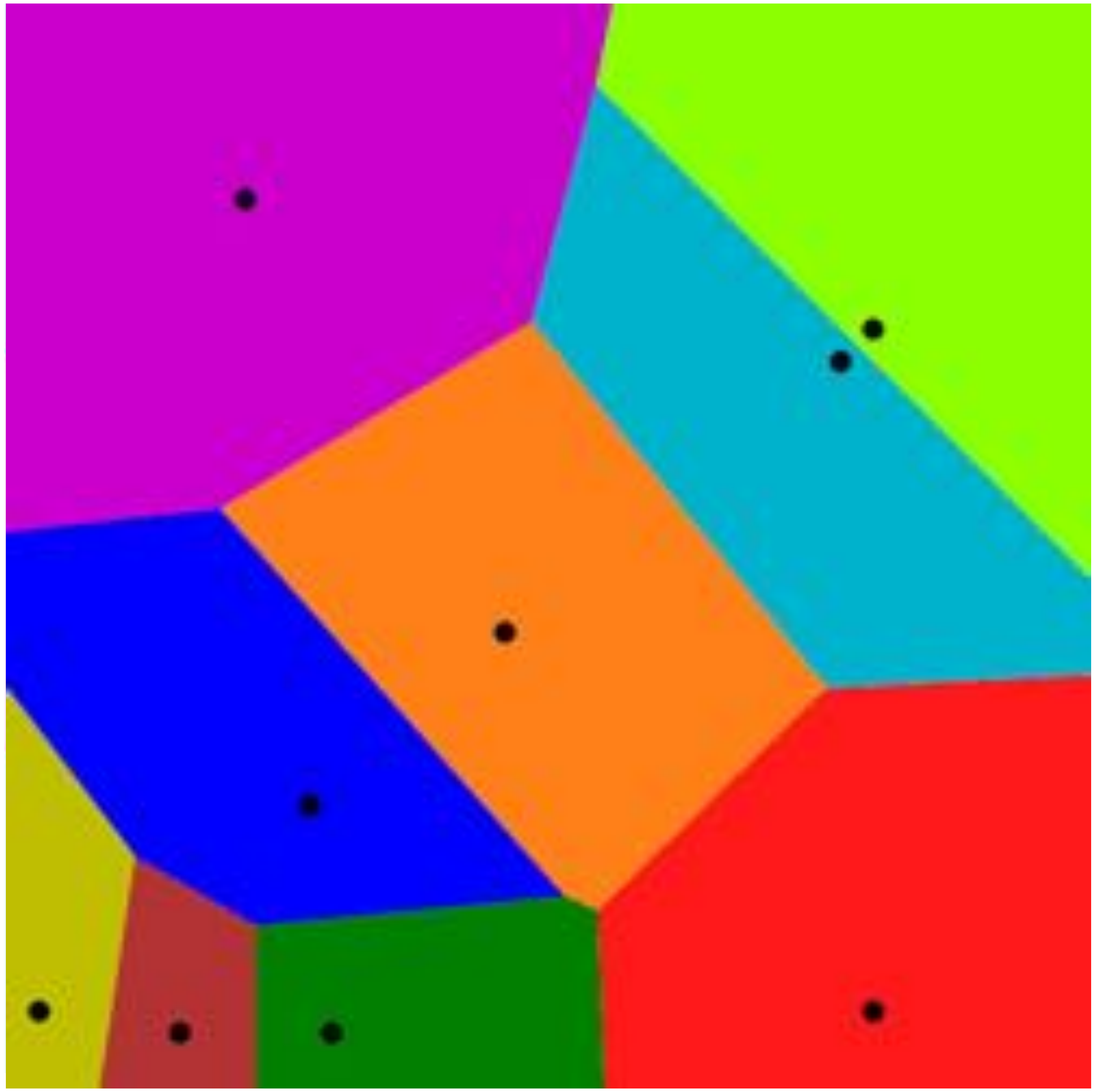
Voronoi Diagram

- Given *N* input points in a *d* dimensional space
- Find region boundaries such that each point on a boundary are equidistant to two or more input points
- Delaunay triangulation is a dual to the Voronoi diagram



https://en.wikipedia.org/wiki/Voronoi_diagram#/media/File:Voronoi_growth_euclidean.gif

2 Approache

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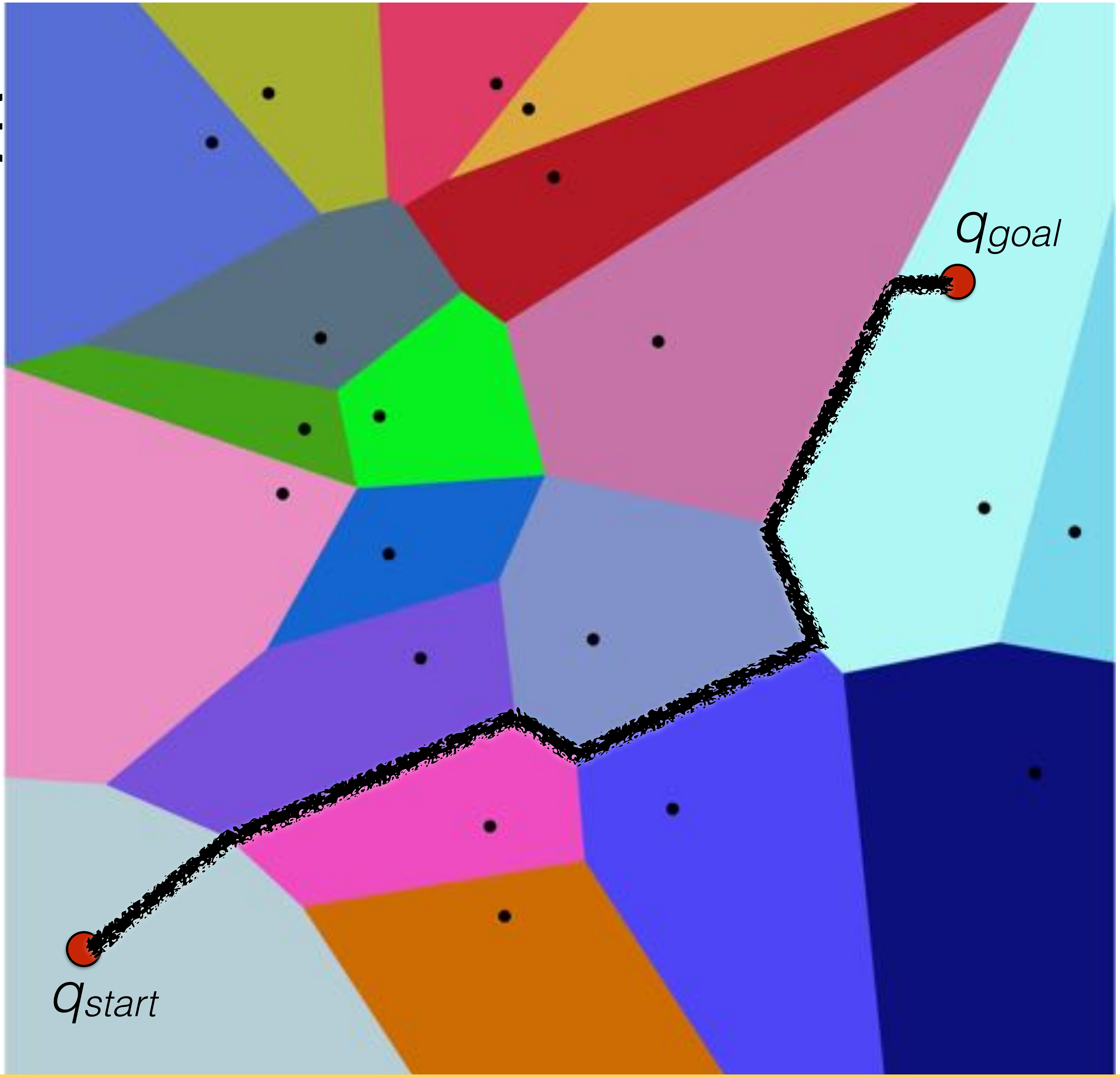

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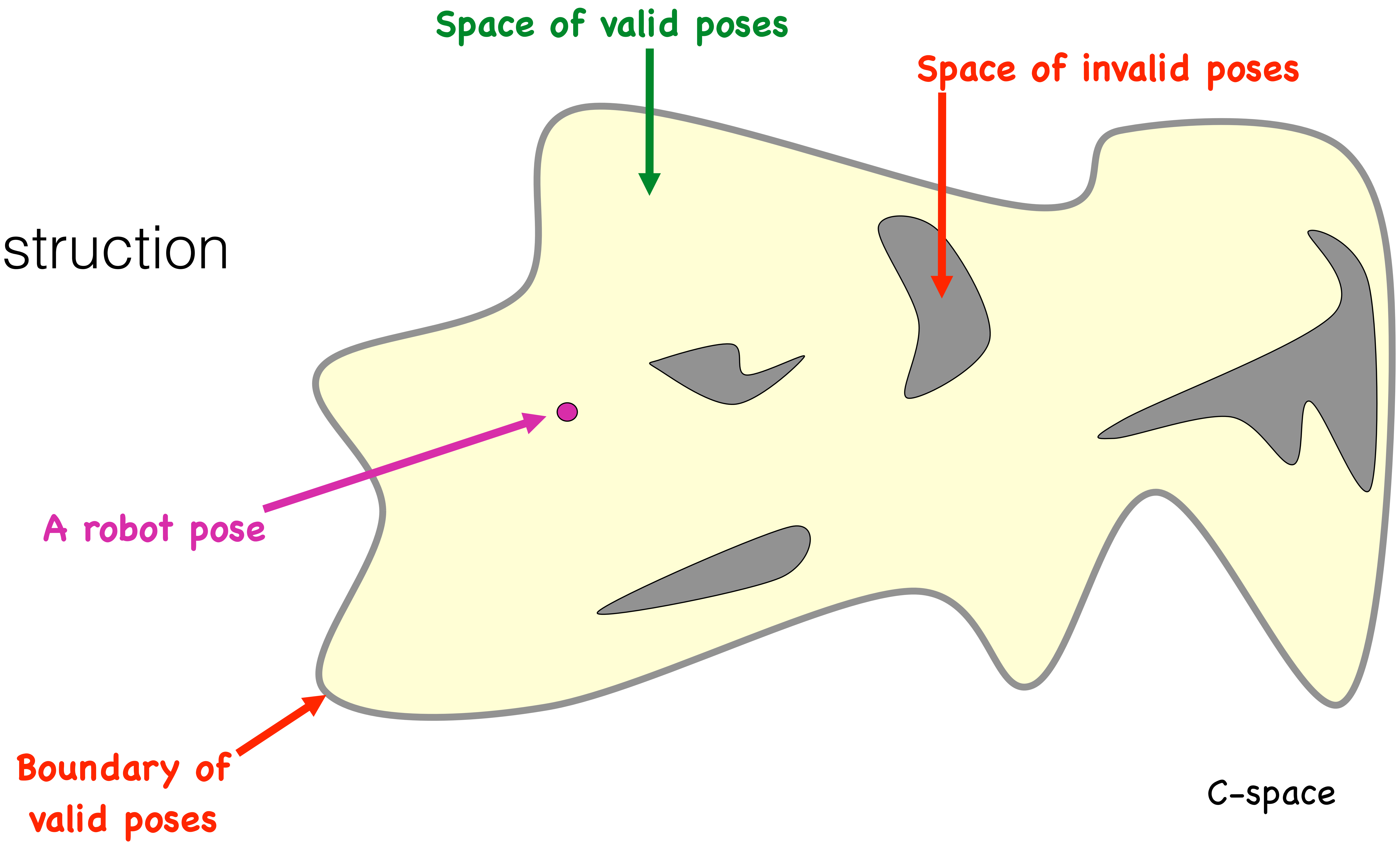
Probabilistic road maps

- Two phases
  - Roadmap construction
  - Path Query

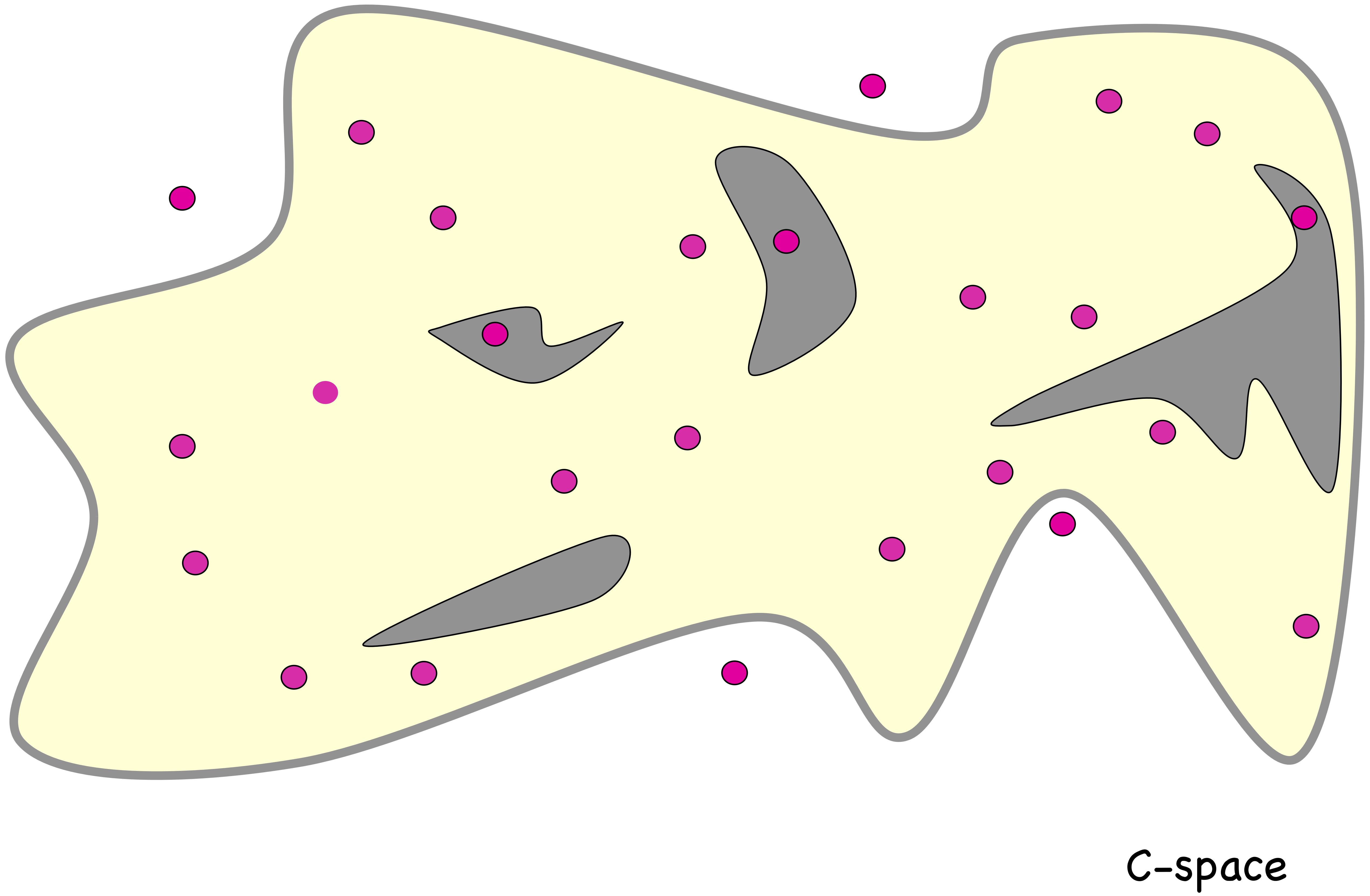


PRM: construction phase

1) **Select N sample poses at random**

2) Eliminate invalid poses

3) Connect neighboring poses



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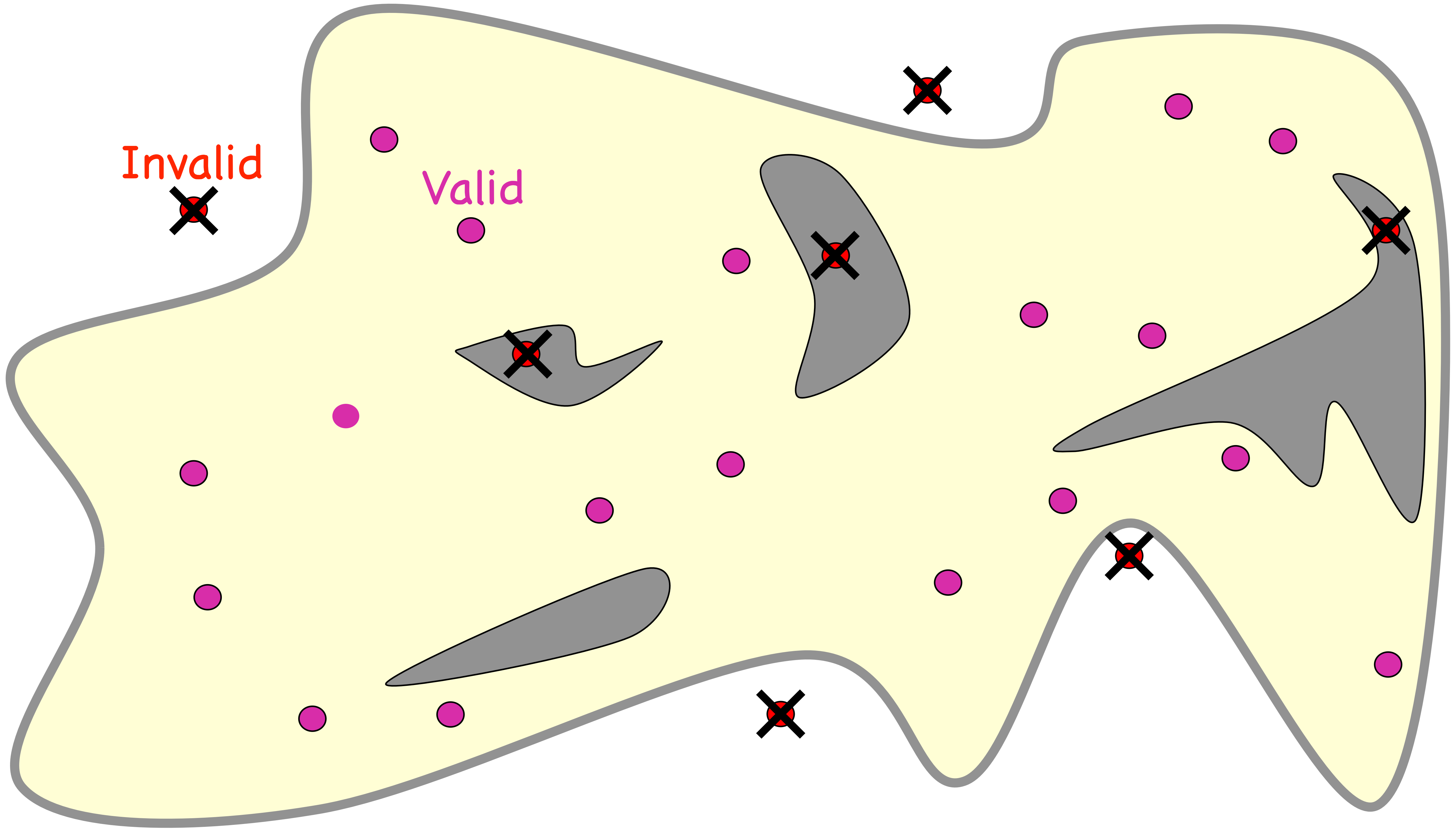

Collision detection will be covered later

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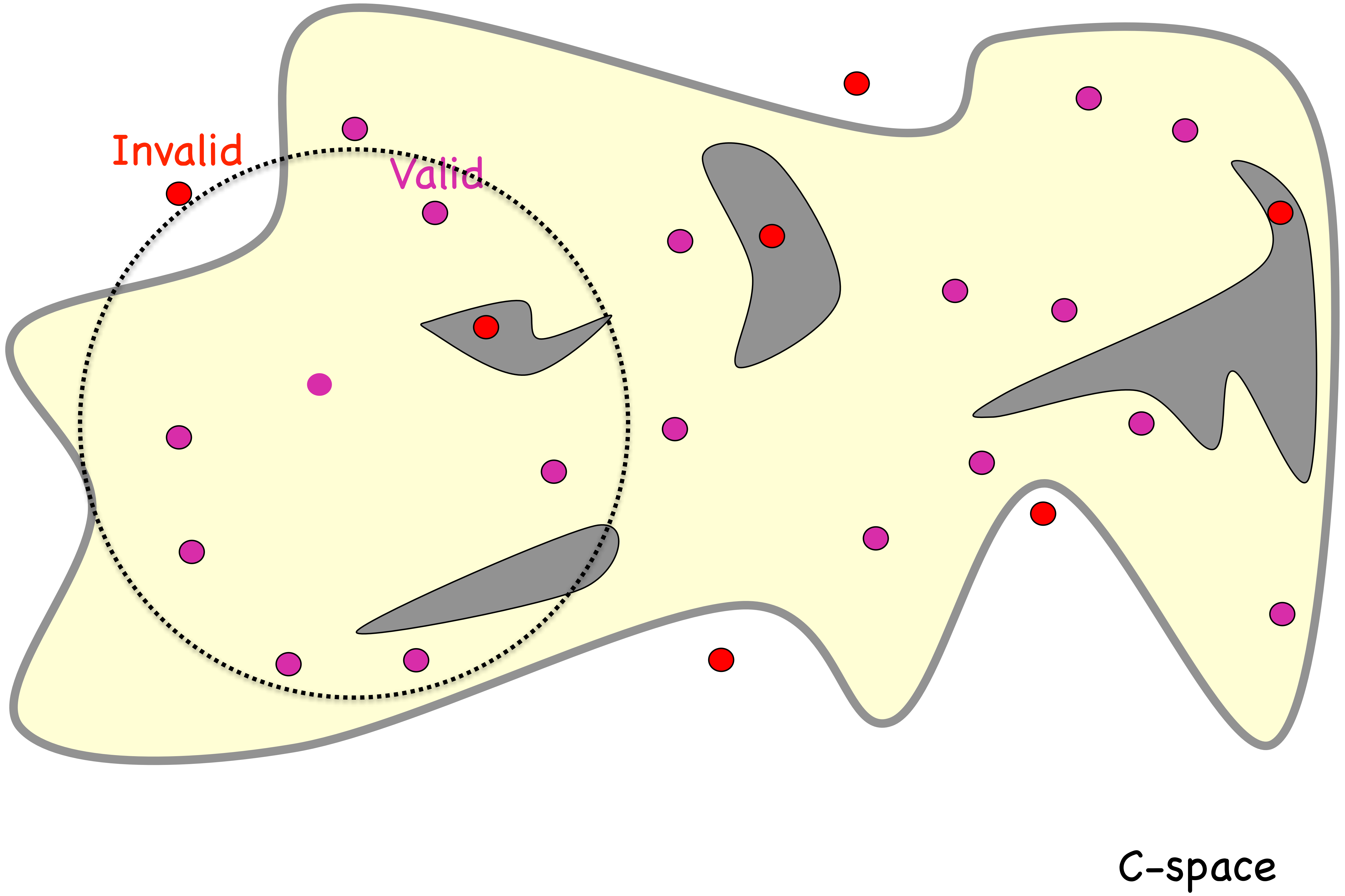

