Lecture 11
Planning - III - Configuration Spaces

Course Logistics

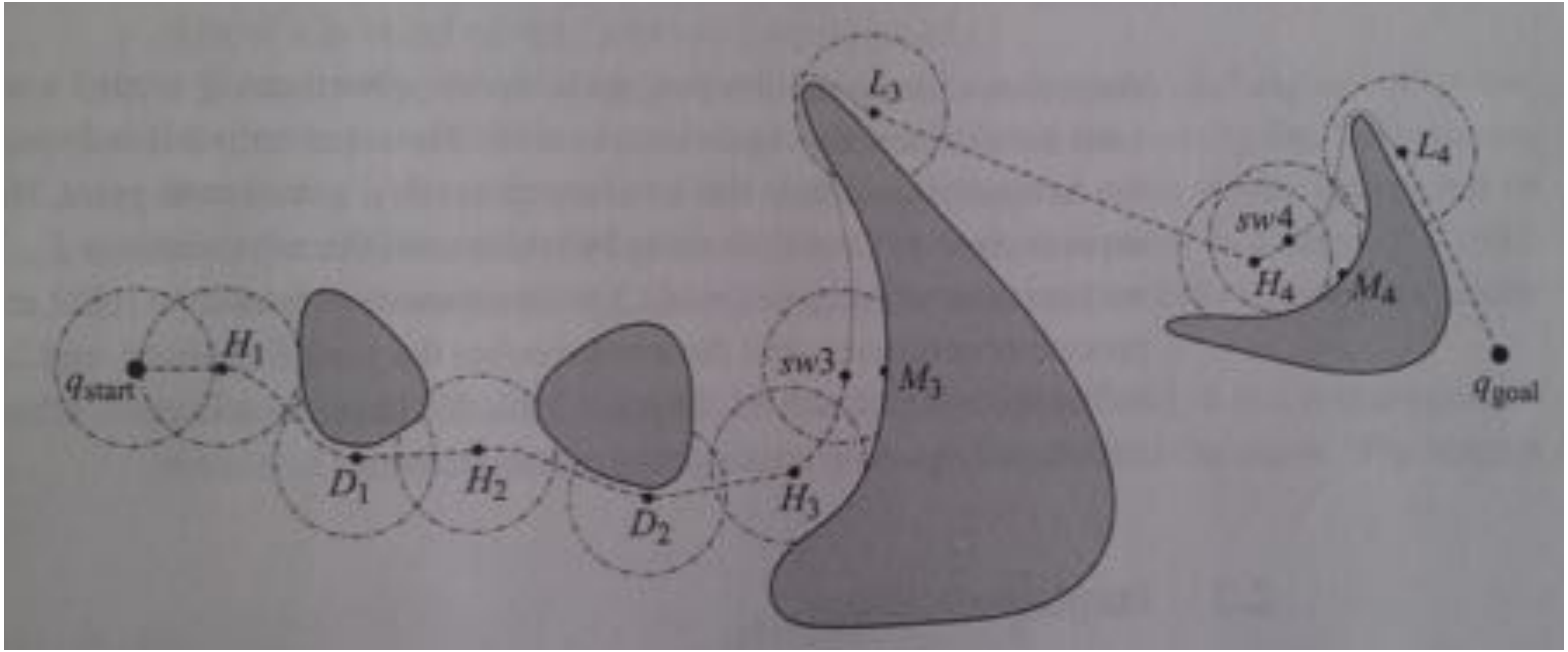
- **Quiz 9 was posted today and was due before the lecture.**
- Project 2 is posted on 10/02 and will be due 10/11 (**today**).
- Project 3 will be posted today 10/11 and will be due 10/25.
  - Start early!

Configuration Spaces

seibertron.net

more than meets the eye

Last time: Tangent Bug



What does BugX assume that Random Walk does not?

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What do search algorithms assume that BugX does not?

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What do search algorithms assume that BugX does not?

A graph of valid locations that can be traversed

Suppose we have or can build such a graph...

What does BugX assume that Random Walk does not?

Localization: knowing the robot's location, at least wrt. distance to goal

What do search algorithms assume that BugX does not?

A graph of valid locations that can be traversed

Suppose we have or can build such a graph...

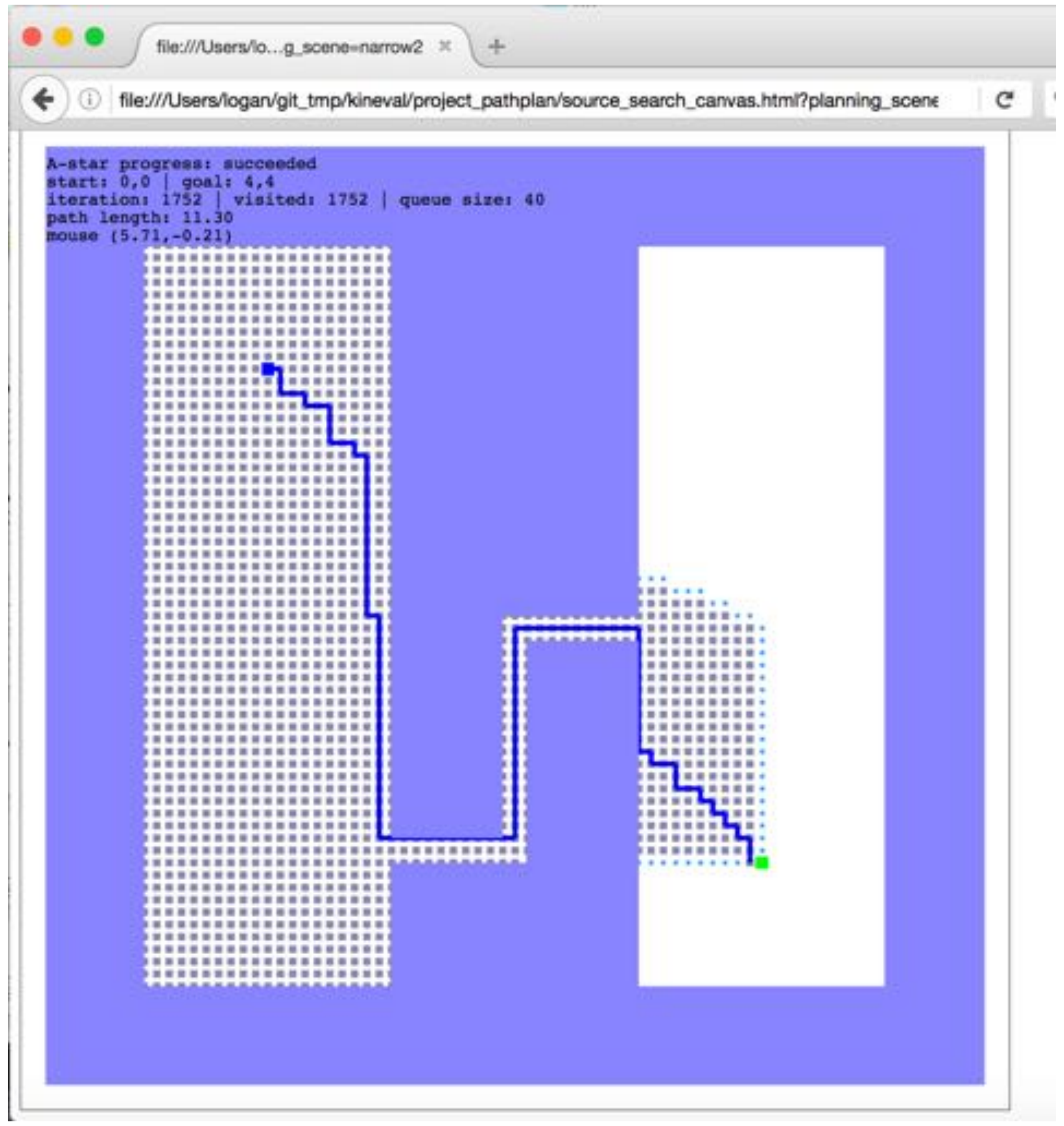
Approaches to motion planning

- Bug algorithms: Bug[0-2], Tangent Bug
- Graph Search (fixed graph)
  - Depth-first, Breadth-first, Dijkstra, A-star, Greedy best-first
- **Roadmap Search (build graph):**
  - **Probabilistic Road Maps, Rapidly-exploring Random Trees**
- Optimization (local search):
  - Gradient descent, potential fields, Wavefront

Will our current search methods apply to this robot?

2D Path Planning

N-dimensional Motion Planning



Will our current search methods apply to this robot?

<u>Assumptions:</u>

- Known graph of traversability
  - How big is this graph? How was this graph built?
- Known localization and map/obstacles
  - How do we detect collisions?
  - Is our robot just a point in workspace?
- Known link geometry
  - Does robot geometry change wrt. configuration?

Configuration Space

(or C-space)

- C-space (Q) is the space of all possible configurations (q) of a system
  - kinematics: geometry of possible configurations, without respect to physics
  - dynamics: evolution of configurations over time wrt. physics
- Each degree of freedom (q_i) is a dimension of C-space
- The span of C-space is constrained by obstacles (QO_i), joint limits, etc.

Consider some examples of configuration spaces

Configuration Space

- Consider a robot d=21 DOFs, where each DOF can take 1 of n=10 angular values
- How many configurations?

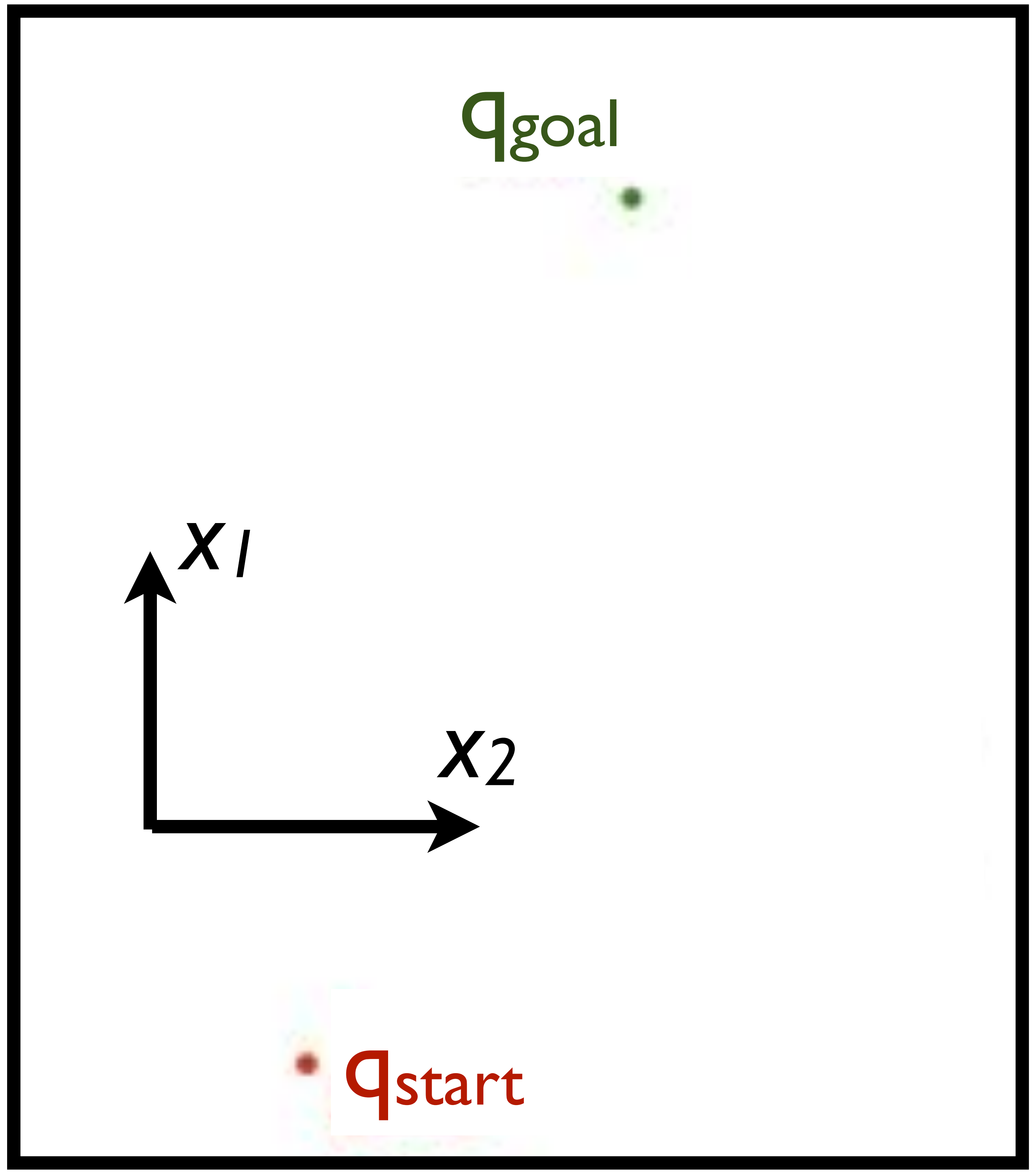
[redacted]

Configuration Space

- Consider a robot d=21 DOFs, where each DOF can take 1 of n=10 angular values
- How many configurations?
  - 10^{21}, n^d in general
- **"Curse of dimensionality"**
  - exponential growth of C-space wrt. number of DOFs
- Obstacles also create discontinuities and nonlinearities in C-space

C-space examples

- How many configurations are in the C-space of a planar point robot in a bounded rectangular world?



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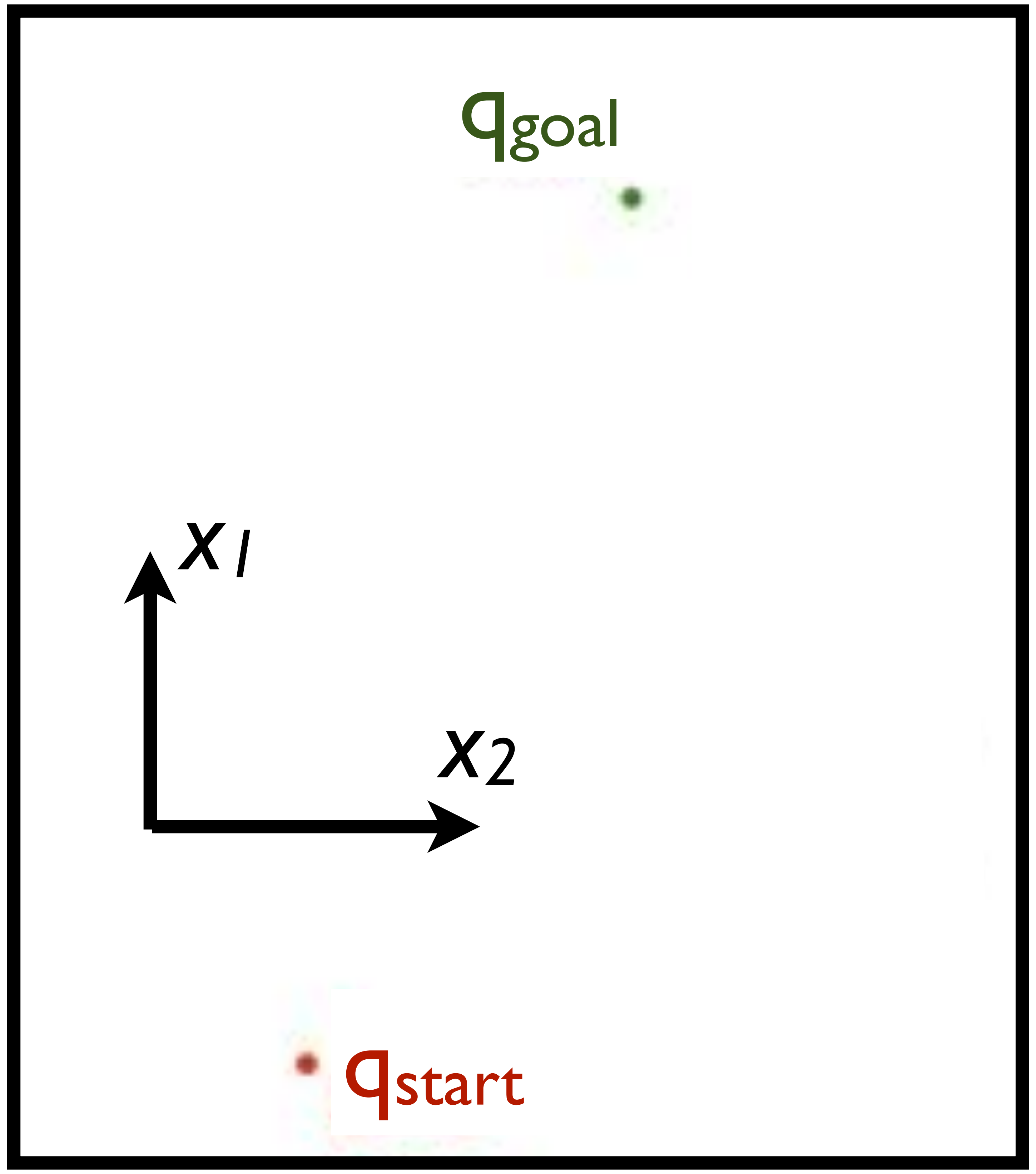
C-space examples

- How many configurations are in the C-space of a planar point robot in a bounded rectangular world?
  - DOFs: 2, $\{x_1, x_2\}$
  - Number of poses is infinite
  - C-space: $\Re^2$

Toplogically, this C-space is a homeomorphism of $\Re^2$



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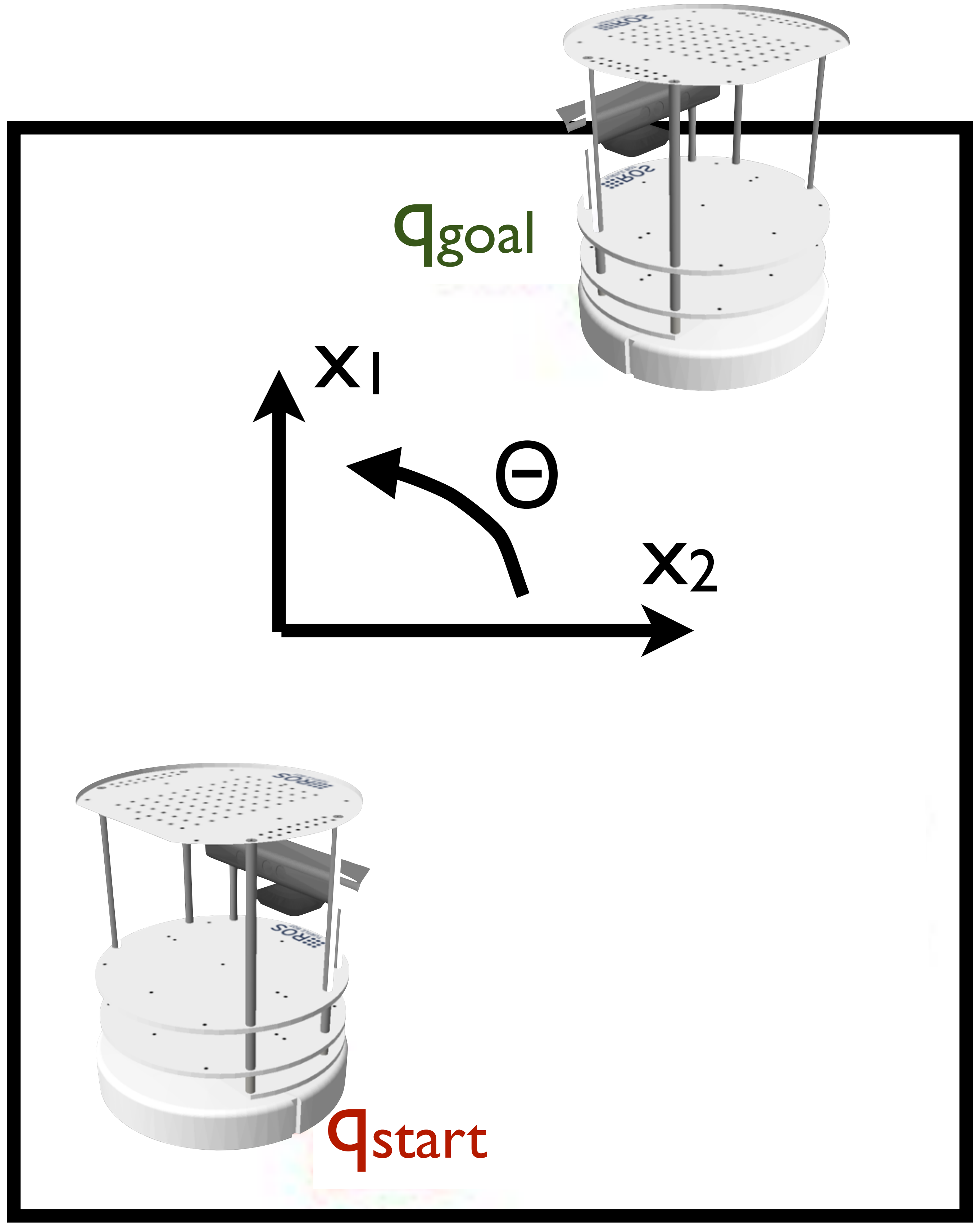
C-space examples

- What is the C-space of a Turtlebot?

[redacted]



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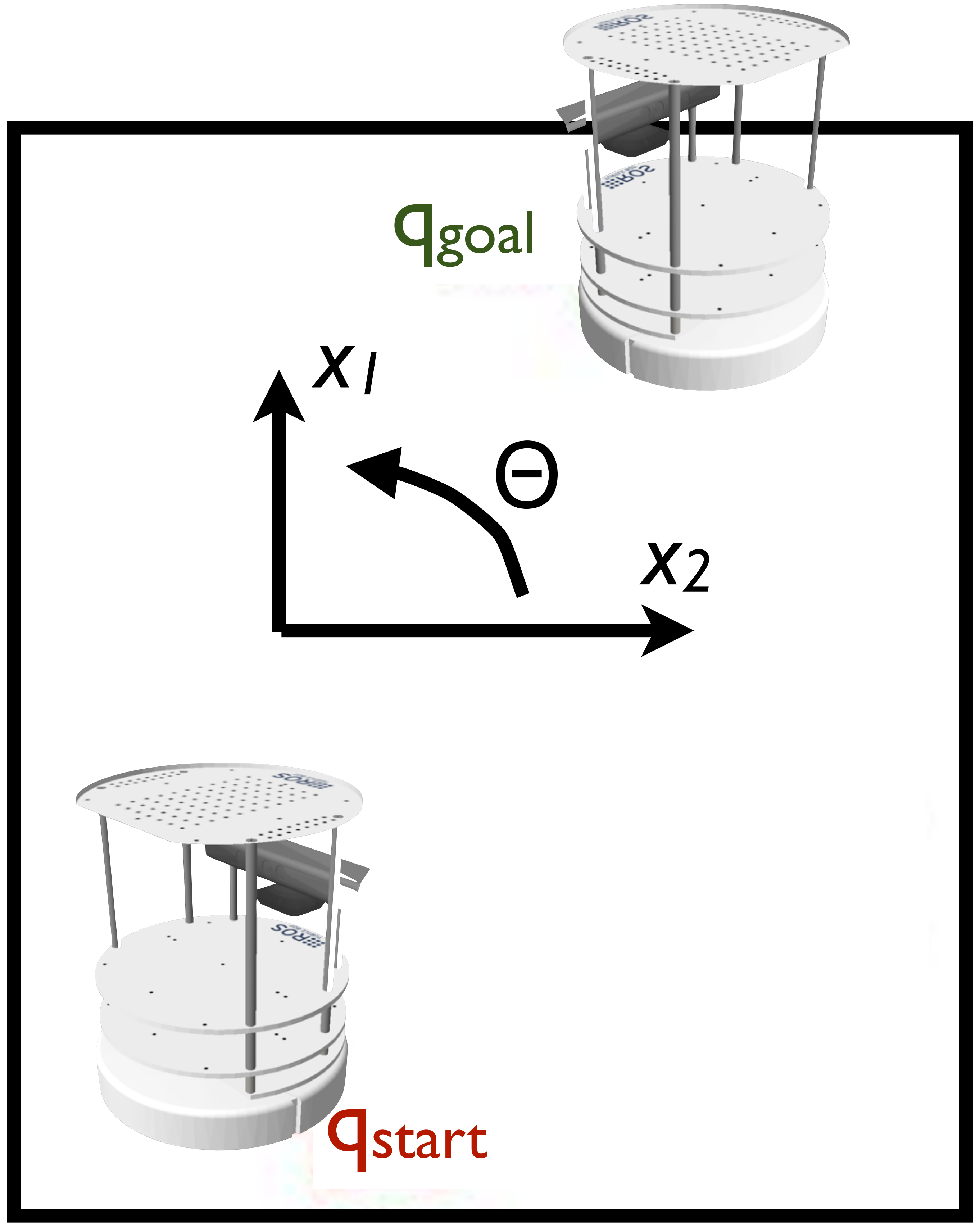
C-space examples

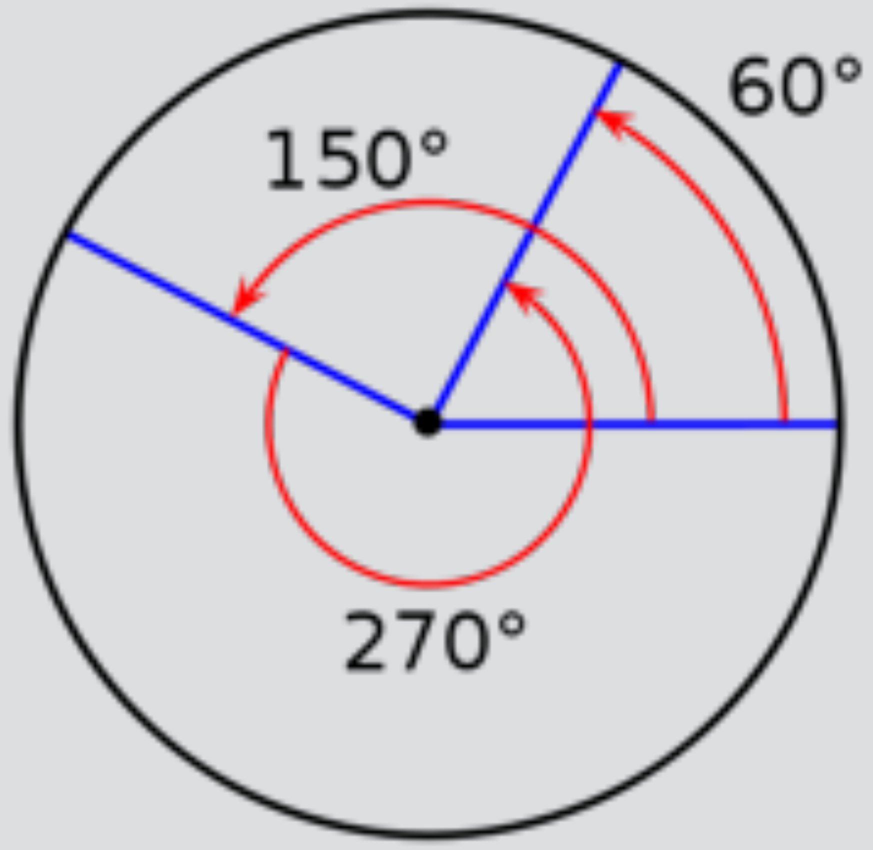
- What is the C-space of a Turtlebot?
  - DOFs: 3, $\{x_1, x_2, \Theta\}$
  - C-space: $\Re^2 \times S^1$



S^1 is the 1-sphere group of 1D rotations

S^n is the n-sphere

$S^1 \times S^1 \neq S^n$



C-space examples

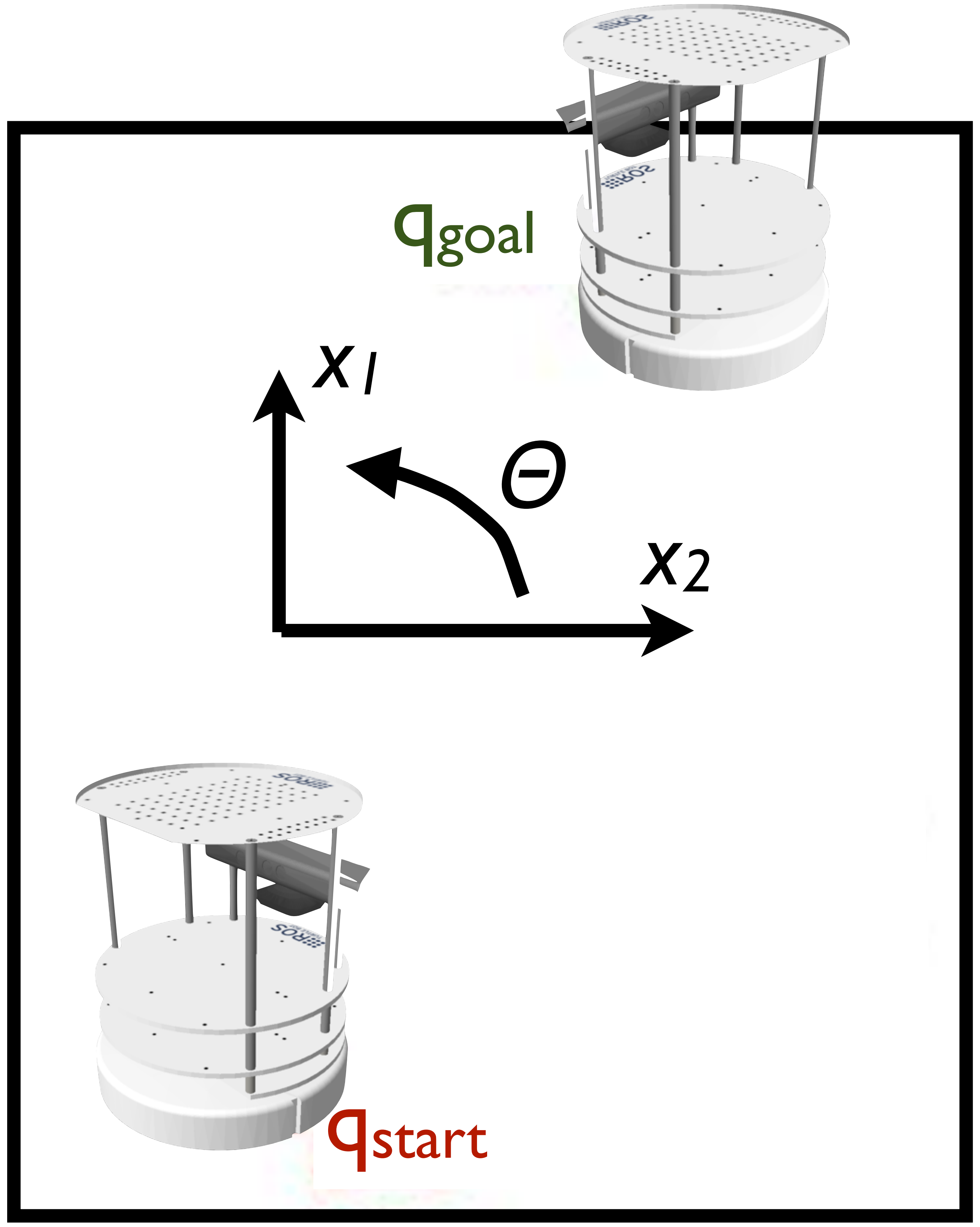
- What is the C-space of a Turtlebot?
  - DOFs: 3, $\{x_1, x_2, \Theta\}$
  - C-space: $\Re^2 \times S^1$

2D translation

rotation in 2D

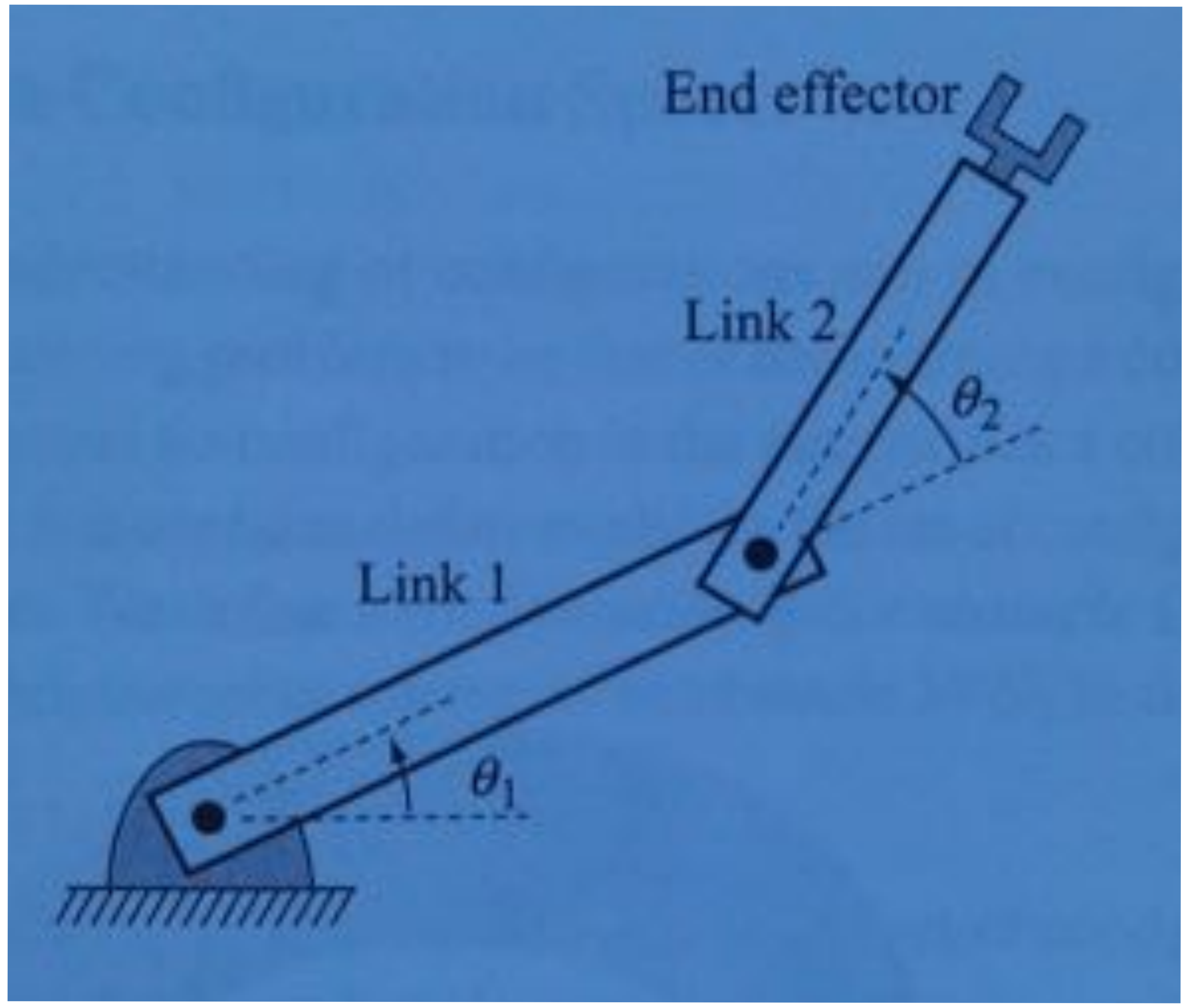
$\Re^2 \times S^1$ is also known as the *SE(2)* group.