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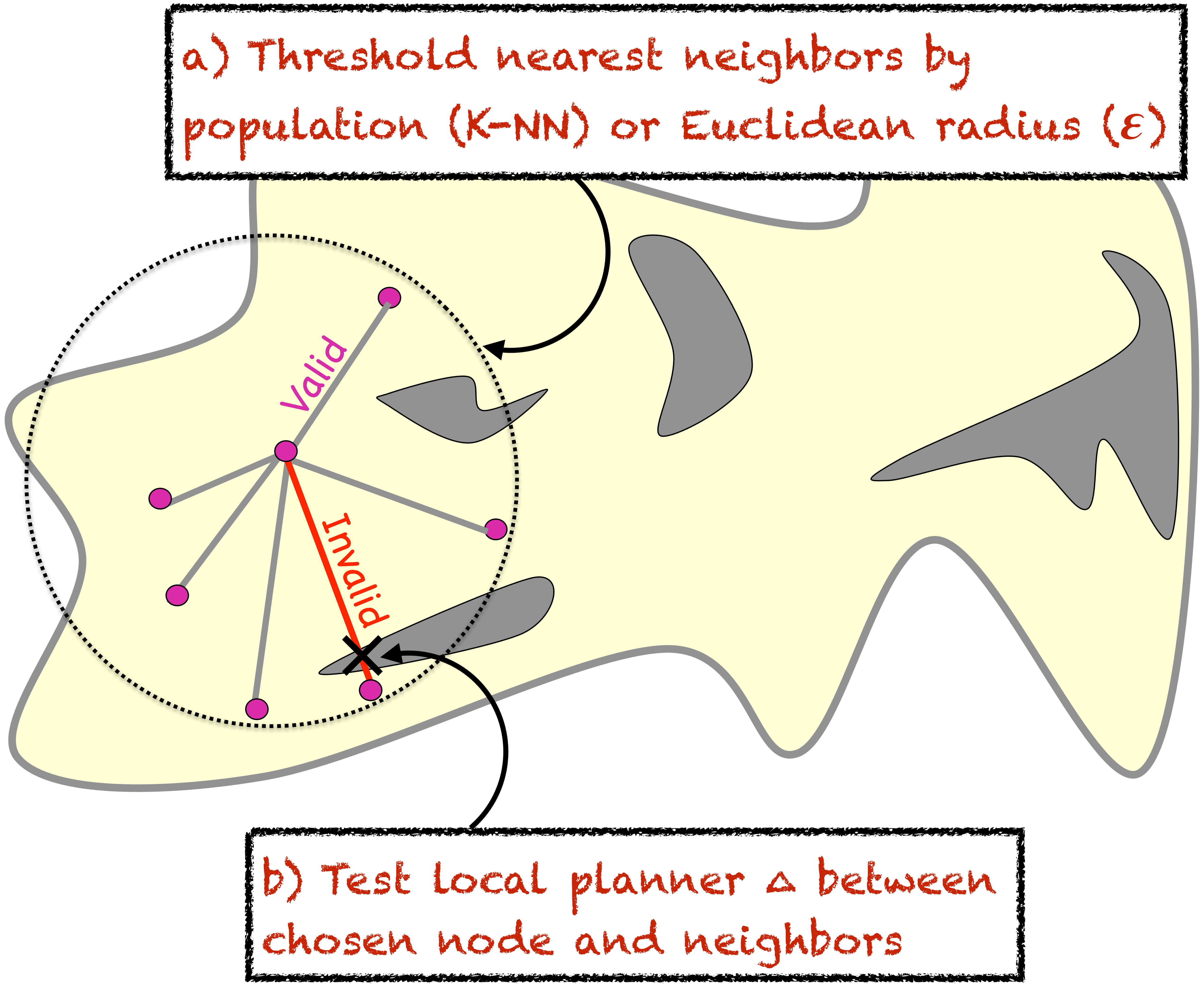


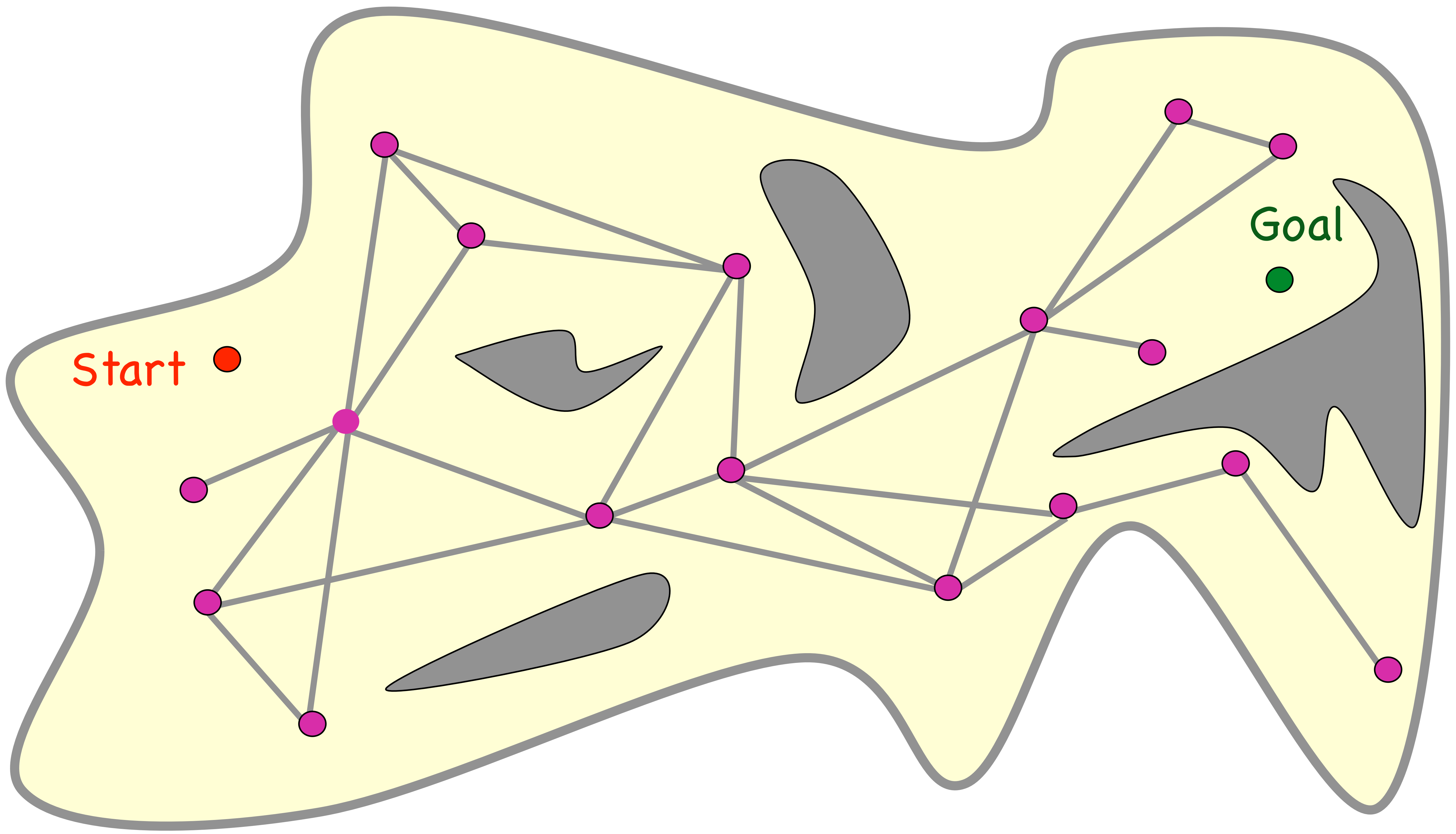
PRM: query phase

1) **Given constructed roadmap, start pose, and goal pose**

2) Attach goal and start to nearest roadmap entry nodes

3) Search for path between roadmap entry nodes

4) Return path with entry and departure edges



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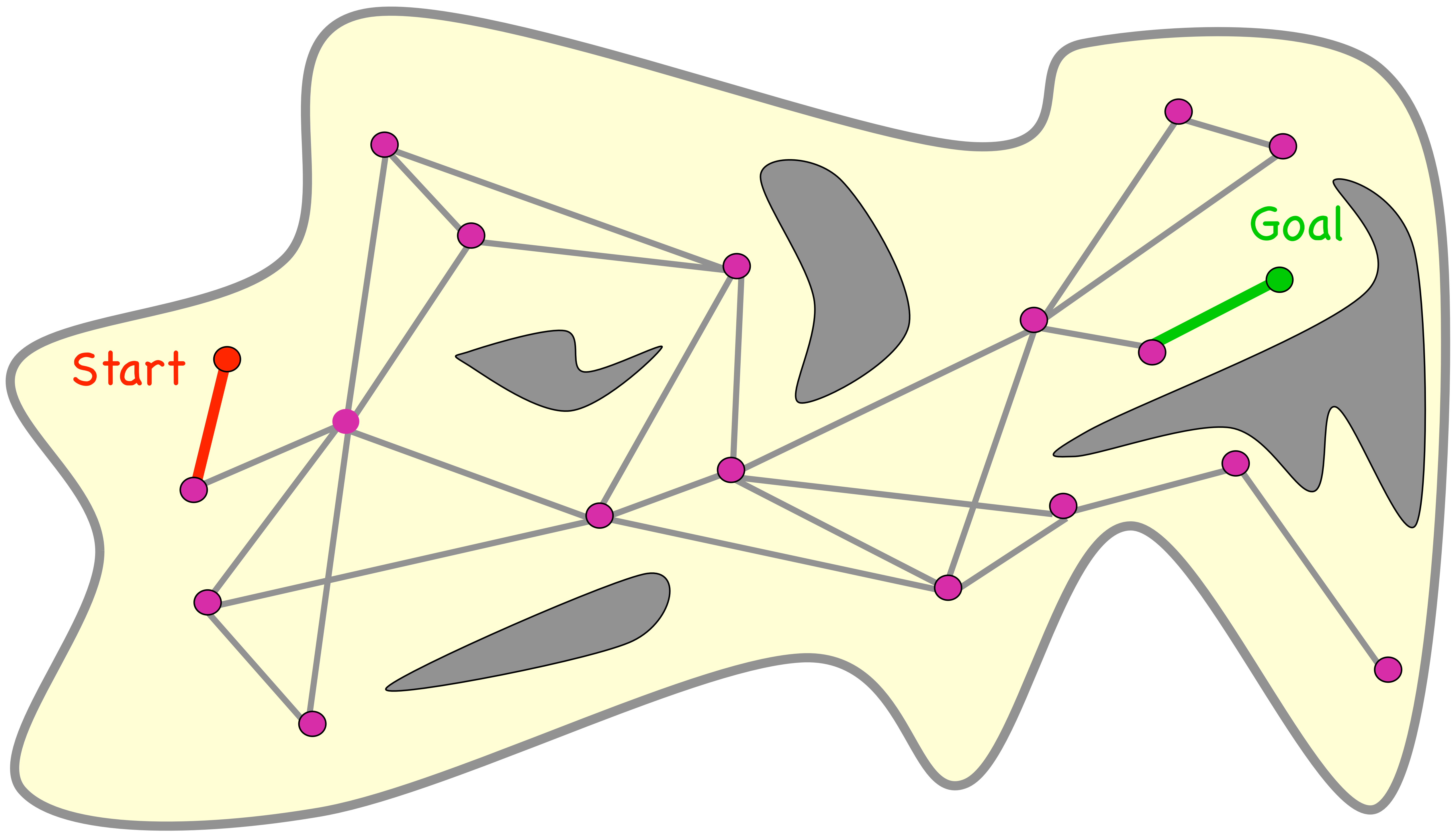

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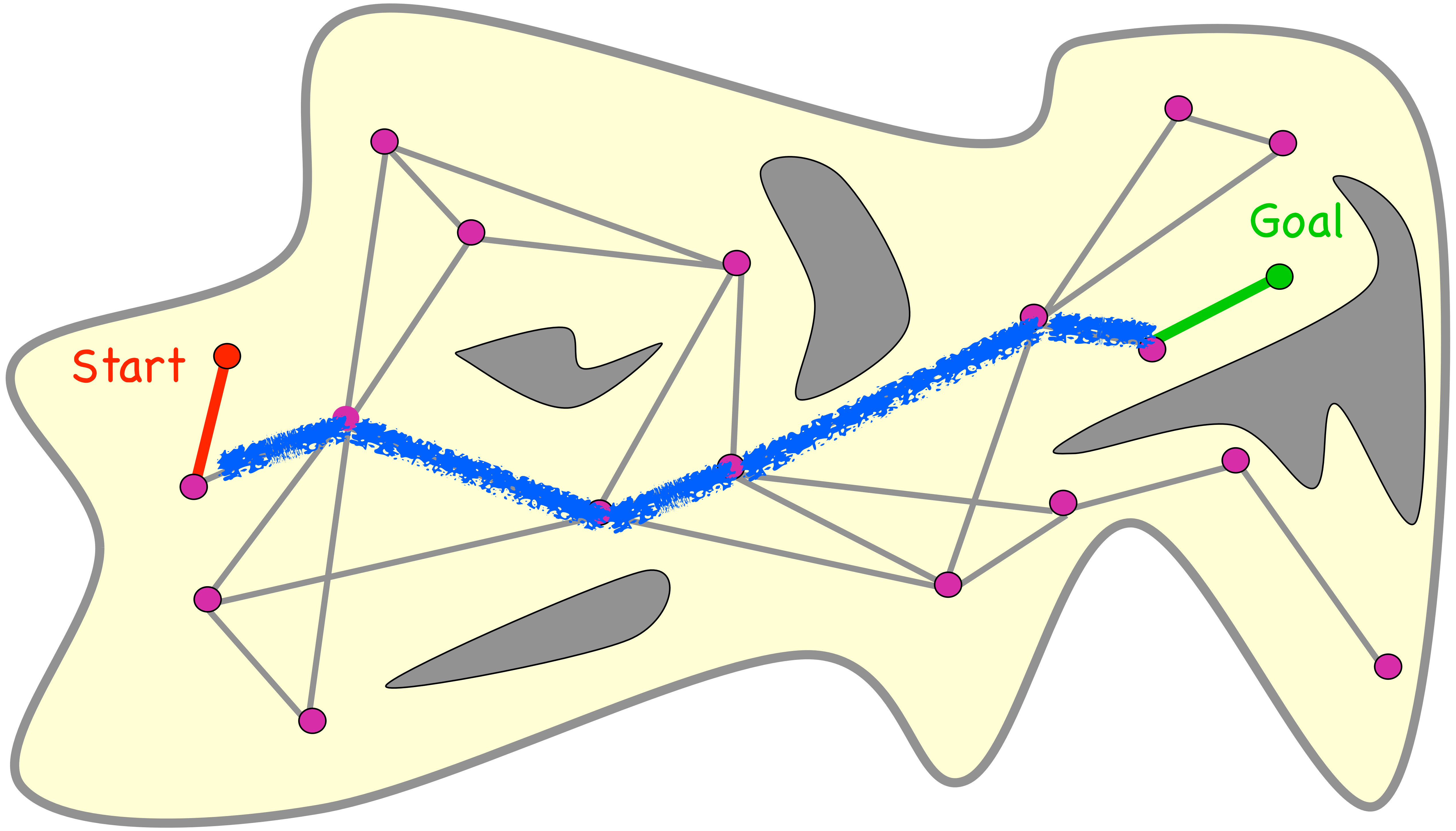

Remember: graph search algorithms
A*, Dijkstra, BFS, DFS

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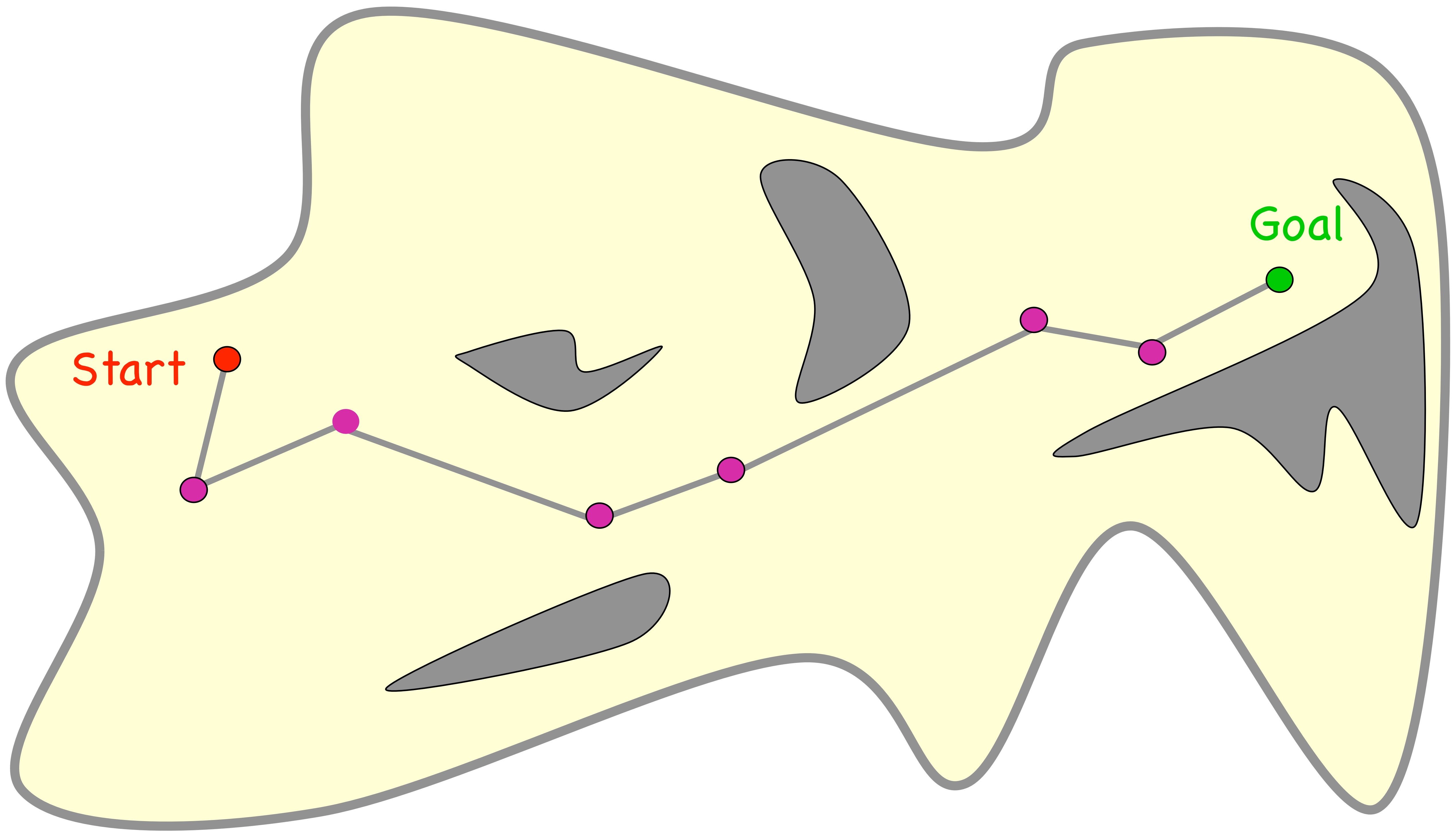

Multi-query planning: Considerations

- Number of samples wrt. C-space dimensionality
- Balanced sampling over C-space
- Choice of distance (e.g., Euclidean)
- Choice of local planner (e.g., line subdivision)
- Selecting neighbors: (e.g., K-NN, kd-tree, cell hashing)

2 Approaches to Roadmaps

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Single Query Planning

- Given specific start and goal configurations
- Grow trees from start and goal towards each other
- Path is found once trees connect
- Focus sampling in unexplored areas of C-space and moving towards start/goal
- Common algorithms:
  - ESTs (expansive space trees)
  - **RRTs (rapidly exploring random trees)**

RRT Algorithm

RRT Algorithm

Extend graph towards a random configuration and repeat

BUILD_RRT(q_{init})
$\mathcal{T}$.init(q_{init});
for $k = 1$ **to** K **do**
    $q_{rand} \leftarrow$ RANDOM_CONFIG();
    EXTEND($\mathcal{T}, q_{rand}$);
Return $\mathcal{T}$

RRT Algorithm

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Figure 3: The EXTEND operation.