Group of homogeneous transformations in 2D



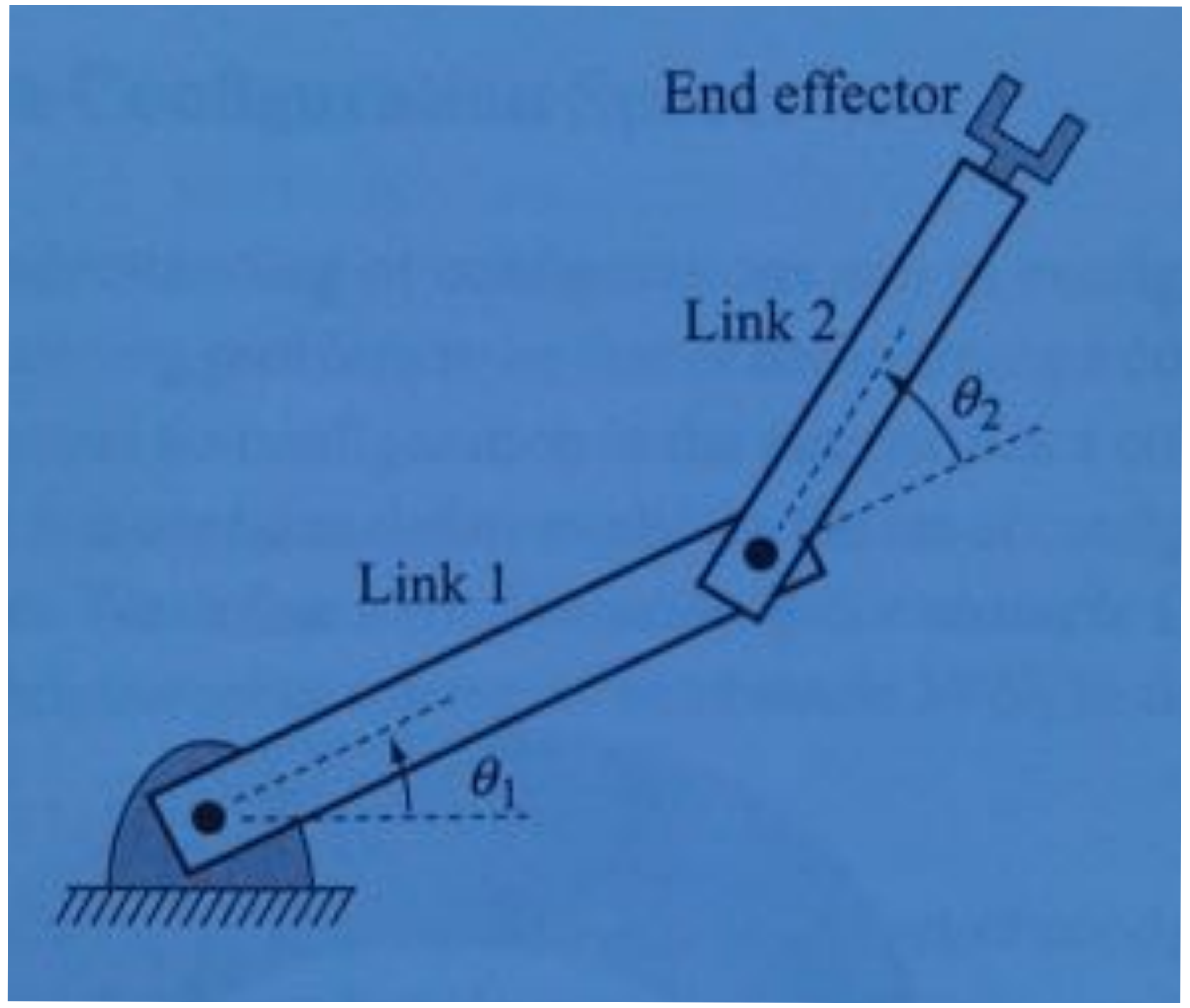
C-space examples

- What is the C-space of a planar arm with 2 rotational joints?



C-space examples

- What is the C-space of a planar arm with 2 rotational joints?
  - DOFs: [redacted]
  - C-space: [redacted]



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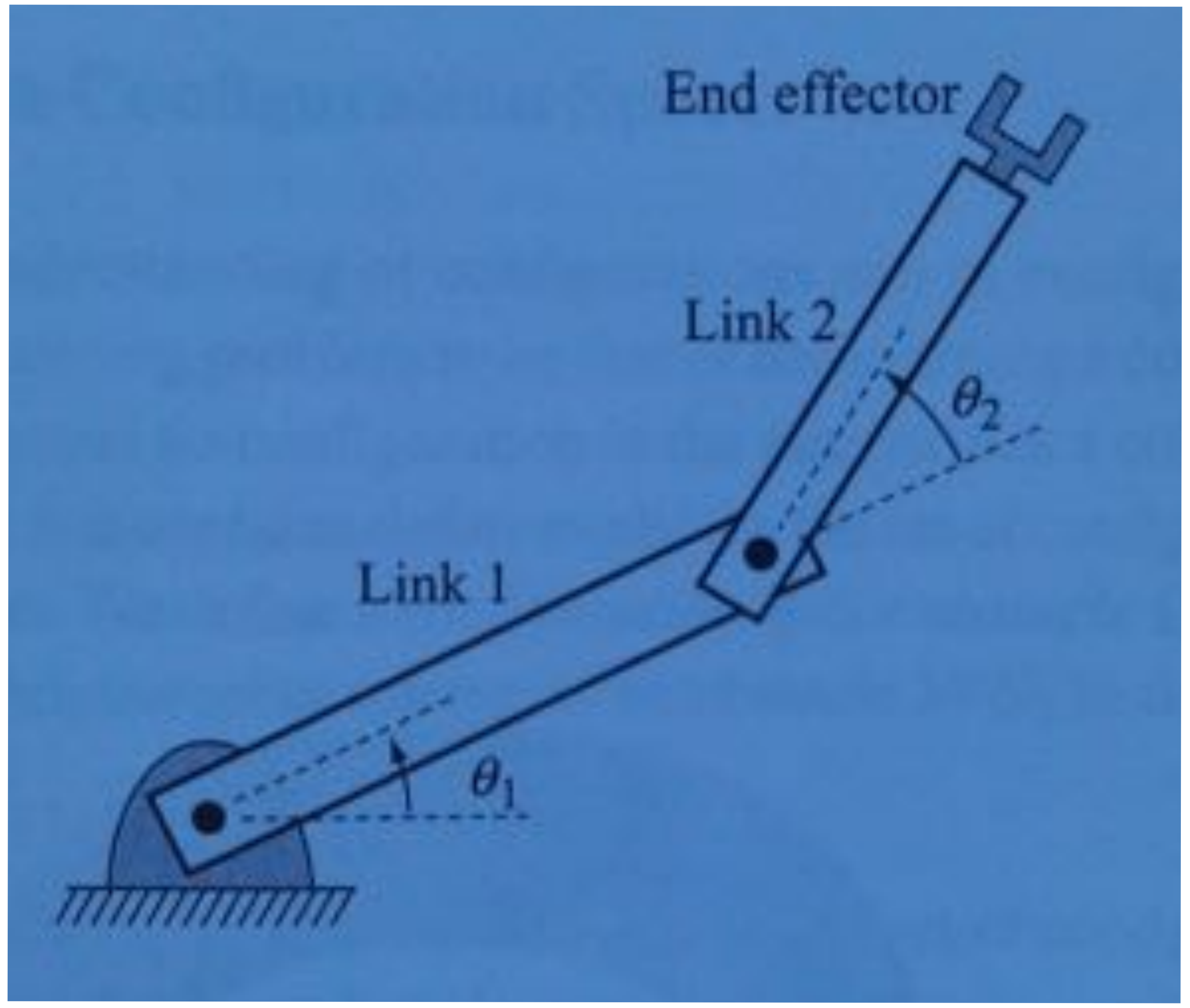
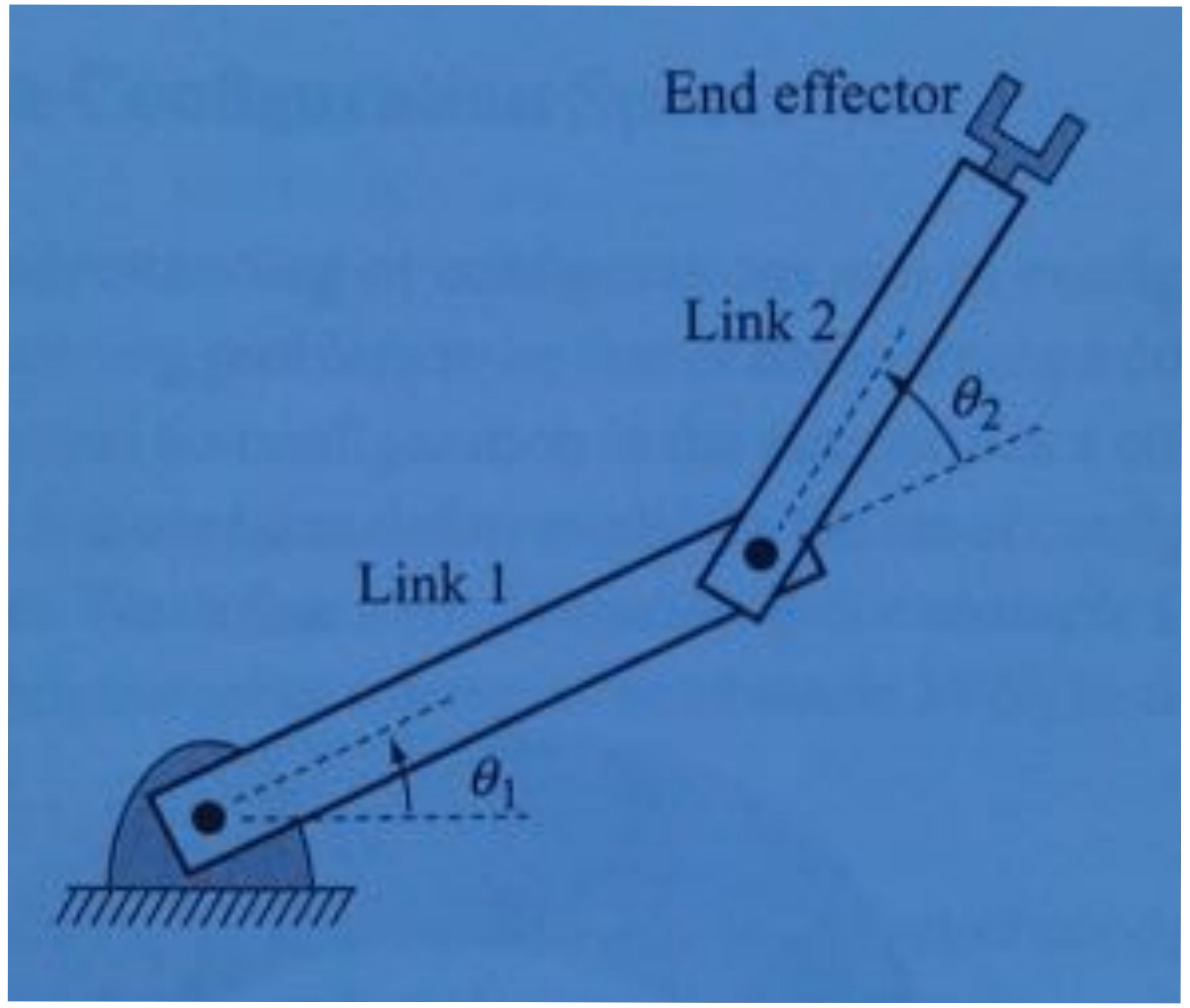
C-space examples

- What is the C-space of a planar arm with 2 rotational joints?
  - DOFs: 2, $\{\Theta_1, \Theta_2\}$
  - C-space: $\Re^2$ or S^2 or $S^1 \times S^1$?

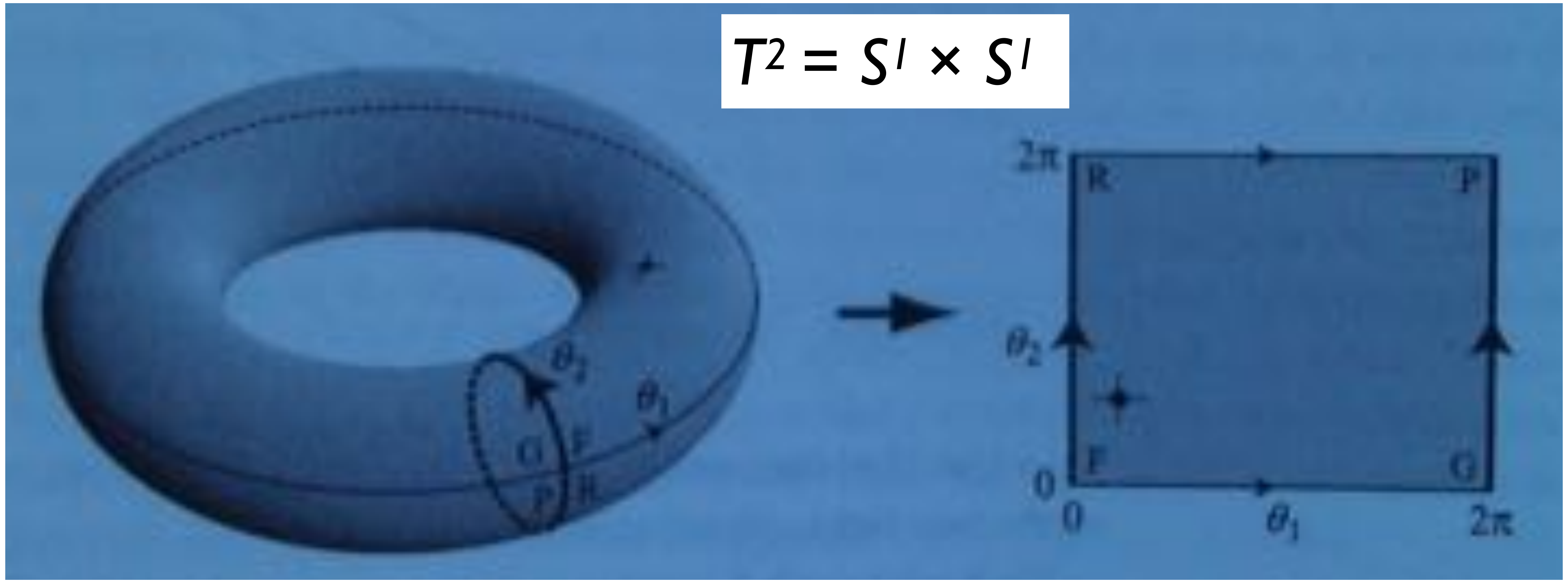
$S^1 \times S^1 = S^2$ when torus axis on surface



T^2 Torus Group



Space must fuse on each DOF where $2\pi = 0$



T^n is the torus group for an N-D rotational system

$$\mathbb{T}^n = \underbrace{S^1 \times S^1 \times \cdots \times S^1}_{n}$$

C-space examples

- What is the C-space of a Barrett WAM arm with 4 rotational joints, not including fingers of gripper?

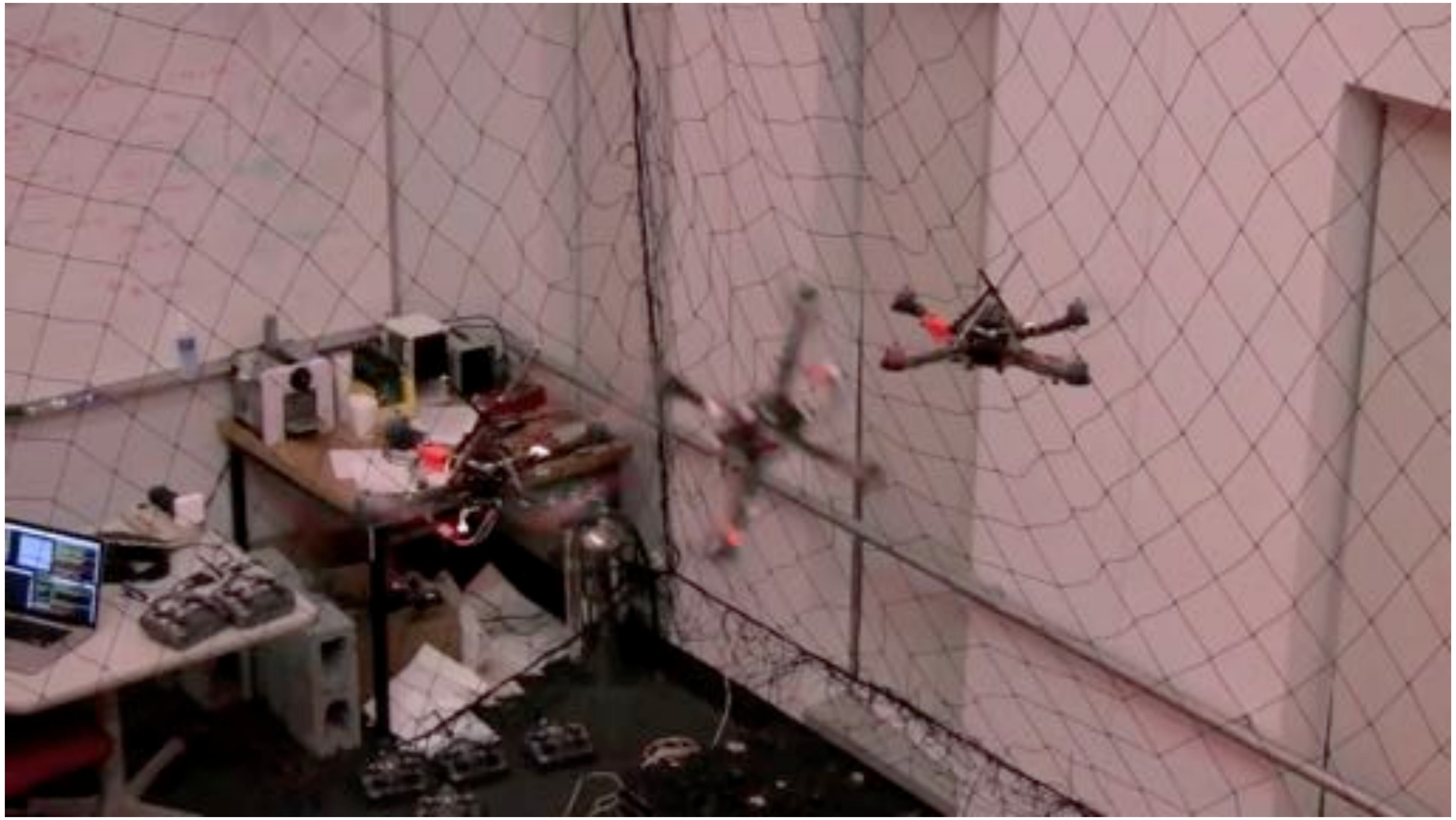
[redacted]

C-space examples

- What is the C-space of a Barrett WAM arm with 4 rotational joints, not including fingers of gripper?
  - DOFs: 4
  - C-space: T^4

C-space examples

- What is the C-space of a quad rotor helicopter?



V. Kumar et al. (2010) - UPenn - https://www.youtube.com/watch?v=MvRTALJp8DM

Drone Racing League (2017) https://www.youtube.com/watch?v=NpgGQCv642o

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IROS 2017 Autonomous Drone Racing Competition
https://www.youtube.com/watch?v=y1DvYkPCnmM

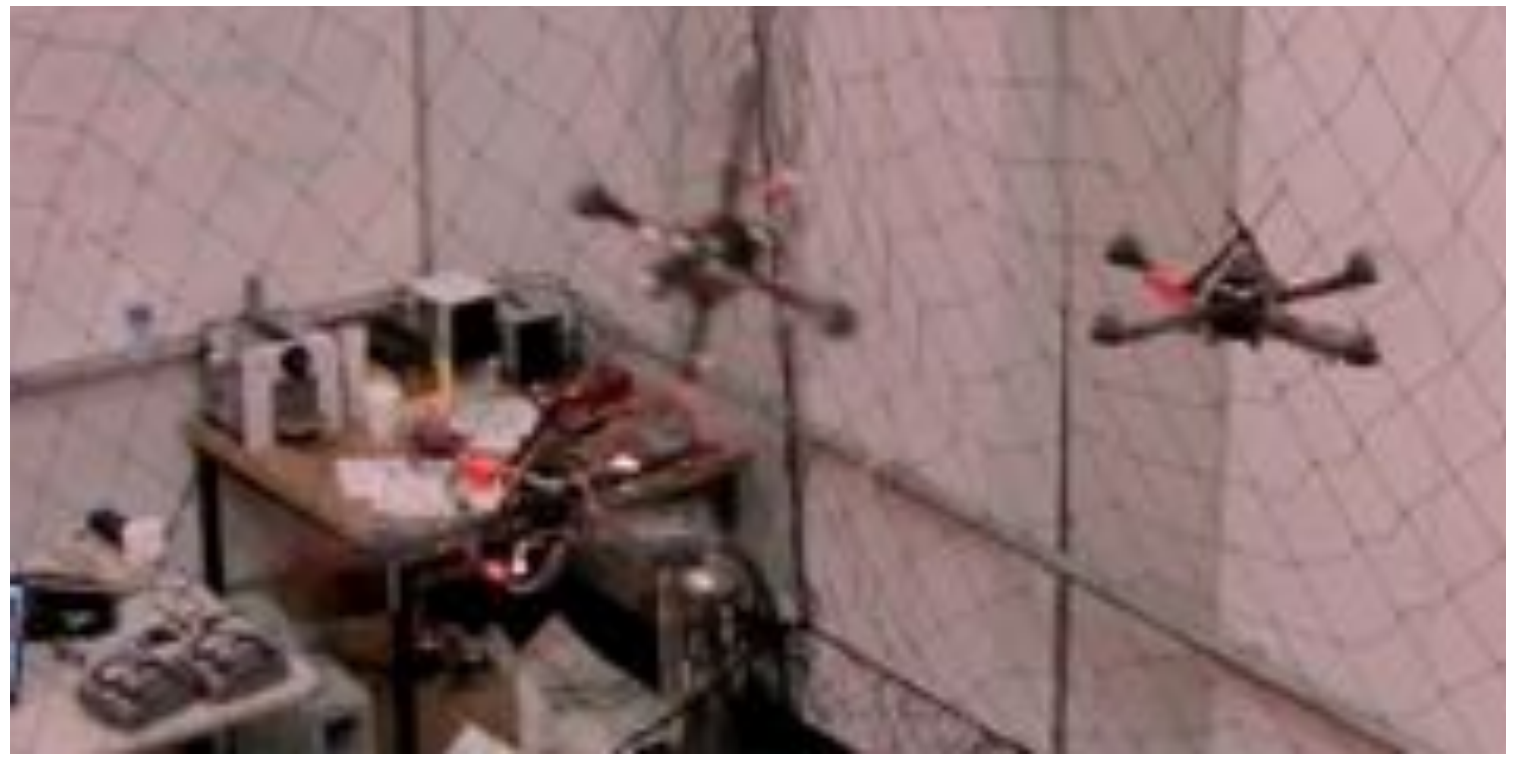
C-space examples

- What is the C-space of a quad rotor helicopter?
- DOFs: 6
- C-space: *SE(3)*,
  - or $\Re^3 \times SO(3)$

Group of homogeneous transformations in 3D

3D translation

3D rotation



V. Kumar et al. - UPenn

SE(3) combines:
$\Re^3$: 3D translation and
SO(3): 3D rotation

$SO(3) = S^1 \times S^1 \times S^1$

C-space examples

- What is the C-space of a Fetch robot, not including grippers?

C-space examples

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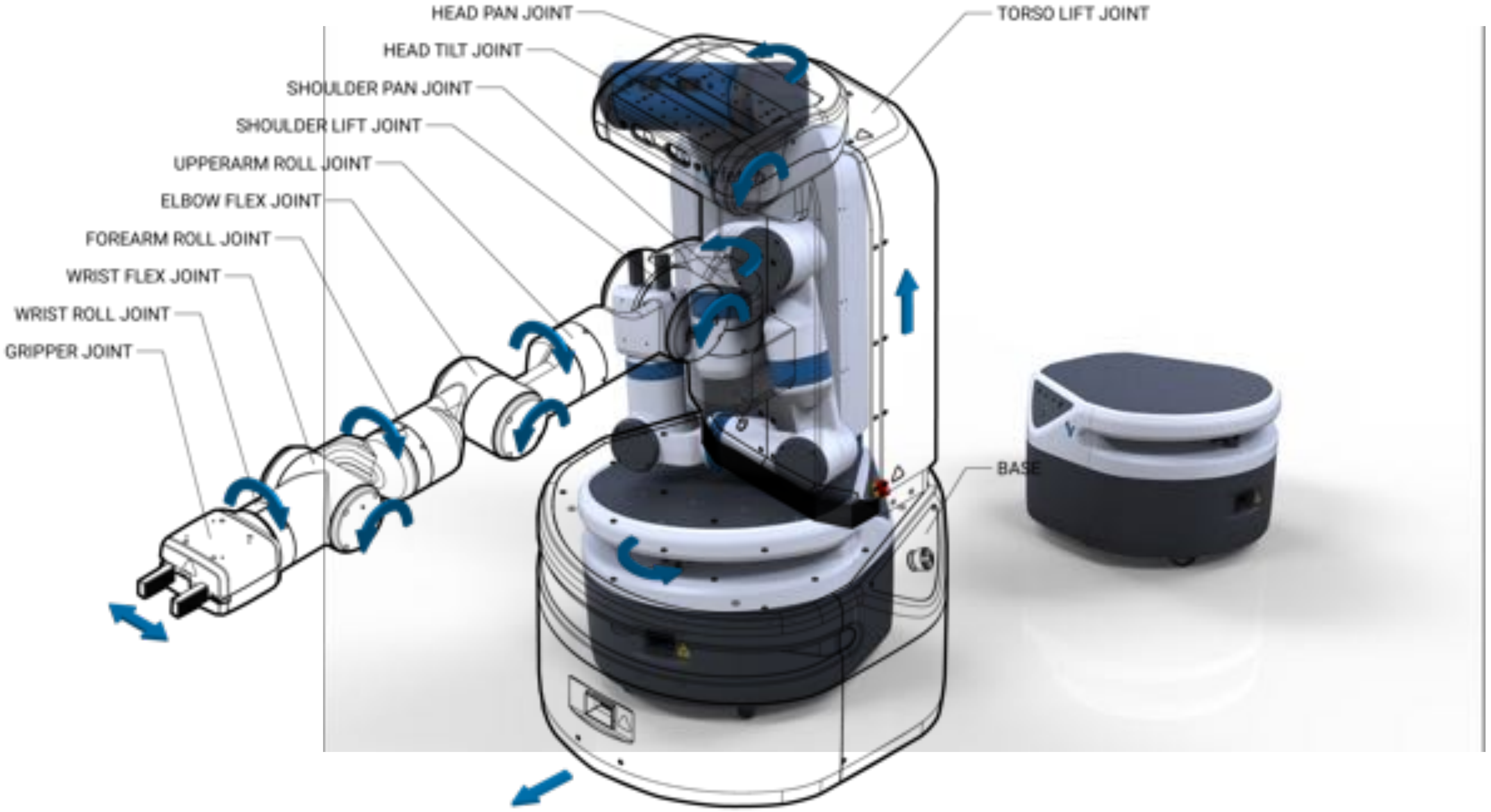

C-space examples

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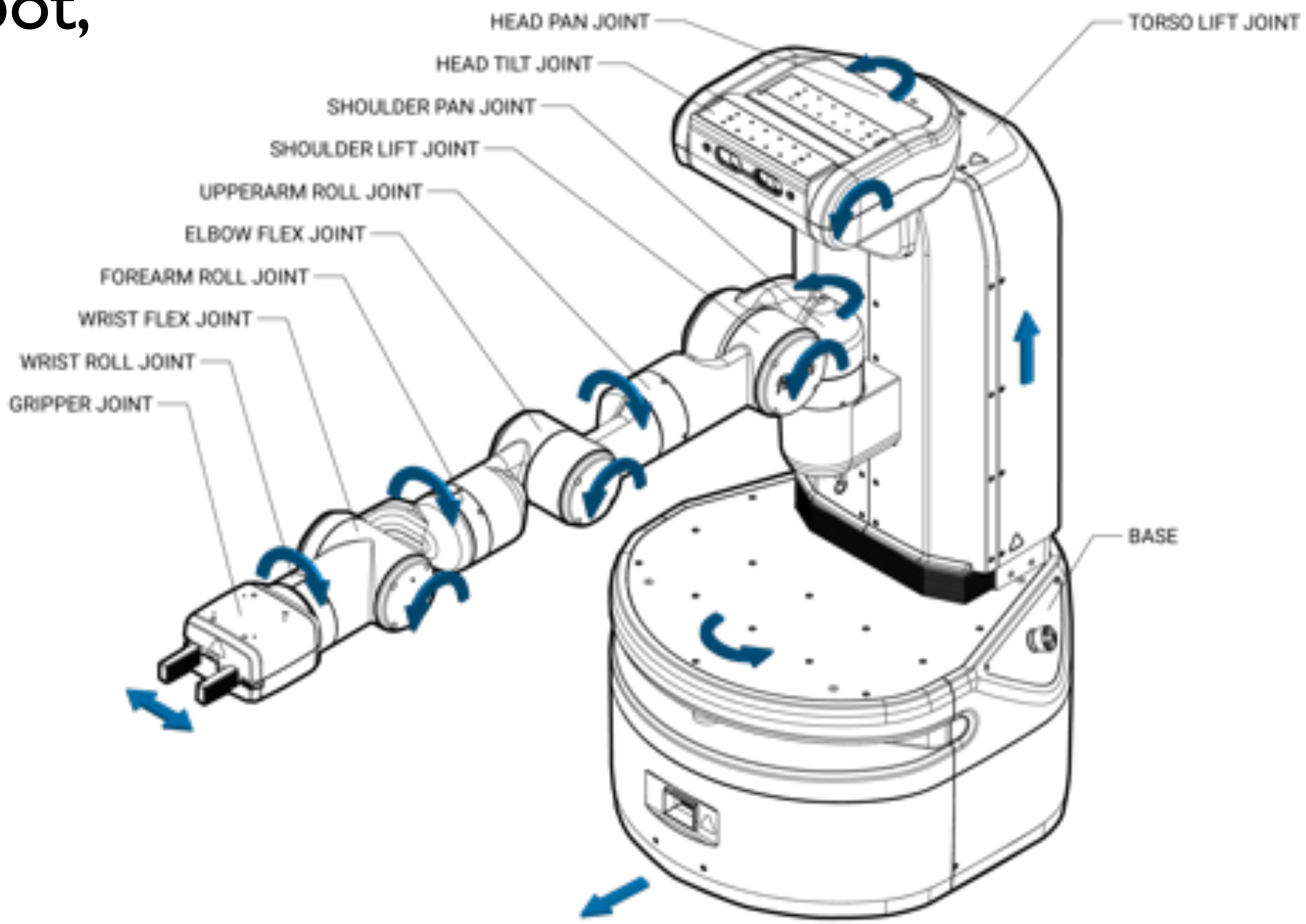

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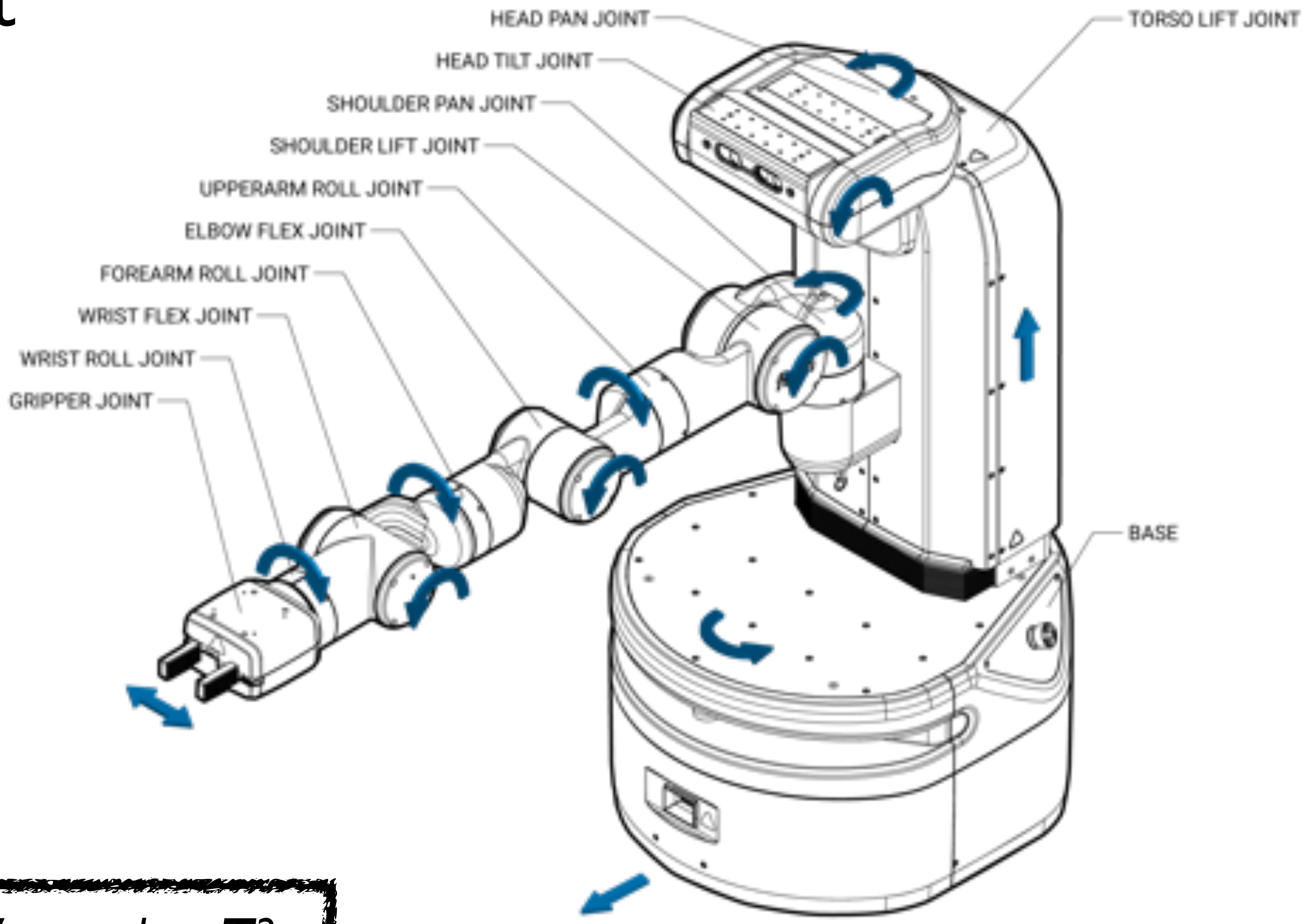
C-space examples

- What is the C-space of a Fetch, not including grippers?
- DOFs: 13
  - 3 in base: $SE(2)$
  - 7 in arm: T^7
  - 1 in the spine: $\Re^1$
  - 2 in neck: T^2

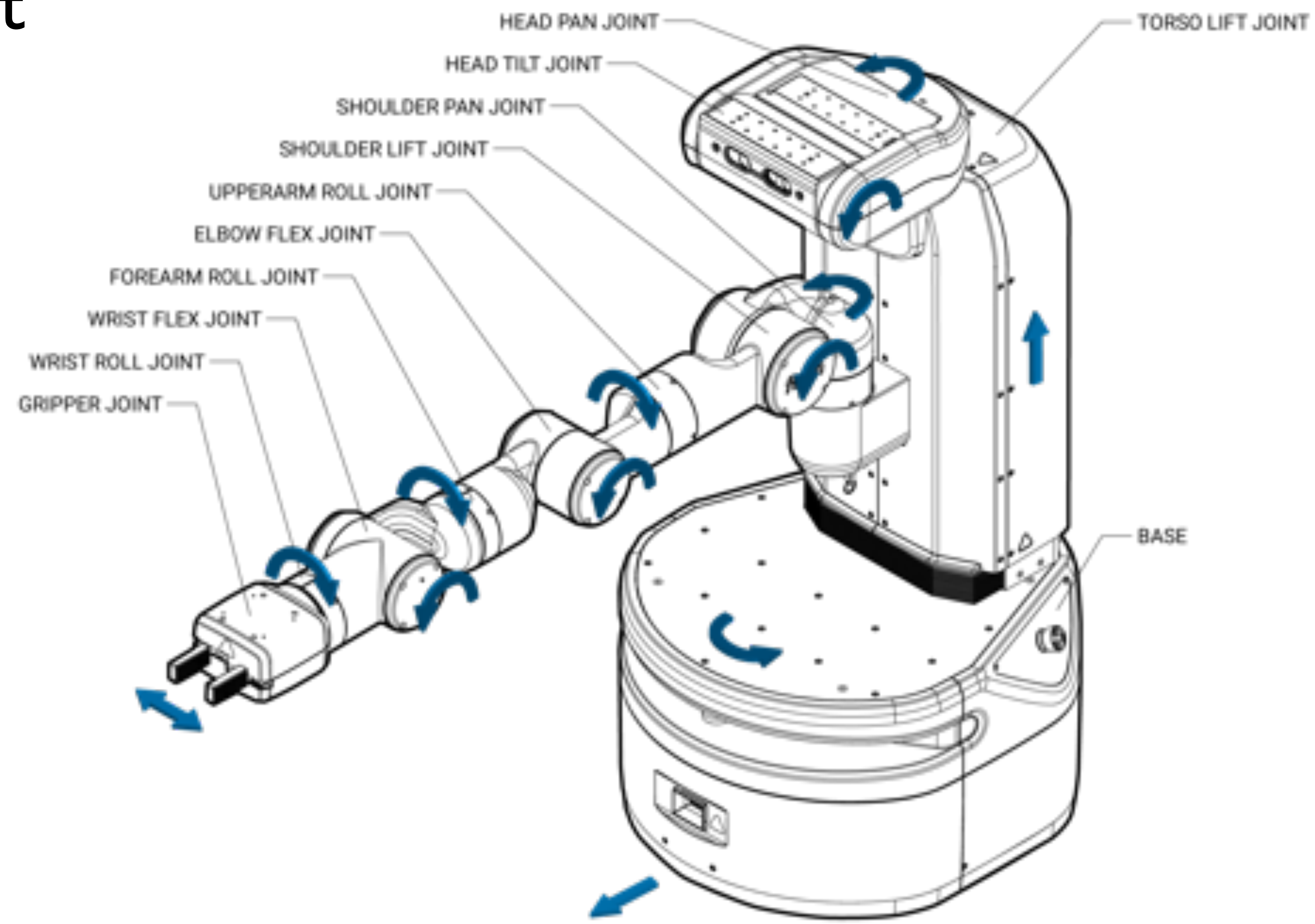


$\boxed{\text{C-space: } SE(2) \times T^7 \times \Re^1 \times T^2}$

Did we get this wrong?

- What is the C-space of a Fetch, not including grippers?
- DOFs: 13
  - 3 in base: $SE(2)$
  - 7 in arm: $\cancel{T^7}$
  - 1 in the spine: $\Re^1$
  - 2 in neck: $\cancel{T^2}$

Consider joint limits



C-space with joint limits

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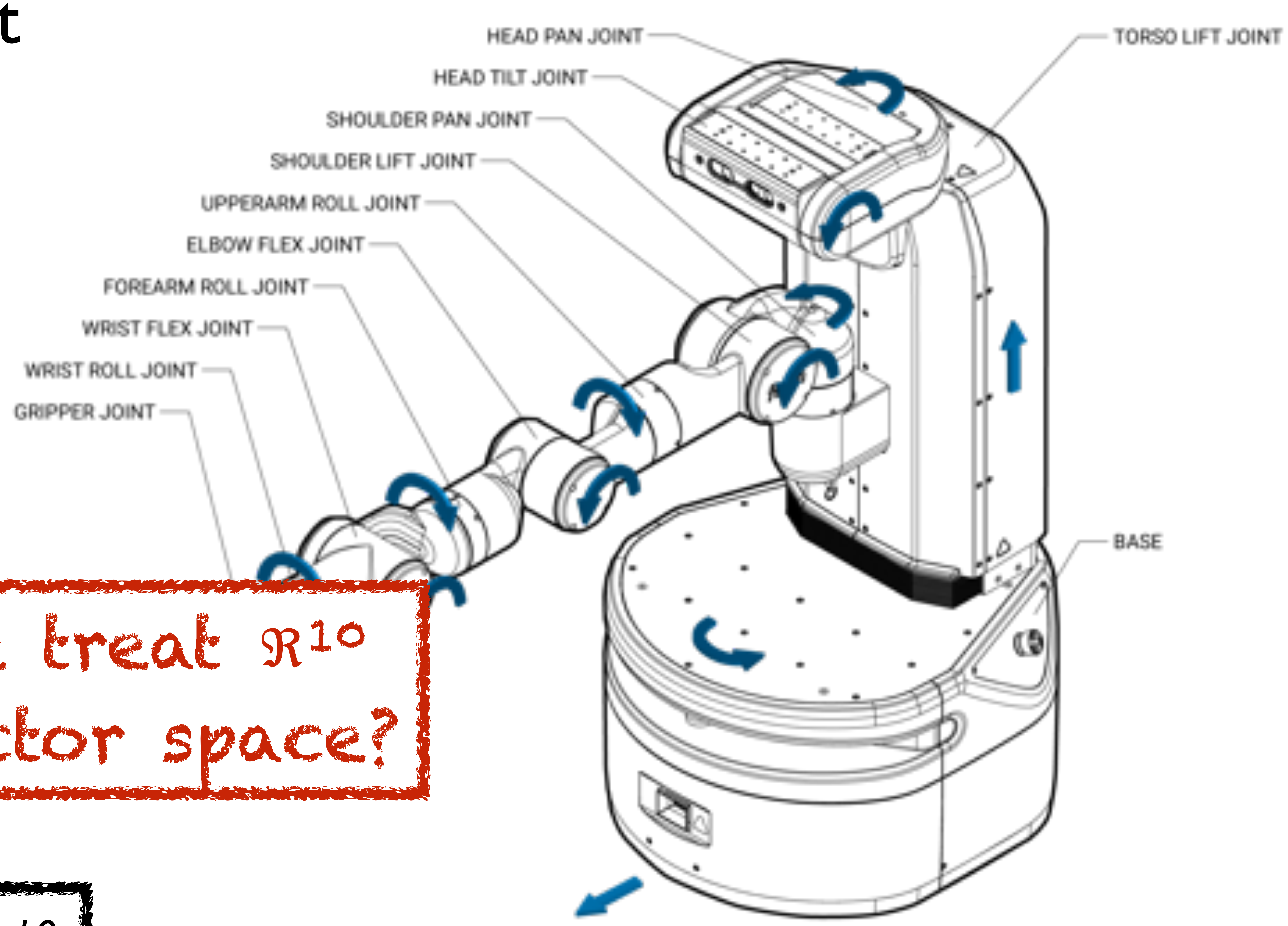
Can we treat $\Re^{10}$ as a vector space?

C-space: $SE(2) \times \Re^{10}$



C-space with joint limits

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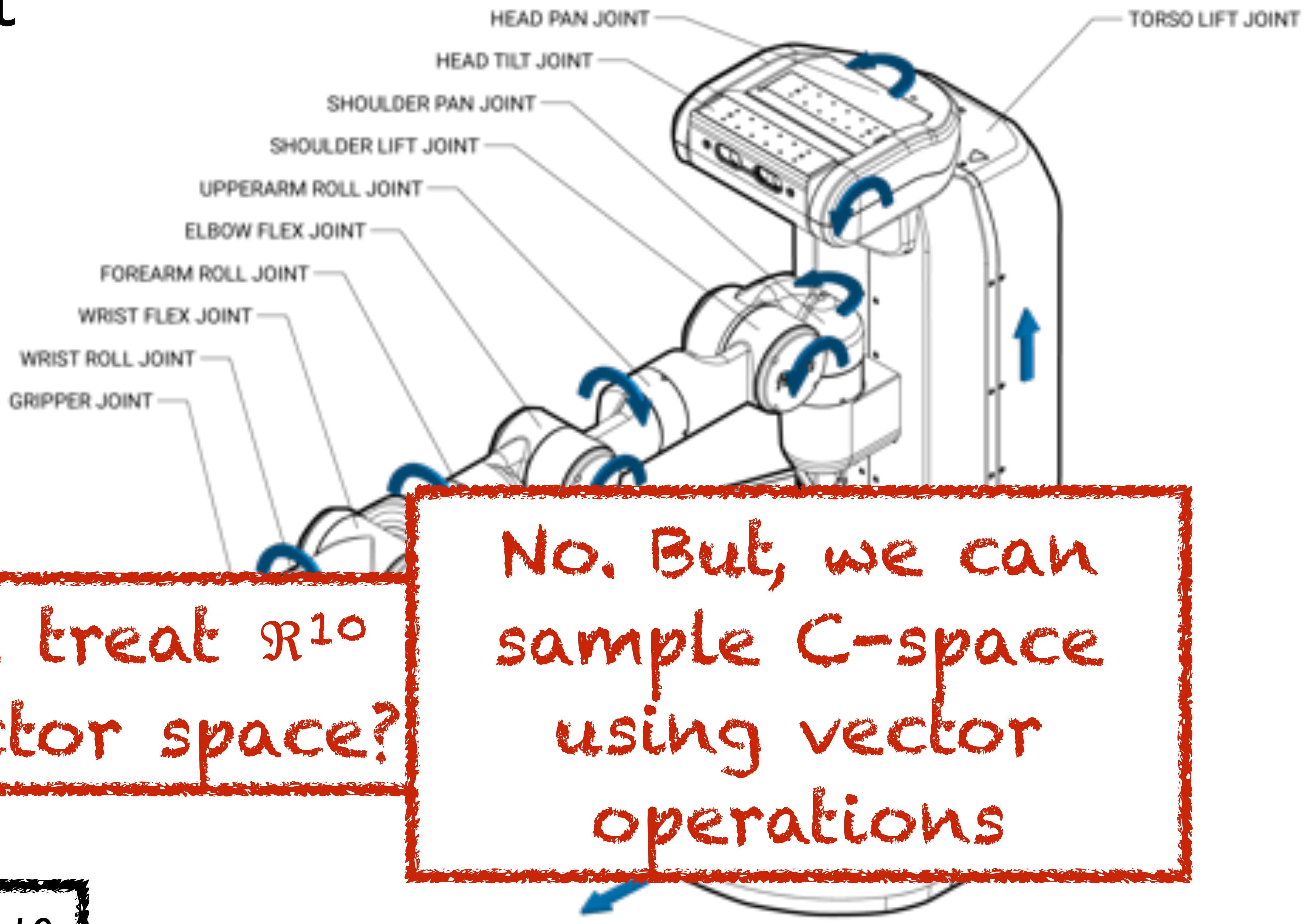
C-space: $SE(2) \times \Re^{10}$

Can we treat $\Re^{10}$ as a vector space?

No. But, we can sample C-space using vector operations



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Still not quite right…

- What is the C-space of a Fetch, not including grippers?
- DOFs: 13
  - 3 in base: *SE(2)*
  - 7 in arm: $\cancel{\Re^7}$
  - 1 in the spine: $\Re^1$
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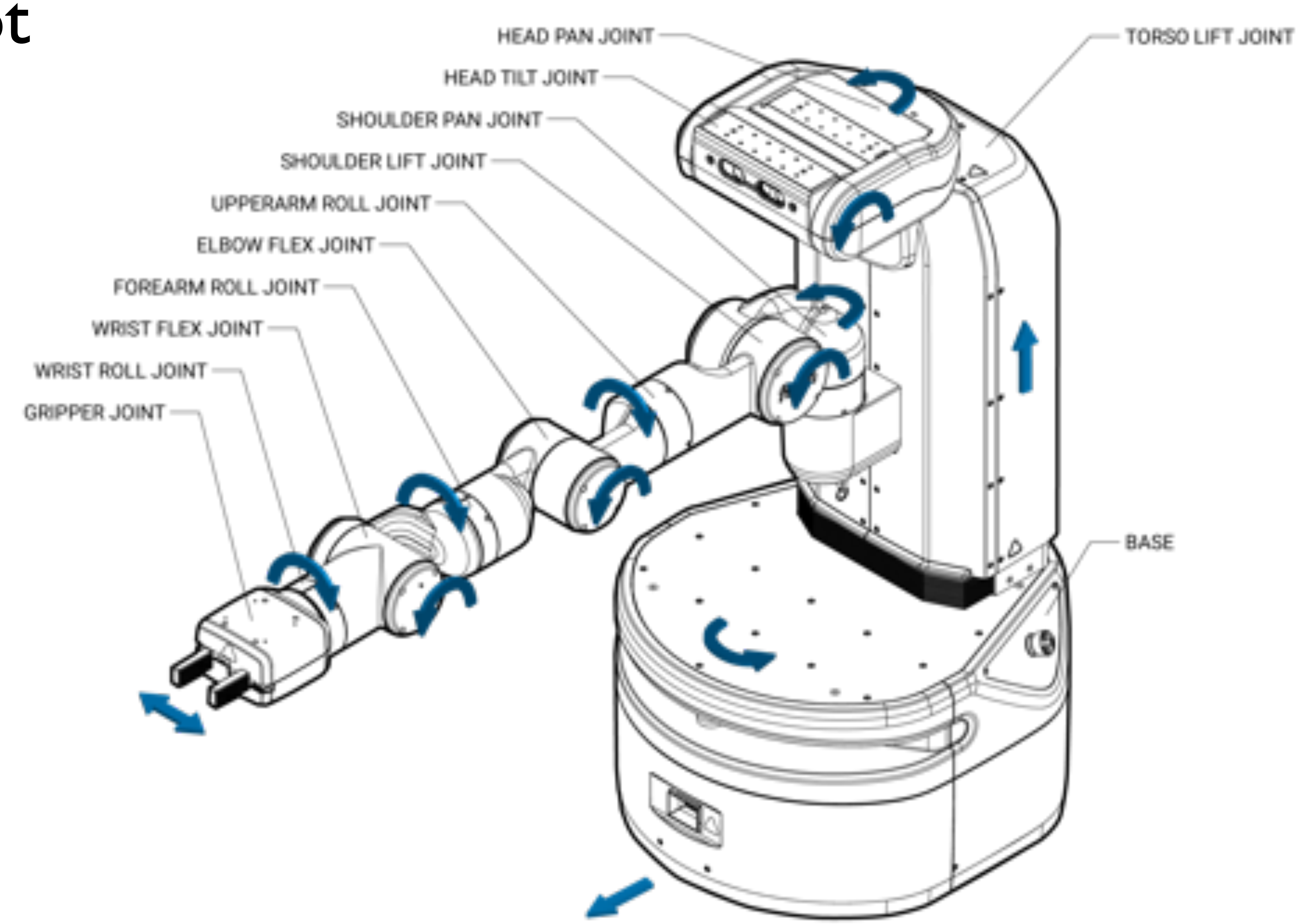

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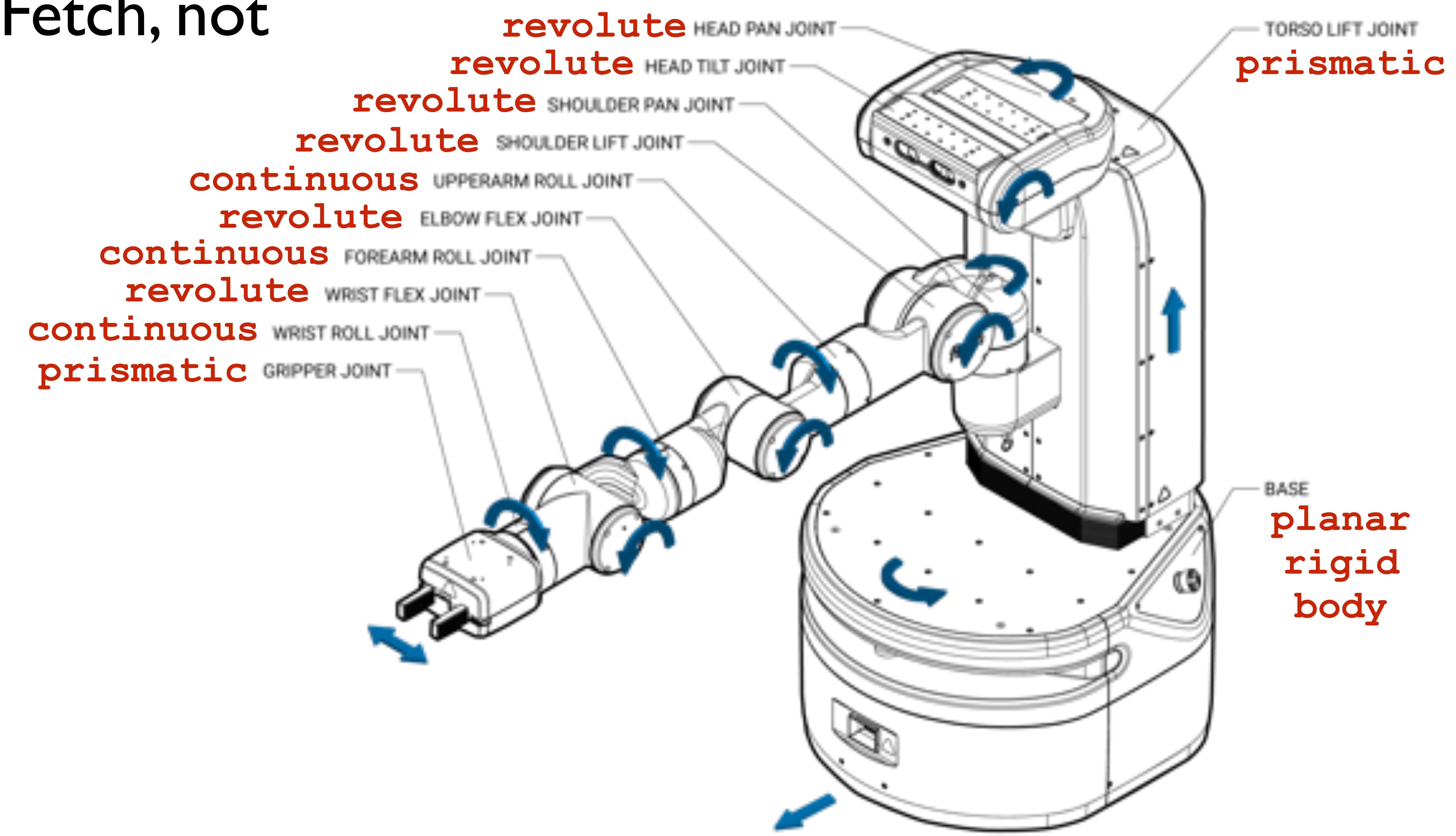
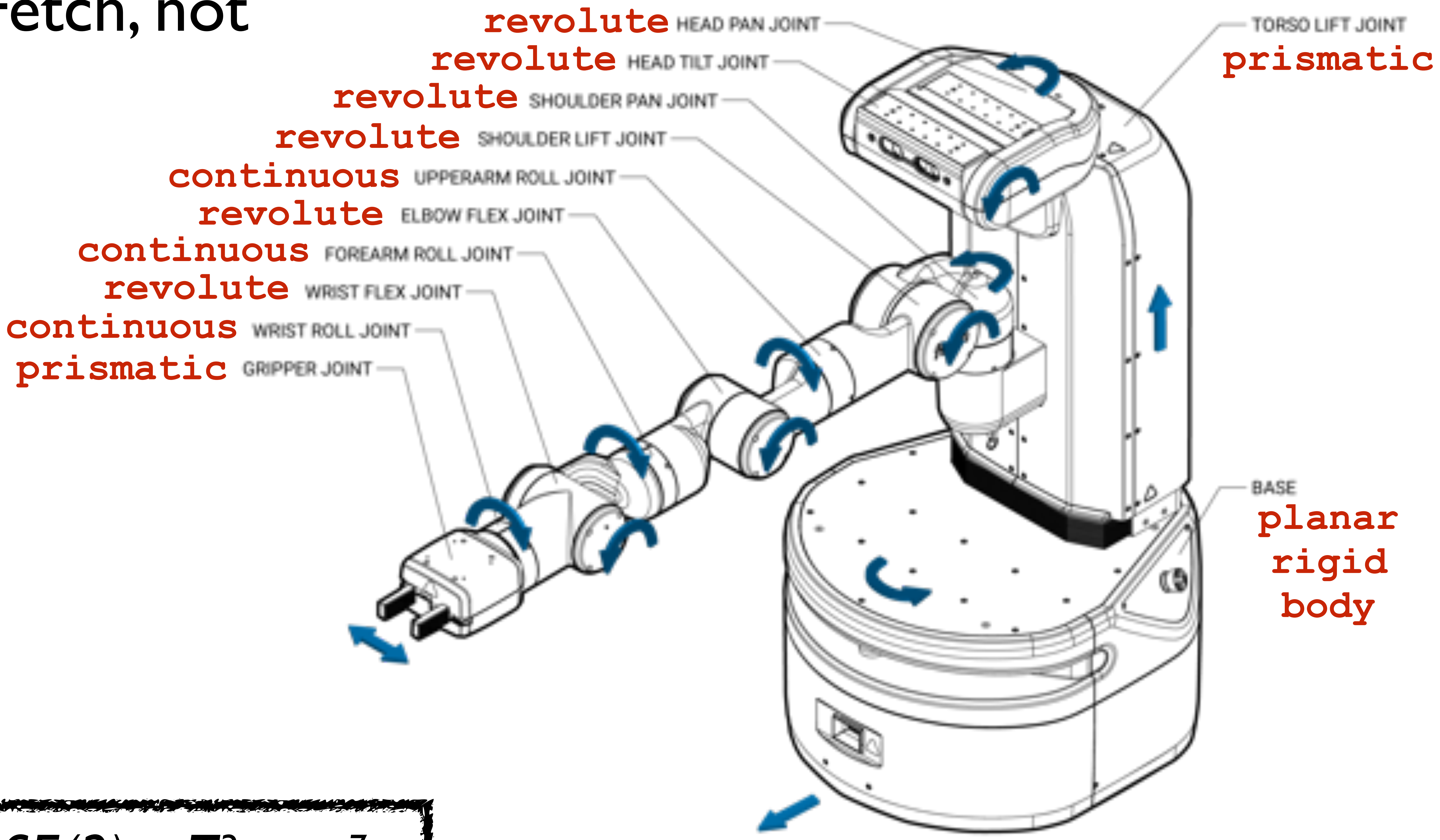


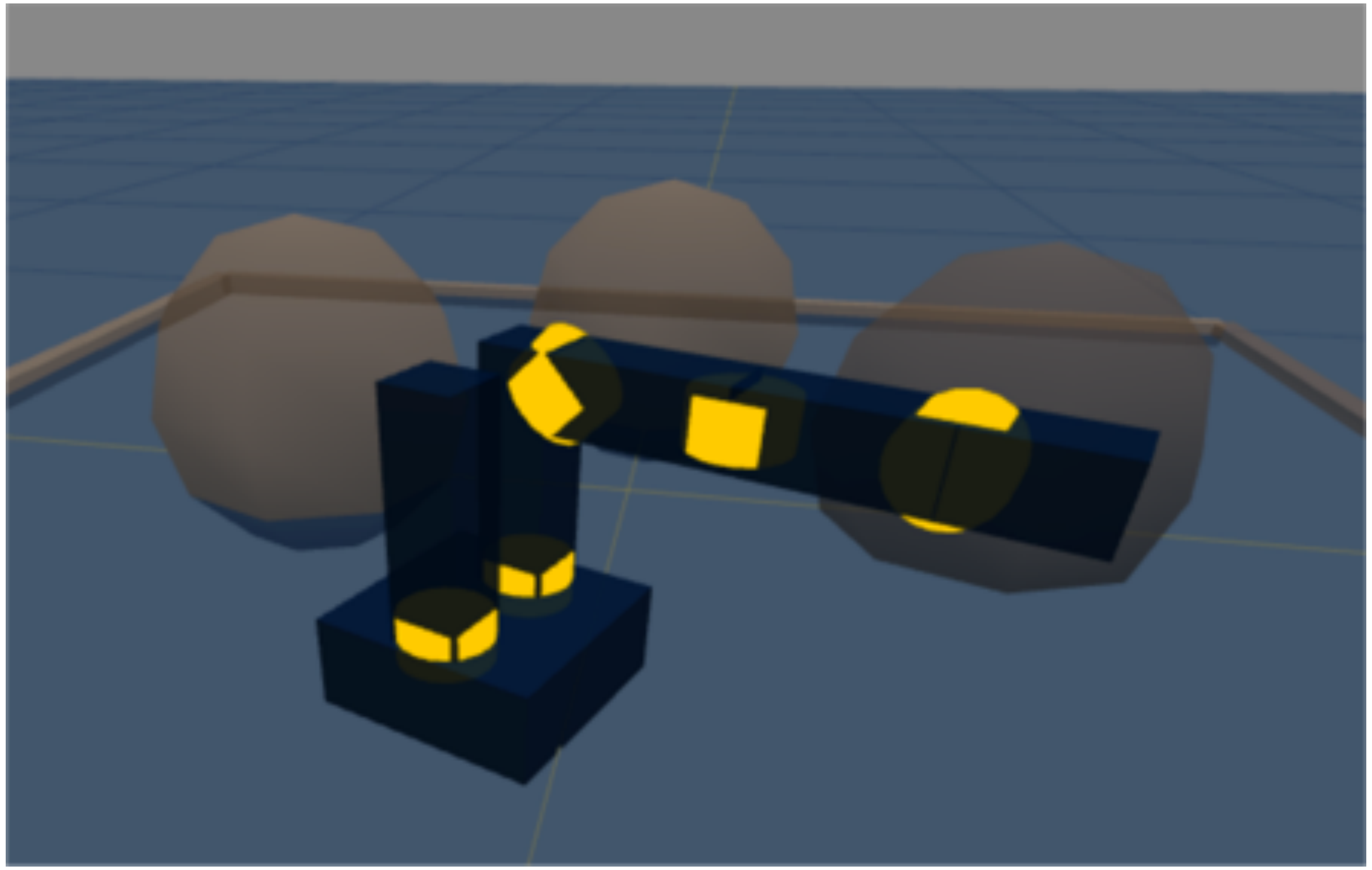
- What is the C-space of a Fetch, not including grippers?
- DOFs: 13
  - 3 in base: $SE(2)$
  - 3 continuous: T^3
  - 1 prismatic: $\Re^1$
  - 6 revolute: $\Re^6$