[Kuffner, LaValle 2000]

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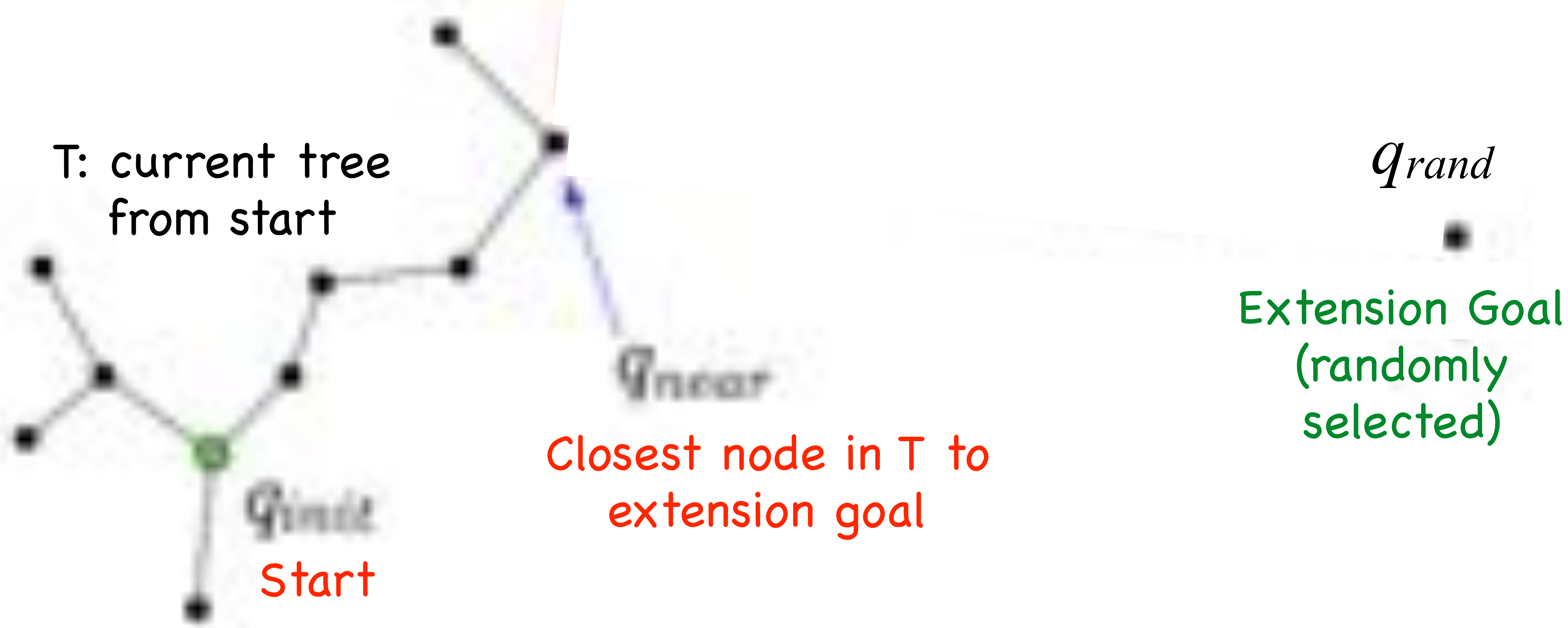
  if NEW_CONFIG(q, q_near, q_new) then
      T.add_vertex(q_new);
      T.add_edge(q_near, q_new);
      if q_new = q then
          Return Reached;
      else
          Return Advanced;
  Return Trapped;
```

Extend graph towards a random configuration



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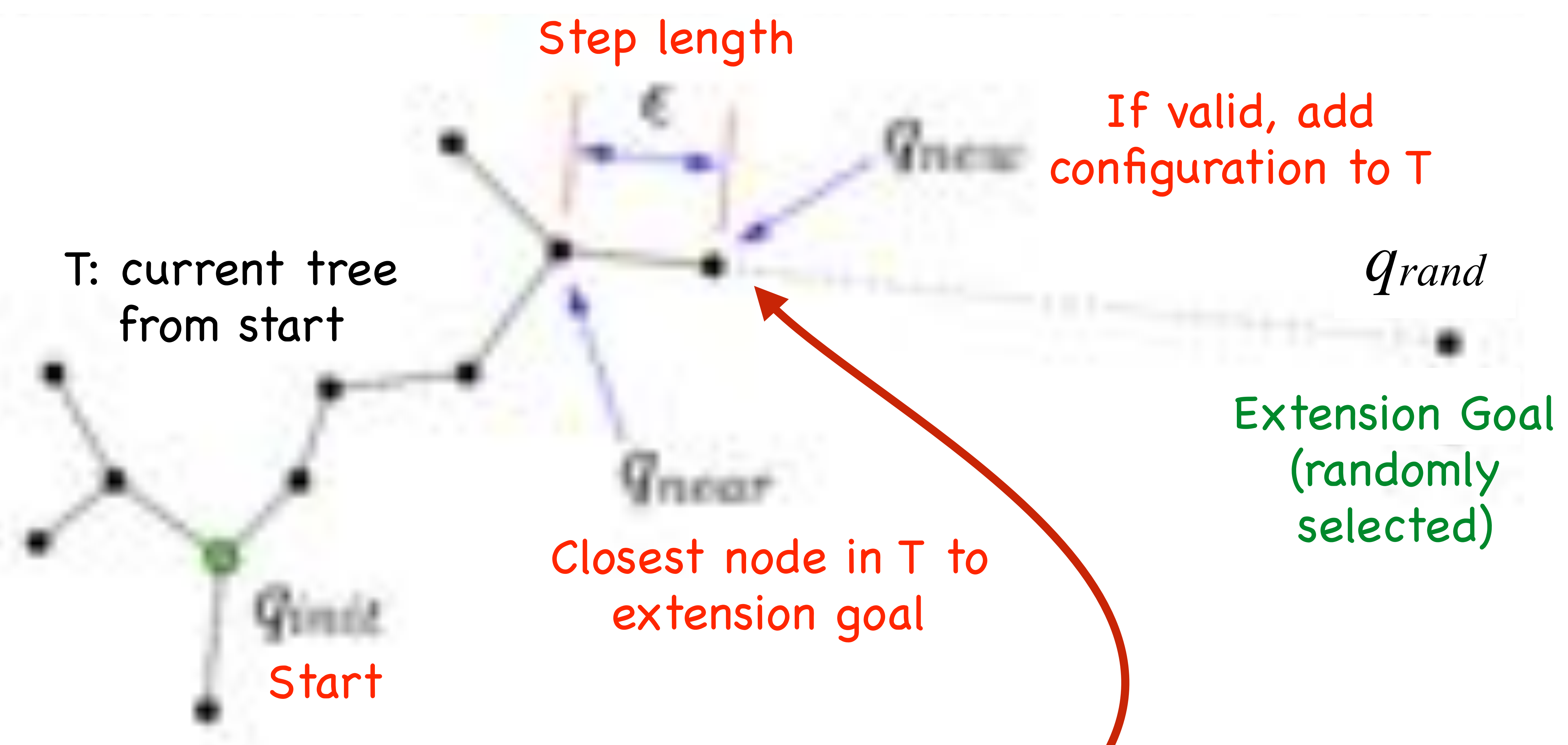

Generate and test new configuration along vector in C-space from q_{near} to q_{rand}

RRT Extend animation

http://www.kuffner.org/james/plan/algorithm.php

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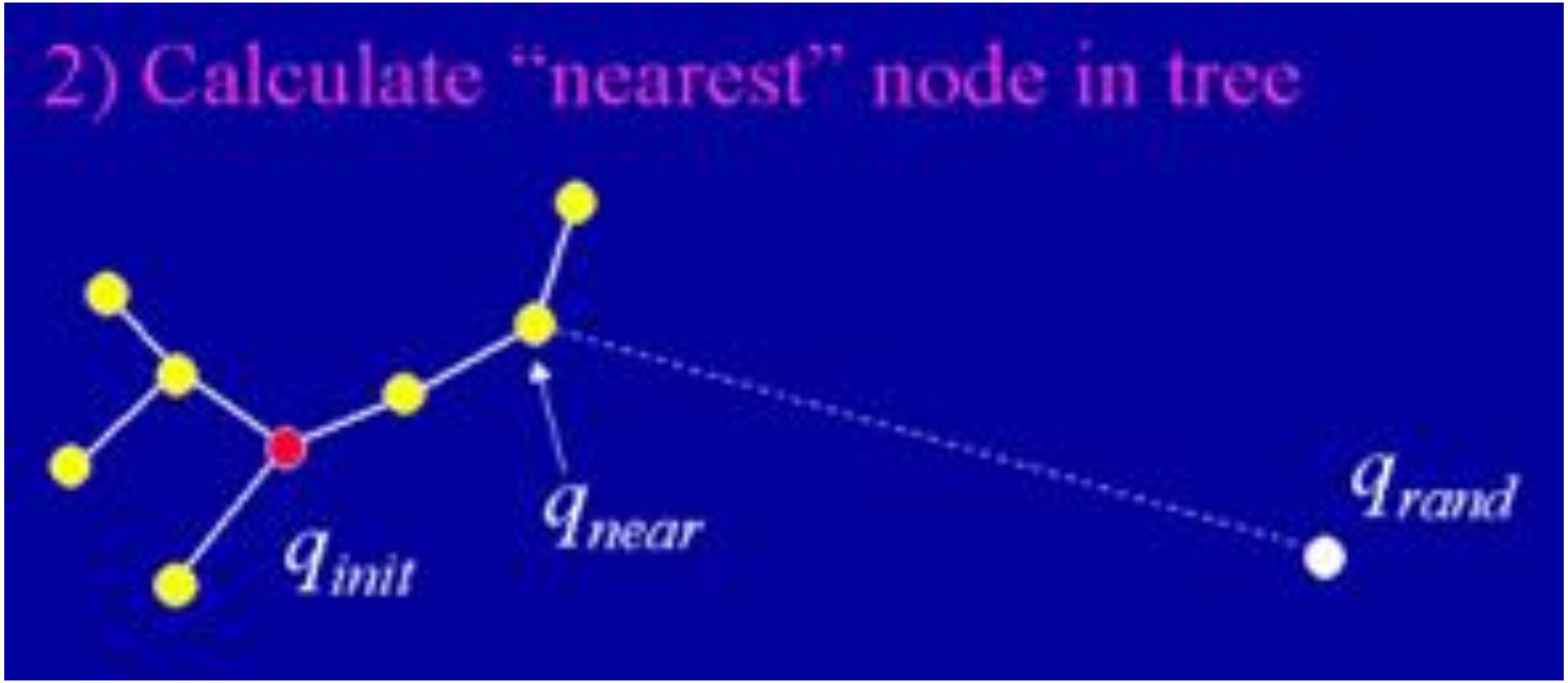
RRT Extend animation



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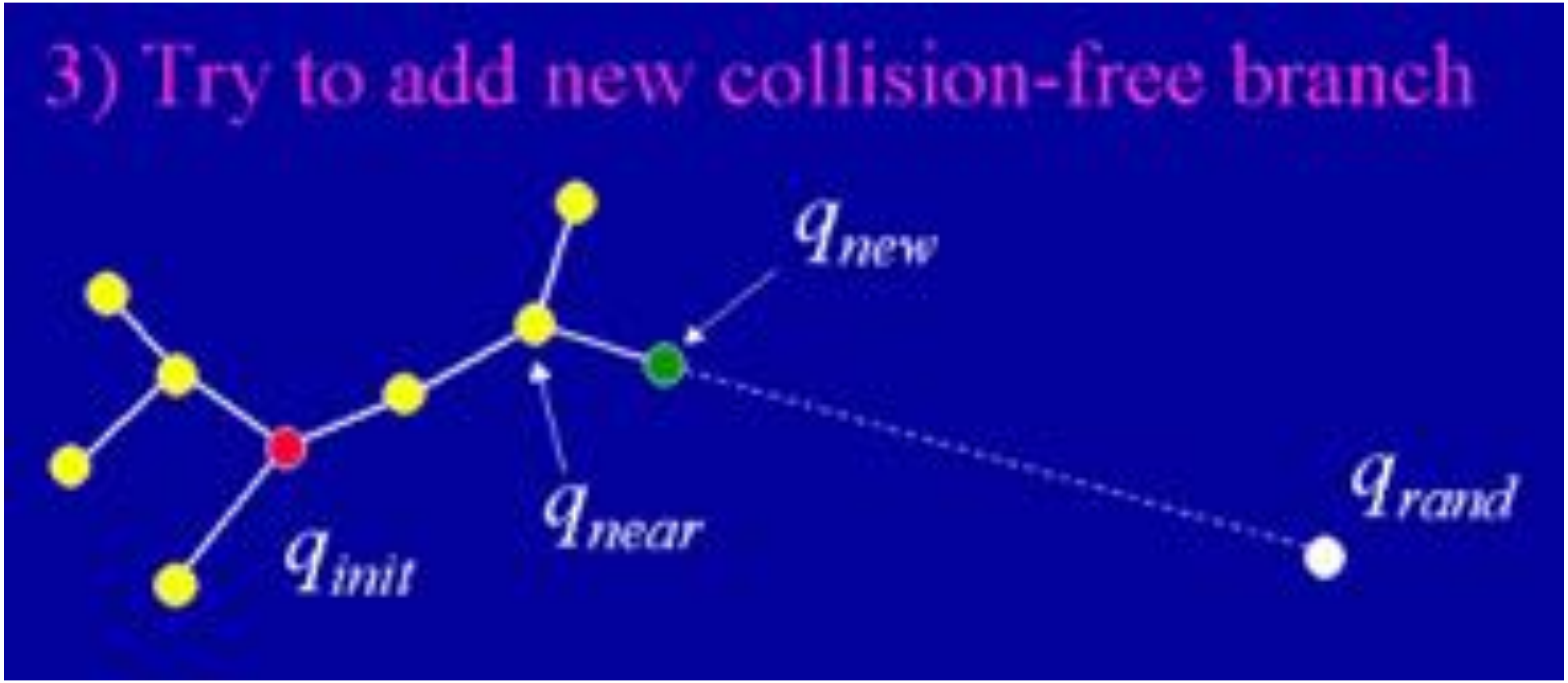
RRT Extend animation



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Demo

RRT Connect

[Kuffner, LaValle 2000]

0) Use 2 trees (A and B) rooted at start and goal configurations

RRT_CONNECT_PLANNER(q_{init}, q_{goal})
$\mathcal{T}_a$.init(q_{init}); $\mathcal{T}_b$.init(q_{goal});
for $k = 1$ **to** K **do**
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    if not (EXTEND($\mathcal{T}_a, q_{rand}$) $=$ *Trapped*) **then**
        if (CONNECT($\mathcal{T}_b, q_{new}$) $=$ *Reached*) **then**