C-space: $SE(2) \times T^3 \times \Re^7$

C-space examples

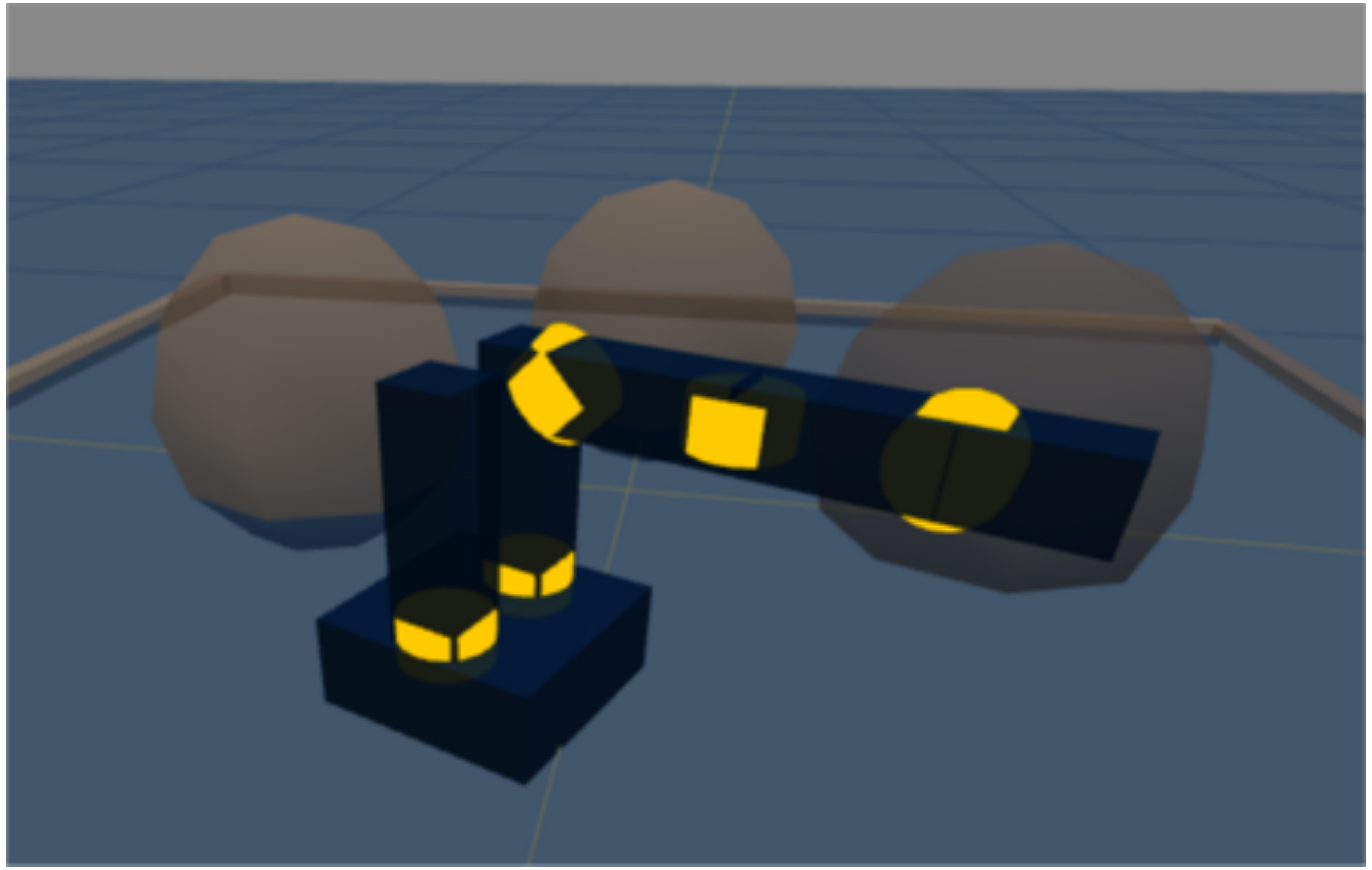
- What is the C-space of a MR2?



C-space examples

- What is the C-space of a MR2?
- DOFs: 14
  - 3 in base: $SE(2)$
  - 5 in arms: T^5

C-space: $SE(2) \times T^5$



C-space examples

- What is the C-space of a Robonaut 2 on the International Space Station?

- What is the C-space of a PR2?

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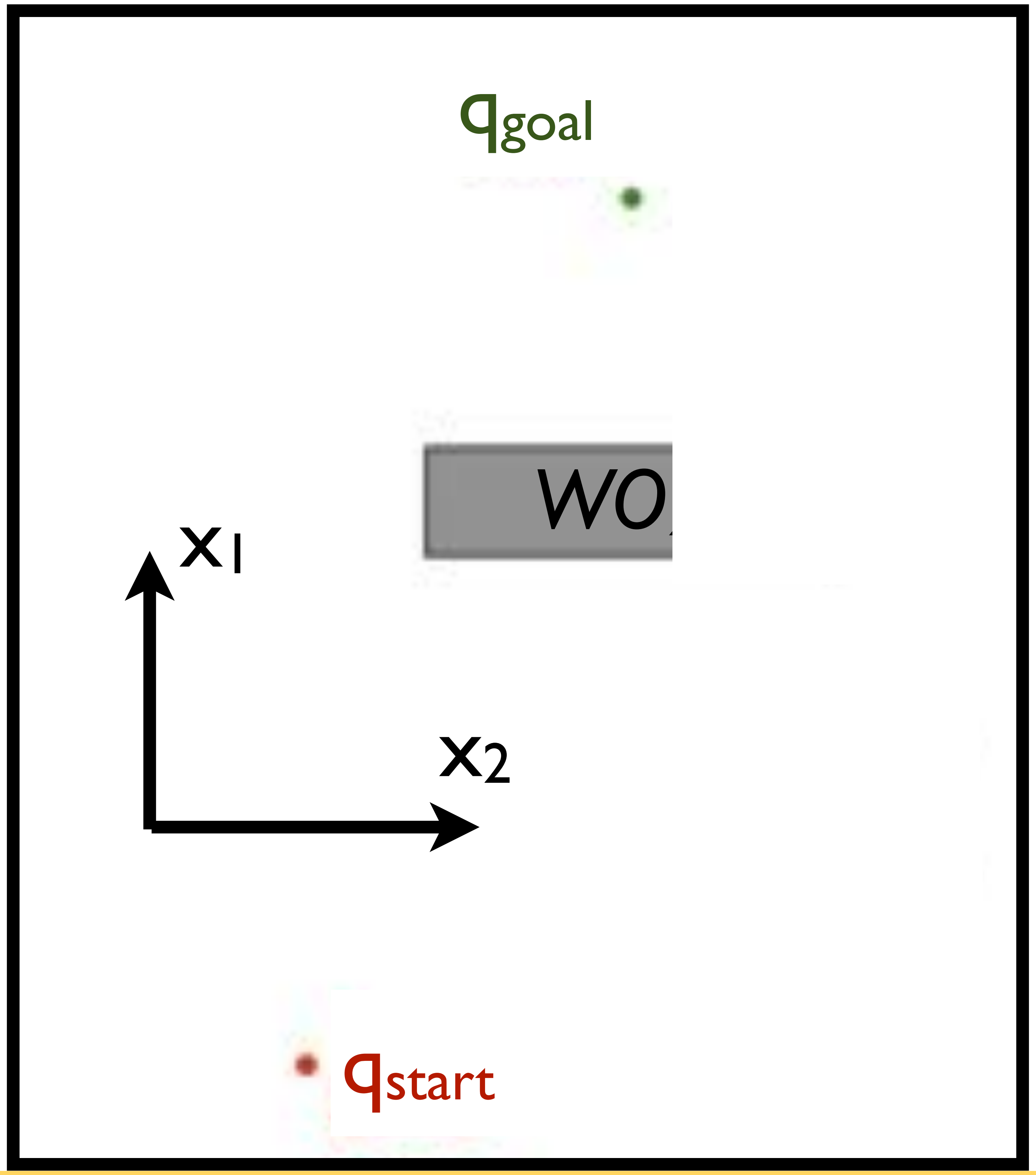
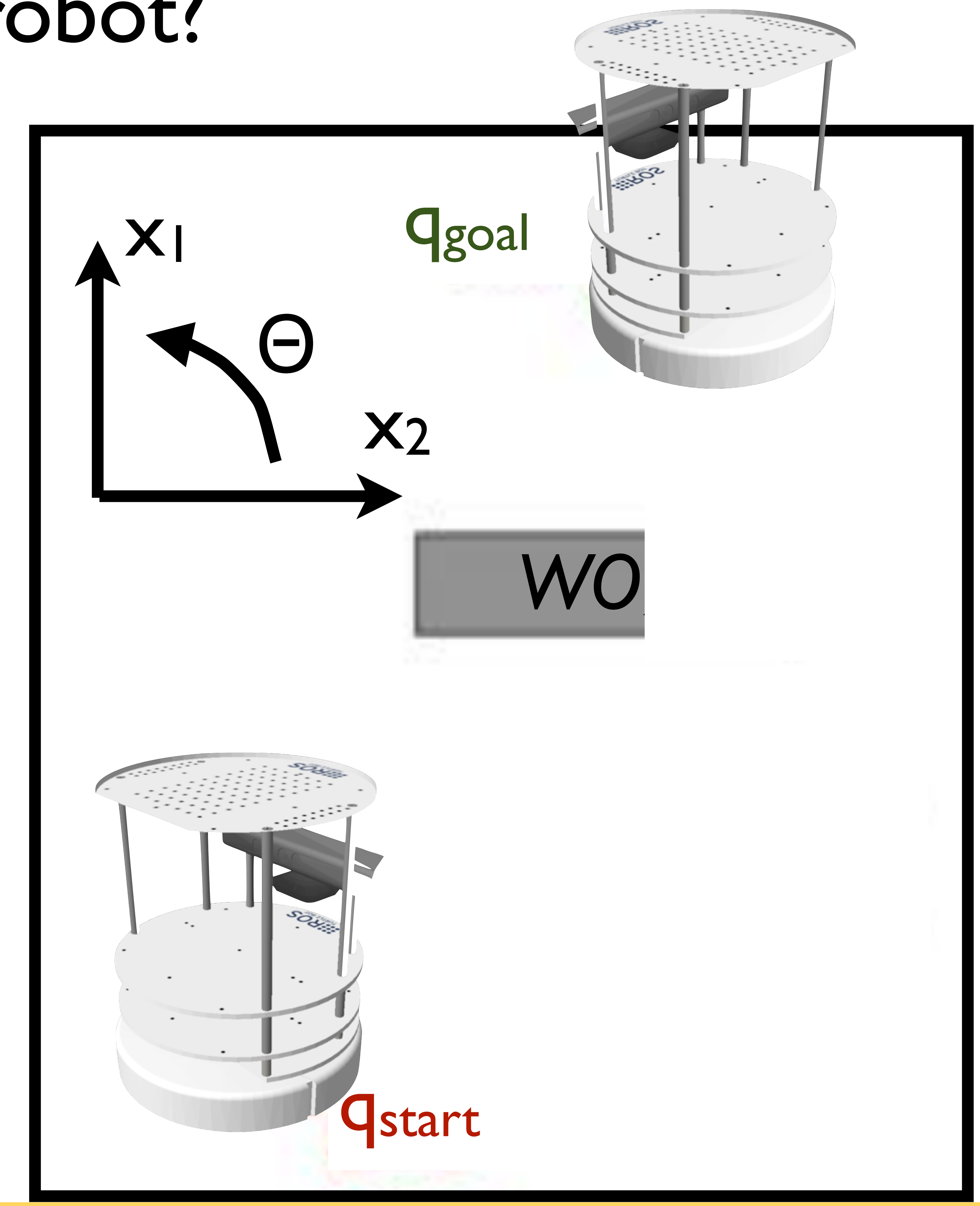
What about the robot's physical geometry?

Configuration v. Workspaces

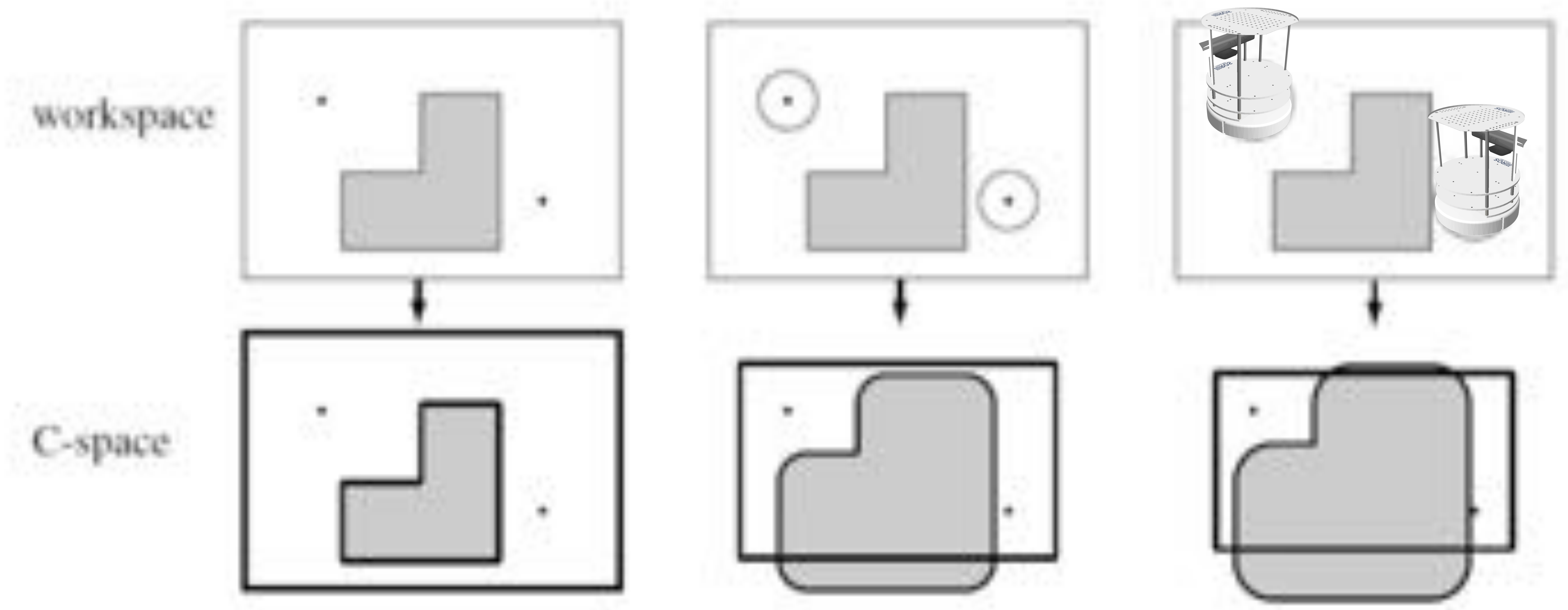
- Other than rotation, how is the Turtlebot different than the point robot?





Robot Geometry

- Turtlebot is larger than a point, having a circular radius in the robot's planar workspace
- As this radius increases, the C-space shrinks



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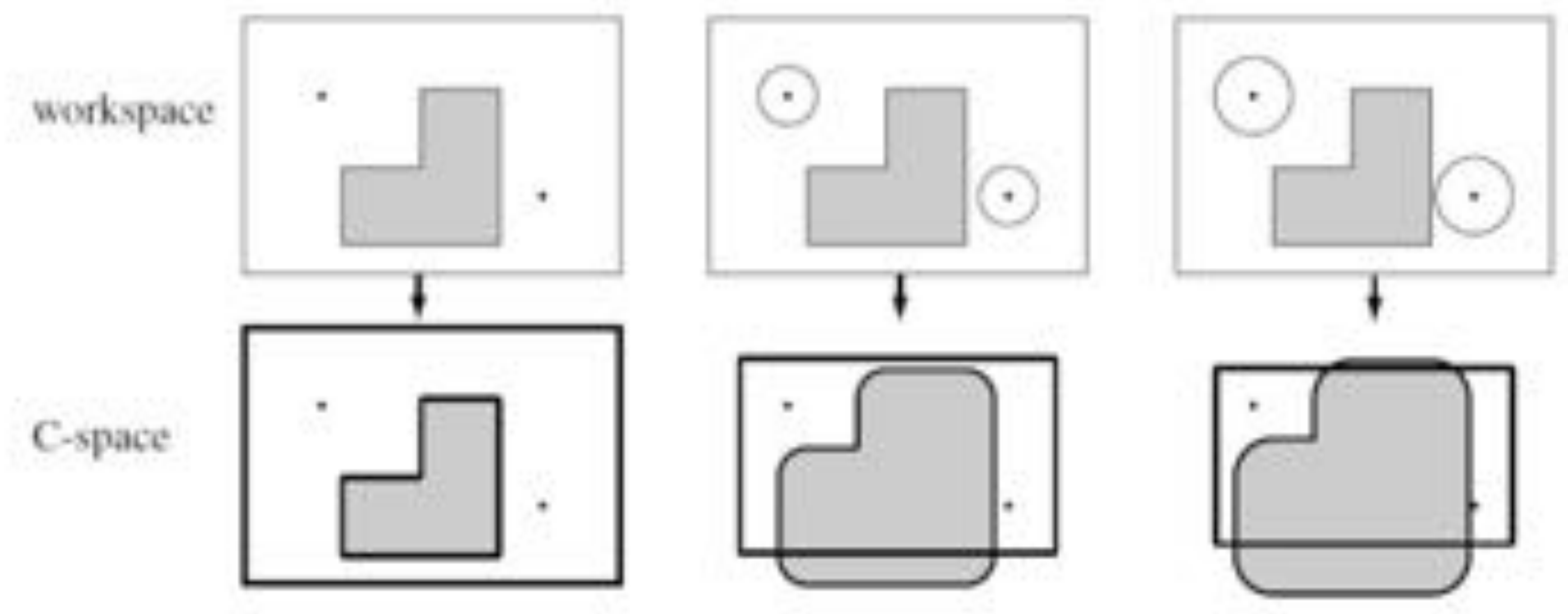
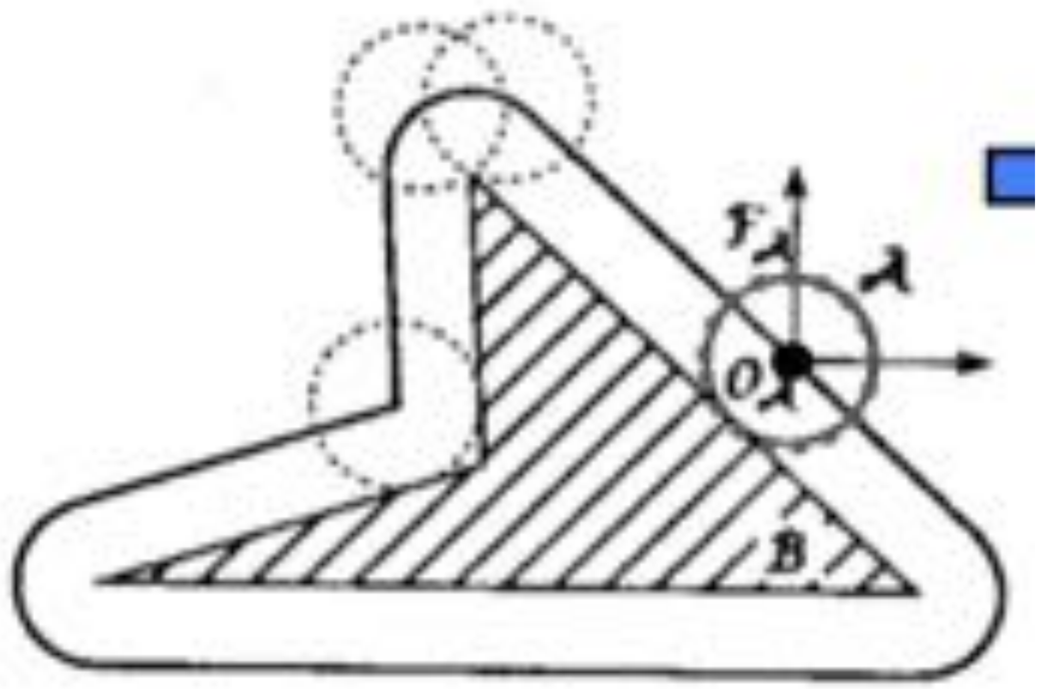


Conversion to point robot C-space

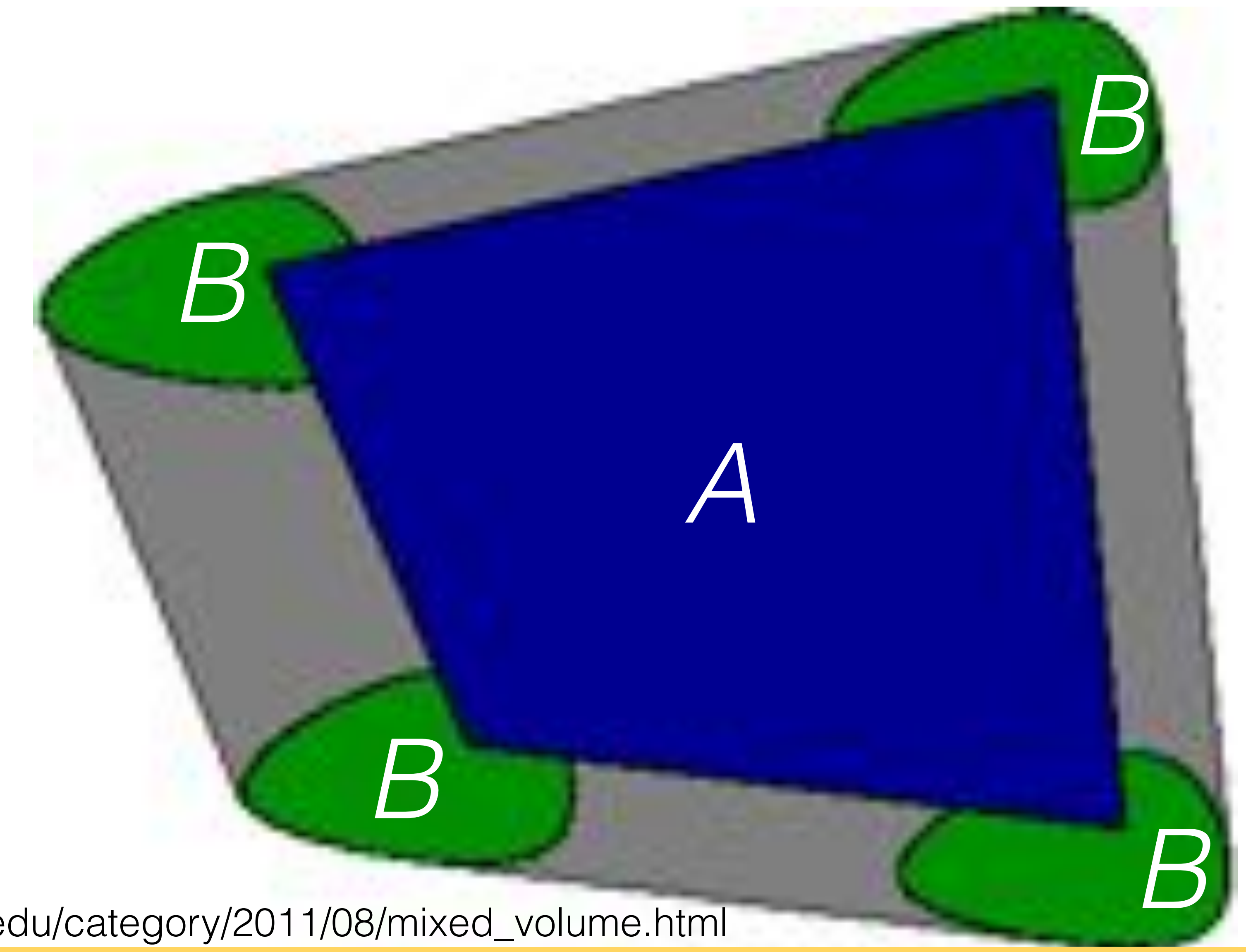


- Workspace for robot can be converted to point robot C-space
- Expand obstacles by tracing robot geometry along boundary
- Computable by Minkowski sum

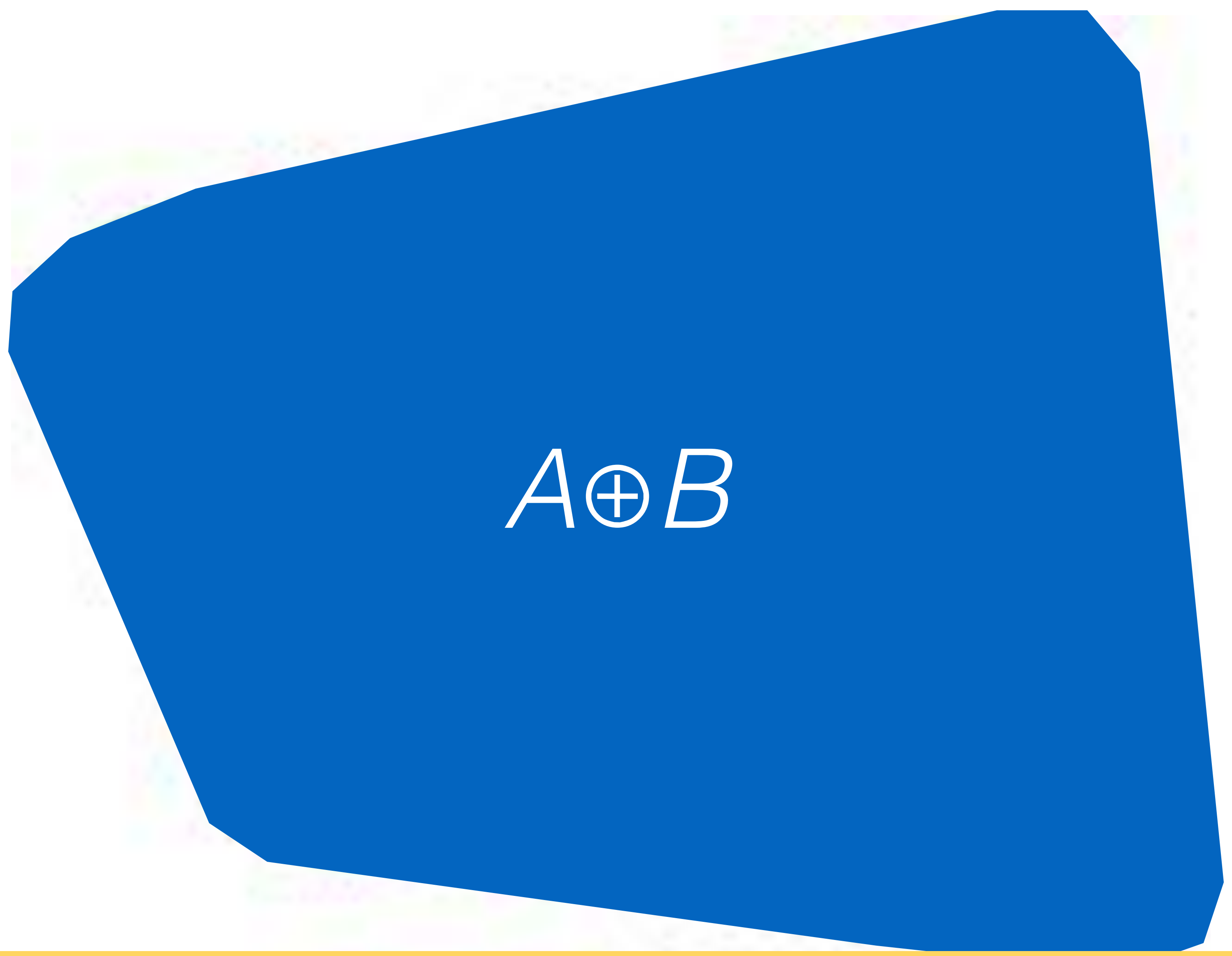
Minkowski Sum (or morphological dilation)

- $A \oplus B$: $\{a+b \mid a \in A, b \in B\}$
  - The result of adding every element of A to every element of B, given two sets A and B in Euclidean space

- Similarly, Minkowski difference (morphological erosion)
  - $A \ominus B$: $\{a-b \mid a \in A, b \in B\}$

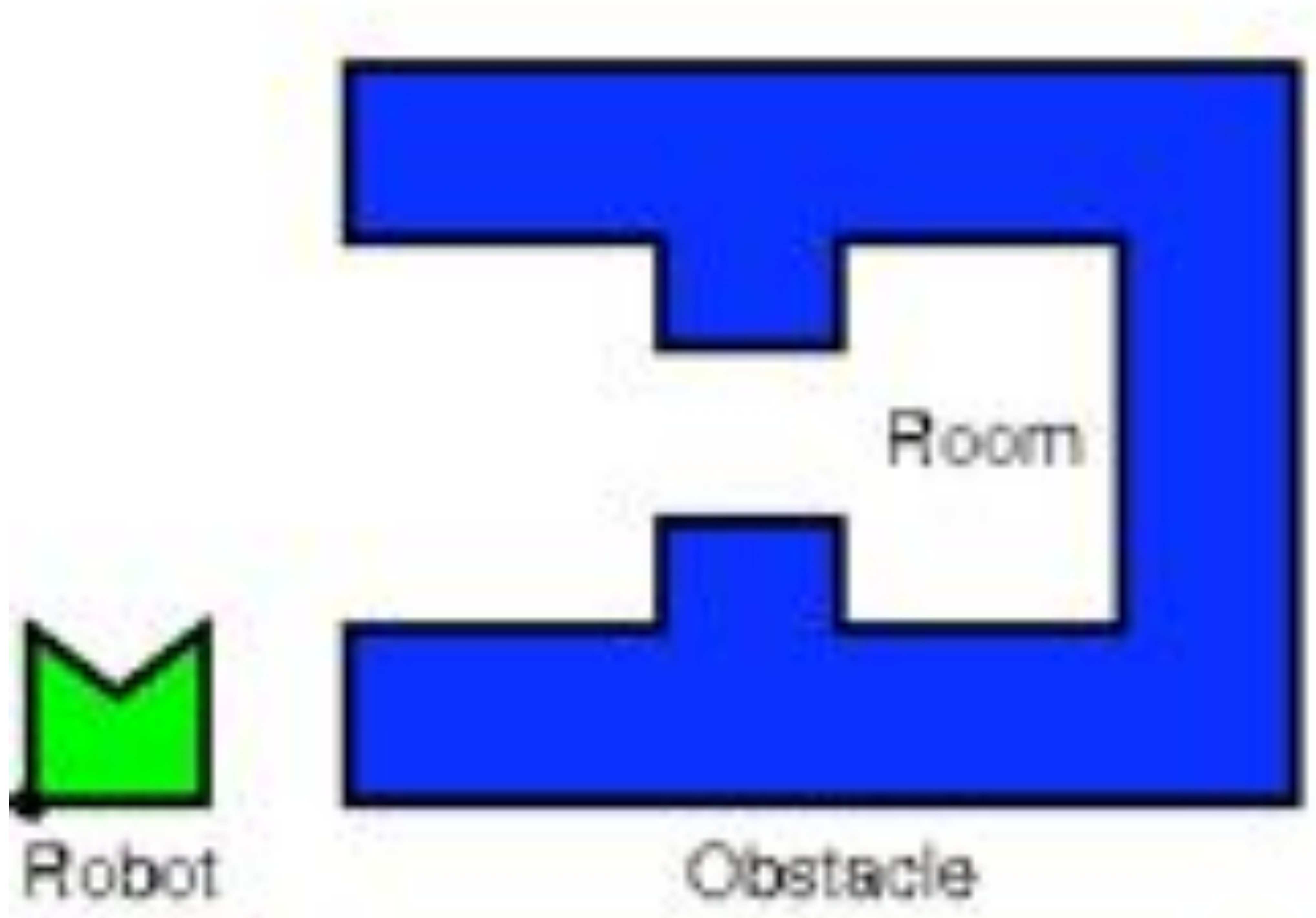


https://golem.ph.utexas.edu/category/2011/08/mixed_volume.html

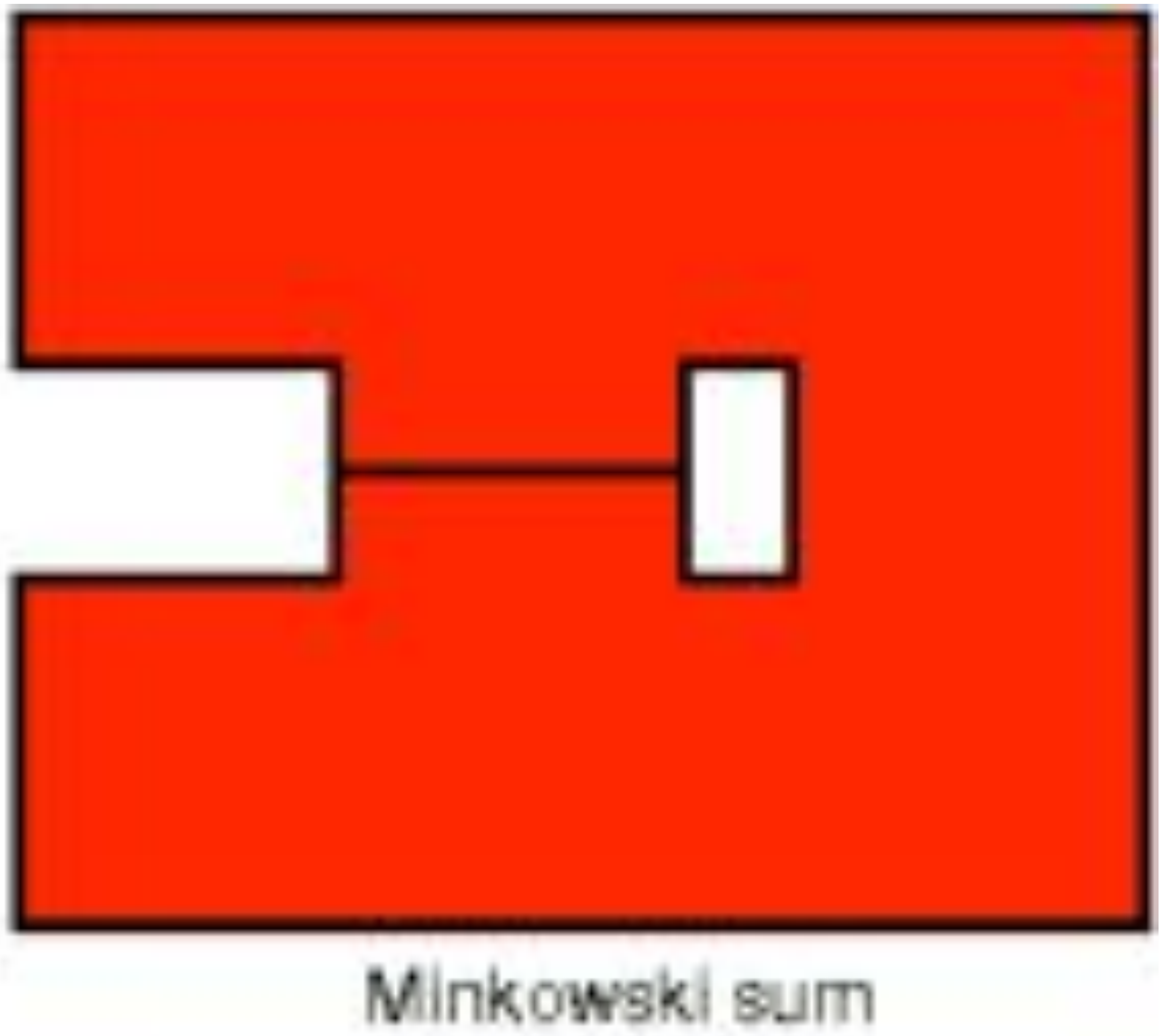


Minkowski Planning

Given



Compute



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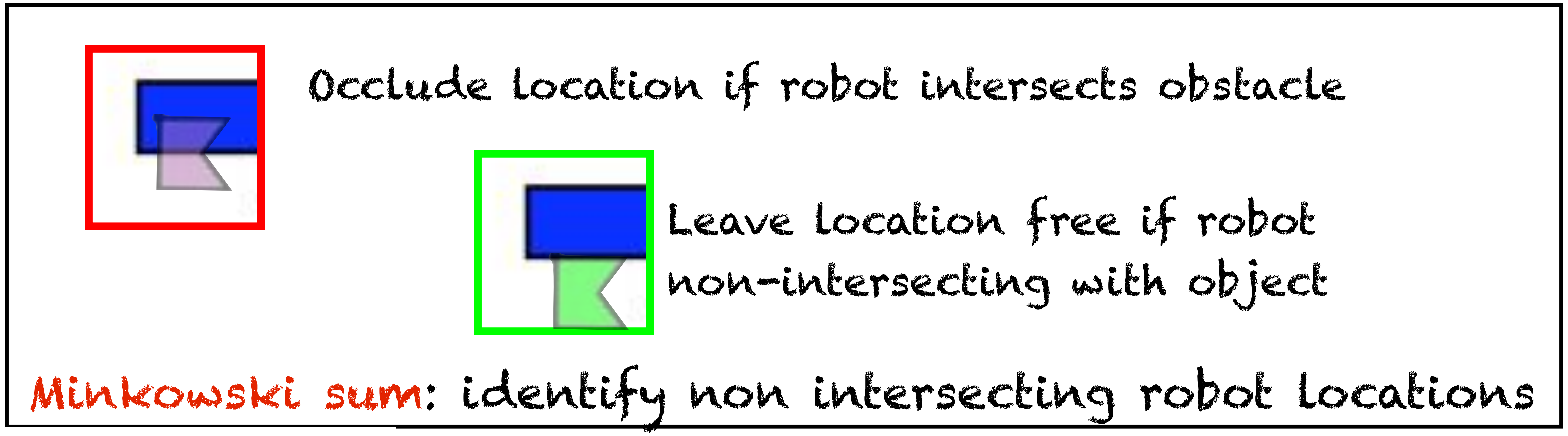
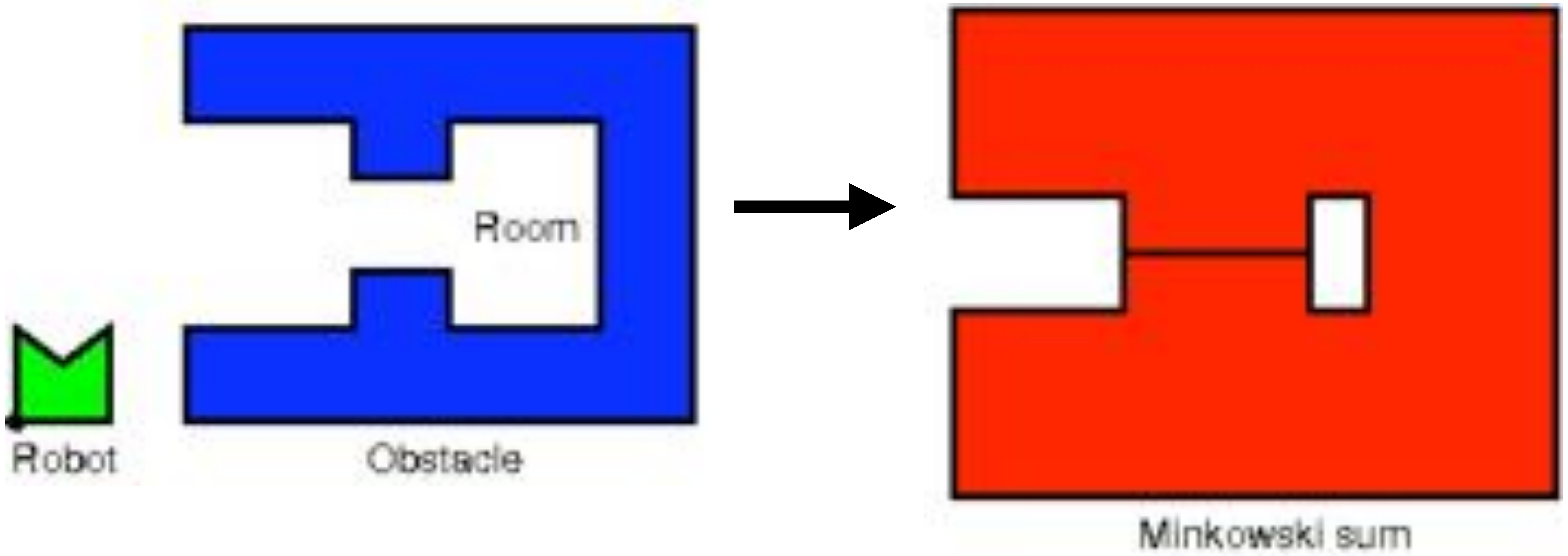
Minkowski Planning





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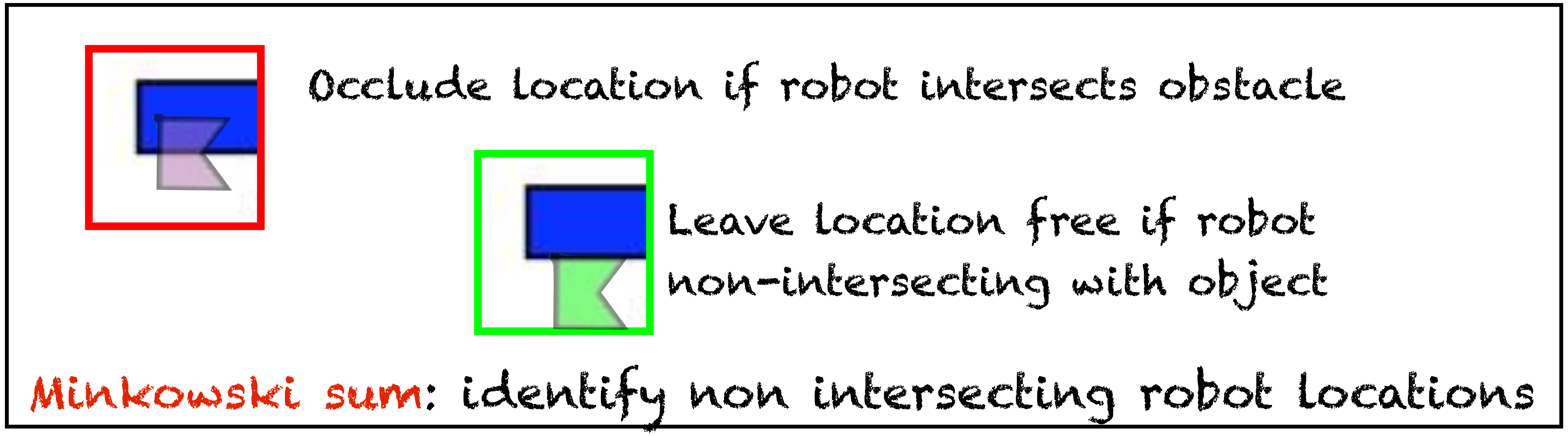
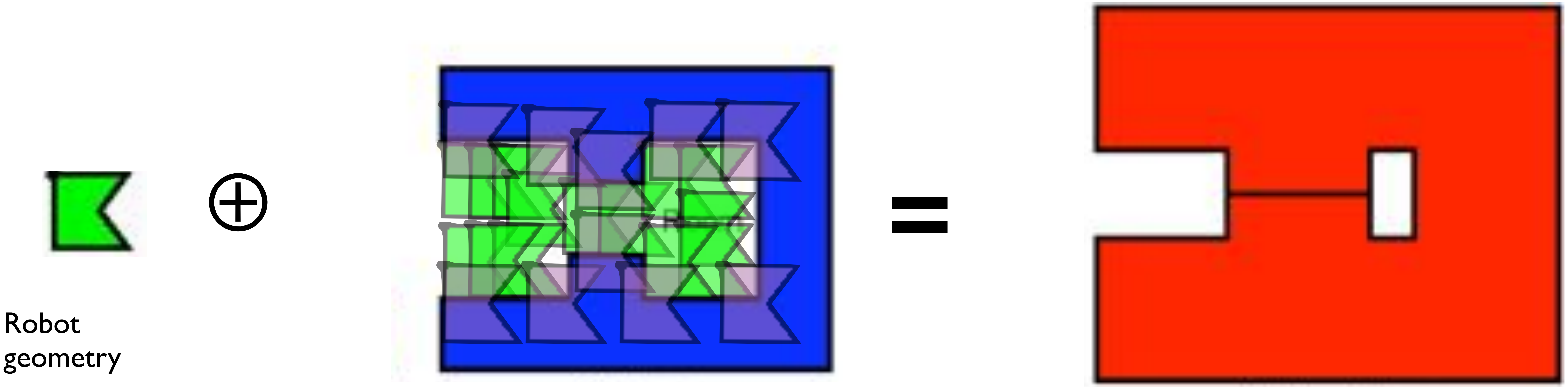
Minkowski Planning





Space of valid paths defined by Minkowski sum

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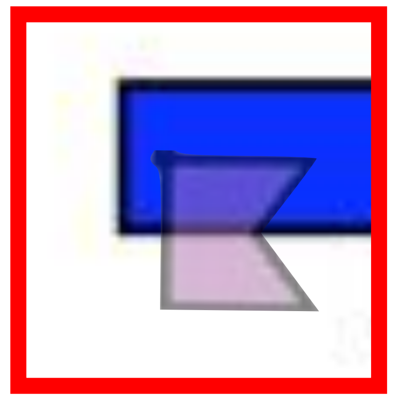
Minkowski Planning



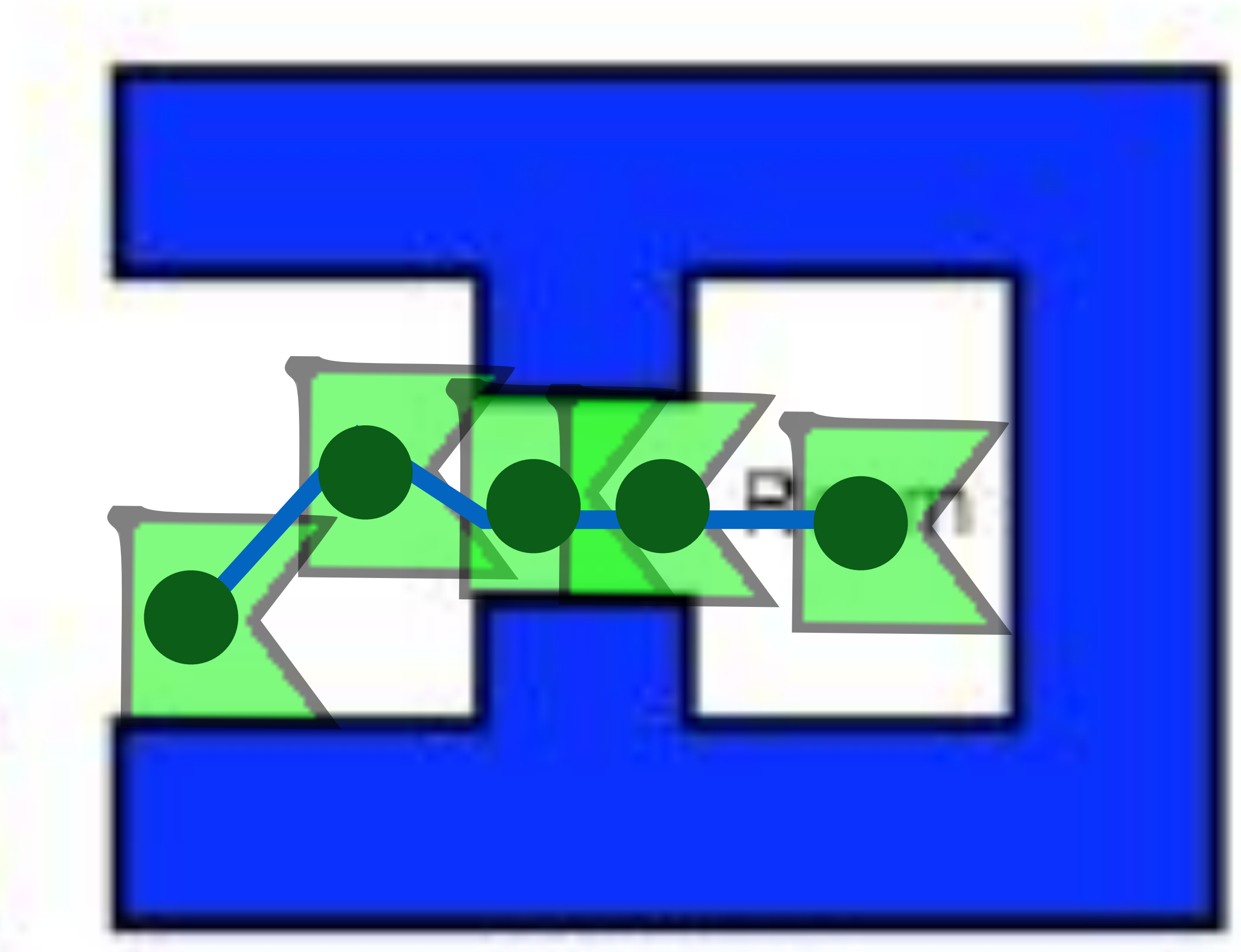
Occlude location if robot intersects obstacle

Leave location free if robot non-intersecting with object

Minkowski sum: identify non intersecting robot locations

Robot geometry

$\oplus$



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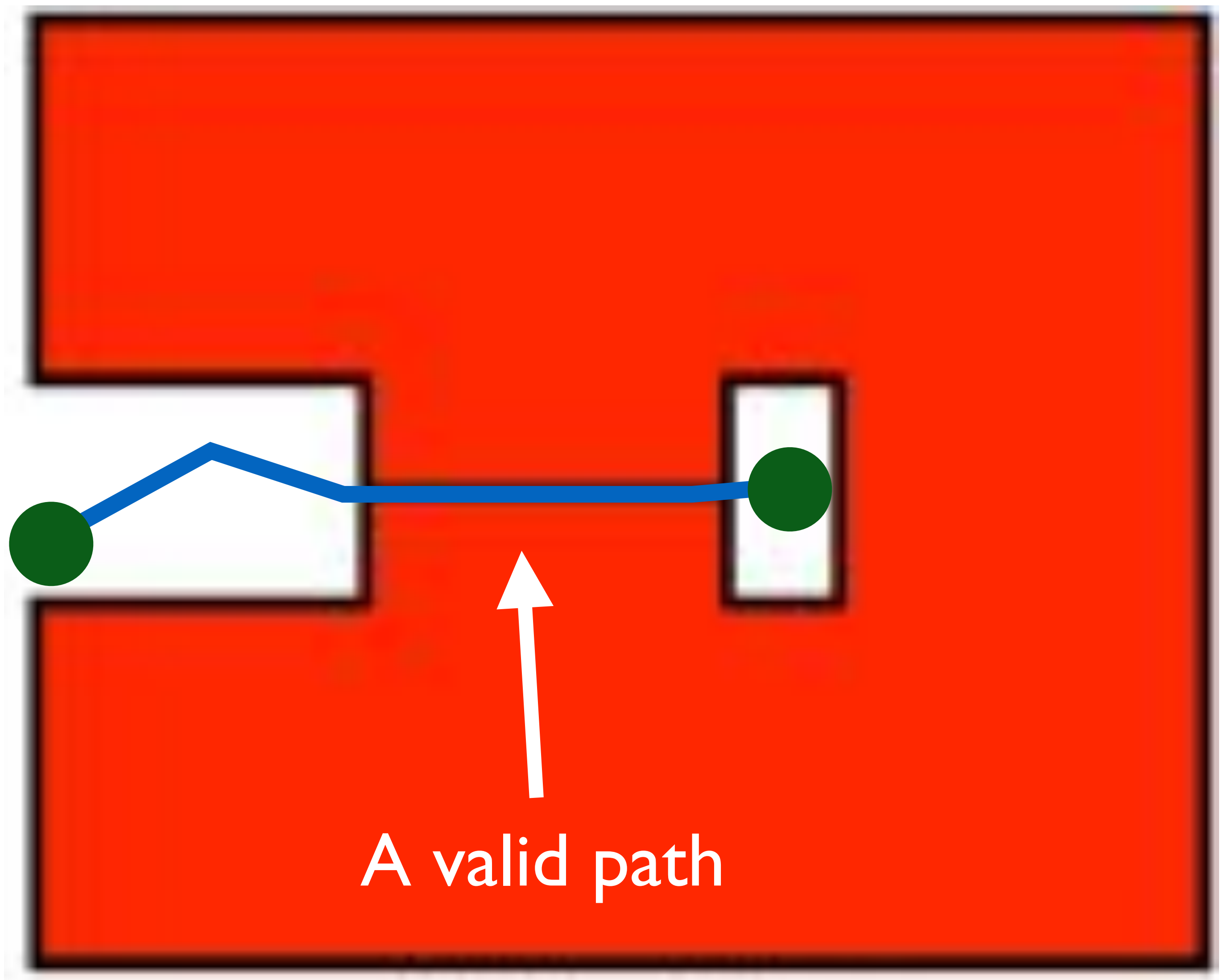
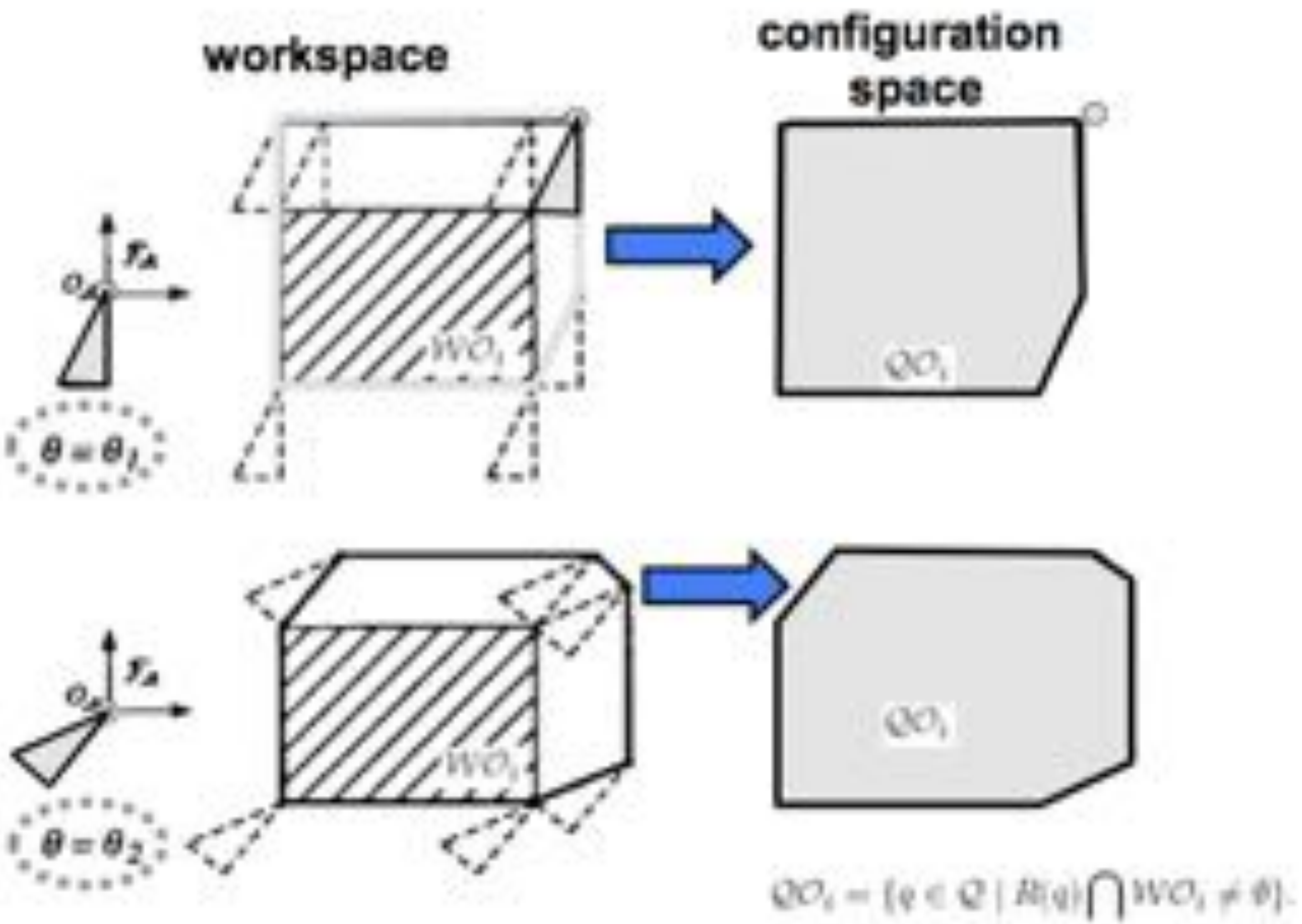


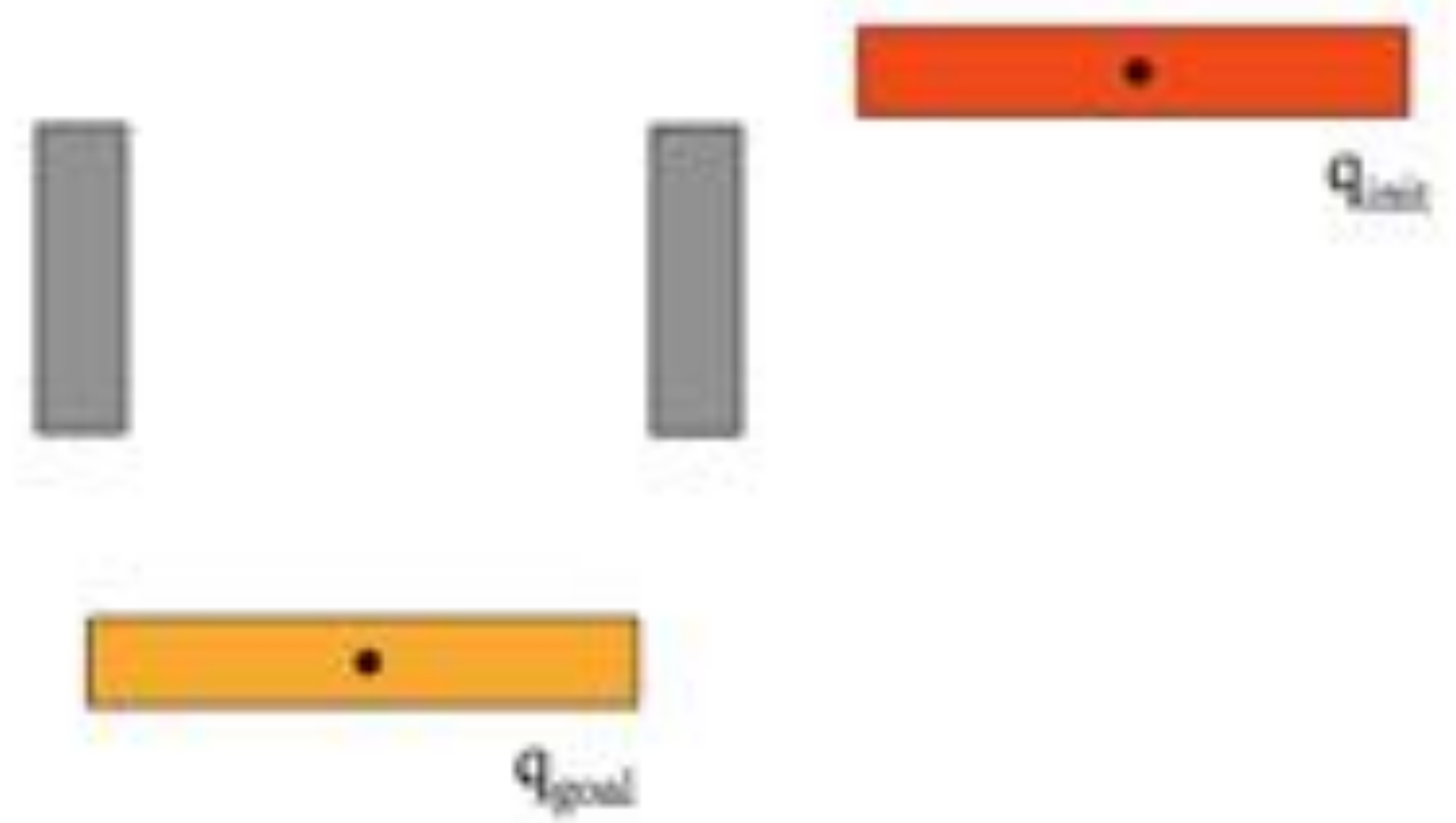
Space of valid paths defined by Minkowski sum

What does an obstacle look like in configuration space?