            Return PATH($\mathcal{T}_a, \mathcal{T}_b$);
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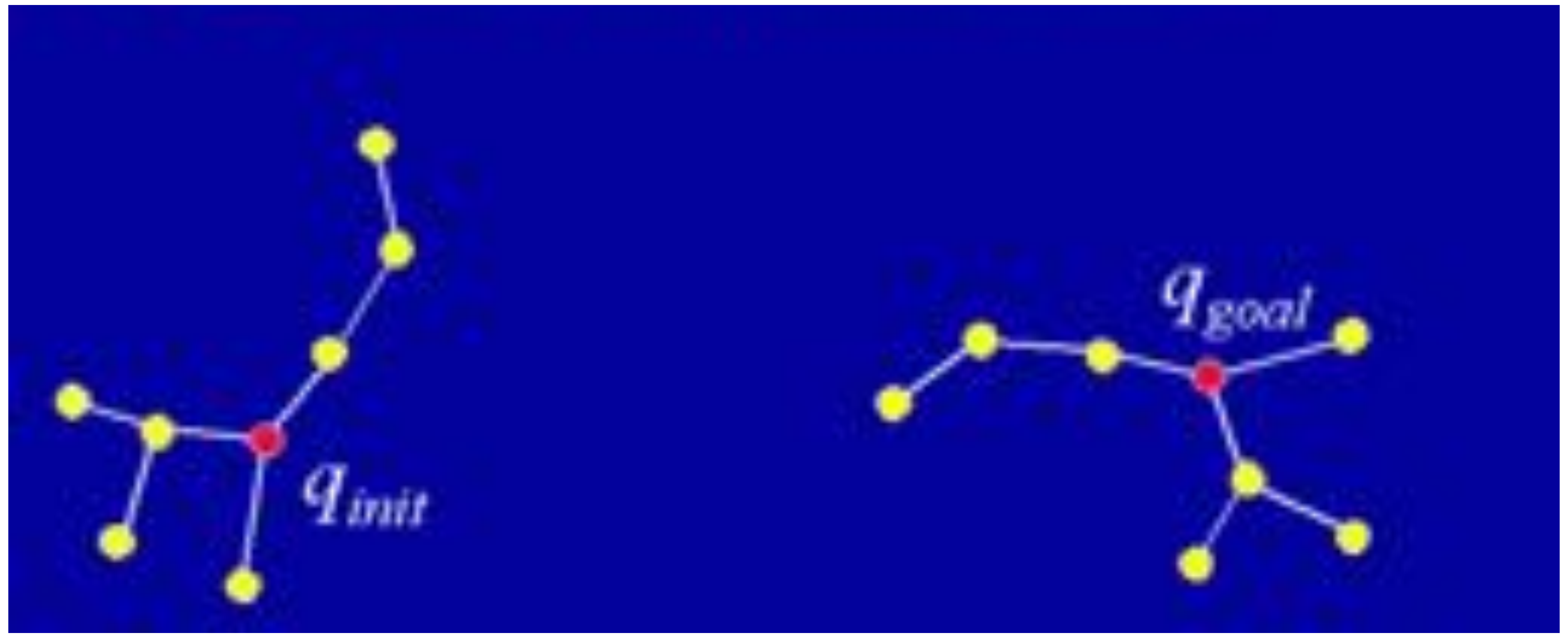

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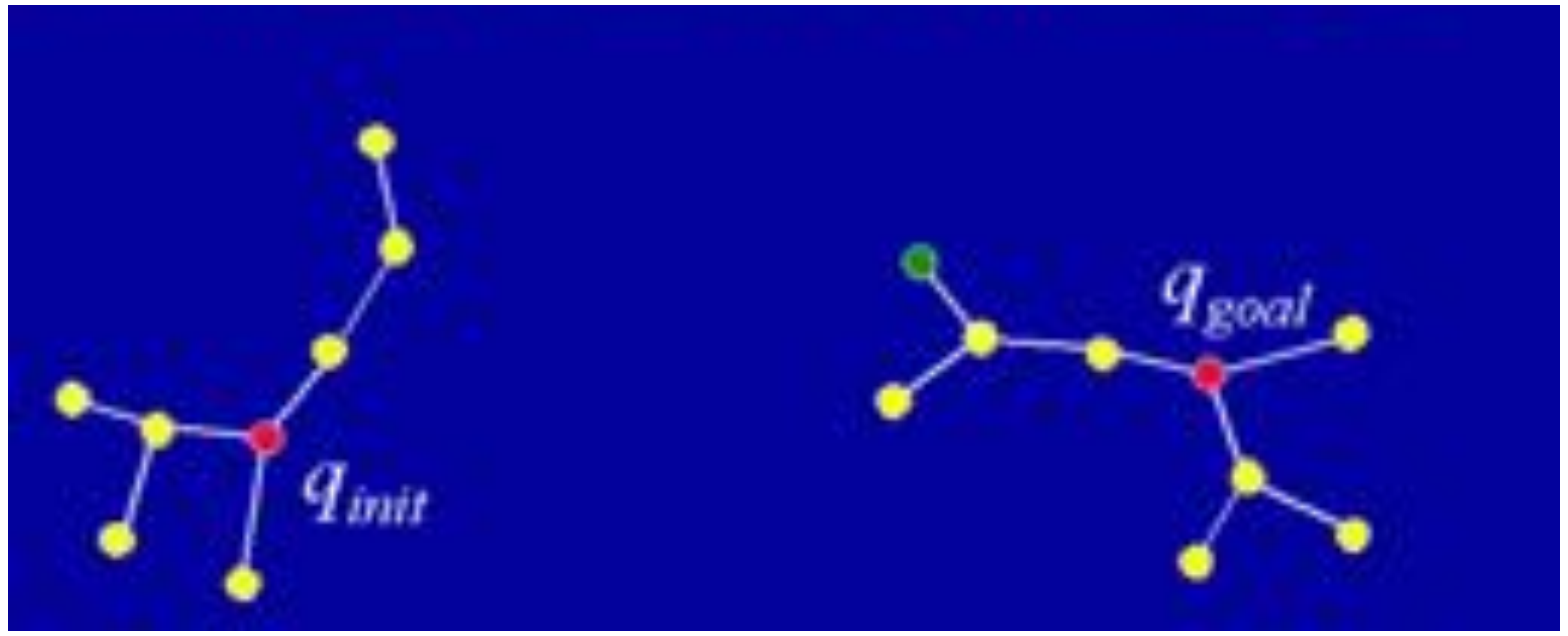
1) Extend tree A towards a random configuration



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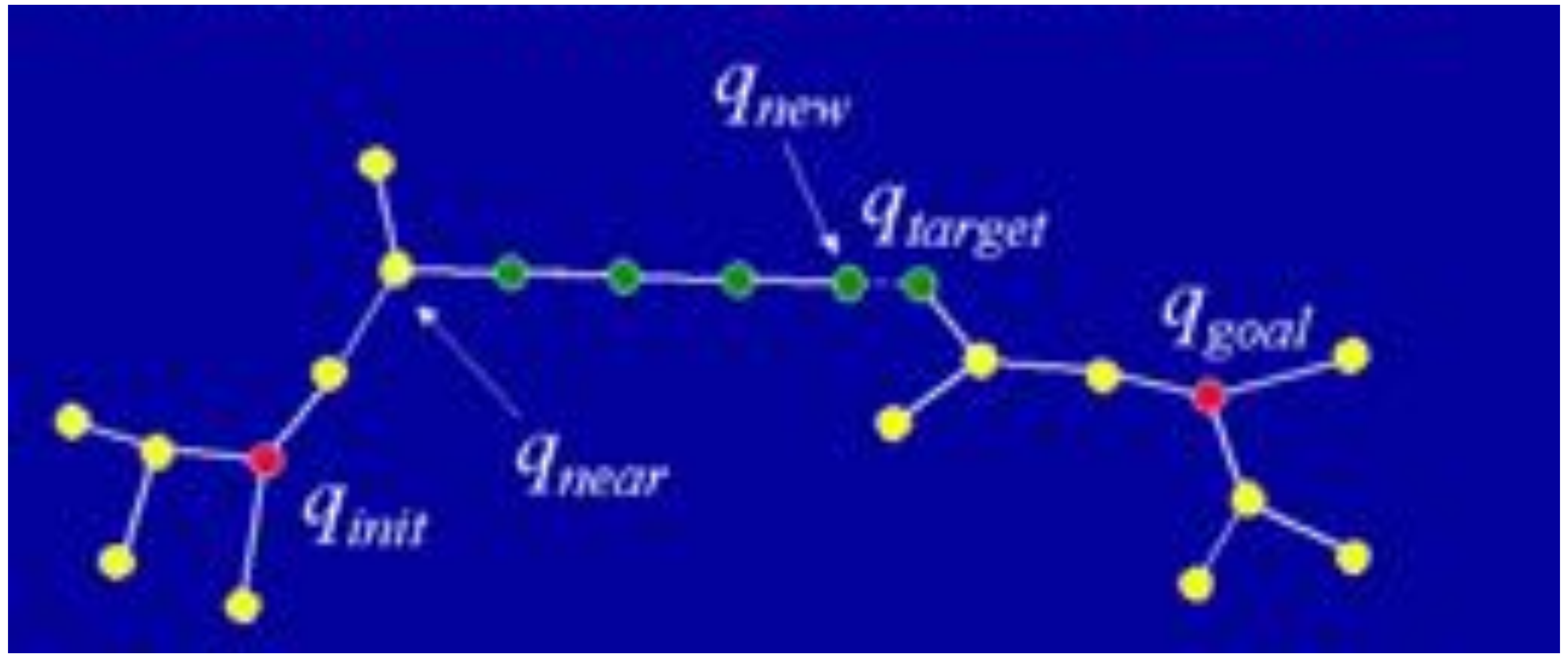

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1) Extend tree A towards a random configuration

CONNECT($\mathcal{T}, q$)
repeat
  $S \leftarrow$ EXTEND($\mathcal{T}, q$);
until not ($S =$ *Advanced*)
Return S;

2) Try to connect tree B to tree A by extending repeatedly from its nearest neighbor

RRT Connect

[Kuffner, LaValle 2000]

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      Return PATH($\mathcal{T}_a, \mathcal{T}_b$);
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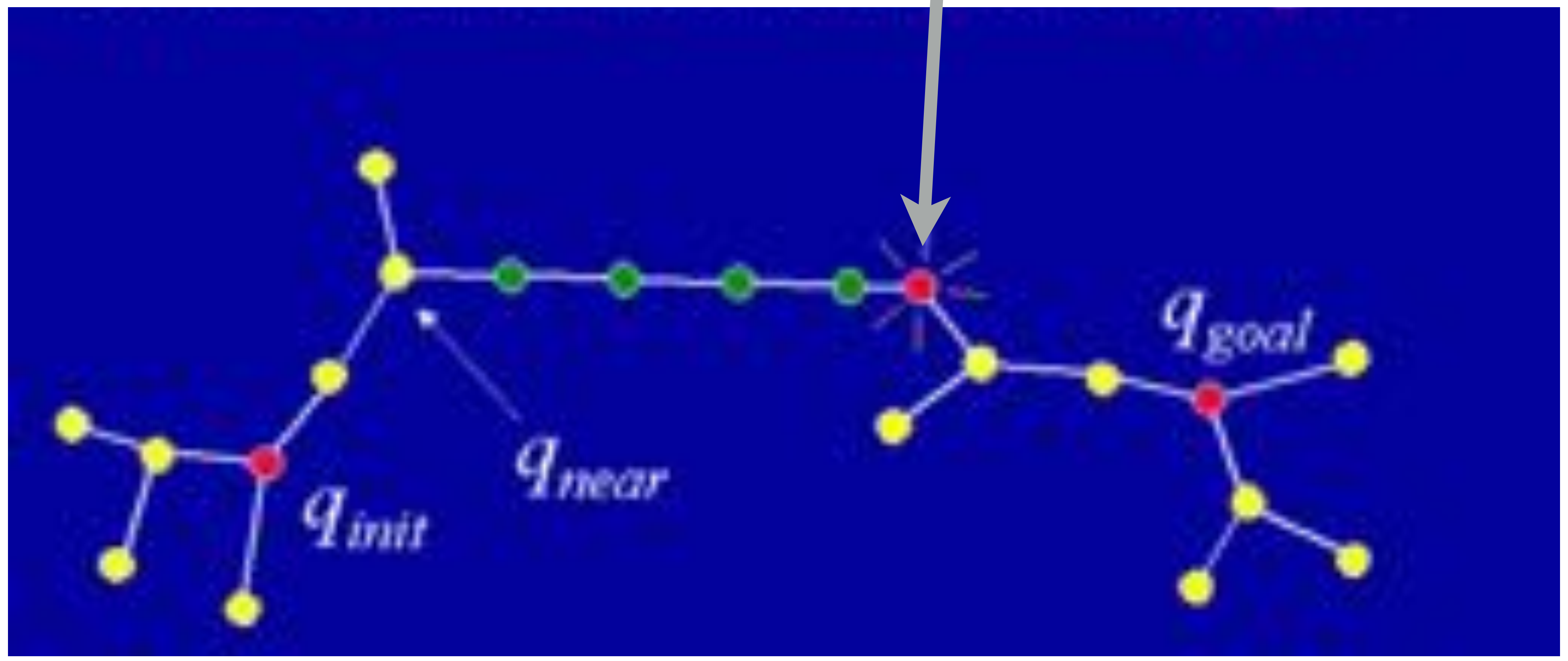
Return *Failure*

search succeeds if trees connect



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1) Extend tree A towards a random configuration

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CONNECT(T, q)
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  until not (S = Advanced)
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```
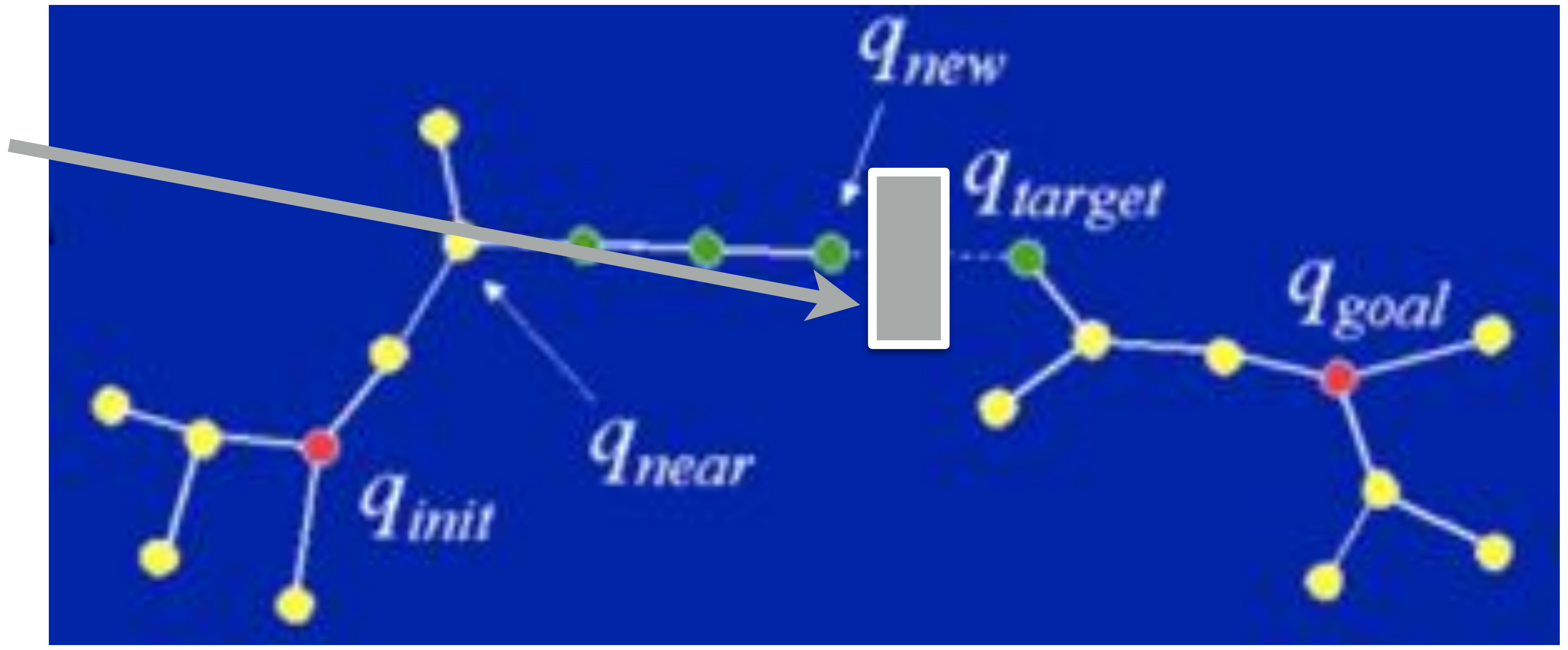

2) Try to connect tree B to tree A by extending repeatedly from its nearest neighbor

3) reverse roles for trees A and B and repeat

collision encountered



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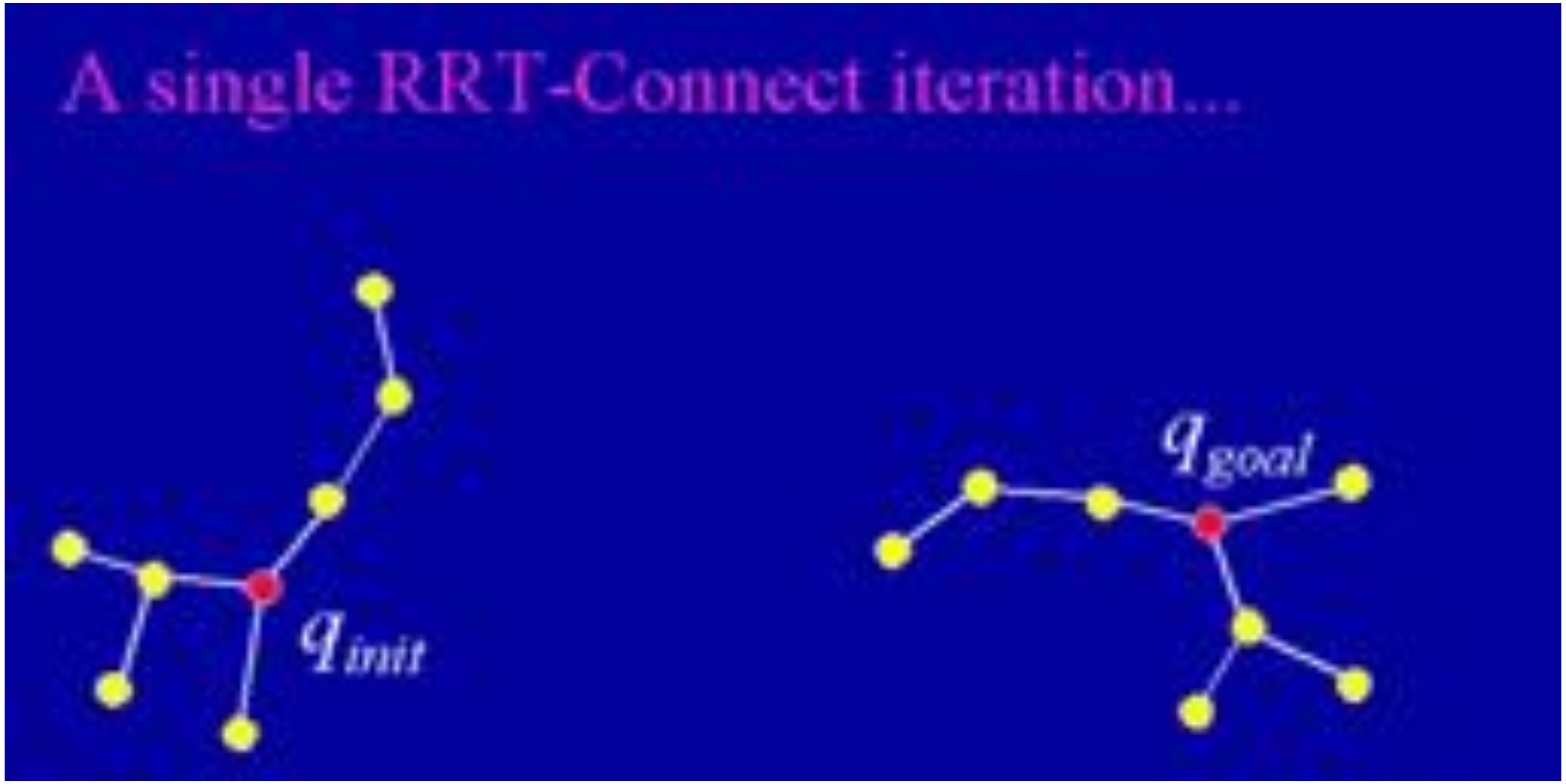
RRT-Connect animation



http://www.kuffner.org/james/plan/algorithm.php

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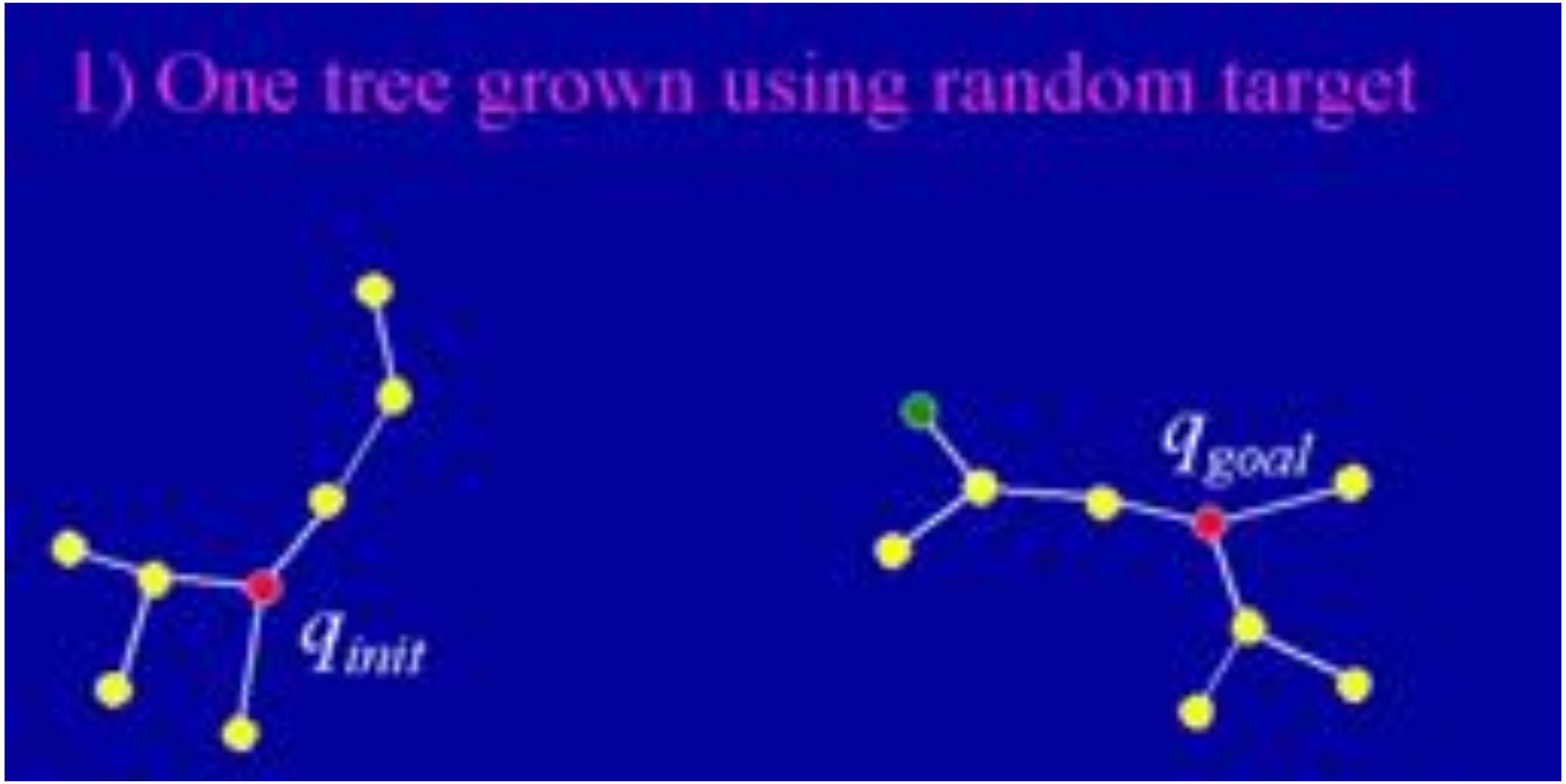

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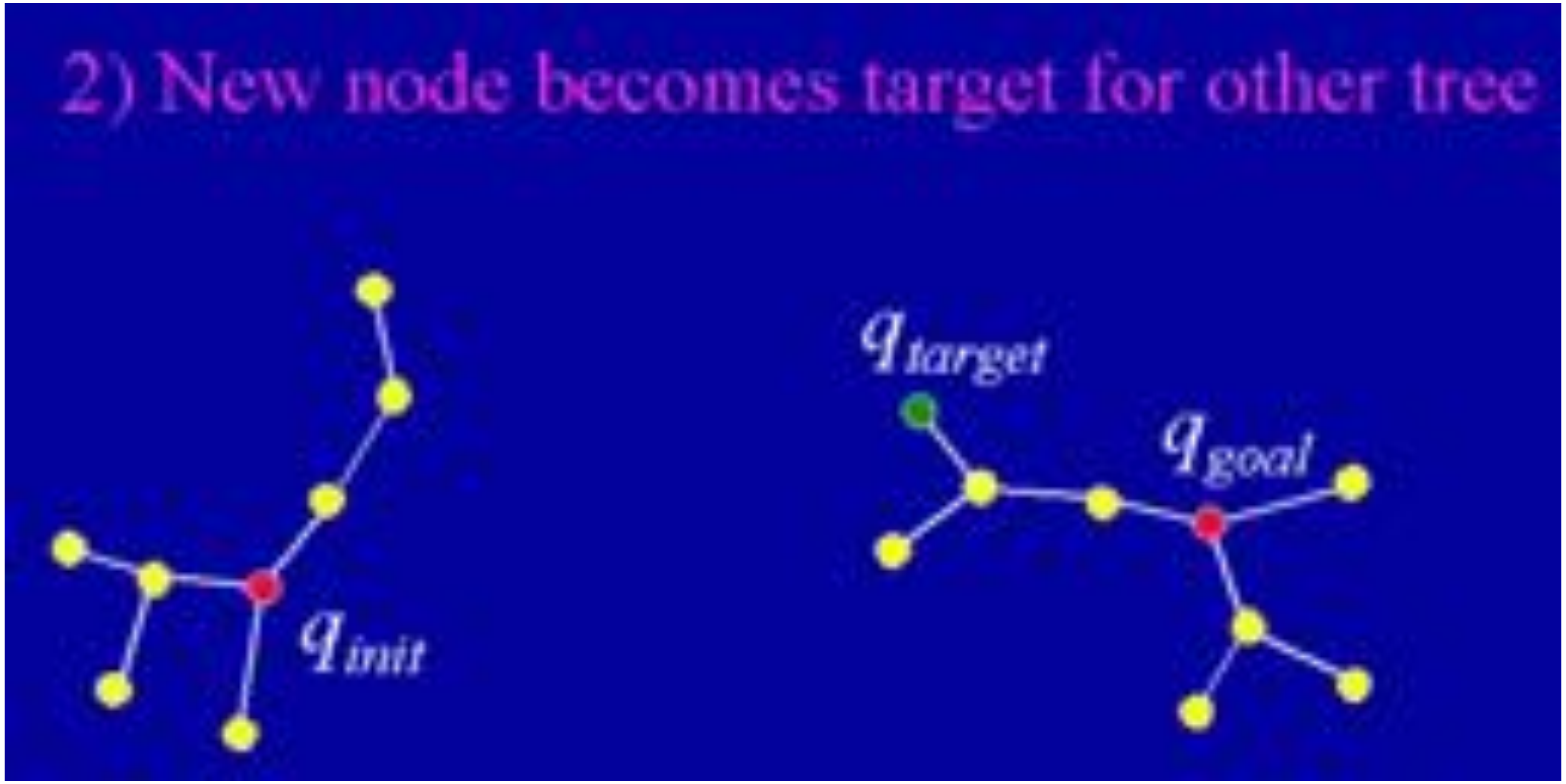

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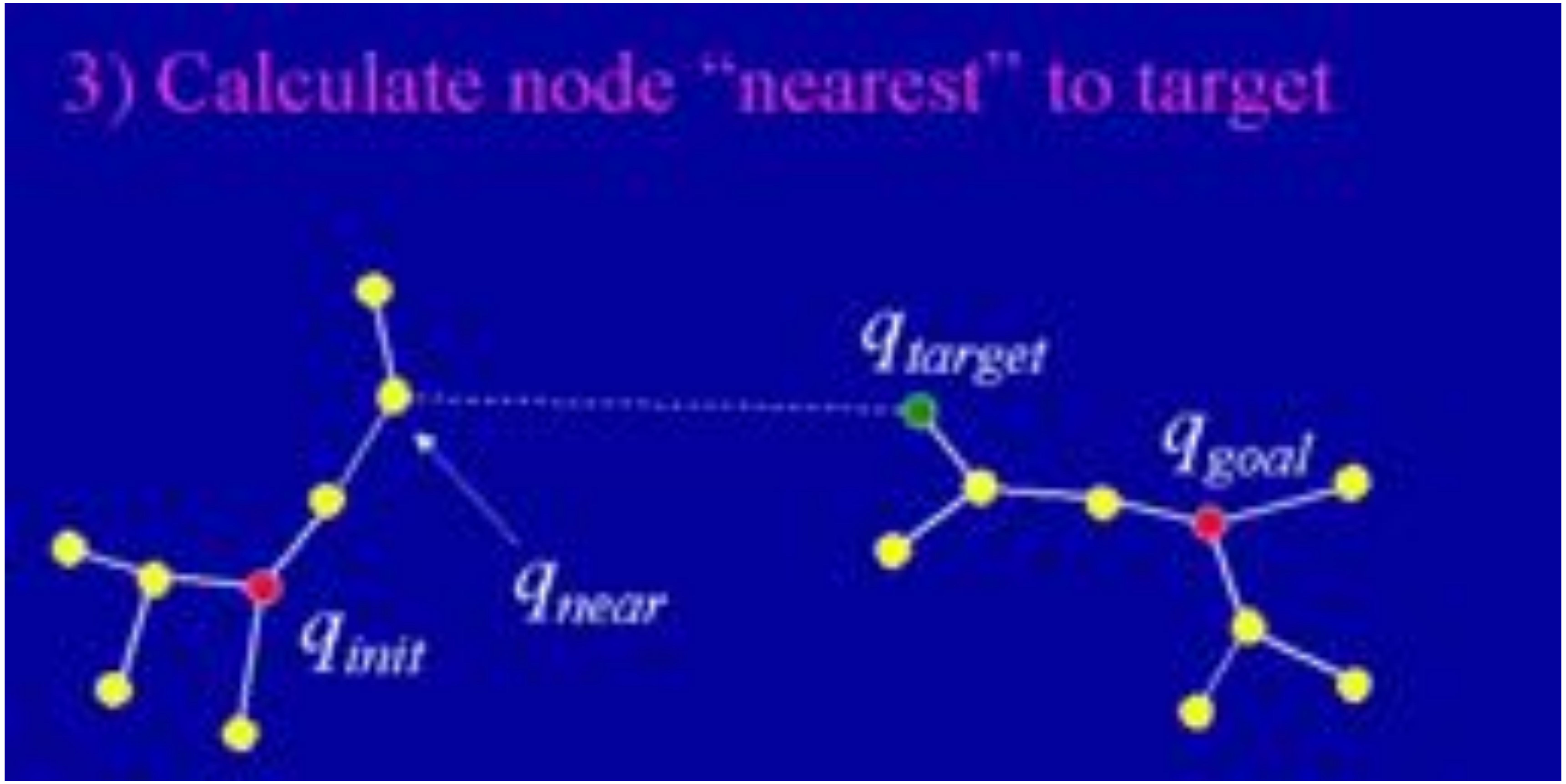

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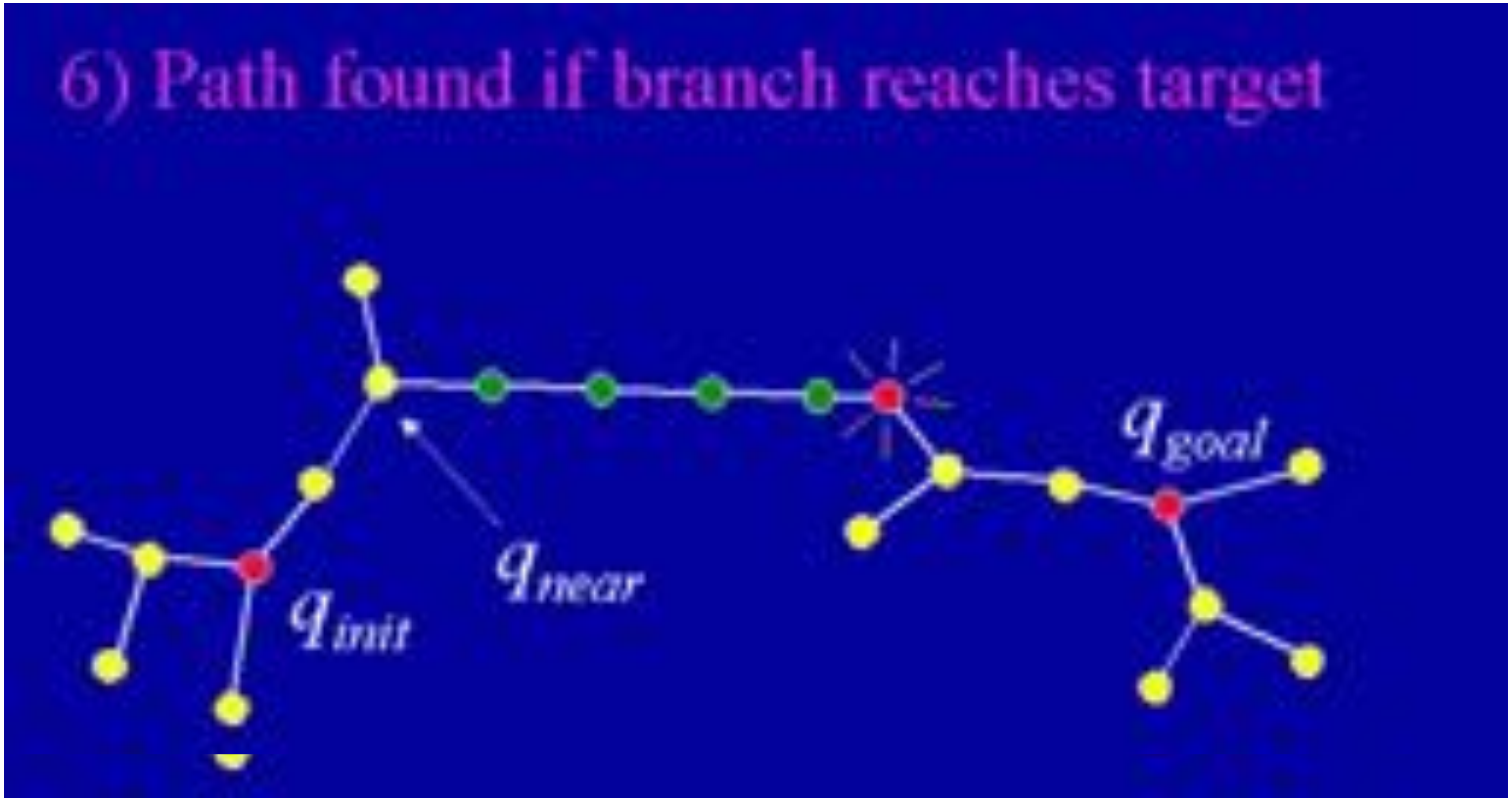
http://www.kuffner.org/james/plan/algorithm.php

RRT-Connect animation



http://www.kuffner.org/james/plan/algorithm.php

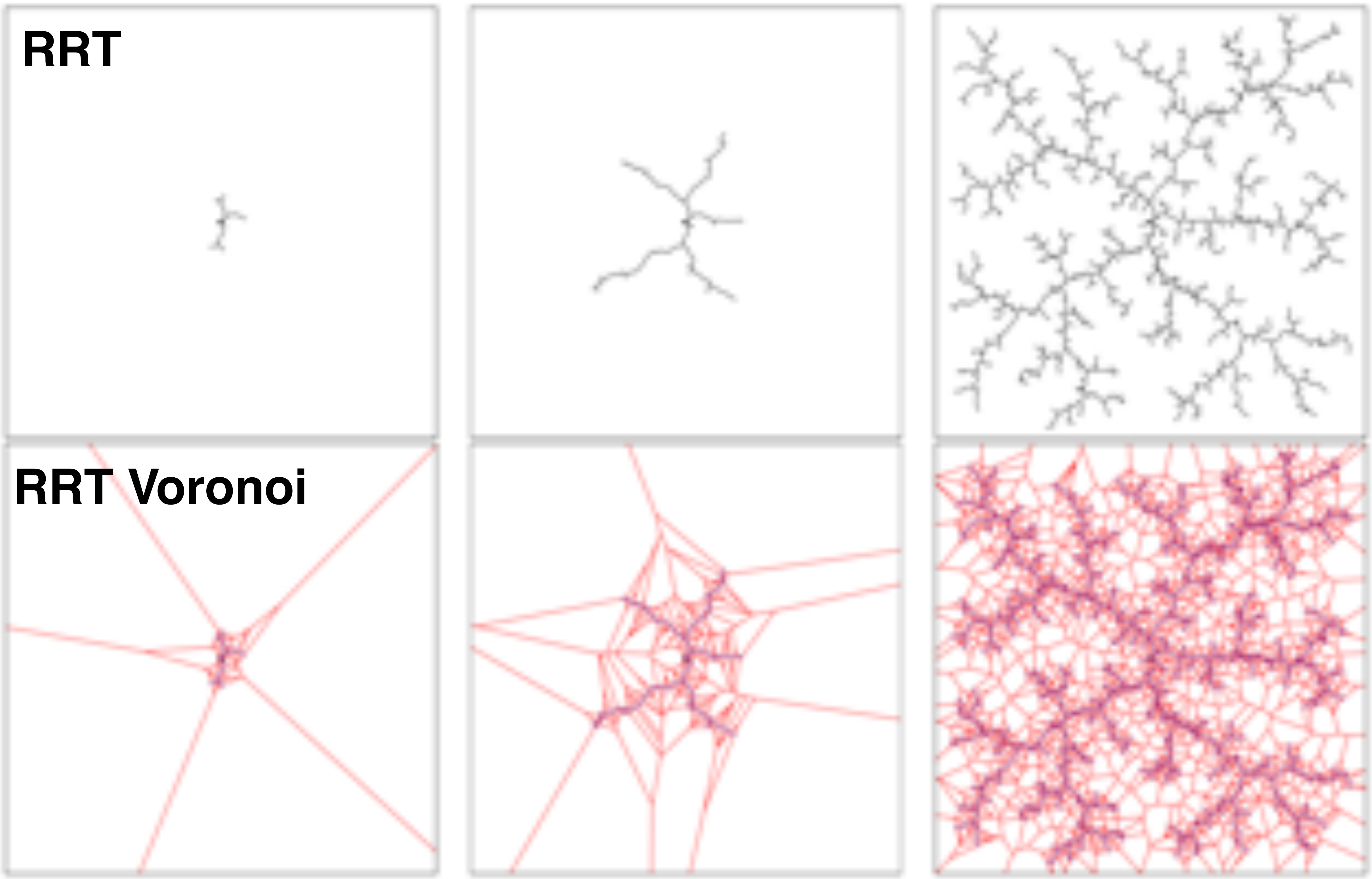
RRT-Connect animation

http://www.kuffner.org/james/plan/algorithm.php

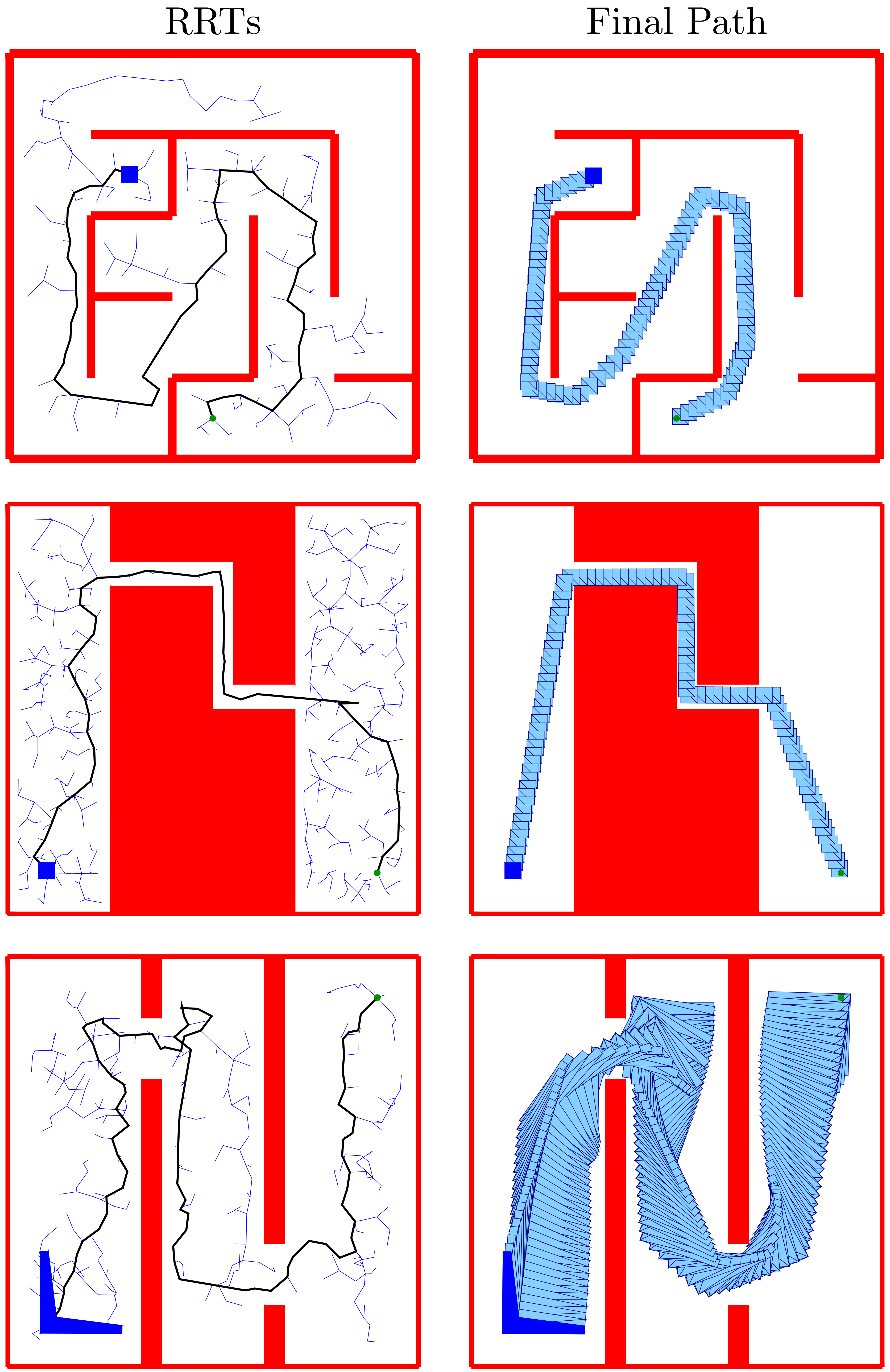
RRT Probabilistic Completeness

- Probability a vertex is selected for extension is proportional to its area in Voronoi diagram

- RRTs converge to a uniform coverage of C-space as the number of samples increases



Examples of RRT-Connect

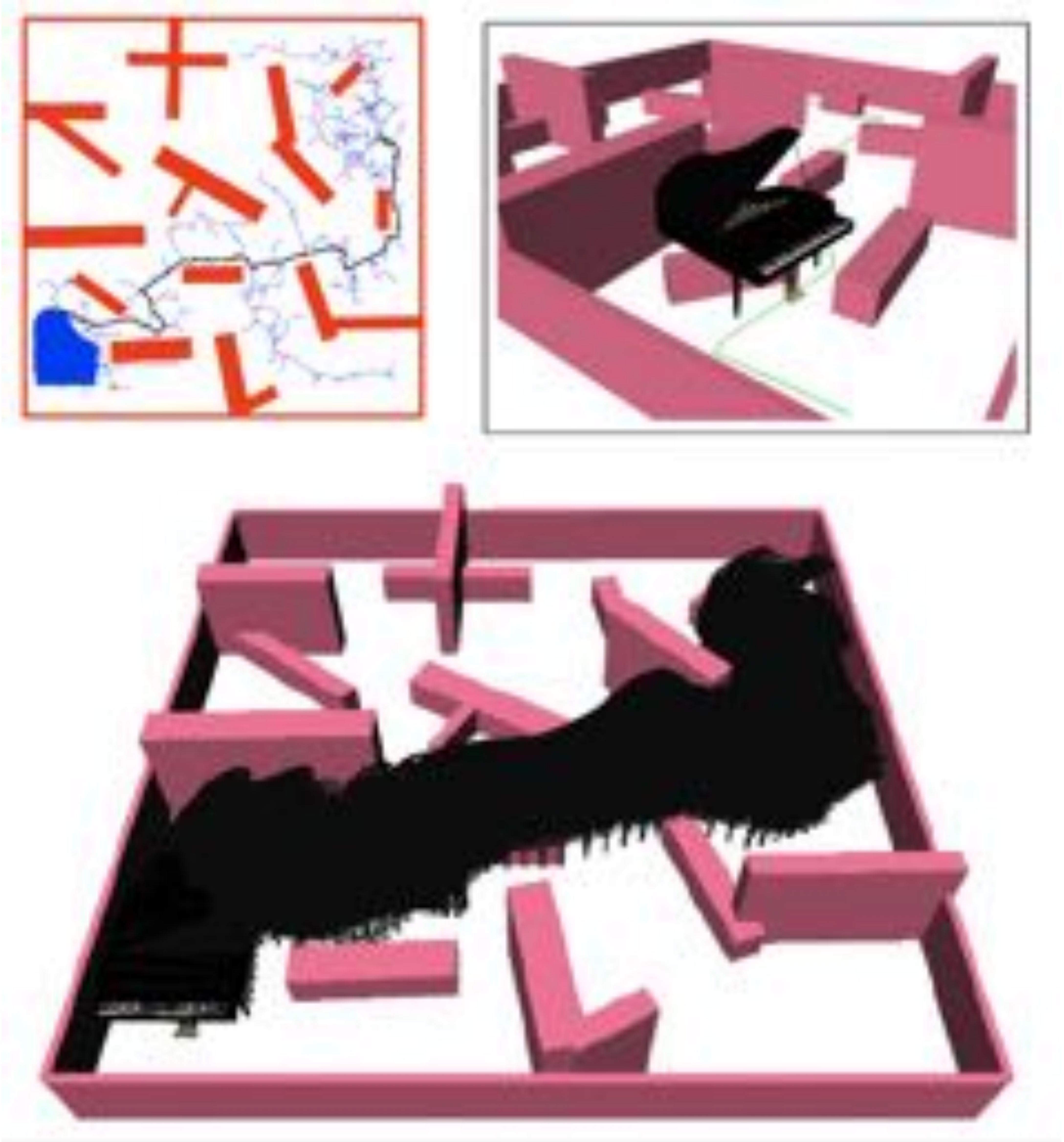


2 DOF maze

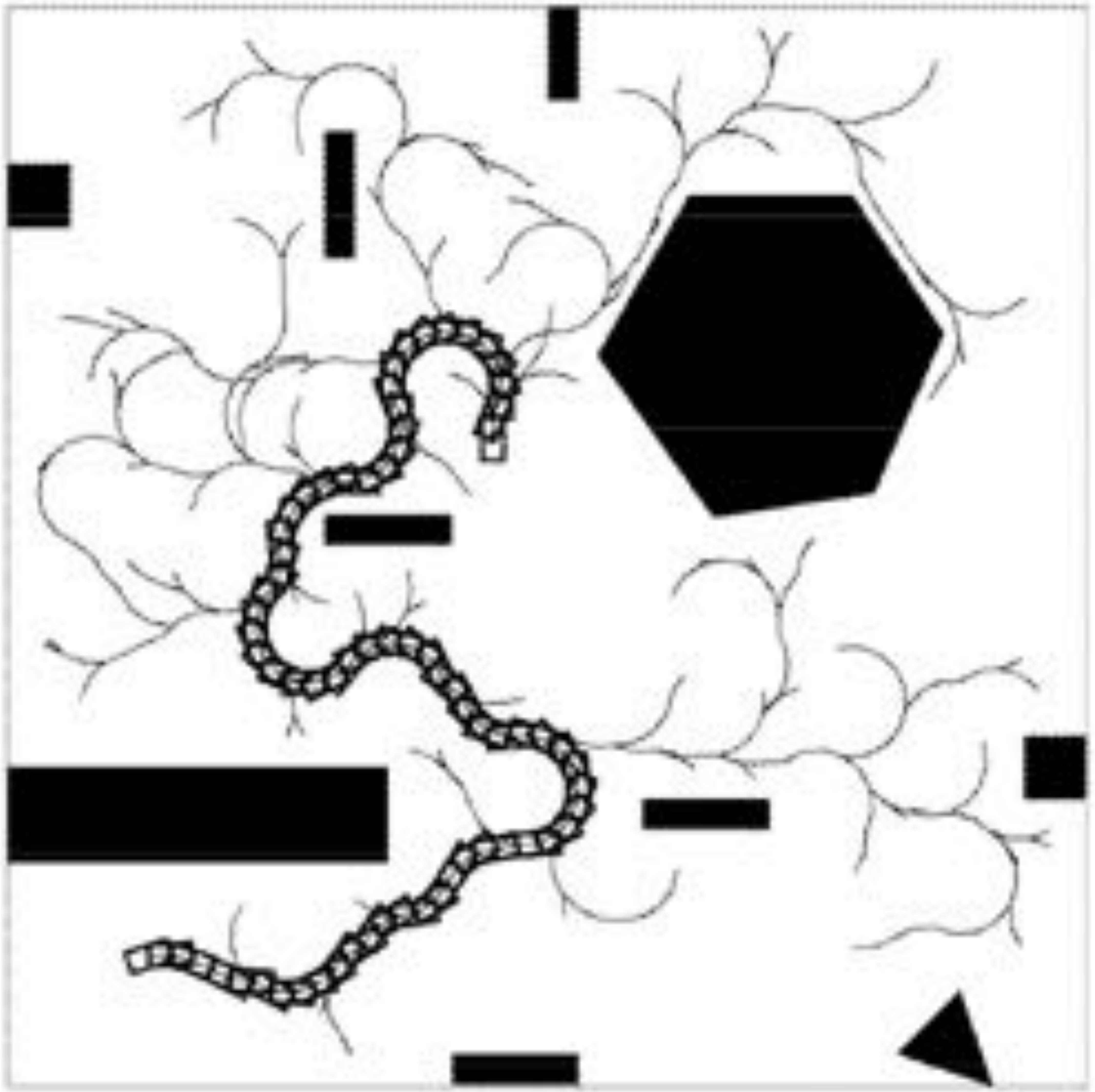
2 DOF single passway

3 DOF single passway
(with non-point geometry)

Piano Mover's Problem