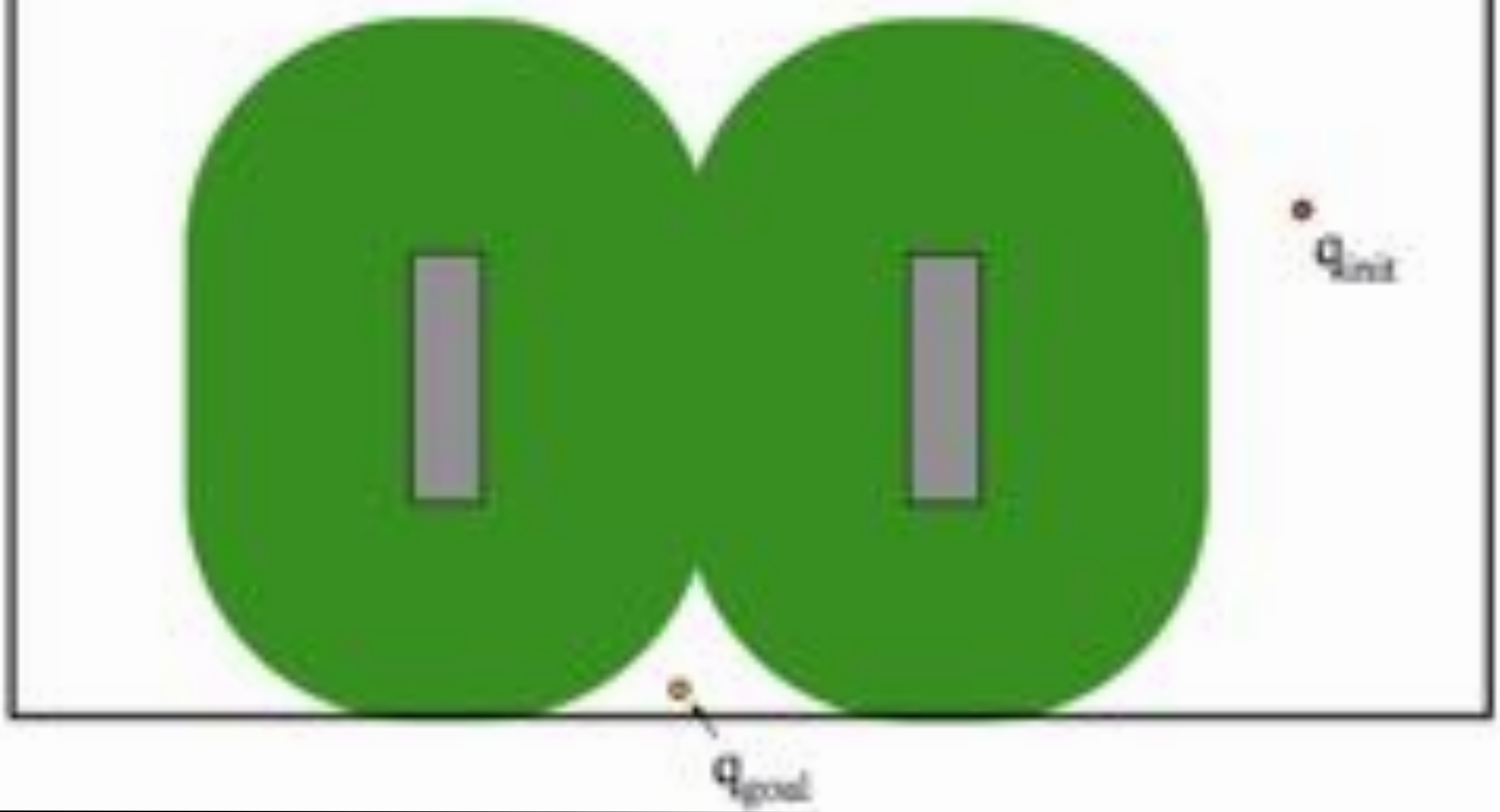
C-space depends on rotation



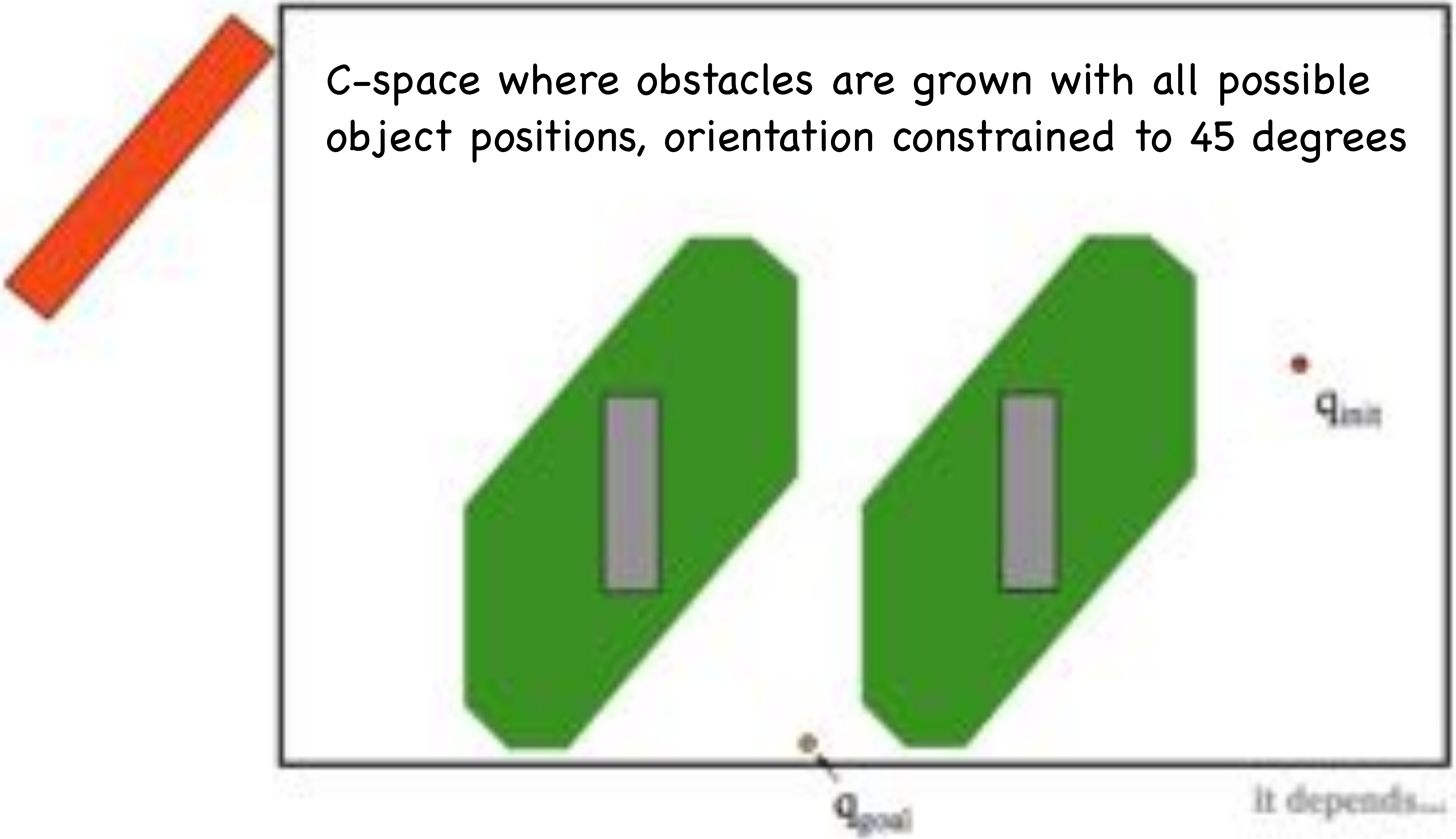
Consider this workspace…



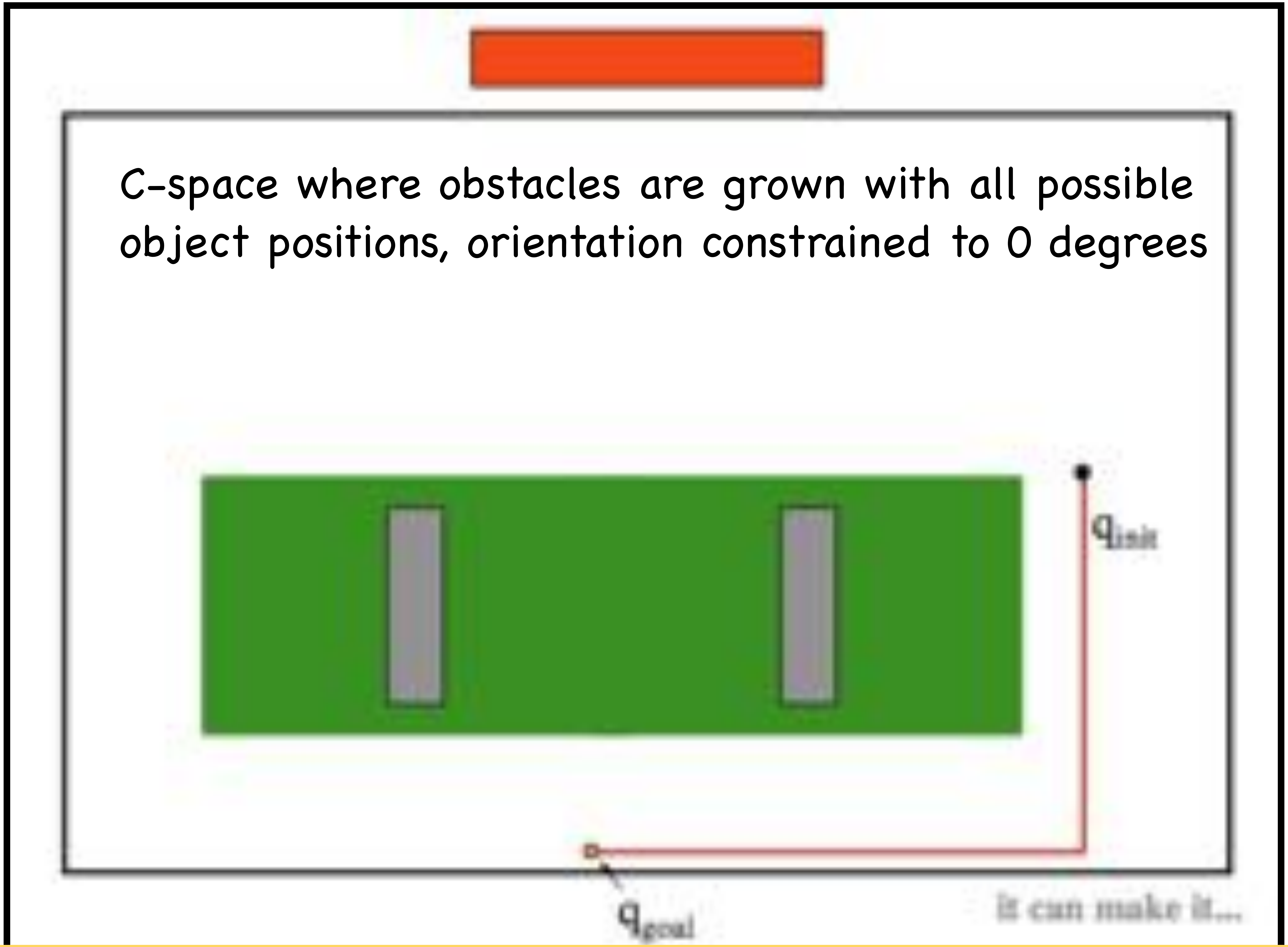
C-space where obstacles are grown with all possible object positions and orientations



C-space where obstacles are grown with all possible object positions, orientation constrained to 45 degrees

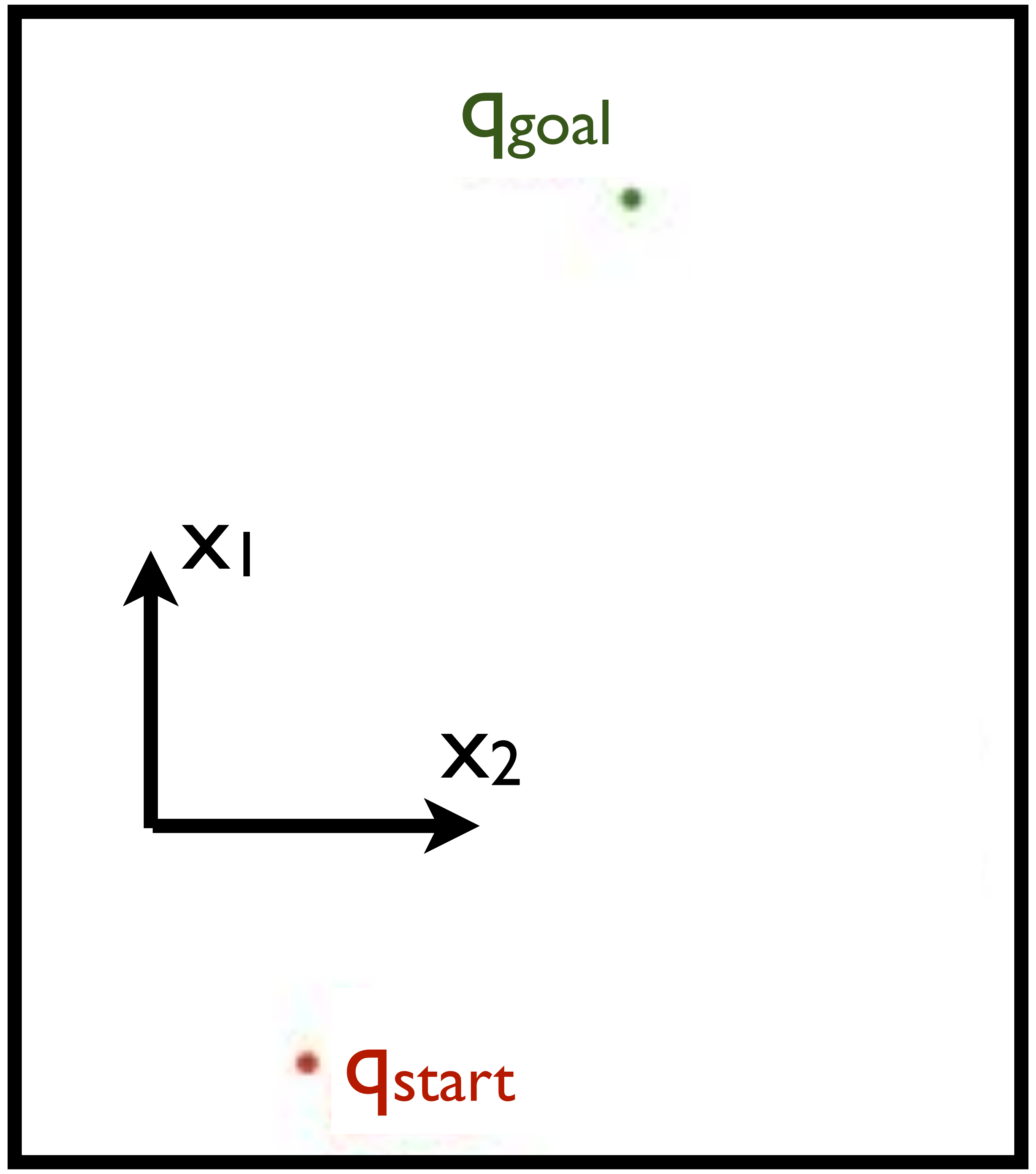


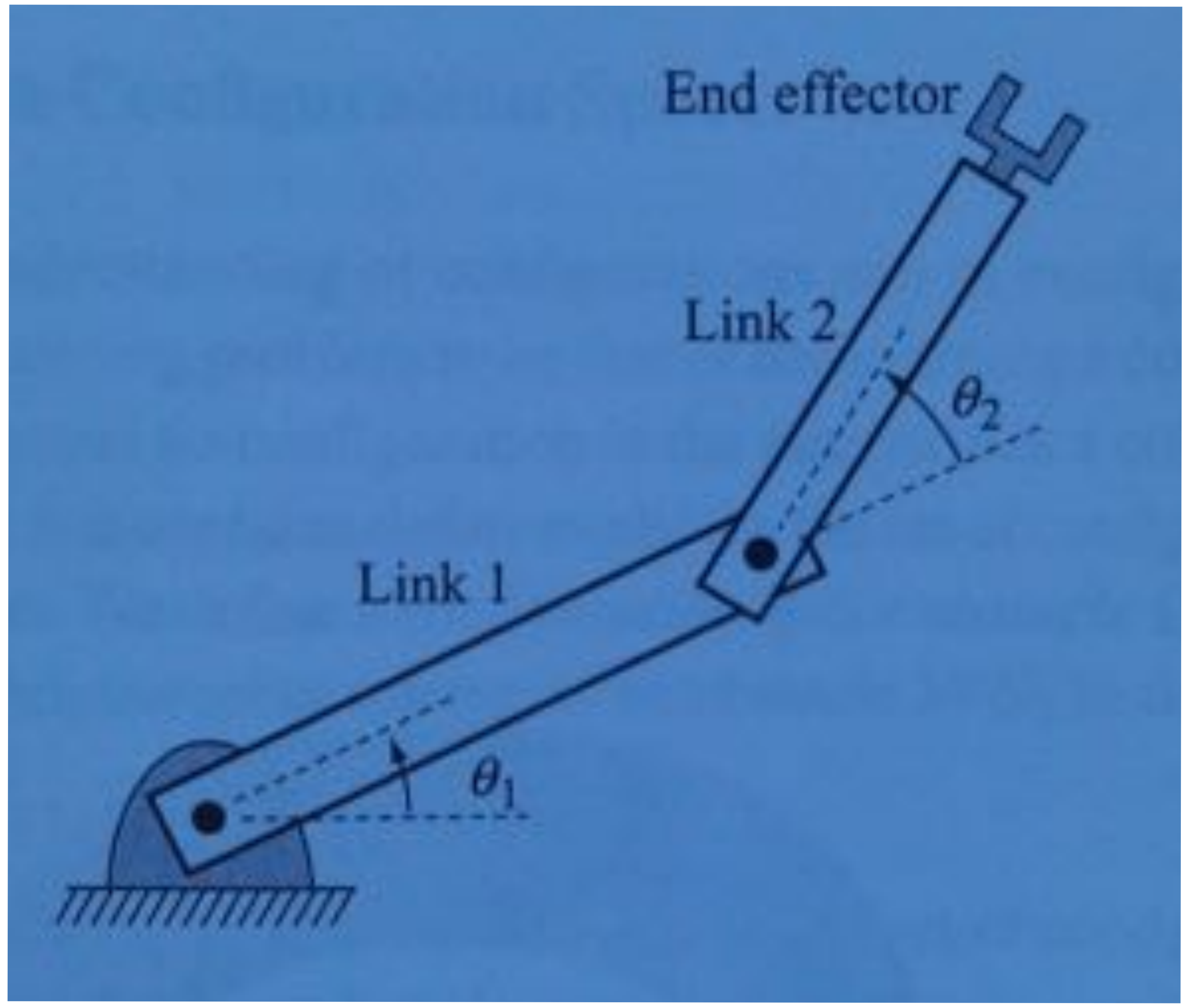
C-space where obstacles are grown with all possible object positions, orientation constrained to 0 degrees



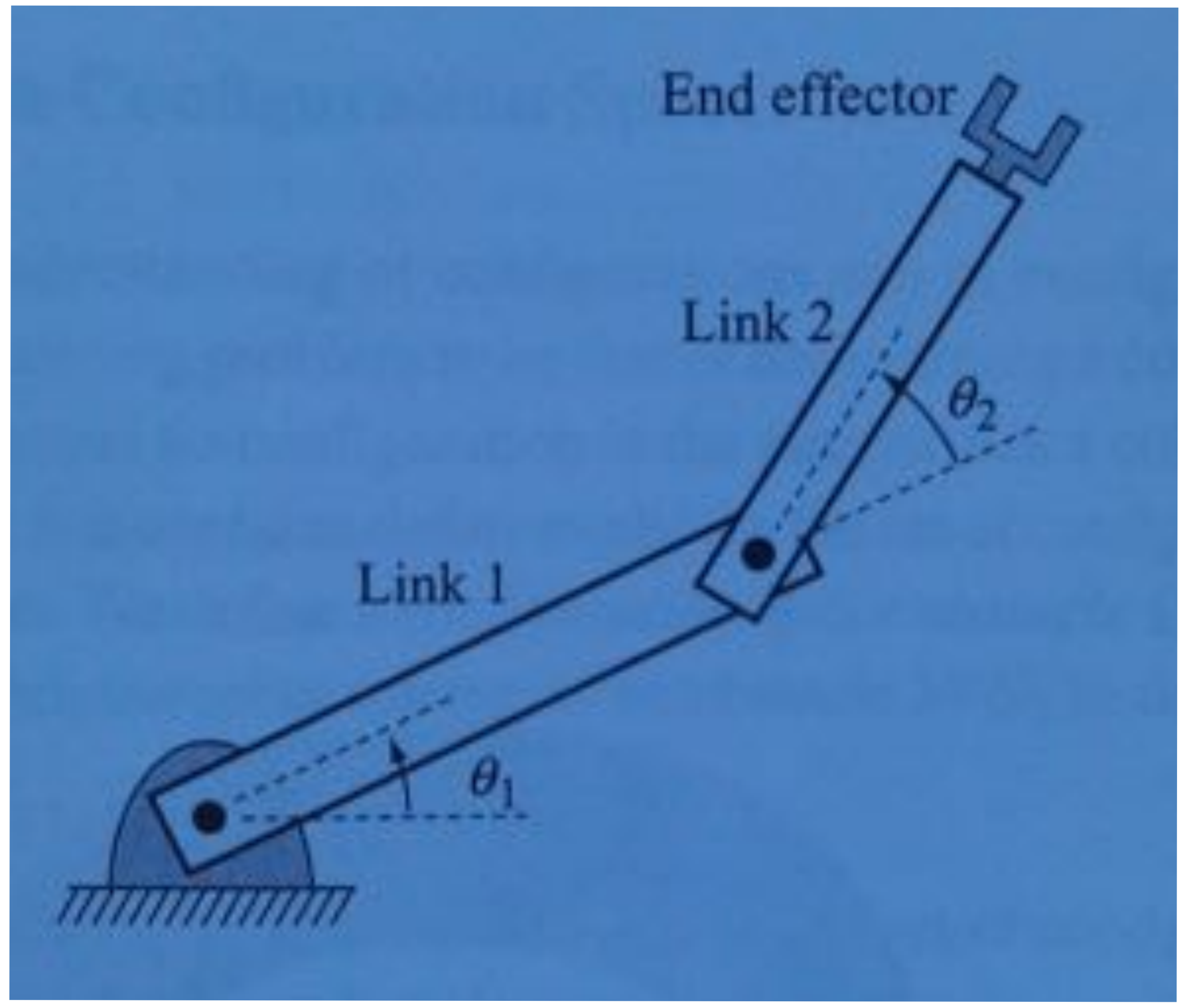
Configuration v. Workspaces

- Other than rotation and geometry, how is the 2-link arm different than the point robot?

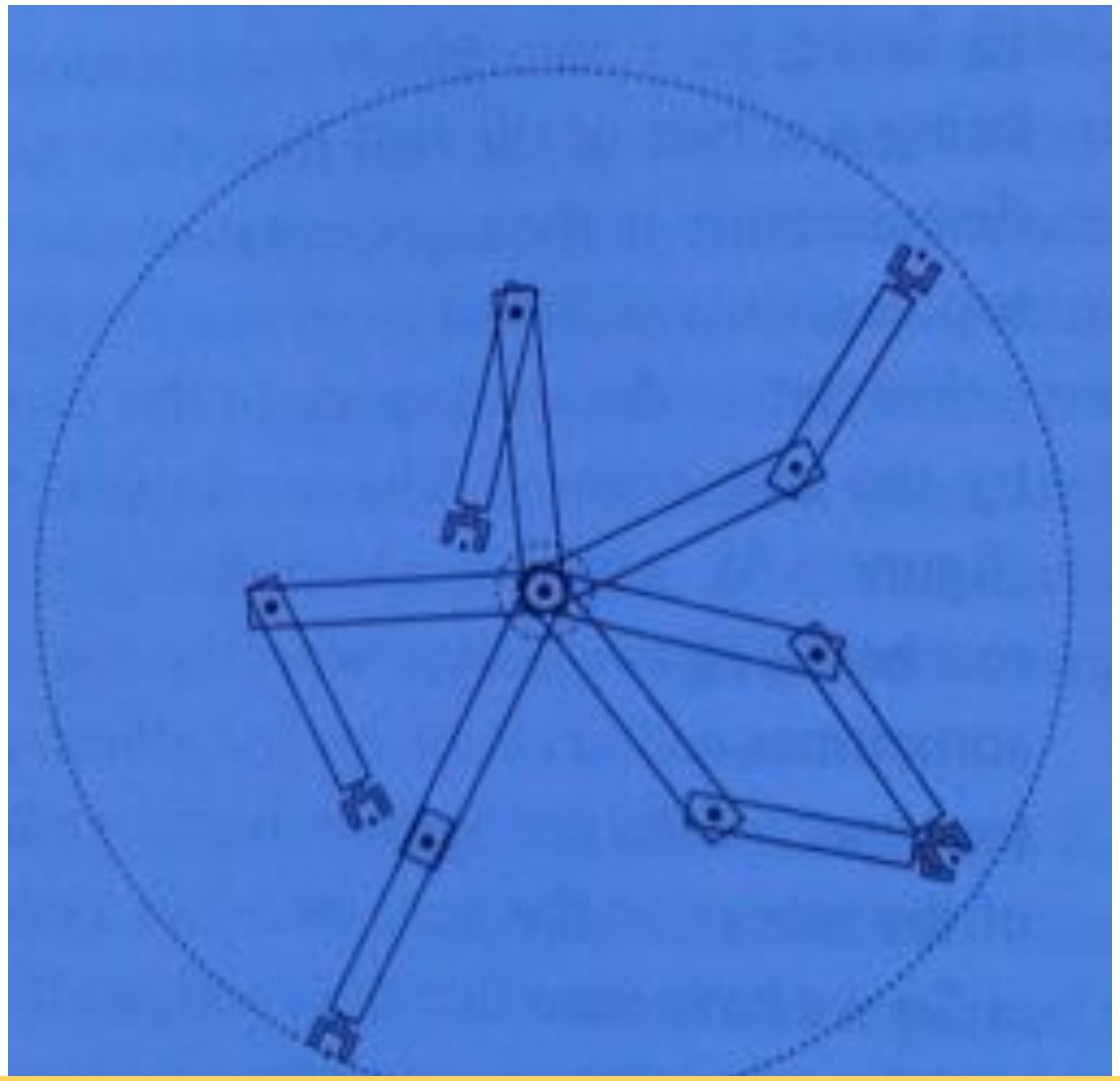




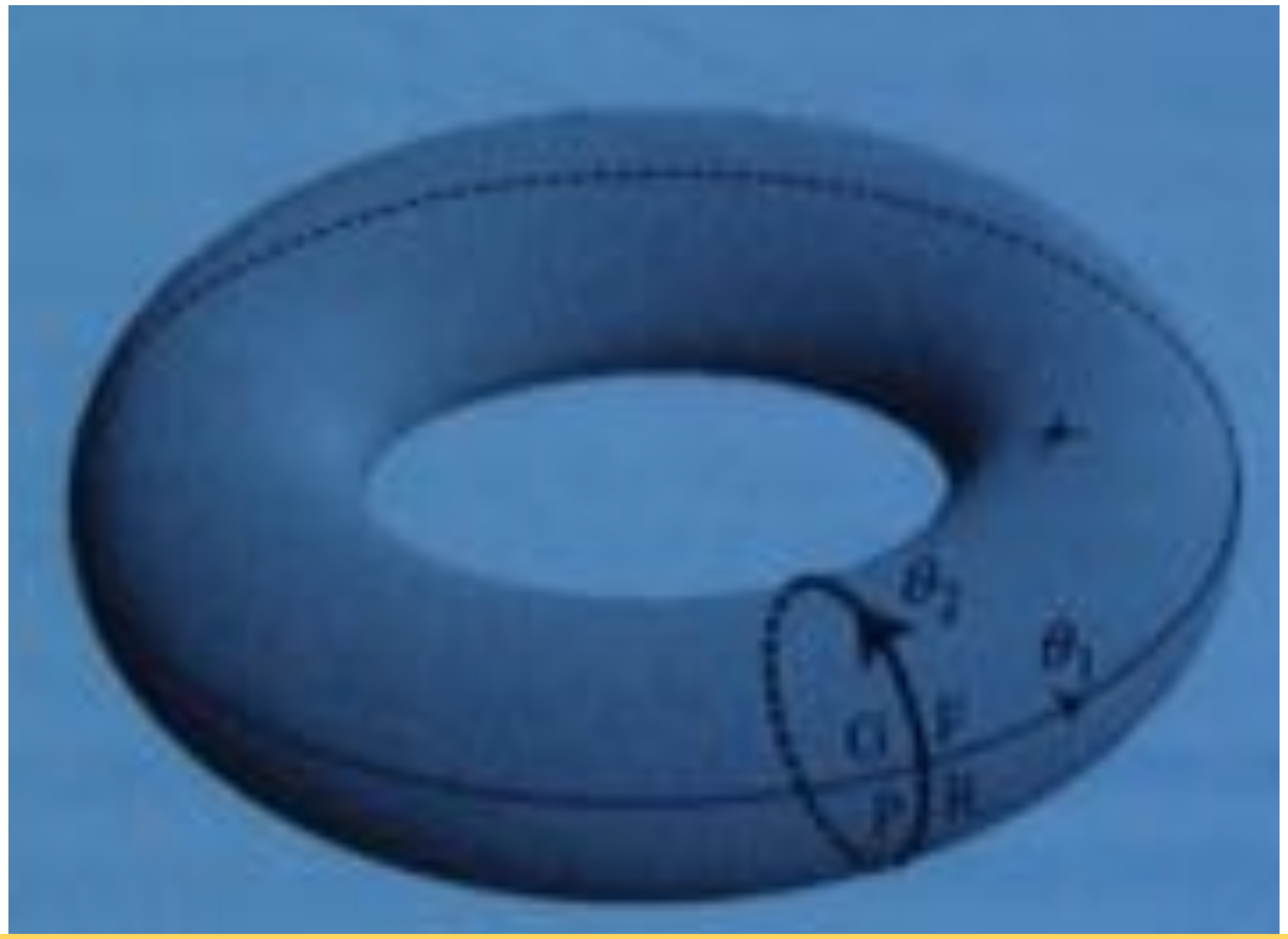
Slide borrowed from Michigan Robotics autorob.org



Workspace is w.r.t. end-effector position *(x,y)*



C-space is w.r.t. joint angles (Θ_1, Θ_2)