A Car-Like Robot



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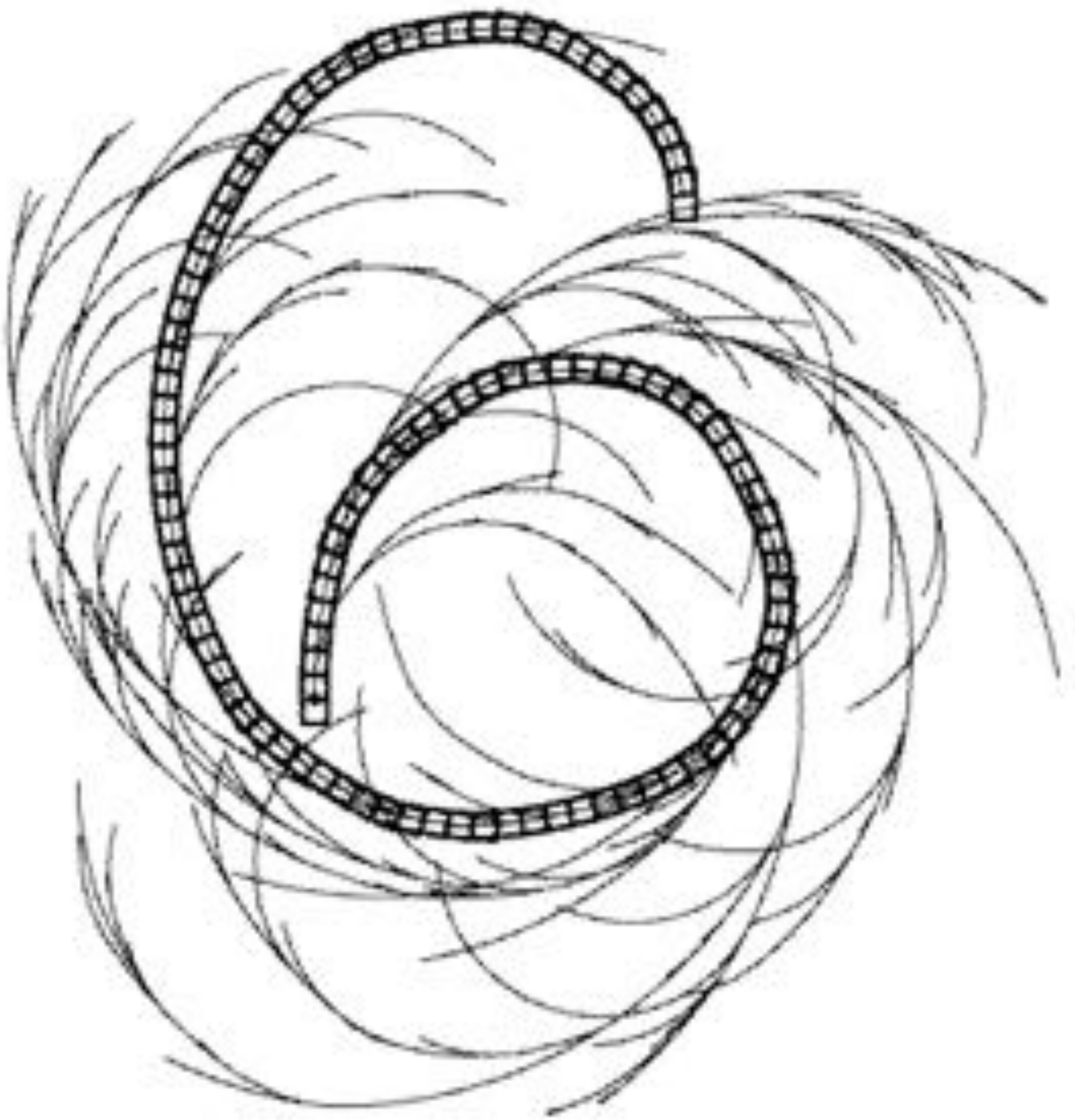
A Right-Turn Only Car



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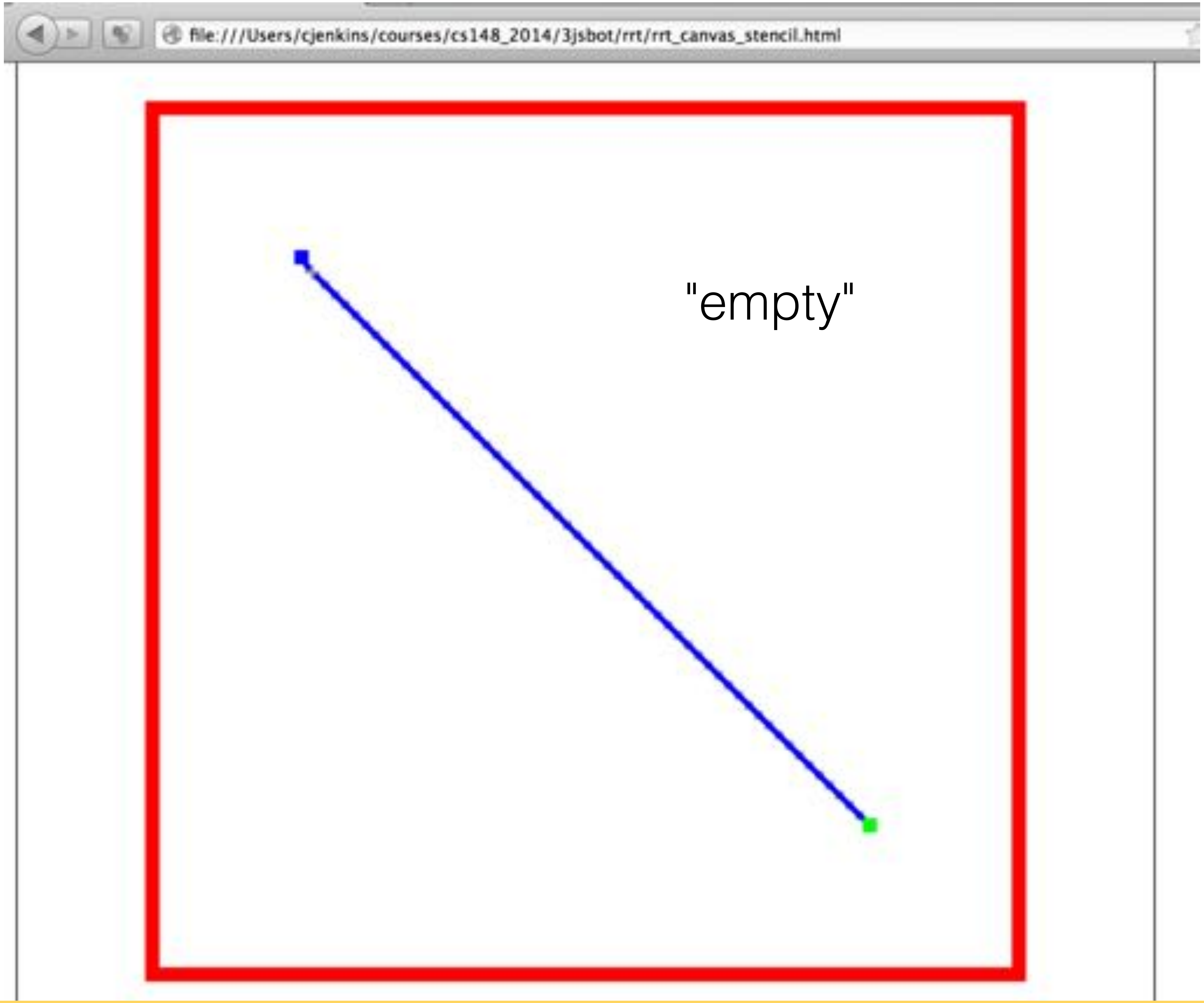
Canvas Stencil Examples



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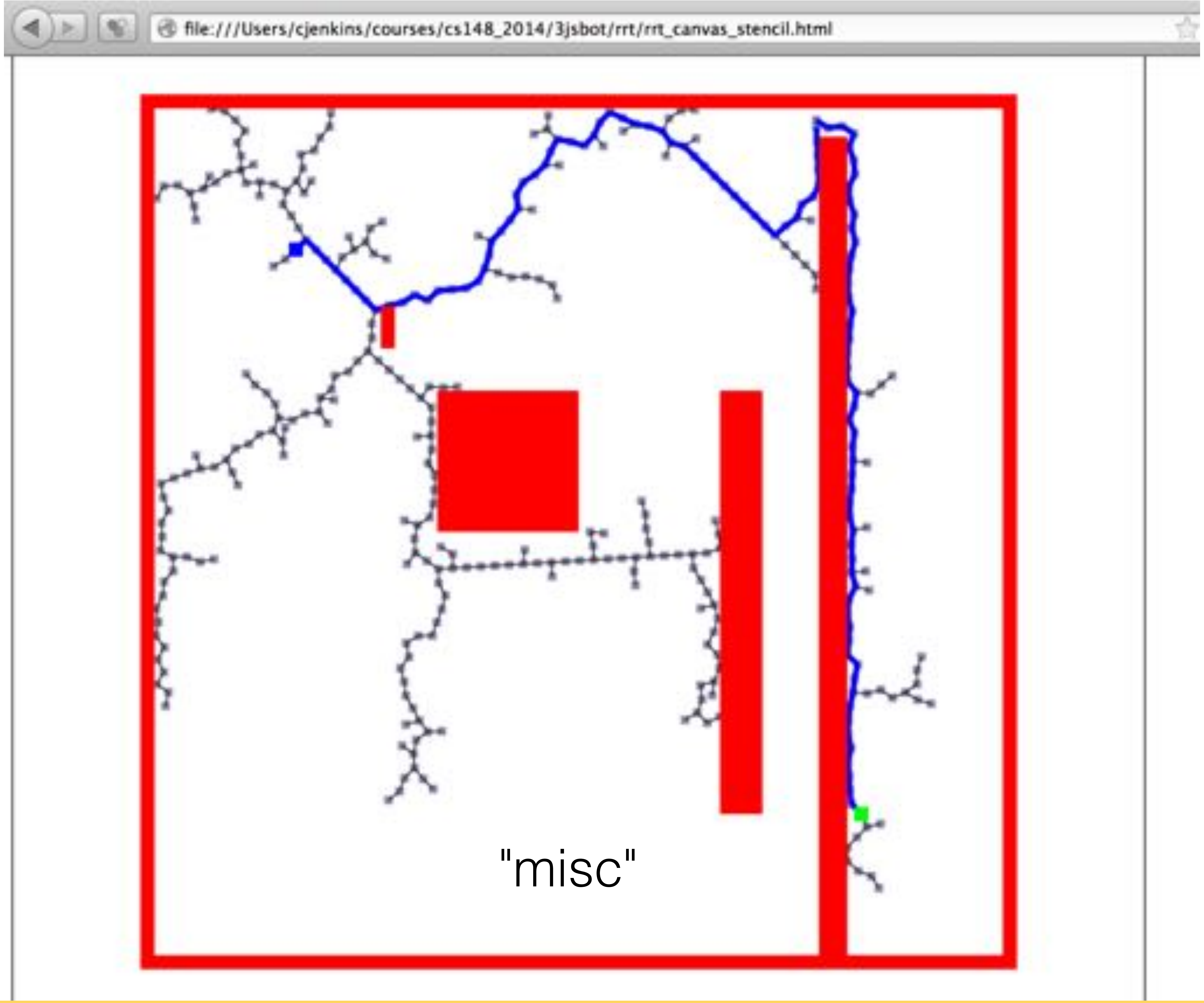
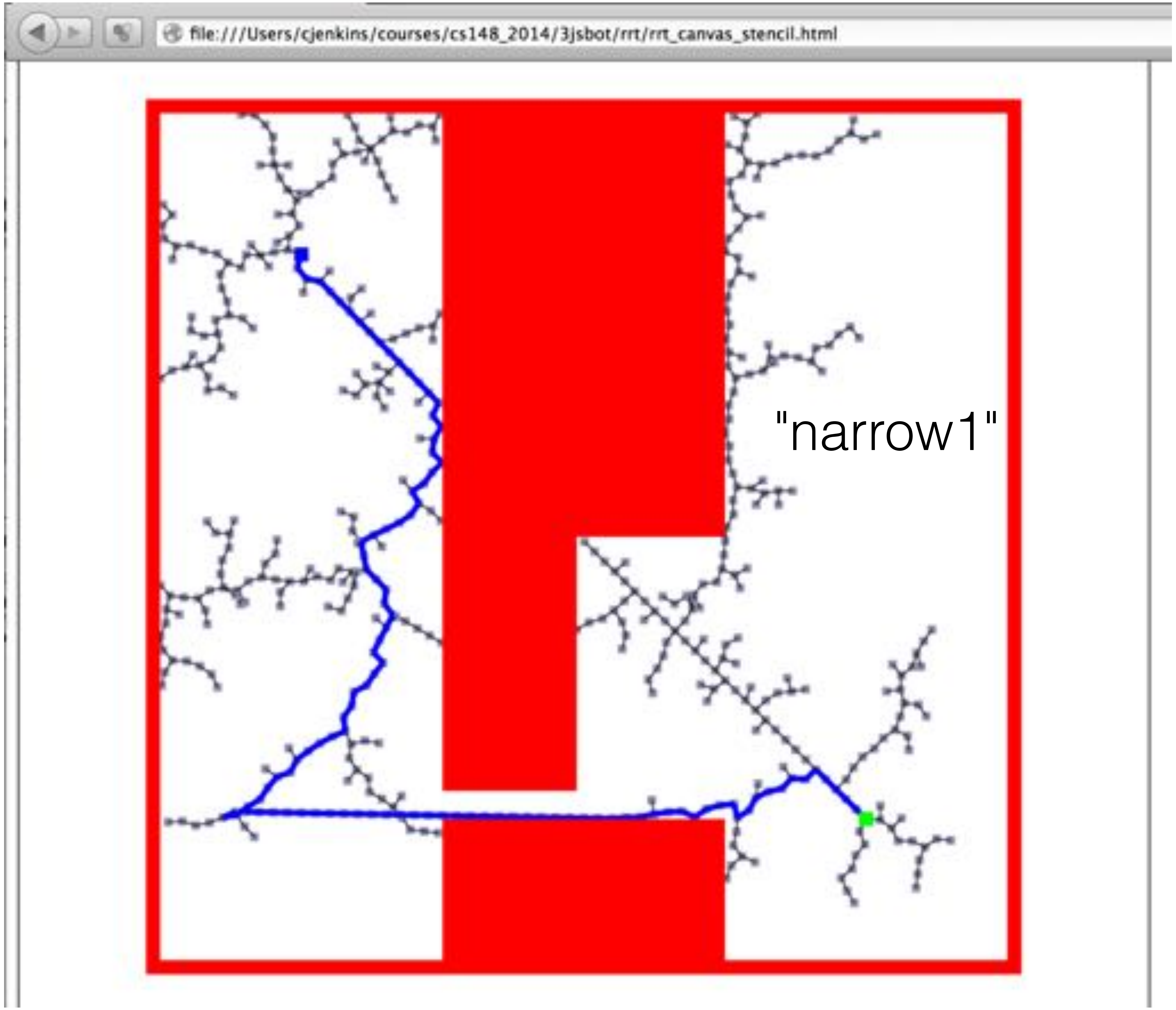
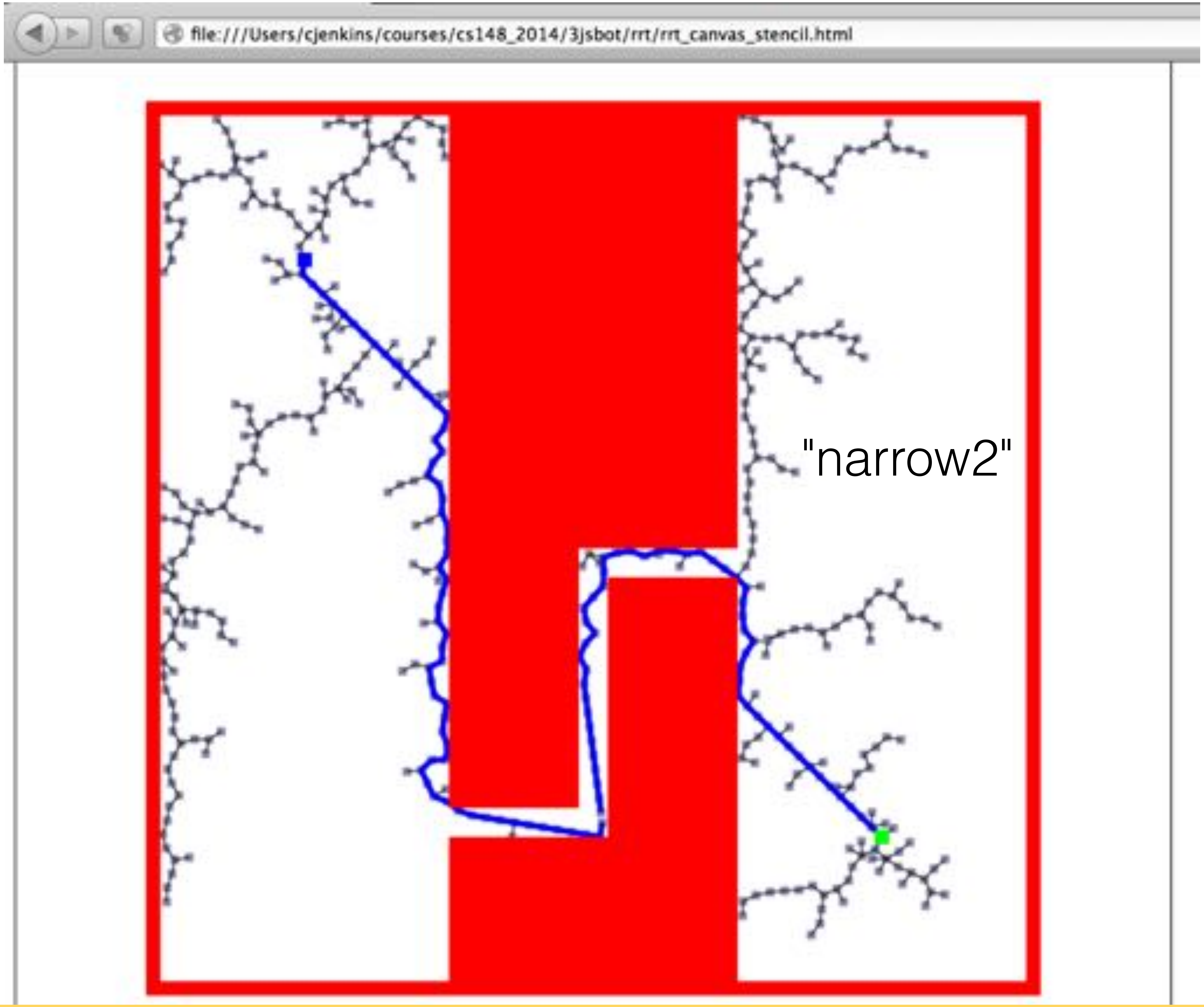
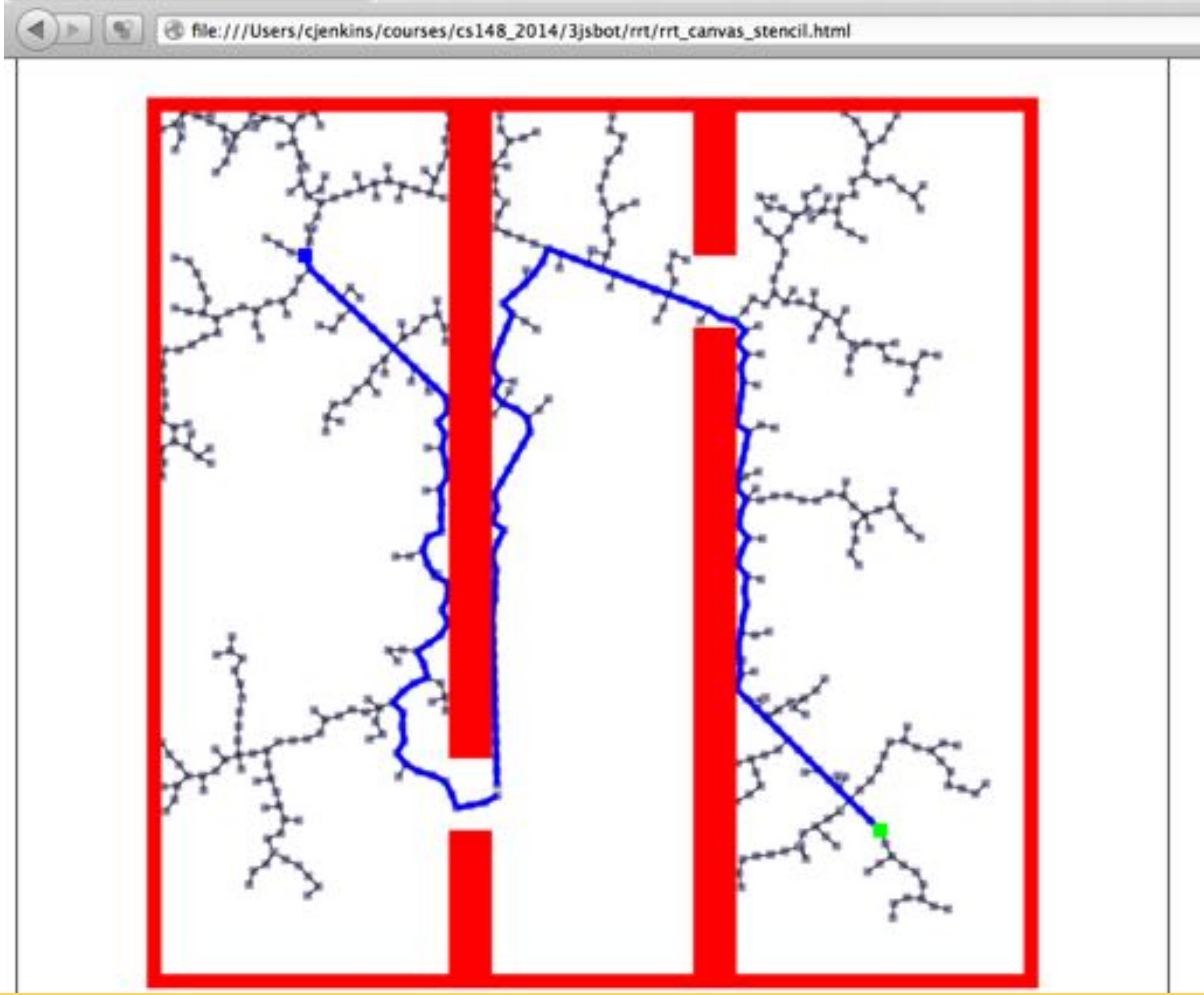

file:///Users/cjenkins/courses/cs148_2014/3jsbot/rrt/rrt_canvas_stencil.html
"misc"


file:///Users/cjenkins/courses/cs148_2014/3jsbot/rrt/rrt_canvas_stencil.html
"narrow1"


file:///Users/cjenkins/courses/cs148_2014/3jsbot/rrt/rrt_canvas_stencil.html
"narrow2"



"three_sections"

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“We’ve made robot history”

Kuffner/Asimo Discovery Channel feature - https://www.youtube.com/watch?v=wtVmbiTfm0Q

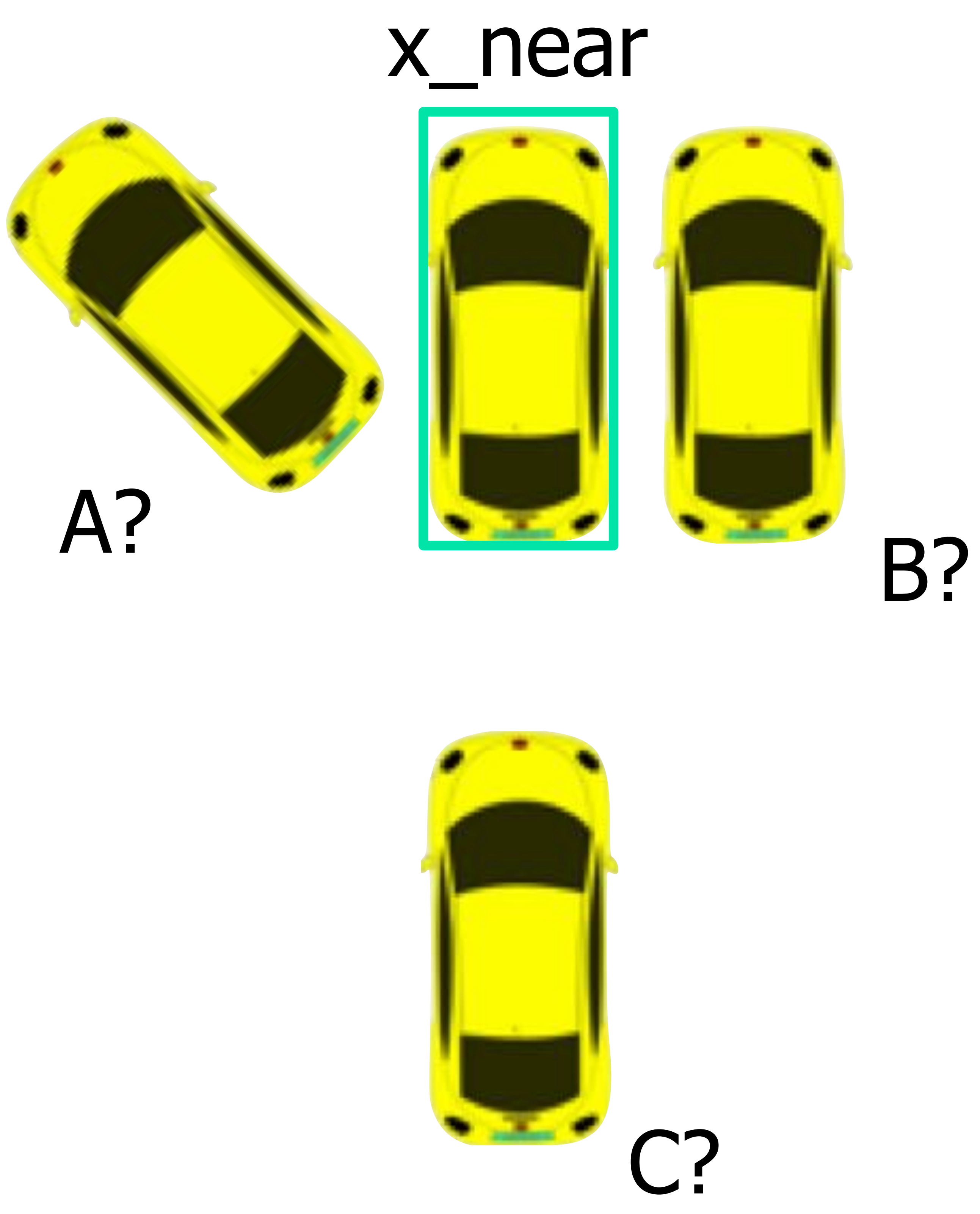
RRT Practicalities

- NEAREST_NEIGHBOR(x_{rand}, T): need to find (approximate) nearest neighbor efficiently
  - KD Trees data structure (upto 20-D) [e.g., FLANN]
  - Locality Sensitive Hashing

- SELECT_INPUT(x_{rand}, x_{near})
  - Two point boundary value problem
    - If too hard to solve, often just select best out of a set of control sequences. This set could be random, or some well chosen set of primitives.

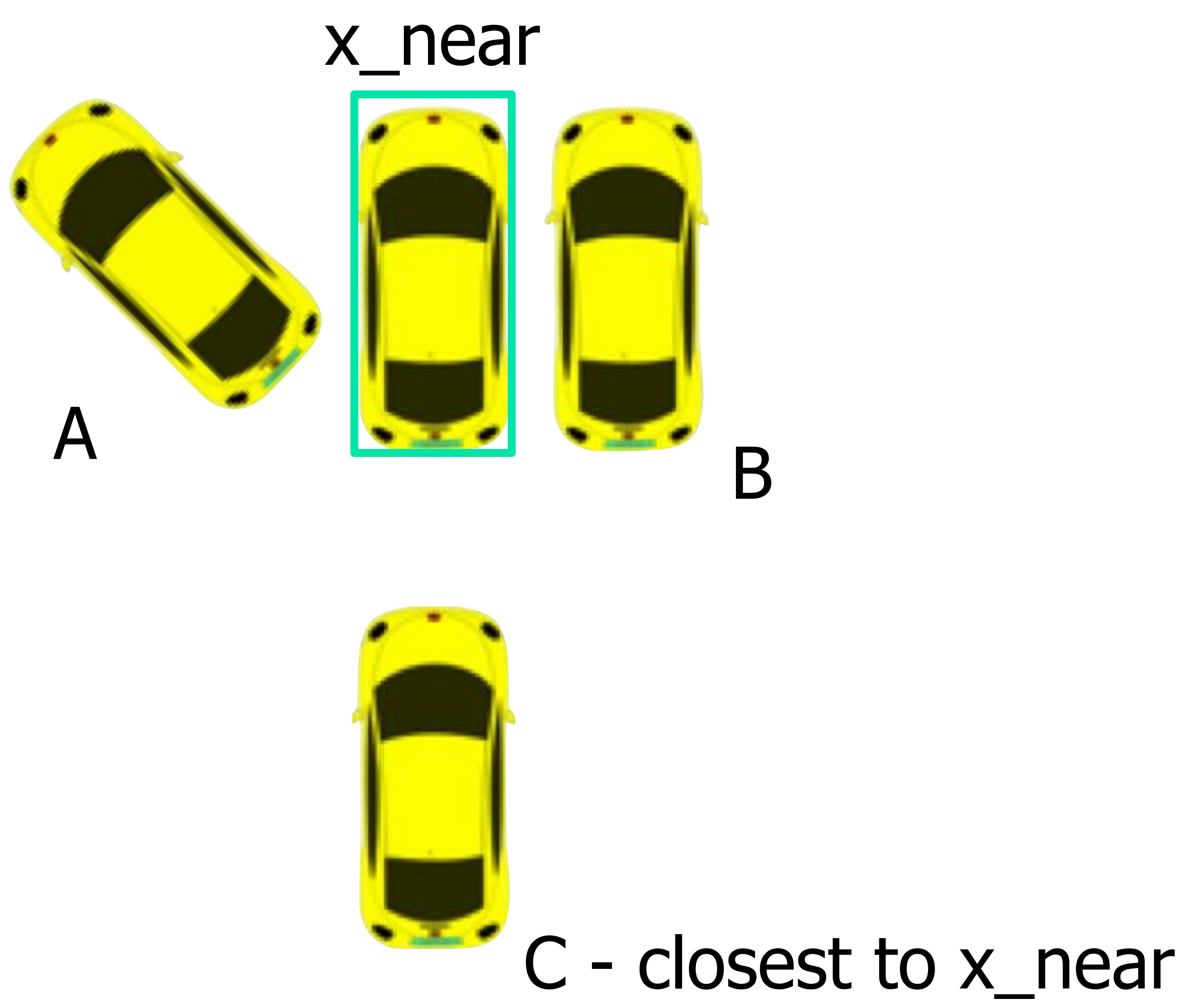
RRT Extension

- Non-holonomic: approximately (sometimes as approximate as picking best of a few random control sequences) solve two-point boundary value problem



RRT Extension

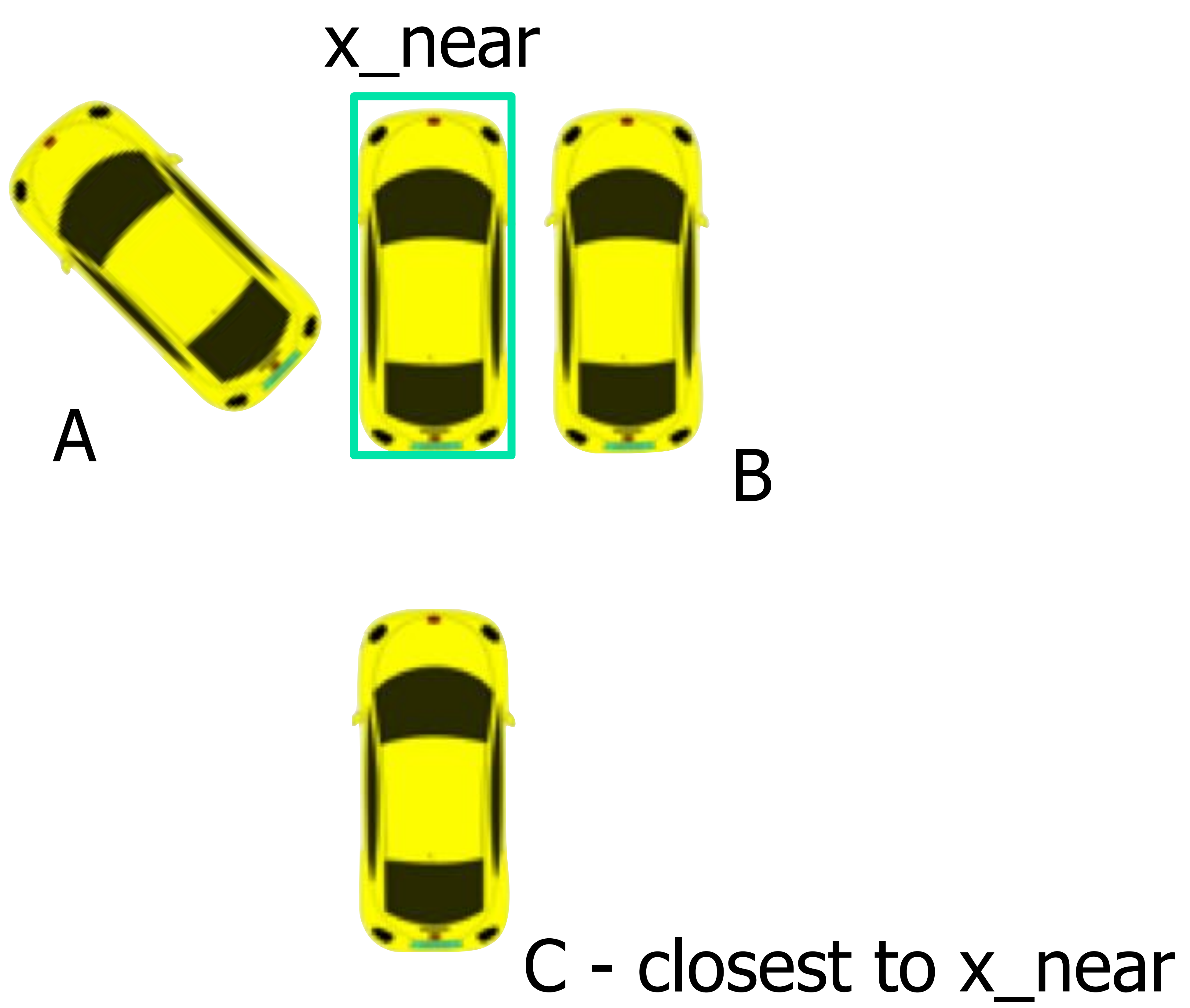
- Non-holonomic: approximately (sometimes as approximate as picking best of a few random control sequences) solve two-point boundary value problem

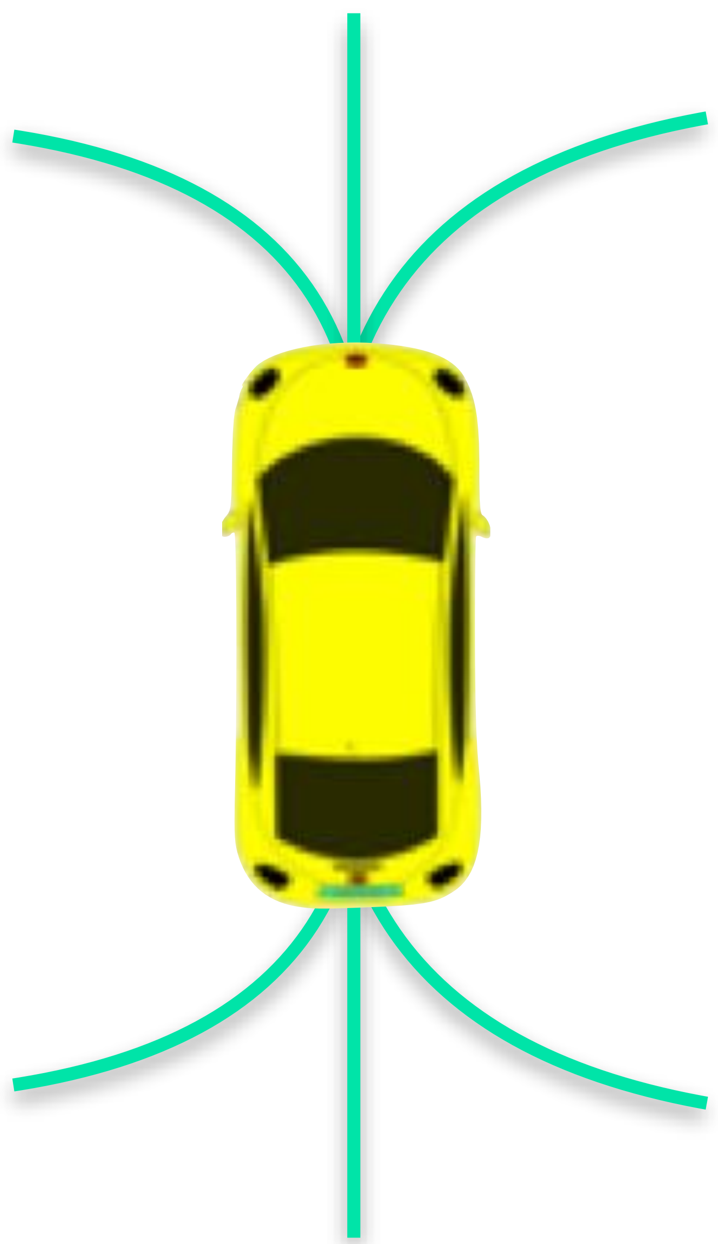


Slide borrowed from Dieter Fox

RRT Extension

- Non-holonomic: approximately (sometimes as approximate as picking best of a few random control sequences) solve two-point boundary value problem

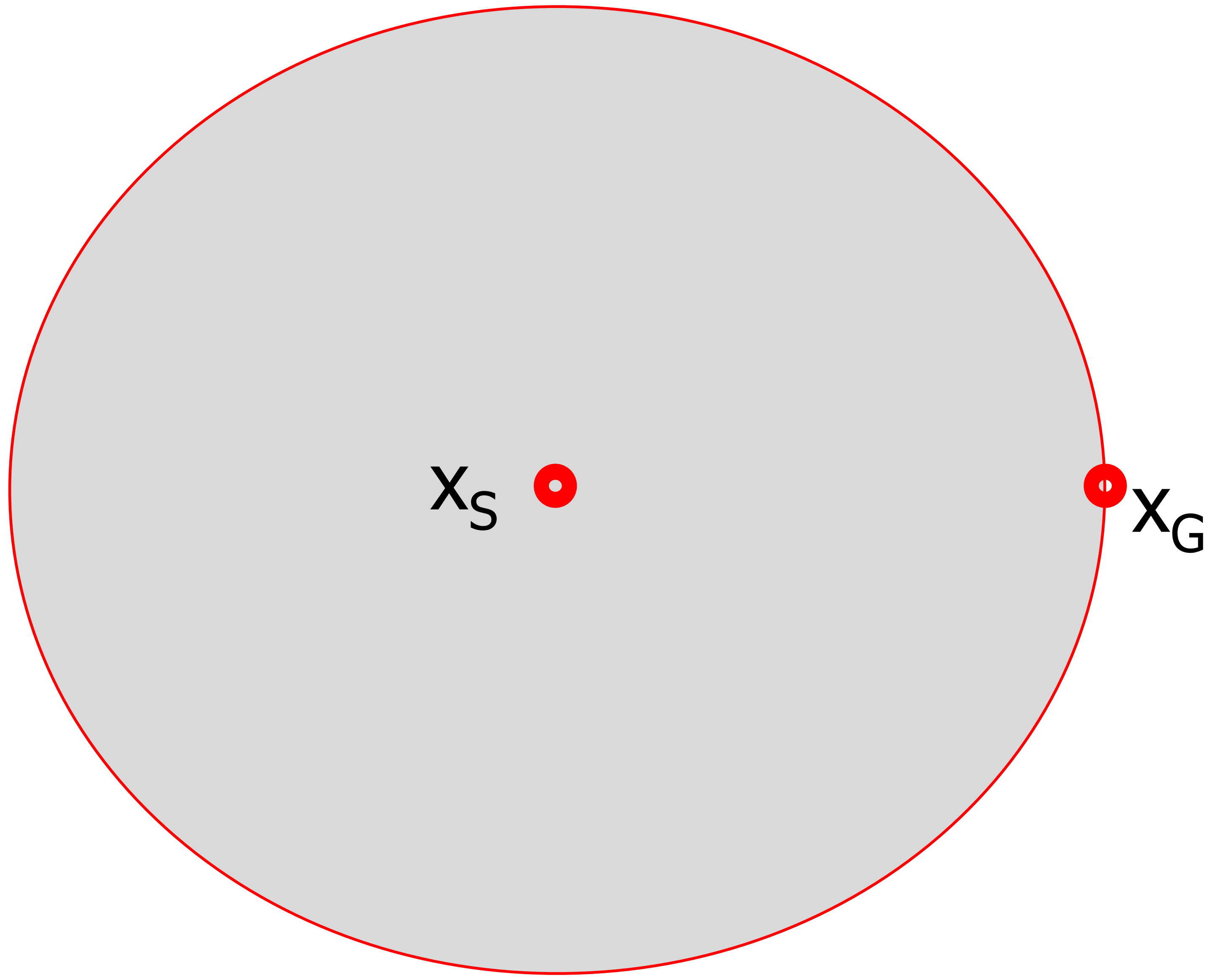




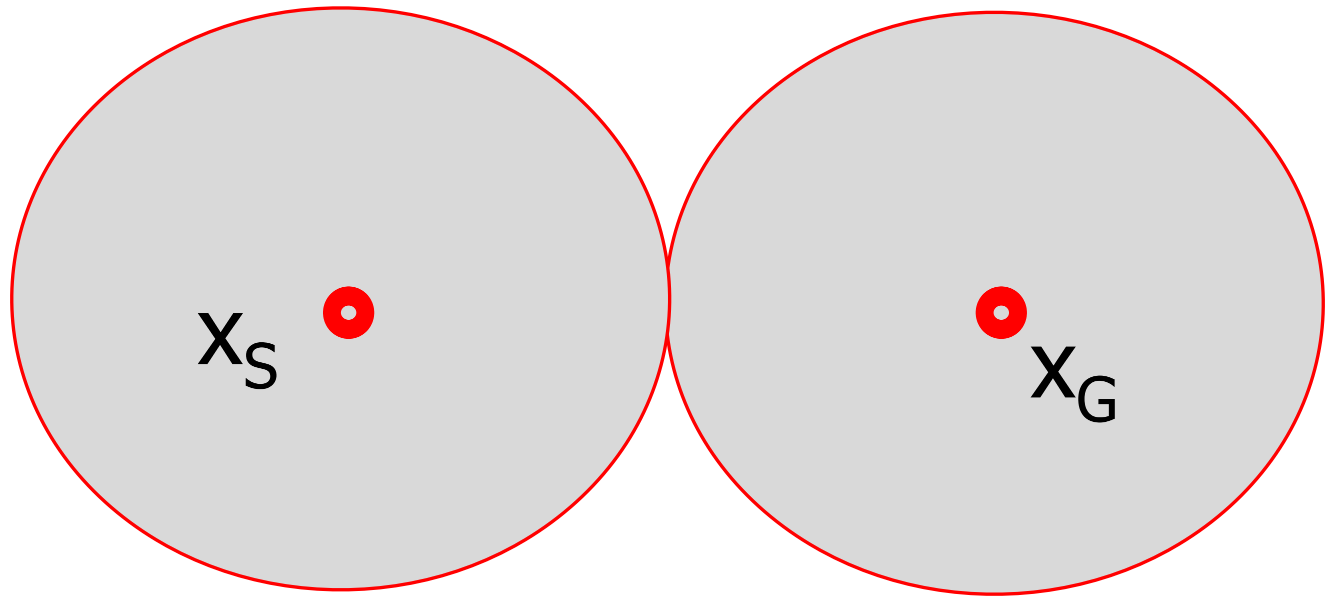
Slide borrowed from Dieter Fox

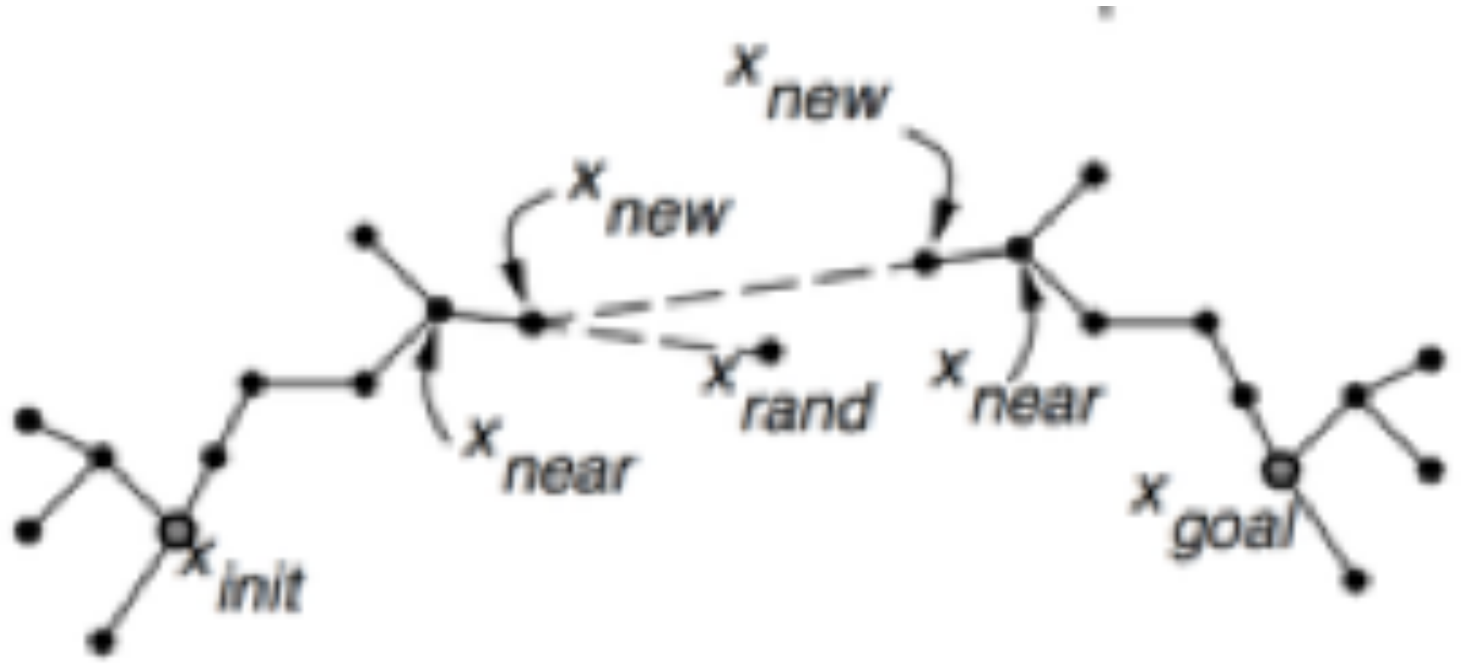
Bi-directional RRT

- Volume swept out by unidirectional RRT:



- Volume swept out by bi-directional RRT:

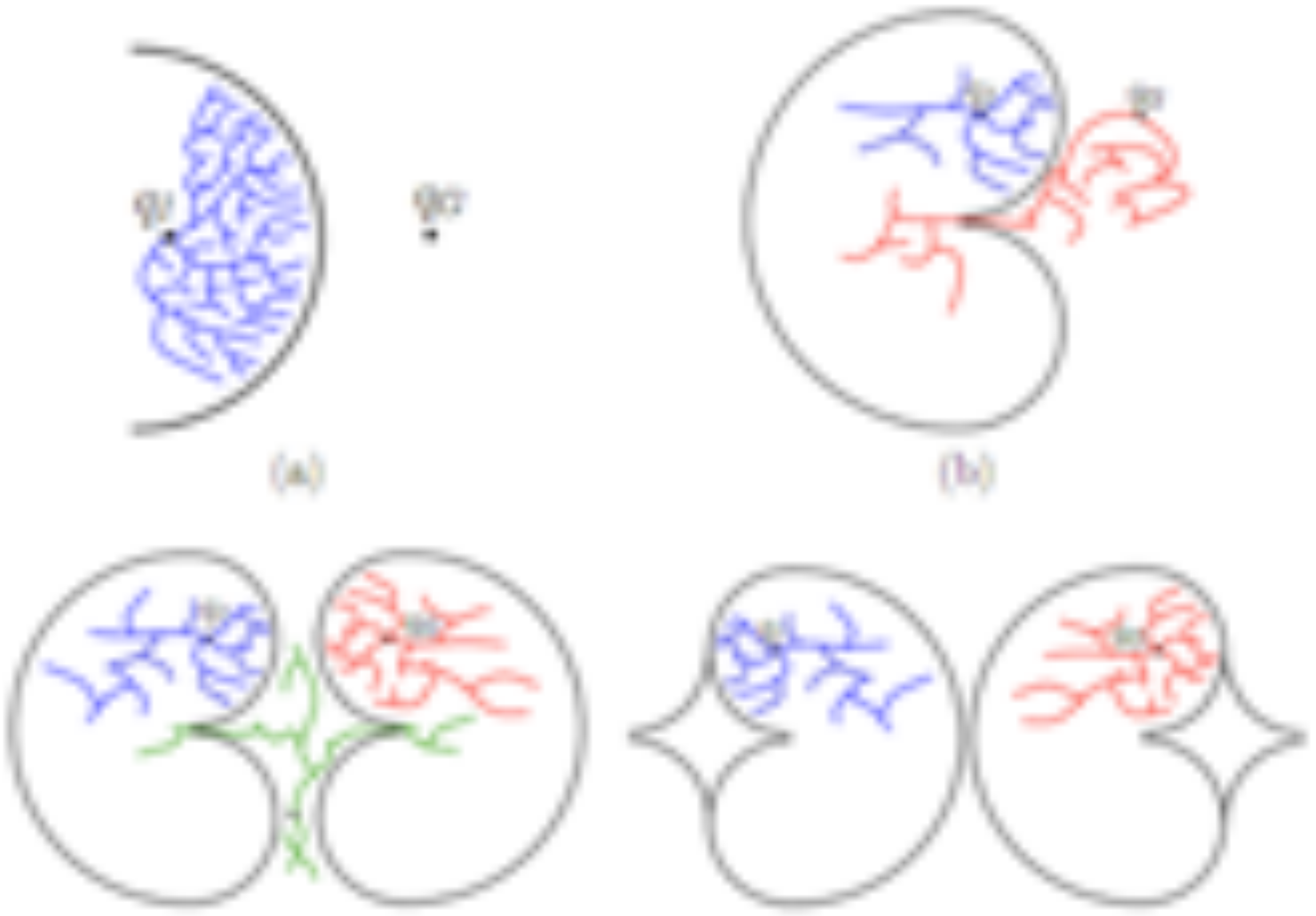




- Difference more and more pronounced as dimensionality increases

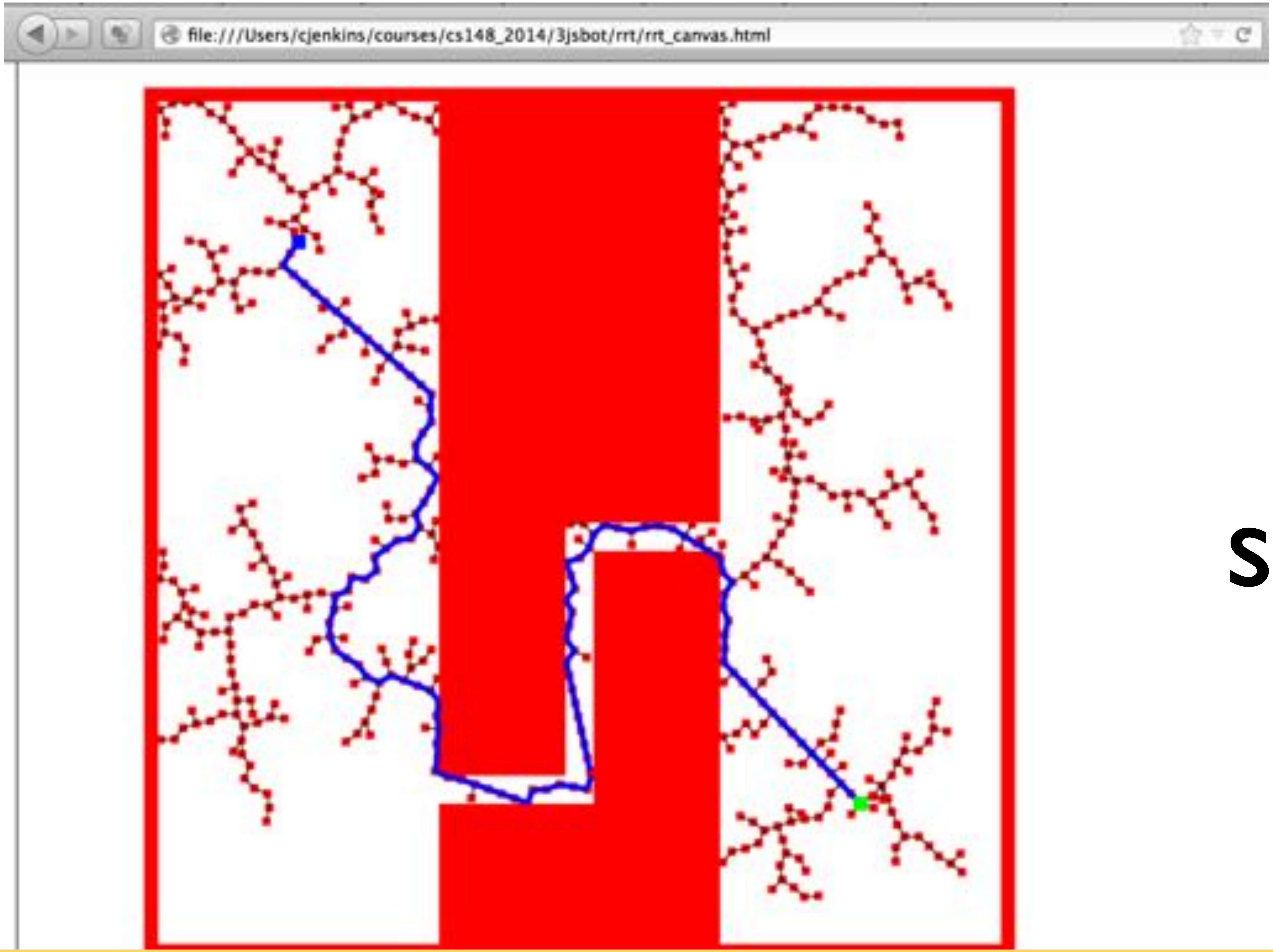
Multi-directional RRT

- Planning around obstacles or through narrow passages can often be easier in one direction than the other



(a) (b) (c) (d)

RRTs can take a lot of time...



Is there a simpler way?

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RRT*

Algorithm 6: RRT*