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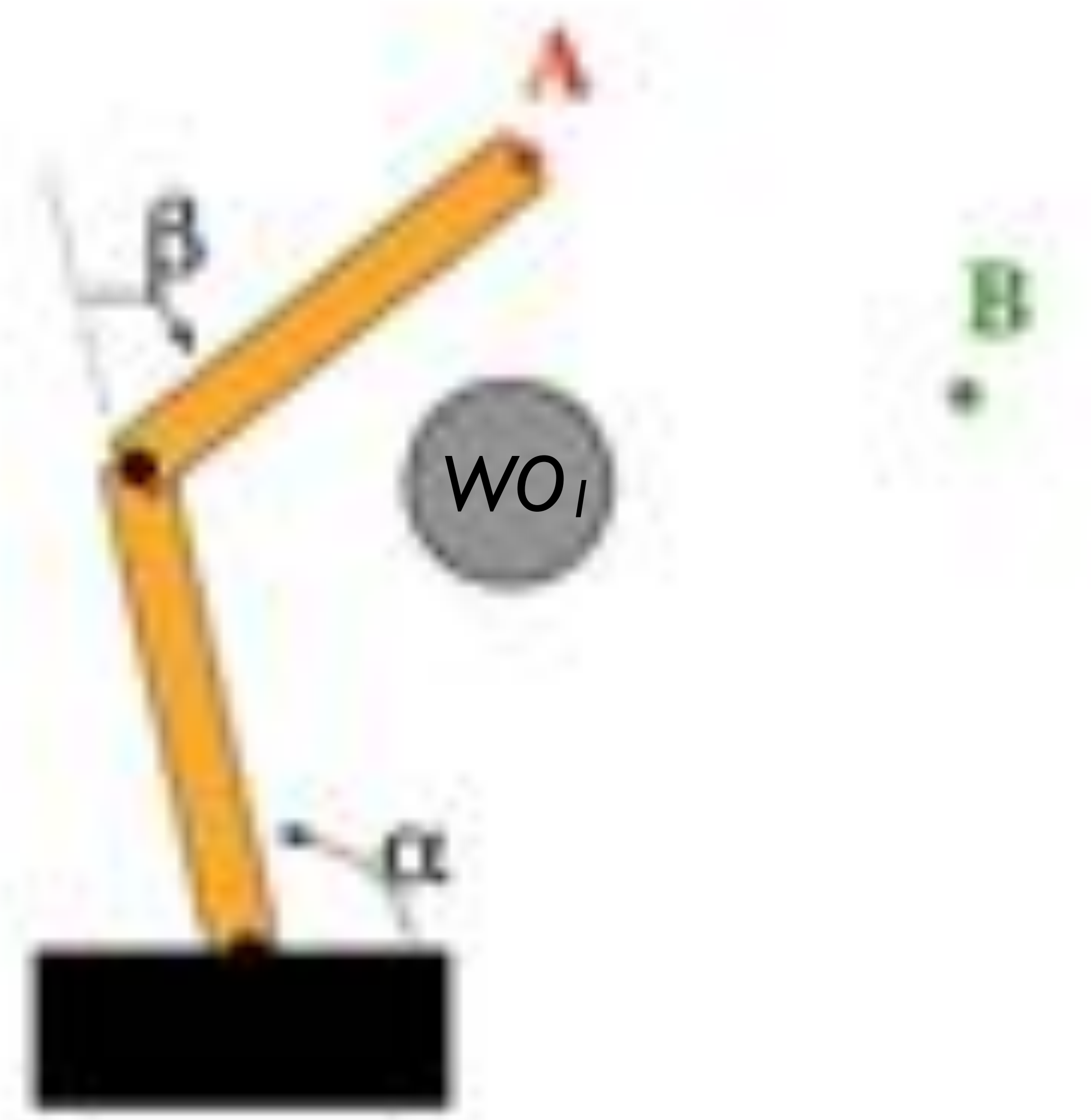
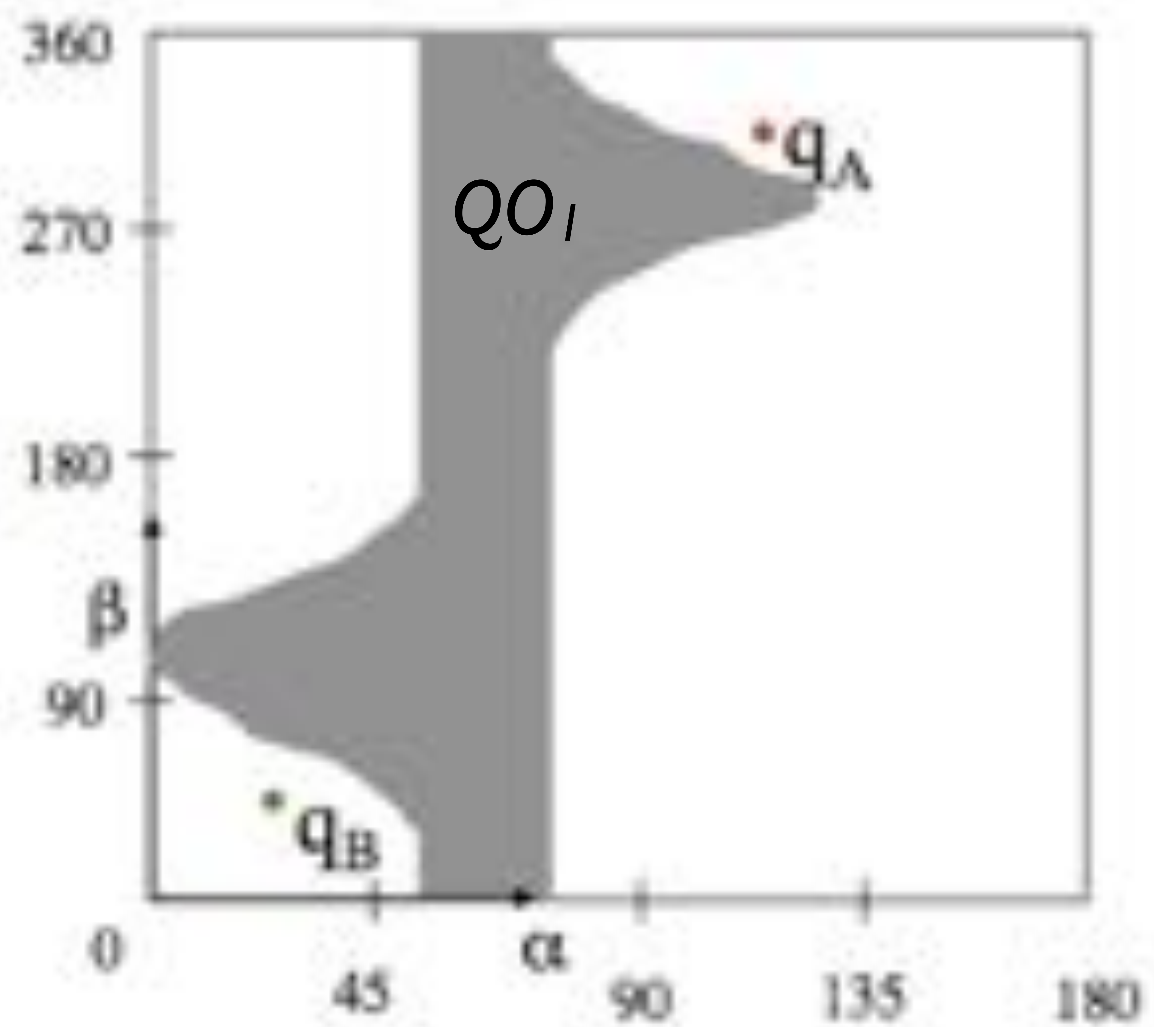
Obstacles in T^2

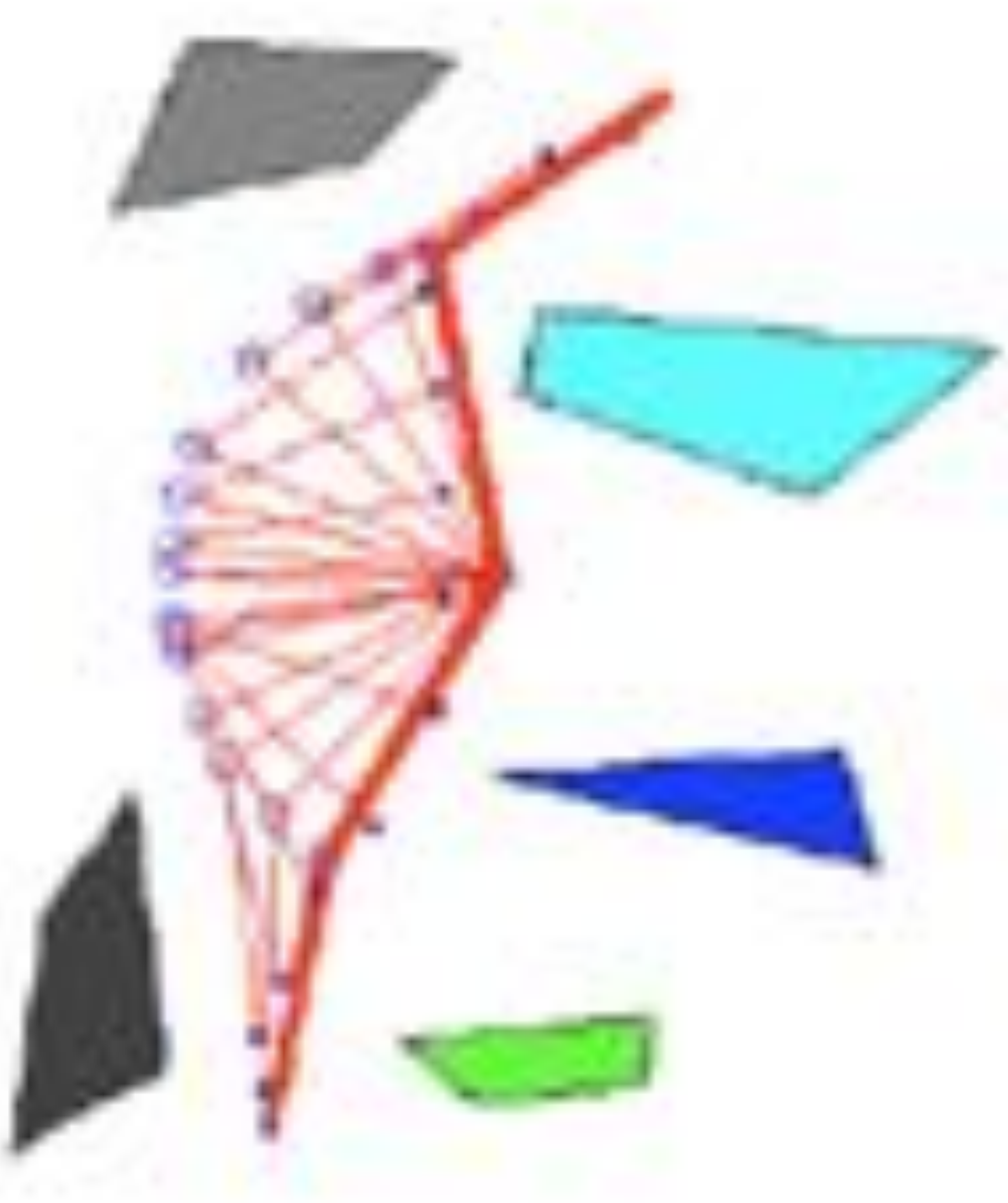


Circular obstacle in workspace

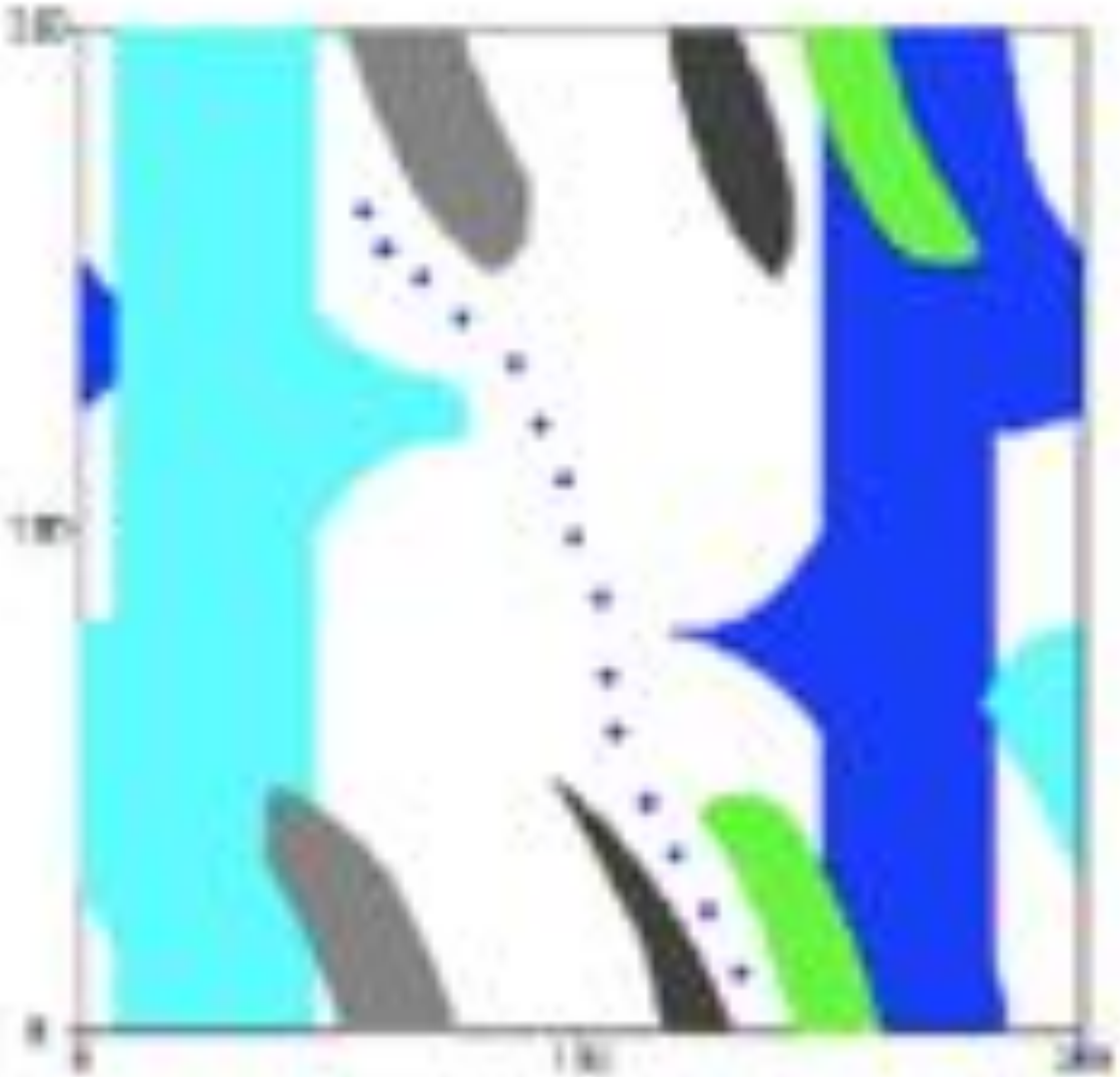


C-space representation

Path in T^2 with several obstacles

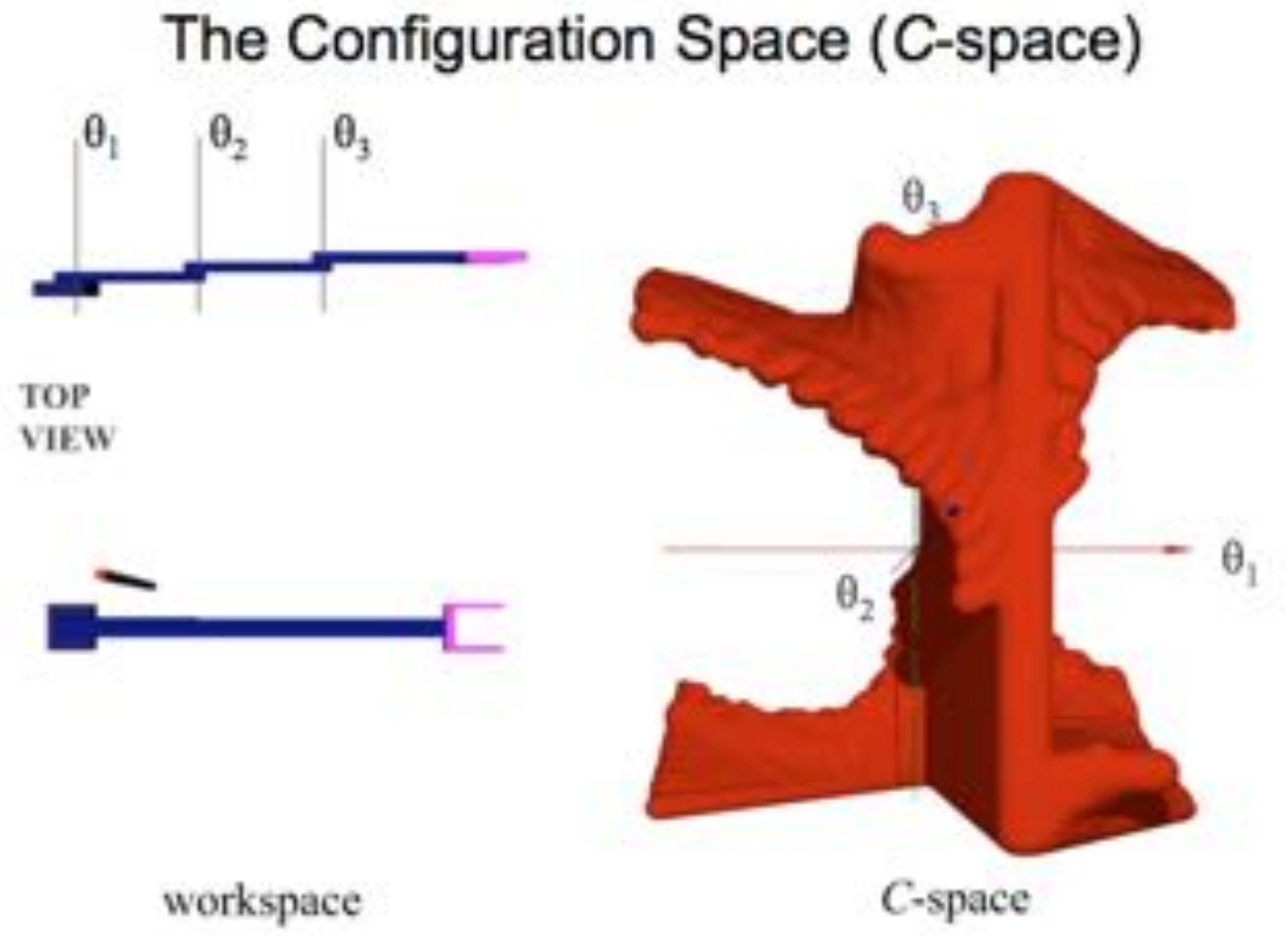


Arm navigation in workspace



C-space representation

C-space for 3-link arm



Generalizing graph search for robot configurations

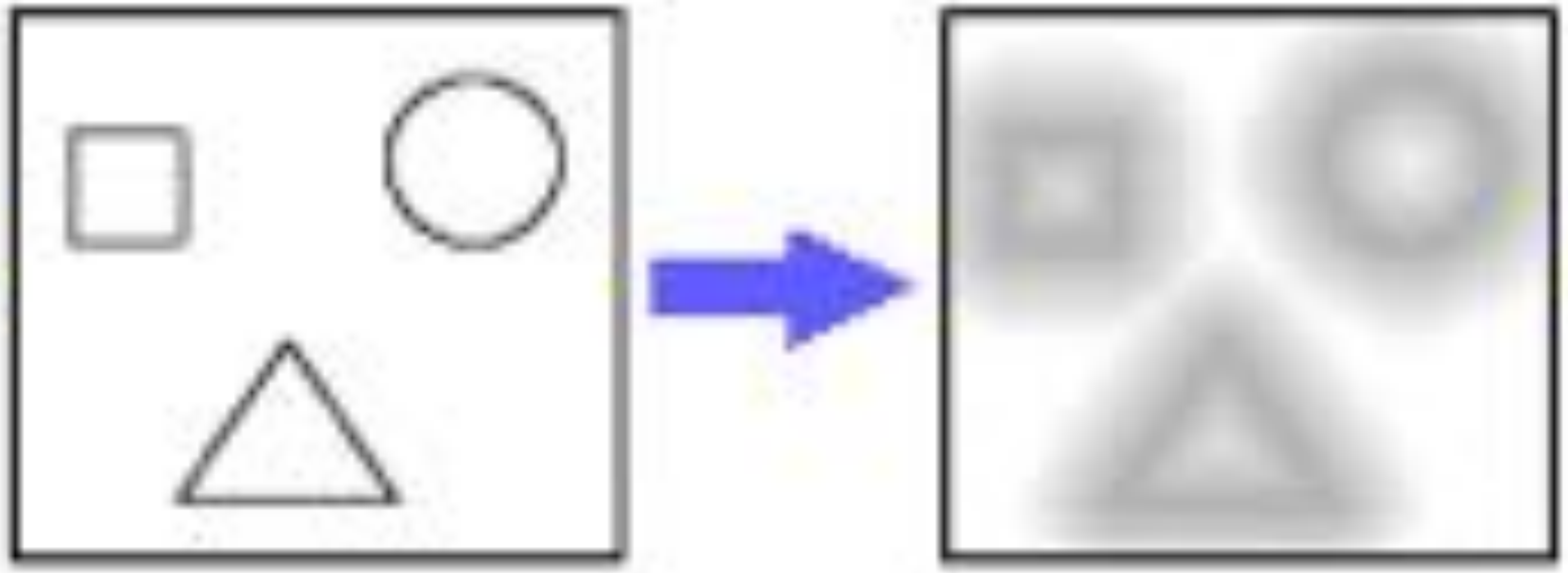
Costmaps: Graph Search Revisited

- Optimality: Path length vs. Path cost?

- **Costmap** provides weights on graph nodes based on cost factors:
  - Robot motion: joint limits, holonomicity, smoothness
  - Collisions and safety: distance from objects, trajectory predictions
  - Environmental conditions: traversability, slip

Distance Transform

Compute distance of each grid cell to nearest obstacle boundary;
Weight grid cell cost higher if closer to a boundary



http://www.gavrila.net/Research/Chamfer_System/chamfer_basics2.gif

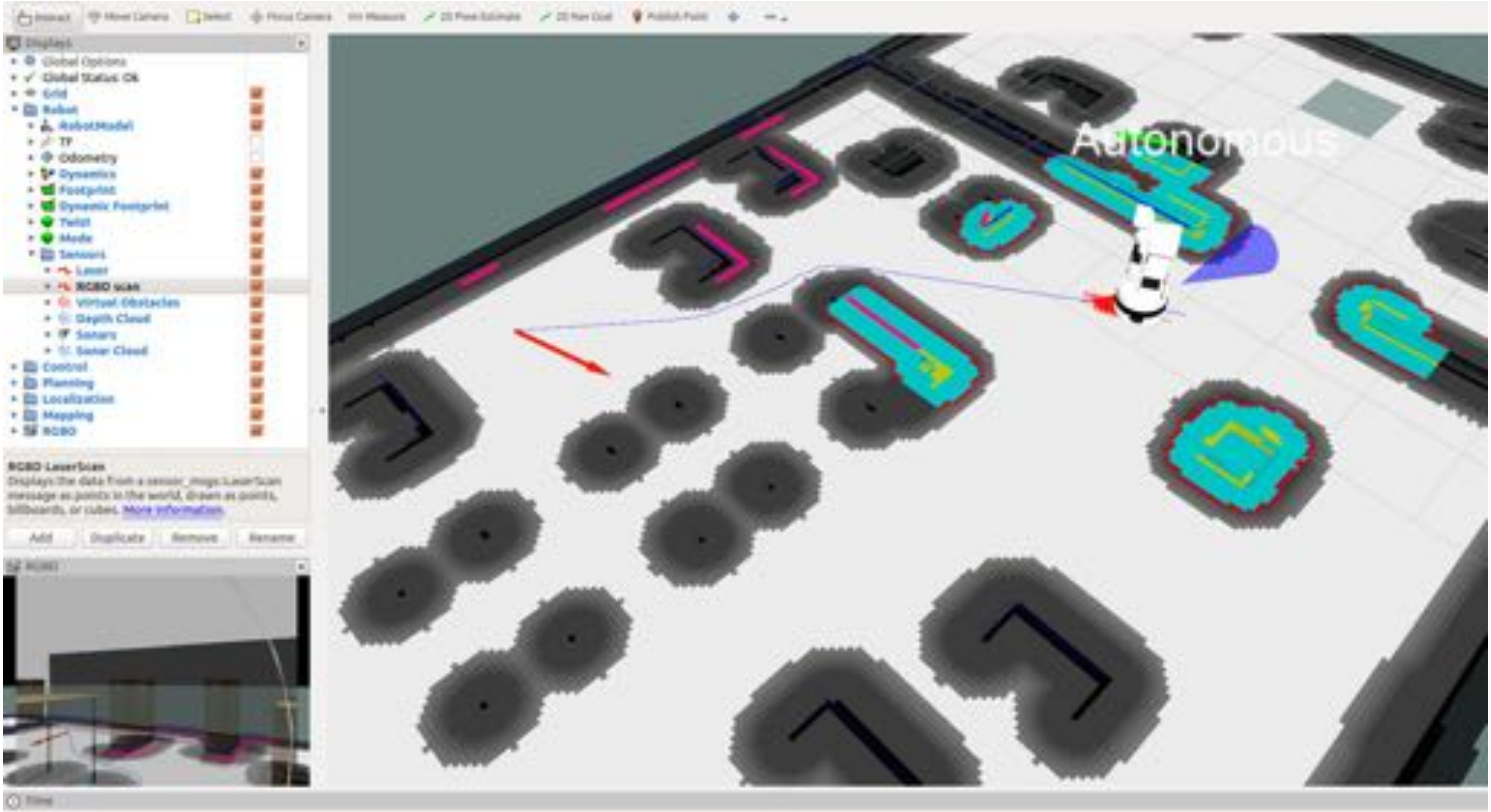
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	1	0	0

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	1	2	4	2	1	0	0
0	0	1	4	8	4	1	0	0
0	0	1	2	4	2	1	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	1	0	0

Nasonov and Krylov 2010
(zero indicates obstacle)

Slide borrowed from Michigan Robotics autorob.org

https://blog.pal-robotics.com/blog/tiago-ros-simulation-tutorial-2-autonomous-robot-navigation/



Search algorithm template

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_list ← start_node

    while visit_list != empty && current_node != goal

      cur_node ← highestPriority(visit_list)

      $visited_{cur\_node}$ ← true

      for each nbr in not_visited(adjacent(cur_node))

        add(nbr to visit_list)

        if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)

          $parent_{nbr}$ ← current_node

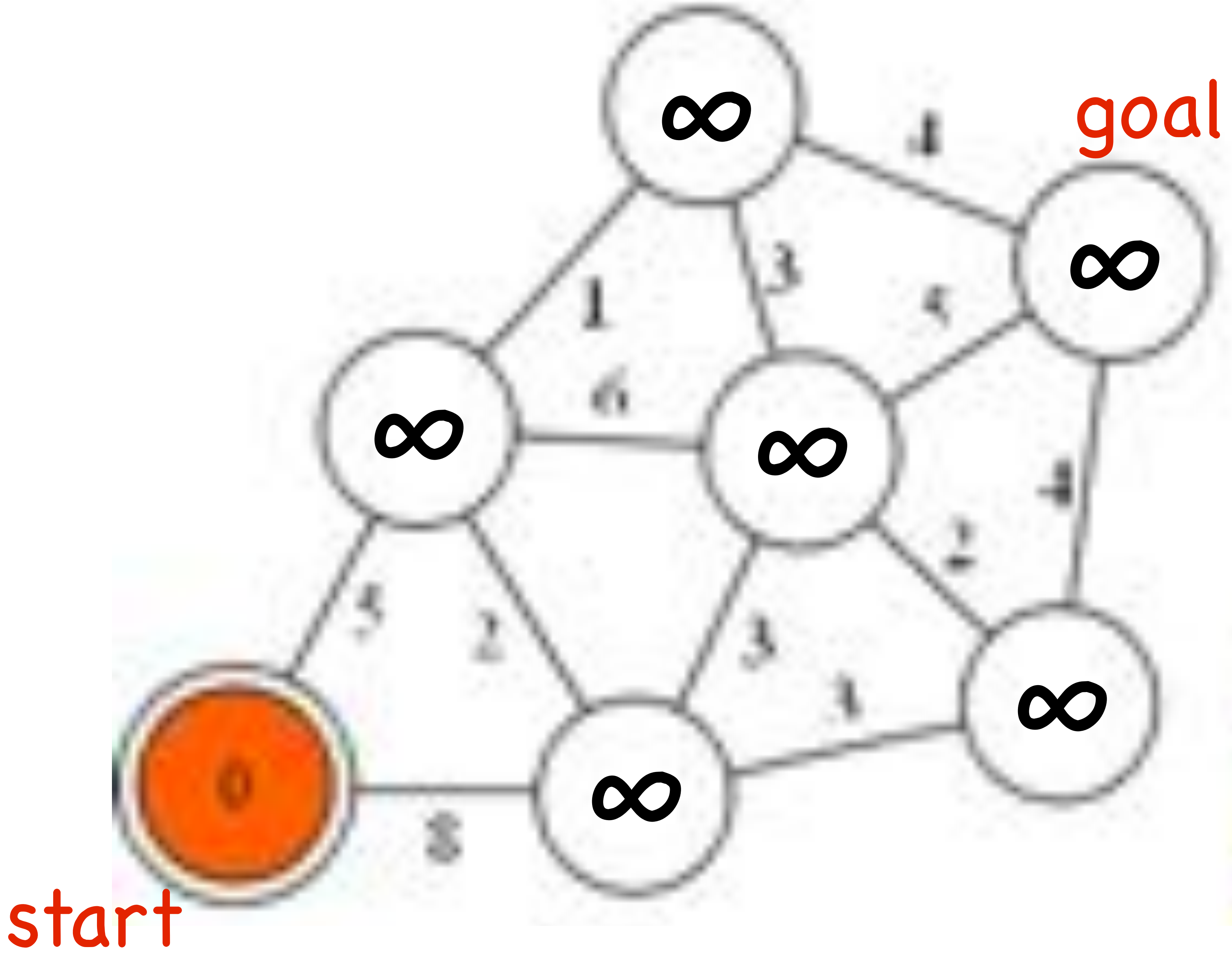
          $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)

        end if

      end for loop

    end while loop

output ← parent, distance



Search algorithm template

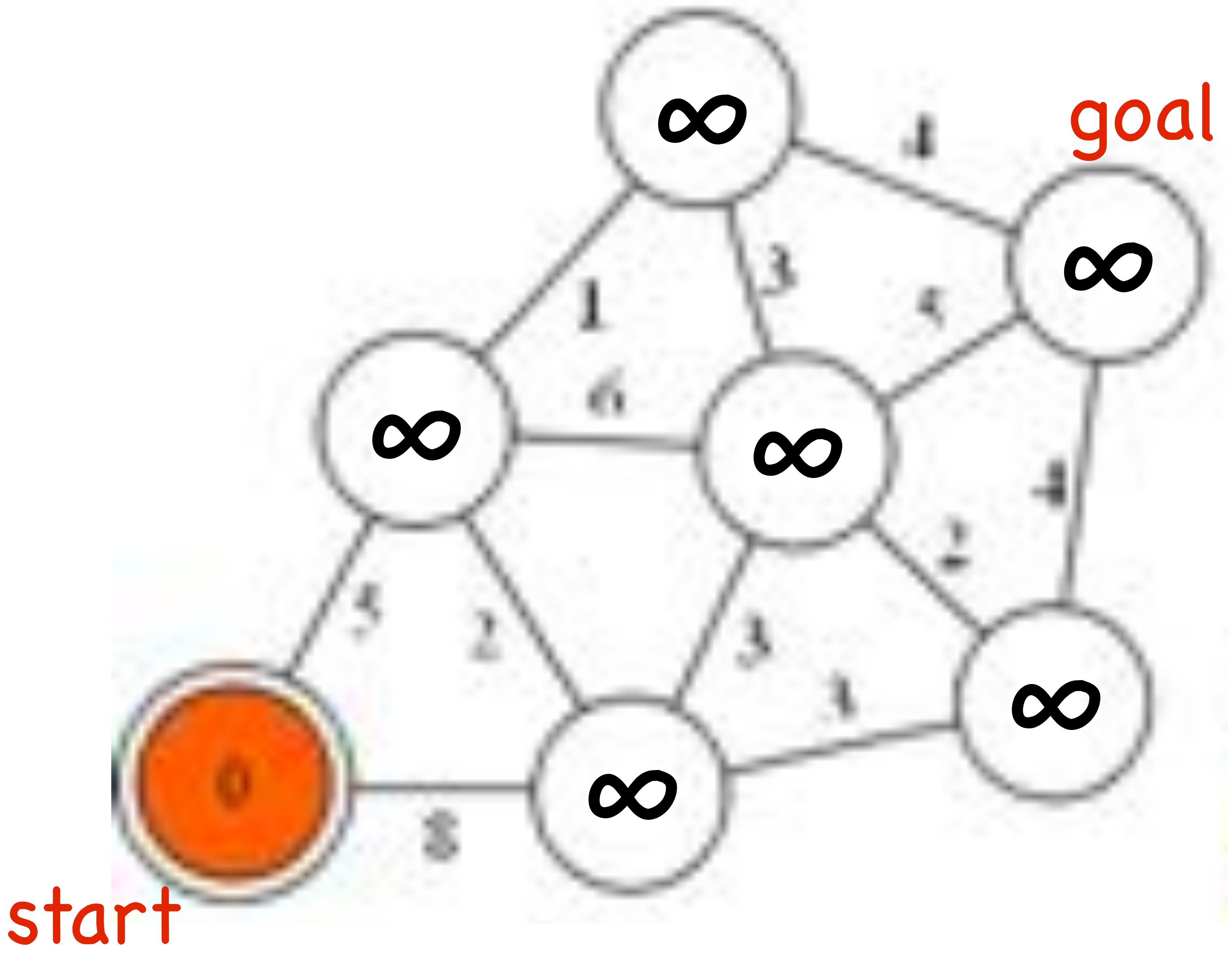
all nodes ← {$cost_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$cost_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_list ← start_node

```
    while visit_list != empty && current_node != goal
        cur_node ← highestPriority(visit_list)
        visited_cur_node ← true
        for each nbr in not_visited(adjacent(cur_node))
            add(nbr to visit_list)
            if cost_nbr > cost_cur_node + cost(nbr)
                parent_nbr ← current_node
                cost_nbr ← cost_cur_node + cost(nbr)
            end if
        end for loop
    end while loop
```

output ← parent, distance



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A-star shortest path algorithm

all nodes ← {$\text{cost}_{\text{start}}$ ← infinity, $\text{parent}_{\text{start}}$ ← none, $\text{visited}_{\text{start}}$ ← false}

start_node ← {$\text{cost}_{\text{start}}$ ← 0, $\text{parent}_{\text{start}}$ ← none, $\text{visited}_{\text{start}}$ ← true}

visit_queue ← start_node

while (visit_queue != empty) && current_node != goal

dequeue: cur_node ← f_score(visit_queue)

$\text{visited}_{\text{cur\_node}}$ ← true

for each nor in not_visited(adjacent(cur_node))

enqueue: nbr to visit_queue

if cost_{nbr} > $\text{cost}_{\text{cur\_node}}$ + cost(nbr)