```
V ← {x_init}; E ← ∅;
for i = 1, ..., n do
    x_rand ← SampleFree_i;
    x_nearest ← Nearest(G = (V, E), x_rand);
    x_new ← Steer(x_nearest, x_rand) ;
    if ObtacleFree(x_nearest, x_new) then
        X_near ← Near(G = (V, E), x_new, min{γ_RRT*(log(card(V))/card(V))^(1/d), η}) ;
        V ← V ∪ {x_new};
        x_min ← x_nearest; c_min ← Cost(x_nearest) + c(Line(x_nearest, x_new));
        foreach x_near ∈ X_near do                // Connect along a minimum-cost path
            if CollisionFree(x_near, x_new) ∧ Cost(x_near) + c(Line(x_near, x_new)) < c_min then
                x_min ← x_near; c_min ← Cost(x_near) + c(Line(x_near, x_new))
        E ← E ∪ {(x_min, x_new)};
        foreach x_near ∈ X_near do                // Rewire the tree
            if CollisionFree(x_new, x_near) ∧ Cost(x_new) + c(Line(x_new, x_near)) < Cost(x_near)
            then x_parent ← Parent(x_near);
                E ← (E \ {(x_parent, x_near)}) ∪ {(x_new, x_near)}
return G = (V, E);
```

FIND x_new

ADD x_new to G
FIND neighbors to x_new in the G

FIND edge to x_new from neighbors with least cost
ADD that to G

REWIRE the edges in the neighborhood if any least cost path exists from the root to the neighbors via x_new

Source: Karaman and Frazzoli

RRT*

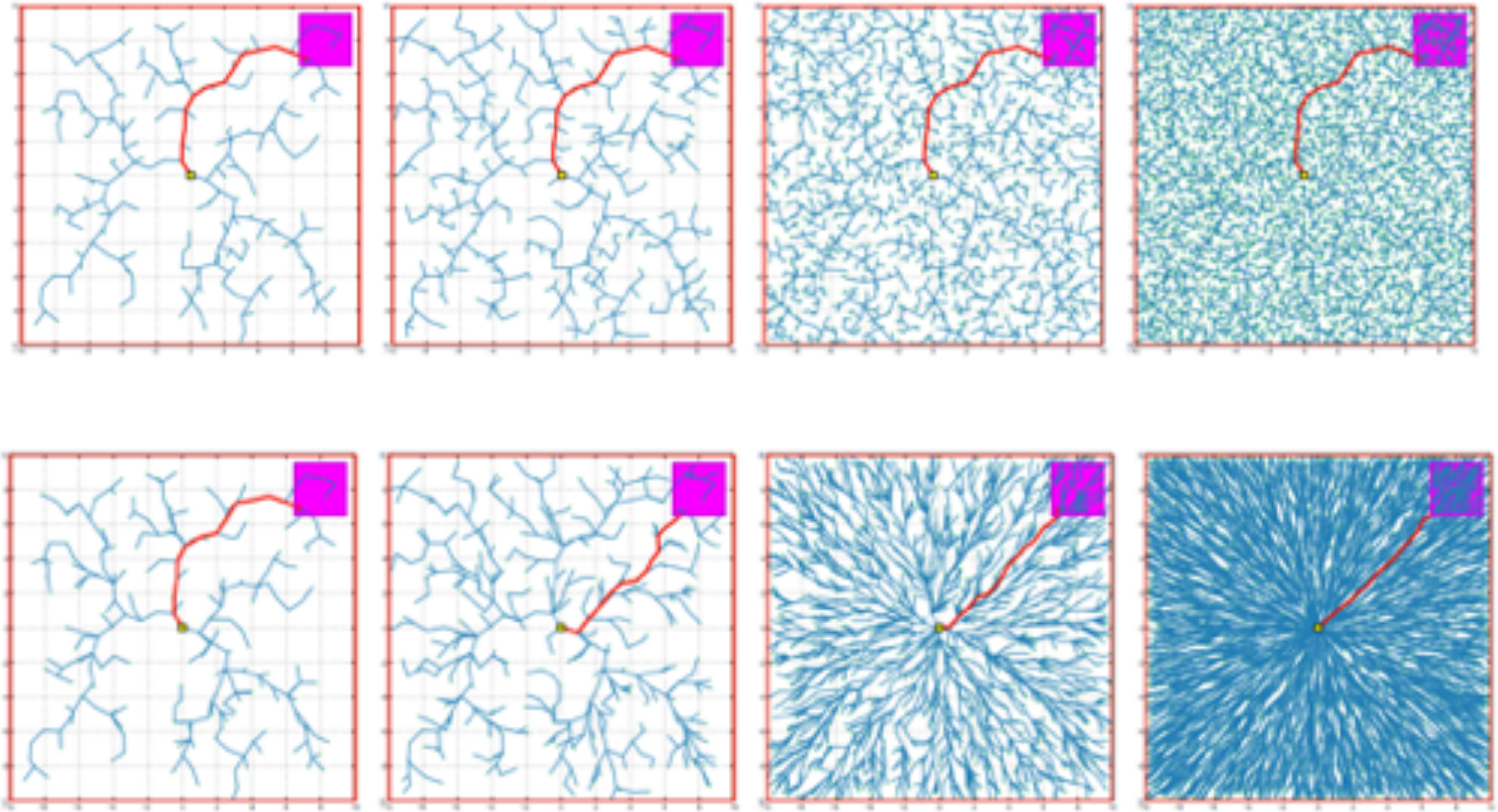
- Asymptotically optimal
- Main idea:
  - Swap new point in as parent for nearby vertices who can be reached along shorter path through new point than through their original (current) parent

Demonstration - https://demonstrations.wolfram.com/RapidlyExploringRandomTreeRRTAndRRT/

RRT*

RRT

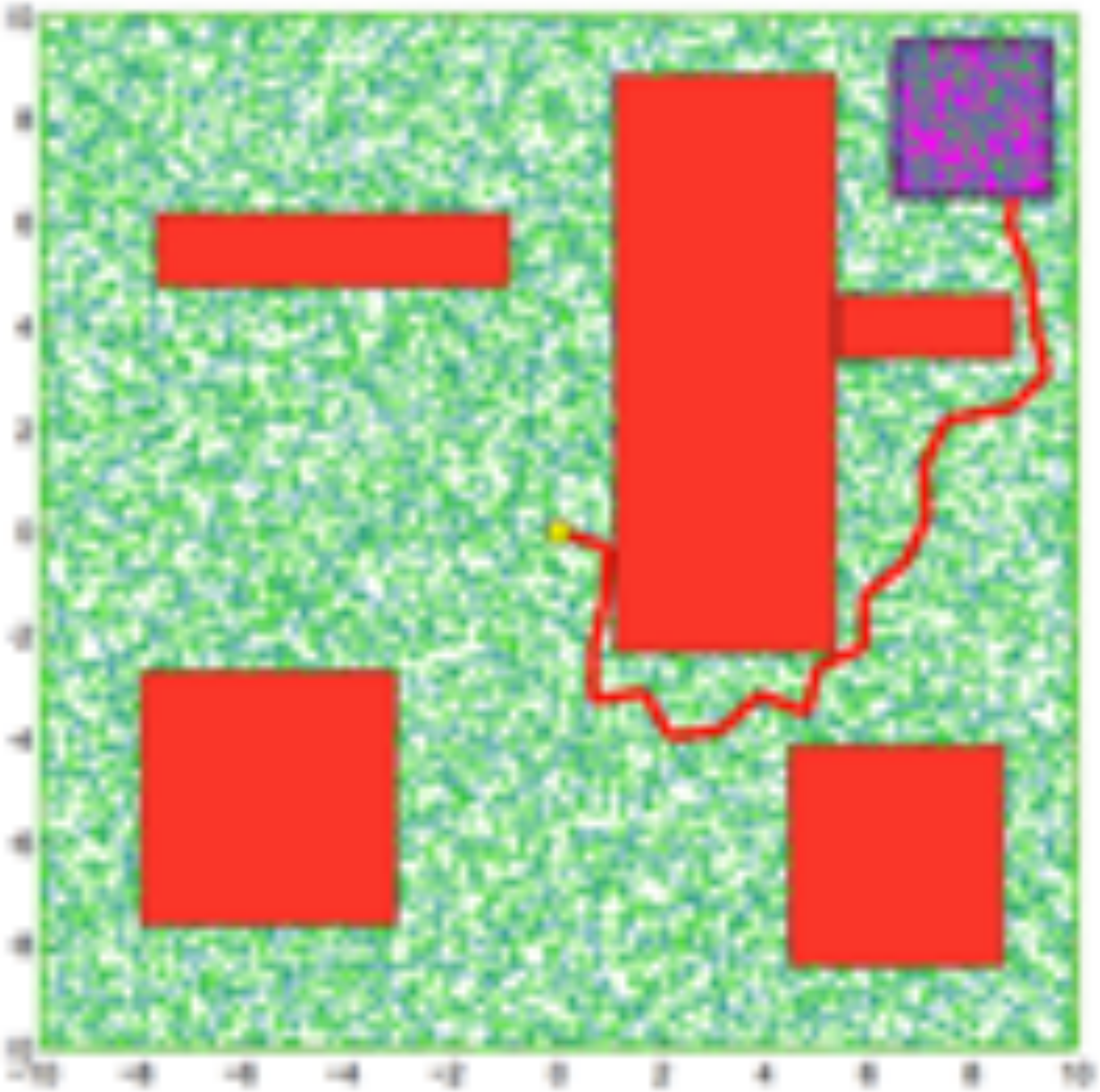
RRT*



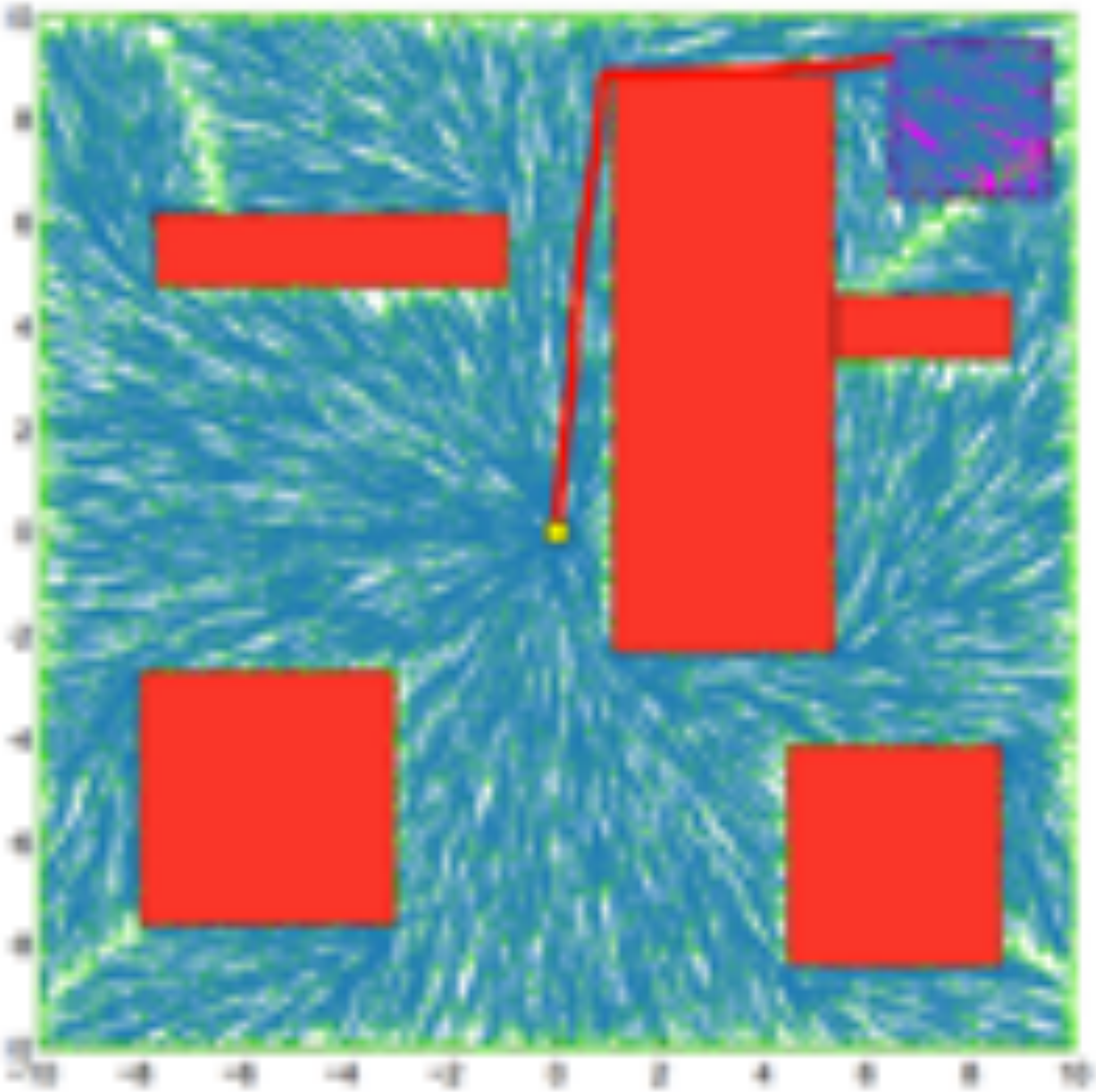
Source: Karaman and Frazzoli

RRT*

RRT



RRT*



Source: Karaman and Frazzoli

Smoothing

Randomized motion planners tend to find not so great paths for execution: very jagged, often much longer than necessary.

→ In practice: do smoothing before using the path

- Shortcutting:
  - along the found path, pick two vertices x_{t1}, x_{t2} and try to connect them directly (skipping over all intermediate vertices)
- Nonlinear optimization for optimal control
  - Allows to specify an objective function that includes smoothness in state, control, small control inputs, etc.

Additional Resources

- Marco Pavone (http://asl.stanford.edu/):
  - Sampling-based motion planning on GPUs: https://arxiv.org/pdf/1705.02403.pdf
  - Learning sampling distributions: https://arxiv.org/pdf/1709.05448.pdf
- Sidd Srinivasa (https://personalrobotics.cs.washington.edu/)
  - Batch informed trees: https://robotic-esp.com/code/bitstar/
  - Expensive edge evals: https://arxiv.org/pdf/2002.11853.pdf
  - Lazy search: https://personalrobotics.cs.washington.edu/publications/mandalika2019gls.pdf
- Michael Yip (https://www.ucsdarclab.com/)
  - Neural Motion Planners: https://www.ucsdarclab.com/neuralplanning
- Lydia Kavraki (http://www.kavrakilab.org/)
  - Motion in human workspaces: http://www.kavrakilab.org/nsf-nri-1317849.html

Next Lecture

Planning - V - Potential Fields