$\text{parent}_{\text{nbr}}$ ← current_node

cost_{nbr} ← $\text{cost}_{\text{cur\_node}}$ + cost(nbr)

f_score ← cost_{nbr} + $\text{line\_distance}_{\text{nbr,goal}}$

end if

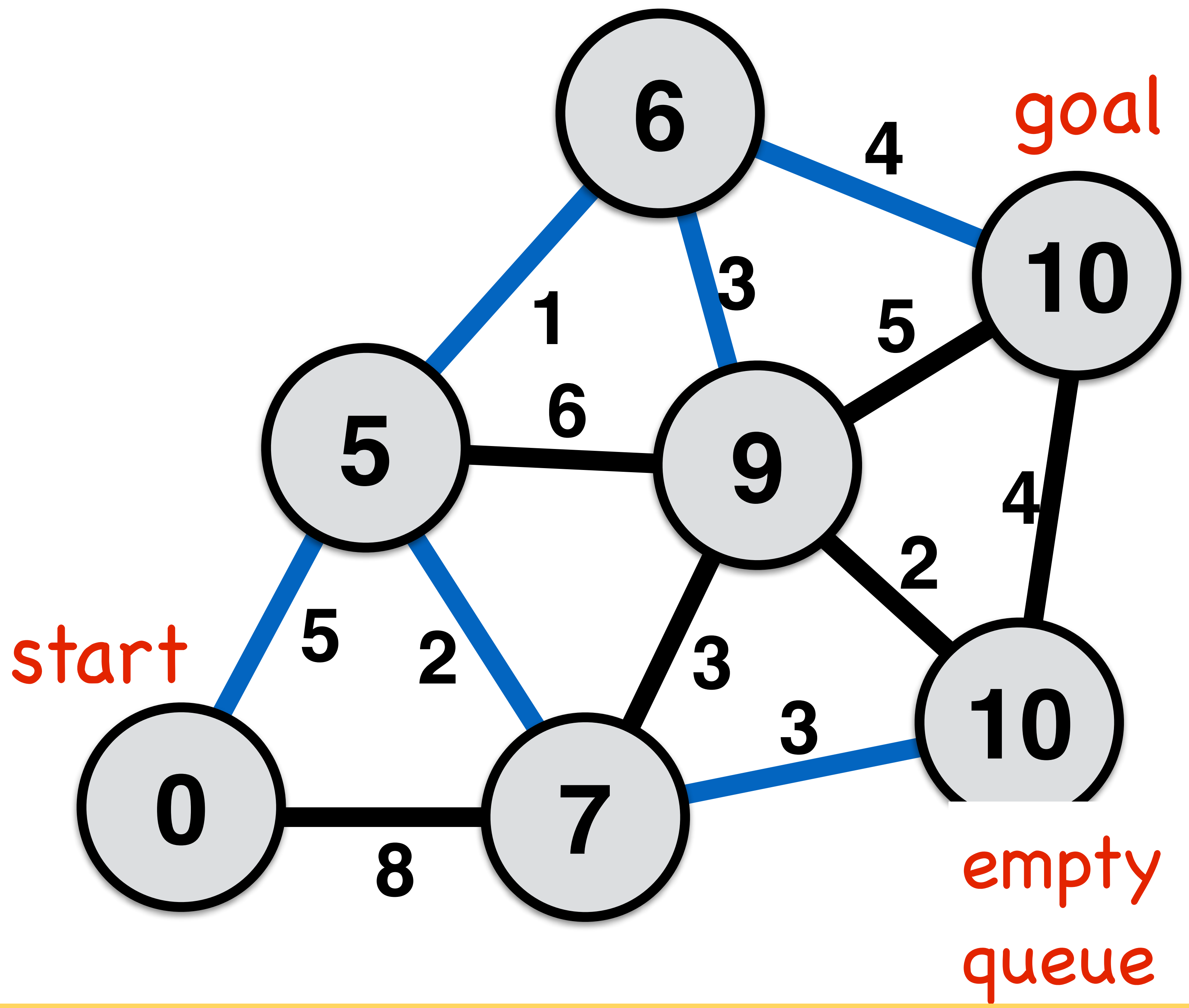
end for loop

end while loop

output ← parent, distance

g_score:
cost from start

h_score:
optimistic cost to goal



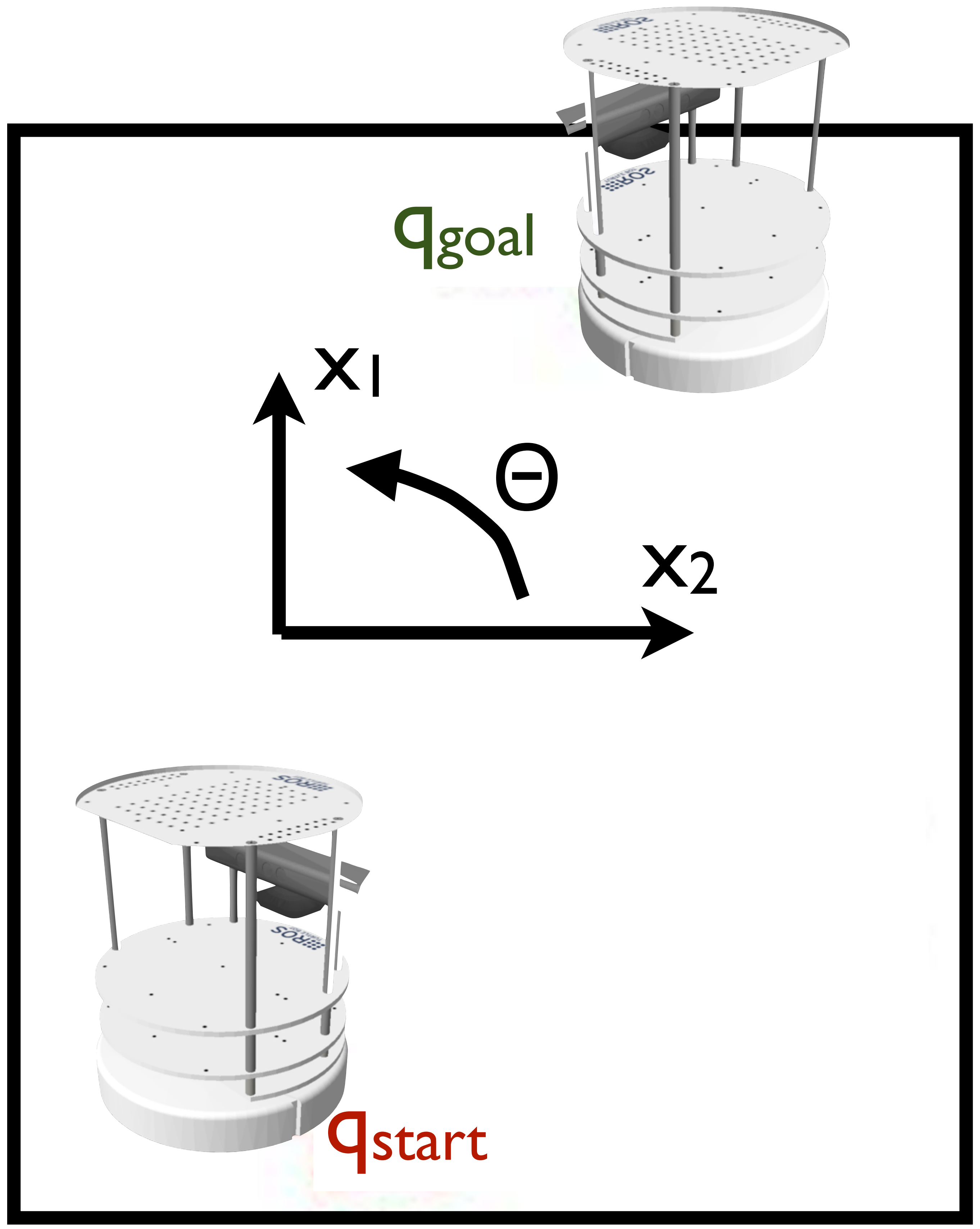
Can a robot move in any direction instantaneously?

Holonomicity

- Does the Turtlebot have 2 DOFs, instead of 3?

- The Turtlebot can only move along 2 axes
  - linear: forward/backward
  - angular: turning

Turtlebot is nonholonomic



Holonomicity

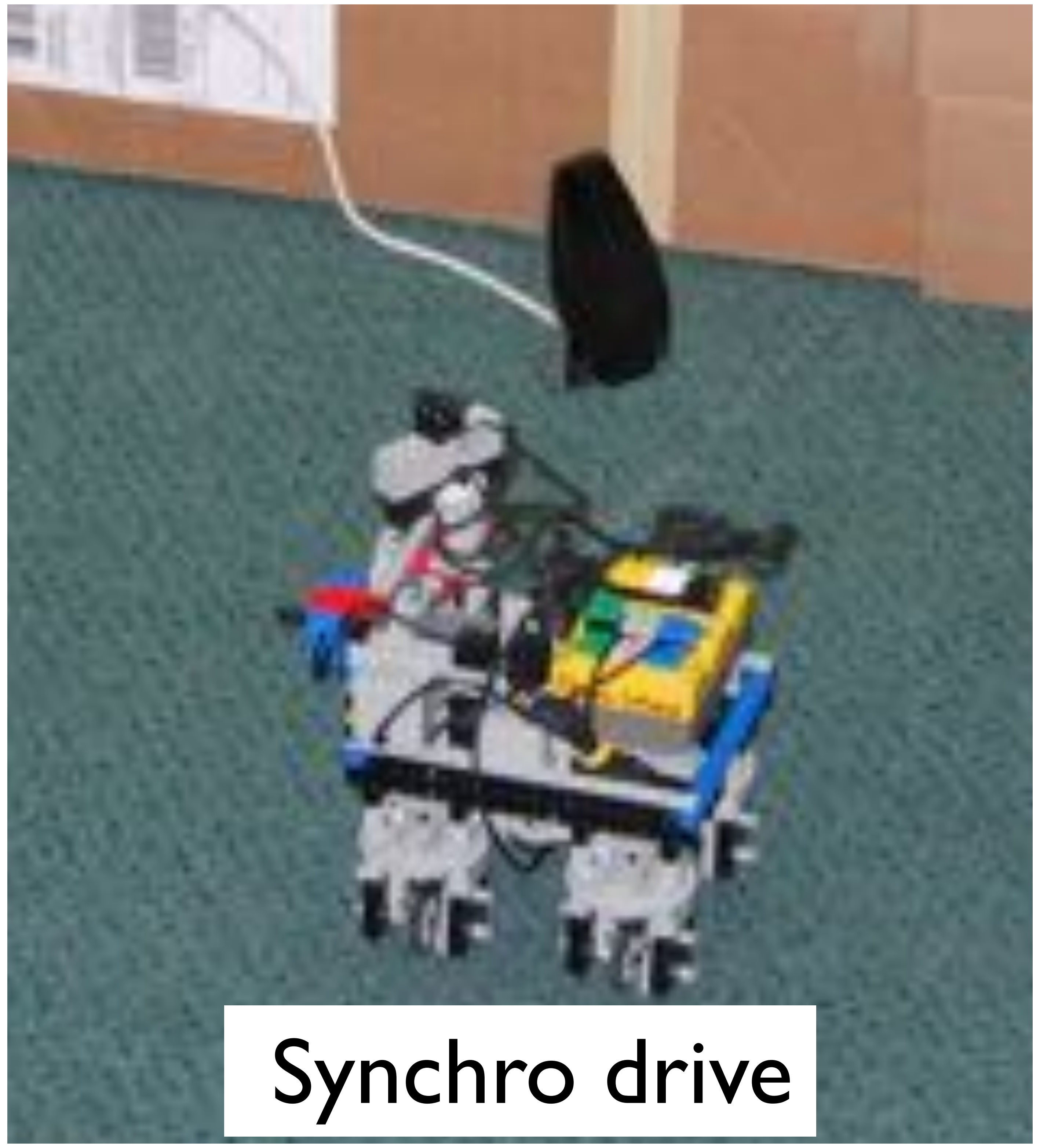
https://www.youtube.com/watch?v=c-lEjVsoiGo

https://www.youtube.com/watch?v=1ak17mdRg5I&t=75s

- A robot is holonomic if it can change its pose instantaneously to move in all directions
- Otherwise, the robot is nonholonomic

Holonomic mobile robot systems

Omni-wheel drive



Synchro drive

E. Leland, Segway, robotthoughts.com

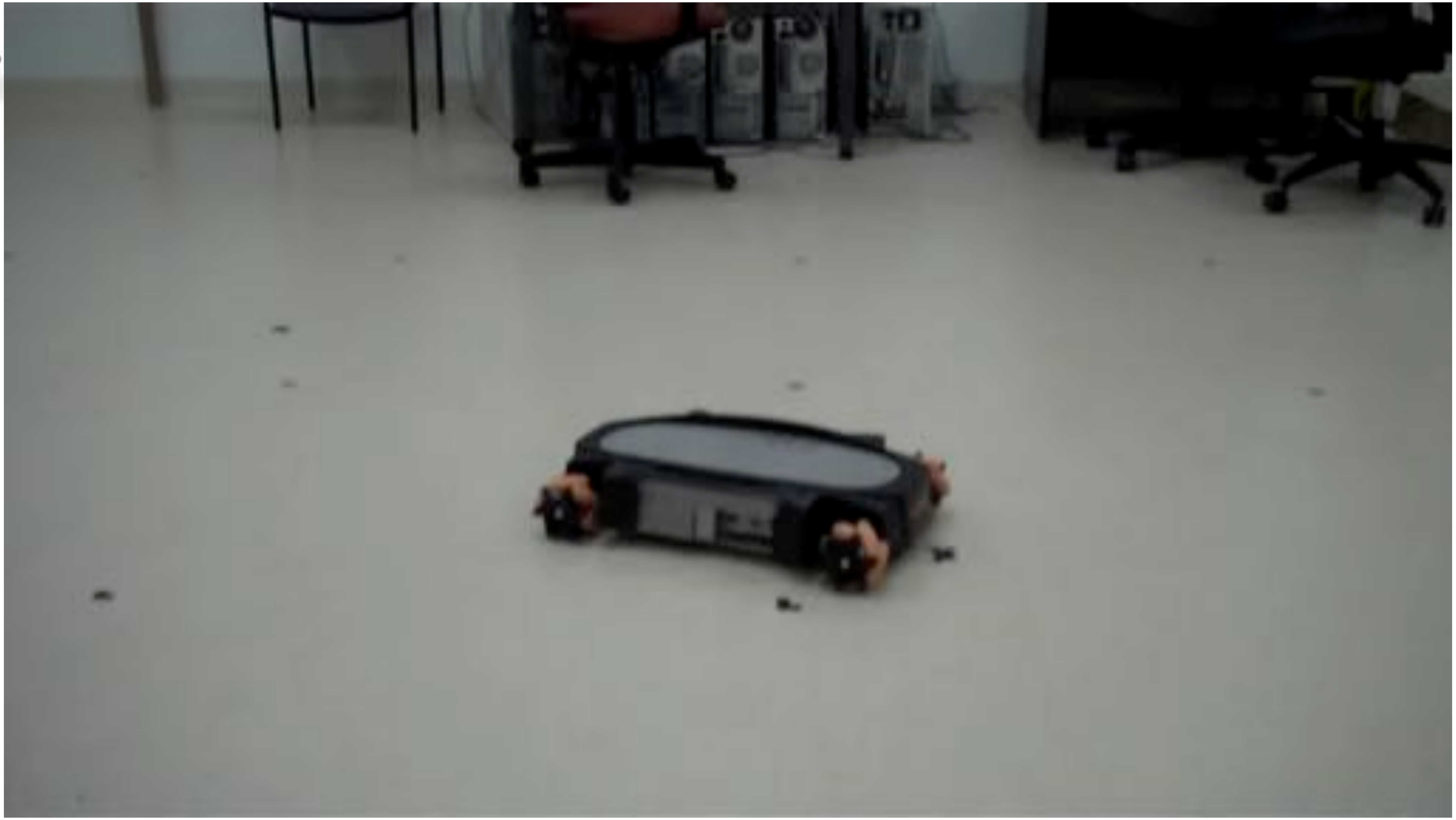
Killough platform

Mecanum wheels

Synchro Drive

markclego, https://www.youtube.com/watch?v=THdu6QD8Roc

KUKA YouBot with Mecanum wheels

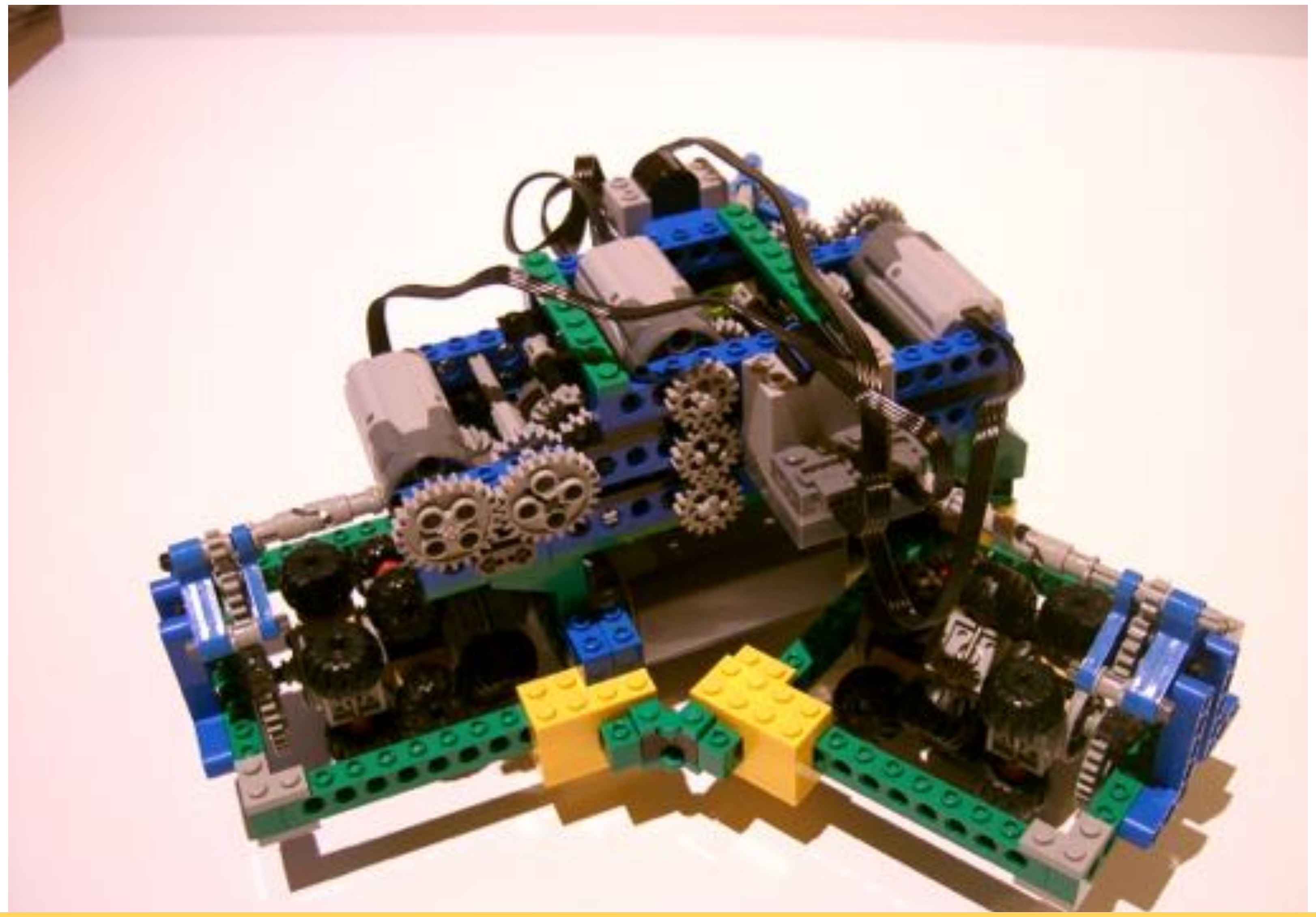


Me teleoperating KUKA YouBot from my house via a web browser, http://youtu.be/sWrRiy0AM_w

DJI Robomaster Racing

Japan Times, https://www.youtube.com/watch?v=52skH4NpnvI

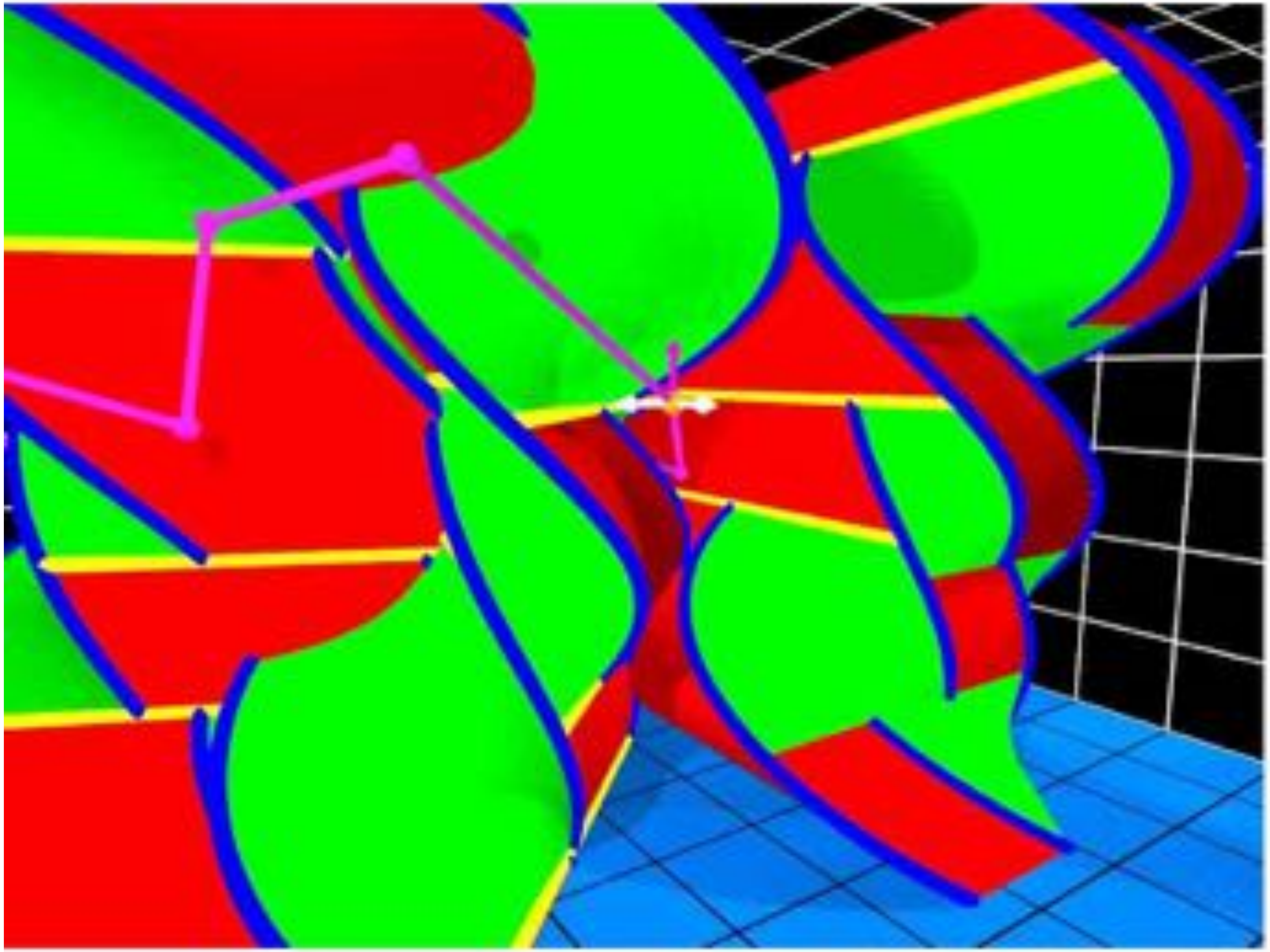
Killough platform



robotthoughts.com; http://technicbricks.blogspot.com/2008/08/going-to-all-places-in-all-directions_29.html

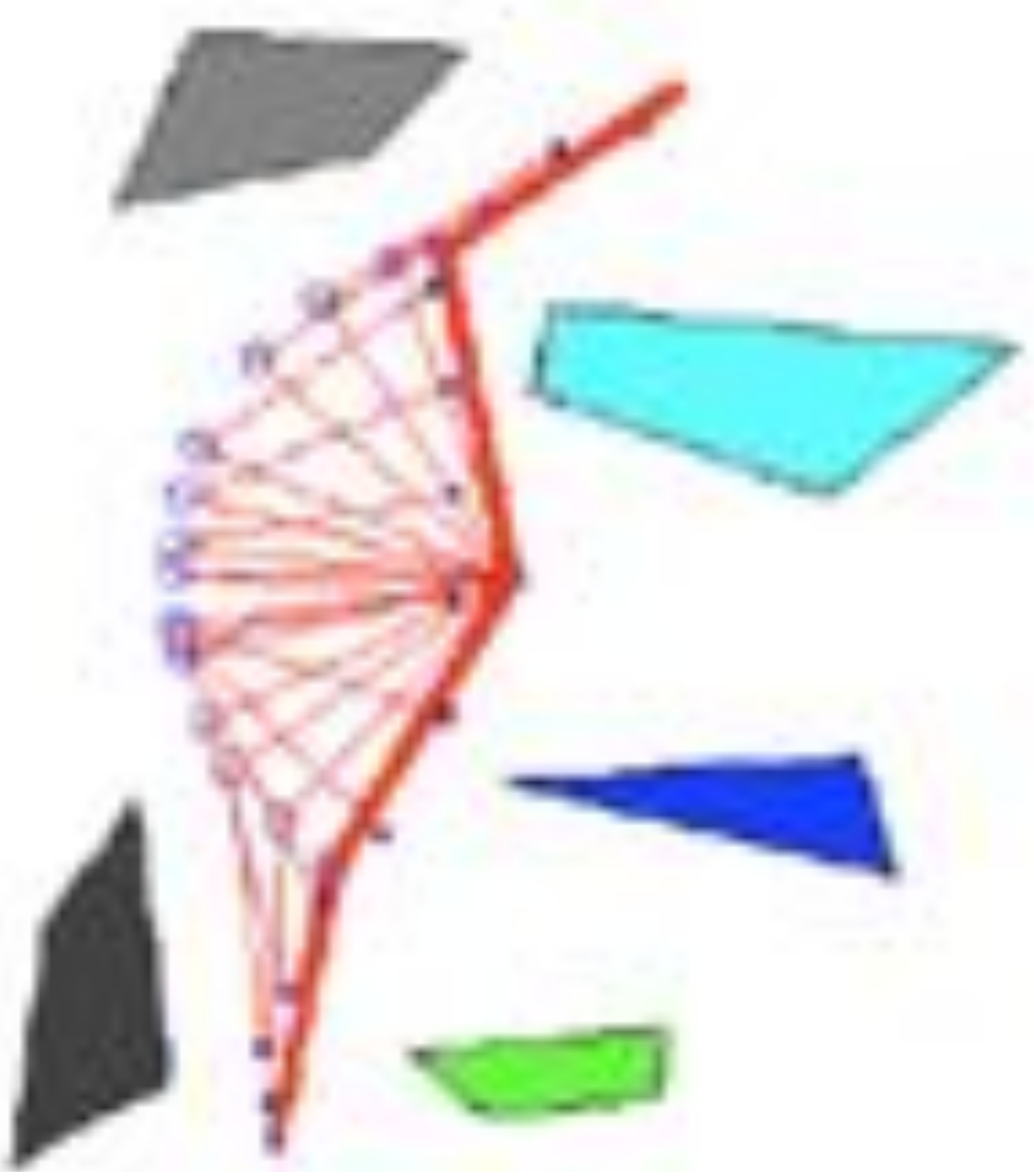
Recommended: D'Andrea on Omni-drive

https://www.youtube.com/watch?v=p_WI-C-ORso

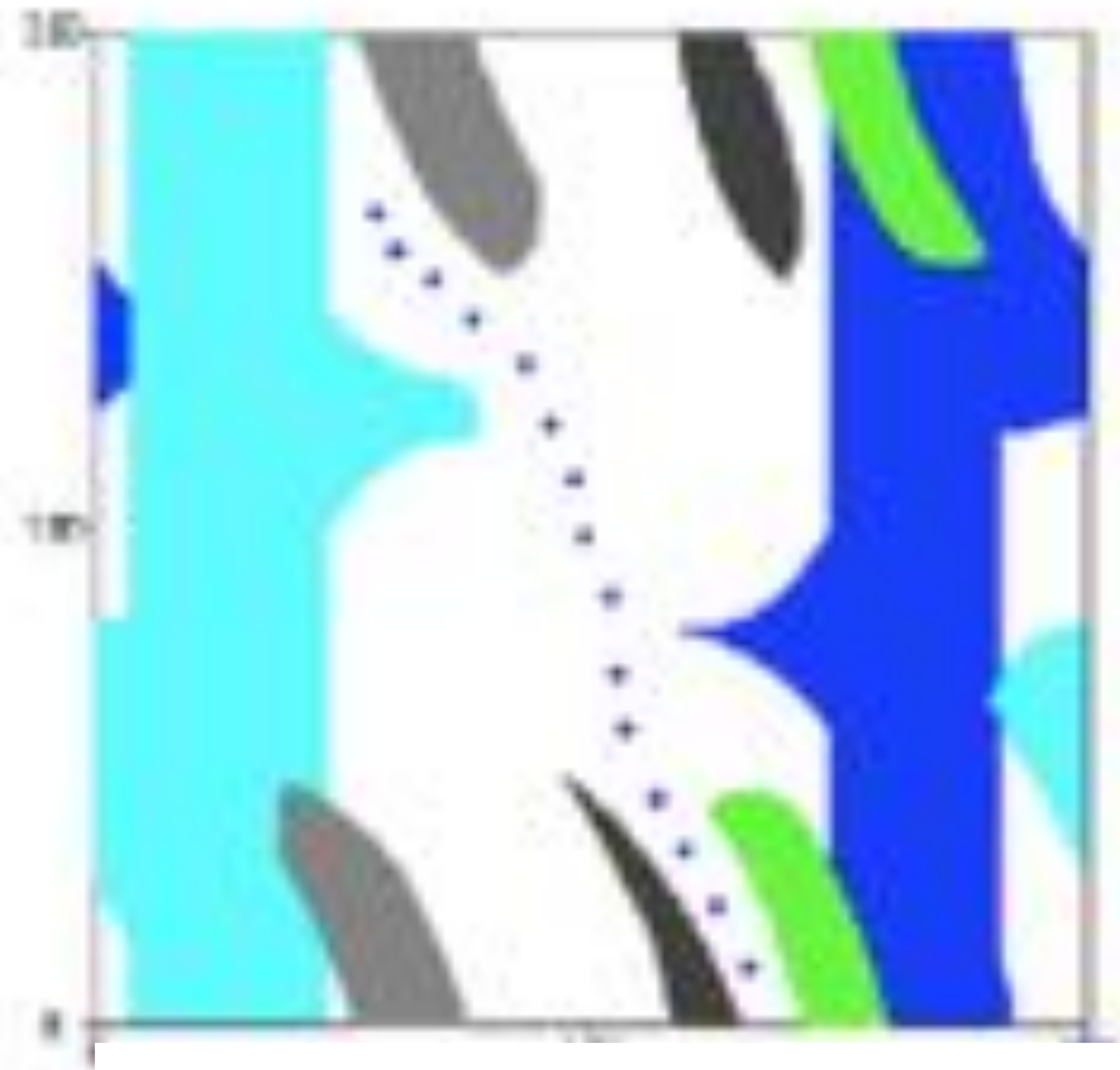


Visualization developed by Dror Atariah and Günter Rote - https://www.youtube.com/watch?v=SBFwgR4K1Gk

How do we search arbitrary C-spaces?



Arm navigation in workspace



C-space representation

How build graphs in arbitrary C-spaces?

Next Lecture

Planning - IV - Sampling-based Planning